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# 000 BEYOND SPEEDUP - UTILIZING KV CACHE FOR SAM- 001 002 PLING AND REASONING 003 004

005 **Anonymous authors**

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## 007 008 ABSTRACT 009

011 KV caches, typically used only to speed up autoregressive decoding, encode con-  
012 textual information that can be reused for downstream tasks at no extra cost. We  
013 propose treating the KV cache as a lightweight representation, eliminating the  
014 need to recompute or store full hidden states. Despite being weaker than dedi-  
015 cated embeddings, KV-derived representations are shown to be sufficient for two  
016 key applications: **(i) Chain-of-Embedding**, where they achieve competitive or  
017 superior performance on Llama-3.1-8B-Instruct and Qwen2-7B-Instruct; and **(ii)**  
018 **Fast/Slow Thinking Switching**, where they enable adaptive reasoning on Qwen3-  
019 8B and DeepSeek-R1-Distil-Qwen-14B, reducing token generation by up to  $5.7 \times$   
020 with minimal accuracy loss. Our findings establish KV caches as a free, effective  
021 substrate for sampling and reasoning, opening new directions for representation  
022 reuse in LLM inference.

## 024 1 INTRODUCTION

026 Large language models (LLMs) rely on key-value (KV) cache to accelerate autoregressive decoding  
027 by reusing past attention states, avoiding costly recomputation. This makes the KV cache indispensable  
028 for low-latency inference in production systems like vLLM (Kwon et al., 2023). However, its  
029 role is typically confined to this speedup. Beyond acceleration, the KV cache is seldom viewed as a  
030 reusable representation—with the notable exception of cache steering, a technique that modifies the  
031 cache’s initial state to guide generation (Belitsky et al., 2025).

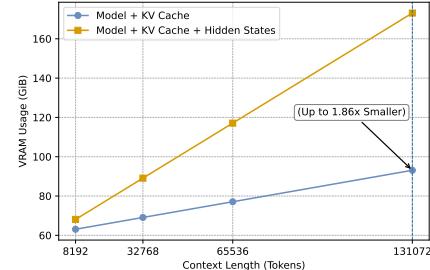
032 While the KV cache has been mostly confined to acceleration, hidden states have been widely exploited  
033 for *self-evaluation* (Wang et al., 2025b; Chen et al.,  
034 2024; Beigi et al., 2024; Zhang et al., 2025a) and  
035 for *adaptive reasoning and control* (Zhang et al.,  
036 2025b; Wang et al., 2025a; Yue et al., 2025). These  
037 methods, however, rely on storing full hidden states,  
038 which is costly in both memory and compute.

039 In this work, we investigate a simple but powerful  
040 question: **Can the KV cache do more than just ac-  
041 celerate decoding?** Since the KV cache is already  
042 computed and stored during inference, using it for  
043 downstream tasks incurs *no additional cost*. This  
044 is a major advantage over storing full hidden states,  
045 which is prohibitively expensive in terms of memory. As shown in Figure 1, the KV cache offers a  
046 significantly more compact and practical alternative for typical decoder-only models.

047 Though the KV cache is not explicitly trained as a general-purpose embedding—its sole objective  
048 is to support next-token prediction—we find it nonetheless encodes rich contextual information  
049 suitable for various downstream tasks. We explore this potential through two applications:

050

- 051 • **Chain-of-Embedding:** We repurpose the KV cache as a lightweight and readily avail-  
052 able embedding. In experiments on Chain of Embedding (CoE) (Wang et al., 2025b)—a  
053 method for selecting optimal reasoning paths without external information—we show that



054 Figure 1: VRAM usage vs. context length  
055 for Qwen3-32B (QwenTeam, 2025), com-  
056 paring Model+KV Cache vs. Model+KV  
057 Cache+Hidden States.

058 As shown in Figure 1, the KV cache offers a  
059 significantly more compact and practical alternative for typical decoder-only models.

054 KV caches achieve classification performance comparable to or even surpassing that of  
055 using the model’s hidden states.  
056

- 057 • **Fast/Slow Thinking Switch:** We leverage the KV cache to implement an adaptive switch-  
058 ing mechanism between fast, low-compute reasoning and slower, deliberate reasoning. By  
059 reusing KV cache, this approach achieves substantial efficiency gains with minimal perfor-  
060 mance loss.

061 Our contributions are fourfold:

- 062 1. We present the first systematic study of KV caches as reusable task representations, show-  
063 ing they can be repurposed at near-zero computational cost. In particular, we propose  
064 simple but effective aggregation techniques that make KV caches directly usable as em-  
065 beddings.
- 066 2. Despite not being designed as general-purpose embeddings, we find that KV cache rep-  
067 resentations when processed with the proposed aggregation strategies, are competitively  
068 effective on certain classification tasks.
- 069 3. We propose **KV-CoE**, a variant of Chain-of-Embedding that reuses the KV cache already  
070 stored during decoding. KV-CoE achieves self-evaluation without extra activation stor-  
071 age, offering nearly zero memory overhead and seamless integration into existing inference  
072 frameworks.
- 073 4. We introduce **KVClassifier**, a fast/slow auto-thinking framework that reuses KV caches  
074 for adaptive reasoning with minimal overhead.

