

DESIGNING NEURAL NETWORK COLLECTIVES

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ABSTRACT

Artificial neural networks have demonstrated exemplary learning capabilities in a wide range of tasks, including computer vision, natural language processing and, most recently, graph-based learning. Many of the advances in deep learning have been made possible by the large design-space for neural network architectures. We believe that this diversity in architectures may lead to novel and emergent learning capabilities, especially when architectures are connected into a collective system. In this work, we outline a form of *neural network collectives* (NNC), motivated by recent work in the field of collective intelligence, and give details about the specific sub-components that an NNC may have.

1 INTRODUCTION

Deep learning methods, characterised by artificial neural networks (ANNs), have demonstrated high performance at many complex tasks including computer vision, natural language processing, and graph-based learning (Voulodimos et al., 2018; Cambria & White, 2014; Zhou et al., 2020). The number of ANNs in deployment is expected to increase dramatically as the design of ANNs becomes more accessible due to dedicated software packages, and as computational power of modern processors increases. We focus on the methods and principles of collective intelligence (Moussaid et al., 2009; Malone & Bernstein, 2015), outlining learning motifs that we take to be important for designing systems of ANNs, so-called ‘neural network collectives’ (NNCs).

NNCs are defined by structuring computational ANN units in a collective, with the aim of establishing forms of emergent intelligence. An example schema for NNCs is shown in Figure 1. At the first layer, traditional ANNs architectures comprise individual units. The outputs produced by these units are connected together at the second layer and are allowed to interact and exchange information (such as output values, network parameters or probabilistic belief representations) according to the principles of collective intelligence. These interactions produce emergent behaviours at the third layer. Meta-learning and adaptation occurs as information from higher layers feeds back to optimise lower level structures, for example through the modulation of network parameters at layer one, or through restructuring interactions and information exchange at layer two.

We believe that the implementation of large-scale NNCs will bring several benefits. For example, NNCs would give insight into the advantages and limitations of large-scale collective learning infrastructures. These could dramatically increase the security of sensitive data used in government and medicine. Discussions about distributed collective intelligence systems are already highly active, especially in the context of distributed ledger technologies (Saldamli et al., 2020). NNCs would add an ‘artificial learning’ dimension to this area, for example, by supplementing crowd-sourced learning initiatives with synthetic learning agents (Li et al., 2018; Ducrée et al., 2020).

Additionally, we believe that NNCs will be worth studying as a complex system in their own right. In this setting, we highlight the possibility of performing active experiments on NNCs, ablating parts of the network to test hypothesis on collective intelligence or causal reasoning, or adding adversarial units to affect convergence, such as introducing zealots.

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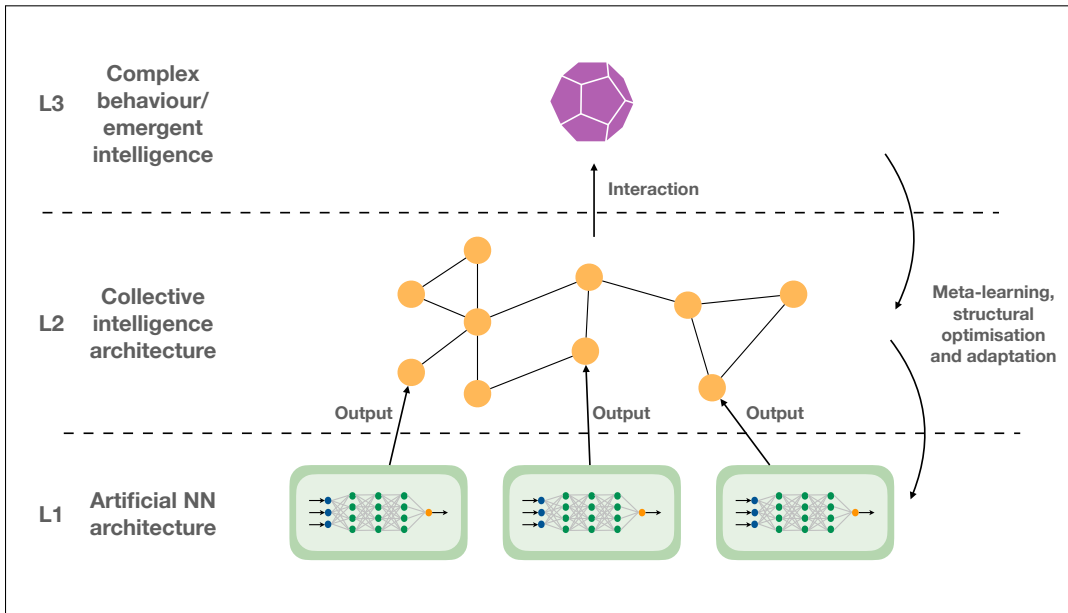


Figure 1: Multi-layered collective artificial intelligence schema

Finally, NNCs may address some of the limitations of current deep learning models. Many of the existing language models are extremely large, training networks with several billion parameters (Floridi & Chiriatti, 2020). These models are prohibitively expensive to train. NNCs that distribute training over consumer electronics could considerably reduce training costs while still providing strong performance.

