SynCode: LLM Generation with Grammar Augmentation

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Abstract

LLMs are widely used in complex AI applications. These applications underscore the need for LLM outputs to adhere to a specific format, for their integration with other components in the systems. Typically the format rules – e.g., data serialization formats such as JSON, YAML, or Code in Programming Language – are expressed as context-free grammar (CFG). Due to the hallucinations and unreliability of LLMs, instructing LLMs to adhere to specified syntax becomes an increasingly important challenge.

We present SynCode, a novel framework for efficient and general syntactical decoding with LLMs, to address this challenge. SynCode ensures soundness and completeness with respect to the CFG of a formal language, effectively retaining valid tokens while filtering out invalid ones. SynCode uses an offline-constructed, efficient lookup table, the *DFA mask store*, created from the DFA (Deterministic Finite Automaton) of the language's grammar for efficient generation. SynCode seamlessly integrates with any language defined by CFG, as evidenced by experiments focusing on generating JSON, SQL, Python, and Go outputs. Our experiments evaluating the effectiveness of SynCode for JSON generation demonstrate that SynCode eliminates all syntax errors and significantly outperforms state-of-the-art baselines. Furthermore, our results underscore how SynCode significantly reduces 96.07% of syntax errors in generated Python and Go code, showcasing its substantial impact on enhancing syntactical precision in LLM generation.

1 Introduction

Recent research has shown that transformer-based large language models (LLMs) can play a pivotal role within compound AI systems, where they integrate with other software tools (Zaharia et al., 2024; Mialon et al., 2023). For example, OpenAI's code interpreter (OpenAI, 2024) generates and executes Python programs automatically while responding to user prompts. Similarly, Wolfram Alpha (wolfram, 2024) translates user queries about mathematical questions into a domain-specific language (DSL) for utilizing various tools. LLMs are utilized in various other applications to translate natural language text into formal languages, such as inputs to logic solvers (Pan et al., 2023; Olausson et al., 2023) and theorem provers (Wu et al., 2022; Yang et al., 2023), among others. In all these applications, the LLM output is expected to follow a certain syntactic structure. However, challenges such as hallucination and non-robustness make LLMs unreliable for such automated systems (Liang et al., 2023). Moreover, recent theoretical (Hahn, 2020; Yang et al., 2024) and empirical (Ebrahimi et al., 2020; Bhattamishra et al., 2020; Delétang et al., 2023) research suggests that language models based on transformers show difficulty in learning basic formal grammars.

The interaction between software tools and LLMs commonly occurs through data serialization formats like JSON or YAML, or code in domain-specific or general-purpose programming languages, such as Python or Go. Despite advancements in techniques such as fine-tuning and prompt engineering, which enhance the model's ability, these approaches fall short of fully addressing the challenge of syntactical accuracy in generated output. This problem is especially prominent in two common scenarios: (1) using open-source models, which are typically relatively small, and (2) generating text for formal languages with relatively modest representation in the LLM's training data.

Modern LLMs generate text sequentially, from left to right, one token at a time. For each prefix, the model computes a probability distribution over a predefined vocabulary to predict the next token. The

LLM's decoding algorithm dictates how these probabilities are used to generate the token sequence. Very recently, researchers have proposed new techniques for grammar-guided generation to enhance the syntactical accuracy of LLMs by modifying the decoding algorithm. Although they ensure that the model consistently selects tokens that adhere to a specified formal language (Scholak et al., 2021; Poesia et al., 2022; Gerganov and et. al., 2024; Willard and Louf, 2023), the existing approaches for grammar-guided generation either suffer from high error rates, resulting in syntactically incorrect output or impose significant run time overhead in the inference:

- **Issues with syntactical accuracy:** The language grammar consists of *the terminals*, fundamental building blocks of the language (e.g., keywords, operators). Typically, a lexer creates lexical tokens from the input, each token associated with a terminal from the grammar. The LLM tokens form part of the model's fixed vocabulary, defined before training, and do not directly correspond to lexical tokens associated with any specific grammar. This discrepancy, known as *token misalignment*, presents a significant challenge in ensuring precise grammar-guided generation (Poesia et al., 2022). Thus, formally showing the soundness of the algorithm poses a challenge for ensuring the precision of the approach.
- **Issues with high computational overhead:** Typically, the computational complexity of additional operations performed for syntactical generation is lower than the standard LLM generation operations needed for propagating the input through LLM layers. However, these syntactical generation operations are typically executed sequentially on a CPU, in contrast to the GPU-accelerated LLM generation, adding to the run time. Achieving low inference overhead faces two primary challenges for syntactical LLM generation. First, the algorithm should facilitate offline computations that minimize the overhead during inference. Second, it should effectively utilize available hardware resources and offload additional computations to modern hardware, such as GPUs, to enable parallel computation.
- **Issues with generality:** Prior works are restricted to specific LLM decoding schemes (Scholak et al., 2021; Lundberg et al., 2023). A major challenge for generality is designing a composable algorithm that can integrate with any decoding strategy such as greedy, beam search, and different types of temperature sampling.

Our goal is to make grammar-guided generation precise and efficient by imposing formal grammar constraints on LLM generations, ensuring the output adheres strictly to the predefined syntax.

SynCode. SynCode is an efficient and general approach for generating syntactically correct output. SynCode takes a context-free grammar (CFG) represented with extended Backus–Naur form (EBNF) rules and ensures that the LLM output follows the provided grammar. SynCode algorithm is general and can be composed with any existing LLM decoding algorithm, including greedy, beam search, and sampling.

During the LLM decoding stage, where LLM selects the next token, SynCode employs a strategic two-step approach. In the initial step, it leverages partial output to generate sequences of terminals that can follow the partial output called *accept sequences*. This reduction to the level of terminals—a closer abstraction to language grammar than LLM tokens—simplifies the problem. Simultaneously, SynCode computes a remainder from the partial output, representing the suffix that may change its terminal type in subsequent generations. In the second step, SynCode algorithm walks over the DFA using the remainder and uses the mask store to compute the mask (a boolean array to filter the vocabulary) specific to each accept sequence. By unifying masks for each accept sequence SynCode gets the set of syntactically valid tokens.

To ensure the efficiency of SynCode's syntactic generation, we propose a novel data structure called *DFA mask store* which is pre-computed offline. DFA mask store is a lookup table derived from Deterministic Finite Automata (DFA) representing the terminals of the language grammar. SynCode algorithm can efficiently compute the syntactically valid next LLM tokens by leveraging this mask store. Moreover, the SynCode algorithm offers the additional benefit of parallelizing a substantial portion of the syntactical LLM generation computations by offloading them to a GPU.

We demonstrate that the SynCode algorithm is *sound* – ensuring it retains all syntactically valid tokens at every generation step. SynCode is also *complete* under specific conditions – affirming it rejects every syntactically invalid token.

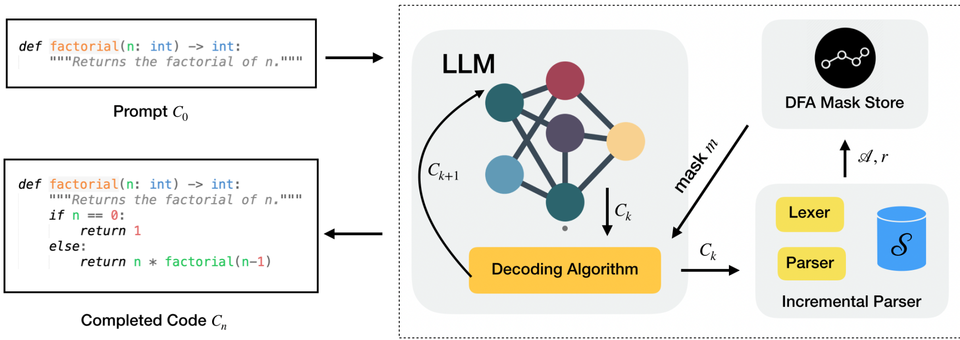

Figure 1: In the SynCode workflow, the LLM takes partial output C_k and generates a distribution for the next token t_{k+1}. The parser processes C_k to produce accept sequences $\mathcal{A}$ and remainder r. These values are used by the DFA mask store to create a token mask, eliminating syntactically invalid tokens. The LLM iteratively generates a token t_{k+1} using the distribution and the mask, appending it to C_k to create the updated code C_{k+1}. The process continues until the LLM returns the final code C_n based on the defined stop condition.