075 Our results suggest that KV caches are a versatile and low-cost foundation for sampling and rea-  
076 soning, moving beyond their traditional role as a mere acceleration component to become a core  
077 resource for effective and efficient LLM-based inference.  
078

## 079 2 RELATED WORK

080 **Hidden-state self-evaluation.** A growing line of work shows that internal activations encode re-  
081 liable signals about answer correctness and hallucination risk. Wang et al. (2025b) propose *Chain-  
082 of-Embedding* (CoE), which models the trajectory of layerwise hidden states during inference and  
083 derives output-free correctness scores from the geometry of this path. Chen et al. (2024) (INSIDE)  
084 introduce *EigenScore*, computed from the eigenspectrum of hidden-state covariance, to assess se-  
085 mantic (in)consistency and detect hallucinations. Beigi et al. (2024) train a contrastive probe on  
086 *internal states* (attention, MLP activations) to produce well-calibrated confidence estimates across  
087 NLU/NLG tasks. Zhang et al. (2025a) further probes hidden states of reasoning models to predict  
088 whether a generated answer will be correct. All of these methods operate directly on hidden states or  
089 logits. **Our study**, by contrast, investigates whether *the KV cache alone*—which is already present  
090 at inference—suffices to support the same families of subtasks.  
091

092 **Adaptive fast/slow reasoning and dynamic control.** To mitigate overthinking on easy inputs  
093 and underthinking on hard ones, recent work explores *adaptive* reasoning depth (Xing et al., 2025).  
094 Zhang et al. (2025b) quantify upper bounds of long- vs. no-thinking modes and propose *Adaptive*  
095 *Self-Recovery Reasoning* (ASRR), adding accuracy-aware length rewards to reduce unnecessary rea-  
096 soning while allowing implicit recovery. PATS (Wang et al., 2025a) performs *process-level* switch-  
097 ing via process reward models with beam search, enabling step-wise fast/slow adaptation with bad-  
098 step penalties. DOTS (Yue et al., 2025) views reasoning as a search over atomic actions and learn  
099 to select dynamic trajectories. These approaches typically require explicit chain-of-thought genera-  
100 tion, external reward models, or re-decoding. **Our contribution** is orthogonal: we show that pooled  
101 *KV-cache* features can drive both one-shot (classification-style) and in-generation (generative-style)  
102 switching through simple control tokens, without storing hidden states or altering model architec-  
103 ture.  
104

105 **KV-cache interventions.** While our work treats the KV cache as a read-only representation for  
106 evaluation and control, a concurrent line of research shows that it can also serve as a *control inter-  
107 face*. Belitsky et al. (2025) introduce *KV Cache Steering*, a one-shot intervention that adds layer-  
108 wise steering vectors—derived from contrastive CoT vs. non-CoT prompts—to the key and value

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108 tensors after prefill, reliably inducing longer and more structured reasoning in small Language Models.  
109 Compared with activation steering, this method offers improved stability and negligible runtime  
110 overhead. **Our approach** is complementary: instead of modifying the cache, we *pool* it to derive  
111 difficulty-aware signals that gate slow reasoning.  
112

113 **3 BACKGROUND**  
114

115 **3.1 TRANSFORMER, HIDDEN STATES, AND KV CACHE**  
116

117 Decoder-only transformers are the architectural foundation of modern large language models  
118 (LLMs) (Vaswani et al., 2017; Brown et al., 2020). During autoregressive generation, each trans-  
119 former layer processes a new token to produce a contextual *hidden state*. A computational bottleneck  
120 arises because standard attention requires recomputing over all previous tokens at each step, result-  
121 ing in  $\mathcal{O}(T^2)$  complexity per step, where  $T$  is the sequence length. To mitigate this, the key–value  
122 (KV) cache (Dao et al., 2022) stores the attention keys and values for all past tokens at every layer.  
123 This allows the model to compute keys and values only for the new token and attend to the cached  
124 history, reducing the complexity to  $\mathcal{O}(T)$  per step and enabling efficient long-sequence generation.  
125

126 Formally, for layer  $l$ , we store  
127

$$\text{KVCache}^{(l)} = \{K_{1:T}^{(l)}, V_{1:T}^{(l)}\},$$

128 where  $K_{1:T}^{(l)}, V_{1:T}^{(l)} \in \mathbb{R}^{T \times H \times d_{\text{head}}}$  are the stacked key and value tensors across all attention heads  $H$ .  
129 The hidden state at step  $t$  is computed via  
130

$$h_t^{(l)} = \text{Attention}\left(Q_t^{(l)}, K_{1:t}^{(l)}, V_{1:t}^{(l)}\right).$$

132 Since  $K_{1:t-1}^{(l)}$  and  $V_{1:t-1}^{(l)}$  are already cached, only  $K_t^{(l)}$  and  $V_t^{(l)}$  need to be computed online.  
133

