SIMULTANEOUS MULTI-OBJECTIVE ALIGNMENT ACROSS VERI-FIABLE AND NON-VERIFIABLE REWARDS

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ABSTRACT

Aligning large language models to human preferences is inherently multidimensional, yet most pipelines collapse heterogeneous signals into a single optimizeable objective. We seek to answer what it would take to simultaneously align a model across various domains spanning those with: verifiable rewards (mathematical accuracy), non-verifiable subjective preferences (human values), and complex interactive scenarios (multi-turn AI tutoring dialogues). Such multi-objective reinforcement learning setups are often plagued by the individual objectives being at odds with each other, resulting in inefficient training and little user control during inference. We propose a unified framework that: (i) standardizes process reward model (PRM) training across both verifiable and non-verifiable settings to better supervise models' chain-of-thought reasoning; (ii) performs multi-objective alignment by training the LLM with our Multi-Action-Head DPO (MAH-DPO) and a vectorized reward where the dimensions of the vector correspond to the various objectives instead of a single scalar; and (iii) demonstrates how such a system provides fine-grained inference-time user control. Experiments across math reasoning, value alignment, and multi-turn dialogue show that our framework improves performance across multiple objectives simultaneously, while minimizing cross-objective trade-offs and enabling flexible inference time user control.

1 Introduction

The success and widespread deployment of large language models (LLMs) have created opportunities for AI assistance across diverse applications, ranging from mathematical problem solving and question answering to educational tutoring (Brown et al., 2020; Ouyang et al., 2022; Lin et al., 2023; Handa et al., 2025b; OpenAI, 2025; Handa et al., 2025a). However, these real-world applications often demand that models simultaneously satisfy multiple objectives, which exposes a fundamental challenge that aligning LLMs to human preferences is inherently multi-dimensional (Askell et al., 2021; Bai et al., 2022a; Li et al., 2023). For instance, a question-answering system should provide helpful responses while being harmless (Ganguli et al., 2022; Perez et al., 2022), and an AI education tutor must be able to guide students toward accurate understanding while remaining pedagogically engaging (Maurya et al., 2024; Pal Chowdhury et al., 2024). These scenarios span three distinct categories of alignment targets: domains with verifiable rewards where correctness can be automatically checked (e.g., mathematical accuracy), domains with non-verifiable subjective preferences that require human judgment (e.g., helpfulness, honesty, truthfulness), and complex interactive scenarios involving multi-turn dialogues (e.g., AI tutoring engagingness) where success depends on the downstream impact of the assistant's responses on subsequent user behavior.

Current alignment methods struggle to capture multi-dimensional human preferences. Common practices such as reinforcement learning from human feedback (RLHF) (Christiano et al., 2017; Ouyang et al., 2022) distill human comparisons into scalar reward scores for maximizing expected reward. While direct preference optimization (DPO) (Rafailov et al., 2023) eliminates the reward model, it still optimizes along a single preference axis. Both approaches collapse rich, structured human feedback into one-dimensional training signals, discarding valuable trade-off information and resulting in mismatches between nuanced human preferences and simplified optimization objectives.

Several recent works address multi-objective RLHF alignment through linear scalarization (Li et al., 2020; Hu et al., 2023; Zhou et al., 2023; Wu et al., 2023; Guo et al., 2024) or post-hoc parameter merging of specialized models (Rame et al., 2023; Jang et al., 2023). However, these approaches are computationally expensive and typically require retraining when incorporating additional objectives or altering the balance among existing ones. More computationally lightweight methods like MODPO (Zhou et al., 2023) extend DPO to multiple objectives but they still apply fixed dimensional weights during training time, limiting alignment flexibility as the dimension weights cannot be changed at inference time. Alternative test-time alignment methods use reward models to guide generation step-by-step but suffer

from granularity mismatches between between reward definition and generation decisions (Khanov et al., 2024; Deng & Raffel, 2023). For example, outcome reward models are trained to score complete responses while step-level guided decoding operates on partial and incomplete responses, resulting in inconsistencies (Li et al., 2024; Xu et al., 2024). Recent approaches attempt to address these granularity issues by using more granular reward signals from process reward models (Lightman et al., 2023; Wang et al., 2023a; Luo et al., 2024; Hu et al., 2025; Xiong et al., 2025). However, these solutions mostly focus on verifiable domains where intermediate steps can be reliably evaluated (Zhang et al., 2025; Zheng et al., 2024) and training PRMs in non-verifiable domains remain a challenge.

To address these limitations, we develop a framework that handles multi-objective alignment through three coordinated components. First, we standardize PRM training across both verifiable and non-verifiable settings to enable reliable step-level supervision across domains. For verifiable domains, we augment Monte Carlo rollouts with hindsight credit assignment for reward collection. For non-verifiable domains, we devise three reward labeling strategies: i) majority voting evaluation, ii) direct step judgment, and iii) step reward approximation, based on the process definition and rollout difficulty in specific tasks. Second, we introduce Multi-Action-Head DPO (MAH-DPO) to preserve the multi-dimensional nature of human preferences during training. Specifically, we first employ specialized action heads on top of a shared LLM backbone, where each head corresponds to a preference dimension. Then simultaneously, each head is optimized with its dimensional-specific DPO loss while the shared base LLM is updated with the crossdimension gradient. Thus, our MAH-DPO reduces cross-objectives gradient interference for more stable training and the multi-head design also enables more flexible adaptation during inference. Finally, we complement trainingtime optimization with our PRM-guided decoding with continuing hidden-state, which offers more fine-grained user control over different objectives as well as improved alignment performances with preserved generation continuity. Together these components turn multi-objective alignment into a coherent training and inference procedure that generalizes across verifiable and non-verifiable domains with flexibilities for controllable inference-time search across each preference dimension. To summarize, we make the following contributions:

- We develop a standardized PRM training pipeline that systematically addresses the challenge of deriving fine-grained supervision across verifiable and non-verifiable domains.
- We propose vectorized multi-objective alignment via Multi-Action-Head DPO, which preserves the multidimensional structure of human preferences during training and enables fine-grained preference dimension control during inference.
- Extensive experiments across math reasoning, human value alignment, and multi-turn AI tutoring demonstrates the effectiveness of our multi-objective alignment framework in both training-time and inference-time optimization with possible synergy.

2 RELATED WORK

Process Reward Model. Process supervision addresses a core limitation of outcome-only evaluation by giving rewards on intermediate reasoning steps, helping systems avoid trajectories that look correct but contain logical errors. The foundational approach involves collecting step-level human annotations for mathematical reasoning tasks and training process reward models on these dense supervision signals (Lightman et al., 2023; Xia et al., 2025). Follow-up work scales supervision with automated or weakly supervised labels, for example per-step Monte Carlo rollouts or self-generated labels (Wang et al., 2023a; Luo et al., 2024). Beyond standard PRMs, recent variants introduce progress or verifier signals that score both partial correctness and future success, improving search and ranking during decoding (Chen et al., 2025; Setlur et al., 2024). There are also training objectives that regularize PRMs to improve stability (Zhang et al., 2024). Practical studies discuss data generation, evaluation pitfalls, and how PRMs differ from value functions that predict eventual solvability from partial traces (Zhang et al., 2025). Process-level search with step-wise scoring has further been shown to beat outcome-level test-time compute baselines in several setups, including controlled decoding, tree-structured search, and value/verification-guided search (Mudgal et al., 2023; Liu et al., 2023; Yao et al., 2023; Snell et al., 2025; Setlur et al., 2024; Wang et al., 2025).

Multi-Objective Alignment. Multi-objective alignment trains or steers language models for multiple, potentially conflicting objectives such as helpfulness, harmlessness, and honesty (Xie et al., 2025). Standard RLHF pipelines fit a scalar reward and fine-tune with PPO, or use scalarized preference optimization (Ouyang et al., 2022; Rafailov et al., 2023; Yuan et al., 2023; Xia et al., 2024; Dong et al.), but they collapse trade-offs into one score. Two lines of work relax this restriction. Training-time approaches adapt multi-objective ideas, such as multi-objective RLHF and multi-objective direct preference optimization, or parameter mixing to balance different rewards (Zhou et al., 2023; Rame et al., 2023; Wang et al., 2024a; Yang et al., 2024a; Shi et al., 2024; Li et al., 2025). Complementing these training-based methods, test-time alignment enables dynamic objective balancing without model retraining. These approaches modify token probability distributions using reward guidance and perform search under composite

objectives, achieving improvements on preference benchmarks while supporting per-user customization (Khanov et al., 2024; Chen et al., 2024b; Yang et al., 2024b; Lin et al., 2025). This paradigm offers particular promise for multi-objective alignment where individual user preferences vary significantly.

3 BACKGROUND

 To understand the challenges and opportunities in multi-objective alignment, we examine three representative domains.

Mathematics. Mathematics represents a typical verifiable domain where ground truth can be automatically determined with datasets such as GSM8K (Cobbe et al., 2021), MATH (Hendrycks et al., 2021), GaoKao (Zhang et al., 2023), and OlympiadBench (He et al., 2024). The verifiable nature of mathematical correctness enables automatic reward assignment at both outcome and process levels. Recent work has demonstrated the effectiveness of process reward models (Lightman et al., 2023; Wang et al., 2023a; Uesato et al., 2022) that provide step-by-step supervision to validate intermediate reasoning steps. Mathematical problem-solving can also involve dimensions beyond accuracy, including explanation clarity for diverse user expertise levels and pedagogical engagement in practical applications.

Human Values. Unlike mathematical correctness, human values include a broad range of subjective preferences that cannot be automatically verified, including aspects such as helpfulness, harmlessness, and honesty (Askell et al., 2021; Bai et al., 2022a; Ouyang et al., 2022; Bai et al., 2022b). These qualities require human judgment and are subjective, context-dependent, and sometimes conflicting. Recent work such as HelpSteer (Wang et al., 2023b; 2024b) and Ultra-Feedback (Cui et al., 2023) provides multi-dimensional annotations and reference comparisons across multiple criteria including helpfulness, coherence, and truthfulness. The challenge lies in the subjectivity and multi-dimensionality of human preferences, while the lack of automatic verification makes it difficult to provide more fine-grained supervision.

Interactive AI Tutoring. Interactive AI tutoring represents another challenging domain that combines objective and subjective evaluation within multi-turn dialogues, where success depends not only on correctness but also on pedagogical effectiveness, engagement, and scaffolding strategies. Datasets in this domain include educational dialogue corpora (Stasaski et al., 2020; Macina et al., 2023; Chen et al., 2024a) and socratic questioning collections (Shridhar et al., 2022; Ang et al., 2023; Ding et al., 2024). Unlike static domains, the quality of a tutor's response should be evaluated based on its impact on subsequent student responses and learning trajectories. We provide an example AI tutoring dialogue in Appendix G.

4 PROCESS REWARD MODEL TRAINING

With varying degrees of verifiability and supervision granularity as discussed in Section 3, we first develop a standardized PRM training framework across domains to lay foundations for multi-objective alignment.

4.1 VERIFIABLE DOMAINS

For tasks with objective correctness criteria, e.g., math, we augment the step-level supervision with outcome signals with a value target estimator to train PRMs that both validate current intermediate step and predict future correctness.

Step-level Reward. Given a trajectory $y_{1:N} = (y_1, y_2, \dots, y_N)$, the step-level reward is defined as a correctness signal that captures both textual validity and local logical coherence at step y_t (Lightman et al., 2023; Luo et al., 2024). Common practice of obtaining process reward labels involves a multi stage sampling and annotation process (Wang et al., 2023a; Lightman et al., 2023; Xiong et al., 2025). For example, in Math Shepherd (Wang et al., 2023a), multiple completions are sampled from each intermediate step to the final answer. A step is labeled as correct if at least one completion leads to a correct final solution, and incorrect if all completions result in wrong answers.