The SynCode framework seamlessly integrates with any language defined by deterministic CFGs and scales efficiently to generate code for general-purpose programming languages (GPLs). We evaluate SynCode's ability to guide the Llama-2-7B-chat and Gemma2-2B-it models with the JSON grammar to generate valid JSON completions to prompts from the JSONModeEval (NousResearch, 2024) dataset. We empirically show that LLMs augmented with SynCode do not generate any syntax errors for JSON and that guiding Gemma2-2B-it generation with SynCode achieves 100% JSON schema validation accuracy. We evaluate SynCode on generating SQL queries from the text in Spider (Yu et al., 2018) and show that SynCode improves both compilation rate and execution accuracy. Further, we evaluate the augmentation of SynCode with a diverse set of state-of-the-art LLMs for the code completion tasks using problems from the HumanEval and MBXP datasets (Athiwaratkun et al., 2023). Our experiments, conducted with CFGs for a substantial subset of Python and Go, demonstrate that SynCode reduces 96.07% of the syntax errors for Python and Go on average. The remaining syntax errors persist because the LLM fails to halt generation before reaching the maximum generation limit defined in our experiments.

Contributions. The main contributions of this paper are:

- ⋆ We present a parsing-based technique for decoding of LLMs by designing novel algorithms that allow us to efficiently generate syntactically correct output.
- ⋆ We implement our approach into a scalable and general framework named SynCode that can work with any formal language with user-provided context-free grammar.
- ⋆ We present an extensive evaluation of the performance of SynCode in generating syntactically correct output for JSON, SQL and two general-purpose programming languages Python and Go.

2 Background

In this section, we provide the necessary background on LLMs and formal language grammar.

Notation. Let the alphabet Σ be a finite set of characters. We use ϵ to denote an empty string. Given a set S, we use S^i to denote the set of all i-length sequences that can be formed by selecting elements from S, and $S^* = \bigcup_{i \in \mathbb{N}} S^i$. Thus Σ^* represents the set of all strings over characters in Σ, including the empty string ϵ. Further, we use Σ^+ to denote $(\Sigma^* - \epsilon)$. Given two strings $w_1, w_2 \in \Sigma^*$, we use $w_1.w_2$ to denote string obtained by concatenating w_2 to w_1. All symbols used in the paper are listed in Appendix A.1.

2.1 Language Models

Current language models (LM) operate on vocabulary $V \subseteq \Sigma^*$ of tokens. A tokenizer takes an input prompt $C_0 \in \Sigma^*$, which is a sequence of characters, as input and converts C_0 into a sequence of tokens $t_1, t_2, \ldots, t_k$. Figure 2 shows a typical tokenization method, where common words (e.g., `def`) have their own token (even with a space in front), while rare words (e.g., `incr_list`) are split into multiple tokens. In order to generate the next token, the LM $M : V^* \to \mathbb{R}^{|V|}$ takes as input the sequence of tokens $t_1, t_2, \ldots, t_k$, and outputs a vector of scores z over the vocabulary: $z = M(t_1, t_2, \ldots, t_k)$. The softmax function $softmax(z_i) = \exp(z_i)/\sum_j(\exp(z_j))$ transforms z into a probability distribution over the vocabulary V.

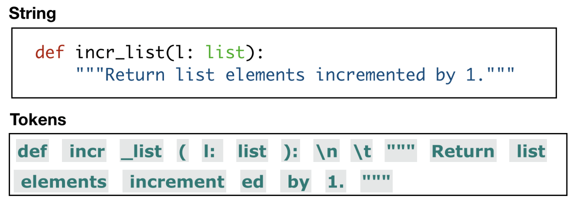

Figure 2: Tokenization of a string.

Decoding. Building upon this, the language model M is recurrently applied to generate a sequence of tokens $t_1, t_2 \ldots t_k$. When choosing the $(k+1)$-th token, the probability distribution for the next token is obtained through softmax$(M(t_1, t_2 \ldots t_k))$. Various approaches for token selection from this distribution have been explored in the literature such as greedy decoding, sampling, and beam search. Each technique is repeated until the prediction of a special end-of-sequence token, `EOS`, or the fulfillment of another stopping criterion. This iterative process is equivalent to sampling from a distribution over V^*, potentially resulting in multiple feasible decodings.

Constrained Masking. In the context of decoding, we encounter scenarios where excluding specific tokens at particular positions becomes crucial (e.g., excluding harmful words). This implies we can disregard these tokens and proceed with decoding based on the remaining set. An algorithm for such masking relies on a function f_m to generate the mask m based on the exact use case. In the mask $m \in \{0, 1\}^{|V|}$, '1' indicates a viable token, and '0' signifies a discarded one. Decoding methods mentioned earlier can be applied to $m \odot softmax(z)$, where $\odot$ represents element-wise multiplication. The resultant vector should be scaled by $1/\sum_i(m \times softmax(z))_i$ to restore correct probabilities. Algorithm 1 presents the steps for masked decoding. In SynCode, we use the constrained masking technique to exclude syntactically invalid tokens.

Algorithm 1 Masked LLM Generation

Inputs: M: LLM, $\mathcal{T}$: tokenizer, C_0: input prompt string, f_m: function that generates mask, n_{max}: maximum generated tokens, D: any decoding algorithm
Output: string C_n

```
function MaskedGenerate(M, T, f_m, C_0)
    T_cur ← Tokenize(T, C_0)
    for i ∈ {1, ... n_max} do
        scores ← M(T_cur)
        m ← f_m(T_cur, T)
        scores ← m ⊙ scores
        t_i ← D(scores)
        if t_i = EOS then
            break
        T_cur ← append(T_cur, t_i)
    C_n ← Detokenize(T, T_cur)
    return C_n
```

2.2 Formal Language Grammar

A formal language syntax is represented by defining a grammar. A formal grammar is essentially a set of production rules that describe all possible strings in a given language. A grammar consists of terminal and nonterminal symbols, where terminal symbols are the actual characters or tokens in the language, while nonterminal symbols are placeholders used to define patterns or structures within the language.

The syntax for most programming languages can be defined using context-free grammar (CFG). CFG is a formal grammar that consists of production rules that can be applied to a nonterminal symbol regardless of its context. In CFG, each production rule is of the form $E \to \beta$ with E a single nonterminal symbol, and β a string of terminals and nonterminals (β can be empty). Regardless of which symbols surround it, the single nonterminal E on the left-hand side can always be replaced by β on the right-hand side.

Terminals. We use Γ to denote the set of terminals in the grammar. Regular expressions are used to describe the terminals. For instance, A regular expression ^[0-9]$^+$ is used for an integer literal: This regular expression describes a sequence of one or more digits (0 to 9). We use ρ to denote a regular expression and $L(\rho) \subseteq \Sigma^*$ to denote the language recognized ρ. Regular expressions are often associated with the creation of Deterministic Finite Automata (DFAs). A DFA is a theoretical construct used to recognize patterns specified by regular expressions.

Definition 1 (DFA)**.** *A deterministic finite automaton (DFA) D is a 5-tuple, $(Q, \Sigma, \delta, q_0, F)$, consisting of a finite set of states Q, a finite set of input symbols called the alphabet Σ, a transition function $\delta : Q \times \Sigma \to Q$, an initial state $q_0 \in Q$ and a set of accept states $F \subseteq Q$.*

Let $w = a_1 a_2 \ldots a_n$ be a string over the alphabet Σ. The DFA computation $\delta^* : Q \times \Sigma^* \to Q$ on a string w is defined as $\delta^*(r_0, w) = r_n$ when $r_{i+1} = \delta(r_i, a_{i+1})$, for $i = 0, \ldots, n-1$. The automaton D accepts the string w if $\delta^*(q_0, w) \in F$.

Lexer. We assume lexical analysis with a 1-character lookahead and no backtracking. This assumption is crucial for the efficiency of SynCode algorithm.

Definition 2 (Lexer)**.** *The function* Lex *is defined to take partial output $C_k \in \Sigma^*$ as input and produce a sequence $l_1, l_2, \ldots, l_f$ of lexical tokens where $l_i \in \Sigma^*$.*

3 Overview

3.1 Illustrative Example

Consider an example grammar in Figure 3 that uses the Lark EBNF syntax for defining the grammar production rules. The grammar represents a Domain-Specific Language (DSL) consisting of arithmetic expressions with basic operations like addition, subtraction, multiplication, and division over integers and floating point numbers. It also includes support for parentheses to specify precedence and allows functions like exponential (math_exp), square root (math_sqrt), sine (math_sin), and cosine (math_cos) to be applied to expressions.

```
start: expr

expr: term
      | expr "+" term
      | expr "−" term

term: factor
      | term "*" factor
      | term "/" factor

factor:  INT | FLOAT | "(" expr ")" | function "(" expr ")"

function: "math_exp" | "math_sqrt" | "math_sin" | "math_cos"

INT: /[0-9]+/
FLOAT: /[0-9]+\.[0-9]+/

%ignore " "
```

Figure 3: Example grammar for illustration.

