DIVERSE OFFLINE IMITATION LEARNING

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ABSTRACT

There has been significant recent progress in the area of unsupervised skill discovery, utilizing various information-theoretic objectives as measures of diversity. Despite these advances, challenges remain: current methods require significant online interaction, fail to leverage vast amounts of available task-agnostic data and typically lack a quantitative measure of skill utility. We address these challenges by proposing a principled offline algorithm for unsupervised skill discovery that, in addition to maximizing diversity, ensures that each learned skill imitates state-only expert demonstrations to a certain degree. Our main analytical contribution is to connect Fenchel duality, reinforcement learning, and unsupervised skill discovery to maximize a mutual information objective subject to KL-divergence state occupancy constraints. Furthermore, we demonstrate the effectiveness of our method on the standard offline benchmark D4RL and on a custom offline dataset collected from a 12-DoF quadruped robot for which the policies trained in simulation transfer well to the real robotic system.[1]

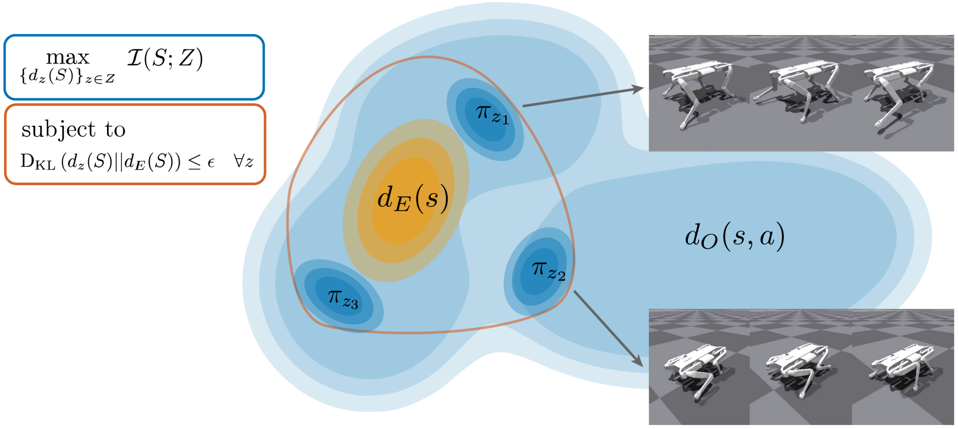
$\max_{\{d_z(S)\}_{z\in Z}} \quad \mathcal{I}(S;Z)$
subject to
$\mathrm{D}_{\mathrm{KL}}\left(d_z(S)||d_E(S)\right) \leq \epsilon \quad \forall z$
$d_E(s)$
$d_O(s,a)$
π_{z_1}
π_{z_2}
π_{z_3}

Figure 1: Diverse Offline Imitation (DOI) maximizes a variational lower bound on the mutual information between skills z and states s subject to a KL-divergence constraint to limit the deviation of the state occupancy $d_z(s)$ of each skill z from that of an expert $d_E(s)$.

1 INTRODUCTION

Recent advancements in reinforcement learning (RL) have included substantial progress in unsupervised skill discovery, aiming to empower autonomous agents with the capability to acquire a diverse set of skills directly from their environment, without relying on predefined human-engineered rewards or demonstrations. These methods have the potential to revolutionize the way RL agents

[1] Project website with videos: https://tinyurl.com/diversity-via-duality

learn to solve complex tasks. The growing interest in unsupervised skill discovery has led to various approaches, typically rooted in information-theoretic concepts, including empowerment (Klyubin et al., 2005; Mohamed and Jimenez Rezende, 2015; Eysenbach et al., 2019), information bottleneck (Tishby et al., 1999; Goyal et al., 2019; Kim et al., 2021a) and information gain (Houthooft et al., 2016; Strouse et al., 2022; Park and Levine, 2023). Despite these advancements, there remains a significant challenge. Current methods demand substantial online interaction with the environment, making exploration in high-dimensional state-action spaces inefficient. Although Zahavy et al. (2022) introduced constraints to enhance skill performance and narrow the exploration space by incentivizing diverse skills to meet a certain utility measure, their approach does not eliminate the need for considerable online interaction with the environment. Meanwhile, there have been significant recent advances in large-scale data collection (Rob, 2020; Walke et al., 2023; Brohan et al., 2023) and in the development of scalable and sample-efficient offline RL algorithms that leverage diverse behaviors of pre-collected experience. However, these approaches struggle with well-known challenges, including off-policy evaluation and the out-of-distribution problem, which have been studied extensively in previous work (Levine et al., 2020; Prudencio et al., 2022).

In this work, we address the aforementioned challenges by introducing a novel problem formulation and complementing it with the first principled “offline” RL algorithm for unsupervised skill discovery that, in addition to maximizing diversity, ensures that each learned skill imitates state expert demonstrations to a certain degree. More specifically, we consider a problem formulation with two datasets: a large one with diverse state-action demonstrations and another much smaller one with only state expert demonstrations. This setting is particularly valuable in robotics scenarios where expert demonstrations are limited and the domain of the expert may be different from that of the agent, such as in human demonstrations. We formulate the problem as a Constrained Markov Decision Process (CMDP) (Altman, 1999; Szepesvári, 2020) that seeks to maximize diversity through a mutual information objective, subject to Kullback-Leibler (KL) divergence state occupancy constraints ensuring that each skill imitates state expert demonstrations to a certain degree. The resulting CMDP has convex objective and constraints, making the optimization problem intractable. We adopt a tractable relaxation approach consisting of an alternating scheme that maximizes a variational lower bound on mutual information, and to handle the constraints it applies Lagrange relaxation. Our method, Diverse Offline Imitation (DOI), overcomes the off-policy evaluation by leveraging the Fenchel-Rockafellar duality in RL (Nachum and Dai, 2020; Kim et al., 2022; Ma et al., 2022) to connect a dual optimal value solution (computed using offline samples) with primal optimal state-action occupancy ratios. These ratios serve as importance weights for offline training of a skill discriminator, skill-conditioned policy, KL-divergence estimators, and Lagrange multipliers. We demonstrate the effectiveness of our method on the standard offline benchmark D4RL (Fu et al., 2020) and on a custom offline dataset collected from a 12-DoF quadruped robot Solo12 (Léziart et al., 2021). In addition, we show that DOI on simulation data transfers well to a real robot system.

2 Preliminaries

We utilize the framework of Markov decision processes (MDPs) (Puterman, 2014), where an MDP is defined by the tuple $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{P}, \rho_0, \gamma)$ denoting the state space, action space, reward mapping $\mathcal{R} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$, stochastic transition kernel $\mathcal{P}(s'|s, a)$, initial state distribution $\rho_0(s)$ and discount factor γ. A policy $\pi : \mathcal{S} \mapsto \Delta(\mathcal{A})$ defines a probability distribution over the action space $\mathcal{A}$ conditioned on the state, where $\Delta(\cdot)$ stands for the probability simplex.

Given a policy π, the corresponding state-action occupancy measure $d^\pi(s, a)$ is defined by $(1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \Pr[s_t = s, a_t = a \,|\, s_0 \sim \rho_0, a_t \sim \pi(\cdot|s_t), s_{t+1} \sim \mathcal{P}(\cdot|s_t, a_t)]$ and its associated state occupancy $d^\pi(s)$ is given by marginalizing over the action space $\sum_{a \in \mathcal{A}} d^\pi(s, a)$.

In the skill discovery setting, the set of skills is defined by Z, which we will treat as a finite set, and the skill-conditioned policy is given by $\pi_z : S \times Z \mapsto \Delta(\mathcal{A})$ with corresponding state occupancy $d_z(s) := d^{\pi_z}(s)$, for each skill $z \in Z$.

We consider an offline setting with access to the following datasets: i) $\mathcal{D}_E$ sampled from an expert state occupancy $d_E(S)$; and ii) $\mathcal{D}_O$ sampled from a state-action occupancy $d_O(S, A)$ generated by a mixture of behaviors. Our analysis makes use of the following coverage assumption on state occupancies.

Assumption 2.1 (Expert coverage)**.** *We assume that* $d_E(s) > 0$ *implies* $d_O(s) > 0$.

3 Method

Given an expert and a coverage dataset as above, we aim to solve *offline* the constrained optimization problem

$$\begin{aligned} \max_{\{d_z(S)\}_{z\in Z}} \quad & \mathcal{I}(S;Z) && (1)\\ \text{subject to} \quad & \mathrm{D}_{\mathrm{KL}}\left(d_z(S)||d_E(S)\right) \leq \epsilon \quad \forall z, && (2) \end{aligned}$$

where $\mathcal{I}(S;Z)$ denotes the mutual information between states and skills. Henceforth, we shall make use of color coding to highlight the diversity signal in blue and the imitation signal in orange. The preceding problem formulation and our algorithmic framework can be easily extended to capture: i) objectives in (1) that combine conditional mutual information (c.f. DADS in (Sharma et al., 2020)) and information gain (c.f. DISDAIN in (Strouse et al., 2022)); and ii) general f-divergence constraints in (2), see Nachum and Dai (2020); Ma et al. (2022). We leave the study of these variants for future work.

Since maximizing the mutual information $\mathcal{I}(S;Z)$ is generally intractable, in line with previous work (Eysenbach et al., 2019) we assume that the skills are sampled uniformly at random, i.e., $p(z) = \frac{1}{|Z|}$, and as a trackable surrogate we consider instead the following variational lower bound

$$\mathcal{I}\left(S;Z\right) \geq \mathbb{E}_{p(z),d_z(s)}\left[\log q(z|s)\right] + \mathcal{H}\left(p(z)\right) = \sum_z \mathbb{E}_{d_z(s)}\left[\frac{\log\left(|Z|q(z|s)\right)}{|Z|}\right]. \qquad (3)$$

Here with $q(z|s)$ we denote a discriminator tasked with distinguishing between skills.

Ma et al. (2022) proposed an offline algorithm (SMODICE) that on input an expert dataset $\mathcal{D}_E \sim d_E(S)$ and a coverage dataset $\mathcal{D}_O \sim d_O(S,A)$ such that $\mathcal{D}_E \subset \mathrm{States}[\mathcal{D}_O]$, trains a policy $\pi_{\widetilde{E}}$ which optimizes the problem

$$\min_\pi \mathrm{D}_{\mathrm{KL}}\left(d^\pi(S)||d_E(S)\right), \qquad (4)$$

and also outputs ratios $\eta_{\widetilde{E}}(s,a) = d_{\pi_{\widetilde{E}}}(s,a)/d_O(s,a)$ for every state-action pair $(s,a) \in \mathcal{D}_O$.

An important observation is that the state constraints (2) can be reduced to state-action constraints, by training an expert policy $\pi_{\widetilde{E}}$, which optimizes eq. (4). More specifically, for each skill z we replace the state constraint (2) with the following state-action constraint

$$\mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_{\widetilde{E}}(S,A)\right) \leq \epsilon, \qquad (5)$$

where $d_{\widetilde{E}}(s,a)$ denotes the state-action occupancy $d_{\pi_{\widetilde{E}}}(s,a)$ induced by the expert policy $\pi_{\widetilde{E}}$.

We focus on a reduction of CMDPs to MDPs using gradient-based techniques, known as Lagrangian methods (Borkar, 2005; Bhatnagar and Lakshmanan, 2012; Tessler et al., 2019). In contrast to prior work on CMDP, which has focused primarily on linear objectives and constraints, we consider the nonlinear setting with convex objectives and constraints. More specifically, we seek to maximize the right-hand side of eq. (3) subject to eq. (5). Solving this problem is equivalent to

$$\max_{\substack{d_z(s,a)\\ q(z|s)}} \min_{\lambda\geq 0} \sum_z \mathbb{E}_{d_z(s)}\left[\frac{\log\left(|Z|q(z|s)\right)}{|Z|}\right] + \sum_z \lambda_z\left[\epsilon - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_{\widetilde{E}}(S,A)\right)\right], \qquad (6)$$

where with λ_z we denote the Lagrange multiplier corresponding to skill z.

