Scaling Textual Gradients via Sampling-Based Momentum

Anonymous Authors[1]

Abstract

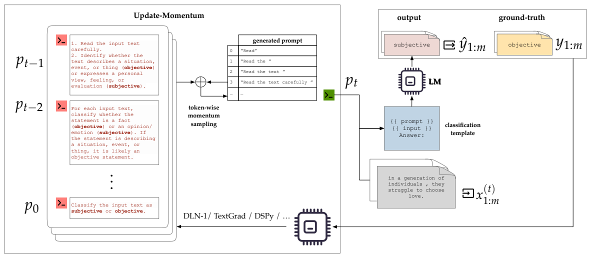
As prompts become central to Large Language Models (LLMs), optimizing them is vital. Textual Stochastic Gradient Descent (TSGD) offers a data-driven approach by iteratively refining prompts using LLM-suggested updates over minibatches. We empirically show that increasing training data initially improves but can later degrade TSGD's performance across NLP tasks, while also raising computational costs. To address this, we propose Textual Stochastic Gradient Descent with Momentum (TSGD-M)—a scalable method that reweights prompt sampling based on past batches. Evaluated on 9 NLP tasks across three domains, TSGD-M outperforms TSGD baselines for most tasks and reduces performance variance.

1. Introduction

Large Language Models (LLMs) are sensitive to prompt design, where minor variations can lead to substantial performance shifts (Ying et al., 2024; Zhou et al., 2022; Sclar et al.; Anagnostidis & Bulian, 2024). Automatic Prompt Engineering (APE) exploits this sensitivity to optimize prompts using LLM feedback. Recent works (Khattab et al., 2024; Sordoni et al., 2023; Wang et al.) propose **Textual Gradient Descent** (TGD), an LLM-driven optimization framework analogous to numerical gradient descent, where prompts are iteratively updated using textual feedback.

In practice, however, context length limitations (Liu et al., 2024; Peng et al., 2023) and the cost of long in-context demonstrations (Hooper et al., 2024; Tang et al., 2024) necessitate stochastic sampling during training. As such, we argue that existing methods more accurately resemble **Textual Stochastic Gradient Descent** (TSGD), which generates textual gradients based on minibatches of examples per iteration—much like SGD in traditional optimization.

This raises fundamental questions: Do LLMs optimize like



1. Read the input text carefully.
2. Identify whether the text describes a situation, event, or thing (**objective**) or expresses a personal view, feeling, or evaluation (**subjective**).

For each input text, classify whether the statement is a fact (**objective**) or an opinion/ emotion (**subjective**). If the statement is describing a situation, event, or thing, it is likely an objective statement.

Classify the input text as **subjective** or **objective**.

	generated prompt
0	"Read"
1	"Read the "
2	"Read the text "
3	"Read the text carefully "
...	...

```
{{ prompt }}
{{ input }}
Answer:
```

in a generation of individuals , they struggle to choose love.

Figure 1. TSGD-M for Subj dataset (Pang & Lee, 2004)

deep neural networks (DNNs)? Unlike DNNs, which benefit from full-batch gradients, LLMs often misestimate aggregate errors across long contexts (Anagnostidis & Bulian, 2024). Thus, simply scaling the number of demonstrations may not guarantee stable performance. We revisit data scaling in TGD/TSGD and assess whether more advanced optimization techniques can help.

We propose **TSGD with Momentum (TSGD-M)**, which draws from momentum-based optimization in deep learning (Rumelhart et al., 1986; Polyak, 1964; Liu et al., 2020). TSGD-M reduces variance by reweighting and sampling from past minibatches at the token level, leading to smoother textual gradients. This approach enables improved training dynamics, particularly under large-scale data.

Across 9 NLP tasks, we find that moderate data scaling improves TGD framework performance (e.g., DSPy (Khattab et al., 2024), DLN1 (Sordoni et al., 2023), and TextGrad (Yuksekgonul et al., 2024)), but aggressive scaling leads to diminishing or negative returns. TSGD-M mitigates this by enabling stable optimization with less variance.

Our contributions:

- We revisit the role of data scaling in TGD/TSGD and highlight its non-monotonic impact on performance.
- We introduce TSGD-M, the first momentum-based method for textual gradient descent, reducing variance in large-scale settings.
- We provide theoretical support (Appendix D) and conduct extensive experiments on BBH, NLU, and reasoning tasks to validate the effectiveness of TSGD-M.

[1]Anonymous Institution, Anonymous City, Anonymous Region, Anonymous Country. Correspondence to: Anonymous Author <anon.email@domain.com>.

Preliminary work. Under review by the International Conference on Machine Learning (ICML). Do not distribute.

2. Preliminaries and Related Work

Problem Setup. Define large language model (LLM) as $\mathbf{LM} : \mathcal{V}^* \to \mathcal{V}^*$, where $\mathcal{V}$ is the vocabulary and $\mathcal{V}^*$ the space of token sequences. Given a prompt $p \in \mathcal{V}^*$ and input $x \in \mathcal{V}^*$, the model outputs $\mathbf{LM}([p, x])$. For a dataset $D = \{(x_i, y_i)\}_{i=1}^N \sim \mathcal{D}$, the goal of prompt engineering is to find p^* that maximizes expected performance:

$$p^* = \arg\max_{p \in \mathcal{V}^*} \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\mathtt{Perf}(\mathbf{LM}([p, x]), y)\right], \quad (1)$$

where `Perf` evaluates output quality.

Textual Gradient Descent (TGD). Manual prompt design is costly and non-scalable. Recent automatic prompt engineering (APE) methods use LLMs to iteratively refine prompts based on observed errors (Sordoni et al., 2023; Yang et al., 2024). More interpretable variants elicit LLMs to summarize prediction errors and apply structured edits to prompts (Pryzant et al., 2023; Yuksekgonul et al., 2024; Ning et al., 2024), forming the basis of the **Textual Gradient Descent (TGD)** framework. However, since these methods sample training examples per iteration, we argue they resemble **Textual Stochastic Gradient Descent (TSGD)**.

At iteration t, a prompt p_t is updated using a minibatch $\{(x_i, y_i)\}_{i=1}^m$ with predictions $\widehat{y}_i = \mathbf{LM}([p_t, x_i])$, which follows

$$p_{t+1} = \mathtt{Update}(\mathbf{LM}, p_t, \{(x_i, y_i, \widehat{y}_i)\}_{i=1}^m),$$

where `Update` analyzes model errors and generates an improved prompt p_{t+1}.

TGD as Two-Stage LLM Calls. TGD typically decomposes `Update` into two LLM queries: (1) error summarization:

$$\nabla p_t = \mathbf{LM}([p_{\text{analyze}}, p_t, \{(x_i, y_i, \widehat{y}_i)\}_{i=1}^m]),$$

and (2) prompt refinement:

$$p_{t+1} = \mathbf{LM}([p_{\text{refine}}, p_t, \nabla p_t]).$$

Here, ∇p_t is a "textual gradient" capturing failure patterns, and $p_{\text{analyze}}, p_{\text{refine}}$ are prompt templates guiding these stages. This structure mirrors gradient descent: compute a direction, then step toward improvement.

For further background and method variations (e.g., using different LLMs for `Update` stages or fusing prompts), we refer readers for related work on Appendix A.

3. Test-time Scaling for TGD and TSGD

Motivation. Prompt optimization frameworks like TGD use full-batch training, while TSGD uses stochastic minibatches to compute "textual gradients." This raises the question: How does scaling full training corpus or batch size affect performance in iterative prompt optimization?

Dataset	Max Tokens (25 ex.)	Avg Tokens / ex.
Subj	∼700	∼28
TREC	∼325	∼13
GSM8K	∼2000	∼80

Table 1. Token statistics across datasets with 25 randomly sampled training examples.

3.1. Scaling Training Data in TGD

We evaluate TextGrad (Yuksekgonul et al., 2024), DLN1 (Sordoni et al., 2023), and DSPy-COPRO (Khattab et al., 2024) on three datasets (Subj (Pang & Lee, 2004), TREC (Lu et al., 2022), GSM8K(Cobbe et al., 2021)) with full-batch TGD using GPT-4o (for TextGrad) or LLaMA3-8B (for DLN1 and DSPy-COPRO), following original setups. We vary the training size $N \in \{3, 5, 10, 20, 25\}$ and observe performance trends (Figure 2 (a), (c), (e)).

Findings. All methods exhibit a performance peak before degradation as N increases. The optimal size varies by task and framework (e.g., TextGrad peaks earlier than DLN1). We hypothesize that degradation arises from reasoning limitations of LLMs over long contexts, consistent with (Levy et al., 2024). We argue that we limit the maximum N to 25 because beyond this point performance declines, and in most real-world tasks—like most tasks in BBH (Suzgun et al., 2022)—golden data are scarce. Moreover, (Levy et al., 2024) systematically evaluated how input length affects reasoning across various models and observed a marked performance drop beyond roughly 500 tokens.

3.2. Scaling Batch Size in TSGD

We fix N and vary minibatch size $m \in \{3, 5, 10, 20, 25\}$ per iteration. TSGD uses each minibatch to estimate gradients. Results across frameworks are shown in Figure 2 (b) (d) (f).

Findings. TextGrad shows high variance and sharp peaks (e.g., TREC maxes at $m = 10$ then drops), whereas DLN1 and DSPy-COPRO are more stable. GSM8K and Subj show consistent improvement up to $m = 20$ before plateauing. These results suggest dataset-specific sensitivity to batch size—no "universal" sweet spot exists.

Conclusion. While moderate scaling improves prompt learning, performance degrades beyond a task-specific threshold. This motivates our Textual Stochastic Gradient Descent-Momentum **(TSGD-M)** in Section 4, stabilizing optimization with adaptive historical reuse of all past meta-prompts (i.e. the prompt selected per iteration).

4. Textual Stochastic Gradient Descent with Momentum (TSGD-M)

We previously observed that simply scaling training data or batch size in TGD and TSGD does not always yield stable improvements due to noisy gradient estimates and the

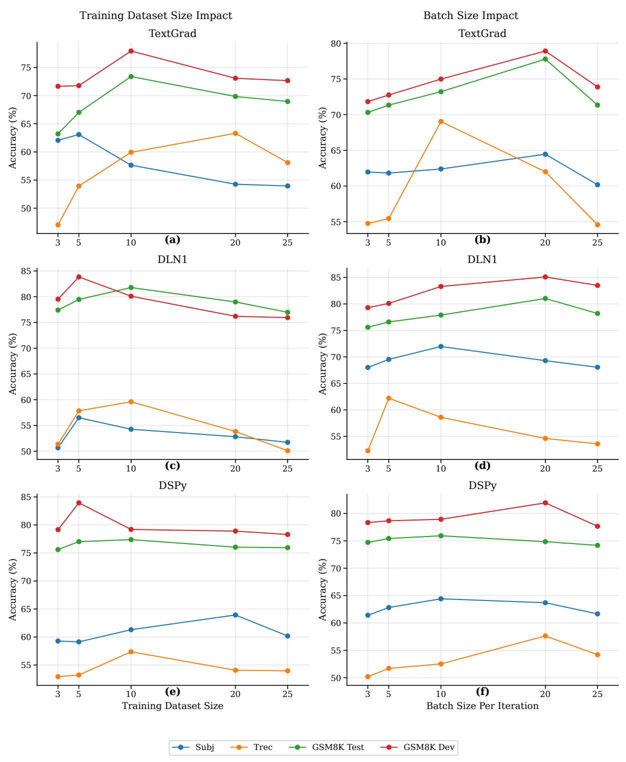


Figure 2. Test performance trends for: (a) TextGrad (full dataset scaling), (c) DLN1 (full dataset scaling), (e) DSPy-COPRO (full dataset scaling). (b) TextGrad (batch size scaling); (d) DLN1 (batch size scaling); (f) DSPy-COPRO (batch size scaling). Dataset size N and batch size m are varied from 3 to 25 examples.

context length limitations of LLMs. Motivated by classical momentum methods in neural optimization (Qian, 1999; Liu et al., 2020), we propose **TSGD with Momentum (TSGD-M)** to stabilize prompt updates and reduce variance. Full Algorithm (TSGD-M) pseudocode is listed in Appendix B.

Key Idea. TSGD-M maintains a buffer of past meta-prompts $\{p_\tau\}_{\tau=0}^{t}$ and uses a momentum-based sampling mechanism to generate the next prompt p_{t+1}. Each new token is generated by sampling from past prompts weighted by an exponential moving average:

$$\mathbb{P}(\text{Token}_{i+1} \mid \text{Token}_{1:i}, \{p_\tau\}_{\tau=0}^{t}) \propto \sum_{\tau=0}^{t} w_\tau \mathbb{P}(\text{Token}_{i+1} \mid \text{Token}_{1:i}, p_\tau) \tag{2}$$

where Token_i denotes the token at position i, and $w_\tau = \frac{\alpha^{t-\tau}}{\sum_{\tau=0}^{t} \alpha^{t-\tau}}$ is the weight on prompt generated at iteration τ. This process adaptively blends information from past prompts, akin to momentum accumulation in SGD:

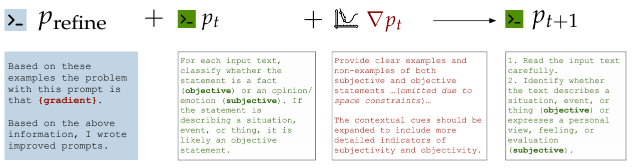
$$m^t = \alpha m^{t-1} + (1-\alpha)\tilde{\nabla} p_t, \quad p_{t+1} = p_t - m^t.$$

Prompt Update Algorithm. We show the core loop of TSGD-M in Algorithm 1 in Appendix B, which supports both momentum and vanilla updates.