The symbols in the grammar such as *expr* and *factor* that can expand into other symbols through the application of production rules are called non-terminals. Symbols such as (or INT cannot be further expanded and are called terminals. Let the set $\Gamma = \{$*lpar, rpar, add, sub, mult, div, int, float, math_exp, math_sqrt, math_sin, math_cos*$\}$ represent the set of all terminals of the grammar. The terminal *int* is defined by the regular expression [0-9]$^+$, and *float* is defined by the regular expression [0-9]$^+$.[0-9]$^+$. We use terminals *lpar, rpar, add, sub, mult, div, math_exp, math_sqrt, math_sin, math_cos*, to denote the strings `(`, `)`, `+`, `*`, `/`, `math_exp`, `math_sqrt`, `math_sin`, `math_cos` respectively.

Task. Consider an LLM that is used to translate a natural language text to an expression in the DSL defined above. Since LLMs are typically not good at mathematical calculations, it is common to instead let the LLM generate intermediate outputs in a certain syntax, and an interpreter of the DSL then computes the LLM's output into accurate results (Mialon et al., 2023). Figure 4 presents the prompt we use for our illustrative example, containing 2 question-answer pairs before the actual question that we want the LLM to answer. Providing question-

```
Question: Can you add sin of 30 degrees and cos of 60 degrees?
Answer: math_sin(30) + math_cos(60)

Question: what is exponent of addition of first 5 prime numbers?
Answer: math_exp(2 + 3 + 5 + 7 + 11)

Question:  what is the area of equilateral triangle with each side 2.27?
Answer:
```

Figure 4: Prompt for the example which is provided as input to the LLM.

answer examples before asking the actual questions is called few-shot prompting (2-shot in this case) and significantly improves the model's accuracy (Brown et al., 2020).

Standard LLM Generation. As described in Section 2, the standard LLM first tokenizes the input and then iteratively predicts the next token from its vocabulary V. Figure 5 presents the output from the LLaMA-7B model and our SynCode when given the Fig. 4 prompt. The output of the model is not a valid program in the DSL; it uses functions math_area and math_side that do not exist in the grammar. Further, LLaMA-7B does not stop after generating the answer to our question and continues to generate more irrelevant question-answer pairs. SynCode on the other hand guarantees the syntactic validity of the LLM's output by excluding syntactically invalid choices when generating each token. For example, after generating `math`, SynCode excludes `_area` and other choices from the LLM's vocabulary. The LLM opts for `_sqrt` which is the top syntactically valid choice and continues the generation from `math_sqrt`.

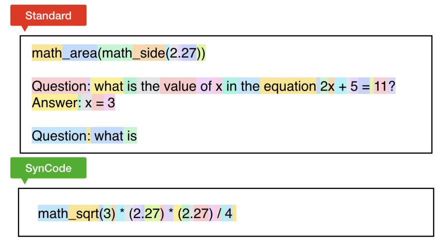

Figure 5: Output from LLM without and with SynCode. The colors represent the tokenization of the output.

Constrained Decoding. Let G denote the grammar in our example and $L(G) \subseteq \Sigma^*$ denote all syntactically valid strings in the grammar. Ideally, we want the final LLM output C_n to be in $L(G)$. Strings such as `math_exp(2 + 3 + 5 + 7 + 11)` and `math_sin(30) + math_cos(60)` belong to $L(G)$ as they are syntactically valid. Let C_k denote the LLM's partial output during the k-th iteration of LLM generation. Suppose $L_p(G)$ denotes all prefixes of $L(G)$, i.e., all strings that can be extended to a syntactically valid output. `math_sin(30` and `math_sin(30) + math` are in $L_p(G)$ as they can be extended to be syntactically valid. By ensuring that at each intermediate step, the invariant that the LLM partial generation C_k is in the set $L_p(G)$ is maintained, we can guarantee that upon completion of the generation process, C_n will indeed be syntactically valid, i.e., $C_n \in L(G)$. This ensures that an intermediate output such as `math_area` which is not in $L_p(G)$ is never generated by the model.

3.2 SynCode Algorithm

A key challenge in syntactic generation is token misalignment, where LLM tokens do not directly correspond to lexical tokens from the grammar. The main reason for the high error rate in syntactic generation in prior works is the lack of formalization in their approaches (Section 6). Our work addresses this challenge by providing an algorithm that is provably sound — retains all syntactically valid tokens and is complete under specific conditions—rejecting every syntactically invalid token at every generation step.

Another significant challenge for efficiency is developing a novel algorithm that facilitates offline computations that minimize the overhead during inference. SynCode tackles this challenge by creating a novel structure called the DFA mask store offline. For a given grammar G and vocabulary V, this mask store is constructed once and can be used across all generations. DFA mask store maps states of DFAs (corresponding to terminals in the grammar G) to boolean masks $m \in \{0,1\}^{|V|}$ over the vocabulary. This approach also benefits from parallelizing a substantial portion of the syntactical LLM generation computations by offloading them to a GPU during inference.

Furthermore, it is challenging to ensure generality with efficiency. Many prior works are restricted to syntactic generation with a specific type of decoding (Scholak et al., 2021; Lundberg et al., 2023). At k-th LLM iteration, for partial LLM output C_k, let $V_k \subseteq V$ denotes the subset of vocabulary such that for any token $t \in V_k$ the intermediate generation continues to maintain the invariant $C_k.t \in L_p(G)$. Our formulation for computing V_k from V is highly general and can be integrated with any decoding algorithm, such as greedy, sampling, or beam-search. Any algorithm that could potentially be applied to V can instead be applied to V_k. The mask store allows more efficient computation of a subset of tokens V_k.

SynCode works in two steps: first, it parses C_k and computes the unparsed remainder $r \in \Sigma^*$ along with the acceptable terminal sequences $\mathcal{A}$ (formally defined in Section 4.2). In the second step, SynCode utilizes

r, $\mathcal{A}$, and the mask store. This step involves traversing the DFA and performing a few lookups within the DFA mask store to obtain a subset of tokens V_k. In the following sections, we elaborate on these steps using our illustrative example.

Parsing Partial Output. SynCode's parsing of partial output C_k begins with lexing C_k. We assume our lexer has a 1-character lookahead and no backtracking. This assumption ensures that LLM's future generations do not alter the lexical types of any previous lexical tokens except for the final lexical token. The remainder r denotes the suffix of C_k that may still change its lexical type in subsequent iterations. We define two cases for assigning r:

- Case 1 is when C_k contains an unlexed suffix u, and here we assign $r = u$. For example, $C_k =$ `math_sqrt(3) * (2.` is lexed as `math_sqrt`, `(`, `3`, `)`, `*`, `(`, `2.`, where `math_sqrt`, `(`, `3`, `)`, `*`, `(` are lexical tokens of type *math_sqrt, lpar, int, rpar, mult, lpar*, respectively. Here `2.` (`2` followed by a `.`) is unlexed suffix which we assign as the remainder r.
- Case 2 is when C_k ends with a complete lexical token, where r is assigned the value of the final lexical token. Hence, $C_k =$ `math_sqrt(3) * (2` is lexed as `math_sqrt`, `(`, `3`, `)`, `*`, `(`, `2`. Where `math_sqrt`, `(`, `3`, `)`, `*`, `(` are lexical tokens of type *math_sqrt, lpar, int, rpar, mult, lpar*, respectively. Although `2` is the complete final lexical token with type *int*, it is assigned as the remainder since in the subsequent iteration it may even change its lexical type to *float*.

In both cases, our lexer assumption ensures that the portion of C_k excluding the remainder r will retain its lexical tokenization in subsequent LLM iterations. The assumption is crucial to enable incremental parsing and ensures that that the remainder r is always small, both of which contribute to reducing time complexity.

Accept Sequences. Given a sequence of lexical tokens $l_1, \ldots l_f$, we use a bottom-up LR parser to compute what types of lexical tokens are acceptable next according to the grammar. If at a certain point in the generation, we have lexical tokens `math_sqrt`, `(`, `3`, `)`, `*`, `(`, `2.27` then the immediate next lexical token can be of type *rpar*, *add* or *mult*. We define an accept sequence as a function of the parsed partial output (excluding the remainder) as a sequence of terminals such that those terminals can follow the currently parsed output (Definition 7). For instance, in the case $C_k =$ `math_sqrt(3) * (2.27`, $\{rpar\}$, $\{add\}$ and $\{mult\}$ all are 1-length accept sequences. $\{add, int\}$ and $\{add, float\}$ are some of the 2-length accept sequences for this example that can follow the current partial output. In Section 4.2, we show how we efficiently compute accept sequences of length 1 and 2 using an LR(1) parser, leveraging its immediate error detection property (Aho and Johnson, 1974). Further, we discuss how an LR(κ) parser can be used to compute accept sequences of length κ efficiently. However, in practice, SynCode can effectively operate with shorter accept sequences while still ensuring the soundness of syntactical generation (see Theorem 1), thereby avoiding the high memory needed for LR(κ) parsers for large values of κ.