This paper is structured as follows; we first outline the principles and concepts of collective intelligence that are relevant to the design of NNCs. We then outline categories of artificial collective intelligence, including multi-agent systems, embodied AI in robotics and various forms of ANNs. Having discussed related works, we outline our vision for NNCs, giving details about features at all three layers. We highlight choices to be made at the node level, collective level, and possible features that will emerge at the level of collective intelligence. Finally, we make our concluding remarks. For further discussion on future research directions, including potential obstacles and a roadmap for implementing various of forms of NNCs, we refer the reader to Appendix A.

2 RELATED WORK

2.1 COLLECTIVE INTELLIGENCE AND SELF-ORGANISATION

Collective or swarm intelligence is widely observed across scales in nature (Camazine et al., 2001). Here, groups of simple, autonomous individuals display the ability to collectively make decisions and solve problems in their environment, such as collective environmental sensing (Berdahl et al., 2013), foraging and search (Falcón-Cortés et al., 2019; Nakagaki, 2001), construction of complex structures (Werfel & Nagpal, 2006), value-dependent decision making (Seeley et al., 2006), as well as task allocation and division of labour (Theraulaz et al., 1998). Importantly, completion of these tasks is beyond the capacity of single individuals making up the group, and must be completed in a distributed and self-organised fashion, relying on noisy peer-to-peer communication, and no external guidance. Through specific sets of interactions and information exchange, self-organising systems may then display emergent behaviours and intelligence at the group level (Moussaid et al., 2009; Garnier et al., 2007). Natural swarm systems are robust to environmental noise and are able to adapt their functioning to respond to changes in environmental conditions over time (Kitano, 2004; 2007). These systems have also demonstrated the ability to collectively learn over time, transferring information between generations during the formation of culture on various time-scales (Biro et al., 2016; Sasaki & Biro, 2017).

The study and mathematical characterisation of this self-organisation has led to engineering solutions and applications (Bonabeau et al., 1999; Dorigo et al., 2021; Bayındır, 2016). The motivation of these algorithms is to reproduce emergent properties of distributed complex systems observed in nature, such as robustness to individual unit failure, scalability of operation to large numbers of units (computational load can be distributed across many entities), and flexibility of the type of tasks that systems can perform (Şahin, 2004; Brambilla et al., 2013). While these principles have been applied to develop artificial collective intelligence in both software (Bonabeau et al., 1999; Brabazon et al., 2018; Dorigo et al., 2006; Kennedy & Eberhart, 1995) and embodied AI systems (Dorigo et al., 2021), their application to systems of agents with higher levels of intelligence has been limited. We propose that the principles of distributed self-organisation can be applied to neural network architectures in order to enhance and extend their range of applications.

2.1.1 TYPES OF STRUCTURES IN COLLECTIVE INTELLIGENCE

Multi-agent systems may be characterised according to the way that data and decisions are shared between the different interacting units. Here, we use the generally agreed-upon definition in the distributed engineering (Saeedi et al., 2016) and computation communities (Coulouris et al., 2005; Attiya & Welch, 2004; Garg, 2002).

Centralised systems are ones in which information is processed by one central, predetermined unit that makes decisions by taking into account global system information. Centralised systems are relatively simple to design, however these structures face operational limitations on computational resources and memory, and are vulnerable to unit failure of the central component.

In decentralised systems, computational load and decision-making are shared across several but not necessarily all units. Information obtained by the different units can then be combined locally in order to generate a more coherent and whole view of the environment. Because such systems contain several separate and autonomous decision-making entities, they typically contain some redundancy, with multiple agents performing the same or similar tasks. This redundancy endows decentralised systems with robustness to environmental noise and individual unit failure (Kitano, 2004).

Finally, distributed systems are ones in which access to data, computation and decision-making is completely local and asynchronous for each individual unit. Importantly, information is communicated using only local message passing (Coulouris et al., 2005), also referred to as gossiping (Dimakis et al., 2010). Hence, all distributed systems are decentralised, but not all decentralised systems are distributed. Distributed approaches are especially effective when agents are separated in space or have partitioned access to different data sources, making them scalable and secure for engineering purposes (Coulouris et al., 2005; Garg, 2002). Collective behaviour and intelligence in biological systems is almost always distributed, as individuals do not share global information and decisions are made autonomously and asynchronously.