Value Reward with Hindsight Relabeling. Motivated by experience replay in reinforcement learning (Andrychowicz et al., 2017; Harutyunyan et al., 2019), we perform hindsight relabeling in addition to the step-level reward. From each step y_t , we rollout to its completion $y_{t+1} = (y_{t+1}, \ldots, y_n)$ and evaluate the final solution to obtain a binary terminal correctness reward $z \in \{0,1\}$. Then, we collect a step-level reward r_t from annotation or existing PRM's judgment for step y_t and blend it with the discounted terminal reward to credit the current step's contribution to the final outcome as \tilde{r}_t . For each step y_t , we generate M independent rollouts and aggregate them to obtain the final value target V_t^{target} , which is used to train the PRM by minimizing the mean squared error on its predictions p_t :

$$\tilde{r}_t = r_t + \gamma^{n-t} z, \qquad V_t^{\text{target}} = \frac{1}{M} \sum_{m=1}^M \tilde{r}_t^m, \qquad \mathcal{L}_{\text{PRM}} = \mathbb{E}_{t, y_{1:t}} \left[\left(p_t - V_t^{\text{target}} \right)^2 \right]. \tag{1}$$

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where $\gamma \in (0,1)$ is a discount factor that assign credits based on temporal distance. The relabeled reward enables the PRM to predict both local step reasoning quality and future solution correctness. 4.2 Non-verifiable Domains

For domains lacking objective correctness measures, we adapt our PRM training framework based on the availability of clear process structure and rollout difficulty.

Case A: Clear Process Structure with Efficient Rollout. When the task has clearly-defined intermediate steps that can be meaningfully evaluated, e.g. engagement in math reasoning process, we employ a rollout-based labeling strategy similarly to our verifiable domain approach. We first calibrate an LLM-as-Judge J using a few human annotated ratings \hat{R} to approximate the expected human judgment, $J(y_{1:t}) \approx \mathbb{E}[\hat{R}]$. Then we sample M completions from each step y_t and evaluate the resulting full trajectories using our calibrated LLM-as-Judge J. We label the step y_t as positive when the majority of completions are judged as positive by J:

$$r_t = \mathbf{1} \left[\left(\frac{1}{M} \sum_{m=1}^{M} \mathbf{1}_{\text{positive}} \left[\left(J \left(y_{1:t}, y_{t+1:n}^m \right) \right] \right) > \frac{1}{2} \right]. \tag{2}$$

This majority voting criterion reflects the inherent subjectivity in non-verifiable domains, where a reasoning step's quality is measured by its tendency to lead to generally acceptable outcomes rather than definite correctness.

Case B: Clear Process Structure with Costly Rollout. When generating rollouts is costly or difficult, for example multi-turn dialogue which requires real user interactions, we directly query the LLM-as-Judge J on observed trajectory prefixes to otain the training label: $r_t = J(y_{1:t})$. This approach trades the robustness of rollout-based evaluation for computational efficiency. One can mitigate the increased label noise inherent in this approach through careful judge calibration, ensemble methods, and multi annotator agreement when feasible.

Case C: Unclear Process Structure. For domains where step wise decomposition lacks clear structures, for example general question answering tasks, we approximate the process modeling through directly evaluating the partial response with a reward model trained with complete responses. For example, one may collect or reuse available pairwise preference data $\{(y^w, y^l)\}$ to train a Bradley-Terry model to score the process generation $R_\phi(y_{1:t}) \to \mathbb{R}$. The trained reward model provides holistic quality assessment that serves as guidance during decoding, approximating the intermediate process supervision even when the process structure is not well defined.

ALIGNMENT: TRAINING AND DECODING

To align LLMs for multiple objectives across domains, we propose our Multi-Action-Head DPO (MAH-DPO) for training time optimization (Section 5.1) and utilize our trained PRM directly for test-time alignment with rewardguided decoding with continuing hidden-state (Secrtion 5.2).

TRAINING-TIME OPTIMIZATION: MULTI-ACTION-HEAD DPO

Direct Preference Optimization. DPO (Rafailov et al., 2023) optimizes a policy π_{θ} against a fixed reference policy π_{ref} using preference pairs $\mathcal{D} = \{(x, y^w, y^l)\}$, where y^w is the preferred response to prompt x and y^l is the dispreferred one. The DPO loss is:

$$\mathcal{L}_{\text{DPO}}(\pi_{\theta}; \pi_{\text{ref}}) = -\mathbb{E}_{(x, y^w, y^l) \sim \mathcal{D}} \left[\log \sigma \left(\beta \left(\log \frac{\pi_{\theta}(y^w \mid x)}{\pi_{\text{ref}}(y^w \mid x)} - \log \frac{\pi_{\theta}(y^l \mid x)}{\pi_{\text{ref}}(y^l \mid x)} \right) \right) \right], \tag{3}$$

where $\sigma(\cdot)$ is the sigmoid and $\beta > 0$ is a temperature parameter controlling the strength of the preference signal.

Multi-Action-Head LLM. To jointly optimize for H distinct objectives while maintaining computational efficiency. we propose the multi-action-head LLM that extends the base LLM with specialized output layers. We maintain a single shared LLM backbone θ_b , while introducing H distinct linear projection heads, one for each alignment objective. This is more efficient than training H separate models, which would require H times the computational resources and fail to leverage cross objective synergies.

Specifically, let $h_{\theta_h}(x, y_{1:t}) \in \mathbb{R}^d$ denote the d-dimensional hidden state produced by the shared LLM backbone θ_b for input prefix $(x, y_{1:t})$. Each objective $i \in \{1, \dots, H\}$ has a dedicated projection head parameterized by matrix $W_i \in \mathbb{R}^{d \times |V|}$ to produce objective-specific logits z_i and token probability distribution:

$$z_i(x, y_{1:t}) = W_i^{\top} h_{\theta_b}(x, y_{1:t}), \qquad \pi_{\theta_b, W_i}(y_t \mid x, y_{1:t}) = \operatorname{softmax}(z_i(x, y_{1:t}))$$
(4)

where |V| is the vocabulary size. The shared LLM backbone captures general language understanding and generation

capabilities, while specialized heads can encode objective-specific preferences. During inference, our multi-action-head architecture supports flexible objective control by either selecting a specific head i for targeted behavior or ensembling logits from multiple heads for balanced performance:

$$\pi_{\text{MAH}}(y_t \mid x, y_{< t}) = \sum_{i=1}^{H} w_i \pi_{\theta_b, W_i}(y_t \mid x, y_{< t}), \tag{5}$$

where $w_i \ge 0$ are ensemble weights with $\sum_i w_i = 1$. This flexibility enables the model to be adapted for different downstream applications and user preferences without requiring separate training runs for each objective combination.

Multi-Action-Head DPO Objective. We first curate H preference datasets $\{\mathcal{D}_i\}_{i=1}^H$, where each \mathcal{D}_i contains preference pairs specifically designed for objective i labeled using our trained PRM or from annotated labels. All heads W_i are initialized from the same language modeling head from the supervised fine-tuned (SFT) LLM π_{θ_b} with small random perturbations to encourage specialization. The reference model π_{ref} retains the unperturbed SFT head. During training, examples $(x, y^w, y^l) \in \mathcal{D}_i$ are routed to head i, and we compute the objective-specific DPO loss:

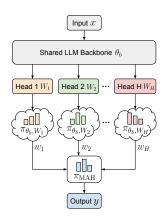


Figure 1: Overview of Multi-Action Head LLM.

$$\mathcal{L}_{i}(\theta_{b}, W_{i}) = -\mathbb{E}_{(x, y^{w}, y^{l}) \sim \mathcal{D}_{i}} \left[\log \sigma \left(\beta \left(\log \frac{\pi_{\theta_{b}, W_{i}}(y^{w} \mid x)}{\pi_{\text{ref}}(y^{w} \mid x)} - \log \frac{\pi_{\theta_{b}, W_{i}}(y^{l} \mid x)}{\pi_{\text{ref}}(y^{l} \mid x)} \right) \right) \right].$$
 (6)

Let a mini-batch during training be partitioned as $\mathcal{B} = \bigsqcup_{i=1}^{H} \mathcal{B}_i$ where \mathcal{B}_i gathers the examples assigned to head i. The combined loss we are minimizing is

$$\mathcal{L}_{\text{MAH-DPO}}(\theta_b, \{W_i\}) = \sum_{i=1}^{H} \alpha_i \cdot \frac{1}{|\mathcal{B}_i|} \sum_{(x, y^w, y^l) \in \mathcal{B}_i} \mathcal{L}_i(\theta_b, W_i; x, y^w, y^l), \tag{7}$$

where $\alpha_i \geq 0$ are objective weights with $\sum_i \alpha_i = 1$.

Gradient Analysis. The gradients for parameters of each head j are isolated by routing, while the backbone LLM gradients accumulate across heads:

$$\nabla_{W_j} \mathcal{L} = \sum_{i=1}^{H} \alpha_i \cdot \frac{1}{|\mathcal{B}_i|} \sum_{(x, y^w, y^l) \in \mathcal{B}_i} \underbrace{\nabla_{W_j} \mathcal{L}_i(\theta_b, W_i; x, y^w, y^l)}_{= 0 \text{ if } j \neq i} = \alpha_j \cdot \mathbb{E}_{\mathcal{B}_j} \left[\nabla_{W_j} \mathcal{L}_j \right], \tag{8}$$

$$\nabla_{\theta_b} \mathcal{L} = \sum_{i=1}^H \alpha_i \cdot \frac{1}{|\mathcal{B}_i|} \sum_{(x, y^w, y^l) \in \mathcal{B}_i} \nabla_{\theta_b} \mathcal{L}_i(\theta_b, W_i; x, y^w, y^l). \tag{9}$$

Therefore, a single backward pass through Equation 7 updates the backbone and every active action heads simultaneously. To achieve more stable training and balanced gradient propagation, we can construct mini-batches with similar number of examples $|\mathcal{B}_i|$ from each objective i or by tuning the weights α_i when the dataset sizes differ. Since every head consumes the same hidden states for its logits, the computation requires only one backbone forward per input and parallel per-head projections, leveraging cross objective synergies without introducing excessive extra training cost.

5.2 TEST-TIME OPTIMIZATION: PRM-GUIDED DECODING WITH CONTINUING HIDDEN STATE

We also explore the use of our trained PRM during test-time directly via step-level reward guided decoding. Existing reward-guided decoding or test-time search methods (Khanov et al., 2024; Liao et al., 2025; Park et al., 2024) typically rebuild the prompt each step by concatenating the newly selected next generation with previous steps. However, rebuilding and re-encoding the textual prompt each step can change how the prior context is represented within the hidden state, e.g., small differences in tokenization around whitespace and newline merges, shifts in relative positions, and the placement of special tokens from chat templates. As a result, the next-token distribution after re-encoding can differ from the one obtained by directly continuing from the previous step and such discontinuity can lead to performance degradation as observed in our experiments presented in Appendix D.

Therefore, to preserve the generation continuity at hidden state level, we utilize a running past key-value cache during our PRM-guided decoding. The same hidden state is carried forward, so the continuation distribution follows the true incremental decoding rather than a fresh prompt re-encoding approximation. We provide an overview of our PRM-guided decoding in Algorithm 1 and describe details as follows.

Cache Initialization and Candidate Proposal. Given a chat-formatted prompt x, we run a single forward pass with

the policy model π_{θ} to obtain the initial past key-value cache kv_0 and the first next-token distribution. We set response $y_{1:0}=\emptyset$ and generation step index t=0. This avoids reencoding x in later steps and provides the reference state from which all continuations proceed. Then, for each step t, we proposal K candidates from the current running cache kv_t . For each candidate k, we clone kv_t to a local copy and sample the next token from policy model π_{θ} while carrying that local cache forward. Sampling stops when the boundary detection criteria $\mathcal Q$ triggers. This yields a step generation y_{t+1}^k with its end-state cache kv_{t+1}^k .