134 **3.2 MODERN LLM FRAMEWORKS AND KV CACHE MANAGEMENT**  
135

136 State-of-the-art LLM serving frameworks carefully manage KV caches to achieve high throughput,  
137 low latency, and efficient GPU memory utilization.  
138

139 **vLLM.** vLLM (Kwon et al., 2023) introduces a *PagedAttention* mechanism that virtualizes the KV  
140 cache in a paging system similar to CPU virtual memory. This allows dynamic allocation and reuse  
141 of KV cache memory, significantly reducing fragmentation and enabling high-throughput serving  
142 for thousands of concurrent sequences. By paging KV blocks in and out efficiently, vLLM supports  
143 fine-grained preemption and scheduling, which is critical for production-scale inference.  
144

145 **Ollama.** Ollama (Ollama Team, 2024) is a lightweight LLM deployment framework that empha-  
146 sizes developer usability and local inference. It manages KV caches per session, allowing multi-turn  
147 conversations to reuse context efficiently without re-computation. Although less focused on extreme  
148 multi-tenant throughput compared to vLLM, Ollama provides a practical demonstration of KV cache  
149 persistence for interactive workloads.  
150

151 Modern frameworks already manage the KV cache as a first-class resource, with sophisticated strate-  
152 gies for allocation, eviction, and sharing. This practice underscores our central claim:  
153

154 “*Since the KV cache is an unavoidable byproduct of efficient inference, repurposing it for down-  
155 stream tasks adds virtually no overhead.*”  
156

157 **4 OBSERVATION**  
158

159 **4.1 CAN KV CACHES SERVE AS AN EMBEDDING SOURCE**  
160

161 The hidden states and attention projections stored in KV caches encode contextualized token repre-  
162 sentations, making them natural candidates for use as embeddings. While recent work has explored  
163 leveraging intermediate representations from LLMs as task-specific embeddings (Liu et al., 2024),  
164 we specifically investigate aggregating KV cache vectors into sentence-level representations.  
165

To evaluate their quality as an embedding source, we construct embeddings by concatenating keys and values at every layer, then averaging across token positions, attention heads, and layers before applying  $\ell_2$  normalization. We benchmark these KV-derived embeddings on the Massive Text Embedding Benchmark (MTEB) (Muennighoff et al., 2023) against a strong, dedicated embedding model (gemini-embedding-001).

Dataset	Llama-3.1-8B-Instruct KV Cache	Gemini-Embedding-001
AmazonCounterfactualClassification	0.3530	0.8820
DBpediaClassification	0.5937	0.9476
FinancialPhrasebankClassification	0.6254	0.8864
TweetTopicSingleClassification	0.3714	0.7111

Table 1: Performance of KV cache-based embeddings vs. a dedicated embedding model on selected MTEB classification tasks. Despite being significantly weaker than trained embeddings, KV-derived embeddings still capture meaningful semantics.

As shown in Table 1, KV-derived embeddings significantly underperform their dedicated counterpart across all datasets, confirming they are *not perfect general-purpose embeddings*. This gap stems from three factors: (i) KV representations are optimized for causal language modeling, not contrastive learning, leading to poor isotropy; (ii) they are inherently token- and position-specific, requiring heuristic pooling for sentence-level use; and (iii) their projection into a lower-dimensional head space ( $d_{\text{head}} \ll d_{\text{model}}$ ) reduces their discriminative power.

Despite these limitations, the results show that KV caches encode substantial semantic information—enough to be competitive on certain classification tasks. This finding motivates our exploration of reusing the KV cache for *Chain-of-Embedding* and *Fast/Slow Thinking Switch*, where global embedding quality is less critical than local, relative separability between candidates.

#### 4.2 WHY KV CACHES ARE SUFFICIENT FOR CHAIN-OF-EMBEDDING AND FAST/SLOW THINKING SWITCH?

While KV caches are suboptimal for general-purpose embedding tasks—as they are trained solely for next-token prediction, are position-specific, and reside in a reduced-dimensional space—we contend they are nevertheless **sufficient** for *Chain-of-Embedding* and *Fast/Slow Thinking Switch*. Their poor performance on global semantic similarity tasks (e.g., 0.35 vs. 0.88 accuracy on Amazon-CounterfactualClassification in our MTEB results) is less critical for these applications, which rely on different criteria.

**Relative Separation via Margin.** Let  $\mathcal{X}$  denote the embedding space and  $f : \mathcal{X} \rightarrow \mathcal{Y}$  a classifier. For a pair of classes  $(y_i, y_j)$ , the empirical margin on embedding  $x \in \mathcal{X}$  is defined as

$$\gamma(x) = f_{y_i}(x) - f_{y_j}(x).$$

While contrastive learning aims to learn embeddings by maximizing  $\mathbb{E}[\gamma(x)]$  globally. In contrast, our target applications (Chain-of-Embedding and Fast/Slow Thinking Switch) only require that for a *restricted candidate set*  $\mathcal{C} \subset \mathcal{Y}$  (e.g., a small label space or a few candidate continuations), the margin satisfies

$$\min_{x \in \mathcal{C}} \gamma(x) > 0.$$

Thus, even with a noisy or anisotropic KV embedding space, preserving correct relative ordering within  $\mathcal{C}$  is sufficient, explaining their competitive performance on tasks with limited labels.