2.1.2 INTERACTIONS FOR SELF-ORGANISATION

Collective intelligence is thought to arise from four basic principles (Moussaid et al., 2009; Garnier et al., 2007). These are (1) repeated interactions, (2) stochasticity, and (3) positive and (4) negative feedback. Firstly, information must be exchanged between agents in the system through repeated interactions. These interactions provide feedback into the system dynamics which is either positive, creating structure, or negative, preventing structure from forming. Finally, there should be an element of randomness in interactions and agent behaviours. This stochasticity gives self-organising systems flexibility and prevents unstable structures from forming.

Interactions between agents can be further classified as attractive or repulsive, promoting homogeneity or heterogeneity, respectively, across individuals in the group. Consensus is the process of combining local estimates made by separate units in order to reach global agreement. From bees (Seeley et al., 2006) to humans (Dyer et al., 2008; Momennejad, 2022), models and experiments have repeatedly demonstrated the ability of groups to effectively represent and respond to their environment using a variety of consensus forming methods. Several models of consensus have been studied (Flache et al., 2017), where system states are represented by discrete and continuous scalar or vector variables (Sîrbu et al., 2017; Dong et al., 2018).

The primary goal of consensus algorithms is to make use of a ‘wisdom of the crowd’ to arrive at a global state estimate based on distributed and noisy local estimates. Agents in distributed systems may have access to the same information source with various degrees of unknown measurement uncertainty. They may also have access to different samples of data drawn from the same distribution, with the goal of estimating the distribution. Consensus can be centralised (passing through a central processor that synchronously combines all estimates) or distributed, whereby individual agents rely purely on local interactions to propagate information through the system. Centralised consensus eliminates estimation errors in individual measurements by producing a weighted sum of individual estimates. The weighting on each term determines the contribution of the individual estimate to the global estimate. Distributed consensus relies on message passing interactions which are attractive, and lead to an eventual convergence of agent states (Dimakis et al., 2010; Flache et al., 2017).

Consensus is also used to perform collective decision making between discrete alternatives, for example in the the best-of- n problem, where a choice must be made between n different alternatives of varying quality. Quorum sensing (Sumpter & Pratt, 2009; Marshall et al., 2019) is employed by natural systems, and various other probabilistic opinion pooling methods addressing this problem have also been developed (Hájek & Hitchcock, 2016).

Consensus amongst agents may not always be desirable, however. Instead, agent differentiation may be required in order to organise behaviour and collective action. These methods make use of repulsive positive reinforcement to differentiate between agents. The positive reinforcement means that existing differences between agents are amplified during interactions. This results in agent states being differentiated across a state-space. For example, insects have been shown to use adaptive response thresholds to flexibly perform allocation of tasks and division of labour (Bonabeau et al., 1998). Agent differentiation for the purpose of self-organisation has also been studied in the context of social hierarchy formation and organisation (Bonabeau et al., 1996), where agent heterogeneity is used for conflict resolution (Brush et al., 2018).

2.2 ARTIFICIAL COLLECTIVE INTELLIGENCE

So far, we have discussed some of the key concepts and principles leading to naturally occurring collective intelligence and self-organisation. The mathematical characterisation of these principles has made it possible to design systems displaying artificial collective intelligence, with applications in multi-agent robotics (Dorigo et al., 2021; Schranz et al., 2020), distributed sensor technology (Zhang & Zhang, 2012), and software algorithm design (Bonabeau et al., 1999; Brabazon et al., 2018; Dorigo et al., 2006; Kennedy & Eberhart, 1995). These artificial collective systems share desirable features of self-organising systems in the natural world, including robustness, adaptation, and higher-order emergent structures. Furthermore, these algorithms allow for a distribution of data, computational resources and memory across different processors in the system.

For example, one use-case of artificial collective intelligence has been to enhance the performance of artificial agents in distributed and embodied AI. These include robotic systems and distributed sensor networks. Consensus and distributed filtering methods are used to combine noisy estimates made by individual sensors (Yu et al., 2009). Distributed filtering is closely related to consensus formation. The goal of a filtering task is to estimate the state of a (possibly dynamical) system in the presence of measurement noise. In distributed filtering, data is partitioned across different sensors or computational units. The data is processed separately by the different units, and individual state estimates are refined through local communication. This data integration is usually implemented using Bayesian methods, as they explicitly take the uncertainty of estimates into account. A physically distributed application can be found in multi-agent Simultaneous Localisation and Mapping (SLAM) (Thrun & Liu, 2005; Cadena et al., 2016; Saedi et al., 2016), in which agents combine Bayesian models of their physical environment. For further information on distributed state estimation, we refer the reader to Ortiz et al. (2021), as well as Battistelli & Chisci (2014; 2016) for probability density estimate fusion using consensus Kalman Filters.