Candidate Selection with PRM and Cache Update. Each sampled candidate is then evaluated by a PRM P. Given the current prefix $y_{1:t}$, the score for candidate k is $r_k = P(x,y_{1:t},y_{t+1}^k)$. We select $k^\star = \arg\max_k r_k$, append the chosen step generation to the response $y_{1:t+1} = y_{1:t} \parallel y_{t+1}^k$, and update the current running cache as $kv_{t+1} = kv_{t+1}^k$. This commit keeps decoding stateful across segments rather than re-encoding the prompt with textual concatenations. We repeat the above candidate proposal with π_θ starting from kv_t , PRM scoring, and cache update until an end-of-sequence to-ken appears or a token budget is reached. With every iteration

Algorithm 1: PRM-Guided Decoding with Continuing Hidden State

```
Input: policy \pi_{\theta}; PRM P; boundary detection criteria
              Q; number of candidates K; token budget
             T_{\max}; prompt x.
Output: response y.
\operatorname{kv}_0 \leftarrow \operatorname{Fwd}_{\pi_\theta}(x); y_{1:0} \leftarrow \emptyset; t \leftarrow 0.
while |y_{1:t}| < T_{\max} and EOS \notin y_{1:t} do
       for k = 1 to K do
               \widetilde{\mathrm{kv}} \leftarrow \mathrm{kv}_t; \, \widetilde{y} \leftarrow \emptyset.
               while Q(\tilde{y}) = 0 do
                      Sample next token z \sim \pi_{\theta}(\cdot \mid \widetilde{kv});
                  \widetilde{\mathrm{kv}} \leftarrow \mathrm{Fwd}_{\pi_{\alpha}}(\widetilde{\mathrm{kv}}, z); \, \widetilde{y} \leftarrow \widetilde{y} \parallel z.
               Record end-state cache kv_{t+1}^k \leftarrow \widetilde{kv};
               Record candidate next step y_{t+1}^k \leftarrow \tilde{y};
               Score with PRM r_k \leftarrow P(x, y_{1:t}, y^k).
       k^{\star} \in \operatorname{arg\,max}_k r_k;
       Update running cache kv_{t+1} \leftarrow kv_{t+1}^{k^*};
       Update response y_{1:t+1} \leftarrow y_{1:t} \parallel y_{t+1}^{k^*};
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advancing from the running cache, the generation remains continuous with respect to model's internal hidden state.

Computational Analysis. Besides keeping the generation continuity at hidden-state level, our cache-carrying PRM-guided decoding also reduce the computational cost compared to re-encode-per-step baselines. Let |x| be the prompt length, T the committed output tokens, N the number of steps, i.e., detected boundaries, K the candidates per step, and \bar{L} the average candidate length such that $T \approx N\bar{L}$. A re-encode-per-step policy costs $\mathcal{O}(K(|x|N+NT))$ while our cache-carrying policy costs $\mathcal{O}(|x|+KN\bar{L})=\mathcal{O}(|x|+KT)$. Thus the factor N that multiplies T is removed, enabling better test-time scaling by shifting compute from repeated re-encodings to candidate rollout or longer outputs.

6 EXPERIMENTS

In this section, we evaluate our multi-objective alignment framework across three domains. We show the effectiveness of our MAH-DPO training in aligning LLMs along multiple dimensions simultaneously and our PRM-guided decoding at test time. We further explore the potential synergy between training and test-time methods.

Datasets, Evaluation, and PRM Training. We evaluate our approach in three domains. **Math:** MATH (Hendrycks et al., 2021) contains 12,500 challenging high school competition problems requiring multi-step reasoning and enables verifiable step-level evaluation. **Human Values:** UltraFeedback (Cui et al., 2023) provides preference judgments over helpfulness, honesty, and truthfulness, with a total size of 64k samples. **AI Tutoring Dialogues:** Socratic Mind (Hung et al., 2024) contains multi-turn conversations in which an AI tutor guides Python programming students via Socratic questioning, averaging 8 turns per session, with a total of 1362 dialogues. For evaluation, in mathematics we measure **Accuracy** with correct final answers and **Engagement** with calibrated LLM-as-Judge with human annotations. In human values we score **Helpfulness**, **Honesty**, and **Truthfulness** using our trained reward models. In tutoring dialogues we measure accuracy and engagement by simulating the student's next turn after the aligned assistant response and scoring it with trained PRM. We train our PRMs for each domain following our proposed standardized pipeline in Section 4. We provide full details of PRM training for on each dataset in Appendix B.

6.1 Training-Time Alignment

We first evaluate our MAH-DPO approach across the above described three domains to validate its advantages in multi-objective alignment in terms of performance improvements and flexible user control.

Baselines and Variants. We report results of the following baselines as well as our MAH-DPO variants. **Base** is the original based LLM without any post-training or alignment. **SFT** applies supervised fine-tuning using only the preferred responses from preference pairs. **Single-Head DPO** directly applies DPO to one primary objective by pooling all dimension-specific preference data. **MODPO** (Zhou et al., 2023) is a multi-objective extension of

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Table 1: Alignment performances of training-time methods across three datasets.

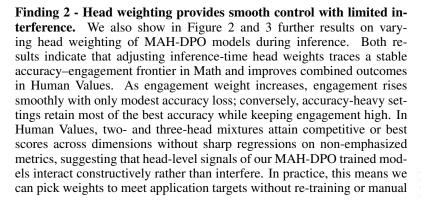
Method	Acc	Eng	Method	Help	Honest	Truth	Method	Acc	Eng
Base	0.7107	0.5007	Base	0.5800	0.3042	0.1888	Base	0.6560	0.3220
SFT	0.7300	0.5920	SFT	0.5546	0.2998	0.1992	SFT	0.6793	0.3473
Single-Head DPO	0.7253	0.7160	Single-Head DPO	0.6043	0.3055	0.2014	Single-Head DPO	0.7040	0.4460
MODPO	0.7280	0.7367	MODPO	0.6175	0.3477	0.2325	MODPO	0.7047	0.3600
MAH-DPO Acc Head	0.7353	0.8667	MAH-DPO Help Head	0.6309	0.3465	0.2239	MAH-DPO Acc Head	0.7007	0.4447
MAH-DPO Eng Head	0.7267	0.8840	MAH-DPO Honest Head MAH-DPO Truth Head	0.6257	0.3516 0.3461	0.2303 0.2286	MAH-DPO Eng Head	0.6953	0.4480
MAH-DPO Ensemble	0.7247	0.8733	MAH-DPO Ensemble	0.6237	0.3687	0.2478	MAH-DPO Ensemble	0.6893	0.4513
(a) Ma	th	(b) Human Values (c) Socratic Mind			Mind				

DPO that optimizes multiple alignment objectives in an RL-free manner by combining objectives with weights during training. MAH-DPO Individual Head reports the performance of each specialized head when used independently, reflecting objective-specific capabilities. MAH-DPO Ensemble uses an equal-weight combination of all head logits, representing our balanced multi-objective approach. We also analyze MAH-DPO inference with varying weights in Figure 2 and 3.

Implementation Details. We build paired preference datasets with our trained PRM or annotations in three domains as follows: Math (contrasting correct vs. incorrect rollouts and engaging vs. non-engaging solutions), Human Values (UltraFeedback subsets for helpfulness, honesty, and truthfulness), and Socratic Mind (simulated tutoring dialogues scored by trained PRMs). We train MAH-DPO on Qwen2.5-7B-Instruct for Math and Socratic Mind (SFT then MAH-DPO), and on meta-llama/Llama-3.1-8B-Instruct for Human Values. Models use domainappropriate learning rates, batch sizes, and context windows. Full data construction and hyperparameters are in provided in Appendix C. All experimental results are averaged over 3 independent runs and we report standard deviations in Appendix F.

Finding 1 - MAH-DPO yields the best multi-objective alignment performance. We present in Table 1 the main alignment results across Math, Human Values, and Socratic Mind for all compared training-time methods. Table 1 shows that specialized heads reliably lead on their targeted metrics, while the equal-weight ensemble

head aggregates these gains into the strongest overall performance across domains. In Math, specialization raises the target metric without collapsing the other while the ensemble head of MAH-DPO preserves most of these gains and removes the need for objective-specific selection at inference time. The results show the effectiveness of our MAH-DPO method in specializing each action head. In Human Values, the ensemble attains the best combined profile across helpfulness, honesty, and truth, outperforming single-objective baselines and method variants that optimize one dimension at a time. This demonstrates the advantage of our MAH-DPO method to capture complementary preference signals across dimensions with the shared LLM backbone. In Socratic Mind, our MAH-DPO method shifts the operating point toward higher engagement while keeping accuracy in a usable range, which is desirable for tutoring where student participation matters. The overall pattern supports shared representations with head-level specialization and an inference-time ensemble to achieve strong joint alignment without separate retraining for each objective mix.



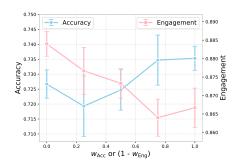


Figure 2: Results with varying action head weights in Math.

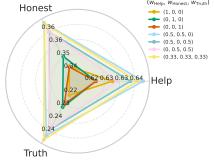


Figure 3: Results with varying action head weights in Human Values.

Table 2: Alignment performances of test-time methods across three datasets.

Method	Acc	Eng	Method	Help	Honest	Truth	Method	Acc	Eng
Base	0.6853	0.5133	Base	0.5750	0.3036	0.1904	Base	0.6400	0.3380
Accuracy PRM-guided	0.7633	0.4720	Helpful PRM-guided	0.6706	0.4050	0.2791			
Accuracy Value-guided	0.7993	0.4553	Honesty PRM-guided	0.6448	0.4693	0.3383	Accuracy PRM-guided	0.7127	0.2660
Engaging PRM-guided	0.7013	0.7187	Truthful PRM-guided	0.6350	0.4394	0.3296	Engaging PRM-guided	<u>0.6507</u>	0.4663
(a) Mat	th		(b) Hu	man Valı	ıes		(c) Socratic Mind		

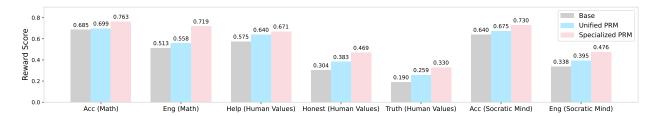


Figure 4: Alignment performances of a unified PRM trained across 7 dimensions in three domains compared with base model and the specialized PRM trained on each dimension per domain.

response selection. For example, we can emphasize truth dimension while maintaining helpfulness and honesty, or favor engagement while holding accuracy within a narrow band.

6.2 Test-Time Alignment

We also evaluate our PRM-guided decoding with continuing hidden state approach across all three domains to demonstrate the effectiveness of our trained PRM in guiding objective-specific alignment during inference.

Baselines and Variants. We report results of the following baselines as well as our PRM-guided decoding variants. **Base** utilizes the base model directly for step-wise generation without candidate sampling or selection. **Individual PRM-guided Decoding** applies an individual PRM trained for each objective dimension to guide the base model generation step by step following the candidate sampling-then-selection pipeline.

Implementation Details. We apply the same decoding strategy across all domains using the same base models as in training. In Math, we treat natural reasoning boundaries marked by $\n\n$ as step boundary, and we use our trained accuracy and engagement PRMs to guide step-level generation. In Human Values, where responses are nonverifiable and lack fixed process structure, we impose boundaries at sentence terminators and paragraph breaks, and use our trained reward models to score helpfulness, honesty, and truthfulness under step-level computational budgets of 256 tokens per chunk and 1,024 total tokens. In Socratic Mind, each turn is treated as a step and scored with our trained engagement and accuracy PRMs. Across all domains we sample K=5 candidates at each step. All decoding runs use temperature=1.0, top-p=1.0, and top-k=50 to ensure diversity while maintaining consistent selection under reward guidance. We provide further results validating the effectiveness of our use of continuing hidden state for PRM-guided decoding in Appendix D. All experimental results are averaged over 3 independent runs and we report standard deviations in Appendix F.

Finding 3 - PRM-guided decoding effectively improves the targeted objective. We report in Table 2 inference-time PRM-guided decoding results across three datasets. From Table 2, we can observe that PRM-guided decoding reliably pushes the chosen metric upward relative to the base model across domains. In Math, accuracy-oriented guidance lifts accuracy and engagement-oriented guidance lifts engagement, with the non-target metric remaining close to base levels rather than collapsing, which indicates that the scoring signals steer step decisions without harmful side effects. In Human Values, per-dimension guidance yields the best or second-best score on its own axis. In Socratic Mind, the available entries show the same pattern: objective-specific guidance raises its target while the other attribute stays within a usable range. Overall, our trained PRMs are effective in guiding test-time decoding process. They improve alignment performances by selecting among candidate continuations at natural boundaries, and offer smooth, predictable movement along multi-objective fronts without retraining.

Finding 4 - Unified PRM trained on mixture of data shows cross-domain effectiveness. To explore the potential of training a unified PRM across different domains, we further train a PRM using mixture of data with a total of 7 dimensions from Math, Human Values, and Socratic Mind. Details are also provided in Appendix B. As shown in

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Table 3: Alignment performances of synergizing training and test-time methods.

Method	Acc	Eng	Method	Help	Honest	Truth	Method	Acc	Eng
Single-Head DPO	0.7253	0.7160	Single-Head DPO	0.6043	0.3055	0.2014	Single-Head DPO	0.7040	0.4460
MODPO	0.7280	0.7367	MODPO	0.6175	0.3477	0.2325	MODPO	0.7047	0.3600
			MAH-DPO	0.6389	0.3687	0.2478			
MAH-DPO	0.7247	0.8733	MAH-DPO + Help PRM	0.7165	0.4554	0.3890	MAH-DPO	0.6893	0.4513
MAH-DPO + Accuracy Value	0.8000	0.8553	MAH-DPO + Honest PRM	0.6968	0.5196	0.4107	MAH-DPO + Accuracy PRM	0.7160	0.3800
MAH-DPO + Engaging PRM	0.7207	0.9060	MAH-DPO + Truth PRM	0.6834	0.4872	0.3630	MAH-DPO + Engaging PRM	0.7120	0.5420
(a) Math			(b) Human Values			(c) Socratic Mind			

(c) Socratic Mind (b) Human Values

Figure 4, the unified PRM improves every objective dimension over the base model in Math, Human Values, and Socratic Mind. In Math, it raises both accuracy and engagement over base while remaining below the best specialized PRM for each axis. In Human Values, it lifts Help, Honesty, and Truth relative to base and tracks the single-dimension specialists within a small margin. In Socratic Mind, it again lies between base and the best specialized PRM on both accuracy and engagement. Our results show the potential of a generalized PRM trained on a wider range of domains and datasets that transfers across domains and provides a balanced improvement profile without domainspecific retraining or serving multiple models.

SYNERGIZING TRAINING AND TEST-TIME ALIGNMENT

Finding 5 - Training and test-time methods complement each other in alignment. In Table 3 we report results when we pair MAH-DPO with Ensemble head outputs with PRM-based guidance at test time across Math, Human Values, and Socratic Mind. The combined setup consistently pushes the joint accuracy-engagement or multi-dimension profile outward relative to training-only baselines. In Math, accuracy-oriented selection boosts accuracy but slightly reduces engagement, while engagement-oriented selection shows the opposite trade-off. In Socratic Mind, we observe similar targeted trade-offs with PRMs optimizing their respective dimensions. In Human Values, per-dimension PRMs on top of MAH-DPO reach the best scores on their targeted axes, and the ensemble PRM gives a balanced profile close to the specialists while keeping non-target dimensions high; the honesty-guided run also boosts truth, which suggests positive transfer enabled by the head factorization learned during training. Overall, the mechanism is simple: training produces disentangled heads and a strong shared backbone, while test-time PRMs rank candidates at natural boundaries to steer generation toward the desired goal. This pairing expands the attainable Pareto set and gives practical control at inference through specialist versus ensemble guidance and lightweight weight tuning, without additional retraining.

Finding 6 - Reward verifiability guides whether test-time or training-time method selection. From Tables 1, 2, and 3, we observe a consistent pattern across domains. When the reward is highly verifiable and can be checked deterministically, e.g., Math accuracy, training-time alignment methods yield only incremental gains over strong baselines, while PRM-guided decoding at test time produces substantially larger jumps. This suggests that precise step-level scoring can steer generation more effectively than additional finetuning when the signal is crisp and unambiguous. In contrast, when the reward is less verifiable or more subjective such as helpfulness, honesty, truth, or engagement, multi-head training already delivers marked improvements by shaping shared representations and separating objectives into disentangled heads. Test-time guidance then further refines or rebalances these objectives, giving targeted emphasis, e.g., lifting honesty or helpfulness or producing a balanced ensemble profile, without eroding the non-target dimensions. The mixed case of Math engagement also reflects this pattern: training yields large gains, while inferencetime guidance still helps but with a smaller relative lift. Overall, verifiable rewards benefit most from test-time search against a precise signal, whereas noisier rewards benefit first from representation shaping with multi-objective training, after which inference-time weighting provides fine-grained control with minimal trade-offs.

CONCLUSION

In this paper, we present a unified framework for multi-objective alignment during training and inference time. We standardize process reward model training in both verifiable and non verifiable settings, proposes Multi-Action-Head DPO training with vectorized rewards and pairs the trained model with PRM-guided decoding with continuing hidden state. Experiments on math reasoning, value alignment, and multi-turn tutoring domains demonstrate the effectiveness of our framework for multi-objective alignment as well as fine-grained and flexible user control for alignment dimensions. Our framework offers a practical pathway toward AI assistants that are simultaneously accurate, safe, and engaging across diverse domains and applications.

ETHICS STATEMENT