DFA Mask Store. SynCode parsing step partitions partial output C_k into lexically fixed part $C_k^\square$ and remainder r. The accept sequences $\mathcal{A}$ are computed using the parser state on parsing $C_k^\square$ and denote the terminals that can follow $C_k^\square$. Thus the problem of obtaining subset V_k of tokens that will lead to syntactical continuation can be reduced to aligning accept sequence $\Lambda \in \mathcal{A}$ with the string $r.t$ obtained by concatenating remainder r and LLM token t in the vocabulary. One approach is to iterate through LLM vocabulary V and verify this alignment for each token t individually. However, this method is inefficient due to the need for matching $|V|$ tokens with $|\mathcal{A}|$ terminal sequences. In SynCode algorithm, the precomputed DFA mask store is crucial for allowing efficient computation of acceptable tokens V_k. Next, we show how the mask store maps the states of DFAs of the terminals and a sequence of terminals to masks over the vocabulary to enable this process.

Given a remainder r and any accept sequence $\Lambda \in \mathcal{A}$, we want to check for a token $t \in V$, if $r.t$ aligns or partially matches with Λ. We formally define this notion of partial match in Definition 8. We establish a connection between the match of a terminal sequence and a string through the DFAs corresponding to the terminals.

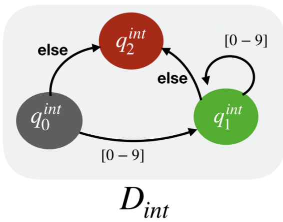

Figure 6: DFA for terminal *int*.

Figure 6 presents a DFA for the terminal int. In this DFA, q_0^{int} is the start state, and q_1^{int} is an accept state. Further, we say that q_0^{int}, q_1^{int} are *live* states since there is a path from those states to an accept state and the state q_2^{int} is not a *live* state.

Consider the partial output $C_k =$ `math_sqrt(3) * (2`. As described above, in this case, the output is split in the parsed part `math_sqrt(3) * (` and the last lexical token `2` which is the remainder. $\{int, add\}$, $\{int, rpar\}$, $\{float\}$ are some of the accept sequences. For each of these accept sequences, we want to compute tokens $t \in V$ such that appending `2` and t i.e. `2`$.t$ partially matches the accept sequence.

Consider an accept sequence $\Lambda = \{float, rpar\}$. Figure 7 displays the DFAs corresponding to the terminals in Λ. If we begin from the initial state q_0^{float} of D_{float} and change the current DFA state according to the characters in r, in our example with $r =$ `2`, the resulting state of the DFA is q_1^{float}. We observe that any token $t \in V$ is acceptable if continuing the DFA walk from q_1^{float} ends on a live state. We also allow a transition from the end state and start state of DFAs of subsequent terminals in the accept sequence as shown by the dotted arrow. The partial match of $r.t$ and Λ can thus be equivalently checked by doing a walk over the DFAs. Tokens such as `11`, `.`, `.1`, and `.27)` are some of the tokens where initiating a walk from q_1^{float} leads to reaching one of the live states. For example, by consuming `.27)`, we reach q_1^{rpar}, which is a live state. Consequently, SynCode approves `.27)` as a valid continuation from $C_k =$ `math_sqrt(3) * (2`.

Our key insight for achieving efficiency is that for each DFA state, we can precompute LLM tokens that will lead to a transition to a live state starting from that state. Precomputing these sets can significantly reduce the computation required during inference. Further, these precomputed set of LLM tokens can be stored as boolean masks for efficiently combining them during inference. Given a DFA state q and any sequence terminals of length α, the mask store maps $\mathcal{M}_\alpha(q, \Lambda) = m$, where $m \in \{0,1\}^{|V|}$ is the mask over vocabulary. During the inference time, for each accept sequence $\Lambda \in \mathcal{A}$, we first consume r and walk over the first DFA in the accept sequence. We then use the map $\mathcal{M}_\alpha$ on the current DFA state to get the mask m_Λ of valid tokens for Λ. Hence, for each accept sequence $\Lambda \in \mathcal{A}$, we require a walk over a DFA and a lookup in the mask store to obtain m_Λ.

Finally, we combine these masks obtained for each acccept sequence to get the masks of all syntactically valid tokens by computing their union $\bigcup_{\Lambda \in \mathcal{A}} m_\Lambda$. In practice, these masks can be stored as tensors and can be combined efficiently using a small number of tensor union operations. We show in Theorem 1 that this combined mask overapproximates the set V_k, ensuring the soundness of our approach. Further, we show that for the LR parser with larger lookahead, our approach is complete and ensures the combined mask is exactly V_k (Theorem 2).

Bringing It All Together. In our example, SynCode improves the LLM's output by guiding the generation. Initially, the LLM produces `math` as C_1. Next, SynCode excludes LLMs top choices such as `_area`, `_tri`, and `_p` from the vocabulary, leading the decoding algorithm to select `_sqrt`. Further, even in the 12th iteration where the LLM outputs $C_{11} =$ `math_sqrt(3)/4 * (2.27`, SynCode filters out

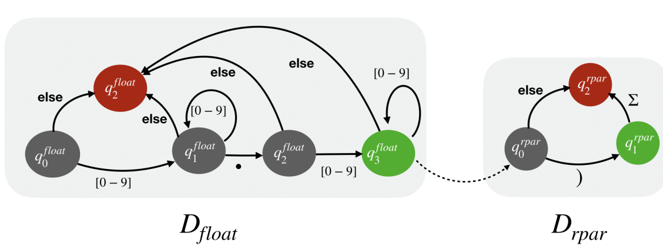

Figure 7: DFAs for accept sequence $\Lambda = \{float, rpar\}$.

the LLM's preferred choice `^` from the vocabulary. Instead, the LLM opts for $*$, eventually generating $C_n =$ `math_sqrt(3)/4 * (2.27) * (2.27)`, which is syntactically correct i.e. $C_n \in L(G)$ and also semantically accurate.

3.3 Time Complexity

At each decoding step in SynCode, the most resource-intensive tasks are computing accept sequences and generating the mask using r and $\mathcal{A}$. In Section 4.6, we demonstrate that our implementation, leveraging LR(1) parsing, efficiently constructs 1 and 2-length accept sequences. We show that the complexity of SynCode at each decoding step is $O(T_\cup \cdot |\mathcal{A}|)$, where $T_\cup$ represents the time needed for boolean mask union operations. Typically, $|\mathcal{A}|$ is small (<10 on average in our experiments) and in the worst case, it equals the size of set of all terminals $|\Gamma|$ in the grammar. For our largest Python grammar, $|\Gamma|$ is 94. Modern hardware, especially with GPUs, can perform these vectorized union operations efficiently (Paszke et al., 2019b), making the SynCode algorithm efficient in practice.

4 Syntactically Correct Generation

This section describes our main technical contributions and the SynCode algorithm.

4.1 Syntactical Decoding Problem

Given a language with grammar G, let $L(G) \subseteq \Sigma^*$ denote the set of all syntactically valid outputs according to the grammar G. For a grammar G, $L_p(G)$ represents the set of all syntactically valid partial outputs. If a string w_1 belongs to $L_p(G)$, then there exists another string w_2 such that appending w_2 to w_1 results in a string that is in the language defined by G. Formally,

Definition 3 (Partial Outputs)**.** *For grammar G, $L_p(G) \subseteq \Sigma^*$ denotes all syntactically valid partial outputs. Formally, if $w_1 \in L_p(G)$ then $\exists w_2 \in \Sigma^*$ such that $w_1.w_2 \in L(G)$*

For a grammar G and a partial output C_k belonging to the set of prefix strings $L_p(G)$, the syntactical decoding problem aims to determine the set V_k of valid tokens from a finite vocabulary V such that appending any token $t \in V_k$ to C_k maintains its syntactic validity according to the grammar G.