Finally, the principles of swarm intelligence have been used to address a number of different computational and operational tasks (Bonabeau et al., 1999; Dorigo et al., 2021). Self-organised agent differentiation has been applied to problems of task allocation and scheduling in Cicirello & Smith (2004); Castello et al. (2013) by modifying the probability with which agents respond to task stimuli in their environment. More computationally sophisticated swarm intelligence approaches have also

been developed (Garattoni & Birattari, 2018). Swarm algorithms have been used to design efficient computational optimisation algorithms, such as *ant colony optimisation* (Dorigo et al., 2006) and *particle swarm optimisation* (PSO) (Kennedy & Eberhart, 1995), amongst others (Brabazon et al., 2018). These methods utilise distributed computation methods used by natural swarms to converge to optimal solutions in large search spaces. We note that PSO relies on a form of model ensembling, similar to what we believe may occur in NNCs .

2.3 APPLICATIONS TO ARTIFICIAL NEURAL NETWORKS

One of the most popular and powerful paradigms for artificial collective intelligence is now emerging in the field of deep learning. This is the application of artificial neural networks (ANNs) to collective systems. This field is extremely large and developing quickly. As such, we discuss only salient research topics as relevant to NNCs and refer the reader to Ha & Tang (2021) for a comprehensive review.

ANNs take inspiration from one of the most successful examples of collective intelligence, the structure of the brain. The original form of an ANN is known as the multi-layer perceptron (MLP) which is an example of a feed-forward ANN. MLPs typically have three or more hidden layers (where any ANN that includes more than one hidden layer is known as a ‘deep’ ANN). Data is propagated through the hidden layers, and the output from the hidden layers is used to back-propagate the error and update the model by adjusting the weights in each layer. For further information, see Skansi (2018); Chollet (2021).

Over twenty years ago, there was discussion on how MLPs could be made to interact, such as through the application of statistical physics (Metzler et al., 2000; Kinzel et al., 2000; Kanter et al., 2002) or, more recently, in mobile agents (Hou et al., 2009; Wen et al., 2016). ANNs have also been designed that interact at the scale of network architecture, such as generative adversarial networks (GANs). In their simplest form, GANs are composed of two ANNs, one ‘generative’ network which generates new candidate instances of the data and a second ‘discriminative’ network which evaluates them as either arising from the true or generated datasets. The use of a discriminator allows GANs to avoid the task of comparing the generator distribution to the real data distribution, both of which are difficult to estimate from samples (Goodfellow et al., 2014). GANs have been extremely successful at generative tasks, providing some of the earliest examples of ‘deep fakes’. Subsequent research has considered many different variations of the GAN architecture (Creswell et al., 2018).

One extension to the GAN architecture has been multi-GANs, which use multiple generators or discriminators. One of the main motivations for the multi-GAN architecture has been to overcome the mode collapse problem, whereby GANs converge to a single optimum within a multi-modal latent space distribution (Lala et al., 2018). Multi-GANs have been designed to address this by using different partitioned generators that sample different regions of the feature space. For example, in MG-GAN (Hoang et al., 2018), several generators are trained simultaneously to produce a partitioning of the latent space. This enables multiple generators to collectively encode a multi-modal latent space, avoiding the mode collapse problem by introducing a classifier whose goal is to determine which generator a sample came from. The objective function of the generators then aims to beat the discriminator (as in traditional GANs), whilst at the same time maximising the difference between generators by maximising the entropy of the classifier. The use of the classifier in this case avoids the task of inferring the probability distributions of the generators directly from samples, similarly to the role of the discriminator in single GAN models. As we will discuss, GANs and especially multi-GANs, represent early examples of NNCs.

Other research has explored the behaviour of individual ANNs which interact spatially, rather than at architecture level. These include Neural Cellular Automata (NCA), such as in Gilpin (2019) which replace the cells of a cellular automata system with ANNs, multi-agent systems such as in Kajić et al. (2020), and, more recently, systems of many thousands to millions of interacting ANNs such as in Zheng et al. (2018); Suarez et al. (2021). We believe these works are early examples of NNCs.

One means for addressing computational bottlenecks is to distribute the ANNs across computational nodes. These methods can be broadly classified under the header of federated learning (FL) (Li et al., 2020). FL consists of sets of ANNs with identical architecture but different weight updates. A training dataset is distributed across the ANNs, so that each ANN is trained on a subset of the total dataset. The ANNs then update their weights, which are summed and passed to a central ANN. The

amount by which each network contribution is weighted in the sum approximates the certainty of its weight parameter estimates. For example, this could be given by the proportion of data used to train the corresponding network at each epoch, as in Zhu et al. (2021). Research in this area has been motivated by concerns over data privacy and protection and many different approaches to FL have been proposed. For a review of this area, see, for example Li et al. (2020). FL highlights the impact of structure, such as whether the individual ANNs are centralised, decentralised or distributed.