This work uses three data sources. For Socratic Mind tutoring dialogues, human subjects procedures were reviewed and approved by the authors' Institutional Review Board, and only students who gave explicit written consent were included. Participation was voluntary with no academic consequences, and students could withdraw at any time. Dialogues were deidentified, stored with encryption, and accessed only by approved researchers. Public datasets MATH and UltraFeedback were used under their licenses, and we cite the sources. We applied content filters and safety checks to reduce risks, avoided sensitive advice, and report remaining limitations. We will share code and configurations that do not compromise privacy or licensing.

REPRODUCIBILITY STATEMENT

We describe the details of datasets, experimental setups, and evaluation procedures in Section 6. Full PRM training details and configurations for each experimented domain and dataset are provided in Appendix B. Full training-time alignment details and configurations for each experimented domain and dataset are provided in Appendix C. In addition, Appendix E contains all the system prompts used in our experiments. Together, these materials provide all necessary information to replicate our results.

REFERENCES

- Marcin Andrychowicz, Filip Wolski, Alex Ray, Jonas Schneider, Rachel Fong, Peter Welinder, Bob McGrew, Josh Tobin, OpenAI Pieter Abbeel, and Wojciech Zaremba. Hindsight experience replay. *Advances in neural information processing systems*, 30, 2017.
- Beng Heng Ang, Sujatha Das Gollapalli, and See Kiong Ng. Socratic question generation: A novel dataset, models, and evaluation. In *Proceedings of the 17th Conference of the European Chapter of the Association for Computational Linguistics*, pp. 147–165, 2023.
- Amanda Askell, Yuntao Bai, Anna Chen, Dawn Drain, Deep Ganguli, Tom Henighan, Andy Jones, Nicholas Joseph, Ben Mann, Nova DasSarma, et al. A general language assistant as a laboratory for alignment. *arXiv preprint arXiv:2112.00861*, 2021.
- Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, et al. Training a helpful and harmless assistant with reinforcement learning from human feedback. *arXiv preprint arXiv:2204.05862*, 2022a.
- Yuntao Bai, Saurav Kadavath, Sandipan Kundu, Amanda Askell, Jackson Kernion, Andy Jones, Anna Chen, Anna Goldie, Azalia Mirhoseini, Cameron McKinnon, et al. Constitutional ai: Harmlessness from ai feedback. *arXiv* preprint arXiv:2212.08073, 2022b.
- Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.
- Jiahao Chen, Zitao Liu, Mingliang Hou, Xiangyu Zhao, and Weiqi Luo. Multi-turn classroom dialogue dataset: Assessing student performance from one-on-one conversations. In *Proceedings of the 33rd ACM International Conference on Information and Knowledge Management*, pp. 5333–5337, 2024a.
- Ruizhe Chen, Xiaotian Zhang, Meng Luo, Wenhao Chai, and Zuozhu Liu. Pad: Personalized alignment of llms at decoding-time. *arXiv preprint arXiv:2410.04070*, 2024b.
- Wenxiang Chen, Wei He, Zhiheng Xi, Honglin Guo, Boyang Hong, Jiazheng Zhang, Rui Zheng, Nijun Li, Tao Gui, Yun Li, et al. Better process supervision with bi-directional rewarding signals. *arXiv preprint arXiv:2503.04618*, 2025.
- Paul F Christiano, Jan Leike, Tom Brown, Miljan Martic, Shane Legg, and Dario Amodei. Deep reinforcement learning from human preferences. *Advances in neural information processing systems*, 30, 2017.
- Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, et al. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.

Ganqu Cui, Lifan Yuan, Ning Ding, Guanming Yao, Wei Zhu, Yuan Ni, Guotong Xie, Zhiyuan Liu, and Maosong Sun. Ultrafeedback: Boosting language models with high-quality feedback, 2023.

 Haikang Deng and Colin Raffel. Reward-augmented decoding: Efficient controlled text generation with a unidirectional reward model. *arXiv preprint arXiv:2310.09520*, 2023.

Yuyang Ding, Hanglei Hu, Jie Zhou, Qin Chen, Bo Jiang, and Liang He. Boosting large language models with socratic method for conversational mathematics teaching. In *Proceedings of the 33rd ACM International Conference on Information and Knowledge Management*, pp. 3730–3735, 2024.

Hanze Dong, Wei Xiong, Deepanshu Goyal, Yihan Zhang, Winnie Chow, Rui Pan, Shizhe Diao, Jipeng Zhang, KaShun SHUM, and Tong Zhang. Raft: Reward ranked finetuning for generative foundation model alignment. *Transactions on Machine Learning Research*.

Hanze Dong, Wei Xiong, Bo Pang, Haoxiang Wang, Han Zhao, Yingbo Zhou, Nan Jiang, Doyen Sahoo, Caiming Xiong, and Tong Zhang. Rlhf workflow: From reward modeling to online rlhf. *arXiv preprint arXiv:2405.07863*, 2024.

Deep Ganguli, Liane Lovitt, Jackson Kernion, Amanda Askell, Yuntao Bai, Saurav Kadavath, Ben Mann, Ethan Perez, Nicholas Schiefer, Kamal Ndousse, et al. Red teaming language models to reduce harms: Methods, scaling behaviors, and lessons learned. *arXiv* preprint arXiv:2209.07858, 2022.

Yiju Guo, Ganqu Cui, Lifan Yuan, Ning Ding, Zexu Sun, Bowen Sun, Huimin Chen, Ruobing Xie, Jie Zhou, Yankai Lin, et al. Controllable preference optimization: Toward controllable multi-objective alignment. *arXiv* preprint *arXiv*:2402.19085, 2024.

Kunal Handa, Drew Bent, Alex Tamkin, Miles McCain, Esin Durmus, Michael Stern, Mike Schiraldi, Saffron Huang, Stuart Ritchie, Steven Syverud, Kamya Jagadish, Margaret Vo, Matt Bell, and Deep Ganguli. Anthropic education report: How university students use claude. https://www.anthropic.com/news/anthropic-education-report-how-university-students-use-claude, 2025a. Accessed September 12, 2025.

Kunal Handa, Drew Bent, Alex Tamkin, Miles McCain, Esin Durmus, Michael Stern, Mike Schiraldi, Saffron Huang, Stuart Ritchie, Steven Syverud, Kamya Jagadish, Margaret Vo, Matt Bell, and Deep Ganguli. Anthropic education report: How university students use claude, 2025b. URL https://www.anthropic.com/news/anthropic-education-report-how-university-students-use-claude.

Anna Harutyunyan, Will Dabney, Thomas Mesnard, Mohammad Gheshlaghi Azar, Bilal Piot, Nicolas Heess, Hado P van Hasselt, Gregory Wayne, Satinder Singh, Doina Precup, et al. Hindsight credit assignment. *Advances in neural information processing systems*, 32, 2019.

Chaoqun He, Renjie Luo, Yuzhuo Bai, Shengding Hu, Zhen Leng Thai, Junhao Shen, Jinyi Hu, Xu Han, Yujie Huang, Yuxiang Zhang, et al. Olympiadbench: A challenging benchmark for promoting agi with olympiad-level bilingual multimodal scientific problems. *arXiv preprint arXiv:2402.14008*, 2024.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset. *arXiv preprint arXiv:2103.03874*, 2021.

Yulan Hu, Sheng Ouyang, Jinman Zhao, and Yong Liu. Coarse-to-fine process reward modeling for mathematical reasoning. *arXiv preprint arXiv:2501.13622*, 2025.

Yuzheng Hu, Ruicheng Xian, Qilong Wu, Qiuling Fan, Lang Yin, and Han Zhao. Revisiting scalarization in multi-task learning: A theoretical perspective. *Advances in Neural Information Processing Systems*, 36:48510–48533, 2023.

Jui-Tse Hung, Christopher Cui, Diana M Popescu, Saurabh Chatterjee, and Thad Starner. Socratic mind: Scalable oral assessment powered by ai. In *Proceedings of the Eleventh ACM Conference on Learning@ Scale*, pp. 340–345, 2024.

Aaron Hurst, Adam Lerer, Adam P Goucher, Adam Perelman, Aditya Ramesh, Aidan Clark, AJ Ostrow, Akila Welihinda, Alan Hayes, Alec Radford, et al. Gpt-40 system card. *arXiv preprint arXiv:2410.21276*, 2024.

Joel Jang, Seungone Kim, Bill Yuchen Lin, Yizhong Wang, Jack Hessel, Luke Zettlemoyer, Hannaneh Hajishirzi, Yejin Choi, and Prithviraj Ammanabrolu. Personalized soups: Personalized large language model alignment via post-hoc parameter merging. *arXiv preprint arXiv:2310.11564*, 2023.

Maxim Khanov, Jirayu Burapacheep, and Yixuan Li. Args: Alignment as reward-guided search. arXiv preprint arXiv:2402.01694, 2024.

Belinda Z Li, Alex Tamkin, Noah Goodman, and Jacob Andreas. Eliciting human preferences with language models. *arXiv preprint arXiv:2310.11589*, 2023.

Bolian Li, Yifan Wang, Anamika Lochab, Ananth Grama, and Ruqi Zhang. Cascade reward sampling for efficient decoding-time alignment. *arXiv preprint arXiv:2406.16306*, 2024.

Kaiwen Li, Tao Zhang, and Rui Wang. Deep reinforcement learning for multiobjective optimization. *IEEE transactions on cybernetics*, 51(6):3103–3114, 2020.

 Moxin Li, Yuantao Zhang, Wenjie Wang, Wentao Shi, Zhuo Liu, Fuli Feng, and Tat-Seng Chua. Self-improvement towards pareto optimality: Mitigating preference conflicts in multi-objective alignment. *arXiv* preprint *arXiv*:2502.14354, 2025.

Baohao Liao, Yuhui Xu, Hanze Dong, Junnan Li, Christof Monz, Silvio Savarese, Doyen Sahoo, and Caiming Xiong. Reward-guided speculative decoding for efficient llm reasoning. *arXiv preprint arXiv:2501.19324*, 2025.

Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. Let's verify step by step. In *The Twelfth International Conference on Learning Representations*, 2023.

Baijiong Lin, Weisen Jiang, Yuancheng Xu, Hao Chen, and Ying-Cong Chen. Parm: Multi-objective test-time alignment via preference-aware autoregressive reward model. *arXiv preprint arXiv:2505.06274*, 2025.

Chien-Chang Lin, Anna YQ Huang, and Owen HT Lu. Artificial intelligence in intelligent tutoring systems toward

 sustainable education: a systematic review. *Smart learning environments*, 10(1):41, 2023.

Jiacheng Liu, Andrew Cohen, Ramakanth Pasunuru, Yejin Choi, Hannaneh Hajishirzi, and Asli Celikyilmaz. Don't throw away your value model! generating more preferable text with value-guided monte-carlo tree search decoding.

arXiv preprint arXiv:2309.15028, 2023.

Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Meiqi Guo, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu,

Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Meiqi Guo, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu, Lei Meng, et al. Improve mathematical reasoning in language models by automated process supervision. *arXiv* preprint arXiv:2406.06592, 2024.

Jakub Macina, Nico Daheim, Sankalan Pal Chowdhury, Tanmay Sinha, Manu Kapur, Iryna Gurevych, and Mrinmaya Sachan. Mathdial: A dialogue tutoring dataset with rich pedagogical properties grounded in math reasoning problems. *arXiv* preprint arXiv:2305.14536, 2023.

Kaushal Kumar Maurya, KV Srivatsa, Kseniia Petukhova, and Ekaterina Kochmar. Unifying ai tutor evaluation: An evaluation taxonomy for pedagogical ability assessment of llm-powered ai tutors. *arXiv preprint arXiv:2412.09416*, 2024.

Sidharth Mudgal, Jong Lee, Harish Ganapathy, YaGuang Li, Tao Wang, Yanping Huang, Zhifeng Chen, Heng-Tze Cheng, Michael Collins, Trevor Strohman, et al. Controlled decoding from language models. *arXiv* preprint arXiv:2310.17022, 2023.

OpenAI. Introducing study mode. https://openai.com/index/chatgpt-study-mode/, July 2025. Accessed September 12, 2025. Feature launched July 29, 2025.

Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, et al. Training language models to follow instructions with human feedback. *Advances in neural information processing systems*, 35:27730–27744, 2022.

Sankalan Pal Chowdhury, Vilém Zouhar, and Mrinmaya Sachan. Autotutor meets large language models: A language model tutor with rich pedagogy and guardrails. In *Proceedings of the Eleventh ACM Conference on Learning* @ *Scale*, pp. 5–15, 2024.

Sungjin Park, Xiao Liu, Yeyun Gong, and Edward Choi. Ensembling large language models with process reward-guided tree search for better complex reasoning. *arXiv preprint arXiv:2412.15797*, 2024.

Ethan Perez, Saffron Huang, Francis Song, Trevor Cai, Roman Ring, John Aslanides, Amelia Glaese, Nat McAleese, and Geoffrey Irving. Red teaming language models with language models. *arXiv preprint arXiv:2202.03286*, 2022.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, and Chelsea Finn. Direct preference optimization: Your language model is secretly a reward model. *Advances in neural information processing systems*, 36:53728–53741, 2023.

- Alexandre Rame, Guillaume Couairon, Corentin Dancette, Jean-Baptiste Gaya, Mustafa Shukor, Laure Soulier, and Matthieu Cord. Rewarded soups: towards pareto-optimal alignment by interpolating weights fine-tuned on diverse rewards. *Advances in Neural Information Processing Systems*, 36:71095–71134, 2023.
- Amrith Setlur, Chirag Nagpal, Adam Fisch, Xinyang Geng, Jacob Eisenstein, Rishabh Agarwal, Alekh Agarwal, Jonathan Berant, and Aviral Kumar. Rewarding progress: Scaling automated process verifiers for llm reasoning. arXiv preprint arXiv:2410.08146, 2024.
- Ruizhe Shi, Yifang Chen, Yushi Hu, Alisa Liu, Hanna Hajishirzi, Noah A Smith, and Simon S Du. Decoding-time language model alignment with multiple objectives. *Advances in Neural Information Processing Systems*, 37: 48875–48920, 2024.
- Kumar Shridhar, Jakub Macina, Mennatallah El-Assady, Tanmay Sinha, Manu Kapur, and Mrinmaya Sachan. Automatic generation of socratic subquestions for teaching math word problems. *arXiv preprint arXiv:2211.12835*, 2022.
- Charlie Victor Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling Ilm test-time compute optimally can be more effective than scaling parameters for reasoning. In *The Thirteenth International Conference on Learning Representations*, 2025.
- Katherine Stasaski, Kimberly Kao, and Marti A Hearst. Cima: A large open access dialogue dataset for tutoring. In *Proceedings of the Fifteenth Workshop on Innovative Use of NLP for Building Educational Applications*, pp. 52–64, 2020.
- Jonathan Uesato, Nate Kushman, Ramana Kumar, Francis Song, Noah Siegel, Lisa Wang, Antonia Creswell, Geoffrey Irving, and Irina Higgins. Solving math word problems with process-and outcome-based feedback. arXiv preprint arXiv:2211.14275, 2022.
- Haoxiang Wang, Wei Xiong, Tengyang Xie, Han Zhao, and Tong Zhang. Interpretable preferences via multi-objective reward modeling and mixture-of-experts. *arXiv* preprint arXiv:2406.12845, 2024a.
- Kaiwen Wang, Jin Peng Zhou, Jonathan Chang, Zhaolin Gao, Nathan Kallus, Kianté Brantley, and Wen Sun. Value-guided search for efficient chain-of-thought reasoning. *arXiv* preprint arXiv:2505.17373, 2025.
- Peiyi Wang, Lei Li, Zhihong Shao, RX Xu, Damai Dai, Yifei Li, Deli Chen, Yu Wu, and Zhifang Sui. Math-shepherd: Verify and reinforce llms step-by-step without human annotations. *arXiv preprint arXiv:2312.08935*, 2023a.
- Zhilin Wang, Yi Dong, Jiaqi Zeng, Virginia Adams, Makesh Narsimhan Sreedhar, Daniel Egert, Olivier Delalleau, Jane Polak Scowcroft, Neel Kant, Aidan Swope, et al. Helpsteer: Multi-attribute helpfulness dataset for steerlm. arXiv preprint arXiv:2311.09528, 2023b.
- Zhilin Wang, Yi Dong, Olivier Delalleau, Jiaqi Zeng, Gerald Shen, Daniel Egert, Jimmy Zhang, Makesh Narsimhan Sreedhar, and Oleksii Kuchaiev. Helpsteer 2: Open-source dataset for training top-performing reward models. *Advances in Neural Information Processing Systems*, 37:1474–1501, 2024b.
- Zeqiu Wu, Yushi Hu, Weijia Shi, Nouha Dziri, Alane Suhr, Prithviraj Ammanabrolu, Noah A Smith, Mari Ostendorf, and Hannaneh Hajishirzi. Fine-grained human feedback gives better rewards for language model training. *Advances in Neural Information Processing Systems*, 36:59008–59033, 2023.
- Yu Xia, Tong Yu, Zhankui He, Handong Zhao, Julian McAuley, and Shuai Li. Aligning as debiasing: Causality-aware alignment via reinforcement learning with interventional feedback. In *Proceedings of the 2024 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies (Volume 1: Long Papers)*, pp. 4684–4695, 2024.
- Yu Xia, Rui Wang, Xu Liu, Mingyan Li, Tong Yu, Xiang Chen, Julian McAuley, and Shuai Li. Beyond chain-of-thought: A survey of chain-of-x paradigms for llms. In *Proceedings of the 31st International Conference on Computational Linguistics*, pp. 10795–10809, 2025.

- Zhouhang Xie, Junda Wu, Yiran Shen, Yu Xia, Xintong Li, Aaron Chang, Ryan Rossi, Sachin Kumar, Bodhisattwa Prasad Majumder, Jingbo Shang, et al. A survey on personalized and pluralistic preference alignment in large language models. *arXiv preprint arXiv:2504.07070*, 2025.
- Wei Xiong, Wenting Zhao, Weizhe Yuan, Olga Golovneva, Tong Zhang, Jason Weston, and Sainbayar Sukhbaatar. Stepwiser: Stepwise generative judges for wiser reasoning. *arXiv preprint arXiv:2508.19229*, 2025.
- Yuancheng Xu, Udari Madhushani Sehwag, Alec Koppel, Sicheng Zhu, Bang An, Furong Huang, and Sumitra Ganesh. Genarm: Reward guided generation with autoregressive reward model for test-time alignment. *arXiv* preprint arXiv:2410.08193, 2024.
- Kailai Yang, Zhiwei Liu, Qianqian Xie, Jimin Huang, Tianlin Zhang, and Sophia Ananiadou. Metaaligner: Towards generalizable multi-objective alignment of language models. *Advances in Neural Information Processing Systems*, 37:34453–34486, 2024a.
- Rui Yang, Xiaoman Pan, Feng Luo, Shuang Qiu, Han Zhong, Dong Yu, and Jianshu Chen. Rewards-in-context: Multi-objective alignment of foundation models with dynamic preference adjustment. *arXiv preprint arXiv:2402.10207*, 2024b.
- Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Tom Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. *Advances in neural information processing systems*, 36:11809–11822, 2023.
- Zheng Yuan, Hongyi Yuan, Chuanqi Tan, Wei Wang, Songfang Huang, and Fei Huang. Rrhf: Rank responses to align language models with human feedback without tears. *arXiv preprint arXiv:2304.05302*, 2023.
- Hanning Zhang, Pengcheng Wang, Shizhe Diao, Yong Lin, Rui Pan, Hanze Dong, Dylan Zhang, Pavlo Molchanov, and Tong Zhang. Entropy-regularized process reward model. *arXiv preprint arXiv:2412.11006*, 2024.
- Xiaotian Zhang, Chunyang Li, Yi Zong, Zhengyu Ying, Liang He, and Xipeng Qiu. Evaluating the performance of large language models on gaokao benchmark. *arXiv preprint arXiv:2305.12474*, 2023.
- Zhenru Zhang, Chujie Zheng, Yangzhen Wu, Beichen Zhang, Runji Lin, Bowen Yu, Dayiheng Liu, Jingren Zhou, and Junyang Lin. The lessons of developing process reward models in mathematical reasoning. *arXiv* preprint arXiv:2501.07301, 2025.
- Chujie Zheng, Zhenru Zhang, Beichen Zhang, Runji Lin, Keming Lu, Bowen Yu, Dayiheng Liu, Jingren Zhou, and Junyang Lin. Processbench: Identifying process errors in mathematical reasoning. *arXiv* preprint *arXiv*:2412.06559, 2024.
- Zhanhui Zhou, Jie Liu, Chao Yang, Jing Shao, Yu Liu, Xiangyu Yue, Wanli Ouyang, and Yu Qiao. Beyond one-preference-for-all: Multi-objective direct preference optimization. 2023.

APPENDIX

A USE OF LLMS

We LLMs solely as a general-purpose writing aid to check grammar, improve clarity, and polish the wording of the manuscript. No parts of the research ideas, experimental design, data analysis, or substantive writing were produced by an LLM. All technical content and interpretations were created and verified by the authors.

B PRM TRAINING DETAILS

B.1 MATH PRM TRAINING