3.1 Approximation Scheme

We use a popular heuristic, known in the literature as *alternating optimization*, to approximately compute a local optimum of Problem (6). More precisely, the method alternates between optimizing each model while holding all others fixed, and iteratively refines the solution until convergence is reached or a stopping criterion is met. Furthermore, as we can guarantee in practice that the Lagrange multipliers λ are always positive, we consider Problem (6) with $\lambda > 0$, that is

$$\max_{\substack{d_z(s,a)\\ q(z|s)}} \min_{\lambda> 0} \sum_z \lambda_z\Big\{\epsilon + \mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right)\Big\}, \qquad (7)$$

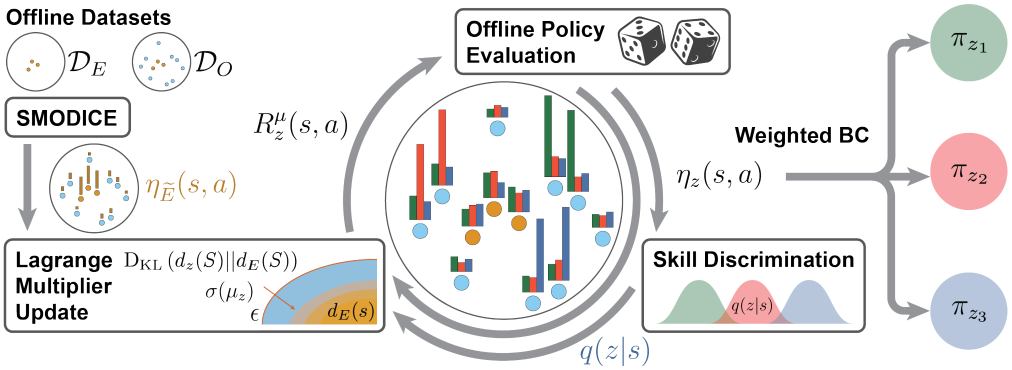

Figure 2: Illustration of Algorithm 1. We compute expert importance ratios $\eta_{\tilde{E}}(s,a)$ by running SMODICE on the offline datasets $\mathcal{D}_E$ and $\mathcal{D}_O$. These expert ratios are then used in the alternating scheme described in Subsec. 3.1 to obtain the importance ratios $\eta_z(s,a)$ (with support in $\mathcal{D}_O$) for each skill z. Specifically, the skill ratios $\eta_z(s,a)$ are computed by a DICE-like offline policy evaluation algorithm on input a reward $R_z^\mu(s,a)$ that balances skill diversity (discriminator $q(z|s)$) and expert imitation (importance ratios $\eta_{\tilde{E}}(s,a)$).

where

$$R_z^\lambda(s,a) := \underbrace{\frac{1}{\lambda_z}}_{\text{Constraint Violation}} \underbrace{\frac{\log\left(q(z|s)|Z|\right)}{|Z|}}_{\text{Skill Diversity}} + \underbrace{\log \eta_{\tilde{E}}(s,a)}_{\text{Expert Imitation}}. \tag{8}$$

The reward in (8) is derived in Supp. B and relies on the following equality (see Supp. C.4) $\mathrm{D}_{\mathrm{KL}}(d_z(S,A)||d_{\tilde{E}}(S,A)) = \mathrm{D}_{\mathrm{KL}}(d_z(S,A)||d_O(S,A)) - \mathbb{E}_{d_z(s,a)}[\log \frac{d_{\tilde{E}}(s,a)}{d_O(s,a)}]$ and the definition of $\eta_{\tilde{E}}(s,a) = d_{\tilde{E}}(s,a)/d_O(s,a)$.

Intuitively, the reward $R_z^\lambda(s,a)$ balances between diversity and KL-closeness to the expert state-action occupancy. The Lagrange multiplier λ_z scales down the log-likelihood of the discriminator, effectively reducing the diversity signal, when the state-action occupancy $d_z(S,A)$ violates the KL-divergence constraint (5), and vice versa. Each term in the reward (8) involves a separate optimization procedure, which will be described in the next section.

3.2 Approximation Phases

Using the alternating optimization scheme, Algorithm 1 decomposes into the following three optimization phases. In Phase 1, we train a value function $V_z^\star$, ratios $\eta_z(s,a)$ and a skill-conditioned policy π_z. In Phase 2, we train a skill discriminator $q(z|s)$. Then in Phase 3, we compute a KL constraint estimator ϕ_z and update accordingly the Lagrange multipliers λ_z. In addition, we perform a preprocessing phase to compute expert ratios $\eta_{\tilde{E}}(s,a)$ with respect to a fixed reward $R(s,a) = \log \frac{d_E(s)}{d_O(s)}$ which ensures KL closeness to the expert state occupancy $d_E(S)$, i.e., optimizing Problem (4).

3.2.1 Phase 1

With fixed skill-discriminator $q(z|s)$ and Lagrange multipliers λ, Problem (6) becomes

$$\max_{\{d_z(s,a)\}_{z\in Z}} \sum_z \lambda_z \Big\{ \mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right) \Big\}, \tag{9}$$

or equivalently for every skill z:

$$\begin{aligned} \max_{d_z(s,a)\geq 0} \quad & \mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right) \\ \text{subject to} \quad & \textstyle\sum_a d_z(s,a) = (1-\gamma)\rho_0(s) + \gamma\mathcal{T}d(s) \qquad \forall s, \end{aligned} \tag{10}$$

where we denote with $\mathcal{T}$ the transition operator: $\mathcal{T}d(s') = \sum_{s,a} \mathcal{P}(s'|s,a)d(s,a)$.

Assumption 3.1 (Strict Feasibility)**.** *We assume there exists a solution such that the constraints* (10) *are satisfied and* $d(s,a) > 0$ *for all states-action pairs* $(s,a) \in \mathcal{S} \times \mathcal{A}$*.*

Using Lagrange duality, Assum. 3.1 (which implies strong duality) and the Fenchel conjugate (see Supp. A), Nachum and Dai (2020, Sec. 6) and Ma et al. (2022, Theorem 2) showed that Problem 10 shares the same optimal value as the following optimization problem

$$V^\star = \arg\min_{V(s)} (1-\gamma)\mathbb{E}_{s\sim\rho_0}\left[V(s)\right] + \log \mathbb{E}_{d_O(s,a)} \exp\left\{R_z^\lambda(s,a) + \gamma\mathcal{T}V(s,a) - V(s)\right\}, \quad (11)$$

where $\mathcal{T}V(s,a) := \mathbb{E}_{\mathcal{P}(s'|s,a)}V(s')$. Moreover, the primal optimal solution is given by

$$\eta_z(s,a) := \frac{d_z^\star(s,a)}{d_O(s,a)} = \operatorname{softmax}\left(R_z^\lambda(s,a) + \gamma\mathcal{T}V_z^\star(s,a) - V_z^\star(s)\right). \quad (12)$$

These ratios allow us to train a skill-conditioned policy π_z by importance-weighted behavior cloning.

Lemma 3.2. *Given a fixed skill-discriminator* $q(z|s)$*, Lagrange multipliers* $\lambda \in \mathbb{R}_{>0}^{|Z|}$ *and (primal) optimal ratios* $\eta_z(s,a)$*, using weighted behavioral cloning, we can compute offline an optimal skill conditioned policy* π_z*. In particular, we optimize by gradient descent the following optimization problem* $\max_\pi \mathbb{E}_{p(z)}\mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)\log\pi_z(a|s)\right]$*.*

The proof of Lem. 3.2 follows the approach of Ma et al. (2022), presented for completeness in Supp. C.2. Given a fixed discriminator $q(z|s)$, we obtain by Lem. 3.2 an optimal policy $\pi_z^\star$. In the next phase, we show how to train off-policy the skill discriminator.

3.2.2 Phase 2

In the following Lemma we give an offline procedure for training an optimal discriminator $q(z|s)$ with respect to the learned policy $\pi_z^\star$. We present the proof in Supp. C.3.

Lemma 3.3. *Given a fixed skill-conditioned policy* $\pi_z^\star$*, Lagrange dual variable* $\lambda \in \mathbb{R}_{>0}^{|Z|}$ *and (primal) optimal ratios* $\eta_z(s,a)$*, using weighted behavioral cloning, we can compute offline an optimal skill-discriminator* $q(z|s)$*. In particular, we optimize by gradient descent the following optimization problem* $\max_{q(z|s)} \mathbb{E}_{p(z)}\mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)\log\left(q(z|s)\right)\right]$*.*

The key insight in Lem. 3.3 is that once we have a skill-conditioned policy π_z, we can train off-policy an optimal discriminator $q(z|s)$ with respect to state-action occupancy $d_z^\star(s,a)$, while sampling from the offline distribution $d_O(s,a)$ and reweighting accordingly by the ratios (12).

In the next phase, we show how to compute offline an estimator of the state-action KL constraint (5).

3.2.3 Phase 3

Here, we fix the discriminator $q(z|s)$ and the skill-conditioned policy $\pi_z^\star(s)$. Then, Problem (6) reduces to

$$\min_{\lambda\geq 0}\sum_z \lambda_z\left[\epsilon - \mathrm{D_{KL}}\left(d_z^\star(S,A)||d_{\widetilde{E}}(S,A)\right)\right] \quad (13)$$

In the offline setting, it is important to note that direct computation of expectations with respect to the occupancy $d_z^\star(S,A)$ is not feasible. Nevertheless, we show next that combining the expert (4) and (primal) optimal (12) ratios are sufficient to design an off-policy estimator of the KL state-action constraint. More specifically, we give the following self-normalizing importance sampling procedure.

Lemma 3.4. *Given the (primal) optimal ratios* $\eta_z(s,a)$ *and the classifier* $c^\star$*, optimizing over the Lagrange multipliers reduces the Problem* (13) *to* $\min_{\lambda>0}\sum_z \lambda_z(\epsilon - \phi_z)$*, where the term* $\phi_z := \mathbb{E}_{d_O(s,a)}[\eta_z(s,a)\log(\eta_z(s,a)/\eta_{\widetilde{E}}(s,a))]$*.*

Proof (Sketch). We show in Supp. C.4 (see Lem. C.5) that $\mathrm{D_{KL}}\left(d_z(S,A)||d_{\widetilde{E}}(S,A)\right) = \mathbb{E}_{d_O(s,a)}[\eta_z(s,a)\log(\eta_z(s,a)/\eta_{\widetilde{E}}(s,a))]$. □

Using Lem. 3.2, Lem. 3.3, and Lem. 3.4, we design our multi-phase algorithm and present it in Sec. 4. In practice, we do not solve the previous three optimization problems to optimality, but rather perform a few gradient descent steps.

4 Algorithm

Our optimization method consists of three phases, each of which optimizes a specific model and fixes the remaining ones. It is important to emphasize that in contrast to prior work, our problem formulation considers an optimization problem with constraints. Furthermore, the reward function in eq. (8) is non-stationary, since it depends on the bounded Lagrange multipliers that balance diversity ($\log q(z|s)$) and expert imitation ($\log \eta_{\widetilde{E}}(s,a)$). This has significant algorithmic implications, as it requires solving a sequence of standard RL problems, each of which admits offline policy evaluation.