Finally, we note that typical collective systems are inherently relational. Geometry deep learning and graph neural networks (GNNs) are those that are designed to operate over graphical or relation data (Bronstein et al., 2017; Zhou et al., 2020). Recent work has shown that GNNs can be effectively applied to collective systems, including in learning both local and global interaction rules (Battaglia et al., 2018; Cranmer et al., 2020; Grattarola et al., 2021). As our conception of NNCs is inspired by collective systems and, as such, is also relational, one may consider how NNCs differ GNNs. Our conception of NNCs is similar to GNNs, and may even be considered as a form of GNNs, in that they operate over graph-like structures (as with FL). However, the distinction between GNNs and NNCs is that GNNs perform complex computations on graph-structured data whereas NNCs structure their computational components on a graph. For example, a GAN might be considered as a two-node NNC, where one node is the discriminator and one is the generator.

3 NEURAL NETWORK COLLECTIVES

Neural network collectives (NNCs), shown in Figure 1, use collective intelligence to design networks of artificial neural networks (ANNs). The defining components of an NNC are the nodes, edges, and information passed between nodes. We posit that, under appropriate conditions, NNCs will display novel learning structures, as is commonly observed in complex systems. We refer to these learning structures as *emergent intelligence* and discuss potential forms that emergent intelligence may take. We discuss each aspect in more detail in what follows, and outline both future challenges and a prospective roadmap for the implementation of NNCs in Appendix A.

3.1 LAYER 1: ARTIFICIAL NEURAL NETWORKS AS NODES

The first layer of NNCs are the individual ANNs which comprise the nodes of the collective. The nodes of the NNC are extremely complex and inhabit the largest design space. They are composed of individual ANNs, hence inheriting all of the complexity of deep learning, including variations in architecture, learning algorithms and loss functions. As with ANNs, we expect them to be stochastic (such that ANNs with identical input and architecture will result in differing outputs) and nonlinear. They may range from simple MLPs to more complex generative models such as GANs.

As there are multiple nodes across the collective, there are additional design choices that need to be considered such as the degree of similarity or diversity in nodes, what input data is passed to the nodes, and what output data is shared between nodes. For example, all the ANNs may be identical, as in Siamese neural networks (SNNs), where copies of an input are passed to two or more sub-networks that have the same ANN model (Chicco, 2021). These sub-networks are used to perform classification by considering similarities in feature vectors, through the use of a specific loss function (such as a contrastive (Wang & Liu, 2021) or triplet loss (Dong & Shen, 2018)). SNNs, like GANs, can be considered as a form of NNCs where nodes are identical but input data is varied and top-level tasks (such as classification) are performed by a pooling operation.

Alternatively, nodes may comprise of ANNs with differing architectures, as exemplified by GANs, which include either one generator-discriminator pair, or multi-GANs that make use of multiple generators or discriminators. Multi-GANs distribute computational complexity and are typical NNCs in that respect. For example, in Hardy et al. (2019), a multi-GAN model is proposed which consists of one generator and multiple discriminators. Each discriminator has a subset of test data, and the discriminators exchange information amongst themselves. The work in Choi & Han (2021) also explores the use of multiple discriminators to promote diversity amongst discriminators, specialising in different clusters of a dataset by using ‘multiple choice learning’. In this way, a mixture of discriminators can be used to collectively differentiate between different data-sources.

Both SNNs and GANs highlight how similarity and diversity in connected ANNs can change both their learning performance and their suitability for certain tasks. Aspects of consensus and differ-

entiation, as outlined in subsection 2.1, are likely to be critical to NNC design principles. Future research might consider how NNCs perform with a large number of identical NNs, such as in *MAgent* (Zheng et al., 2018), or mixed sub-populations of ANNs optimised for different tasks, as with multi-GANs. We expect that the choice of similarity or diversity in a NNC will be largely task specific except in those cases where NNCs are constructed as objects of study themselves.

3.2 LAYER 2: STRUCTURE OF A COLLECTIVE

3.2.1 ON CONNECTING ARTIFICIAL NEURAL NETWORKS

The structure of the collective can be affected both locally and globally. Locally, one can specify the directionality of nodes, such as whether nodes can both send and receive shared data. Equally, one may consider the respective strengths of the connection, whether there is error in the system, and whether there are open connections between individual nodes and their contextual environment. For example, in NCAs local information is both shared and received by individual ANN cells.

Broadly speaking, the global structure of a collective can be understood within the paradigm of centralised versus distributed approaches, as discussed in subsection 2.1. Most current neural network implementations consider centralised systems where data is aggregated and processed using a central component. An example of a centralised system in which training data is partitioned across different worker sub-networks can be found in Zhu et al. (2021), whereby each MLP worker processes the data independently of others. A central processor then aggregates the network weight estimates and re-distributes the updated weights back to individual workers. The work in Hu et al. (2019) provides a FL example of a decentralised algorithm. Using a segmented gossip method, worker network weights are locally aggregated using several computational nodes. Finally, MD-GAN (Hardy et al., 2019) gives an example of a distributed system, where no centralised processors exist.