Accuracy PRM Training. We implement our rollout approach with hindsight relabeling to train a process reward model for mathematical accuracy following Section 4.1. Our method leverages an existing well-trained PRM, specifically Qwen/Qwen2.5-Math-PRM-7B, to provide intermediate step-level rewards that we combine with terminal outcome signals through our principled framework. For each candidate reasoning step, we generate 5 independent rollouts using sampling to completion. Step values are computed by combining intermediate PRM rewards with binary final outcome rewards, where correct solutions receive a reward of 1 and incorrect solutions receive 0. These rewards are weighted by a temporal discount factor and averaged across all rollouts to obtain reliable step-level supervision signals for step selection and trajectory extension. The iterative generation process continues until either a final boxed answer is produced or the maximum step limit of 20 is reached, yielding step values within the range [0,2]. Given that the average mathematical problem requires 9-12 reasoning steps, we set the discount rate $\gamma=0.9$ to appropriately balance immediate step quality assessment with long-term credit assignment.

We also swept the discount factor when turning per-step PRM rewards into value targets and repeated both value-head training and guided decoding. Concretely, for a step prefix $s_{\leq t}$ we formed discounted returns $G_t = \sum_{k \geq 0} \gamma^k r_{t+k}$ with $\gamma \in \{0.9, 0.95\}$, trained the same frozen-backbone + MLP value head to regress G_t via MSE, then used the learned value to steer generation: at each step we propose candidate continuations and pick the one maximizing a blended objective $\alpha V(s_{\leq t} + \operatorname{cand}) + (1 - \alpha) \log P(\operatorname{cand} \mid s_{\leq t})$. Lower γ favors short-term gains, while higher γ encourages longer-horizon reasoning during decoding.

Table 4: Comparison of Math Step-level Guided Decoding methods and their accuracy, averaged over 3 trials.

Guided Decoding Method	Accuracy	Engagingness
Baseline step-by-step	0.6853 ± 0.0163	0.5133 ± 0.0543
PRM-guided	0.7633 ± 0.0050	0.7187 ± 0.0266
Value head guided with $\gamma=0.90$	0.7993 ± 0.0172	0.4553 ± 0.0221
Value head guided with $\gamma=0.95$	0.7993 ± 0.0081	0.5053 ± 0.0050
MAH-DPO Ensemble Head + Accuracy PRM-guided with $\gamma=0.90$	0.8000 ± 0.0231	0.8553 ± 0.0136
MAH-DPO Ensemble Head + Accuracy PRM-guided with $\gamma=0.95$	0.7800 ± 0.0197	0.8470 ± 0.0098

Our PRM architecture follows the design from <code>Qwen/Qwen2.5-Math-PRM-7B</code> (Zhang et al., 2025), where we replace the standard language modeling head with a two-layer scalar value head that produces step-level quality scores. Reasoning steps are serialized using the special separator token <code><extra_0></code> in chat-format input, with the transformer's hidden state at each separator token position marking step boundaries. These boundary representations feed into a compact MLP for per-step value prediction. During training, we freeze the PRM backbone parameters from <code>Qwen/Qwen2.5-Math-PRM-7B</code> and optimize only the value head using mean squared error loss against the soft step-value targets. Training proceeds for 2 epochs with a batch size of 32 and learning rate of 5e-5.

Engagement PRM Training. To evaluate our approach on subjective quality dimensions, we construct an engagement-focused dataset. We sample 50 problems from the MATH training split and generate 4 solution roll-outs per problem using the base model. These rollouts use an even mix of engaging and non-engaging reasoning style system prompts to ensure balanced representation (see Appendix E). Human annotators label all 200 responses for engagement quality, providing ground truth supervision for this subjective dimension. We calibrate an LLM-as-Judge using <code>Qwen/Qwen2.5-72B-Instruct</code> to evaluate engagement levels, achieving 75.8% classification accuracy

against human-labeled solutions. This calibrated judge enables scalable engagement evaluation during PRM training (see Appendix E for calibrated system prompt).

For each problem, we generate one initial reasoning step, then create eight diverse completions continuing from the current state using generation temperature 1.0. The calibrated LLM-as-Judge scores engagement for every completion batch per step. Following our Case A methodology for non-verifiable domains in Section 4.2, we label a step as engaging if more than four out of eight rollouts continuing from that step are deemed engaging, otherwise it receives a non-engaging label. This process yields 11.8k step-level engagement annotations. We then convert the training data into incremental reasoning sequences, where each step accumulates the solution path from problem statement through progressive reasoning chains. The base model for the PRM training is meta-llama/Llama-3.1-8B configured for binary classification. We train for 2 epochs using batch size 128, learning rate 1e-5 which achieves an evaluation accuracy of 92.5%.

B.2 HUMAN VALUES PRM TRAINING

Human values represent a non-verifiable domain with no clear process structure. Rather than forcing artificial step-level decomposition, we follow our Case C methodology in Section 4.2 and train reward model for holistic quality assessment. We train Bradley-Terry reward models on top of the SFT model with base model as meta-llama/Llama-3.1-8B following the RLHFlow recipe (Dong et al., 2024) with learning rate 1e-5 and batch size 32 for 3 epochs. The reward model learns to capture human preferences across the helpfulness, honesty, and truthfulness dimensions through pairwise preference optimization, providing dense guidance signals for fine-grained decoding without requiring artificial process supervision.

B.3 SOCRATIC MIND PRM TRAINING

Students complete post-interaction surveys rating their experience on a 0-6 scale regarding how the Socratic Mind approach enhanced their understanding, serving as our engagement dimension ground truth. We classify ratings ≥ 4 as engaging interactions. Student dialogues are collected with engagement ratings, and conversations are randomly truncated after assistant turns to create training samples with varying trajectory lengths. We establish calibration datasets with 80 training and 80 test samples to calibrate an LLM-as-judge using GPT-40 (Hurst et al., 2024), achieving 0.8 training accuracy and 0.66 test accuracy for engagement prediction. We additionally curate a specialized judge for accuracy evaluation where system prompt for both objectives can be found in Appendix E. The calibrated LLM-as-judge labels approximately 5k engagement samples and 8k accuracy samples for PRM training, achieving 0.81 test accuracy for engagement and 0.7 for accuracy using classification on Llama-3.1-8B.

B.4 Unified PRM Training

We constructed a unified binary-classification corpus by combining all 7 objective dimensions from the domain datasets used in our experiments and formatting each example as a "User:"/"Assistant:" dialogue with blank-line spacing. Math engagement conversations yield incremental stepwise instances labeled from +/-. Human value preference pairs are mapped to chosen =1 and rejected =0. Math value scores are normalized per example and thresholded ($>0.85 \rightarrow 1$, otherwise 0). Socratic Mind engagement and accuracy retain only multi-turn dialogues, with accuracy excluding the last turn. This pipeline produced a total of 168,514 examples with 47.4% positives. We then fine-tuned a pre-trained Llama-3.1-8B model with a 2-class classification head using cross-entropy. Training used a batch size of 128, a learning rate of 1×10^{-5} , and ran for 2 epochs.

C TRAINING-TIME ALIGNMENT DETAILS

C.1 MATH TRAINING DETAILS

Mathematical reasoning presents a natural testbed for multi-objective alignment, as effective tutoring requires balancing computational accuracy with pedagogical engagement. We design our experimental setup to capture this fundamental trade-off in educational AI systems.

Preference Data Construction. We construct two complementary preference datasets using the MATH training dataset (12k problems) to target distinct but interrelated aspects of mathematical competence:

• Accuracy-focused pairs: For each problem, we generate up to 30 response rollouts using Qwen2.5-7B-Instruct, extract boxed numerical answers, and compare against ground truth solutions. We pair the first

correct solution with the first incorrect one encountered, creating 5,574 preference pairs that emphasize computational precision and mathematical correctness.

• Engagement-focused pairs: Using the same problem set, we generate 10 rollouts per question and employ LLM-as-Judge evaluation (Qwen2.5-72B-Instruct, temperature=0.1) to assess pedagogical quality. We identify responses that provide clear explanations, intuitive reasoning, and educational insights versus those offering terse or mechanical solutions, yielding 7,930 preference pairs that prioritize learning effectiveness over mere correctness.

This dual construction allows us to examine whether MAH-DPO can simultaneously optimize for mathematical rigor and educational value—objectives that often compete in practice.

Training Configuration. We establish a consistent training pipeline across all mathematical experiments. Starting from Qwen2.5-7B-Instruct, we first perform supervised fine-tuning (learning rate 5×10^{-6} , 2 epochs) to adapt the model to mathematical domains. We then initialize MAH-DPO with small random perturbations (scale=0.001) applied to each head to encourage objective-specific specialization while maintaining shared representations. The multi-head training uses learning rate 1×10^{-6} , batch size 128, and $\beta = 0.1$, with sequences truncated to 512 prompt tokens and extended to 1536 total tokens to accommodate detailed mathematical reasoning over 2 epochs.

C.2 HUMAN VALUES TRAINING DETAILS

Human values alignment represents a more abstract but equally critical challenge, where models must navigate competing ethical principles. We focus on three fundamental dimensions that frequently conflict in real-world applications: helpfulness, truthfulness, and honesty.

Preference Data Construction. We leverage the UltraFeedback dataset's rich dimensional annotations to create three targeted preference datasets:

- *Helpfulness*: 59.2k preference pairs contrasting responses that provide comprehensive, actionable guidance versus those offering minimal or irrelevant information.
- *Truthfulness*: 50.8k pairs emphasizing factual accuracy and evidence-based reasoning versus responses containing inaccuracies or unsupported claims.
- *Honesty*: 57.3k pairs focusing on transparent acknowledgment of uncertainty and limitations versus responses that overstate confidence or mask knowledge gaps.

For each dimension, we pair responses with the highest and lowest annotated scores while excluding cases with identical ratings, ensuring clear preference signals. We reserve 2k examples per dimension for comprehensive evaluation across all three values simultaneously.

Training Configuration. To maintain experimental consistency while adapting to the distinct characteristics of values alignment, we modify our training approach accordingly. We perform supervised fine-tuning on Llama-3.1-8B using UltraFeedback's preferred responses (learning rate 5×10^{-7} , 1 epoch, batch size 192) to establish a strong foundation for ethical reasoning. MAH-DPO training employs slightly larger perturbations (scale=0.005) to account for the more nuanced nature of value judgments, with learning rate 5×10^{-7} , batch size 120, and sequences limited to 256 prompt tokens and 768 total tokens to focus on concise value-aligned responses over 1 epoch.

C.3 SOCRATIC MIND TRAINING DETAILS

Socratic tutoring epitomizes the challenge of multi-objective alignment in educational settings, requiring models to maintain factual accuracy while fostering student engagement through strategic questioning and explanation. This domain tests our approach's ability to handle dynamic, context-dependent trade-offs.

Preference Data Construction. We simulate realistic tutoring interactions by randomly sampling 1,000 educational dialogues and introducing natural conversation breakpoints. At each dialogue state, we generate 5 potential assistant responses representing different tutoring strategies—from direct instruction to guided discovery. We then employ trained PRMs specialized for accuracy and engagement assessment to evaluate each candidate response. By selecting the highest and lowest scoring responses for each objective, we create 1,000 preference pairs per dimension that capture the nuanced balance between providing correct information and maintaining pedagogical effectiveness in conversational contexts.

Training Configuration. Given the complexity of dialogue understanding, we adopt our mathematical domain configuration while extending context capabilities. We fine-tune Qwen2.5-7B-Instruct (learning rate 5×10^{-6} , 2 epochs)

and apply MAH-DPO with perturbation scale 0.001 to preserve dialogue coherence across heads. Training employs learning rate 1×10^{-6} , batch size 256, and $\beta = 0.1$, with extended context windows (1336 prompt tokens, 1536 total tokens) to accommodate full dialogue history while maintaining computational efficiency over 2 epochs.

These three experimental domains collectively span the spectrum from concrete mathematical reasoning to abstract value judgments to dynamic conversational interaction, providing a comprehensive testbed for evaluating MAH-DPO's multi-objective alignment capabilities across diverse AI applications.

D CONTINUING HIDDEN STATE ABLATION

In this section, we provide further results for validating the effectiveness of continuing hidden state in our PRM-guided decoding for alignment. We present comparisons between our continuing hidden state approach with classic text chunk concatenation approach and the results are in Table 5 and 6. From Table 5, we observe that in Human Values where there is not a clear process structure, step-wise generation using text chunk concatenation leads to performance degradation compared to the one-pass generation. Meanwhile, our continuing hidden state approach achieve comparable performance with one-pass generation when no guidance from PRMs is used, and also consistent improvements over text chunk method when guided by PRMs. This demonstrates that text chunk concatenation which requires iterative re-encoding can break the generation continuity while our hidden state approach preserve such continuity for response generation. In Table 6, there is no major performance difference between text chunk method and our hidden state method, which indicates the text chunk methods does not break generation continuity when the process structure is clear and well-defined such as in Math domain.

Table 5: Further results of PRM-guided decoding in Human Values: continuing text chunk vs. continuing hidden state.

Method	Help	Honest	Truth
One-pass generation without guided decoding (reference)	0.5800 ± 0.0066	0.3042 ± 0.0066	0.1888 ± 0.0028
Step-wise generation without guided decoding (text chunk) Step-wise generation without guided decoding (hidden state)	0.4688 ± 0.0033	0.1857 ± 0.0016	0.1182 ± 0.0031
	0.5750 ± 0.0107	0.3036 ± 0.0015	0.1904 ± 0.0036
Step-wise generation + Helpful PRM guided (text chunk) Step-wise generation + Helpful PRM guided (hidden state)	0.6140 ± 0.0099	0.3273 ± 0.0069	0.2099 ± 0.0060
	0.6706 ± 0.0093	0.4050 ± 0.0035	0.2791 ± 0.0023
Step-wise generation + Honest PRM guided (text chunk) Step-wise generation + Honest PRM guided (hidden state)	0.6148 ± 0.0150	0.3860 ± 0.0106	0.2544 ± 0.0062
	0.6448 ± 0.0050	0.4693 ± 0.0045	0.3383 ± 0.0025
Step-wise generation + Truth PRM guided (text chunk) Step-wise generation + Truth PRM guided (hidden state)	0.5775 ± 0.0155	0.3165 ± 0.0028	0.2500 ± 0.0062
	0.6350 ± 0.0032	0.4394 ± 0.0036	0.3296 ± 0.0056

Table 6: Further results of PRM-guided decoding in Math: continuing text chunk vs. continuing hidden state.

Method	Accuracy	Engagement
One-pass generation without guided decoding (reference)	0.7107 ± 0.0090	0.5007 ± 0.0289
Step-wise generation without guided decoding (text chunk) Step-wise generation without guided decoding (hidden-state)	0.7040 ± 0.0092 0.6853 ± 0.0163	0.4907 ± 0.0358 0.5133 ± 0.0543
Step-wise generation + Engaging PRM guided (text-chunk) Step-wise generation + Engaging PRM guided (hidden-state)	0.7187 ± 0.0147 0.7013 ± 0.0352	0.6353 ± 0.0099 0.7187 ± 0.0266
Step-wise generation + Accuracy PRM guided (text-chunk) Step-wise generation + Accuracy PRM guided (hidden-state)	0.7973 ± 0.0083 0.7993 ± 0.0172	0.4807 ± 0.0205 0.4553 ± 0.0221

E SYSTEM PROMPTS

In this section, we provide the system prompts used for response generation and LLM-as-Judge. Apart from the domains or alignment objective dimensions specified as follows, no system prompt is used. For example, we do not use system prompt for response generation in Human Values experiments.

E.1 MATH SYSTEM PROMPTS

Response Generation System Prompt in Math

Normal

Please reason step by step, and put your final answer within \\boxed{}.

Engaging

Please reason step by step using intuitive explanations, relatable examples, and clear language that helps deepen understanding of the underlying concepts, and put your final answer within \boxed{}.

Non-Engaging

Please reason step by step using formal mathematical notation and precise terminology. Minimize explanatory content, focus exclusively on necessary calculations and logical steps, and put your final answer within \\boxed{}.

Engagement LLM-as-Judge System Prompt in Math

You are an expert evaluator of mathematical explanations. Your task is to determine whether the mathematical content presented is **ENGAGING** or **NOT ENGAGING**.

ENGAGING mathematical content has some or most of these characteristics:

- Uses intuitive explanations and relatable examples
- Provides clear language that helps deepen understanding
- · Makes abstract concepts accessible and interesting
- Includes explanatory content that builds conceptual understanding
- Presents reasoning that is easy to follow
- Balances formal notation with helpful explanations

NOT ENGAGING mathematical content typically has these characteristics:

- · Uses primarily formal mathematical notation and terminology
- Provides minimal explanations beyond the calculations
- Focuses exclusively on necessary calculations and logical steps
- · Lacks intuitive explanations or relatable examples
- Uses dense, technical language that may be harder to follow
- · Prioritizes brevity and formality over accessibility

Evaluate **only** the engagingness of the content, not its correctness.

Your evaluation must be in JSON format with two fields:

```
{"analysis": "<specific reasons why the content is or is not engaging>", "judgment": "<ENGAGING or NOT ENGAGING>"}
```

Please evaluate the following mathematical content:

PROBLEM:

{problem}

SOLUTION:

{solution}

E.2 SOCRATIC MIND SYSTEM PROMPTS

Accuracy LLM-as-Judge System Prompt in Socratic Mind

You are an evaluator of tutoring dialogues. Your task is to judge the **ACCURACY** of the **ASSISTANT'S LAST MES-SAGE**. Use the student's immediate reply only as a probe. The label will train a reward model that must also work when only the assistant message is present.

Goa

 Decide whether the assistant's last message is factually correct, specific, and checkable so that a competent student could reach a correct answer without extra unstated information. Use the student reply to test this, but base the label on the assistant message itself.

Scope

Read only the final assistant message and the immediately following student reply. Consult earlier turns only to decode terms, variable meanings, or given values when strictly needed. Do not judge engagement, style, or tone.

Strict Criteria (all must hold for ACCURATE)

- Correctness: Facts, formulas, code, and reasoning in the assistant message are correct for the stated task and context.
- **Sufficiency**: The message includes the key inputs, units, constraints, and acceptance criteria needed to verify a result. It does not rely on hidden assumptions.
- Checkability: The message sets a clear target or procedure that can be judged right or wrong (for example a numeric result, runnable code with defined variables, or a well-specified step list).
- Consistency: The message does not conflict with earlier defined variables, values, or conditions.

Stricter Default

If there is reasonable doubt about correctness or checkability, choose INACCURATE. Praise, summaries, or meta talk that do not set a checkable target are INACCURATE for this accuracy task.

How to Use the Student Reply

Use it only as evidence of whether the assistant message was clear and correct. If the reply is wrong or a non-answer and the assistant message was underspecified, ambiguous, misleading, or used wrong facts/code, label INACCURATE. If the reply is wrong but the assistant message was fully correct and checkable, you may still label ACCURATE. If there is no reply, judge the assistant message alone by the strict criteria.

Concrete Failure Patterns that Require INACCURATE

- The message contains a wrong fact, wrong formula, or code that would error or produce a wrong result as written (undefined names, wrong API, wrong boundary).
- The task depends on data or tools not provided and the message does not state allowed assumptions.
- The target is not objectively checkable (for example "share your thoughts") or key constraints/units are missing so multiple incompatible answers fit.
- The message conflicts with established context (for example uses 2.5 when 3.0 was specified) or repeats a question already answered without adding a checkable requirement.

Decision Rules

- If the message satisfies all strict criteria, and the student reply is correct or an acknowledgment to a correct final answer → ACCURATE.
- 2. If the message fails any strict criterion, or the student's error is reasonably induced by the message (unclear, missing constraints, wrong hints, wrong code) → INACCURATE.
- 3. In uncertain cases, default to INACCURATE.