We consider two settings for applying momentum:

- **Case 1: One-step Prompt Refinement** (Sordoni et al., 2023; Khattab et al., 2024) — the updated prompt is generated directly from p_t via sampling (Figure 5 in Appendix C).
- **Case 2: Two-stage Gradient Refinement** (Yuksekgonul et al., 2024) — textual gradients ∇p_t are generated first, then used to refine p_t (Figure 3).

In both cases, we apply momentum sampling token-wise from the prompt or prompt-gradient pairs.



```
Based on these
examples the problem
with this prompt is
that {gradient}.

Based on the above
information, I wrote
improved prompts.
```

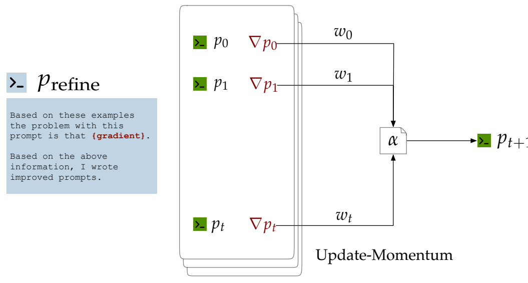
```
For each input text,
classify whether the
statement is a fact
(objective) or an opinion/
emotion (subjective). If
the statement is
describing a situation,
event, or thing, it is
likely an objective
statement.
```

```
Provide clear examples and
non-examples of both
subjective and objective
statements …(omitted due to
space constraints)…

The contextual cues should be
expanded to include more
detailed indicators of
subjectivity and objectivity.
```

```
1. Read the input text
carefully.
2. Identify whether
the text describes a
situation, event, or
thing (objective) or
expresses a personal
view, feeling, or
evaluation
(subjective).
```

(a) Standard TSGD update



```
Based on these examples
the problem with this
prompt is that {gradient}.

Based on the above
information, I wrote
improved prompts.
```

(b) TSGD-M: token-wise momentum sampling

Figure 3. Comparison of TSGD and TSGD-M update mechanisms.

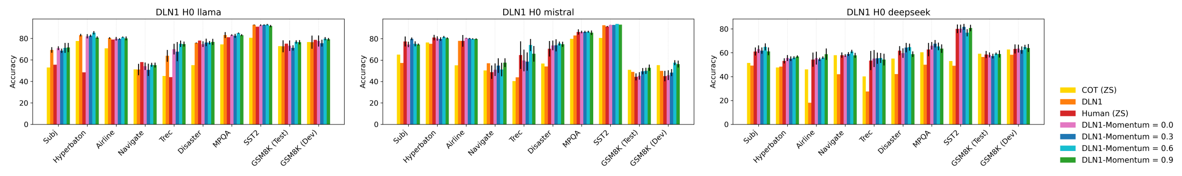
Comparison with Momentum (Yuksekgonul et al., 2024). Yuksekgonul et al. (2024) define momentum as simple prompt concatenations. Our method instead uses exponential decay weights to sample from past prompts per token, enabling more memory-efficient and dynamic prompt evolution. See Appendix F.2 for ablation study on our method compared to momentum version in Textgrad.

5. Experiments

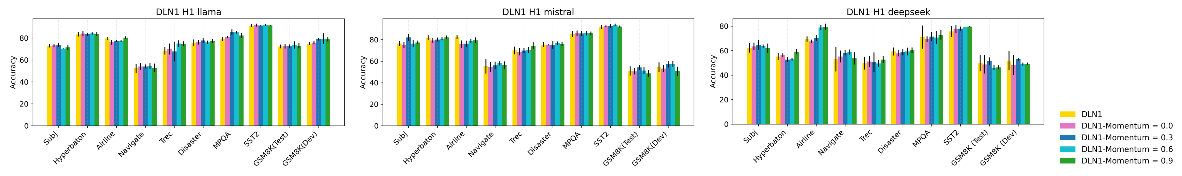
We compare TSGD-M against TSGD across reasoning, BBH, and NLU tasks. TSGD-M consistently improves task accuracy across model scales. All experiments reported in this section is mean with standard deviation over 10 runs. Full details on the tasks used are given in Appendix E.1.

5.1. Setup

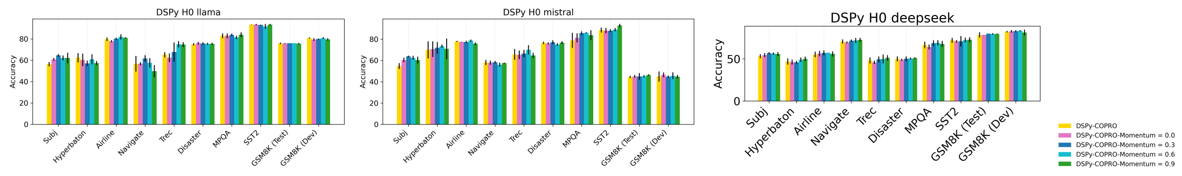
Tasks: We evaluate on BBH (Navigate, Hyperbaton), NLU (Mpqa, Trec, Subj, Disaster, Airline), and GSM8K for math reasoning. For each NLU dataset, we use 400/250/250 splits. GSM8K uses a 200/300/1319 split as (Yuksekgonul et al., 2024; Khattab et al., 2024).



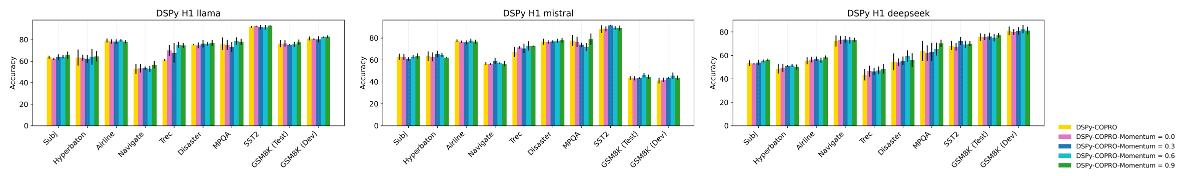
(a) DLN1, H_0, Llama3-8B **(b)** DLN1, H_0, Mistral-7B **(c)** DLN1, H_0, Deepseek 1.5B



(d) DLN1, H_1, Llama3-8B **(e)** DLN1, H_1, Mistral-7B **(f)** DLN1, H_1, Deepseek 1.5B



(g) DSPy-COPRO, H_0, Llama3-8B **(h)** DSPy-COPRO, H_0, Mistral-7B **(i)** DSPy-COPRO, H_0, Deepseek 1.5B



(j) DSPy-COPRO, H_1, Llama3-8B **(k)** DSPy-COPRO, H_1, Mistral-7B **(l)** DSPy-COPRO, H_1, Deepseek 1.5B

Figure 4. Performance of TSGD-M applied to DLN1 (Sordoni et al., 2023), DSPy-COPRO (Khattab et al., 2024) under H_0 and H_1.

Baselines: We compare TSGD-M to TextGrad (Yuksekgonul et al., 2024), DSPy-COPRO(Khattab et al., 2024), and DLN1(Sordoni et al., 2023). Human (ZS) and CoT (ZS) serve as zero-shot references for initial prompts (Appendix E.2) and Chain-of-thoughts (Kojima et al., 2022).

Models: We test Llama3-8B (Grattafiori et al., 2024), Mistral-7B (Jiang et al., 2023), and DeepSeek-R1-Distill-Qwen-1.5B (denoted as Deepseek 1.5B) (Guo et al., 2025). For TextGrad, we follow original settings with GPT-4o (gradient generation) and GPT-3.5-Turbo (inference).

Hypotheses We test two hypotheses for TSGD-M against the baselines described above and use them as an ablation study to demonstrate that TSGD-M remains robust across different prompt-generation temperatures and total iterations of prompt refinements.

- H_0 follows the same settings as Sordoni et al. (2023) with a generation temperature of 0.7 and early stopping after 2 iterations no improvement for holdout accuracy.
- H_1 raises the generation temperature to 1.1 and allows more iterations with early stopping after 5 iterations for no improvement on holdout accuracy, hypothesizing that increased stochasticity and longer optimization will produce more diverse prompts and presumably better performance (Yu et al., 2023; Wang et al., 2023; Renze, 2024).

Results: Figure 4 presents all results and all the original statistics is presented in Appendix F.1. H_0**:** Moderate momentum ($\alpha = 0.6$) consistently improves accuracy. Llama3-8B achieves top results on 8/10 tasks (e.g., Hyperbaton 85.33% vs. 83.07%, SST2 92.87% vs. 92.67%). Mistral-7B and Deepseek-1.5B also perform best at $\alpha = 0.6$, with Deepseek showing the largest relative gains (e.g., Subj 65.02% vs. 61.07%). This suggests TSGD-M is particularly effective for smaller models. Higher momentum ($\alpha = 0.9$) often degrades performance (e.g., Navigate: 61% to 57.80%). H_1**:** Under higher temperature and longer refinement (up to 5 iterations), all models benefit most from $\alpha = 0.3$–0.6. Llama3-8B and Mistral-7B peak at $\alpha = 0.6$, while Deepseek-1.5B performs best at $\alpha = 0.9$, likely due to higher stochasticity tolerance. Sample prompts generated by DLN1 and DLN1-α=0.6 are provided in Appendix F.4.

References

Amin, K., Bie, A., Kong, W., Kurakin, A., Ponomareva, N., Syed, U., Terzis, A., and Vassilvitskii, S. Private prediction for large-scale synthetic text generation. In *Findings of the Association for Computational Linguistics: EMNLP 2024*, pp. 7244–7262, 2024.

Anagnostidis, S. and Bulian, J. How susceptible are llms to influence in prompts? In *COLM*, 2024.

Cobbe, K., Kosaraju, V., Bavarian, M., Chen, M., Jun, H., Kaiser, L., Plappert, M., Tworek, J., Hilton, J., Nakano, R., et al. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.

Cui, A., Nandyalam, P., Cheung, E., and Zhu, K. Introducing mapo: Momentum-aided gradient descent prompt optimization. *arXiv preprint arXiv:2410.19499*, 2024.

DSPy. Optimization / optimizers. https://dspy.ai/learn/optimization/optimizers/, 2025. Accessed: 2025-05-14.

Grattafiori, A., Dubey, A., Jauhri, A., Pandey, A., Kadian, A., Al-Dahle, A., Letman, A., Mathur, A., Schelten, A., Vaughan, A., et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024.

Guo, D., Yang, D., Zhang, H., Song, J., Zhang, R., Xu, R., Zhu, Q., Ma, S., Wang, P., Bi, X., et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025.

Hooper, C., Kim, S., Mohammadzadeh, H., Maheswaran, M., Paik, J., Mahoney, M. W., Keutzer, K., and Gholami, A. Squeezed attention: Accelerating long context length llm inference. *arXiv preprint arXiv:2411.09688*, 2024.

Jiang, A. Q., Sablayrolles, A., Mensch, A., Bamford, C., Chaplot, D. S., de las Casas, D., Bressand, F., Lengyel, G., Lample, G., Saulnier, L., Lavaud, L. R., Lachaux, M.-A., Stock, P., Scao, T. L., Lavril, T., Wang, T., Lacroix, T., and Sayed, W. E. Mistral 7b, 2023. URL https://arxiv.org/abs/2310.06825.

Khattab, O., Singhvi, A., Maheshwari, P., Zhang, Z., Santhanam, K., Haq, S., Sharma, A., Joshi, T. T., Moazam, H., Miller, H., et al. Dspy: Compiling declarative language model calls into state-of-the-art pipelines. In *The Twelfth International Conference on Learning Representations*, 2024.

Kojima, T., Gu, S. S., Reid, M., Matsuo, Y., and Iwasawa, Y. Large language models are zero-shot reasoners. *Advances in neural information processing systems*, 35: 22199–22213, 2022.

Levy, M., Jacoby, A., and Goldberg, Y. Same task, more tokens: the impact of input length on the reasoning performance of large language models. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 15339–15353, 2024.

Liu, N. F., Lin, K., Hewitt, J., Paranjape, A., Bevilacqua, M., Petroni, F., and Liang, P. Lost in the middle: How language models use long contexts. *Transactions of the Association for Computational Linguistics*, 12:157–173, 2024.

Liu, Y., Gao, Y., and Yin, W. An improved analysis of stochastic gradient descent with momentum. *Advances in Neural Information Processing Systems*, 33:18261–18271, 2020.

Lu, Y., Bartolo, M., Moore, A., Riedel, S., and Stenetorp, P. Fantastically ordered prompts and where to find them: Overcoming few-shot prompt order sensitivity. In *Proceedings of the 60th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 8086–8098, 2022.

Ning, X., Wang, Z., Li, S., Lin, Z., Yao, P., Fu, T., Blaschko, M., Dai, G., Yang, H., and Wang, Y. Can llms learn by teaching for better reasoning? a preliminary study. *Advances in Neural Information Processing Systems*, 37: 71188–71239, 2024.