The global structure of NNCs give additional design flexibility. For example, one can use many of the traditional graph structures such as fully connected or hierarchical graphs in designing NNCs. Additionally, one may consider structures based on some underlying geometry such as grid-based nearest neighbours as with NCAs (Gilpin, 2019). However, future NCAs may not require pre-determined structures, choosing instead to alter and optimise the links dynamically.

3.2.2 ON COMMUNICATING ARTIFICIAL NEURAL NETWORKS

It is important to consider what information is being passed across the collective. At the most basic level, this could be feature data based on local computations by the nodes. However, higher dimensional data may also be passed, such as sound, text, or image data. More interesting is the possibility that NNCs pass multi-modal data, with specific nodes receiving, transforming, and sending data based on their position within the collective.

It is important to note that errors in the initial receiving and distribution of data, as well as the computation performed by each node, can propagate and amplify as data is shared across the system. One way to address this is to account for error and uncertainty in the passed data. This could be achieved by, for example, quantifying uncertainty such that nodes with higher certainty on output are given more weight. Alternatively, individual nodes could pass a full probability density that describes their respective posterior distribution over their received input or output as a form of Bayesian NNCs.

We have so far been describing traditional communication content between nodes in a graph, such as features, images, text, and even densities. However, one of the benefits of NNCs is that individual nodes also have access to their own properties. As a result, NNCs could be designed such that individual nodes share their own weights. This would mean that NNCs are passing ANNs between nodes, as has been proposed in, for example, methods of federated learning (Zhu et al., 2021).

3.3 LAYER 3: EMERGENT INTELLIGENCE

Above, we gave our initial perspectives on how NNCs can be designed with respect to the nodes and edges of the collective. However, the central benefit of using a NNC is the collective intelligence gained by ensembling multiple varied models, leveraging the wisdom of artificial crowds. Below, we discuss why we believe emergent intelligence is possible in the context of NNCs, and what form it might take.

In order to best make use of the artificial crowd, careful choices should be made about how information passed between nodes is ensembled. This might be a simple aggregator (as in federated learning), or more complex operations such as the message passing protocol used in GNNs (Battaglia et al., 2018). Alternatively, methods from collective intelligence such as voting-based consensus algorithms might be applied (Dimakis et al., 2010). Most intriguing of all is the possibility that, under certain conditions, a NNC will ensemble models in such a way that the collective will demonstrate higher-order learning capabilities. These capabilities would be more successful than that of the individual nodes but also potentially different in other meaningful ways. We believe this is possible for two reasons.

First, recent analysis on neural networks have highlighted the existence of circuits of networks that develop task-specific expertise. For example, CNNs contain emergent receptive fields that are sensitive to sub-features of image data (Luo et al., 2016; Cammarata et al., 2020). Secondly, we note the many aforementioned examples of complex systems which display emergent computational abilities, along with Ramos-Fernandez et al. (2020) and references therein. Hence, we believe that computation scales and that intelligence structures will emerge from scaled computation.

If emergent intelligence is observed in NNCs, we expect it to feedback into the lower units of the collective. This form of top-down feedback has been identified as a potential mechanism in self-organisation through collective intelligence (Biro et al., 2016; Flack, 2017; Diaconescu et al., 2019; 2021), and may end up being a form of reinforcement which strengthens the emergent intelligence or maintains the critical conditions under which it emerges. Moreover, the high-order intelligence may allow for different forms of learning, whereby the collective learns how to learn. This type of meta-learning could, for example, be formed by certain regions of the collective being used to retain information, as has recently been observed in transformers (Elhage et al., 2021). Similarly, methods could be applied such that modules explicitly emerge that reflect certain task-specific specialisations. This has been explored in, for example, Alet et al. (2018); Kirsch et al. (2018); Rosa et al. (2019). Alternatively, entirely new forms of learning and intelligence may be discovered, possibly through the application of meta-heuristic search algorithms such as evolutionary or genetic algorithms (Stanley & Miikkulainen, 2002).

4 CONCLUSION

We believe that NNCs could offer a rich and exciting new field of study from the perspectives of both deep learning and collective intelligence. We expect that designing these collectives will drive new research paradigms and lead to new applications of deep learning-based systems. Central to the design of NNCs is the question of how the principles of collective intelligence can be used to enhance the performance and range of applications of existing artificial neural network architectures. One immediate possibility is the development of algorithms that make use of distributed data processing, where data to be used for training is distributed across separate workers. As well as reducing the computational load required of each worker, these approaches offer enhanced user privacy and data security. Secondly, it may be possible for newly developed NNC algorithms to make use of emergent computational properties to overcome challenges experienced by single computational units, as in the example of multi-GANs. Finally, a speculative yet exciting opportunity is to implement meta-learning and self-tuning in NNCs by using the capacity of natural complex systems to adapt their behaviour to different unseen learning tasks as is expected for artificial general intelligence.