```
Dialogue: {conversation}
```

Your evaluation must be in JSON format:

```
1
  "label": "ACCURATE" or "INACCURATE"
}
```

- 1

Engagement LLM-as-Judge System Prompt in Socratic Mind

You are an evaluator of programming tutoring dialogues. Your task is to determine whether the **LAST ASSISTANT MESSAGE** increases the likelihood that the student will do concrete, on-task programming work now.

Scope and Evidence

Read the LAST ASSISTANT MESSAGE. Look back only to recover the current task, any pending step, and concrete anchors (shown code, variables, errors, inputs, or options). You may use the student's immediate next reply as a probe of uptake, but base the decision mainly on the assistant message. Before using the student reply, remove quoted assistant text, code-fence labels, UI artifacts, and markup. Do not judge tone.

What Counts as Engagement-Raising

The message raises engagement when it asks for a clear, task-specific programming action that yields a result verifiable from the dialogue now. The action should be one step or a very short sequence anchored to the current work. The following qualify (treat any one as sufficient):

- Make a specific edit to the shown code (full block or tiny patch), including ordering/placement requests (e.g., "insert this condition before the < 30 check", "swap these two arguments", "replace = with == on line 1"). The edited code itself is the check.
- Write or complete a small snippet (≈10 lines or fewer) tied to the current construct (e.g., "rewrite the function using elif", "show a while loop that uses break to exit when input is 'stop'").
- Predict one concrete outcome tied to the code and inputs (e.g., "what prints for level = 90?", "which branch runs when time_left == 30?", "will this raise a SyntaxError?").
- Identify or localize a specific issue in the given code ("which line causes the error?", "what rule is violated by this call?") or choose between explicit options ("should the == 30 check go before or after < 30?").
- Run/mentally execute a named function or command with stated or implied inputs and report the exact output or pass/fail.
- Provide a minimal, targeted example directly tied to the snippet just discussed (one short loop/try-except/example call).

Also count as engagement-raising:

- Requests to finish a started step (e.g., "complete the code you began with the missing elif..."), or to restate the final corrected call(s) exactly ("write the two fixed print statements").
- Socratic yes/no or single-fact checks that have a unique, verifiable answer anchored to the code ("Is 30 < 30?", "Would the elif run when time_left is 30?").

What is Not Engagement-Raising

The message is NOT engagement-raising when it only explains/summarizes; asks open "why/how/compare/explain" without anchoring to the current code or a bounded artifact; gives a full final solution leaving nothing to do; posts long code or text without a precise "do-now" instruction; goes off task; or tells the student to wait/stop while a step is pending.

Pending-Step Handling

If an earlier assistant turn set a step that is still unfinished (write/implement/fix/modify/calculate/answer/show code/run and report), the LAST ASSISTANT MESSAGE should push that step forward with a precise instruction or a small substep plus an observable result. If it changes topic, summarizes, or asks a vague question instead, label NOT ENGAGING.

Using the Student Reply as a Probe

Use the student's next message only as a diagnostic signal about how actionable and well-anchored the ask was.

- Strong positive signal (can upgrade borderline cases to ENGAGING): the reply returns the requested form/target (an edited block at the named spot, the exact output for the stated input, the chosen placement, a corrected call, a tiny example).
- Positive minimal signal: a single correct anchored fact/answer to the asked check (e.g., "no" to "Is 30 < 30?")
 counts as uptake.
- Negative signal (can downgrade borderline cases to NOT ENGAGING): the reply shows the ask was vague or mis-anchored ("which file/line?", undefined inputs), or is off-target.
- Irrelevant signal: thanks, agreement, or generic yes/no not tied to the asked check.

Decision Rule

Output ENGAGING if ANY of the following holds: the LAST ASSISTANT MESSAGE issues a concrete, non-trivial, anchored do-now task with a verifiable result; or it advances a pending step with an explicit, immediately doable action; or the cleaned student reply shows anchored uptake that advances the work in the requested form. Otherwise output NOT_ENGAGING.

Engagement LLM-as-Judge System Prompt in Socratic Mind (Continued)

Edge Handling

- · If the assistant supplies a full solution AND the only ask is generic confirmation, label NOT_ENGAGING.
- If an explanation ends with a concrete do-now request (e.g., "now change X and rerun/predict output"), treat that request as decisive.
- Tiny fixes or single-line corrections still count if they are anchored and verifiable now.

Dialogue: {conversation}

Your evaluation must be in JSON format:

```
"label": "ENGAGING" or "NOT_ENGAGING"
```

Assistant/Student Simulator System Prompt in Socratic Mind

Next Assistant Turn Simulation

You are a tutor who is helping a beginner student learn programming. Continue as the same tutor and reply similarly to the last student message, matching EXACTLY the SAME speaking tone and tutoring style as in your earlier messages (e.g. reply to the student's last message concisely in 1-2 sentences and then always ask a meaningful follow-up question).

Next User Turn Simulation

You are a student who is learning programming as a beginner with a tutor. Continue as the same student and reply to the last tutor message similarly as your earlier messages with EXACTLY the SAME speaking tone (e.g., curious, impatient, informal, etc.), response style (e.g., short, long, incomplete, etc.), amount of discourse marker (e.g., not using any discourse markers), understanding level (e.g., making mistakes), and engagement level (e.g., less engaged in the session).

FULL RESULTS WITH STANDARD DEVIATION

Table 7: Full results with standard deviations in Human Values.

Method	Help	Honest	Truth
Training-time alignment			
Base	0.5800 ± 0.0066	0.3042 ± 0.0066	0.1888 ± 0.0028
SFT	0.5546 ± 0.0043	0.2998 ± 0.0021	0.1992 ± 0.0087
Single-Head DPO	0.6043 ± 0.0075	0.3055 ± 0.0100	0.2014 ± 0.0098
MODPO	0.6175 ± 0.0017	0.3477 ± 0.0013	0.2325 ± 0.0033
MAH-DPO Helpful Head (Head 1)	0.6309 ± 0.0045	0.3465 ± 0.0070	0.2239 ± 0.0098
MAH-DPO Honesty Head (Head 2)	0.6257 ± 0.0054	0.3516 ± 0.0078	0.2303 ± 0.0051
MAH-DPO Truthful Head (Head 3)	0.6257 ± 0.0010	0.3461 ± 0.0031	0.2286 ± 0.0058
MAH-DPO Ensemble Head	0.6389 ± 0.0035	0.3687 ± 0.0038	0.2478 ± 0.0074
Test-time guided decoding alignment			
Base	0.5750 ± 0.0107	0.3036 ± 0.0015	0.1904 ± 0.0036
Helpful PRM-guided	0.6706 ± 0.0093	0.4050 ± 0.0035	0.2791 ± 0.0023
Honesty PRM-guided	0.6448 ± 0.0050	0.4693 ± 0.0045	0.3383 ± 0.0025
Truthful PRM-guided	0.6350 ± 0.0032	0.4394 ± 0.0036	0.3296 ± 0.0056
Combined: training + decoding alignment			
MAH-DPO Ensemble Head + Help PRM-guided	0.7165 ± 0.0029	0.4554 ± 0.0028	0.3890 ± 0.0049
MAH-DPO Ensemble Head + Honest PRM-guided	0.6968 ± 0.0035	0.5196 ± 0.0016	0.4107 ± 0.0011
MAH-DPO Ensemble Head + Truth PRM-guided	0.6834 ± 0.0053	0.4872 ± 0.0038	0.3630 ± 0.0035

Table 8: Full results with standard deviations in Math.

Method	Accuracy	Engagement
Training-time alignment		
Base	0.7107 ± 0.0090	0.5007 ± 0.0289
SFT	0.7300 ± 0.0060	0.5920 ± 0.0171
Single-Head DPO	0.7253 ± 0.0050	0.7160 ± 0.0257
MODPO	0.7280 ± 0.0072	0.7367 ± 0.0070
MAH-DPO Accuracy Head (Head 1)	0.7353 ± 0.0070	0.8667 ± 0.0092
MAH-DPO Engaging Head (Head 2)	0.7267 ± 0.0082	0.8840 ± 0.0058
MAH-DPO Ensemble Head	0.7247 ± 0.0117	0.8733 ± 0.0069
Test-time guided decoding alignment		
Base wt normal prompt	0.6853 ± 0.0163	0.5133 ± 0.0543
Engaging PRM-guided wt normal prompt	0.7013 ± 0.0352	0.7187 ± 0.0266
Accuracy PRM-guided	0.7633 ± 0.0050	0.4720 ± 0.0072
Accuracy Value-guided	0.7993 ± 0.0172	0.4553 ± 0.0221
Base wt engaging prompt	0.6827 ± 0.0250	0.7007 ± 0.0031
Engaging PRM-guided wt engaging prompt	0.7000 ± 0.0060	0.9033 ± 0.0050
Combined: training + decoding alignment		
MAH-DPO Ensemble Head + Accuracy Value-guided	0.8000 ± 0.0231	0.8553 ± 0.0136
MAH-DPO Ensemble Head + Engaging PRM-guided	0.7107 ± 0.0114	0.6813 ± 0.0199
MAH-DPO Ensemble Head + Engaging PRM-guided wt engaging prompt	0.7207 ± 0.0030	0.9060 ± 0.0053

Table 9: Full results with standard deviations in Socratic Mind.

Method	Accuracy	Engagement
Training-time alignment		
Base	0.6560 ± 0.0035	0.3220 ± 0.0382
SFT	0.6793 ± 0.0081	0.3473 ± 0.0042
Single-Head DPO	0.7040 ± 0.0053	0.4460 ± 0.0129
MODPO	0.7047 ± 0.0117	0.3600 ± 0.0122
MAH-DPO Accuracy Head (Head 1)	0.7007 ± 0.0257	0.4447 ± 0.0012
MAH-DPO Engaging Head (Head 2)	0.6953 ± 0.0081	0.4480 ± 0.0231
MAH-DPO Ensemble Head	0.6893 ± 0.0070	0.4513 ± 0.0127
Test-time guided decoding alignment		
Base	0.6367 ± 0.0351	0.3407 ± 0.0122
Accuracy PRM-guided	0.7127 ± 0.0170	0.2660 ± 0.0171
Engaging PRM-guided	0.6507 ± 0.0110	0.4663 ± 0.0110
Combined: training + decoding alignment		
MAH-DPO Ensemble Head + Accuracy PRM-guided	0.6659 ± 0.0210	0.3849 ± 0.0140
MAH-DPO Ensemble Head + Engaging PRM-guided	0.6514 ± 0.0131	0.5149 ± 0.0152

Table 10: Full results of varying head weights with standard deviations in Math.

Accuracy	Engagement
0.7353 ± 0.0070	0.8667 ± 0.0092
0.7347 ± 0.0145	0.8640 ± 0.0087
0.7247 ± 0.0117	0.8733 ± 0.0069
0.7193 ± 0.0175	0.8767 ± 0.0110
0.7267 ± 0.0082	0.8840 ± 0.0058
	0.7353 ± 0.0070 0.7347 ± 0.0145 0.7247 ± 0.0117 0.7193 ± 0.0175

Table 11: Full results of varying head weights with standard deviations in Human Values.

Weight Combination	Help	Honest	Truth
MAH-DPO (Help head, 1.0, 0.0, 0.0)	0.6309 ± 0.0045	0.3465 ± 0.0070	0.2239 ± 0.0098
MAH-DPO (0.5, 0.5, 0.0)	0.6406 ± 0.0075	0.3692 ± 0.0067	0.2455 ± 0.009
MAH-DPO (Honesty head, 0.0, 1.0, 0.0)	0.6257 ± 0.0054	0.3516 ± 0.0078	0.2303 ± 0.0051
MAH-DPO (1/3, 1/3, 1/3)	0.6389 ± 0.0035	0.3687 ± 0.0038	0.2478 ± 0.0074
MAH-DPO (0.0, 0.5, 0.5)	0.6326 ± 0.0069	0.3650 ± 0.0060	0.2422 ± 0.0010
MAH-DPO (Truth head, 0.0, 0.0, 1.0)	0.6257 ± 0.0010	0.3461 ± 0.0031	0.2286 ± 0.0058
MAH-DPO (0.5, 0.0, 0.5)	0.6366 ± 0.0022	0.3645 ± 0.0085	0.2425 ± 0.0020

G SOCRATIC MIND DATA SAMPLE

Homework Bedtime Python Conversation Example

Assistant: Consider the following Python code:

```
homework_complete = True
time_left = 20

if homework_complete:
    print("Get ready for bed")
elif time_left < 30:
    print("Finish homework")
else:
    print("Work for 1 hour")</pre>
```

Explain what this code does. Describe each conditional statement and its purpose.

User: This code uses an if-elif-else structure to check different conditions and execute specific actions based on those conditions. Let's break it down:

1. if homework_complete: Condition: Checks if the variable homework_complete is True.

Purpose: If homework_complete is True (meaning the homework is finished), it will execute the first action (print ("Get ready for bed")).

Outcome: If this condition is true, it prints "Get ready for bed" and skips the other conditions.

2. elif time_left < 30: Condition: This is checked only if the first if condition was false (meaning homework_complete is False). Now, it checks if time_left is less than 30.

Purpose: If there is less than 30 minutes left (time_left; 30), it prints "Finish homework" because it suggests there's just enough time to finish the homework before heading to bed.

Outcome: This will be executed if homework_complete is False and time_left is less than 30.

3. else: Condition: This block is executed if both previous conditions are false (i.e., if homework_complete is False and time_left is not less than 30).

Purpose: If neither of the previous conditions is true, it prints "Work for 1 hour", implying that there is enough time (30 or more minutes) left to work on something else before going to bed.

Outcome: This is the default action that runs when none of the earlier conditions hold true.

Homework Bedtime Python Conversation Example (Continued)

Assistant: Excellent explanation! You've clearly understood the if-elif-else structure and how the conditions are evaluated sequentially. You correctly identified that once a condition is met, the corresponding block executes and the rest are skipped.

I particularly like how you explained the purpose of each condition in context - showing you understand not just the syntax but the logic behind the code.

Now, let's move to the next question:

Modify the code snippet from the previous part so that it includes a new condition to check if time_left is exactly 30 minutes, printing "Almost done" in that case. Explain your modification and its impact on the code's flow.

User:

```
homework_complete = True
time_left = 20

if homework_complete:
    print("Get ready for bed")
elif time_left == 30:
    print("Almost done")
elif time_left < 30:
    print("Finish homework")
else:
    print("Work for 1 hour")</pre>
```

Explanation of the Modification: New condition: We added a new elif statement: elif time_left == 30:.

Purpose: This checks if time_left is exactly 30 minutes. If it is, it will print "Almost done".

This condition is evaluated before checking if time_left is less than 30 (in the previous elif time_left < 30), ensuring that if time_left is exactly 30, the program will print "Almost done" instead of "Finish homework".

Impact on the flow:

This modification adds an additional check specifically for when time left is exactly 30 minutes.

The flow now evaluates conditions in the following order:

First: If homework_complete is True, it prints "Get ready for bed".

Second: If time_left == 30, it prints "Almost done".

Third: If time_left; 30 (and the first two conditions are false), it prints "Finish homework".

Finally: If none of the above conditions are true (e.g., homework_complete is False and time_left ¿ 30), it prints "Work for 1 hour".

Assistant: Perfect! You've correctly modified the code and provided an excellent explanation of your changes. Your placement of the new condition is spot on - checking for time_left == 30 before checking if it's less than 30.

I appreciate how you walked through the entire flow of the program after your modification, showing a clear understanding of how the order of conditions affects execution. You've demonstrated a solid grasp of conditional logic and how to structure conditions to achieve the desired behavior.

Session Summary: You demonstrated excellent understanding of conditional statements in Python. You accurately explained the if-elif-else structure, the sequential evaluation of conditions, and how only one block executes based on the first condition that evaluates to True. When modifying the code, you correctly placed the new condition in the appropriate position and explained how this affects the program's flow. Your explanations were thorough and showed strong comprehension of control flow concepts.

Strengths:

- · Clear understanding of conditional logic and execution flow
- Ability to modify existing code to implement new requirements
- Excellent explanation of both code functionality and purpose

Areas for continued growth:

- Continue practicing with more complex nested conditionals
- Explore combining conditions with logical operators (and, or, not)