To smooth the transition of the reward signal between successive iterations, we enforce a slow change of the Lagrange multipliers. More specifically, we use the technique of bounded Lagrange multipliers (Stooke et al., 2020; Zahavy et al., 2022), which applies a Sigmoid transformation $\lambda = \sigma(\mu)$ component-wise to unbounded variables $\mu \in \mathbb{R}^{|Z|}$, so that the effective reward is a convex combination of a diversity term and an expert imitation term. In practice, this transformation ensures that $\lambda > 0$. Hence, the reward for each skill z becomes

$$R_z^{\mu}(s,a) := (1-\sigma(\mu_z))\,\frac{\log\left(q(z|s)|Z|\right)}{|Z|} + \sigma(\mu_z)\log \eta_{\widetilde{E}}(s,a). \tag{14}$$

We now present the resulting multi-phase optimization procedure in Algorithm 1. For a practical implementation, we leverage the power of neural networks and deep learning techniques for accurate function approximation. More specifically, we train an expert policy $\pi_{\widetilde{E}}$, a skill-conditioned policy $\{\pi_z\}_{z\in Z}$ and a value function $\{V_z\}_{z\in Z}$. While practically convenient, this means that each phase of Algorithm 1 is only approximately solved.

Algorithm 1 Diverse Offline Imitation (DOI)

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Pre-compute a discriminator $c^\star : \mathcal{S} \to (0,1)$ via optimizing the following objective with the gradient penalty in (Gulrajani et al., 2017) $\min_c \mathbb{E}_{d_E(s)}[\log c(s)] + \mathbb{E}_{d_O(s)}[\log(1-c(s))]$
Apply Phase 1 with reward $R(s,a) = \log \frac{c^\star(s)}{1-c^\star(s)}$ to compute ratios $\eta_{\widetilde{E}}(s,a) := \frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}$ for all $s,a \in \mathcal{D}_O$

Repeat until convergence:
  Phase 1. (Fixed Lagrange multipliers $\sigma(\mu)$ and discriminator values $q^\star(z|s)$)
  For each skill $z$:
    compute a value function $V_z^\star$ optimizing Equation (11) and
    ratios $\eta_z(s,a) := \frac{d_z^\star(s,a)}{d_O(s,a)} = \mathrm{softmax}\left(R_z^\mu(s,a) + \gamma\mathcal{T}V_z^\star(s,a) - V_z^\star(s)\right)$ for all $s,a \in \mathcal{D}$

  Phase 2. (Fixed ratios $\eta_z(s,a)$ and bounded Lagrange multipliers $\sigma(\mu)$)
  Train a discriminator $q^\star = \arg\max_{q(\cdot|s)} \mathbb{E}_{p(z)}\mathbb{E}_{d_O(s,a)}[\eta_z(s,a)\log q(z|s)]$
  Train a skill-conditioned policy $\pi_z^\star = \arg\max_{\pi_z} \mathbb{E}_{d_O(s,a)}[\eta_z(s,a)\log \pi_z(a|s)]$ for all $z \in Z$

  Phase 3. (Fixed ratios $\eta_{\widetilde{E}}(s,a)$ and $\eta_z(s,a)$)
  Compute for each skill $z$ an estimator $\phi_z := \mathbb{E}_{d_O(s,a)}[\eta_z(s,a)\log(\eta_z(s,a)/\eta_{\widetilde{E}}(s,a))]$
  Optimize the loss $\min_\mu \sum_z \sigma(\mu_z)(\epsilon - \phi_z)$
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5 Related Work

In the context of skill discovery Achiam et al. (2018) and Campos et al. (2020) showed that methods like DIAYN (Eysenbach et al., 2019) can struggle to learn large numbers of skills and have a poor coverage of the state space. Strouse et al. (2022) observed that when a novel state is visited, the discriminator lacks sufficient training data to accurately classify skills, which results in a low intrinsic reward for exploration. They address this by introducing an information gain objective (involving an ensemble of discriminators) as a bonus term. Kim et al. (2021b) gave a skill discovery approach based on an information bottleneck that leads to disentangled and interpretable skill representations. Park et al. (2022; 2023) proposed a Lipschitz-constrained skill discovery method based on a distance-maximizing and controllability-aware distance function to overcome the bias toward static skills and to allow the agent to learn complex and far-reaching behaviors. Sharma et al. (2020) developed a method that simultaneously discovers predictable skills and learns their dynamics. In a follow-up work, Park and Levine (2023) addresses the problem of errors in predictive models by learning a transformed MDP, whose action space contains only easy to model and predictable actions. A large body of research has focused on successor features (Dayan, 1993; Barreto et al., 2016), a powerful technique in RL for transfer of knowledge across tasks by capturing environmental dynamics, particularly promising for skill discovery when coupled with variational intrinsic motivation (Gregor et al., 2017; Barreto et al., 2018; Hansen et al., 2020) to enhance feature controllability, generalization, and task inference. Zahavy et al. (2022) cast the task of learning diverse skills, each achieving a near-optimal performance with respect to a given reward, into a constrained MDP setting with physics-inspired diversity objective. More specifically, diversity is measured by the minimum ℓ_2 distance between the successor features of different skills.

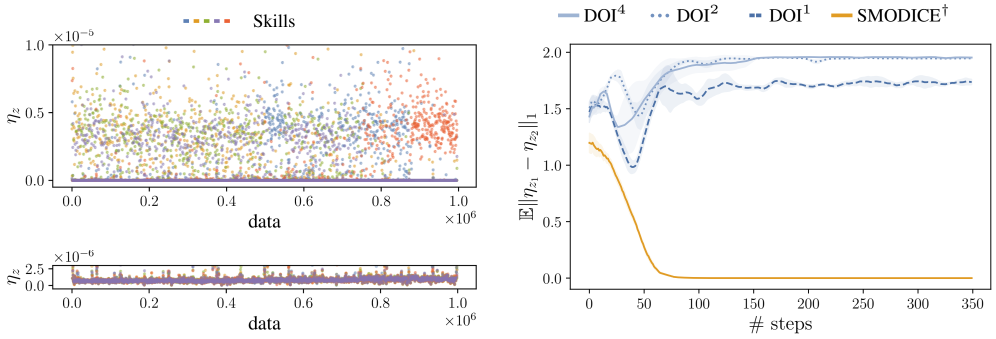
Naive importance sampling approaches for off-policy estimation are known to suffer from unbounded variance in the infinite horizon setting, a problem known in the literature as "the curse of horizon". (Liu et al., 2018; Mousavi et al., 2020) addressed this challenge by providing theoretical foundations and a principled off-policy algorithm, using a backward Bellman operator, that avoids exploding variance by applying importance sampling to state-visitation distributions, and by providing practical solutions in Reproducing Kernel Hilbert Spaces. An alternative research direction in off-policy estimation, referred to as "Distribution Correction Estimation (DICE)", has introduced innovative techniques, with Nachum et al. (2019a) mitigating variance with importance sampling, Nachum et al. (2019b) enabling policy gradient from off-policy data without importance weighting, Kim et al. (2022) stabilizing offline imitation learning with imperfect demonstrations, Zhang et al. (2020) improving density ratio estimation, Dai et al. (2020) providing high-confidence off-policy evaluation. Subsequently, Xu et al. (2021) applied this approach to offline RL and demonstrated its effectiveness in continuous control tasks. Several practical algorithms for offline RL have been proposed in the literature, including methods based on advantage-weighted behavioral cloning Nair et al. (2020); Wang et al. (2020) and using conservative strategies to stay close to the original data distribution Kumar et al. (2020); Kostrikov et al. (2022); Cheng et al. (2022).

6 Experiments

For evaluation of our method we consider 12 degree-of-freedom quadruped robot, Solo12 (Grimminger et al., 2020), on a simple locomotion task in both *simulation* and the *real* system. We provide further evaluation on the Ant, Walker2D, HalfCheetah and Hopper environments from the D4RL benchmark (Fu et al., 2020).

For the Solo12 evaluation we collected domain-randomized offline and expert data from simulation in the Isaac Gym (Makoviychuk et al., 2021) using saved checkpoints obtained by training the robot to track a certain velocity of the base with a version of DOMiNO (Zahavy et al., 2022). We defer the training procedure of the policies used for data collection to the Supp. D. The *expert dataset* was collected by using the best deterministic skill from the last checkpoint of the training procedure, which was trained to track forward velocity only. In contrast, the *offline dataset* was acquired by employing stochastic policies gathered from various checkpoints throughout the training of the expert, featuring multiple skill latents. More than half of the *offline dataset* was collected by a random Gaussian policy. In line with previous approaches by Kim et al. (2022) and Ma et al. (2022), our practical implementation aims to fulfill the expert coverage Assum. 2.1. To achieve this, we create the coverage dataset $\mathcal{D}_O$ by adding a small number of expert trajectories to the offline dataset, resulting in an (unlabeled) expert fraction of 1/160 in $\mathcal{D}_O$. To ensure that our algorithm does not have access to labeled expert actions, we discard them from the expert dataset. The resulting *expert dataset* $\mathcal{D}_E$ is used to learn a state classifier, in order to compute the ratios $\eta_{\widetilde{E}}(s,a)$. We trained the policy for 350 steps, where each step involves the stages described in Sec. 4. In each stage, we execute 200 epochs of batched training over the data. For the computation of the skill ratios $\eta_z(s,a)$, we choose a projection Π of the expert state (see Supp. H) that yields 3-dimensional planar and angular velocities of the robot's base in the base frame.

We have found that fitting the discriminator $q(z|s)$ is prone to collapse to the uniform distribution. To alleviate this issue, in addition to the variational lower bound objective (3), we add the DISDAIN information gain term, proposed in (Strouse et al., 2022). This bonus term is an entropy-based disagreement penalty that estimates the epistemic uncertainty of the discriminator, and is implemented in practice by an ensemble of randomly initialized discriminators. Due to the high initial disagreement on unvisited states, this intrinsic reward provides a strong exploration signal and leads to the discovery of more diverse skills. Intuitively, for states with small epistemic uncertainty, the discriminator (averaged over the ensemble members) should reliably discriminate between skills, thus making the intrinsic reward of the discriminator's log-likelihood more accurate. In all figures, we denote with DOI$^\epsilon$ the different constraint levels. We defer further experiment details to Supp. J.



(a) (b)

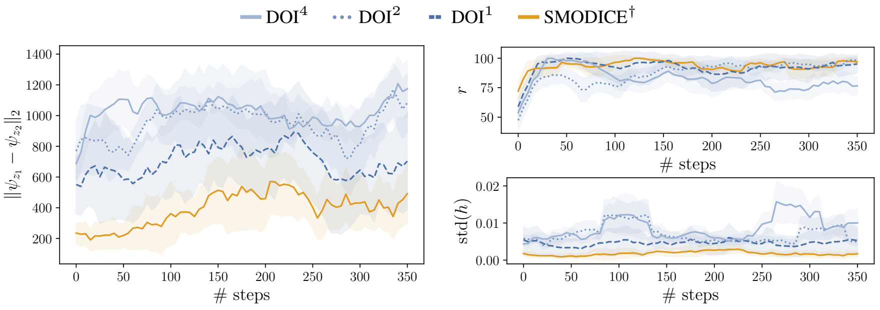
Figure 3: Data points separation by importance ratios $\eta_z(s, a)$, given different levels of ϵ in SOLO12. (a) Distribution of importance ratios $\eta_z(s, a)$ over the offline dataset $\mathcal{D}_O$ for different skills with DOI^4 ($\epsilon = 4$) (upper) and a skill-conditioned variant of SMODICE (lower). (b) Average ℓ_1 distance of ratios η_z belonging to different skills, depending on ϵ. The higher the value of ϵ, the greater the ℓ_1 distance.