Opsahl-Ong, K., Ryan, M., Purtell, J., Broman, D., Potts, C., Zaharia, M., and Khattab, O. Optimizing instructions and demonstrations for multi-stage language model programs. In *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 9340–9366, 2024.

Pang, B. and Lee, L. A sentimental education: Sentiment analysis using subjectivity summarization based on minimum cuts. In *Proceedings of the 42nd Annual Meeting of the Association for Computational Linguistics (ACL-04)*, pp. 271–278, Barcelona, Spain, July 2004. doi: 10.3115/1218955.1218990. URL https://aclanthology.org/P04-1035/.

Peng, H., Wang, X., Chen, J., Li, W., Qi, Y., Wang, Z., Wu, Z., Zeng, K., Xu, B., Hou, L., et al. When does in-context learning fall short and why? a study on specification-heavy tasks. *arXiv preprint arXiv:2311.08993*, 2023.

Polyak, B. T. Some methods of speeding up the convergence of iteration methods. *Ussr computational mathematics and mathematical physics*, 4(5):1–17, 1964.

Pryzant, R., Iter, D., Li, J., Lee, Y. T., Zhu, C., and Zeng, M. Automatic prompt optimization with" gradient descent" and beam search. *arXiv preprint arXiv:2305.03495*, 2023.

Qian, N. On the momentum term in gradient descent learning algorithms. *Neural networks*, 12(1):145–151, 1999.

Renze, M. The effect of sampling temperature on problem solving in large language models. In *Findings of the Association for Computational Linguistics: EMNLP 2024*, pp. 7346–7356, 2024.

Rumelhart, D. E., Hinton, G. E., and Williams, R. J. Learning representations by back-propagating errors. *nature*, 323(6088):533–536, 1986.

Sclar, M., Choi, Y., Tsvetkov, Y., and Suhr, A. Quantifying language models' sensitivity to spurious features in prompt design or: How i learned to start worrying about prompt formatting. In *The Twelfth International Conference on Learning Representations*.

Sordoni, A., Yuan, E., Côté, M.-A., Pereira, M., Trischler, A., Xiao, Z., Hosseini, A., Niedtner, F., and Le Roux, N. Joint prompt optimization of stacked llms using variational inference. *Advances in Neural Information Processing Systems*, 36:58128–58151, 2023.

Suzgun, M., Scales, N., Schärli, N., Gehrmann, S., Tay, Y., Chung, H. W., Chowdhery, A., Le, Q. V., Chi, E. H., Zhou, D., , and Wei, J. Challenging big-bench tasks and whether chain-of-thought can solve them. *arXiv preprint arXiv:2210.09261*, 2022.

Tang, J., Zhao, Y., Zhu, K., Xiao, G., Kasikci, B., and Han, S. Quest: Query-aware sparsity for efficient long-context llm inference. *arXiv preprint arXiv:2406.10774*, 2024.

Tang, X., Shin, R., Inan, H. A., Manoel, A., Mireshghallah, F., Lin, Z., Gopi, S., Kulkarni, J., and Sim, R. Privacy-preserving in-context learning with differentially private few-shot generation. In *The Twelfth International Conference on Learning Representations*.

Together AI. Chat api overview – long-running conversations. https://docs.together.ai/docs/chat-overview, 2025. Accessed April 2025.

Wan, X., Sun, R., Nakhost, H., and Arik, S. Teach better or show smarter? on instructions and exemplars in automatic prompt optimization. *Advances in Neural Information Processing Systems*, 37:58174–58244, 2024.

Wang, C., Liu, X., and Awadallah, A. H. Cost-effective hyperparameter optimization for large language model generation inference. In *International Conference on Automated Machine Learning*, pp. 21–1. PMLR, 2023.

Wang, X., Li, C., Wang, Z., Bai, F., Luo, H., Zhang, J., Jojic, N., Xing, E., and Hu, Z. Promptagent: Strategic planning with language models enables expert-level prompt optimization. In *The Twelfth International Conference on Learning Representations*.

Xie, C., Lin, Z., Backurs, A., Gopi, S., Yu, D., Inan, H. A., Nori, H., Jiang, H., Zhang, H., Lee, Y. T., et al. Differentially private synthetic data via foundation model apis 2: Text. *arXiv preprint arXiv:2403.01749*, 2024.

Yang, C., Wang, X., Lu, Y., Liu, H., Le, Q. V., Zhou, D., and Chen, X. Large language models as optimizers. In *The Twelfth International Conference on Learning Representations*, 2024.

Ying, H., Zhang, S., Li, L., Zhou, Z., Shao, Y., Fei, Z., Ma, Y., Hong, J., Liu, K., Wang, Z., et al. Internlm-math: Open math large language models toward verifiable reasoning. *arXiv preprint arXiv:2402.06332*, 2024.

Yu, X. and Gen, M. *Introduction to evolutionary algorithms*. Springer Science & Business Media, 2010.

Yu, Y., Zhuang, Y., Zhang, J., Meng, Y., Ratner, A. J., Krishna, R., Shen, J., and Zhang, C. Large language model as attributed training data generator: A tale of diversity and bias. *Advances in Neural Information Processing Systems*, 36:55734–55784, 2023.

Yuksekgonul, M., Bianchi, F., Boen, J., Liu, S., Huang, Z., Guestrin, C., and Zou, J. Textgrad: Automatic" differentiation" via text. *arXiv preprint arXiv:2406.07496*, 2024.

Zhou, Y., Muresanu, A. I., Han, Z., Paster, K., Pitis, S., Chan, H., and Ba, J. Large language models are human-level prompt engineers. In *The Eleventh International Conference on Learning Representations*, 2022.

Scaling Textual Gradients via Sampling-Based Momentum

Technical Appendices and Supplementary Material

A. Extended Related Works

A.1. Automatic Prompt Engineering Workflow

We will revisit several **Automatic Prompt Engineering** frameworks below.

1. **APE** (Zhou et al., 2022) is a seminal work in leveraging LLMs for instruction optimization. In each iteration, a set of instructions is evaluated on a validation set, and the optimizer generates a new set by paraphrasing the highest-performing instructions. This iterative process continues until convergence. However, we argue that APE does not fall under the category of Textual Gradient Descent (TGD) but instead aligns more closely with evolutionary algorithms (Yu & Gen, 2010), as it is inherently gradient-free. Rather than utilizing textual gradients for optimization, APE explicitly prompts LLMs to generate variations of instructions while preserving their semantic meaning, replacing lower-performing prompts through mechanisms akin to random variation (e.g., mutation or crossover), a hallmark of evolutionary strategies. Therefore, we exclude it for our evaluation.
2. **DLN1** (Sordoni et al., 2023) views prompt optmization as learning a distribution $p_{LM}(y|x, \pi)$ in which x, y are inputs or outputs separately, and π is learnable prompt. The iterative process is similar to APE but can include a verbalization of difficult examples from the task: the final prompts shall combine both instructions and task examples, which mimic a mix of zero-shot learning and in-context learning.
3. **OPRO** (Yang et al., 2024) optimizes instructions by presenting the trajectory of previously generated prompts with their corresponding training set accuracy, together with randomly extracted demonstrations from the training set to denote the task of interest. The algorithm only keep instructions with highest scores in the meta-prompt in consideration of LLM context length limit. The iterative process only asks LLMs to generate one more new prompt per iteration. Note here Yang et al. (2024) typically runs much longer iterations compared to DLN1 (Sordoni et al., 2023) and DLN1 shall serve as a shorter version of Yang et al. (2024). We argue that due to its similarity to DLN1, we use DLN1 as a representative method to evaluate our TSGD-M algorithm.
4. **TextGrad** (Yuksekgonul et al., 2024) backpropogates textual feedbacks provided by the proposal and view the textual feedbacks as gradients to perform descent or improve upon. For every iteration, they randomly extract several demonstrations and generate only one new prompt. They also present a momentum version by simply concatenating previously generated past gradients within certain window length.
5. **DSPy** (Khattab et al., 2024). As we limit our study into zero-shot prompt optimization, in which we solely focus on *instruction optimization* rather than *example optimization* or jointly optimize both of them (Wan et al., 2024; Opsahl-Ong et al., 2024), we only discuss COPRO module in Khattab et al. (2024). As our tasks are APE with zero-shot demonstrations needed to optimize, we use COPRO for automatic instruction optimization and exclude MIPROv2 as our baselines do not involve optimizing the set of few shots demonstrations. Similar to DLN-1, COPRO leverages Signatures (structured prompts) to optimize Signatures themselves. We refer readers for further discussions on different optimizers (DSPy, 2025).
6. **PromptAgent** (Wang et al.) views prompt optimization as a more advanced planning agent using Monte Carlo Tree Search (MCTS). We argue that Wang et al. does not fall under TGD framework also. The MCTS algorithm itself is not a gradient based algorithm as it relies on a search based approach rather than differentiable optimization techniques and MCTS does not compute or apply gradients. Even though MCTS shall be combined with gradient base learning where a policy network is trained using policy gradients and used to guide tree search, it is beyond our paper's research scope. Thus, we exclude this method.
7. **ProTeGi** (Pryzant et al., 2023) was among the first methods to incorporate gradient descent principles into automatic prompt generation. Our TSGD-M framework can be naturally extended to **ProTeGi**. Specifically, we propose performing token-wise sampling over batches of *meta-prompts*. Rather than applying momentum sampling to individual meta-prompts from p_0 to p_{t-1}, we instead sample across batches of prompts, denoted by $\cup_{\tau=1}^{i} B_\tau$. From this union, we select a batch B_i

and apply uniform weights to all prompts within that batch. We exclude this method for evaluation due to double sampling but this method shall be viewed as further research direction. We are also aware of a concurrent line of work on momentum integration in ProTeGi (Cui et al., 2024), where a history of past gradients is maintained and *a single gradient is randomly sampled to generate a new prompt pool at each iteration*. In contrast, our approach performs adaptive sampling with decayed weights (defined by momentum parameter) over past gradients (or meta-prompts) at the token level, continuing until the maximum token limit is reached.

A.2. Synthetic Text Generation

Recent work in differentially private (DP) language model training has explored synthetic generation as a mechanism to protect sensitive data while enabling downstream utility (Tang et al.; Amin et al., 2024; Xie et al., 2024). Notably, prior approaches such as those in DP-Few-Shot Generation (Tang et al.) construct synthetic datasets by prompting an LLM to generate tokens one at a time, with differential privacy applied via logit-level mechanisms such as clip-and-aggregate, Gaussian noise, or report-noisy-max (Amin et al., 2024). These methods often rely on carefully controlled sampling from private logits or fallback to public logits when logit similarity allows, in order to minimize privacy cost. In contrast, our approach focuses on momentum-based prompt synthesis, where token-by-token generation is guided not by privacy constraints, but by a trajectory of previously optimized prompts—analogous to a gradient descent path in prompt space. While our framework does not aim for differential privacy, it shares structural similarities with the above methods in generating synthetic text autoregressively under external constraints (e.g., past prompt trajectories). This connection highlights the broader utility of iterative prompt conditioning in synthetic data pipelines, whether for privacy-preserving learning or for optimizing instruction-following behavior via momentum sampling.

B. Additional algorithm details

Below, Algorithms 1, 2, and 3 present the full pseudocode for the TSGD and TSGD-M workflows. We separate the core loop (Algorithm 1) from the per-iteration update routines for vanilla TSGD (Algorithm 2) and TSGD with momentum (Algorithm 3) for clarity.