Definition 4 (Syntactical Decoding)**.** *For grammar G, given partial output $C_k \in L_p(G)$ and finite token vocabulary $V \subset \Sigma^*$, the syntactical decoding problem is to compute the set $V_k \subseteq V$ such that for any $t \in V_k, C_k.t \in L_p(G)$*

We next present SynCode's key aspects to solve this problem:

- In the initial step, it parses C_k and computes the unparsed remainder $r \in \Sigma^*$ along with the acceptable terminal sequences $\mathcal{A}$ (Section 4.2).
- In the second step, SynCode utilizes r, $\mathcal{A}$, and the precomputed mask store. This phase involves traversing the DFA and performing a few lookups within the DFA mask store to obtain the set of syntactically valid tokens t capable of extending C_k (Section 4.3).
- Consequently, SynCode efficiently computes the set of syntactically valid tokens. We show the soundness and completeness of our approach in Section 4.4.
- We further discuss the theoretical complexity of SynCode in Section 4.6 and the SynCode framework in Section 4.7.

4.2 Parsing Partial Output

In this section, we describe the remainder r and accept sequences $\mathcal{A}$ returned by the parsing step.

Remainder. SynCode uses a lexer to convert C_k to sequence of lexical tokens $l_1, l_2 \dots l_f \in \Sigma^*$. Each lexical token l_i is associated with a terminal type τ_i, where $l_i \in L(\rho_{\tau_i})$ (ρ_{τ_i} is the regular expression for terminal

τ_i). We assume our lexer uses a 1-character lookahead without backtracking. This ensures that the lexical types of previous tokens in C_k remain unchanged, except for the final token. The remainder r represents the suffix of C_k that could potentially change its lexical type in future iterations. Thus the remainder r is assigned such that it is either unlexed because it does not match any terminal, or has been lexed but might undergo a different lexing in subsequent iterations when C_k is extended by the LLM by appending tokens. This assumption is crucial for enabling incremental parsing and ensures that the remainder r remains small, which contributes to reducing overall time complexity. SynCode assigns the remainder according to the following two cases:

Case 1: $C_k = l_1.l_2 \ldots l_f$ Assuming a standard lexer with 1-character lookahead and no backtracking, all lexical tokens $l_1, l_2, \ldots, l_{f-1}$ remain unchanged upon extending C_k. However, the final lexical token l_f may change. For example, in Python partial output in the k-th LLM iteration, if the final lexical token is $l_f = \boxed{\texttt{ret}}$ and the language model generates the token $\boxed{\texttt{urn}}$ in the next iteration, the updated code results in the final lexical token becoming $l_f = \boxed{\texttt{return}}$. This transition reflects a transformation from an identifier name to a Python keyword in the subsequent iterations. Thus, r is assigned the value l_f, i.e., $r = \boxed{\texttt{ret}}$ for k-th iteration in our example.

Case 2: $C_k = l_1.l_2 \ldots l_f.u$**:** Here, $u \in \Sigma^*$ is the unlexed remainder of C_k. In this case, considering the 1-character lookahead of the lexer, the types of $l_1, l_2, \ldots, l_f$ do not change upon extending C_k. Consequently, r is assigned value u of the suffix that remains unlexed.

SynCode parsing step partitions partial output C_k into lexically fixed part $C_k^\square$ and remainder r. Given a sequence $\Lambda = \tau_0, \tau_1, \ldots, \tau_f$, we simplify notation by using $L(\Lambda) = L(\rho_{\tau_0} \cdot \rho_{\tau_1} \ldots \rho_{\tau_f})$ throughout the rest of the paper.

Definition 5 (Partial Parse)**.** *Given the partial output* $C_k \in \Sigma^*$*, the partial parse function* $pparse : \Sigma^* \to \Gamma^* \times \Sigma^*$ *returns a terminal sequence* $\Lambda^\square$ *and remainder* r *such that* $C_k = C_k^\square.r$ *and* $C_k^\square$ *is parsed as* $\Lambda^\square$*. i.e.* $C_k^\square \in L(\Lambda^\square)$*.*

Accept Sequences. A sentence is a sequence of terminals. A grammar G describes a (possibly infinite) set of sentences, that can be derived by using the production rules of the grammar. We use $L^\Gamma(G) \subseteq \Gamma^*$ to denote the valid sequences of terminals that can be derived from the rules of G. Further, $L_p^\Gamma(G)$ denotes all syntactically valid partial sentences of terminals. Formally,

Definition 6 (Partial Sentences)**.** *We define a set of all syntactically valid partial sentences* $L_p^\Gamma(G) \subseteq \Gamma^*$ *such that* $\Lambda \in L_p^\Gamma(G)$ *if and only if* $\exists \Lambda_1 \in \Gamma^*$ *such that* $\Lambda.\Lambda_1 \in L^\Gamma(G)$*.*

Note that $L(G)$ and $L_p(G)$ are defined over alphabet Σ, whereas $L^\Gamma(G)$ and $L_p^\Gamma(G)$ over terminals Γ. Nevertheless, if a program C is parsed to obtain terminal sequence Λ, then $C \in L(G)$ is equivalent to $\Lambda \in L^\Gamma(G)$. The SynCode parsing algorithm obtains $\Lambda^\square = \tau_1, \tau_2 \ldots \tau_f$ by parsing C_k corresponding to the parserd part of partial output $C_k^\square$. Given a partial sentence $\Lambda_\square$, an accept sequence is a sequence over Γ such that when appended to $\Lambda^\square$ the result is still a partial sentence.

Definition 7 (Accept Sequence)**.** *Given partial output* $C_k \in L_p(G)$*, and* $\Lambda^\square, r = pparse(C_k)$*,* $\Lambda_1 \in \Gamma^*$ *is an accept sequence if* $\Lambda^\square.\Lambda_1 \in L_p^\Gamma(G)$*.*

Consider a Python partial program $C_k = \boxed{\texttt{def is}}$ and let *def*, *name*, *lpar* and *rpar* be the terminals in Python grammar. we get $\{def\}, \boxed{\texttt{is}} = pparse(\boxed{\texttt{def is}})$, where $\Lambda^\square = \{def\}$ and $r = \boxed{\texttt{is}}$. $\Lambda_1 = \{name, lpar, rpar\}$ is an accept sequence in this case as the sequence of terminals $\Lambda^\square.\Lambda_1 = \{def, name, lpar, rpar\}$ is a valid partial sentence. The parser state on parsing the partial output C_k can be utilized to compute a set of accept sequences denoted as $\mathcal{A}$. The soundness and completeness of the SynCode algorithm depend on the length of these accept sequences in $\mathcal{A}$. In theory, using longer accept sequences enhances the precision of the SynCode algorithm at the cost of increased computational complexity. In Section 4.5, we show our method for obtaining 1 and 2-length accept sequences that are efficient and precise in practice.

4.3 Grammar Mask

This section outlines the utilization of the set of acceptable terminal sequences $\mathcal{A}$ and the remainder r in the creation of a boolean mask using the DFA mask store which is subsequently used for constraining the LLM output. The DFA mask store is constructed offline and makes SynCode efficient during the LLM generation. Given partial output C_k, our objective is to identify tokens $t \in V$ such that appending them to C_k leads to syntactical completion. Given remainder r and set of sequences $\mathcal{A}$, the goal is to determine whether $r.t$ partially matches the regular expression derived from any of the sequences in $\mathcal{A}$. To characterize the notion of strings partially matching a regular expression, we next introduce the function *pmatch*.

Definition 8 (*pmatch*)**.** *The function pmatch takes a word $w \in \Sigma^*$, a regular expression ρ and returns a boolean. $pmatch(w, \rho) = true$ if either of the following conditions holds:*

1. $\exists w_1 \in \Sigma^*, w_2 \in \Sigma^+$ *such that* $w = w_1.w_2$ *and* $w_1 \in L(\rho)$ *or*
2. $\exists w_1 \in \Sigma^*$ *such that* $w.w_1 \in L(\rho)$

Thus *pmatch*(w, ρ) is true when either a prefix of w matches ρ or w can be extended to match ρ. The consequence of allowing *pmatch* to be defined such that it is true even when prefix matches, is that SynCode will conservatively accept all tokens for which the prefix matches the accept sequence. Hence, we overapproximate the precise set of syntactically valid tokens. We make this choice to ensure that SynCode is sound for any length of accept sequences. Next, we give definitions related to DFAs. These definitions are useful for describing the construction of the DFA mask store and proving properties related to its correctness in the SynCode algorithm. In particular, we first define the live states of DFA. We say state q is live if there is a path from q to any final states in F. Formally,

Definition 9 (DFA *live* states)**.** *Given a DFA $D(Q, \Sigma, \delta, q_0, F)$, let $live(Q) \subseteq Q$ denote the set of live states such that*

$$q \in live(Q) \textit{ iff } \exists w \in \Sigma^* \textit{ s.t. } \delta^*(w, q) \in F$$

We use $D_\tau(Q_\tau, \Sigma_\tau, \delta_\tau, q_0^\tau, F_\tau)$ to denote a DFA corresponding to a terminal $\tau \in \Gamma$. Next, we establish the definition of *dmatch* for DFA, which is an equivalent concept to *pmatch* with regular expressions. *dmatch* is recursively defined such that its computation can be performed by walking over the DFAs of a sequence of terminals.