As a baseline, we consider a skill-conditioned variant of (Ma et al., 2022), denoted SMODICE†, which does not have access to the discriminator. This is equivalent to DOI with $\epsilon = 0$. In Figure 3, we measure the state-action occupancy $d_z(s, a)$ for each skill z through the proxy of importance ratios $\eta_z(s, a)$, for different values of ϵ. As expected, a higher value of ϵ increases diversity, resulting in different importance ratios per skill for individual data points. We aggregate this difference by computing an average across different skills ℓ_1 norm of the importance ratios $\mathbb{E}\|\eta_{z_i} - \eta_{z_j}\|_1$ and report it in Figure 3. We note that the looser the constraint (lighter color), the easier it is to "diversify" in the sense of η_z. In Figure 3a, we observe diversification across the dataset assignment to skills when using DOI, whereas training an ensemble of skills with only expert imitation reward (i.e., $\sigma(\mu_z) = 1$) collapses to nearly the same importance per skill per data point. Figure 3b shows the average ℓ_1 distance between skill importance vectors η_z over the dataset for $\epsilon \in \{0.0, 1.0, 2.0, 4.0\}$ (lighter color indicates higher ϵ). Moreover, the tighter the constraint (smaller ϵ), the smaller the difference between the different skill importance ratios.

We have further evaluated diversity on the Monte Carlo estimates of the expected successor feature of the initial state, based on 30 policy rollouts per skill. The γ-discounted successor features (SFs) for state s are defined as $\psi_z(s) = \mathbb{E}_{d_z(s)}[\phi(s)]$, where $d_z(s)$ is the γ-discounted state occupancy for a skill policy π_z. With slight abuse of notation, we define $\psi_z = \mathbb{E}_{\rho_0(s)}[\psi_z(s)]$, the expected SFs over the initial state distribution. As a diversity metric, we take the average over different skills ℓ_2 norm between SFs, i.e., $\mathbb{E}\|\psi_{z_1} - \psi_{z_2}\|_2$. The results are presented in Figure 4 and show an alignment with the proxy diversity metric, i.e. the separation of the data indicated by the importance ratios η_z shows a higher distance between the expected SFs ψ_z. In terms of performance, DOI is able to achieve a forward velocity comparable to the expert (see Figure 4a) while diversifying the behavior in terms of base height h (Figure 4b). We also observed that the multipliers $\sigma(\mu_z)$ are non-zero for all skills, indicating that the constraint is active. In addition, they stabilize at reasonable levels as training progresses, which we show in Supp. F for both the SOLO12 and ANT.

For D4RL environments, we consider the case where we have offline data generated from a random policy mixed with a small amount of expert trajectories.[2]. Figure 5 shows the results for both the expected average SFs distance (Figure 5a) and the average importance ratio η_z distance across skills (Figure 5b). We normalize the state feature $\phi(s)$ when comparing ψ_z across environments in Figure 5a. As expected, there is a trade-off between the average skill return and the respective diversity metric across skills in most cases. Furthermore, the diversity distance that is more controllable by ϵ corresponds to the importance ratios η_z. This observation is in line with expectations, since η_z is part of the constraint. Nonetheless, in Figure 5a we show that ϵ retains some controllability over

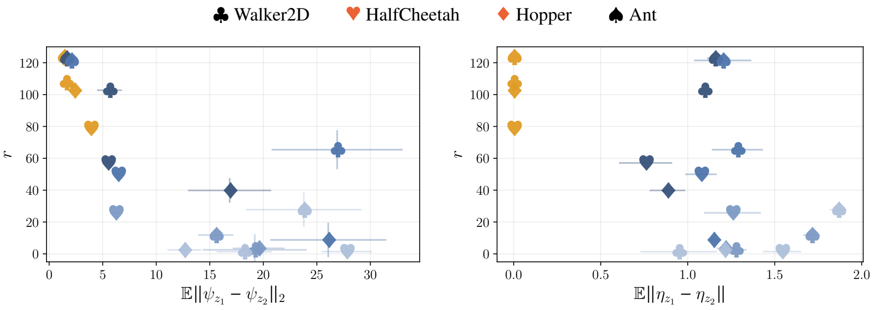
[2] The same setting was considered by Ma et al. (2022).



(a) (b)

Figure 4: Average ℓ_2 distance between MC estimated successor features ψ_z of different skills (a), return r as $\%$ of expert return and standard deviation of base height $\mathrm{std}_z(h)$ (b), depending on ϵ for the SOLO12.

diversity. The WALKER2D is particularly sensitive to relaxation of the occupancy constraint with respect to performance. We hypothesize that this is due to the fact that the space of policies that achieve a stable gait is very restrictive, resulting in a significant loss of task return for even slight skill diversification. In contrast, the ANT exhibits high stability, with multiple clusters achieving close to expert performance in terms of r. These results are also consistent with SMODICE expert policies used for computing $\eta_{\widetilde{E}}$ (see Supp. E).



(a) (b)

Figure 5: Results on D4RL environments with offline data collected from a random policy for $\epsilon = 0.0$, 0.5, 1.0, 2.0, 4.0. In figure (a) we observe the tradeoff between average skill return and average successor features distance over skills. In figure (b), we report the tradeoff w.r.t. average ℓ_1 distance of importance ratios η_z.

7 Conclusion

We proposed DOI, a principled offline RL algorithm for unsupervised skill discovery that, in addition to maximizing diversity, ensures that each learned skill imitates state-only expert demonstrations to a certain degree. Our main analytical contribution is to connect Fenchel duality, reinforcement learning, and unsupervised skill discovery to maximize a mutual information objective subject to KL-divergence state occupancy constraints. We have shown that DOI can diversify offline policies for a 12-DoF quadruped robot (in simulation and in reality) and for several environments from the standard D4RL benchmark in terms of both ℓ_2 distance of expected successor features and ℓ_1 distance of importance ratios, which is visible from the data separation induced by $\eta_z(s, a)$ amongst skills. The importance ratio distance, computed offline, is a robust indicator of diversity, which aligns with the online Monte Carlo diversity metric of expected successor features. The resulting skill diversity naturally entails a trade-off in task performance. We can control the amount of diversity via a KL constraint level ϵ, which ensures that different skills remain close to the expert in terms of state-action occupancy, which also indirectly controls task performance loss. A promising direction for future research is to impose constraints on the value function of each skill to ensure near-optimal task performance.

8 REPRODUCIBILITY

For implementation of DOI we have used the PyTorch autograd framework. For the SOLO12 training we made use of Isaac Gym for data collection and evaluation of the learned skill policies. For the D4RL experiments we evaluated the policies using the Mujoco v2.1 rigid body simulator. The training of the skill policies with evaluation and pre-training of the SMODICE expert ratios takes about 4 hours on an NVIDIA GeForce RTX 4080 graphics card with a batch size of 512. We plan on opensourcing the code and the SOLO12 data post conference acceptance. The SOLO12 robot has been developed as part of the Open Dynamic Robot Initiative (Grimminger et al., 2020), and a full assembly kit is available at a cheap price in order to reproduce the real system experiments from Supp. G.

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REFERENCES

Robohive – a unified framework for robot learning. https://sites.google.com/view/robohive, 2020. URL https://sites.google.com/view/robohive.

J. Achiam, H. Edwards, D. Amodei, and P. Abbeel. Variational option discovery algorithms. *CoRR*, abs/1807.10299, 2018. URL http://arxiv.org/abs/1807.10299.

E. Altman. *Constrained Markov decision processes*, volume 7. CRC Press, 1999.

A. Barreto, W. Dabney, R. Munos, J. J. Hunt, T. Schaul, H. Van Hasselt, and D. Silver. Successor features for transfer in reinforcement learning. *arXiv preprint arXiv:1606.05312*, 2016.

A. Barreto, D. Borsa, J. Quan, T. Schaul, D. Silver, M. Hessel, D. Mankowitz, A. Zidek, and R. Munos. Transfer in deep reinforcement learning using successor features and generalised policy improvement. In *International Conference on Machine Learning*, pages 501–510. PMLR, 2018.

S. Bhatnagar and K. Lakshmanan. An online actor–critic algorithm with function approximation for constrained markov decision processes. *Journal of Optimization Theory and Applications*, 153 (3):688–708, 2012.

V. S. Borkar. An actor-critic algorithm for constrained markov decision processes. *Systems & control letters*, 54(3):207–213, 2005.

S. P. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

A. Brohan, N. Brown, J. Carbajal, Y. Chebotar, X. Chen, K. Choromanski, T. Ding, D. Driess, A. Dubey, C. Finn, P. Florence, C. Fu, M. G. Arenas, K. Gopalakrishnan, K. Han, K. Hausman, A. Herzog, J. Hsu, B. Ichter, A. Irpan, N. J. Joshi, R. Julian, D. Kalashnikov, Y. Kuang, I. Leal, L. Lee, T. E. Lee, S. Levine, Y. Lu, H. Michalewski, I. Mordatch, K. Pertsch, K. Rao, K. Reymann, M. S. Ryoo, G. Salazar, P. Sanketi, P. Sermanet, J. Singh, A. Singh, R. Soricut, H. Tran, V. Vanhoucke, Q. Vuong, A. Wahid, S. Welker, P. Wohlhart, J. Wu, F. Xia, T. Xiao, P. Xu, S. Xu, T. Yu, and B. Zitkovich. RT-2: vision-language-action models transfer web knowledge to robotic control. *CoRR*, abs/2307.15818, 2023. doi: 10.48550/arXiv.2307.15818. URL https://doi.org/10.48550/arXiv.2307.15818.

V. Campos, A. Trott, C. Xiong, R. Socher, X. Giró-i-Nieto, and J. Torres. Explore, discover and learn: Unsupervised discovery of state-covering skills. In *Proceedings of the 37th International Conference on Machine Learning, ICML 2020, 13-18 July 2020, Virtual Event*, volume 119 of *Proceedings of Machine Learning Research*, pages 1317–1327. PMLR, 2020. URL http://proceedings.mlr.press/v119/campos20a.html.

C. Cheng, T. Xie, N. Jiang, and A. Agarwal. Adversarially trained actor critic for offline reinforcement learning. In *International Conference on Machine Learning, ICML 2022, 17-23 July 2022, Baltimore, Maryland, USA*, volume 162 of *Proceedings of Machine Learning Research*, pages 3852–3878. PMLR, 2022.

B. Dai, N. He, Y. Pan, B. Boots, and L. Song. Learning from conditional distributions via dual embeddings. In *Artificial Intelligence and Statistics*, pages 1458–1467. PMLR, 2017.

B. Dai, O. Nachum, Y. Chow, L. Li, C. Szepesvári, and D. Schuurmans. Coindice: Off-policy confidence interval estimation. *Advances in neural information processing systems*, 33:9398–9411, 2020.

P. Dayan. Improving generalization for temporal difference learning: The successor representation. *Neural Computation*, 5(4):613–624, 1993. doi: 10.1162/neco.1993.5.4.613.

B. Eysenbach, A. Gupta, J. Ibarz, and S. Levine. Diversity is all you need: Learning skills without a reward function. In *7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019*. OpenReview.net, 2019. URL https://openreview.net/forum?id=SJx63jRqFm.

J. Fu, A. Kumar, O. Nachum, G. Tucker, and S. Levine. D4rl: Datasets for deep data-driven reinforcement learning. *arXiv preprint arXiv:2004.07219*, 2020.

I. J. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. C. Courville, and Y. Bengio. Generative adversarial networks. *CoRR*, abs/1406.2661, 2014. URL http://arxiv.org/abs/1406.2661.

A. Goyal, R. Islam, D. Strouse, Z. Ahmed, H. Larochelle, M. M. Botvinick, Y. Bengio, and S. Levine. Infobot: Transfer and exploration via the information bottleneck. In *7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019*. OpenReview.net, 2019. URL https://openreview.net/forum?id=rJg8yhAqKm.