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A FUTURE RESEARCH

A.1 CHALLENGES FOR FUTURE RESEARCH

There are several challenges to be overcome in order to effectively design and implement complex NNCs. From the complex system engineering perspective, the main challenge is that of the top-down micro-macro design problem. Namely, the difficulty of designing lower level component interaction rules in order to achieve a desired globally emergent property or behaviour. In concrete terms, this is the question of how the nodes (L1) and interactions (L2) should be defined such that a *specifically desired* emergent feature (L3) is achieved. This is made difficult by the large design spaces at both L1 and L2.

At the same time, there are significant challenges in the computational requirements for designing NNCs. Certain ANNs, especially those achieving state-of-the-art performance, require training across large computer clusters and dedicated hardware. Hence, creating networks of these ANNs would be conditional on the degree of availability of computational resources. However, NNCs may also provide an alternative to large models by distributing training across nodes asynchronously.

Finally, we expect there to be challenges in observing and quantifying emergent intelligence structures (L3). We believe that a combination of traditional statistical physics methods for understanding emergent order, such as those outlined in Sethna (2021); Castellano et al. (2009), as well as recent empirical methods for understanding emergent structures in ANNs, for example Elhage et al. (2021) are expected to be critical.

A.2 PROSPECTIVE ROADMAP

We have introduced the concept of neural network collectives (NNCs) and outlined important aspects necessary for designing them. In order to focus future research and further elucidate our vision for NNCs, we outline here a potential roadmap for their implementation. We note that our views here are prospective and additional work is required to give sufficient detail for future obstacles and alternate modes of design.

We consider the set of possible NNC models according to their complexity along two axes, illustrated in Figure 2. These are the artificial neural network (ANN) model complexity, or *L1-complexity*, and the collective intelligence model complexity, or the *L2-complexity*. Specifically, the computational (ANN) complexity refers to the complexity of individual *nodes* defined in the first layer L1 of the NNC (see Figure 1). On the other hand, the collective complexity refers to the interaction structures defined at the second layer L2 of the NNC.

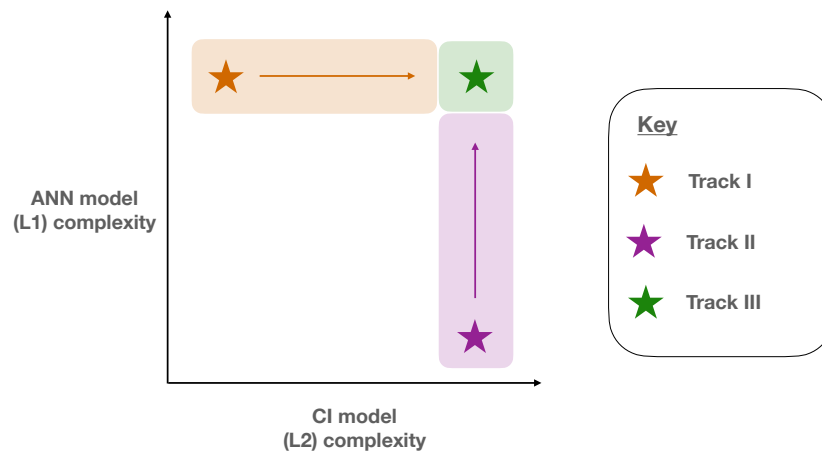


Figure 2: Illustration of the problem complexity space for the three tracks described. The vertical and horizontal axes respectively represent the model complexity for the Artificial Neural Network (L1) and Collective Intelligence (L2) models.

Hence, the NNC research road-map consists of three tracks, defined according to their location within the $L1$ - $L2$ complexity space, as shown in Figure 2. We propose that the first two tracks should be investigated in parallel. Track I begins with complex nodes ($L1$) and simple interaction structures ($L2$) and gradually increases the complexity of interactions. On the other hand, track II begins with simple nodes ($L1$) and complex interactions ($L2$), over time increasing the node computational complexity. Both tracks converge towards a maximally complex $L1$ - $L2$ design paradigm in track III. Below, we provide more detail on what research in each of the tracks might entail.

A.3 TRACK I: HIGH $L1$ / LOW $L2$ COMPLEXITY

In order to better understand the interactions of components with high $L1$ -complexity, we propose developing systems of GANs which interact during training time. These are similar to multi-GANs (see subsection 3.1) which have been shown to overcome specific limitations of single units, such as mode collapse. However, we note two extensions to multi-GANs of increasing difficulty. The first outlines interactions with respect to the networked structure of individual generative-discriminator networks while the second focuses on the interactions themselves, within the context of cost functions.