**Contextual Conditioning.** The pooled KV embedding  $e = \text{Pool}(K, V)$  in an instruction-tuned model can be expressed as  $e = g(x, \iota)$ , where  $x$  is the input and  $\iota$  is the instruction. Unlike general-purpose embeddings, this representation is contextually conditioned on the task, which inherently shapes the relevant decision boundaries. This conditioning reduces the need for a globally isotropic embedding space.

**Efficiency Constraint.** Let  $C_{\text{hidden}}$  and  $C_{\text{KV}}$  denote the memory cost of storing hidden states and reusing the KV cache, respectively, with  $C_{\text{hidden}} \gg C_{\text{KV}} \approx 0$ . The expected utility of reusing the KV cache is

$$U = \text{Acc}_{\text{KV}} - \lambda C_{\text{KV}} \approx \text{Acc}_{\text{KV}},$$

216 while storing hidden states incurs  
217

$$U' = \text{Acc}_{\text{hidden}} - \lambda C_{\text{hidden}},$$

219 for a resource trade-off parameter  $\lambda > 0$ . In latency-sensitive regimes where  $\lambda$  is large,  $U > U'$   
220 holds even when  $\text{Acc}_{\text{KV}} < \text{Acc}_{\text{hidden}}$ .  
221

222 **Local Decision Adequacy.** Both target applications require only *local* discrimination—such as  
223 ranking a few candidates or deciding whether to engage in deeper reasoning—rather than globally  
224 faithful embeddings. Formally, if  $\epsilon$  is the probability of local ranking error under KV embeddings,  
225 and  $\epsilon \ll$  the baseline task error rate, the final performance degradation is bounded by

$$|R_{\text{task}}^{\text{KV}} - R_{\text{task}}^{\text{ideal}}| \leq \epsilon,$$

228 where  $R$  is task risk. Our experiments confirm that  $\epsilon$  remains sufficiently small for KV embeddings  
229 to maintain competitive performance..  
230

## 231 5 CHAIN OF EMBEDDING WITH KV CACHE

### 232 5.1 BACKGROUND

233 LLMs exhibit emergent reasoning capabilities, though their internal decision processes remain  
234 largely opaque. To address this, Wang et al. (2025b) introduce *Chain-of-Embedding* (CoE), a  
235 method that probes the model’s latent space by tracking the evolution of sentence-level representa-  
236 tions across layers. Formally, for an LLM  $\mathcal{M}$  with  $L$  layers, let  $h_l^{(t)}$  denote the hidden representation  
237 of token  $t$  at layer  $l$ . The sentence-level representation at layer  $l$  is obtained by averaging over  
238 the sequence length  $T$ :

$$241 s_l = \frac{1}{T} \sum_{t=1}^T h_l^{(t)}, \quad l = 0, 1, \dots, L. \quad (1)$$

242 The CoE trajectory is then defined as the sequence of these layer-wise representations:  
243

$$244 \text{CoE} = \{s_0, s_1, \dots, s_L\}. \quad (2)$$

245 CoE characterizes this trajectory by measuring both magnitude and directional changes of embed-  
246 dings between consecutive layers:  
247

$$248 \Delta r_l = \|s_{l+1} - s_l\|_2, \text{ and } \Delta \theta_l = \arccos\left(\frac{s_{l+1} \cdot s_l}{\|s_{l+1}\| \|s_l\|}\right). \quad (3)$$

249 These features are aggregated into self-evaluation scores. For instance, the real-space combination  
250 (CoE-R) is  
251

$$252 \text{CoE-R} = \frac{1}{L-1} \sum_{l=0}^{L-1} (\alpha \Delta r_l + \beta \Delta \theta_l), \quad (4)$$

253 where  $\alpha, \beta$  are weighting coefficients. A more robust complex-space variant (CoE-C) treats each  
254  $(\Delta r_l, \Delta \theta_l)$  pair as a complex number  $z_l = \Delta r_l + i \Delta \theta_l$  and computes the magnitude of their average:  
255

$$256 \text{CoE-C} = \left| \frac{1}{L-1} \sum_{l=0}^{L-1} z_l \right|. \quad (5)$$

257 CoE has demonstrated strong discriminative power in distinguishing correct from incorrect model  
258 generations, achieving state-of-the-art performance on self-evaluation benchmarks.  
259

### 260 5.2 METHODOLOGY

261 Our key innovation is to adapt the CoE framework to use the KV cache, eliminating its primary  
262 computational overhead. While vanilla CoE constructs trajectories from hidden states  $h_l^{(t)}$ , re-  
263 quiring expensive activation storage or re-computation, we instead leverage the key-value tensors  
264  $K^{(l,t)}, V^{(l,t)}$  that are already maintained by autoregressive decoders. This modification preserves  
265 the CoE analytical framework while rendering it virtually cost-free.  
266