K. Gregor, D. J. Rezende, and D. Wierstra. Variational intrinsic control. In *5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Workshop Track Proceedings*. OpenReview.net, 2017. URL https://openreview.net/forum?id=Skc-Fo4Yg.

F. Grimminger, A. Meduri, M. Khadiv, J. Viereck, M. Wüthrich, M. Naveau, V. Berenz, S. Heim, F. Widmaier, T. Flayols, J. Fiene, A. Badri-Spröwitz, and L. Righetti. An open torque-controlled modular robot architecture for legged locomotion research. *IEEE Robotics and Automation Letters*, 5(2):3650–3657, 2020.

I. Gulrajani, F. Ahmed, M. Arjovsky, V. Dumoulin, and A. C. Courville. Improved training of wasserstein gans. In *Advances in Neural Information Processing Systems 30: Annual Conference on Neural Information Processing Systems 2017, December 4-9, 2017, Long Beach, CA, USA*, pages 5767–5777, 2017. URL https://proceedings.neurips.cc/paper/2017/hash/892c3b1c6dccd52936e27cbd0ff683d6-Abstract.html.

S. Hansen, W. Dabney, A. Barreto, D. Warde-Farley, T. V. de Wiele, and V. Mnih. Fast task inference with variational intrinsic successor features. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL https://openreview.net/forum?id=BJeAHkrYDS.

R. Houthooft, X. Chen, Y. Duan, J. Schulman, F. De Turck, and P. Abbeel. Vime: Variational information maximizing exploration. In *Advances in Neural Information Processing Systems*, 2016.

G.-H. Kim, S. Seo, J. Lee, W. Jeon, H. Hwang, H. Yang, and K.-E. Kim. DemoDICE: Offline imitation learning with supplementary imperfect demonstrations. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=BrPdX1bDZkQ.

J. Kim, S. Park, and G. Kim. Unsupervised skill discovery with bottleneck option learning. In *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139, pages 5572–5582. PMLR, 2021a. URL http://proceedings.mlr.press/v139/kim21j.html.

J. Kim, S. Park, and G. Kim. Unsupervised skill discovery with bottleneck option learning. In *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139 of *Proceedings of Machine Learning Research*, pages 5572–5582. PMLR, 2021b. URL http://proceedings.mlr.press/v139/kim21j.html.

A. Klyubin, D. Polani, and C. Nehaniv. Empowerment: a universal agent-centric measure of control. In *IEEE Congress on Evolutionary Computation*, volume 1, pages 128–135 Vol.1, 2005. URL https://ieeexplore.ieee.org/document/1554676.

I. Kostrikov, A. Nair, and S. Levine. Offline reinforcement learning with implicit q-learning. In *The Tenth International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29, 2022*. OpenReview.net, 2022. URL https://openreview.net/forum?id=68n2s9ZJWF8.

A. Kumar, A. Zhou, G. Tucker, and S. Levine. Conservative q-learning for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 33:1179–1191, 2020.

S. Levine, A. Kumar, G. Tucker, and J. Fu. Offline reinforcement learning: Tutorial, review, and perspectives on open problems. *CoRR*, abs/2005.01643, 2020.

P.-A. Léziart, T. Flayols, F. Grimminger, N. Mansard, and P. Souères. Implementation of a reactive walking controller for the new open-hardware quadruped solo-12. In *2021 IEEE International Conference on Robotics and Automation (ICRA)*, pages 5007–5013. IEEE, 2021.

Q. Liu, L. Li, Z. Tang, and D. Zhou. Breaking the curse of horizon: Infinite-horizon off-policy estimation. In *Advances in Neural Information Processing Systems 31: Annual Conference on Neural Information Processing Systems 2018, NeurIPS 2018, December 3-8, 2018, Montréal, Canada*, pages 5361–5371, 2018. URL https://proceedings.neurips.cc/paper/2018/hash/dda04f9d634145a9c68d5dfe53b21272-Abstract.html.

Y. J. Ma, A. Shen, D. Jayaraman, and O. Bastani. Versatile offline imitation from observations and examples via regularized state-occupancy matching. In *International Conference on Machine Learning, ICML 2022, 17-23 July 2022, Baltimore, Maryland, USA*, volume 162 of *Proceedings of Machine Learning Research*, pages 14639–14663. PMLR, 2022. URL https://proceedings.mlr.press/v162/ma22a.html.

V. Makoviychuk, L. Wawrzyniak, Y. Guo, M. Lu, K. Storey, M. Macklin, D. Hoeller, N. Rudin, A. Allshire, A. Handa, et al. Isaac gym: High performance gpu-based physics simulation for robot learning. *arXiv preprint arXiv:2108.10470*, 2021.

S. Mohamed and D. Jimenez Rezende. Variational information maximisation for intrinsically motivated reinforcement learning. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2015. URL https://proceedings.neurips.cc/paper/2015/hash/e00406144c1e7e35240afed70f34166a-Abstract.html.

A. Mousavi, L. Li, Q. Liu, and D. Zhou. Black-box off-policy estimation for infinite-horizon reinforcement learning. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL https://openreview.net/forum?id=S1ltg1rFDS.

O. Nachum and B. Dai. Reinforcement learning via fenchel-rockafellar duality. *arXiv preprint arXiv:2001.01866*, 2020.

O. Nachum, Y. Chow, B. Dai, and L. Li. Dualdice: Behavior-agnostic estimation of discounted stationary distribution corrections. *Advances in Neural Information Processing Systems*, 32, 2019a.

O. Nachum, B. Dai, I. Kostrikov, Y. Chow, L. Li, and D. Schuurmans. Algaedice: Policy gradient from arbitrary experience, 2019b.

A. Nair, M. Dalal, A. Gupta, and S. Levine. Accelerating online reinforcement learning with offline datasets. *CoRR*, abs/2006.09359, 2020.

S. Park and S. Levine. Predictable MDP abstraction for unsupervised model-based RL. In *International Conference on Machine Learning, ICML 2023, 23-29 July 2023, Honolulu, Hawaii, USA*, volume 202 of *Proceedings of Machine Learning Research*, pages 27246–27268. PMLR, 2023. URL https://proceedings.mlr.press/v202/park23i.html.

S. Park, J. Choi, J. Kim, H. Lee, and G. Kim. Lipschitz-constrained unsupervised skill discovery. In *The Tenth International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29, 2022*. OpenReview.net, 2022. URL https://openreview.net/forum?id=BGvt0ghNgA.

S. Park, K. Lee, Y. Lee, and P. Abbeel. Controllability-aware unsupervised skill discovery. *CoRR*, abs/2302.05103, 2023. doi: 10.48550/arXiv.2302.05103. URL https://doi.org/10.48550/arXiv.2302.05103.

R. F. Prudencio, M. R. O. A. Maximo, and E. L. Colombini. A survey on offline reinforcement learning: Taxonomy, review, and open problems. *CoRR*, abs/2203.01387, 2022.

M. L. Puterman. *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons, 2014.

A. Sharma, S. Gu, S. Levine, V. Kumar, and K. Hausman. Dynamics-aware unsupervised discovery of skills. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL https://openreview.net/forum?id=HJgLZR4KvH.

A. Stooke, J. Achiam, and P. Abbeel. Responsive safety in reinforcement learning by PID lagrangian methods. In *Proceedings of the 37th International Conference on Machine Learning, ICML 2020, 13-18 July 2020, Virtual Event*, volume 119 of *Proceedings of Machine Learning Research*, pages 9133–9143. PMLR, 2020. URL http://proceedings.mlr.press/v119/stooke20a.html.

D. Strouse, K. Baumli, D. Warde-Farley, V. Mnih, and S. S. Hansen. Learning more skills through optimistic exploration. In *The Tenth International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29, 2022*. OpenReview.net, 2022. URL https://openreview.net/forum?id=cU8rknuhxc.

C. Szepesvári. Constrained mdps and the reward hypothesis. *Musings about machine learning and other things (blog)*, 2020. URL https://readingsml.blogspot.com/2020/03/constrained-mdps-and-reward-hypothesis.html.

C. Tessler, D. J. Mankowitz, and S. Mannor. Reward constrained policy optimization. In *International Conference on Learning Representations*, 2019. URL https://openreview.net/forum?id=SkfrvsA9FX.

N. Tishby, F. C. Pereira, and W. Bialek. The information bottleneck method. In *Proc. of the 37-th Annual Allerton Conference on Communication, Control and Computing*, pages 368–377, 1999. URL https://arxiv.org/abs/physics/0004057.

H. Walke, K. Black, A. Lee, M. J. Kim, M. Du, C. Zheng, T. Zhao, P. Hansen-Estruch, Q. Vuong, A. He, V. Myers, K. Fang, C. Finn, and S. Levine. Bridgedata v2: A dataset for robot learning at scale. In *Conference on Robot Learning (CoRL)*, 2023.

Z. Wang, A. Novikov, K. Zolna, J. S. Merel, J. T. Springenberg, S. E. Reed, B. Shahriari, N. Siegel, C. Gulcehre, N. Heess, et al. Critic regularized regression. *Advances in Neural Information Processing Systems*, 33:7768–7778, 2020.

H. Xu, X. Zhan, J. Li, and H. Yin. Offline reinforcement learning with soft behavior regularization. *CoRR*, abs/2110.07395, 2021. URL https://arxiv.org/abs/2110.07395.

T. Zahavy, Y. Schroecker, F. M. P. Behbahani, K. Baumli, S. Flennerhag, S. Hou, and S. Singh. Discovering policies with domino: Diversity optimization maintaining near optimality. *CoRR*, abs/2205.13521, 2022. doi: 10.48550/arXiv.2205.13521. URL https://doi.org/10.48550/arXiv.2205.13521.

S. Zhang, B. Liu, and S. Whiteson. Gradientdice: Rethinking generalized offline estimation of stationary values. In *International Conference on Machine Learning*, pages 11194–11203. PMLR, 2020.

Supplementary for Diverse Offline Imitation Learning

A Fenchel Conjugate

The Fenchel conjugate $f_\star$ of a function $f : \Omega \to \mathbb{R}$ is given by

$$f_\star(y) = \sup_{x \in \Omega} \langle x, y \rangle - f(x), \tag{S1}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product defined on a space Ω. For any proper, convex and lower semi-continuous function f the following duality statement holds $f_{\star\star} = f$, that is

$$f(x) = \sup_{y \in \Omega_\star} \langle x, y \rangle - f_\star(y), \tag{S2}$$

where $\Omega_\star$ denotes the domain of $f_\star$. For any probability distributions $p, q \in \Delta(S)$ with $p(s) > 0$ implying $q(s) > 0$, we define for convex continuous functions f the family of f-divergences

$$\mathrm{D}_f(p||q) = \mathbb{E}_q \left[f\left(\frac{p(x)}{q(x)} \right) \right]. \tag{S3}$$

The Fenchel conjugate of an f divergence $\mathrm{D}_f(p||q)$ at a function $y(s) = p(s)/q(s)$ is, under certain conditions[3], given by

$$\mathrm{D}_{\star,f}(y) = \mathbb{E}_{q(s)} \left[f_\star(y(s)) \right]. \tag{S4}$$

Furthermore, its maximizer satisfies

$$p^\star(s) = q(s) f'_\star(y(s)). \tag{S5}$$

In the important special case where $f(x) = x \log(x)$, we obtain the well-known Kullback-Leibler (KL) divergence

$$\mathrm{D}_{\mathrm{KL}}(p||q) = \sum_s p(s) \log \frac{p(s)}{q(s)}. \tag{S6}$$