Algorithm 1 Textual Stochastic Gradient Descent with Momentum (TSGD-M)

```
1: Input: LM: Language model, $p_0$: initial prompt, $\mathcal{D}$: data distribution, $m$: batch size , $T$: total iterations, use momentum flag, $\alpha$: momentum parameter, $T_{\max}$: max tokens, $k$: number of candidate prompts need to generate, $S$: Score Function, $p_{\text{refine}}$: Template to generate new prompts
2: for $t = 0, 1, \dots, T-1$ do
3:     Sample batch $\{(x_i^{(t)}, y_i^{(t)})\}_{i=1}^m \sim \mathcal{D}$
4:     Compute predictions $\widehat{y}_i^{(t)} = \mathbf{LM}([p_t, x_i^{(t)}])$ for $i = 1, \dots, m$
5:                                                  ▷ Generate next prompt below
6:     if use momentum then
                                                     ▷ Update prompt with momentum
           $p_{t+1} \leftarrow \texttt{Update-Mom}\left(\mathbf{LM}, \alpha, \{p_\tau\}_{\tau=0}^t, \bigcup_{\tau=0}^{t} \{(x_i^{(\tau)}, y_i^{(\tau)}, \widehat{y}_i^{(\tau)})\}_{i=1}^m, T_{max}, k, S, p_{\text{refine}}\right)$ (Alg 3)
7:     else
8:                                                   ▷ Update prompt without momentum
           $p_{t+1} \leftarrow \texttt{Update}\left(\mathbf{LM}, p_t, \{(x_i^{(t)}, y_i^{(t)}, \widehat{y}_i^{(t)})\}_{i=1}^m, T_{max}, k, S, p_{\text{refine}}\right)$ (Alg 2)
9:     end if
10: end for
11: Output: Optimized prompt $p_T$
```

Algorithm 2 `Update`

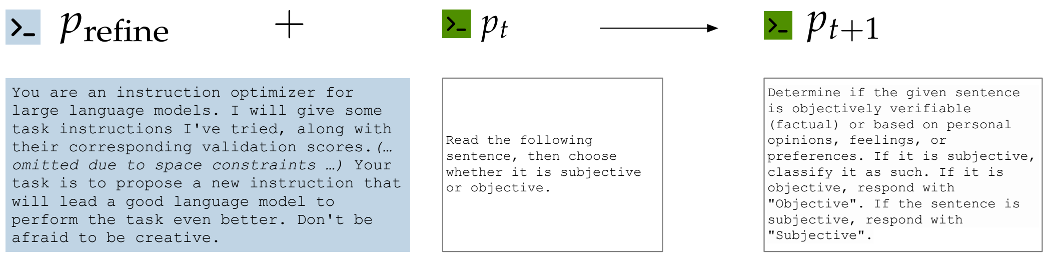
```
1:  Input: Language model LM, current prompt $p_t$, $\{(x_i^{(t)}, y_i^{(t)}, \widehat{y}_i^{(t)})\}_{i=1}^m$, max tokens $T_{max}$, number of candidate prompts
    need to generate $k$, Score Function $S$, LLM Template to generate new prompts $p_{\text{refine}}$
2:  $\tilde{Z} \leftarrow \emptyset$
3:  for $j = 1$ to $k$ do
4:      $\tilde{z} \leftarrow \emptyset$
5:      for $i = 1$ to $T_{max}$ do
6:          Generate one more token $t_i$ using $\mathbf{LM}(p_{\text{refine}} + p_t)$
7:          $\tilde{z} \leftarrow \tilde{z} + [t_i]$
8:      end for
9:      $\tilde{Z} \leftarrow \tilde{Z} \bigcup \tilde{z}$
10: end for
11: if $k = 1$ then
12:     $p_{t+1} \leftarrow \tilde{Z}$
13: else
14:     $p_{t+1} \leftarrow_{z \in \tilde{Z}} S(z)$
15: end if
16: Output: $p_{t+1}$
```

Algorithm 3 `Update-Mom`

```
1:  Input: LM, $T_{\max}$, $k$, $S$, $p_{\text{refine}}$, $\alpha$, $\{p_\tau\}_{\tau=0}^t$, $\bigcup_{\tau=0}^t \{(x_i^{(\tau)}, y_i^{(\tau)}, \hat{y}_i^{(\tau)})\}_{i=1}^m$
2:  Generate weights: $\{w_\tau\}_{\tau=0}^t = \left[\frac{\alpha^{t-\tau}}{\sum_{\tau=0}^t \alpha^{t-\tau}}\right]$; Initialize $\tilde{Z} \leftarrow \emptyset$
3:  for $j = 1$ to $k$ do
4:      $\tilde{z} \leftarrow \emptyset$
5:      for $i = 1$ to $T_{\max}$ do
6:          Sample $p_i$ from $\mathbb{P}$, where $\mathbb{P}(p_\tau) = w_\tau$                          ▷ Momentum sampling
7:          Generate token $t_i$ using LM$(p_{\text{refine}} + p_i)$ or LM$(p_{\text{refine}} + p_i + \nabla p_i)$    ▷ Token-wise prompt generation
8:          $\tilde{z} \leftarrow \tilde{z} + [t_i]$
9:      end for
10:     $\tilde{Z} \leftarrow \tilde{Z} \cup \tilde{z}$
11: end for
12: if $k = 1$ then
13:     $p_{t+1} \leftarrow \tilde{Z}$
14: else
15:     $p_{t+1} \leftarrow \arg\max_{z \in \tilde{Z}} S(z)$
16: end if
17: Output: $p_{t+1}$
```

C. Illustration of Momentum Sampling (Case 1)

Figure 5 illustrates the momentum sampling process described in Section 4 for Case 1, where textual gradient descent operates in a single step (Sordoni et al., 2023; Khattab et al., 2024). In this example, the refinement prompt p_{refine} is instantiated using a textual example from the DSPy-COPRO framework and is concatenated with the current meta prompt p_t to generate the next round of candidate prompts.

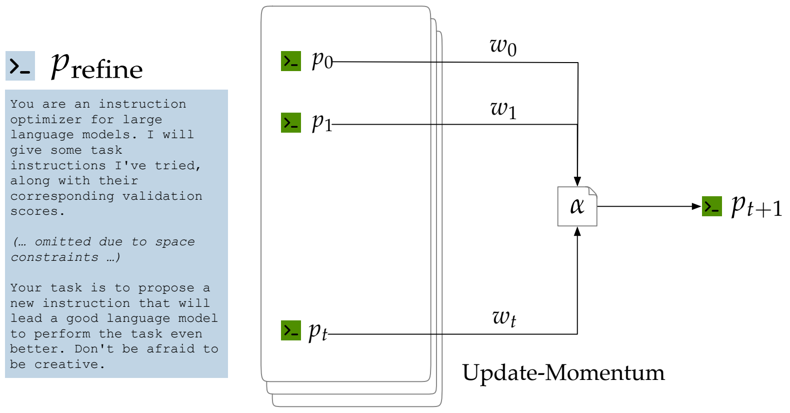


```
You are an instruction optimizer for
large language models. I will give some
task instructions I've tried, along with
their corresponding validation scores. (…
omitted due to space constraints …) Your
task is to propose a new instruction that
will lead a good language model to
perform the task even better. Don't be
afraid to be creative.
```

```
Read the following
sentence, then choose
whether it is subjective
or objective.
```

```
Determine if the given sentence
is objectively verifiable
(factual) or based on personal
opinions, feelings, or
preferences. If it is subjective,
classify it as such. If it is
objective, respond with
"Objective". If the sentence is
subjective, respond with
"Subjective".
```

(a) TSGD: new prompts are generated using p_{refine} prompt template to instruct the LLM in updating the prompt p_t meta prompt from last iteration.



```
You are an instruction
optimizer for large
language models. I will
give some task
instructions I've tried,
along with their
corresponding validation
scores.

(… omitted due to space
constraints …)

Your task is to propose a
new instruction that will
lead a good language model
to perform the task even
better. Don't be afraid to
be creative.
```

(b) Momentum on meta-prompts

Figure 5. An illustration of the momentum sampling in Case 1.

D. Theoretical Justification

Setting. We consider a simplified setting where the optimal prompt is a scalar $\mu \in \mathbb{R}$. In each iteration, the LLM samples from $\mu + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is independent noise. Prompt performance is measured by mean squared error (MSE) $\mathbb{E}[(p - \mu)^2]$.

Let the baseline approach generate prompts $\{X_t\}$ where:

$$X_t = \text{LLM} \tag{3}$$

and the alternative approach (momentum) generate prompts $\{Y_t\}$ using exponential moving average

$$Y_t = \alpha \cdot \text{LLM} + (1 - \alpha) \cdot Y_{t-1}, \quad 0 < \alpha < 1 \tag{4}$$

for all $t \geq 1$. We note that $Y_0 = \text{LLM}$.

Theorem D.1 (Variance Reduction due to Exponential Moving Average)**.** *Then for all* $t \geq 1$*, we have* $\mathbb{E}[X_t] = \mathbb{E}[Y_t] = \mu$*, and*

$$\mathbb{E}[(X_t - \mu)^2] = \mathbb{E}[\epsilon_t^2] = \sigma^2 \tag{5}$$

and

$$\begin{aligned}\mathbb{E}[(Y_t - \mu)^2] &= \alpha^2 \sum_{k=0}^{t-1} (1 - \alpha)^{2k} \sigma^2 \\ &= \sigma^2 \left[\frac{\alpha}{2 - \alpha} + \frac{2}{2 - \alpha} (1 - \alpha)^{2t+1} \right]\end{aligned}$$

Therefore, the momentum approach achieves strictly lower MSE.

Proof. The baseline process follows $X_t = \mu + \epsilon_t$ with $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. At iteration t:

$$\mathbb{E}[(X_t - \mu)^2] = \mathbb{E}[\epsilon_t^2] = \sigma^2 \tag{6}$$

The momentum approach can be expanded recursively as follows:

$$Y_t - \mu = (1 - \alpha)^t \epsilon_0 + \alpha \sum_{k=1}^{t} (1 - \alpha)^{t-k} \epsilon_k$$

where $\epsilon_k \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2)$. We can easily see that $\mathbb{E}[Y_t - \mu] = 0$. Hence, the mean squared error becomes

$$\begin{aligned}\mathbb{E}[(Y_t - \mu)^2] = \text{Var}(Y_t - \mu) &= (1 - \alpha)^{2t} \sigma^2 + \alpha^2 \sum_{k=1}^{t} (1 - \alpha)^{2(t-k)} \sigma^2 \\ &= \sigma^2 \left[(1 - \alpha)^{2t} + \alpha^2 \frac{1 - (1 - \alpha)^{2t}}{1 - (1 - \alpha)^2} \right] \\ &= \sigma^2 \left[(1 - \alpha)^{2t} + \frac{\alpha}{2 - \alpha} (1 - (1 - \alpha)^{2t}) \right] \\ &= \sigma^2 \left[\frac{\alpha}{2 - \alpha} + \frac{2}{2 - \alpha} (1 - \alpha)^{2t+1} \right] \\ &< \sigma^2\end{aligned}$$

and as $t \to \infty$, we have

$$\mathbb{E}[(Y_t - \mu)^2] \to \frac{\alpha}{2 - \alpha} \sigma^2$$

□

E. More Experiment Details

E.1. Task Description

In Table 2, we provide brief descriptions and dataset statistics for all tasks used in our experiments. For GSM8K (Cobbe et al., 2021), we adopt the same data split as Khattab et al. (2024); Yuksekgonul et al. (2024), using 200 examples for training, 300 for validation/development, and 1319 for testing.

Task	|train|	|valid|	|test|	|class|	Description
Mpqa	400	256	250	2	Sentiment analysis.
Trec	400	256	250	6	Question type classification.
Subj	400	256	250	2	Determine whether a sentence is subjective or objective.
Disaster	400	250	250	2	Determine whether a sentence is relevant to a disaster.
Airline	400	250	250	3	Airline tweet sentiment analysis.
Hyperbaton	400	1000	250	2	Order adjectives correctly in English sentences.
Navigate	375	375	250	2	Spatial reasoning given navigation instructions.
SST2	67349	872	872	2	Sentiment analysis.
GSM8K	200	300	1319	N/A	Reasoning Task.

Table 2. Tasks used in this work.

E.2. Prompt Initialization

We initialize the task description as reported in Table 3 for all tasks and evaluations. For GSM8K(Cobbe et al., 2021), we use the default system prompt (Khattab et al., 2024) as initialization.

Task	Initialization
Mpqa	Read the following review, then choose whether it is negative or positive.
Trec	Read the following question, then choose whether it is about a description, entity, expression, human, location or number.
Subj	Classify the input text as subjective or objective.
Disaster	Read the following sentence, then choose whether it is relevant to a disaster.
Airline	Read the following sentence, then choose whether it is positive, negative, or neutral.
Hyperbaton	Which sentence has the correct adjective order.
Navigate	Read the following sentence, then determine whether you return to the starting point.
SST2	Classify the input text as positive or negative.
GSM8K	Your input fields are: 1. ‘question‘ (str) Your output fields are: 1. ‘reasoning‘ (str) 2. ‘answer‘ (str) All interactions will be structured in the following way, with the appropriate values filled in. [[## question ##]] question [[## reasoning ##]] reasoning [[## answer ##]] answer [[## completed ##]] In adhering to this structure, your objective is: Given the fields ‘question‘, produce the fields ‘answer‘.

Table 3. Prompt initializations.

E.3. Templates

For DLN1, we adopt the same forward classification template as used in Sordoni et al. (2023) for computing predictions of mini-batches (forward pass). For DSPy-COPRO (Khattab et al., 2024) and TextGrad (Yuksekgonul et al., 2024), we follow the prompt templates provided in their respective papers.

Classification Template Forward Pass

```
{{ prompt }}
{{ input }}
Answer:
```

General API Templates As an example of an API template, we provide the following configuration for DSPy_COPRO, which uses the Together AI platform (Together AI, 2025) as the API provider and Llama-3 8B (Grattafiori et al., 2024) as the selected language model.