270     **Embedding Construction.** For each token  $t$  and layer  $l$ , we start with the cached key-value tensors  
 271      $K^{(l,t)}, V^{(l,t)} \in \mathbb{R}^{H \times d}$ . We flatten the head and key/value dimensions and average across layers to  
 272     produce a compact per-token embedding:  
 273

$$274 \quad e_t = \frac{1}{L} \sum_{l=1}^L \text{flatten}(K^{(l,t)}, V^{(l,t)}) \in \mathbb{R}^{H \cdot d}. \quad (6)$$

277     The resulting token-wise trajectory is defined as:  
 278

$$279 \quad \text{KV-CoE} = \{e_1, e_2, \dots, e_T\}, \quad (7)$$

280     which directly parallels the structure of vanilla CoE but operates along the token dimension.  
 281

282     **Trajectory Characterization.** We characterize this trajectory using the established CoE metrics,  
 283     simply substituting the token index  $t$  for the layer index  $l$ :  
 284

$$285 \quad \Delta r_t = \|e_{t+1} - e_t\|_2, \text{ and } \Delta \theta_t = \arccos\left(\frac{e_{t+1} \cdot e_t}{\|e_{t+1}\|_2 \|e_t\|_2}\right), \quad (8)$$

$$287 \quad \text{KV-CoE-R} = \frac{1}{T-1} \sum_{t=1}^{T-1} (\alpha \Delta r_t + \beta \Delta \theta_t), \text{ and } \text{KV-CoE-C} = \left| \frac{1}{T-1} \sum_{t=1}^{T-1} (\Delta r_t + i \Delta \theta_t) \right|. \quad (9)$$

290     These formulations maintain the analytical rigor of CoE-R and CoE-C with minimal conceptual  
 291     alteration.

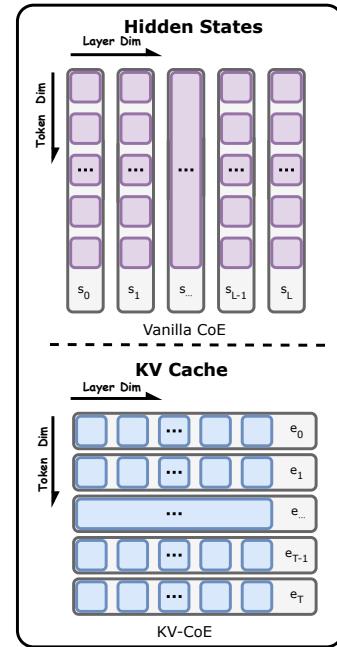
292     **Contributions and Advantages.** As illustrated in Figure 2, which compares vanilla CoE and our **KV-CoE**, our  
 293     method offers two main advantages:  
 294

- 296     **No extra activation cost.** Since the KV cache  
 297     is already computed and stored during standard  
 298     autoregressive decoding, reusing it for trajectory  
 299     analysis incurs virtually no additional activation  
 300     cost. The required reductions are computa-  
 301     tionally negligible compared to a full forward pass,  
 302     resulting in  $\Delta M \approx 0$  extra memory and mini-  
 303     mal FLOPs.
- 304     **Deployment-friendly.** The approach works  
 305     directly with standard inference stacks (e.g.,  
 306     `past_key_values` in Transformers or  
 307     vLLM). It requires no architectural changes,  
 308     re-forwarding, or activation hooks, making it  
 309     immediately deployable in production LLM  
 310     serving systems.

### 311     5.3 EXPERIMENTAL RESULTS

313     We evaluate **KV-CoE** on two reasoning benchmarks  
 314     from the original CoE work: MATH (Hendrycks et al.,  
 315     2021) for multi-step arithmetic and TheoremQA (Chen  
 316     et al., 2023) for theorem proving. Experiments are  
 317     conducted on two popular instruction-tuned models:  
 318     Llama-3.1-8B-Instruct (LlamaTeam, 2024) and Qwen2-  
 319     7B-Instruct (QwenTeam, 2024).

320     We construct embeddings directly from the KV cache  
 321     by extracting value vectors at every layer, concatenating  
 322     across attention heads, and averaging over layers to obtain one embedding per token, all without  
 323     storing hidden states. This reuse of cached information introduces negligible VRAM overhead and  
 324     incurs minimal FLOPs consumption compared to vanilla CoE.



325     Figure 2: Comparison between vanilla  
 326     CoE (top) and **KV-CoE** (bottom). Vanilla CoE aggre-  
 327     gates hidden states across the token dimension to con-  
 328     struct a representation for each layer, whereas  
 329     **KV-CoE** aggregates KV Cache across the  
 330     layer dimension to construct a rep-  
 331     resentation for each token.