The Fenchel conjugate $\mathrm{D}_{\star,\mathrm{KL}}$ of the KL-divergence at a function $y(s) = p(s)/q(s)$ has a closed-form (Boyd and Vandenberghe, 2004, Example 3.25)

$$\mathrm{D}_{\star,\mathrm{KL}}(y) = \log \mathbb{E}_{q(s)}[\exp y(s)], \tag{S7}$$

and its maximizer $p^\star$ satisfies

$$p^\star(s) = q(s) \mathrm{softmax}(y(s)). \tag{S8}$$

B Lagrange Relaxation

The Lagrange relaxation is given by

$$\max_{d_z(s,a), q(z|s)} \min_{\lambda > 0} \sum_z \mathbb{E}_{d_z(s)} \left[\frac{\log(|Z| q(z|s))}{|Z|} \right] + \sum_z \lambda_z \left[\epsilon - \mathrm{D}_{\mathrm{KL}} \left(d_z(S,A) || d_{\widetilde{E}}(S,A) \right) \right].$$

By combining Lem. C.4 and the definition of $\eta_{\widetilde{E}}(s,a) = \frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}$, we have

$$\mathrm{D}_{\mathrm{KL}} \left(d_z(S,A) || d_{\widetilde{E}}(S,A) \right) = \mathrm{D}_{\mathrm{KL}} \left(d_z(S,A) || d_O(S,A) \right) - \mathbb{E}_{d_z(s,a)} \left[\log \eta_{\widetilde{E}}(s,a) \right]$$

and thus

$$\max_{d_z(s,a), q(z|s)} \min_{\lambda > 0} \sum_z \lambda_z \left[\epsilon + \mathbb{E}_{d_z(s,a)} \left[R_z^\lambda(s,a) \right] - \mathrm{D}_{\mathrm{KL}} \left(d_z(S,A) || d_O(S,A) \right) \right], \tag{S9}$$

where the reward is given by

$$R_z^\lambda(s,a) := \frac{\log(|Z| q(z|s))}{\lambda_z |Z|} + \log \eta_{\widetilde{E}}(s,a).$$

[3] f needs to satisfy certain regularity conditions (Dai et al., 2017)

C Algorithmic Phases

C.1 Value Function Training

With fixed skill-discriminator $q(z|s)$ and Lagrange multipliers $\lambda > 0$, the Problem S9 becomes:

$$\max_{\{d_z(s,a)\}_{z\in Z}} \sum_z \lambda_z \left\{\mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(s,a)||d_O(s,a)\right)\right\}$$

or equivalently for every skill z:

$$\begin{aligned} \max_{d_z(s,a)\geq 0} \quad & \mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right) \\ \text{s.t.} \quad & \textstyle\sum_a d_z(s,a) = (1-\gamma)\rho_0(s) + \gamma\mathcal{T}d(s) \quad \forall s. \end{aligned} \tag{S10}$$

We note that the preceding problem formulation involves state-action occupancy.

The strict feasibility in Assumption 3.1 implies strong duality, and thus Problem (S10) shares the same optimal value as the following dual minimization problem (for details see (Nachum and Dai, 2020, Section 6) and (Ma et al., 2022, Theorem 2)):

$$\begin{aligned} V^\star = \quad & \arg\min_{V(s)} (1-\gamma)\mathbb{E}_{s\sim\rho_0}\left[V(s)\right] \\ & + \log\mathbb{E}_{d^{\pi_O}(s,a)} \exp\left\{R_z^\lambda(s,a) + \gamma\mathcal{T}V(s,a) - V(s)\right\}, \end{aligned} \tag{S11}$$

where

$$\mathcal{T}V(s,a) = \mathbb{E}_{\mathcal{P}(s'|s,a)}V(s').$$

Moreover, the optimal primal solution reads

$$\frac{d_z^\star(s,a)}{d_O(s,a)} = \mathrm{softmax}\left(R_z^\lambda(s,a) + \gamma\mathcal{T}V_z^\star(s,a) - V_z^\star(s)\right). \tag{S12}$$

C.2 Skill-Conditioned Policy Training

With fixed skill-conditioned policy $\pi_z^\star$ and Lagrange multipliers $\lambda > 0$, the Problem S9 becomes

$$\max_{d_z(s,a)} \sum_z \left\{\mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right)\right\}$$

and reduces to

$$\max_{\pi_z} \mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)\log\pi_z(a|s)\right] \quad \forall z\in Z$$

The proof of Lem. 3.2 follow the approach in Ma et al. (2022), which is briefly summarized below for completeness. We pretrain a state discriminator $c^\star(s)$, by optimizing an objective and a gradient penalty as in Goodfellow et al. (2014) and Gulrajani et al. (2017), that distinguishes between expert and offline states. The Bayes optimal classifier $c^\star$ satisfies $\frac{c^\star(s)}{1-c^\star(s)} = \frac{d_E(s,a)}{d_O(s,a)}$, and thus $\log\frac{c^\star(s)}{1-c^\star(s)} = \log\frac{d_E(s)}{d_O(s)}$. Solving Problem (11) with fixed rewards $R_z^\lambda(s,a)$, yields the dual optimal value function $V^\star$. Using Fenchel duality, see (12), we compute (primal) optimal ratios $\eta_z(s,a)$ which we further use for training off-policy, via importance-weighted behavior cloning, the skill-conditioned policy π_z.

C.3 Skill Discriminator Training

With fixed skill-conditioned policy $\pi_z^\star$ and Lagrange multipliers $\lambda > 0$, the Problem S9 becomes

$$\max_{q(z|s)} \sum_z \left\{\mathbb{E}_{d_z(s,a)}\left[R_z^\lambda(s,a)\right] - \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right)\right\}$$

and reduces to

$$\max_{q(z|s)} \mathbb{E}_{p(z)}\mathbb{E}_{d_z(s,a)}\log q(z|s).$$

Lemma C.1. *Given a fixed skill-conditioned policy $\pi^\star_z$, Lagrange dual variable $\lambda > 0$ and ratios $\eta_z(s,a)$, using weighted behavioral cloning, we can train offline an optimal skill-discriminator $q(z|s)$. In particular, we optimize by gradient descent the following optimization problem*

$$\max_{q(z|s)} \mathbb{E}_{p(z)}\mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)\log q(z|s)\right].$$

Proof. The statement follows by combining Lem. C.2 and Lem. C.3. □

Lemma C.2 (Discriminator Gradient)**.** *It holds that*

$$\nabla_\phi \mathbb{E}_{p(s)}\left[\mathrm{D}_{\mathrm{KL}}\left(p(Z|s)||q_\phi(Z|s)\right)\right] = -\mathbb{E}_{p(z)}\mathbb{E}_{p(s|z)}\left[\nabla_\phi \log q_\phi(z|s)\right].$$

Proof. Observe that

$$\begin{aligned}\nabla_\phi \mathrm{D}_{\mathrm{KL}}\left(p(Z|s)||q(Z|s)\right) &= \nabla_\phi \mathbb{E}_{p(z|s)} \log \frac{p(z|s)}{q_\phi(z|s)} \\ &= -\mathbb{E}_{p(z|s)}\nabla_\phi \log q_\phi(z|s),\end{aligned}$$

where the second equality follows by

$$\nabla_\phi \log \frac{p(z|s)}{q_\phi(z|s)} = -\frac{q_\phi(z|s)}{p(z|s)}p(z|s)\frac{\nabla_\phi q_\phi(z|s)}{[q_\phi(z|s)]^2} = -\frac{\nabla_\phi q_\phi(z|s)}{q_\phi(z|s)} = -\nabla_\phi \log q_\phi(z|s).$$

□

Lemma C.3 (Importance Sampling)**.** *Given ratios $\eta_z(s,a)$, it holds for any function $f(s)$ that*

$$\mathbb{E}_{d^\star_z(s)}\left[f(s)\right] = \mathbb{E}_{d_O(s)}\left[\eta_z(s,a)f(s)\right].$$

Proof. Observe that

$$\begin{aligned}\mathbb{E}_{d^\star_z(s)}\left[f(s)\right] &= \mathbb{E}_{d^\star_z(s)\pi^\star_z(a|s)}\left[f(s)\right] = \mathbb{E}_{d^\star_z(s,a)}\left[f(s)\right] \\ &= \mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)f(s)\right].\end{aligned}$$

□

C.4 Estimating State KL Constraint Violation

Lemma C.4 (Structural)**.** *We have*

$$\mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_{\widetilde{E}}(S,A)\right) = \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right) - \mathbb{E}_{d_z(s,a)}\left[\log\frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}\right].$$

Proof. We have

$$\begin{aligned}\mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_{\widetilde{E}}(S,A)\right) &= \mathbb{E}_{d_z(s,a)}\left[\log\left(\frac{d_z(s,a)}{d_O(s,a)}\cdot\frac{d_O(s,a)}{d_{\widetilde{E}}(s,a)}\right)\right] \\ &= \mathrm{D}_{\mathrm{KL}}\left(d_z(S,A)||d_O(S,A)\right) - \mathbb{E}_{d_Z(s,a)}\left[\log\frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}\right].\end{aligned}$$

□

Lemma C.5 (State-Action KL Estimator)**.** *Suppose we are given offline datasets $\mathcal{D}_O(S,A) \sim d_O$, $\mathcal{D}_E(S) \sim d_E$ and optimal ratios $\eta_\pi(s,a) = \frac{d_\pi(s,a)}{d_O(s,a)}$ and $\eta_{\widetilde{E}}(s,a) = \frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}$ for all $(s,a) \in \mathcal{D}_O$, where the state-action occupancy $d_{\widetilde{E}}$ is induced by a policy $\pi_{\widetilde{E}}$ agreeing on the state occupancy of an expert π_E, i.e.*

$$\pi_{\widetilde{E}} \in \arg\min_\pi \mathrm{D}_{\mathrm{KL}}\left(d_\pi(S)||d_E(S)\right).$$

Then, we can compute offline *an estimator of*

$$\phi_z = \mathbb{E}_{d_O(s,a)}\left[\eta_z(s,a)\log\frac{\eta_z(s,a)}{\eta_{\widetilde{E}}(s,a)}\right].$$

Proof. By Claim 1, we have

$$\mathrm{D}_{\mathrm{KL}}\left(d_\pi(S,A)||d_{\widetilde{E}}(S,A)\right) = \mathrm{D}_{\mathrm{KL}}\left(d_\pi(S,A)||d_O(S,A)\right) - \mathbb{E}_{d_\pi(s,a)}\left[\log\frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}\right].$$

For the first term, we have

$$\begin{aligned}\mathrm{D}_{\mathrm{KL}}\left(d_\pi(S,A)||d_O(S,A)\right) &= \mathbb{E}_{d_\pi(s,a)}\log\frac{d_\pi(s,a)}{d_O(s,a)}\\ &= \mathbb{E}_{d_O(s,a)}\left[\eta_\pi(s,a)\log\eta_\pi(s,a)\right].\end{aligned}$$

The second term reduces to

$$\mathbb{E}_{d_\pi(s,a)}\left[\log\frac{d_{\widetilde{E}}(s,a)}{d_O(s,a)}\right] = \mathbb{E}_{d_O(s,a)}\left[\eta_\pi(s,a)\log\eta_{\widetilde{E}}(s,a)\right].$$

□

D Solo-12 Dataset Collection

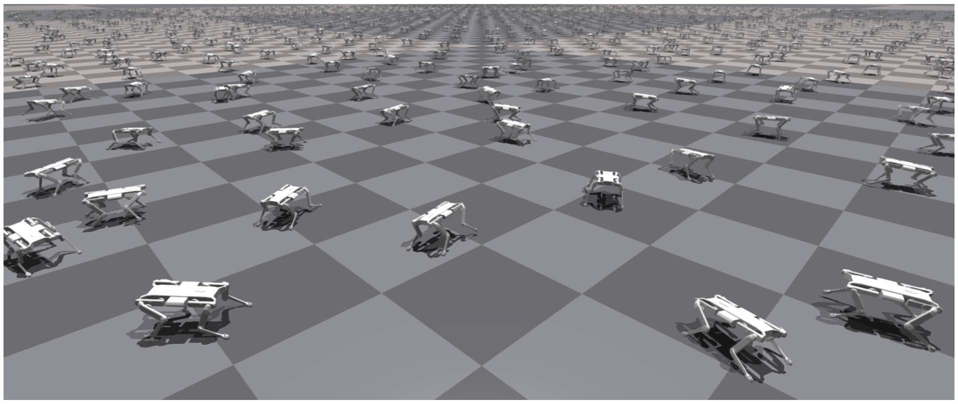


Figure S1: Solo-12 datasets are collected with 4000 environments in parallel using IsaacGym.