Definition 10 (*dmatch*)**.** *Given a DFA $D(Q, \Sigma, \delta, q_0, F)$, a string $w \in \Sigma^*$, a DFA state $q \in Q$ and any sequence of terminals $\Lambda = \{\tau_{f+1}, \tau_{f+2} \dots \tau_{f+d}\}$, $dmatch(w, q, \Lambda) = true$, if either of the following conditions hold:*

1. $\delta^*(w, q) \in live(Q)$ *or*
2. $\exists w_1 \in \Sigma^*, w_2 \in \Sigma^+$ *such that* $w_1.w_2 = w$, $\delta^*(w_1, q) \in F$ *and* $\Lambda = \{\}$ *or*
3. $\exists w_1 \in \Sigma^*, w_2 \in \Sigma^*$ *such that* $w_1.w_2 = w$, $\delta^*(w_1, q) \in F$,
   and $dmatch(w_2, q_0^{\tau_{f+1}}, \{\tau_{f+2} \dots \tau_{f+d}\}) = true$ *where* $q_0^{\tau_{f+1}}$ *is the start state corresponding to the DFA for* τ_{f+1}

Given an accept sequence $\Lambda = \{\tau_{f+1}, \tau_{f+2} \dots \tau_{f+d}\} \in \mathcal{A}$, our objective is to compute the set of tokens $t \in V$ such that *pmatch*$(r.t, \rho_\Lambda)$ holds, where $\rho_\Lambda = (\rho_{f+1}.\rho_{f+2}.\dots.\rho_{f+d})$ is the regular expression obtained by concatenating regular expressions for terminals. If Λ^p denotes the sequence $\{\tau_{f+2}, \dots \tau_{f+d}\}$, Lemma 1 simplifies this problem to finding *dmatch*$(r.t, q_0^{\tau_1}, \Lambda^p)$. Furthermore, utilizing Lemma 2, this can be further reduced to computing $q = \delta^*_{\tau_1}(r, q_0^{\tau_1})$ and *dmatch*(t, q, Λ^p). It's important to note that *dmatch*(t, q, Λ^p) does not depend on C_k and can be computed offline. While the computation of q for *dmatch*(t, q, Λ^p) is relatively inexpensive, evaluating *dmatch*(t, q, Λ^p) can be computationally expensive both offline and online, as it requires considering numerous potential accept sequences offline, and where it needs to iterate over all tokens in V online. We observe that if we consider sequences of smaller lengths, we can efficiently precompute the set of tokens satisfying *dmatch*(t, q, Λ^p) for all q, t and Λ^p offline. We later establish the soundness of SynCode when using accept sequences of length at least 1 (Theorem 1) and completeness for accept sequences of the length greater than maximum length of tokens in the vocabulary (Theorem 2). Typically, LLM tokens are small in size, allowing us to obtain these guarantees.

Lemma 1. *Given* $\Lambda = \{\tau_{f+1}, \tau_{f+2} \dots \tau_{f+d}\}$*,* $\Lambda^p = \{\tau_{f+2} \dots \tau_{f+d}\}$ *and* $\rho_\Lambda = (\rho_{f+1}, \rho_{f+2}, \dots, \rho_{f+d})$*,* $dmatch(w, q_0^{\tau_1}, \Lambda^p) \iff pmatch(w, \rho_\Lambda)$*.*

Lemma 2. *If* $q = \delta^*_\tau(r, q_0^\tau)$ *and no prefix of* r *is in* $L(\tau)$ *i.e.* $\nexists w_1 \in \Sigma^*, w_2 \in \Sigma^*$ *such that* $w_1.w_2 = r$ *and* $\delta^*_\tau(w_1, q_0^\tau) \in F_\tau$ *then* $dmatch(t, q, \Lambda) \iff dmatch(r.t, q_0^\tau, \Lambda)$*.*

The proofs of both the lemmas are in Appendix A.2.

Illustrative Example: Consider the scenario with C_k = `def is`, r = `is`, and an accept sequence $\Lambda = \{name, lpar, rpar\}$ in $\mathcal{A}$, where *name*, *lpar*, and *rpar* are terminals in Γ. Our objective is to determine all $t \in V$ such that `def is`.t forms a valid partial program. This can be achieved by finding tokens t that satisfy *pmatch*(`is`.t, ρ_Λ), where $\rho_\Lambda = [a\text{-}z, A\text{-}Z, \_]^*()$. Let's consider a token t = `_prime():`. We observe that $r.t$ = `is_prime():` can be decomposed into `is_prime` (*name*), `(` (*lpar*), `)` (*rpar*), and `:`. Consequently, it partially matches ρ_Λ as defined by *pmatch*. In Figure 9, we present the DFAs for Λ used in computing *dmatch*. The reduction $dmatch(r.t, q_0^{name}, lpar, rpar)$ = *dmatch*(`is_prime():`, $q_0^{name}, lpar, rpar$) simplifies successively to *dmatch*(`():`, $q_0^{lpar}, rpar$), then to *dmatch*(`):`, $q_0^{rpar},$), and finally to *dmatch*(`:`, $q_1^{rpar},$). As q_1^{rpar} is a final state, according to condition 2 of Definition 10, *dmatch*(`:`, $q_1^{rpar},$) holds true. Next, we define a mask over vocabulary

Definition 11 (Vocabulary mask)**.** *Given vocabulary* $V \subseteq \Sigma^*$*,* $m \in \{0,1\}^{|V|}$ *is a mask over the vocabulary. We also use* $set(m) \subseteq V$ *to denote the subset represented by* m*.*

DFA Mask Store For an integer α, we define a DFA table $\mathcal{M}_\alpha$ as the mask store over the DFA states with α lookahead. Given the set of all DFA states $Q_\Omega = \bigcup_{\tau \in \Gamma} Q_\tau$, the table stores binary masks of size $|V|$, indicating for token string t, for any DFA state $q \in Q_\Omega$ and a sequence of α terminals Λ_α if $dmatch(t, q, \Lambda_\alpha) = true$. The lookahead parameter α signifies the number of subsequent terminals considered when generating the mask stored in the table. Choosing a larger value for α enhances the precision of SynCode algorithm, but it comes at the cost of computing and storing a larger table. We next formally define the DFA mask store,

Definition 12 (DFA mask store)**.** *For an integer* α*, the DFA mask store* $\mathcal{M}_\alpha$ *is a function defined as* $\mathcal{M}_\alpha : Q_\Omega \times \Gamma^\alpha \to \{0,1\}^{|V|}$*, where* $Q_\Omega = \bigcup_{\tau \in \Gamma} Q_\tau$ *represents the set of all DFA states and* Γ^α *is a set of* α*-length terminal sequences. Then* $\mathcal{M}_\alpha(q, \Lambda) = m$ *is a binary mask such that* $t \in set(m)$ *if* $dmatch(t, q, \Lambda)$

For our illustrative example if $m = \mathcal{M}_2(q_1^{name}, \{lpar, rpar\})$ then t = `_prime():` should be contained in $set(m)$. The grammar mask for a set of accept sequences $\mathcal{A}$ can be computed by combining masks for each $\Lambda \in \mathcal{A}$. The DFA mask store $\mathcal{M}_0$ maps each DFA state to all tokens such that they *pmatch* without considering any following accept sequence (0-length sequence). In this case, the table maps each state with a single mask denoting the tokens that match the regular expression of the corresponding DFA.

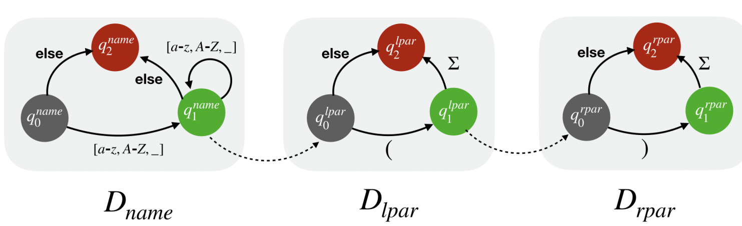

Figure 8: DFAs in accept sequence $\Lambda = \{name, lpar, rpar\}$ for example. The start state, final states, and dead states are in gray, green, and red respectively. The dashed arrows link the final states of one DFA to the starting state of the next DFA, adhering to condition 3 in Definition 10. This illustrates the sequential traversal across DFAs during the computation of *dmatch*.