Model	Method	MATH		TheoremQA	
		AUROC $\uparrow$	FPR95 $\downarrow$	AUROC $\uparrow$	FPR95 $\downarrow$
Llama-3.1-8B-Instruct	MaxProb	59.16	87.96	45.41	98.60
	PPL	60.82	86.42	46.45	97.82
	Entropy	62.74	84.14	47.37	97.82
	CoE-R $^\dagger$ (Llama3-8B)	72.54	75.61	63.12	89.83
	CoE-C $^\dagger$ (Llama3-8B)	73.08	79.60	55.85	90.14
	<b>KV-CoE-R (ours)</b>	<b>64.36</b>	<b>63.82</b>	74.74	62.93
Qwen2-7B-Instruct	MaxProb	12.40	99.34	4.92	99.87
	PPL	12.43	99.50	5.11	100.00
	Entropy	16.19	99.42	5.28	99.87
	CoE-R	75.75	65.95	66.68	85.84
	CoE-C	76.68	64.48	62.70	87.42
	KV-CoE-R (ours)	76.92	49.83	<b>88.87</b>	<b>54.30</b>
	<b>KV-CoE-C (ours)</b>	<b>84.12</b>	<b>44.82</b>	83.27	58.35

Table 2: Self-evaluation results on reasoning tasks. KV-CoE consistently improves AUROC and reduces FPR95 relative to MaxProb, PPL, and Entropy. Bold indicates the best value per model–dataset pair except CoE baselines. CoE-R and CoE-C results are taken from the original CoE paper (Wang et al., 2025b).  $^\dagger$ These baseline results are reported on Llama3-8B-Instruct, while our experiments use the updated Llama3.1-8B-Instruct, so the numbers may not perfectly align.

**Analysis.** As shown in Table 2, KV-CoE substantially outperforms baselines such as MaxProb, PPL, and Entropy on both MATH and TheoremQA. This demonstrates that the Chain-of-Embedding approach retains its strong discriminative power even when using KV cache-derived trajectories instead of hidden states. The token-level evolution captured by the KV cache provides a rich signal for identifying correct reasoning paths, especially in multi-step problems, all while adding negligible overhead since the cache is inherently available.

## 6 FAST/SLOW THINKING SWITCHING WITH KV CACHE

### 6.1 BACKGROUND

Large Reasoning Models (LRMs) can operate in two modes: *fast thinking*, which produces short, direct answers, and *slow thinking*, which generates detailed, step-by-step reasoning chains (Yao et al., 2023; Lightman et al., 2024). Although slow thinking enhances reliability on complex tasks, it incurs substantial computational overhead by producing significantly more tokens. For example, on GSM8K (Cobbe et al., 2021), Qwen3-32B (QwenTeam, 2025) slow thinking yields a marginal improvement in accuracy (0.95 vs. 0.94) while generating nearly four times the tokens, drastically increasing latency and cost as shown in Figure 3. This inefficiency motivates **adaptive reasoning**, where slow thinking is triggered selectively based on problem difficulty.

### 6.2 METHODOLOGY

We propose a method for adaptive reasoning that selects between fast and slow thinking on a per-instance basis to minimize unnecessary computation while maintaining accuracy. Our approach leverages the **KV cache** from the prompt encoding phase to make this decision, eliminating the need for additional forward passes.

**Key Idea.** Instead of predicting a binary “slow or fast” mode, we estimate a continuous difficulty score  $d \in [0, 100]$  from the pooled KV cache representation:

$$d = f_\theta \left( \text{Pool} \left( KV_{1:T}^{(1:L)} \right) \right),$$

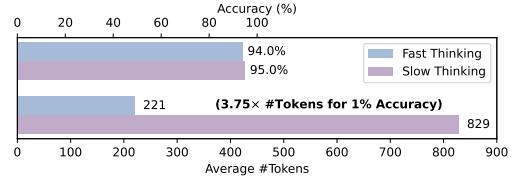


Figure 3: Comparison of efficiency and effectiveness of fast vs. slow thinking on GSM8K using Qwen3-32B. Slow thinking achieves slightly higher accuracy but at a much higher token cost.

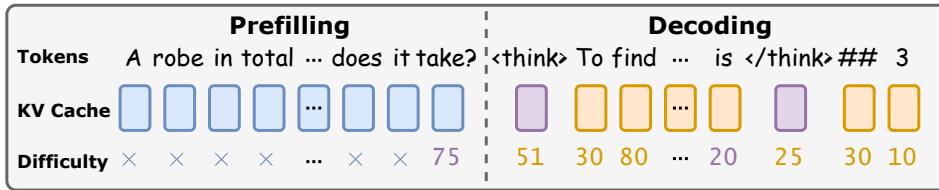


Figure 4: KVClassifier: special tokens are dynamically inserted to perform thinking-mode switching based on KV-derived difficulty scores.

where  $\text{Pool}(\cdot)$  aggregates keys and values across layers, heads, and token positions via mean pooling, and  $f_\theta(\cdot)$  is a lightweight MLP classifier. This score determines whether to engage slow thinking.

