Observer-Based Fuzzy Fault-Tolerant Control for Nonlinear Systems in the Presence of General Noise

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***Abstract*—This paper addresses the observer-based fuzzy fault-tolerant control problem for nonlinear systems subject to general noise. The presence of component faults and general noise can significantly alter system behavior, potentially leading to instability. Stochastic differential equations theory is an effective tool for managing control problems in interval type-2 T-S fuzzy systems influenced by white noise. However, compared to stochastic differential equations, random differential equations are more suitable when random disturbances are better described by stationary processes rather than white noise. In this paper, the problem of observer-based fault-tolerant control for nonlinear systems with general noise is solved using a mixed design method that combines an interval type-1 observer with an interval type-2 controller. Furthermore, by describing the effects of stationary processes on the system using general noise and applying random differential equation theory, the probabilistic stabilization of the system is analyzed, and LMIs-based stability criteria are derived in the observer design process. Similar results are also obtained in the controller design process. The proposed method not only reduces the computational burden of designing fault-tolerant controllers in the presence of general noise but also provides results with lower conservatism, allowing for greater uncertainty handling. Finally, the effectiveness of the proposed method is demonstrated through an example.**

***Index Terms*—IT2 T-S fuzzy system; fault-tolerant control; fault estimation; non-white noise; random differential equations**

I. Introduction

Systems in practical industrial applications are vulnerable to stochastic changes such as noise, external disturbances, random faults, and more. Therefore, studying the problem of fuzzy fault-tolerant control for nonlinear systems under the influence of noise is of great significance. In constantly changing environments, various control systems are often affected by uncertain factors like faults [1], perturbations [2], disturbances [3], and noise [4]. Additionally, actuators, sensors, and other components may fail during system operation, and such faults can lead to deterioration in system performance or even system instability. To restore system performance and stabilize the system, appropriate measures must be taken to mitigate the impact of these faults. Fault-tolerant control (FTC) has emerged as an effective method for this purpose, drawing considerable attention, and leading to extensive research on compensating for faults [5]-[11]. Currently, existing FTC methods can be classified into two categories: passive FTC approaches [12]-[14] and active FTC approaches [15]-[18]. While passive FTC approaches have the advantage of being easier to implement, their conservative nature due to limited fault estimation (FE) can result in unsatisfactory control performance. In contrast, active FTC approaches can achieve more accurate FE, leading to better performance in compensating for the impact of faults.

Since nonlinear characteristics are very common in practical systems [19], linear analysis theory cannot be directly applied to analyze FTC problems in nonlinear systems. Therefore, it is essential to find an appropriate method that effectively captures nonlinear characteristics and allows the use of existing linear analysis theory to address FTC problems. The literature [20] proposed the Takagi-Sugeno (T-S) fuzzy modeling method to solve this issue, leveraging fuzzy set theory and the fuzzy summation of multiple linear subsystems. By using the summation of multiple linear subsystems to describe the original nonlinear system, appropriate linear analysis techniques can be employed to analyze and solve FTC problems in nonlinear systems. Consequently, the T-S fuzzy modeling method has become a powerful tool for addressing and solving nonlinear FTC problems. Over the past few decades, the T-S fuzzy model method has gained widespread attention and led to significant achievements in areas such as system stability analysis [21]-[22], tracking control [23], compensation control [24]-[26], and filter design [27]-[28]. However, when a system contains perturbations, type-1 fuzzy sets cannot effectively capture the uncertainty. To address this, the interval type-2 (IT2) T-S fuzzy modeling method has been proposed, which extends traditional fuzzy modeling methods to handle parameter uncertainties arising from imprecise fuzzy rules and membership functions [29]. In this method, perturbations in the system are better described by constructing lower and upper membership functions to replace the traditional fuzzy model's membership functions. To date, the IT2 T-S fuzzy modeling method has yielded several important research results, including advancements in filtering design [30]-[31], controller synthesis [32]-[33], and system stabilization [34].

Due to the inherently noisy nature of real-world environments, the dynamic characteristics of systems are highly susceptible to environmental noise. Currently, the primary tools for effectively describing noisy environments are stochastic differential equations (SDEs). Recently, research on IT2 T-S fuzzy stochastic systems has produced notable results in various areas, including fault-tolerant control [35]-[37], dynamic event-based control [38], fault detection [39]-[40], Hankel-norm-based model reduction [41], and fuzzy pinning control [42]. Sun et al. proposed a novel observer designed to achieve rapid fault verification and fault-tolerant control (FTC), effectively preventing system collapse due to sudden faults [35]. In [41], the model reduction problem is addressed, and a Hankel-norm-based model reduction approach is proposed to reduce the conservatism of the results.