Together API Chat Example for DSPy-COPRO

```
import os
from together import Together
client = Together()
response = client.chat.completions.create(
model="together_ai/meta-llama/Meta-Llama-3-8B-Instruct-Turbo",
messages=[ {"role": "system", "content": "Your input fields are:
1.attempted_instructions (str)
Your output fields are:
1. ‘proposed_instruction‘ (str): The improved instructions for the language model
2. ‘proposed_prefix_for_output_field‘ (str): The string at the end of the prompt, which will help the model start solving
the task.
All interactions will be structured in the following way, with the appropriate values filled in.
[[## attempted_instructions ##]]
{attempted_instructions} [[## proposed_instruction ##]] {proposed_instruction }
[[## proposed_prefix_for_output_field ##]]
proposed_prefix_for_output_field
[[## completed ##]]
In adhering to this structure, your objective is:
You are an instruction optimizer for large language models. I will give some task instructions I’ve tried, along with their
corresponding validation scores. The instructions are arranged in increasing order based on their scores, where higher
scores indicate better quality.
Your task is to propose a new instruction that will lead a good language model to perform the task even better. Don’t be
afraid to be creative." },
"role": "user", "content": [##attempted_instructions##]
[1] Instruction #1 : Analyze the sentiment of the given sentence by considering the tone, language, and context, ...
[2] Prefix #1 : The sentiment of the sentence is: [3] Resulting Score #1 : 40.0
[4] Instruction #2 : Analyze the given sentence carefully, considering the context, tone, and language used to express
the sentiment. Evaluate the emotional undertones, such as excitement, sadness, or frustration, to determine the overall
sentiment of the sentence. Classify the sentiment as positive if it expresses happiness, satisfaction, or approval, negative
if it expresses dissatisfaction, anger, or sadness, and neutral if it states a fact or shows no emotional tone. [5] Prefix #2 :
The sentiment of the given sentence is:
[6] Resulting Score #2 : 43.3... },
{" "role": "assistant", "content": "Analyze the sentiment of the given sentence by considering the language, } ▷
Comment: We concatenate all previously generated tokens here.
],
"temperature": 0.7, "api_key": XXX, "n": 1, "max_tokens": 10 ▷ Comment: We choose 10 as max new tokens
generated for every momentum sampling round.
)
print(response.choices[0].message.content)
```

In our experiments, we observed that many platforms recommended by DSPy (Khattab et al., 2024) and TextGrad (Yuksekgonul et al., 2024) do not support token-wise generation at the granularity of every single token. For example, the OpenAI platform does not allow true token-by-token generation. Similarly, the Together API also lacks support for generating tokens one at a time. We experimentally found that if using deepseek models, together AI API platform would output "<think ><think ><think >" consecutively; if using Llama models, together AI API platform would output "[[[[[[" based on the above template as our template explicitly asks model to output [1] Instruction. As a practical workaround, we perform momentum sampling every 10 tokens (refer to as "max_tokens": 10), which we found to be effective in practice. Specifically, we find that extending the generation length to 10 tokens reduces token-level repetition observed in single-token generation, while still preserving the benefits of momentum sampling.

E.4. Implementation Details

Same as Sordoni et al. (2023), for all tasks, we set max tokens to be generated $T_{\max} = 100$ for all tasks except GSM8K. For DLN1 and DSPy-COPRO, the number of candidate prompts need to generate $k = 20$ and batch size $m = 20$ for all tasks. We set the total iterations T is 20 while most of our iterations end in 10 as we set the early stopping.

E.5. Runtime

For both TextGrad and DSPy-COPRO experiments, across all datasets, the total runtime remains under 1 hour for both the momentum and non-momentum variants when executed on CPU. For DLN1, we report runtime using LLaMA3-8B as a representative model, using 4 NVIDIA A40 GPUs.

Table 4. Llama3-8B: Averaged Runtime across 10 trials (in hours) of DLN1 and DLN1-Momentum across datasets under H_0 (no prompt optimization) and H_1 (with prompt optimization).

Dataset	**DLN1** (H_0)	**DLN1-M** (H_0)	**DLN1** (H_1)	**DLN1-M** (H_1)
Subj	1	1.5	2	4
Hyperbaton	4	5	2	3.5
Airline	0.3	0.5	0.5	2.5
Navigate	0.1	1	0.2	4
Trec	1	2.5	0.5	4
Disaster	0.1	0.6	0.3	3
MPQA	0.2	0.5	1.2	3
SST2	1	2	1.5	3
GSM8K	3	8	6	7.5

F. More Experiments

F.1. Result Tables for Figure 4

Below we provide the original statistics for Figure 4 in Section 5 with various α values. The reported statistics is averaged across 10 trials with standard deviation in parentheses. We provide DLN1 and DSPy-COPRO under both hypotheses for H_0 and H_1. Under H_0, we report only the results of CoT (ZS) and Human (ZS) for DLN1, as these are the only methods that do not involve any form of prompt optimization or iterative generation. All other reported methods operate in an iterative manner and incorporate prompt refinement or optimization strategies. Both CoT (ZS) and Human (ZS) are evaluated on the test set using greedy decoding (temperature $= 0$), therefore no standard deviation for these two methods. DLN1 and DSPy-COPRO use the same dataset setup so we put COT (ZS) and Human (ZS) under DLN1. We emphasize that incorporating momentum into existing modules reduces variance for both DLN1 and DSPy-COPRO. Similar to SGD with momentum, where a typical setting like $\alpha = 0.9$ is commonly used but not universally optimal, TSGD-M achieves variance reduction across a range of $\alpha > 0$ values (though dataset dependent), without requiring fine-tuning for peak performance. Table 17 denotes the momentum addition compared to vanilla Textgrad (Yuksekgonul et al., 2024) under the original settings with gpt-3.5-turbo-0125 (inferencing for p_{refine}) and gpt-4o (generating gradients for $p_{analyze}$). For TextGrad, we repeat each experiment over 10 random seeds and find that the improved test/development accuracy trajectory is not consistently reproducible (especially for GSM8K). Interestingly, for tasks such as Subj, Hyperbaton, SST2, and GSM8K, the prompts obtained without the TextGrad optimization framework sometimes outperform those generated through the iterative process. Among the configurations tested, a momentum value of $\alpha = 0.3$ tends to yield performance that ranks among the top two across most datasets.

Dataset	**DSPy-COPRO**	**DSPy-COPRO - Momentum = 0.0**	**DSPy-COPRO - Momentum = 0.3**	**DSPy-COPRO - Momentum = 0.6**	**DSPy-COPRO - Momentum = 0.9**
Subj	54.80(2.70)	60.47(2.10)	**63.83(0.72)**	62.50(1.87)	60.40(3.21)
Hyperbaton	69.96(8.07)	70.65(7.40)	71.88(5.37)	**73.45(1.56)**	70.93(9.64)
Airline	77.90(0.56)	77.40(0.17)	77.30(1.15)	**78.63(1.07)**	75.73(1.25)
Navigate	58.13(2.46)	57.93(1.99)	**58.40(1.20)**	56.07(1.93)	57.47(0.46)
Trec	65.61(5.1)	65.33(4.05)	66.45(3.42)	**70.05(4.38)**	64.68(2.46)
Disaster	76.57(1.16)	76.07(1.01)	**77.13(1.79)**	74.93(1.21)	76.73(1.42)
MPQA	78.87(7.05)	81.23(4.30)	85.70(1.65)	**86.00(0.53)**	83.77(4.65)
SST2	88.57(2.35)	88.0(2.52)	87.93(1.46)	88.93(1.57)	**92.83(1.52)**
GSM8K (Test)	44.53(1.01)	45.07(1.28)	45.03(3.11)	**45.17(1.11)**	**46.33(0.81)**
GSM8K (Development)	44.87(4.83)	**46.57(2.23)**	44.77(1.08)	45.63(3.21)	44.70(1.73)

Table 9. DSPy-COPRO: H_0 Hypothesis for Mistral-7B

Dataset	COT(ZS)	DLN1	Human(ZS)	DLN1 -Momentum = 0.0	DLN1-Momentum = 0.3	DLN1-Momentum = 0.6	DLN1-Momentum = 0.9
Subj	53.0	69.03(2.57)	55.5	70.93(1.83)	68.57(2.01)	71.20(4.63)	**71.57(3.29)**
Hyperbaton	77.6	83.07(1.15)	48.4	82.00(1.96)	82.53(1.10)	**85.53(1.53)**	80.8(1.18)
Airline	70.8	80.20(0.79)	79.0	79.73(1.72)	79.47(0.66)	**81.47(0.62)**	80.10(1.79)
Navigate	51.20	51.07(5.28)	**58.0**	54.13(3.46)	50.80(5.72)	55.47(1.86)	55.20(2.29)
Trec	45.00	63.80(5.49)	44.0	70.00(4.98)	67.73(8.96)	**75.00(2.97)**	74.73(2.29)
Disaster	55.10	75.83(0.42)	**78.0**	74.67(2.71)	76.27(3.47)	76.07(1.53)	76.83(2.85)
MPQA	74.45	83.13(2.81)	81.2	83.13(0.79)	82.47(1.85)	**85.02(0.58)**	83.12(0.88)
SST2	80.6	92.67(0.37)	90.94	92.37(0.70)	92.60(0.50)	**92.87(0.69)**	91.57(1.02)
GSM8K (Test)	72.90	72.67(5.56)	75.12	73.2(4.89)	71.2(2.4)	**76.75(1.6)**	76.45(2.0)
GSM8K (Development)	76.70	76.53(6.16)	78.73	77.60(5.12)	75.6(3.2)	**79.80(1.5)**	79.23(1.2)

Table 5. DLN1: H_0 Hypothesis for Lllama3-8B

Dataset	COT(ZS)	Human(ZS)	DLN1	DLN1 - Momentum = 0.0	DLN1 - Momentum = 0.3	DLN1 - Momentum = 0.6	DLN1 - Momentum = 0.9
Subj	65.10	57.35	77.4(4.67)	74.6(2.50)	**79.87(1.21)**	75.2(2.32)	74.6(0.85)
Hyperbaton	76.40	75.2	81.04(2.52)	80.13(2.05)	79.73(1.71)	**81.6(0.95)**	80.4(0.85)
Airline	55.10	78.00	77.97(5.45)	**80.5(0.36)**	80.1(0.54)	79.80(0.64)	79.67(0.58)
Navigate	50.40	57.0	48.72(6.29)	51.08(5.89)	54.88(6.82)	51.20(6.23)	**57.6(4.07)**
Trec	40.40	44.0	64.70(12.9)	59.6(10.1)	58.6(8.52)	**74.07(5.62)**	65.9(7.34)
Disaster	56.8	54.0	70.60(7.26)	73.73(4.56)	73.8(5.67)	**75.45(1.81)**	74.8(2.4)
MPQA	79.85	83.0	86.3(2.41)	86.23(1.11)	86.34(1.00)	**86.5(0.86)**	85.67(2.05)
SST2	80.73	92.55	91.37(0.45)	92.9(0.04)	92.8(0.03)	**93.8(0.02)**	93.1(0.03)
GSM8K (Test)	50.9	49.0	44.37(3.34)	45.29(3.45)	49.6(2.9)	50.0(2.84)	**52.9(2.98)**
GSM8K (Development)	55.3	50.0	45.1(4.83)	45.91(4.6)	48.3(3.3)	**57.6(2.3)**	56.3(3.2)

Table 6. DLN1: H_0 Hypothesis for Mistral-7B

Dataset	COT(ZS)	Human(ZS)	DLN1	DLN1 - Momentum = 0.0	DLN1 - Momentum = 0.3	DLN1 - Momentum = 0.6	DLN1 - Momentum = 0.9
Subj	51.50	49.25	61.07(3.41)	62.98(3.20)	61.78(2.65)	**65.0(2.04)**	61.19(3.22)
Hyperbaton	47.60	48.4	53.2(2.14)	55.6(2.50)	55.04 (1.99)	55.89(1.22)	**56.8(0.99)**
Airline	46.10	18.1	54.23(5.90)	55.5(5.40)	54.47(1.35)	55.87(1.07)	**58.83(4.75)**
Navigate	58.00	42.0	58.0(2.23)	57.64(1.17)	58.9(1.58)	**61.07(1.41)**	57.8(2.1)
Trec	40.20	27.6	53.48(8.07)	55.15(7.10)	55.4(4.72)	**55.6(4.17)**	54.3(4.89)
Disaster	55.10	42.1	61.78(3.82)	59.71(4.02)	64.32(3.89)	**64.7(2.9)**	58.80(2.34)
MPQA	60.35	50.00	62.70(5.59)	65.87(3.15)	**67.58(3.10)**	65.35(3.18)	63.50(3.54)
SST2	53.10	49.08	80.07(3.58)	80.17(3.38)	**81.67(2.53)**	76.77(2.98)	80.77(2.73)
GSM8K (Test)	59.12	56.50	58.53(3.02)	58.20(2.1)	57.23(1.98)	**59.2(1.02)**	58.96(2.99)
GSM8K (Development)	62.7	58.3	63.23(3.77)	63.46(3.29)	62.26(3.04)	**64.82(1.89)**	63.96(3.28)

Table 7. DLN1: H_0 Hypothesis for Deepseek 1.5B

Dataset	DSPy-COPRO	DSPy-COPRO Momentum = 0.0	DSPy-COPRO Momentum = 0.3	DSPy-COPRO Momentum = 0.6	DSPy-COPRO Momentum = 0.9
Subj	56.37(1.96)	60.86(1.27)	**64.70(1.10)**	62.2(2.36)	62.23(4.92)
Hyperbaton	**62.40(4.33)**	60.40(5.89)	57.07(2.60)	60.93(4.60)	57.33(2.01)
Airline	79.73(1.68)	77.8(1.02)	80.2(1.04)	**81.9(2.42)**	81.13(0.45)
Navigate	56.67(7.38)	56.80(1.44)	**61.73(3.03)**	57.60(4.45)	49.87(5.75)
Trec	65.13(2.54)	62.40(4.02)	67.73(8.96)	**75.00(2.97)**	74.73(2.29)
Disaster	74.97(1.15)	**76.03(1.22)**	75.77(1.04)	75.43(0.90)	75.47(0.94)
MPQA	82.93(2.29)	82.93(2.29)	83.83(1.16)	81.47(1.57)	**83.93(2.21)**
SST2	**93.47(0.25)**	93.43(0.68)	93.10(0.35)	91.70(2.34)	93.40(0.70)
GSM8K (Test)	75.86(0.87)	75.90(0.29)	**76.07(0.05)**	76.02(0.04)	75.6(0.71)
GSM8K (Development)	80.86(0.51)	79.43(1.22)	79.97(0.47)	**80.87(0.80)**	79.53(1.08)