**Switching Mechanism.** We control the reasoning mode by injecting special control tokens ( $\langle\text{think}\rangle$  and  $\langle/\text{think}\rangle$ ) during decoding:

- **Initial Decision:** Before generation starts,  $d$  is compared to a predefined threshold  $\tau$ :
  - If  $d > \tau$ , prepend  $\langle\text{think}\rangle$  to trigger slow thinking.
  - Otherwise, proceed with fast thinking.
- **Dynamic Adjustment During Decoding:** During generation, the difficulty score is recomputed from the updated KV cache at predefined checkpoints:
  - If  $d < \tau_{\text{fast}}$  during slow thinking, append  $\langle/\text{think}\rangle$  to switch back to fast mode.
  - If  $d > \tau_{\text{slow}}$  during fast mode, inject  $\langle\text{think}\rangle$  to re-engage slow thinking and continue decoding with step-by-step reasoning.

This approach enables a fine-grained, difficulty-aware control over reasoning depth. Since the KV cache is already available from prompt encoding, both initial and ongoing difficulty assessments add negligible overhead. This significantly reduces token generation and latency for easy queries while allocating more resources to challenging problems. The overall workflow is illustrated in Figure 4.

**Training Data Construction.** To train the difficulty estimator  $f_\theta(\cdot)$ , we construct supervision signals from public reasoning datasets (training splits of GSM8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021)). For each training question, we generate two candidate solutions using the base model: a *fast thinking* response (no chain-of-thought) and a *slow thinking* response (with chain-of-thought). We then extract the final answers and compare them against the ground truth.

Based on the outcomes, we assign a difficulty label  $d \in \{0, 25, 75, 100\}$  reflecting the required reasoning depth:

- $d = 0$  (Very easy): Fast-thinking answer is correct and short ( $< 128$  tokens).
- $d = 25$  (Moderate): Fast-thinking answer is correct but long ( $\geq 128$  tokens).
- $d = 75$  (Hard): Fast-thinking answer is wrong, Slow-thinking answer is correct.
- $d = 100$  (Very hard): Both answers are incorrect.

This labeling scheme creates a natural difficulty progression, enabling  $f_\theta(\cdot)$  to learn a smooth score that correlates with both correctness and reasoning effort. The token-length criterion distinguishes trivial questions from those needing lengthy outputs without explicit reasoning. The trained estimator provides the continuous difficulty score needed for our adaptive switching mechanism.

### 6.3 EXPERIMENTAL RESULTS

**Setup.** We evaluate our KV-cache-based fast/slow thinking mechanism on two reasoning benchmarks: GSM8K evaluation split (Cobbe et al., 2021) and MATH500 (OpenAI / HuggingFaceH4 / Vals AI, 2025). Our experiments compare two switching strategies:

- **One-step switch (KV-Classification):** This strategy makes a single decision at generation start based on the predicted difficulty score, committing to either slow or fast thinking for the entire decoding process. It functions as a *classification-style* controller.

Dataset	Method	DeepSeek-R1-14B	Qwen3-8B
GSM8K	Fast Thinking	0.845 / 218	0.904 / 211
	Reasoning	0.847 / 432	0.933 / 1632
	KV-Classification	0.845 / 218 <small>-49.5%</small>	0.914 / 554 <small>-66.1%</small>
	KV-Generative	0.835 / 242 <small>-44.0%</small>	0.892 / 273 <small>-83.3%</small>
MATH500	Fast Thinking	0.536 / 540	0.568 / 616
	Reasoning	0.590 / 1839	0.610 / 4150
	KV-Classification	0.578 / 1506 <small>-18.1%</small>	0.604 / 3963 <small>-4.5%</small>
	KV-Generative	0.566 / 657 <small>-64.3%</small>	0.578 / 727 <small>-82.5%</small>

Table 3: Comparison of accuracy and average token usage for fast thinking, full reasoning, and our KV-cache-based switching methods on GSM8K and MATH500. For each KV-based method, we report the result from best hyper-parameter configuration identified in Appendix B.

- **Two-step switch (KV-Generative):** This method performs an initial mode selection and continuously monitors difficulty during decoding. If difficulty drops below  $\tau_{\text{fast}}$  during slow thinking, it appends  $\langle\text{think}\rangle$  to terminate reasoning early; if difficulty exceeds  $\tau_{\text{slow}}$  during fast thinking, it injects  $\langle\text{think}\rangle$  to engage slow thinking mid-generation. This implements a *generative-style* controller that dynamically adjusts reasoning depth.

We deploy both strategies on two open-weight models: DeepSeek-R1-14B (DeepSeek-AI, 2025) and Qwen3-8B (QwenTeam, 2025), evaluating their ability to selectively trigger slow thinking and reduce unnecessary token generation.

We construct representations by concatenating key and value tensors across all heads, summing over selected token positions, and averaging across selected layers without normalization, then feed the result into a two-layer MLP (hidden dimension 512, ReLU activation) for difficulty prediction. This design directly reuses the KV cache available during decoding, introduces negligible VRAM or FLOPs overhead, and functions as a modular component that can be seamlessly integrated into existing inference pipelines without modifications to the base model.