As shown in Figure S1, both *expert dataset* and *offline dataset* are collected using locomotion policies trained to track certain velocity in Isaac Gym (Makoviychuk et al., 2021). The policies are trained using an on-policy version of DOMiNO (Zahavy et al., 2022) to exhibit diverse behaviors while maintaining a certain level of tracking. Even trained with randomly sampled velocity, the policies are fed with forward velocity of 1 m/s when collecting both datasets. Both datasets contain 4000 trajectories with an episode length of 250 steps, or 1 million transitions each.

We summarize the main ideas of the training procedure, for details see (Zahavy et al., 2022). Using DOMiNO, we train policies that are conditioned on discrete skill latents and present different behaviors across different skills. Each skill-conditioned policy has a designated skill which is trained with only extrinsic reward and is maintained as the target in the constraint formulation in (Zahavy et al., 2022). We use this target skill from the last training checkpoint (iteration 2000) as the expert of our method. For each skill-conditioned policy, all skills except the target, are trained to balance between extrinsic and intrinsic reward, so as to generate diverse behaviours while being aligned to some degree to the target skill, i.e., maintaining a certain level of tracking velocity. The intrinsic reward is designed to maximize the ℓ_2 distance of the successor features (Barreto et al., 2016) between different skills, where in our setting the feature space includes: the base height velocity, base roll and pitch velocities, and feet height velocities.

We collected the *offline dataset* using these skill-conditioned policy from different checkpoints during training. The *offline dataset* is composed of 1/2 data from checkpoint 0, 1/4 data from checkpoint 50, 1/8 data from checkpoint 100, 1/16 data from checkpoint 500, 1/32 data from checkpoint 1500 and 1/32 data from checkpoint 2000. For each policy checkpoint, we collect data from the 5 corresponding skills, including the target skill. It is worth noting that more than half of the data from the *offline dataset* comes from the nearly random policies from the start of the training (checkpoint 0 and 50).

Furthermore, in the data collection process, we use a deterministic policy for the *expert dataset*, while for the *offline dataset* we use a stochastic policy. Randomizing the action selection in the latter case, results in more diverse interactions with the environment. In addition, we use domain randomization during training and data collection, in order to tackle the sim-to-real transfer and to simulate more diverse environment interaction. Specifically, we randomize the friction coefficient between $[0.5, 1.5]$ and additional base mass between $[-0.5, 0.5]$ kg, as well as simulate the observation noise and an actuator lag of 15 ms.

E SMODICE Expert Return

In table S1 we show the performance of the evaluated policies trained by SMODICE(Ma et al., 2022) on the Walker2D and HalfCheetah. The results are consistent with the performance that we obtain with DOI in Figure 5. We also note here the importance of having expert state coverage in the offline data that is reflected in the performance of the policies.

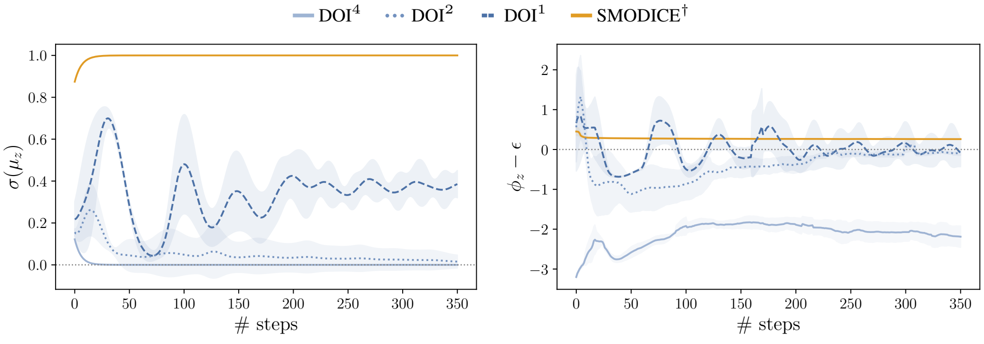
Environment	dataset	N	r
halfcheetah	medium-expert	25	81.25
		50	80.47
		200	73.56
	medium-replay	25	29.28
		50	36.73
		200	60.67
	random	25	10.89
		50	27.71
		200	78.94
walker2d	medium-expert	25	3.98
		50	19.22
		200	4.10
	medium-replay	25	15.09
		50	3.60
		200	0.95
	random	25	52.62
		50	103.52
		200	108.20

Table S1: Expected return for SMODICE-learned expert policies in the Walker2D and Ant environments for N expert trajectories mixed-in.

F Lagrange Multiplier Stability

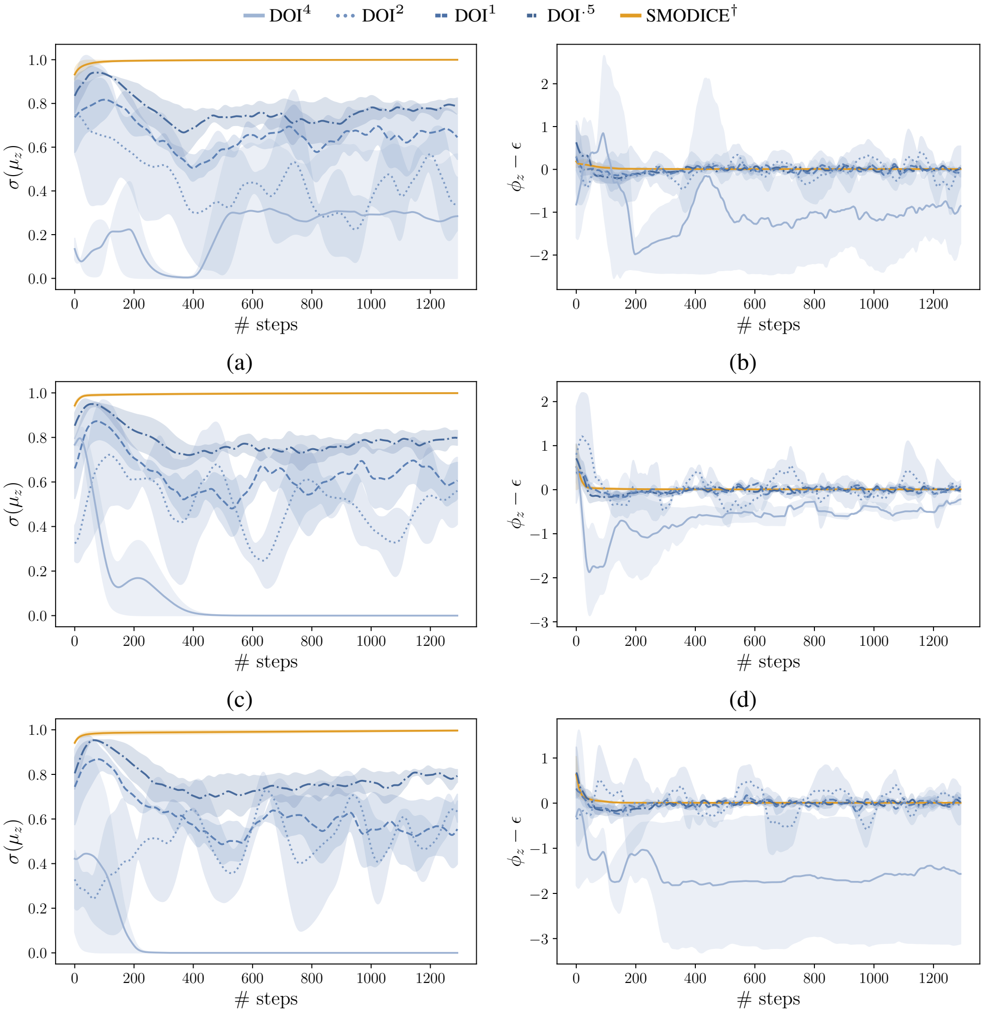
In Figure S2 we observe the behavior of the Lagrange multipliers for different levels of ϵ for a specific skill z in the Solo12 experiment. In case of $\epsilon \in \{1.0, 2.0\}$, the multipliers fluctuate around a specific level that strikes the balance between diversity and expert imitation. This can also be validated when observing the violation level in Figure S2b of the constraint given estimator ϕ_z, which is for $\epsilon \in \{1.0, 2.0\}$ around 0. On the other hand, if we introduce a strong constraint on the KL-divergence ($\epsilon = 0.0$), which is constantly violated, hence $\sigma(\mu_z) = 1$. Similarly, if the constraint is too weak, only diversity is optimized, in which case there is a significant degradation in performance (see figure Figure 4).

In Figure S3 we show the bounded lagrange multiplier values for three skills and the resulting violations for different ϵ levels for the Ant experiment. Again, the multiplier values fluctuate around appropriate levels ensuring the the violation of the constraint remains close to 0.



(a) (b)

Figure S2: Behavior of Lagrange multipliers. (a) Evolution of $\sigma(\lambda_z)$ for one skill ($z = 1$ chosen arbitrarily), (b) violation of the constraint for different ϵ. Negative $\phi_z - \epsilon$ indicates no violation. Means and standard deviation across restarts.



(a) (b)

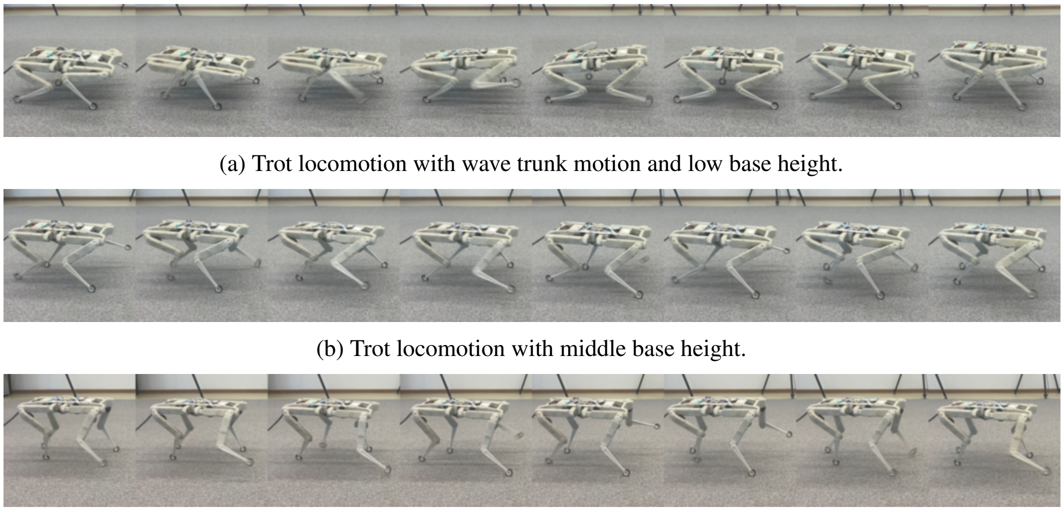
(c) (d)

(e) (f)

Figure S3: Behavior of Lagrange multipliers. (a) Evolution of $\sigma(\lambda_z)$ for one skill ($z = 1$ chosen arbitrarily), (b) violation of the constraint for different ϵ. Negative $\phi_z - \epsilon$ indicates no violation. Means and standard deviation across restarts.