Table 8. DSPy-COPRO: H_0 Hypothesis for Lllama3-8B

Dataset	DSPy-COPRO	DSPy-COPRO - Momentum = 0.0	DSPy-COPRO - Momentum = 0.3	DSPy-COPRO - Momentum = 0.6	DSPy-COPRO - Momentum = 0.9
Subj	53.23(2.5)	54.53(2.37)	**56.90(1.25)**	56.37(1.44)	55.77(2.24)
Hyperbaton	47.0(3.62)	46.23(2.98)	45.97(1.63)	49.0(2.60)	**50.10(2.64)**
Airline	55.23(3.30)	56.33(3.27)	57.19(2.91)	**57.23(0.21)**	55.91(2.98)
Navigate	70.67(2.54)	69.77(1.05)	71.60(1.39)	71.97(3.03)	**72.67(2.31)**
Trec	48.2(3.63)	46.00(1.97)	49.48(3.35)	49.93(5.31)	**51.28(3.20)**
Disaster	50.1(2.89)	49.03(1.18)	50.20(2.81)	50.50(1.28)	**50.97(1.07)**
MPQA	66.57(3.89)	64.23(2.92)	68.70(3.39)	**69.17(3.19)**	67.88(3.41)
SST2	72.1(2.96)	70.87(1.42)	71.0(6.15)	72.47(2.80)	**72.87(2.91)**
GSM8K (Test)	78.47(3.21)	78.43(0.12)	**79.97(0.21)**	79.70(0.92)	79.77(0.38)
GSM8K (Development)	82.47(0.88)	82.77(1.50)	83.00(1.18)	**83.47(0.68)**	81.33(3.45)

Table 10. DSPy-COPRO: H_0 Hypothesis for Deepseek 1.5B

Hypothesis 1

Dataset	DLN1	DLN1 - Momentum = 0.0	DLN1 - Momentum = 0.3	DLN1 - Momentum = 0.6	DLN1 -Momentum = 0.9
Subj	72.97(1.56)	73.2(1.39)	**73.67(1.68)**	70.25(0.32)	71.57(2.37)
Hyperbaton	83.44(2.07)	84.0(2.47)	83.46(1.32)	**84.2(1.07)**	83.67(1.98)
Airline	79.50(1.27)	76.2(2.24)	77.5(0.94)	77.3(0.78)	**80.27(1.21)**
Navigate	52.40(4.17)	54.00(3.12)	54.1(1.78)	**54.53(2.78)**	52.80(3.89)
Trec	68.47(3.55)	70.00(4.98)	67.73(8.96)	**75.00(2.97)**	74.73(2.29)
Disaster	75.58(3.12)	76.16(2.17)	**77.73(2.02)**	76.07(1.53)	77.23(1.88)
MPQA	79.12(1.54)	80.73(0.98)	**85.32(2.59)**	85.27(1.28)	82.35(2.20)
SST2	91.32(1.27)	91.77(1.30)	91.40(0.70)	**91.97(0.84)**	91.53(0.34)
GSM8K(Test)	72.3(1.74)	72.5(2.05)	72.5(1.63)	**73.6(3.33)**	72.9(2.32)
GSM8K(Development)	74.90(1.40)	75.80(1.52)	79.0(1.21)	**79.33(4.93)**	78.9(2.02)

Table 11. DLN1: H_1 Hypothesis for Llama3-8B

Dataset	DLN1	DLN1 - Momentum = 0.0	DLN1 - Momentum = 0.3	DLN1 - Momentum = 0.6	DLN1 - Momentum = 0.9
Subj	76.33(2.40)	75.15(2.88)	**82.13(3.38)**	76.3(3.37)	77.4(1.71)
Hyperbaton	81.87(2.24)	79.2(2.04)	80.10(1.82)	80.8(1.39)	**82.13(1.79)**
Airline	**82.57(1.92)**	75.83(3.43)	76.27(2.82)	78.87(2.00)	79.37(2.66)
Navigate	55.33(6.8)	54.6(4.8)	56.2(3.34)	**58.2(2.21)**	56.3(3.45)
Trec	69.93(3.59)	68.73(3.34)	69.90(2.42)	70.46(2.84)	**74.4(3.46)**
Disaster	75.3(2.34)	75.17(0.60)	75.08(3.81)	**76.63(1.66)**	75.90(1.87)
MPQA	85.2(2.58)	85.87(2.59)	85.78(2.32)	**85.97(2.18)**	85.93(1.62)
SST2	91.87(1.70)	92.40(0.78)	92.60(1.85)	**93.77(1.04)**	92.17(0.87)
GSM8K(Test)	50.93(4.25)	50.56(2.85)	**54.2(2.37)**	51.2(3.2)	48.67(3.27)
GSM8K(Development)	54.33(4.47)	53.2(3.14)	57.2(3.12)	**57.3(2.9)**	50.67(4.25)

Table 12. DLN1:H_1 Hypothesis for Mistral-7B

Dataset	DLN1	DLN1 - Momentum = 0.0	DLN1 - Momentum = 0.3	DLN1 - Momentum = 0.6	DLN1 - Momentum = 0.9
Subj	62.37(4.03)	63.42(3.03)	**64.67(3.9)**	63.67(1.25)	62.0(3.72)
Hyperbaton	55.20(2.88)	56.40(1.39)	52.67(1.97)	52.93(1.08)	**59.06(2.17)**
Airline	69.47(2.1)	67.57(1.46)	70.2(2.47)	78.87(2.00)	**79.37(2.66)**
Navigate	52.93(9.91)	55.20(4.87)	58.30(2.23)	**58.77(2.10)**	53.67(5.00)
Trec	49.8(5.26)	51.12(4.67)	50.48(8.03)	49.48(3.03)	**52.70(2.90)**
Disaster	59.27(3.32)	57.83(2.54)	58.70(2.65)	59.3(3.16)	**60.33(2.21)**
MPQA	71.07(9.43)	69.3(2.27)	71.43(3.85)	70.70(5.52)	**72.8(3.84)**
SST2	75.70(4.52)	77.6(3.32)	77.93(1.69)	79.27(0.2)	**79.67(0.47)**
GSM8K (Test)	49.78(6.71)	48.72(7.45)	**51.28(3.21)**	46.07(2.08)	46.33(1.68)
GSM8K (Development)	51.73(7.82)	48.3(8.2)	**53.0(1.21)**	49.0(1.21)	49.2(1.02)

Table 13. DLN1: H_1 Hypothesis for Deepseek 1.5B

Dataset	DSPy-COPRO	DSPy-COPRO Momentum = 0.0	DSPy-COPRO Momentum = 0.3	DSPy-COPRO Momentum = 0.6	DSPy-COPRO Momentum = 0.9
Subj	63.7(1.47)	62.10(1.17)	63.87(2.49)	64.07(1.59)	**65.60(3.35)**
Hyperbaton	63.33(7.49)	63.27(2.81)	62.0(3.49)	64.0(7.21)	**64.5(4.68)**
Airline	79.23(1.86)	78.33(1.99)	78.17(2.06)	**79.37(0.99)**	78.03(1.42)
Navigate	52.8(4.57)	53.32(3.92)	53.60(1.42)	52.81(2.67)	**56.67(3.40)**
Trec	61.2(0.91)	70.00(4.98)	67.73(8.96)	**75.00(2.97)**	74.73(2.29)
Disaster	75.4(0.57)	74.67(2.71)	76.27(3.47)	76.07(1.53)	**76.83(2.85)**
MPQA	76.20(5.81)	75.10(4.82)	73.30(4.43)	**78.57(3.54)**	77.93(3.01)
SST2	91.8(0.80)	92.47(0.46)	91.60(2.04)	91.47(1.98)	**92.90(0.44)**
GSM8K (Test)	76.27(3.37)	76.50(2.90)	75.27(0.55)	75.57(2.40)	**77.53(2.57)**
GSM8K (Development)	81.13(1.82)	80.39(0.8)	80.33(2.81)	82.33(0.65)	**82.67(1.35)**

Table 14. DSPy-COPRO: H_1 Hypothesis for Lllama3-8B

Dataset	DSPy-COPRO	DSPy-COPRO - Momentum = 0.0	DSPy-COPRO - Momentum = 0.3	DSPy-COPRO - Momentum = 0.6	DSPy-COPRO - Momentum = 0.9
Subj	62.97(2.82)	62.53(2.81)	60.93(1.55)	62.90(1.47)	**63.57(2.55)**
Hyperbaton	63.33(4.44)	62.77(4.03)	**65.33(3.05)**	64.50(1.98)	62.10(0.57)
Airline	77.57(1.14)	76.53(1.15)	76.03(1.79)	**77.53(1.94)**	76.73(1.81)
Navigate	56.53(1.36)	56.13(0.83)	**59.07(2.66)**	57.20(0.57)	56.53(2.37)
Trec	67.2(4.55)	71.43(1.10)	70.53(4.27)	**72.77(4.21)**	72.63(0.57)
Disaster	76.70(2.73)	76.27(1.56)	76.83(1.18)	77.43(1.87)	**77.93(2.09)**
MPQA	77.73(4.86)	76.40(4.68)	73.97(1.89)	71.70(3.56)	**78.90(5.15)**
SST2	88.07(3.52)	88.37(2.23)	**91.43(0.46)**	89.27(1.50)	89.07(2.22)
GSM8K (Test)	43.6(2.02)	43.23(1.86)	43.27(0.81)	**46.00(2.04)**	44.57(2.04)
GSM8K (Development)	41.2(2.71)	41.77(2.04)	43.77(0.81)	**45.67(2.87)**	43.67(2.03)

Table 15. DSPy-COPRO: H_1 Hypothesis for Mistral-7B

Dataset	DSPy-COPRO	DSPy-COPRO - Momentum = 0.0	DSPy-COPRO-Momentum = 0.3	DSPy-COPRO-Momentum = 0.6	DSPy-COPRO-Momentum = 0.9
Subj	53.23(2.49)	53.03(0.76)	53.93(2.37)	55.30(1.23)	**56.17(1.07)**
Hyperbaton	48.67(4.18)	49.47(3.49)	50.87(0.96)	**51.42(1.08)**	50.06(2.17)
Airline	55.4(2.97)	56.40(2.32)	57.10(1.89)	55.80(2.34)	**58.33(1.52)**
Navigate	72.30(4.71)	73.01(3.51)	**73.60(2.83)**	72.90(2.81)	73.20(1.89)
Trec	43.53(4.90)	46.60(4.65)	46.27(3.13)	47.20(3.28)	**48.50(3.98)**
Disaster	54.46(7.23)	54.07(3.32)	55.43(3.63)	**59.4(4.89)**	55.9(5.89)
MPQA	63.66(8.57)	61.93(6.61)	62.8(7.78)	65.38(5.52)	**70.27(3.18)**
SST2	68.33(3.92)	67.30(3.19)	**72.30(3.21)**	69.33(3.23)	69.97(2.15)
GSM8K (Test)	75.52(3.29)	75.67(2.61)	76.17(3.17)	75.13(3.20)	**77.20(1.97)**
GSM8K (Development)	80.87(3.74)	79.93(2.43)	80.90(3.13)	**82.35(2.59)**	81.20(3.20)

Table 16. DSPy-COPRO: H_1 Hypothesis for Deepseek 1.5B

Textgrad

Dataset	COT(ZS)	Human(ZS)	Textgrad	Textgrad - Momentum = 0.0	Textgrad - Momentum = 0.3	Textgrad - Momentum = 0.6	Textgrad - Momentum = 0.9
Subj	**70.45**	64.1	63.20(7.28)	64.23(6.76)	66.99(3.89)	64.80(6.41)	62.92(6.12)
Hyperbaton	85.2	**88.0**	69.28(11.22)	70.80(7.97)	74.26(10.97)	71.52(11.25)	73.36(6.67)
Airline	84.30	83.30	83.70(0.78)	84.0(0.42)	**84.53(0.47)**	83.17(1.11)	83.15(3.10)
Navigate	48.8	37.6	61.10(8.57)	62.04(8.23)	62.30(2.95)	57.28(5.35)	**70.80(10.61)**
Trec	53.40	45.40	52.0(17.21)	48.9(10.24)	62.45(9.23)	**63.91(8.55)**	61.92(6.12)
Disaster	73.8	69.0	74.48(4.36)	72.58(5.35)	74.50(1.22)	**76.10(0.46)**	74.28(3.97)
MPQA	59.10	51.44	**70.42(7.39)**	68.43(9.06)	58.22(15.19)	66.73(6.89)	61.64(8.06)
SST2	91.17	**93.12**	90.14(1.95)	91.40(0.69)	90.84(1.41)	90.27(1.89)	92.72(1.08)
GSM8K (Test)	67.25	**79.31**	70.32(14.63)	71.27(2.71)	71.82(8.32)	70.92(10.59)	70.83(8.58)
GSM8K (Development)	70.0	**82.0**	71.83(13.72)	72.48(2.45)	71.32(10.94)	72.12(8.37)	72.11(8.11)

Table 17. Textgrad: gpt-3.5-turbo-0125(inferencing for p_{refine}) + gpt-4o(gradient for $p_{analyze}$)

F.2. Various Momentum Interpretations

As Yuksekgonul et al. (2024) noted that an alternative interpretation of momentum is earlier iterations of the gradient variable when making the *descent* step. Specifically, Yuksekgonul et al. (2024) simply concatenating all the textual gradients together by the prompt template below.