Computing Grammar Mask The mask store is constructed offline by enumerating all DFA states Q_Ω, considering all possible terminals in Γ, and all tokens in V. The DFA mask store depends on the set of terminals Γ and the model's vocabulary V. As a result, a unique mask store is created for each grammar and tokenizer combination, and to enhance efficiency, we cache and reuse this table for future inferences.

Algorithm 2 presents our approach for computing the grammar mask during LLM generation. It computes a grammar mask based on the sets of current accept sequences $\mathcal{A}$, and the remainder string (r). It iterates over $\mathcal{A}$, considering each sequence Λ. The algorithm initializes an empty mask m. It iterates over each acceptable sequence, considering the first terminal τ_1 in each. It computes the resulting state q_r by processing τ_1 from an initial state $q_0^{\tau_1}$ and the remainder string r. If q_r is in a live state, the algorithm updates the grammar mask by unifying the mask cached in $\mathcal{M}_\alpha$.

Algorithm 2 Computing Grammar Mask

Inputs: $\mathcal{A}$: set of accept sequences, r: remainder

function GrammarMask($\mathcal{A}, r$)
  $m \leftarrow \{\}$
  for $\Lambda \in \mathcal{A}$ **do**
    $\tau_1 \leftarrow \Lambda[0]$
    $q_r \leftarrow \delta^*(q_0^{\tau_1}, r)$
    if $q_r \in \mathit{live}(Q_{\tau_1})$ **then**
      $\Pi \leftarrow \mathit{len}(\Lambda) - 1$
      $m \leftarrow m \cup \big(\mathcal{M}_\Pi(q_r, \Lambda[1:])\big)$
  return m

4.4 Soundness and Completeness

This section establishes the soundness and completeness of the SynCode algorithm. Algorithm 3 presents the LLM generation algorithm with SynCode. It takes as inputs an LLM represented by M, a tokenizer denoted by $\mathcal{T}$, an input prompt string C_0, the maximum number of generated tokens n_{max}, and a base decoding strategy D. The algorithm begins by tokenizing the input prompt using the tokenizer. It then iteratively generates tokens using the LLM, decodes the current token sequence, and performs parsing to obtain acceptable terminal sequences $\mathcal{A}$, and a remainder r (line 6). A grammar mask is applied to the logit scores based on these values (line 7). The algorithm subsequently selects the next token using the decoding strategy, and if the end-of-sequence token (EOS) is encountered, the process terminates. The final decoded output is obtained, incorporating the generated tokens, and is returned as the result of the MaskedGenerate algorithm.

Given partial output $C_k \in L_p(G)$, SynCode generates a corresponding mask m. If, for a token $t \in V$, the concatenation $C_k.t$ results in a syntactically valid partial output, i.e. $C_k.t \in L_p(G)$, our soundness theorem ensures that t is indeed a member of the set defined by the generated mask m. The subsequent theorem formally states this soundness property.

Theorem 1. *Let $C_k \in L_p(G)$ be the partial output and any integer $d \geq 1$, let $\mathcal{A}_d \subseteq \Gamma^d$ contain all possible accept terminal sequences of length d and $r \in \Sigma^*$ denote the remainder. If $m = \mathit{GrammarMask}(\mathcal{A}, r)$ then for any $t \in V$, if $C_k.t \in L_p(G)$ then $t \in \mathit{set}(m)$*

The proof of the theorem is in Appendix A.2.

Next, we give a definition that establishes a partial order on sets of terminal sequences, where given two sets $\mathcal{A}_1$ and $\mathcal{A}_2$, we say sets $\mathcal{A}_1 \preccurlyeq \mathcal{A}_2$ if every sequence in $\mathcal{A}_2$ has a prefix in $\mathcal{A}_1$.

Definition 13 ($\preccurlyeq$). *We define a partial order $\preccurlyeq$ on set of terminal sequences $\mathcal{P}(\Gamma^*)$ such that $\mathcal{A}_1 \preccurlyeq \mathcal{A}_2$ when $\forall \Lambda_2 \in \mathcal{A}_2 \exists \Lambda_1 \in \mathcal{A}_1 \exists \Lambda_3 \in \Gamma^*$ s.t. $\Lambda_2 = \Lambda_1.\Lambda_3$*

We further state the lemma that shows the relation in the grammar masks generated by two accept sequences satisfying relation $\preccurlyeq$.

Lemma 3. *Given $\mathcal{A}_1$ and $\mathcal{A}_2$ are set of accept sequences such that $\mathcal{A}_1 \preccurlyeq \mathcal{A}_2$ and $m_1 = \mathit{GrammarMask}(\mathcal{A}_1, r)$ and $m_2 = \mathit{GrammarMask}(\mathcal{A}_2, r)$ then $\mathit{set}(m_2) \subseteq \mathit{set}(m_1)$*

The proof of the lemma is in Appendix A.2.

Theorem 1 proves soundness for accept sequences $\mathcal{A}_d$ of length d, while Lemma 3 extends this proof to any set of accept sequences $\mathcal{A}$ where $\mathcal{A} \preccurlyeq \mathcal{A}_d$. Our implementation, employing sequences of varying lengths, can be proven sound based on this extension.

The completeness theorem ensures that, under specified conditions, each token $t \in \mathit{set}(m)$ guarantees $C_k.t$ as a syntactically valid partial output. An implementation of SynCode with a short length of accept sequences although sound, may not guarantee completeness. To illustrate, let's take the example where $\Lambda = \tau_{f+1}, \tau_{f+2} \in \mathcal{A}$ with simple singleton regular expressions $\rho_{\tau_{f+1}} = \boxed{\texttt{(}}$ and $\rho_{\tau_{f+2}} = \boxed{\texttt{(}}$. In this case, our algorithm conservatively treats all tokens $t \in V$ as syntactically valid, whenever $\boxed{\texttt{((}}$ is a prefix of those tokens (e.g., $\boxed{\texttt{(((}}$, $\boxed{\texttt{(()}}$)) even though some tokens may not meet syntactic validity. However, by assuming that the accept sequences are long enough, we can establish the completeness of the approach.

Algorithm 3 SynCode Generation

Inputs: M: LLM, $\mathcal{T}$: tokenizer, C_0: input prompt, n_{max}: maximum generated tokens, D: decoding strategy

```
function MaskedGenerate(M, T, C_0, n_max, D)
    T_cur ← Tokenize(T, C_0)
    for i ∈ {1, ... n_max} do
        scores ← M(T_cur)
        C_k ← decode(T, T_cur)
        A, r ← Parse(C_k)
        m ← GrammarMask(A, r)
        scores ← m ⊙ scores
        t_i ← D(scores)
        if t_i = EOS then
            break
        T_cur ← append(T_cur, t_i)
    output ← decode(T, T_cur)
    return output
```

Theorem 2. *Let $C_k \in L_p(G)$ be the partial output, let $\mathcal{A}_d \subseteq \Gamma^d$ contain all possible accept terminal sequences of length d and $r \in \Sigma^*$ denote the remainder. Suppose for any $t \in V, d > \mathit{len}(t)$ and $m = \mathit{GrammarMask}(\mathcal{A}_d, r)$ such that $t \in \mathit{set}(m)$ then $C_k.t \in L_p(G)$*

The proof of the theorem is in Appendix A.2. While our completeness theorem ensures the SynCode consistently extends syntactically correct partial outputs, it does not guarantee termination with a correct and complete output. The focus of the theorem is on generating syntactically valid partial outputs, and the theorem does not address whether the process converges to a syntactically correct whole output. Termination considerations go beyond the completeness theorem's scope.

4.5 SynCode Implementation

Base LR parser: Bottom-up LR parsers, including LR(1) and LALR(1) parsers, process terminals generated from the lexical analysis of the code sequentially and perform shift or reduce operations (Aho et al., 1986). LR(κ) parsers have the immediate error detection property, ensuring they do not perform shift or reduce operations if the next input κ terminals on the input tape is erroneous (Aho and Johnson, 1974). Consequently, every entry in the parsing table corresponding to κ terminals that maps to a shift or reduce operation indicates that the terminal is acceptable. This property allows us to use LR(1) parsing tables to efficiently compute accept sequences at any intermediate point, making them preferable for SynCode applications. Thus, computing acceptable terminals with LR(1) parsers has a complexity of $O(|\Gamma|)$. Although LALR(1) parsers are more commonly used due to their smaller memory requirements and faster construction, computing acceptable terminals with them requires iterating over all terminals leading to a complexity of $O(T_P \cdot |\Gamma|)$ due to the need for multiple reduce operations before confirming the validity of each terminal. Furthermore, while for $\kappa > 1$, LR(κ) parsers can compute accept sequences of length κ immediately, they incur extremely high memory requirements. Additionally, while we can use LL(κ) parsing tables to compute the next κ accept terminals, LR(κ) parsers offer a higher degree of parsing power. Therefore, we employ LR parsers in SynCode. Our evaluation indicates that LR(1) parsers suffice for eliminating most syntax errors, making them a practical choice for SynCode. We discuss how the implementation of how parsing is performed *incrementally* to obtain the accept sequences and remainder in the Appendix A.3.