Although SDEs have been remarkably successful in analyzing the stability of IT2 T-S fuzzy systems in noisy environments, it is not entirely accurate to describe the noise affecting the dynamic characteristics of actual systems using white noise. Most noise in practical environments is non-white, and its spectral power distribution may be uneven across the frequency domain, making white noise an inappropriate model for such noise. For example, waves that influence the trajectory of ship motion can be considered non-Wiener processes, which are not effectively described by white noise. Therefore, it is crucial to investigate the influence of general noise on the dynamic characteristics of IT2 T-S fuzzy systems. From this perspective, random differential equations (RDEs) are considered more suitable analytical tools for modeling general noise. Under certain assumptions [43]-[44], RDEs can be used to prove the stability and asymptotic stability of systems. Furthermore, by applying the theoretical framework established through Lyapunov's second method, the stability conditions of RDEs can be relaxed [45]. In this paper, we design a fault observer and a fault-tolerant controller to address the fuzzy FTC problem for nonlinear systems with general noise using a mixed approach [2]. Additionally, the LMIs-based stability criteria for the probabilistic stabilization of nonlinear systems with general noise are provided.

Based on the aforementioned discussion, this paper investigates the fuzzy FTC problem for nonlinear systems affected by general noise. The main contributions of this paper are as follows:

1) Utilizing the mixed design approach, a fault observer and fuzzy fault-tolerant controller for nonlinear systems with perturbations in a general noisy environment are designed to enhance fault estimation performance and reduce computational burden. By considering both cases where noise is white and where the noise spectral density function is non-constant, the models developed during the design process are more broadly applicable.

2) Leveraging RDEs theory, the system with general noise is effectively described, and the probabilistic stability of the system is demonstrated. Additionally, using results from improved Lyapunov stability theory for RDEs, LMIs-based criteria for the observer and controller design of nonlinear systems with general noise are provided. Finally, the probabilistic stability region of the system, considering noise intensity and the H_∞ index, is also established.

In this paper, 0 and I are used to denote the zero matrix and the identity matrix with appropriate dimensions, respectively; $\mathfrak{P} < (>)\ 0$ is a negative (positive) definite matrix; $\mathfrak{B}^{-1}$ represents the inverse of matrix $\mathfrak{B}$ and $\mathfrak{B}^T$ represents the transpose of matrix $\mathfrak{B}$; $\mathfrak{G}^\dagger$ is the generalized inverse of $\mathfrak{G}$. $\mathfrak{R}^n$ is n-dimensional Euclidean space. $He\{\mathfrak{E}\}$ represents $He\{\mathfrak{E}\} = \mathfrak{E} + \mathfrak{E}^T$. $diag\{\mathfrak{c}_1, ..., \mathfrak{c}_n\}$ is matrix which the elements on the diagonal line.

II. System Description and advance preparation

Considering the nonlinear system with general noise, and the system can be described as follows:

$$\begin{aligned}\dot{x}(t) &= \mathfrak{W}\big(x(t), u(t), f(t), d(t), \tau\big) + \mathfrak{g}(x)\varpi(t),\\ y(t) &= \mathfrak{w}\big(x(t)\big)\end{aligned} \tag{1}$$

where $\mathfrak{W}\big(x(t), u(t), f(t), d(t), \tau\big)$ represents the nonlinear function. $x(t) \in \mathbb{R}^{n_{\mathbb{X}}}$ is the vector of the nonlinear system and the initial condition $x(t_0)$ is defined as x_0; $u(t)$ stands for the input vector; $y(t)$ represents the output signal; $f(t)$ denotes the fault signal which occurs in the actuator; $d(t)$ is the external disturbance considered to exist in the above system and τ is the perturbation. $\varpi(t)$ denotes the stochastic process, and $\mathfrak{g}(x)$ stands for the intensity matrix of $\varpi(t)$. The potential complete probability space can be regarded as the quartet $(\Omega, \mathfrak{F}, \mathfrak{F}_t, \mathfrak{P})$ with a filtration $\mathfrak{F}_t$ satisfying the usual conditions (i.e., if $\mathfrak{F}_0$ contains all $\mathfrak{P}$-null sets, it can be thought of as right continuous and increasing).

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