TextGrad Momentum prompt

```
Here are the past iterations of this variable:
<PAST_ITERATIONS>{past_values}</PAST_ITERATIONS>
```

To investigate the role of momentum in TextGrad (Yuksekgonul et al., 2024), we compared our DLN1 approach with a baseline that mimics the momentum strategy used in TextGrad—specifically, concatenating all past textual gradients (i.e., meta-prompts) (See $B_{\pi}^{ConcatPrompts}$ below), which adapts from Appendix $D.2$ in Sordoni et al. (2023)). We present results for both Llama3-8B (See Table 18, and Table 19) (Grattafiori et al., 2024), Mistral-7B (See Table 20 and Table 21) (Jiang et al., 2023) and DeepSeek-R1-Distill-Qwen-1.5B (See Table 22 and Table 23) (Guo et al., 2025).

This concatenation-based strategy was tested under both H_0 and H_1 prompt selection conditions. Across both settings for Llama3-8B, DLN1 consistently outperformed the concatenation method on a wide range of tasks. For instance, under H_0 setting, DLN1 achieved significantly better performance in Subj (69.33 vs. 66.22), Airline (80.40 vs. 78.3), and Disaster (75.83 vs. 74.33), among others. Under H_1, the advantages of DLN1 were even more pronounced, such as on Hyperbaton (83.44 vs. 70.67) and MPQA (79.12 vs. 78.40). These results demonstrate that simply concatenating all previous gradient prompts, as done in TextGrad, is not an effective mechanism for leveraging momentum in APE workflow. By contrast, vanilla DLN1 setting leads to consistently stronger performance across diverse language understanding tasks. Note here we did not list our methods of momentum performance here as most of our methods of momentum outperform vanilla DLN1.

Under the H_1 hypothesis using the Mistral-7B model, DLN1 consistently outperformed the concatenated meta-prompt baseline across a wide variety of datasets. This supports our intuition that simply appending all past prompts—as done in the momentum interpretation of TextGrad—can degrade performance, particularly as the sequence becomes longer and noisier. DLN1 led to stronger results on core language understanding tasks such as Airline (82.57 vs. 64.7), Trec (69.93 vs. 47.07), and MPQA (85.2 vs. 69.37), showing substantial margins of improvement. Notably, DLN1 also demonstrated better performance on reasoning-heavy datasets like GSM8K (Test) (50.93 vs. 48.26) and GSM8K (Dev) (54.33 vs. 52.2), even though the differences were subtler. These results reinforce our claim that raw accumulation of past meta prompts yields less effective generalization and task performance, particularly when scaling up to stronger models like Mistral-7B.

With the DeepSeek-R1-Distill-Qwen-1.5B model (Guo et al., 2025) —representing the smallest model in our evaluation suite—we observed that DLN1 still outperformed the concatenation-based baseline across most datasets, under both H_0 and H_1 hypotheses. Notably, the performance gains with DLN1 were accompanied by higher variance compared to larger models like Mistral-7B and LLaMA3-8B, which is expected given the reduced capacity and stability of smaller language models. For instance, under H_1 hypothesis , DLN1 surpassed the concatenation method on challenging tasks such as Airline (69.47 vs. 53.43), MPQA (71.07 vs. 59.68), and GSM8K (Dev) (51.73 vs. 49.2), but also exhibited substantial standard deviations (e.g., MPQA ±9.43, Disaster ±7.54). These results further substantiate that prompt concatenation, as used in TextGrad-style momentum, fails to scale down effectively to lower-capacity models. DLN1's performance, while more variable, remains more robust and reliable, suggesting that even for compact models, simply concatenating past meta prompts or accumulating past gradients would bring in performance degradation, which aligns with other research findings that LLMs might not be able to effectively perform summarizations for long context tasks (Liu et al., 2024; Peng et al., 2023).

Dataset Name	DLN1	DLN1 Concat Prompts
Subj	**69.33(2.57)**	66.22(1.2)
Hyperbaton	**83.07(1.15)**	81.19(2.28)
Airline	**80.40(0.79)**	78.3(0.6)
Navigate	**51.07(5.28)**	50.13(8.57)
Trec	**63.80(5.49)**	59.0(7.44)
Disaster	**75.83(0.42)**	74.33(5.63)
MPQA	**83.13(2.81)**	80.40(2.82)
SST2	**92.67(0.37)**	92.13(1.31)
GSM8K (Test)	**72.67(5.56)**	71.62(3.34)
GSM8K (Development)	**76.53(6.16)**	74.23(7.21)

Table 18. Llama3-8B: H_0 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Dataset Name	DLN1	DLN1 Concat Prompts
Subj	**72.97(1.56)**	66.55(3.90)
Hyperbaton	**83.44(2.07)**	70.67(4.78)
Airline	**79.50(1.27)**	71.8(0.64)
Navigate	**52.40(4.17)**	50.03(4.08)
Trec	68.47(3.55)	**72.47(3.73)**
Disaster	**75.58(3.12)**	68.83(3.76)
MPQA	**79.12(1.54)**	78.40(2.82)
SST2	**91.32(1.27)**	78.06(7.04)
GSM8K (Test)	**72.3(1.74)**	70.82(2.31)
GSM8K (Development)	**74.90(1.40)**	71.3(3.56)

Table 19. Llama3-8B: H_1 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Dataset Name	DLN1	DLN1 Concat Prompts
Subj	**77.4(4.67)**	66.75(11.01)
Hyperbaton	**81.04(2.52)**	76.0(2.80)
Airline	**77.97(5.45)**	75.23(7.51)
Navigate	48.72(6.29)	**56.53(1.40)**
Trec	**64.70(12.9)**	57.07(5.47)
Disaster	**74.97(1.15)**	59.33(8.03)
MPQA	**86.3(2.41)**	73.73(9.59)
SST2	**91.37(0.45)**	90.33(5.51)
GSM8K (Test)	**44.37(3.34)**	42.26(2.7)
GSM8K (Development)	**45.1(4.83)**	43.2(2.3)

Table 20. Mistral-7B: H_0 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Dataset Name	DLN1	DLN1 Concat Prompts
Subj	**76.33(2.40)**	75.67(6.66)
Hyperbaton	**81.87(2.24)**	75.23(1.26)
Airline	**82.57(1.92)**	64.7(3.33)
Navigate	**55.33(6.8)**	47.67(9.81)
Trec	**69.93(3.59)**	47.07(0.02)
Disaster	**75.3(2.34)**	57.37(0.91)
MPQA	**85.2(2.58)**	69.37(5.65)
SST2	**91.87(1.70)**	90.33(5.51)
GSM8K (Test)	**50.93(4.25)**	48.26(2.7)
GSM8K (Development)	**54.33(4.47)**	52.2(2.3)

Table 21. Mistral-7B: H_1 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Dataset Name	**DLN1**	**DLN1 Concat Prompts**
Subj	**61.07(3.41)**	54.67(2.63)
Hyperbaton	**53.2(2.14)**	48.47(0.41)
Airline	54.23(5.90)	**61.3(2.48)**
Navigate	**58.0(2.23)**	58.0(3.23)
Trec	**53.48(8.07)**	39.7(5.25)
Disaster	**61.78(3.82)**	56.97(1.01)
MPQA	**62.70(5.59)**	58.33(3.41)
SST2	**80.07(3.58)**	72.33(3.10)
GSM8K (Test)	**60.53(1.88)**	58.53(2.3)
GSM8K (Development)	**63.23(2.77)**	60.2(3.3)

Table 22. DeepSeek-R1-Distill-Qwen-1.5B: H_0 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Dataset Name	**DLN1**	**DLN1 Concat Prompts**
Subj	**62.37(4.03)**	54.83(2.48)
Hyperbaton	**55.20(2.88)**	53.27(3.83)
Airline	**69.47(2.1)**	53.43(2.58)
Navigate	52.93(9.91)	**56.67(0.19)**
Trec	**49.8(5.26)**	43.8(5.62)
Disaster	**59.27(3.32)**	55.87(7.54)
MPQA	**71.07(9.43)**	59.68(4.67)
SST2	**75.70(4.52)**	72.33(3.10)
GSM8K (Test)	**49.78(6.71)**	47.53(5.32)
GSM8K (Development)	**51.73(7.82)**	49.2(4.83)

Table 23. DeepSeek-R1-Distill-Qwen-1.5B: H_1 hypothesis for DLN1 vs. DLN1 Concatenating Meta Prompts

Prompt Proposal Template $B_{\pi}^{ConcatPrompts}$ for DLN1 Concat Prompts

template:

```
A student is completing a task that requires producing a text output from a text input. The student
receives an instruction that describes how to produce the output given each input.
The student has made some errors. Your task is to improve the instruction such that the student can fix
the errors.

Here are the past iterations of the instruction.
## Instructions
> {{ past_prompts }}
[END] ▷ Comment: We concatenate all past meta prompts together here.

# Student successes
{% for backward_info in backward_infos %} {% if backward_info.loss == 0.0 %}
## Input:
> {{ backward_info.input }}
## Correct Output:
> {{ backward_info.target }}
{% endif %} {% endfor %}

# Student errors
{% for backward_info in backward_infos %} {% if backward_info.loss > 0.0 %}
## Input:
> {{ backward_info.input }}
## Student Output:
> {{ backward_info.output }}
## Correct Output:
> {{ backward_info.target }}
{% endif %} {% endfor %}

Improve the instruction to fix the student errors. {{ message }}
## Instruction
>
```

message_alternatives:

- Clarify the instruction by adding few words or a short sentence. Be concise.
- Improve the instruction by providing examples on how to solve the task. Be concise.
- Shorten the instruction by removing superfluous words or sentences.
- Rewrite the instruction by providing detailed information to avoid ambiguity. Be concise.

F.3. Optimized Prompts by Momentum Based Sampling

We use DLN1 as a representative method and Llama3-8B as a representative model to illustrate the impact of our momentum-based tokenwise sampling approach. The prompts generated using this strategy consistently incorporate more detailed explanations and illustrative guidance compared to those produced by non-momentum baselines. In this section, we present optimized prompts across all tasks, highlighting how momentum-based tokenwise sampling yields qualitatively different prompts from both human-written and original DLN1-generated prompts. We restrict the number of maximum tokens to be generated by 100. For GSM8k task, we restrict max generated tokens to be 500 and the starting prompt is the default systematic prompt provided by DSPy (Khattab et al., 2024). Since the hyperparameter for momentum parameter $\alpha = 0.6$ seems to outperform other hyperparameters settings for DLN1-Momentum, we only present prompts for DLN1-Momentum=0.6. For simplicity, we only present prompts generated under H_0 hypothesis.

Approach	Optimized Prompt	Acc.
Human	Classify the input text as subjective or objective.	0.491
DLN1	1. Carefully read the input text. 2. Identify the type of language used in the text. 3. Determine if the text includes words that express the author's opinion, emotion, or perspective. Look for words such as "I", "me", "my", "we", "us", "our", "believe", "think", "feel", "opinion", "perspective", "view", "attitude", "emotion", etc. If it does...	0.713
DLN1-Momentum = 0.6	Classify each input text as subjective or objective. Subjective texts express a personal opinion, emotion, or experience.They often use words and phrases like: - "I think", "I believe", "I feel", "my opinion", "my experience", "I love", "I hate", etc. - Use of first-person pronouns (I, me, my) - Words that describe emotions (e.g., beautiful, sad, excited) - Use of evaluative language (e.g "smart and alert , thirteen conversations about one thing is a small gem .	0.770

Table 24. Prompt comparison for the **Subj** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance. DLN1-Momentum improves the prompt by keeping the task description after iterative refinement.

Approach	Optimized Prompt	Acc.
Human	Which sentence has the correct adjective order.	0.750
DLN1	1. Read the sentence carefully. 2. Identify the adjective(s) in the sentence. 3. Arrange the adjectives in the correct order: descriptive adjective (if any), size adjective, color adjective, shape adjective, material adjective, and finally the noun. 4. Check if the adjective order is correct for each sentence. This improved instruction should help the student to identify and fix the errors in the adjective order. The student can follow the steps and learn to produce the correct output given each input.	0.816
DLN1-Momentum = 0.6	1. Follow the correct order of adjectives: opinion, size, shape, material, color, and origin. 2. Start with an opinion adjective if it is present. If not, start with a size adjective. 3. Then, add a shape adjective, followed by a material adjective, a color adjective, and finally, an origin adjective. 4. If an adjective is missing, leave a space for it. 5. Remember, each adjective should be separated by a space.	0.872

Table 25. Prompt comparison for the **Hyperbaton** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis.