G Real Robot Deployment

We successfully deployed policies exhibiting diverse skills extracted from the *offline dataset* while being able to track a certain velocity similar to the expert on real hardware. Our skill-conditioned policy exhibits different walking behaviors with diverse base motions. Snapshots of these diverse behaviors can be seen in Figure S4.



(a) Trot locomotion with wave trunk motion and low base height.

(b) Trot locomotion with middle base height.

(c) Trot locomotion with high base height.

Figure S4: Snapshots of the trained policy exhibiting different skills on hardware. From above to bottom, the policy has low, middle and high base positions while moving forward.

H Observation Projection

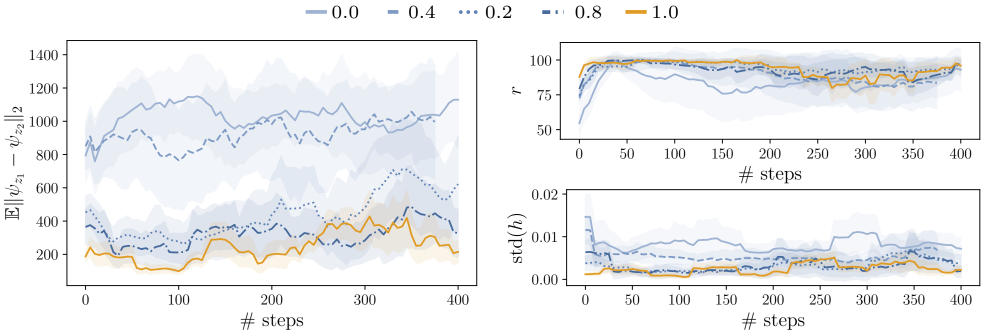
Imitation learning is of particular interest when the agent's and the target expert policy's state spaces do not necessarily match, but overlap in certain parts, as is often the case when learning from demonstrations. Our framework naturally accounts for this. If we consider $\mathcal{S}'$ to be the state space of the expert and $\mathcal{S}$ the state space of the agent, we assume that there exists a simple projection mapping $\Pi : \mathcal{S}' \mapsto \mathcal{O}$, where $\mathcal{O} := \{o : o \subset s, s \in \mathcal{S}\}$ is the power set of observations, allowing us to potentially imitate beyond expert policies with the same state space as the agent. Note that the agent still observes its full state s, however the projected state $\Pi(s)$ is observed by the expert classifier and skill discriminator. The projection Π can be selected to specify which parts of the state we want to diversify and constrain in terms of occupancy, depending on the task at hand.

I Limitations

The DOI method also comes with certain caveats. Maximizing the mutual information, as a diversity objective, poses a hard optimization problem due to its convexity. Thus, designing alternative diversity objectives can be beneficial. Furthermore, closeness in state-action occupancy can be quite restrictive in terms of availability of diverse behaviors that satisfy the constraint. Replacing this with constraints on the return of the policy would allow more freedom to optimize diversity in cases where the optimal policy may be multimodal. The above challenges are promising directions for future work.

J Additional Experiments

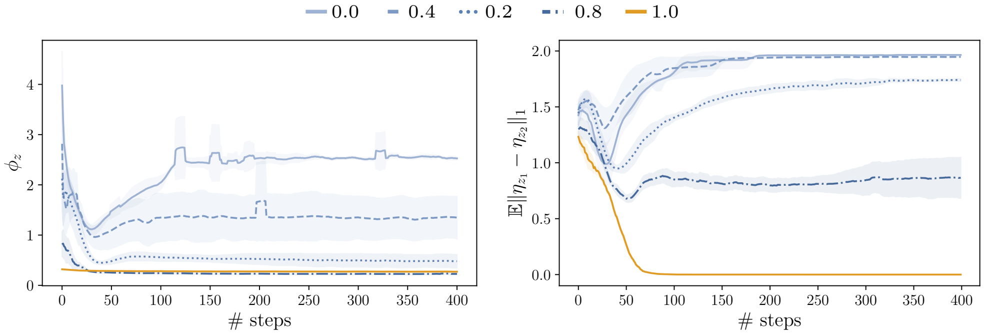
Instead of learning the Lagrange multipliers λ_z via KL estimators ϕ_z, we can also fix λ_z at a certain level, making it a hyperparameter. In our setting, this also works well, and we demonstrate a tradeoff between diversity and task reward optimization, see Figures S5 and S6. However, in this case we lose the possiblity to enforce a certaint constraint on the KL divergence between the skill state-action occupancy and expert state-action occupancy.



(a)

(b)

Figure S5: (a) Average ℓ_2 distance between Monte Carlo estimated successor representations ψ_z of different skills, (b) return r as $\%$ of expert return and standard deviation of base height $\mathrm{std}_z(h)$, depending on a fixed $\sigma(\lambda_z)$ (see legend).



(a)

(b)

Figure S6: Divergence estimate and η_z distance for the case of fixed $\sigma(\lambda_z)$. (a) Value of divergence estimator ϕ_z for a specific skill over the course of training ($z = 1$ chosen arbitrarily), (b) average ℓ_1 distance of η_z's of skills. Means and standard deviation across restarts.

We further provide results of applying DOI to different levels of expert trajectory mix-in to the *medium-replay* and *random* datasets of Walker2D and HalfCheetah in tables S2 and S3.

dataset # expert mixin	ϵ	$\mathbb{E}\|\eta_{z_1} - \eta_{z_2}\|$	r	$\mathbb{E}\|\psi_{z_1} - \psi_{z_2}\|$	
medium-replay	25	0.0	0.00 ± 0.00	46.00 ± 1.46	6.16 ± 0.30
		0.5	0.21 ± 0.08	0.33 ± 0.48	3.54 ± 2.14
		1.0	1.40 ± 0.05	2.33 ± 0.51	6.09 ± 2.40
		2.0	1.30 ± 0.03	0.64 ± 0.11	7.67 ± 4.27
		4.0	1.54 ± 0.08	2.30 ± 1.64	19.26 ± 2.29
	50	0.0	0.00 ± 0.00	54.29 ± 2.13	5.53 ± 0.14
		0.5	0.82 ± 0.28	31.31 ± 7.03	14.13 ± 1.86
		1.0	1.21 ± 0.15	4.33 ± 0.75	0.42 ± 0.05
		2.0	1.37 ± 0.03	1.61 ± 0.41	13.85 ± 2.50
		4.0	1.48 ± 0.12	1.11 ± 0.36	22.02 ± 1.33
	200	0.0	0.00 ± 0.00	98.33 ± 0.44	2.67 ± 0.26
		0.5	0.45 ± 0.11	74.59 ± 8.96	6.22 ± 1.17
		1.0	1.20 ± 0.09	2.52 ± 1.50	12.97 ± 4.33
		2.0	1.30 ± 0.03	2.07 ± 0.65	3.23 ± 2.02
		4.0	1.59 ± 0.06	1.43 ± 0.64	19.48 ± 1.43
random	25	0.0	0.00 ± 0.00	36.49 ± 11.54	15.70 ± 0.48
		0.5	0.93 ± 0.02	20.48 ± 7.90	16.81 ± 3.14
		1.0	1.30 ± 0.12	3.72 ± 1.38	8.16 ± 5.43
		2.0	1.45 ± 0.09	1.22 ± 0.32	20.47 ± 3.08
		4.0	1.27 ± 0.05	0.60 ± 0.26	20.60 ± 4.17
	50	0.0	0.00 ± 0.00	103.16 ± 0.69	3.32 ± 0.07
		0.5	1.03 ± 0.13	33.60 ± 6.64	18.27 ± 2.50
		1.0	1.37 ± 0.09	5.05 ± 2.66	20.16 ± 3.05
		2.0	1.46 ± 0.06	0.77 ± 0.29	10.46 ± 3.77
		4.0	1.23 ± 0.09	0.26 ± 0.11	14.33 ± 1.97
	200	0.0	0.00 ± 0.00	107.43 ± 0.26	1.84 ± 0.08
		0.5	1.29 ± 0.07	103.29 ± 1.38	6.75 ± 0.77
		1.0	1.26 ± 0.22	2.43 ± 0.30	7.30 ± 4.86
		2.0	1.46 ± 0.10	0.47 ± 0.15	15.39 ± 1.56
		4.0	1.29 ± 0.01	1.91 ± 0.57	19.66 ± 3.36

Table S2: WALKER2D metrics across random and medium-replay variants with varying number of mixed-in trajectories of the expert to satisfy the coverage assumption.

dataset	# expert mixin	ϵ	$\mathbb{E}\|\eta_{z_1} - \eta_{z_2}\|$	r	$\mathbb{E}\|\psi_{z_1} - \psi_{z_2}\|$
medium-replay	25	0.0	0.00 ± 0.00	37.64 ± 0.30	3.22 ± 0.06
		0.5	0.83 ± 0.12	36.95 ± 0.63	3.02 ± 0.10
		1.0	1.36 ± 0.09	24.30 ± 6.28	13.34 ± 4.84
		2.0	1.44 ± 0.06	6.73 ± 3.65	22.09 ± 8.15
		4.0	1.27 ± 0.09	2.68 ± 0.72	21.68 ± 1.87
	50	0.0	0.01 ± 0.01	45.40 ± 0.22	3.26 ± 0.27
		0.5	1.14 ± 0.02	42.89 ± 0.19	2.94 ± 0.12
		1.0	1.41 ± 0.12	37.28 ± 2.41	6.18 ± 1.21
		2.0	1.32 ± 0.11	8.60 ± 4.66	13.66 ± 1.97
		4.0	1.24 ± 0.16	1.72 ± 0.18	28.74 ± 7.84
	200	0.0	0.00 ± 0.00	73.60 ± 0.39	3.65 ± 0.09
		0.5	1.16 ± 0.08	69.91 ± 1.14	3.67 ± 0.10
		1.0	1.28 ± 0.13	23.74 ± 12.94	13.47 ± 1.73
		2.0	1.49 ± 0.10	15.52 ± 4.29	32.03 ± 0.56
		4.0	1.42 ± 0.07	2.16 ± 0.04	11.92 ± 2.28
random	25	0.0	0.00 ± 0.00	2.80 ± 0.36	5.55 ± 1.18
		0.5	1.12 ± 0.04	3.03 ± 0.28	4.30 ± 0.85
		1.0	1.14 ± 0.12	2.24 ± 0.09	10.45 ± 3.30
		2.0	1.24 ± 0.08	1.73 ± 0.33	25.01 ± 8.78
		4.0	1.44 ± 0.03	1.60 ± 0.30	35.08 ± 8.27
	50	0.0	0.00 ± 0.00	31.89 ± 1.14	9.97 ± 0.58
		0.5	1.14 ± 0.11	10.29 ± 3.13	17.90 ± 6.01
		1.0	1.42 ± 0.15	6.45 ± 2.95	23.30 ± 0.96
		2.0	1.41 ± 0.08	2.73 ± 0.43	23.91 ± 6.98
		4.0	1.68 ± 0.06	1.44 ± 0.27	35.07 ± 8.08
	200	0.0	0.00 ± 0.00	68.35 ± 1.25	5.20 ± 0.31
		0.5	1.30 ± 0.08	50.85 ± 17.30	9.80 ± 3.68
		1.0	1.21 ± 0.12	15.06 ± 5.58	29.57 ± 4.26
		2.0	1.03 ± 0.10	2.10 ± 1.99	10.84 ± 7.57
		4.0	1.20 ± 0.20	2.16 ± 0.05	16.90 ± 5.95

Table S3: HALFCHEETAH metrics across random and medium-replay variants with varying number of mixed-in trajectories of the expert to satisfy the coverage assumption.