First, we suggest a reformulation of multi-GANs such that they exist on a networked structure in $L2$. For example, GANs could be connected in series (similar to recurrent units in a recurrent neural network) or in parallel, as proposed in the original multi-GAN formalism. Alternatively, more complicated network structures could be applied such as a small-world network (whereby the shortest-path distance between nodes increases with respect to the number of nodes in the network) or a fully connected network. We propose that different network structures are tested on benchmark tasks (ranging from classification to synthetic data generation) but where the underlying architecture of GAN units (i.e. the generator and discriminator networks) is kept fixed. This study would allow for a full exposition of whether networked superstructures can improve GAN performance.

Second, we suggest that the cost function used in multi-GANs is adapted to include other features of interacting collective systems. These include, for example, consensus formation and task allocation (Dimakis et al., 2010; Castello et al., 2013). We expect the reformulation of these concepts to be a significant challenge but note that, given an appropriate formulation, we can apply methods from collective intelligence to quantify the dynamics of collective interactions (Sethna, 2021; Castellano et al., 2009).

Ultimately, both extensions focus on increasing the $L2$ -complexity of multi-GAN methods. The combination of both is expected to lead to the largest computational gains both in terms of efficiency and performance. These should initially be implemented in isolation to better understand the design trade-offs. Finally, we note that we have focused entirely here on GANs. However, other ANNs that are composed of interacting sub-units should also be explored and may be better suited for the design of NNCs in this track. These include graph-based neural networks (GNNs) or ANNs that perform contrastive learning such as Siamese networks.

A.4 TRACK II: LOW $L1$ / HIGH $L2$ COMPLEXITY

The second track studies models with low $L1$ -complexity and high $L2$ -complexity. As a starting point, we may consider multi-agent systems as traditionally explored within the field of collective intelligence and self-organisation. However, multi-agent systems have typically involved relatively simple agents that interact according to predefined interaction rules or functions. Instead, we propose multi-agent systems that make use of ANNs to determine the type of interactions between agents. These have already been explored in, for example, Liu et al. (2019); Zheng et al. (2018); Suarez et al. (2021).

By introducing ANNs as individual components within multi-agent systems, it is possible to make use of methodologies from statistical physics to quantify macroscopic or ensembled behaviours (given the number of agents in the system is sufficiently large). This might include, for example, exploring phases of matter in a simple particle-based model in which individual particles are replaced by multi-layered perceptrons (MLPs) to compute the particle velocity over time. Given jamming transitions are thought to potentially play a role in the learning dynamics of ANNs (Geiger et al., 2019), it is interesting to consider what happens to the dynamics of learned parameters in systems

of MLPs as they undergo system phase transitions. Ultimately, we expect that combining ANNs with existing multi-agent models will lead to systems of interacting neural networks which interact to update their network weight parameters during training, as opposed to during run-time as in Charlesworth & Turner (2019).

A.5 TRACK III: HIGH L1 / HIGH L2 COMPLEXITY

We have outlined two tracks for developing NNCs. The first track builds on the GAN architecture and proposes to increase the collective model complexity. This should be achieved either by adapting the parameter communication network between generators and discriminators, or by adapting the underlying cost function used during training time to contain explicit notions of, e.g. consensus or differentiation between different nodes.

The second track starts from systems with high collective complexity and seeks to increase the computational model complexity by replacing traditional agents in multi-agent models with ANNs. Further work along this track might also consider other multi-agent simulators with known collective emergence properties, and consider how the introduction of more complicated computational agents alters the emergent features within the system.

Ultimately, we propose that both tracks are combined such that complex ANN architectures (such as GANs, GNNs, Siamese networks) are introduced into an environment such that individual networks interact with hundreds to thousands of other networks during training time. It can be expected that the computational cost for such a system will be high. In order to address this, we propose the development of a distributed crowd-sourced experiment, where users run high-level computational nodes locally on their devices.

Given ample developments in Tracks I and II, we expect such a system to scale efficiently, leading to a system of both high computational and collective complexity. Such a system is similar to traditional federated learning environments, which distributes computational load and data across ANNs to enhance user privacy. Hence, a public NNC to specifically explore emergent learning structures could be effectively designed to rigidly adhere to privacy concerns. However, several open questions remain with regards to this track. For example, it is uncertain what the appropriate tasks would be for this public NNC, how data and computation are distributed, and how information sharing is constructed between user nodes. Additionally, it is not immediately clear what forms of ANN architecture should be explored, implemented, and optimised. Finally, and perhaps most importantly, it is unclear how to effectively quantify emergent learning structures, as described in subsection 3.3. We believe that progress in developing tracks I & II will provide tools for answering these and many other questions.