Approach	Optimized Prompt	Acc.
Human	Read the following sentence, then choose whether it is positive, negative, or neutral.	0.701
DLN1	1. Read the sentence carefully. 2. Identify the tone of the sentence. 3. Determine if the tone is positive, negative, or neutral. 4. Write the correct output (positive, negative, or neutral) based on your analysis. Additional Tips: - Pay attention to the words used in the sentence, such as "thanks", "I love", "good", "bad", "happy", "sad", etc. - Look for phrases that indicate a problem or issue.	0.802
DLN1-Momentum = 0.6	1. Read the tweet carefully and identify the key phrases, words, or sentences that convey the sentiment. 2. Determine if the tweet is expressing a positive, negative, or neutral sentiment. 3. Consider the tone and language used in the tweet. 4. Identify the correct output by classifying the sentiment as positive, negative, or neutral. 5. Remember, even if the tweet is mentioning a specific airline, the sentiment expressed may not necessarily be about the airline itself.	0.817

Table 26. Prompt comparison for the **Airline** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. DLN1-Momentum improves the prompt quality by inferring the training examples are tweets rather than sentence.

Approach	Optimized Prompt	Acc.
Human	Read the following question, then choose whether it is about a description, entity, expression, human, location or number.	0.451
DLN1	1. Read the question carefully and identify the main topic. 2. Determine if the topic is a person, place, thing, or idea. 3. Ask yourself if the topic is a living thing (human, animal, or plant), a location, a concept, an object, or an event. 4. Choose the correct category (human, location, entity, description, expression, or number) from the options. 5. Write the chosen category as the output.	0.586
DLN1-Momentum = 0.6	1. Read the question carefully and identify the type of information being asked. 2. Determine whether the question is seeking: * A specific quantity (number) * A person's name or a group of people (human) * A concept, expression, or idea that is not a person (entity) * A description or explanation of something (description) * A location or geographical area (location) 3. Use the guidelines to categorize the correct output type for each problem.	0.772

Table 27. Prompt comparison for the **Trec** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. DLN1-Momentum improves the prompt by presenting a description for each type of answer for Term Clarification

.

Approach	Optimized Prompt	Acc.
Human	Read the following sentence, then choose whether it is relevant to a disaster.	0.621
DLN1	Read the following sentence carefully and decide whether it is relevant to a disaster. Look for information in the sentence that indicates a potential catastrophe or serious harm to people, property, or the environment. If the sentence is about a disaster or could be related to one, answer "yes". Otherwise, answer "no". Remember, the focus is on disasters and the potential harm they can cause.	0.725
DLN1-Momentum = 0.6	Read the sentence carefully and determine if it's relevant to a disaster or crisis, such as a natural disaster, accident, or serious event. Ignore any non-disaster related information, such as personal opinions, jokes, or advertisements. Consider only the main topic or event mentioned in the sentence. If the sentence is about a disaster or crisis, answer "yes", otherwise, answer "no".	0.807

Table 28. Prompt comparison for the **Disaster** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. DLN1-Momentum improves the prompt by providing longer Solution Guidance and more detailed Term Clarification on disaster descriptions.

Approach	Optimized Prompt	Acc.
Human	Read the following review, then choose whether it is negative or positive.	0.785
DLN1	Read the review and identify whether it is a negative or positive review based on the language used. Consider the tone and the words used in the review. Determine if the review uses words that convey a sense of disapproval, criticism, or negativity. If the review uses words that convey a sense of approval, praise, or positivity, then the review is positive. Conversely, if the review uses words that convey a sense of disappointment, frustration, or dissatisfaction, then the review is negative.	0.810
DLN1-Momentum = 0.6	1. Read the review carefully and identify the words or phrases that convey a positive or negative sentiment. 2. Consider the meaning of the words or phrases in the context of the review. 3. Determine if the overall tone of the review is positive or negative. 4. If you're unsure, look for specific words or phrases that clearly indicate a positive or negative sentiment. 5. Write your answer as 'positive' or 'negative'.6. Make sure to consider the context.	0.856

Table 29. Prompt comparison for the **Mpqa** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. DLN1-Momentum generates prompts with assertive tone Emphasis on answering the question.

Approach	**Optimized Prompt**	**Acc.**
Human	Classify the input text as positive or negative.	0.840
DLN1	1. Read the input text carefully. 2. Determine the overall tone of the text. 3. Classify the tone as positive, negative, or neutral. If the text expresses a positive sentiment, such as praise, admiration, or enthusiasm, classify it as positive. If the text expresses a negative sentiment, such as criticism, dislike, or frustration, classify it as negative. If the text does not express a clear sentiment, classify it as neutral. 4. Write the corresponding output.	0.922
DLN1-Momentum = 0.6	1. Read the input text carefully and identify the main idea or sentiment expressed. 2. Determine whether the text expresses admiration, criticism, or a neutral opinion. If it expresses admiration, write ”positive”. If it expresses criticism, write ”negative”. If it is neutral, write ”neutral”. 3. Be cautious not to misinterpret the text, especially when it uses sarcasm or irony. 4. Practice reading between the lines and considering the tone and context of the text.	0.935

Table 30. Prompt comparison for the **SST2** task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. DLN1-Momentum generates prompts with more direct and accurate Solution Guidance

.

Approach	Optimized Prompt	Acc.
Human	Your input fields are: 1. ‘question‘ (str) Your output fields are: 1. ‘reasoning‘ (str) 2. ‘answer‘ (str) All interactions will be structured in the following way, with the appropriate values filled in. [[## question ##]] question [[## reasoning ##]] reasoning [[## answer ##]] answer [[## completed ##]] In adhering to this structure, your objective is: Given the fields ‘question‘, produce the fields ‘answer‘.	0.6
DLN1	Your task is to write a text output given a text input. The input will be in the format of a question, and you need to produce a reasoning and an answer. The structure of the output is: [[## question ##]] question [[## reasoning ##]] reasoning [[## answer ##]] answer To complete this task, you will need to follow these steps: 1. Read the input question carefully. 2. Understand what is being asked and what information is needed to solve the problem. 3. Write a clear and concise reasoning for your answer. Make sure your reasoning is logical and easy to follow. 4. Calculate the answer based on the given information. 5. Write the answer in the format of a sentence or a number. Remember to use proper mathematical operations and follow the given instructions carefully. If you are unsure about any step, you can try breaking it down into smaller parts or using a calculator to help you with the calculation. Also, double-check your answer to ensure it is reasonable and accurate. By following these steps, you should be able to produce accurate and well-structured outputs for the given inputs. Good luck!	0.69
DLN1-Momentum = 0.6	1. ‘question‘ (str) - Your output fields are: - 1. ‘reasoning‘ (str) - 2. ‘answer‘ (str) - All interactions will be structured in the following way, with the appropriate values filled in. [[## question ##]] question [[## reasoning ##]] reasoning [[## answer ##]] answer [[## completed ##]] - In adhering to this structure, your objective is: - Given the input ‘question‘, produce the fields ‘answer‘ by applying logical and mathematical operations to the provided information. - Ensure that your output is accurate and well-supported by your reasoning. - Note: - For calculations involving percentages, ratios, and proportions, use decimal equivalents. For example, 10% can be converted to 0.1. - For questions involving time, ensure accurate calculations of time periods and convert them to the required units. For example, if a time period is given in hours, convert it to days or minutes as needed. - Check your units and make sure they match the required units in the problem. - By following these guidelines, you will be able to produce accurate and well-supported answers. - Remember to: - Clearly explain your steps in the ‘reasoning‘ field. - Provide accurate calculations and assumptions. - Avoid ambiguity in your answers. - Ensure that your answer is concise and easy to understand. - For the ‘answer‘ field, provide the numerical value or solution to the problem. - For the ‘reasoning‘ field, provide a clear and concise explanation of your steps and calculations. - In the ‘completed‘ field, indicate that your task is finished and your output is ready for review. - By following these guidelines and the structure provided, you will be able to produce accurate and well-supported answers. - Note: - If you are unsure about any part of the problem or calculation, it is recommended that you review the problem and recalculate your answer before submitting it. - It is also important to double-check your units and ensure that they match the required units in the problem. - If you have any questions or concerns, please don’t hesitate to ask for help.	0.81

Table 31. Prompt comparison for the **GSM8K**(test) task, showing human, DLN1, and DLN1-Momentum prompts. Colored text highlights components such as Task Description, Term Clarification, Solution Guidance and Priority/Emphasis. Prompts generated by DLN1-Momentum are more detailed, structured, and pedagogically grounded, often mixing Solution Guidance and Priority/Emphasis to better support task completion and reduce ambiguity.

F.4. Iterative Workflow with Momentum based sampling

To illustrate the effectiveness of our momentum sampling module as an enhancement to the vanilla TextGrad framework, we use DLN1 as a representative method. We report all meta prompts generated by both DLN1 and DLN1-Momentum ($= 0.6$) throughout the iterative optimization process on the subjective classification task. Specifically, we present the evolving meta prompts—defined as the best-performing prompts selected from a candidate pool at each iteration. Only the changing meta prompts are shown under the H_1 setting, where the prompt generation temperature is set to 1.1 and early stopping is triggered if the best test accuracy does not improve over 5 consecutive iterations.

Compared to the standard DLN1 framework, DLN1-Momentum$= 0.6$ demonstrates a more gradual and stable improvement in test set accuracy across iterations. While DLN1 achieves a rapid performance boost in early iterations—jumping from 0.491 to 0.750 at iteration 1—its accuracy gains begin to plateau shortly thereafter. In contrast, DLN1-Momentum exhibits a slower but smoother trajectory, progressing from 0.491 to 0.66 in the first iteration and gradually increasing to 0.856 by iteration 5. This extended optimization process allows the momentum-based approach to explore a wider space of meta prompts and refine them more cautiously. As a result, DLN1-Momentum tends to converge toward stronger final prompts with higher accuracy, suggesting that the momentum mechanism effectively stabilizes prompt updates and reduces premature convergence on suboptimal prompt candidates.

Meta Prompt #	Meta Prompt	Test Acc.
0	Classify the input text as subjective or objective.	0.491
1	For each input text, classify whether the statement is a fact (objective) or an opinion/emotion (subjective). If the statement is describing a situation, event, or thing, it is likely an objective statement. If the statement is expressing a personal view, feeling, or evaluation, it is likely a subjective statement. Consider the language used, tone, and context to make your decision. Pay attention to words such as ”is”, ”seems”, ”feels”, ”right”, and ”wrong.”	0.750
2	1. Read the input text carefully. 2. Identify whether the text describes a situation, event, or thing (objective) or expresses a personal view, feeling, or evaluation (subjective). 3. Pay attention to words like ”is,” ”seems,” ”feels,” ”right,” and ”wrong.” 4. Consider the language used and the context in which it is used.	0.756
3	1. Read the input text carefully. 2. Identify whether the text describes a situation, event, or thing (objective) or expresses a personal view, feeling, or evaluation (subjective). Pay attention to words like ”is,” ”seems,” ”feels,” ”right,” and ”wrong.” 3. Consider the language used and the context in which it’s used.	0.8125
4	1. Read the text carefully. 2. Determine whether the text is describing a situation, event, or thing (objective) or expressing a personal view, feeling, or evaluation (subjective). Pay attention to words like ”is,” ”seems,” ”feels,” ”right,” and ”wrong.” 3. Use context clues like language, tone, and the presence of emotions to help determine the text type.	0.825

Table 32. DLN1 Iterative Prompt Workflow for Subj Task

Meta Prompt #	Meta Prompt	Test Acc.
0	Classify the input text as subjective or objective.	0.491
1	1. Read the input text carefully. 2. Determine if the text is describing an opinion, a feeling, or an evaluation about something. If it is, classify it as subjective. If it is describing a fact, situation, or event without expressing a personal opinion, classify it as objective. 3. Check if the text includes words or phrases that indicate an evaluation, such as ”good”, ”bad”, ”interesting”, ”amazing”, or phrases that express a personal point of view	0.66
2	1. Read the input text carefully and identify the purpose of the text. 2. Ask yourself, ”Is the text giving information about a fact, situation, or event without expressing a personal opinion?” If yes, classify it as objective. If not, consider whether the text is expressing an evaluation, opinion, or personal point of view. 3. Look for words or phrases that indicate an evaluation, such as ”good”, ”bad”, ”interesting”, ”amazing”, or phrases that	0.78
3	1. Read the text carefully and identify the purpose of the text. Does the text present a fact, situation, or event without expressing a personal opinion? Look for words or phrases that indicate an evaluation, such as ”good”, ”bad”, ”interesting”, or phrases that imply a subjective perspective. 2. Ask yourself, ”Is the text describing a situation or event without making a value judgment?” If yes, classify it as objective. If not, consider whether the text is expressing an evaluation	0.8
4	1. Determine whether the text presents a fact, situation, or event without expressing a personal opinion. Look for words or phrases that indicate an objective description, such as ”the film is a comedy” or ”the main character is a woman”. 2. Check for words or phrases that imply a subjective perspective, such as ”good”, ”bad”, ”interesting”, or ”brilliant”. 3. Ask yourself, ”Does the text describe the event or situation without expressing an opinion?”	0.85
5	1. Determine whether the text presents a fact, situation, or event without expressing a personal opinion. Look for objective words or phrases that describe a description, such as ”the film is a comedy” or ”the main character is a woman”. Avoid subjective words or phrases that convey a personal perspective, such as ”good”, ”bad”, ”interesting”, or ”brilliant”. If you find a subjective phrase, it likely expresses an opinion and is not a simple fact or description.	0.856

Table 33. DLN1-Momentum=0.6 Iterative Prompt Workflow for Subj Task