**Analysis.** As shown in Table 3, our KV-cache-based switching approach achieves an effective balance between accuracy and efficiency. For instance, on MATH500 using Qwen3-8B, two-step generative switching reduces average token count from 4,150 (full reasoning) to 727 (5.7 $\times$  reduction) with only a minimal 3.2% accuracy drop. The one-step classification strategy is more conservative, using more tokens but achieving near-full-reasoning accuracy (0.604 vs. 0.610). Similar trends are observed on GSM8K, where KV-cache-based switching maintains high accuracy (up to 0.914) while significantly reducing token consumption compared to full reasoning. These results demonstrate that difficulty scores derived from the KV cache generalize well across tasks and models, enabling efficient and effective adaptive reasoning with negligible overhead.

## 7 CONCLUSION

This work repurposes the KV cache, moving beyond its conventional role in decoding acceleration to unlock its potential as a versatile, cost-free representation. We demonstrate that although not designed as general-purpose embeddings, KV caches encode rich contextual information that can be effectively utilized for downstream tasks without incurring additional computational overhead. Our experiments establish two practical applications: (i) **Chain-of-Embedding**, where KV-derived embeddings match or surpass the performance of hidden-state embeddings, and (ii) **Fast/Slow Thinking Switching**, which uses KV-cache-based difficulty scores to enable adaptive reasoning-reducing token usage by up to 5.7 $\times$  with minimal accuracy loss. These findings position the KV cache as a deployment-friendly substrate for advanced inference techniques, opening new avenues for reusing inference-time artifacts to improve efficiency and controllability in LLMs.

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648            **A THE USE OF LARGE LANGUAGE MODELS (LLMs)**  
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650            We acknowledge the use of a Large Language Model (LLM) to support the preparation of this  
651            manuscript. The LLM was employed exclusively for editorial purposes, such as refining the clarity  
652            of exposition, improving grammar and readability, and polishing the overall presentation. At times,  
653            it was also used to suggest alternative phrasings for technical descriptions in order to make the  
654            arguments more accessible.

655            Importantly, the LLM did not contribute to the conceptual development, methodology, or exper-  
656            imental design of this work. All ideas, including the proposal to treat the KV cache as a reusable  
657            representation, the development of KV-CoE for output-free self-evaluation, and the design of KV-  
658            based adaptive Fast/Slow Thinking Switching for token-efficient reasoning, were conceived and  
659            implemented solely by the authors. The LLM was not used to generate research results, proofs, or  
660            derivations.

661            All scientific claims, analyses, and conclusions presented in this paper remain the full responsibility  
662            of the authors. Any text initially produced with LLM assistance was carefully reviewed, revised,  
663            and verified prior to inclusion.

664

665            **B HYPER-PARAMETER SELECTION FOR KVCLASSIFIER**  
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667            To better understand the effect of hyper-parameters on KV-based classification, we conduct a sys-  
668            tematic study by varying the number of layers pooled and the number of tokens selected from the  
669            end of the sequence. Importantly, we fix the *total amount of KV data* to be approximately constant  
670            across configurations (256 token  $\times$  layer units). This ensures a fair comparison: for example, select-  
671            ing 8 layers with 32 tokens, 4 layers with 64 tokens, or 2 layers with 128 tokens all yield the same  
672            KV budget.

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674 <b>Model</b>	675 <b>Dataset</b>	676 <b>Method</b>	677 <b>8L, Len=32</b>	678 <b>4L, Len=64</b>	679 <b>2L, Len=128</b>
680            DeepSeek-14B	681            GSM8K	KV-Classification	0.845 / 218	0.845 / 218	<b>0.845</b> / 218
		KV-Generative	<b>0.835</b> / 242	0.825 / 232	0.805 / 217
	682            MATH500	KV-Classification	0.536 / 540	0.550 / 905	<b>0.578</b> / 1506
		KV-Generative	0.538 / 524	0.550 / 544	<b>0.566</b> / 657
683            Qwen3-8B	684            GSM8K	KV-Classification	0.904 / 211	0.904 / 217	<b>0.914</b> / 554
		KV-Generative	<b>0.892</b> / 273	0.886 / 276	0.881 / 257
	685            MATH500	KV-Classification	0.570 / 736	0.598 / 3673	<b>0.604</b> / 3963
		KV-Generative	<b>0.578</b> / 727	0.524 / 933	0.550 / 837

682            Table 4: Hyper-parameter selection results for KV-Classification and KV-Generative. Values are  
683            reported as Accuracy / #Tokens. Best accuracy for each dataset–method pair is in bold.  
684            Table 4 summarizes the results on GSM8K and MATH500 for both DeepSeek-R1-14B and Qwen3-  
685            8B, under KV-Classification and KV-Generative settings. We observe that while performance varies  
686            slightly with the allocation of layer vs. token depth, the overall trends are consistent: (i) accuracy  
687            remains competitive across different allocations, and (ii) increasing token coverage (e.g., 2L  $\times$  128)  
688            tends to favor more complex datasets such as MATH500, whereas shallow but wider layer coverage  
689            (e.g., 8L  $\times$  32) can suffice for GSM8K.

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