Accept Sequences: In our implementation, we focus on generating accept sequences of length 1 or 2, as they can be efficiently obtained from LR(1) parser. While this approach incurs some loss of precision, it leads to sound but incomplete syntactical decoding. Further, our evaluation demonstrates that this strategy is efficient and precise in practical scenarios. We note that *pmatch r.t* with a 2-length sequence is equivalent to *dmatch* with a 1-length sequence, as stated in Lemma 1. Consequently, in our work, we precompute mask stores $\mathcal{M}_0$ and $\mathcal{M}_1$. On parsing the partial output C_k, the parser state of LR(1) parsers can be used to directly obtain syntactically acceptable terminals for the current completion (A_0) and the next completion (A_1). We utilize A_0 and A_1 to construct the accept sequences $\mathcal{A}$, considering two cases:

Case 1: $C_k = l_1.l_2 \dots l_f$**:** Let τ_f represent the type of the final lexical token. In many instances, a token may be extended in the subsequent generation step, such as when an identifier name grows longer or additional words are appended to a comment. In those cases if $A_1 = \tau_1^1, \tau_2^1, \dots, \tau_n^1$, we include all 2-length sequences $\{\tau_f, \tau_i^1\}$ for each i. As previously discussed, the type of the final lexical token may change from τ_f. Consequently, when $A_0 = \{\tau_1^0, \tau_2^0, \dots, \tau_n^0\}$, we add 1-length sequences Λ_i for each terminal sequence $\{\tau_i\}$ from A_0, excluding τ_f. This method ensures the generation of sequences accounting for potential extensions of the same token and changes in the type of the final lexical token.

Case 2 $C_k = l_1.l_2 \dots l_f.u$**:** In this scenario, the current terminal is incomplete, leading to a lack of information about subsequent terminals. Consequently, when $A_1 = \{\tau_1, \tau_2, \dots, \tau_n\}$, we define $\mathcal{A}$ as a set of sequences: $\{\Lambda_1, \Lambda_2, \dots, \Lambda_n\}$, where each Λ_i corresponds to a single terminal sequence $\{\tau_i\}$ from A_1. Specifically, $\Lambda_1 = \{\tau_1\}$, $\Lambda_2 = \{\tau_2\}$, and so forth.

4.6 Time Complexity

In this section, we analyze the time complexity of the SynCode algorithm. We focus on the cost of creating the mask at each iteration of the LLM generation loop. The key computations involved in this process are the parsing carried out by the incremental parser to compute $\mathcal{A}$ and the lookup/unification operations performed through the DFA mask store.

The incremental parser parses $O(1)$ new tokens at each iteration and computes $\mathcal{A}$. Let T_A represent the time taken by the parser to compute the accepted terminals and T_P denote the time the parser takes to parse a new token and update the parser state. Hence, in each iteration, the parser consumes $O(T_A + T_P)$ time to generate $\mathcal{A}$. The DFA mask store lookup involves traversing $|\mathcal{A}|$ DFA sequences, with the number of steps in this walk bounded by the length of the remainder r. As $\mathcal{A}$ can have a maximum of $|\Gamma|$ sequences, the DFA walk consumes $O(|\Gamma| \cdot \mathit{len}(r))$ time. We employ a hashmap to facilitate efficient lookups at each DFA node, ensuring that all lookups take constant time. Consequently, this step takes $O(|\Gamma|)$ time. Let $T_\cup$ denote the time taken for computing the union of binary masks. With potentially $|\Gamma|$ union operations to be performed, the mask computation takes $O(T_\cup \cdot |\Gamma|)$ time. Therefore, the overall time complexity at each step during generation is given by $O(T_A + T_P + |\Gamma| \cdot \mathit{len}(r) + T_\cup \cdot |\Gamma|)$.

For an incremental LR(1) parser, the complexity of our algorithm at each step of LLM token generation is $O(|\Gamma| \cdot \mathit{len}(r) + T_\cup \cdot |\Gamma|)$. With our lexer assumption, we ensure that the remainder r is small, allowing us to further simplify our complexity analysis to $O(T_\cup \cdot |\Gamma|)$ by treating $\mathit{len}(r)$ as constant.

Offline cost: The cost of computing the mask store $\mathcal{M}_\alpha$ offline involves considering all DFA states $q \in Q_\Omega$, all possible terminal sequences of length α, and all tokens $t \in V$. Given that we need to traverse the DFA for $\mathit{len}(t)$ steps for each entry in the store, the time complexity for computing the mask store is $O(max_{t \in V}(\mathit{len}(t)).|Q_\Omega|.|V|.|\Gamma|^\alpha)$. Typically, $\mathit{len}(t)$ is small, allowing us to simplify this to $O(|Q_\Omega|.|V|.|\Gamma|^\alpha)$. In our implementation, the use of $\mathcal{M}_0$ and $\mathcal{M}_1$ results in a cost of $O(|Q_\Omega|.|V|.|\Gamma|)$. The size of $|Q_\Omega|$ depends on the complexity of regular expressions for the terminals, which may vary for each grammar. However, as demonstrated in our evaluation section, these mask stores can be computed within 10 minutes for each combination of grammar and LLM. This computation is a one-time cost that can be amortized over all generations performed for the given LLM and grammar.

4.7 SynCode Framework

Figure 9 shows how SynCode framework can be used in practice by selecting a grammar. We next discuss other important features of the framework.

Adding a New Grammar. Our Python-based SynCode framework is shipped with several built-in grammars such as JSON, Python, Go, etc. A user can apply SynCode for arbitrary grammar by providing the grammar rules in EBNF syntax with little effort. The grammar needs to be unambiguous LALR(1) or LR(1) grammar for using the respective base parsers.

Ignore Terminals. Our EBNF syntax adopted from Lark allows one to provide *ignore terminals* as part of the grammar. Lark ignores those terminals while parsing. In the case of Python, this includes *comments*

Figure 9: The upper section displays erroneous output from a standard LLM generation, failing to produce the intended JSON format. The lower segment showcases the fix achieved through the use of the SynCode framework.

and *whitespaces*. SynCode handles these ignore terminals by adding a trivial 1-length accept sequence for each of these ignore terminals.

Parsers. SynCode supports both LALR(1) and LR(1) as base parsers. We adapt Lark's (Lark,) LALR(1) parser generator for SynCode. Since Lark does not implement the LR(1) parser generator, we implemented the LR(1) parser generator on top of the Lark. The generation of LR(1) parser which is performed offline may take longer time compared to the LALR(1) parser (e.g., up to 2 mins for our Python grammar), however, it is more efficient at inference time in computing the accept sequences. Further, since the Lark-generated parser is non-incremental, we build the incremental parser on top of it by caching the parser state as described in Appendix A.3.

Non-CFG Fragments of PLs. SynCode can handle non-context-free fragments of PLs, such as *indentation* in Python and end-of-scope markers in Go. To support languages with indentation, such as Python and YAML, SynCode has a mechanism that tracks acceptable indentation for the next token, effectively masking tokens that violate indentation constraints at a given point. This indentation constraint feature can be enabled with any new grammar. Similarly, for handling other custom parsing rules beyond CFGs, users can add additional constraints to the generation by overriding specific SynCode functions. For instance, in Go, semicolons are optional and may be automatically inserted at the end of non-blank lines. Implementing such constraints in SynCode programmatically requires minimal effort. However, SynCode currently does not support enforcing semantic constraints. (e.g, if a variable in a program is defined before it is used.)

5 Experimental Methodology

Models. In our evaluation, we select a diverse set of state-of-the-art open-weight LLMs of varying sizes. Since closed-source LLMs, such as GPT-4 or Gemini, do not expose generation logits through their APIs, applying a constrained generation approach in SynCode is not feasible. Therefore, we focus on enhancing smaller, open-source models in our evaluation. We select the state-of-the-art models Llama-2-7B-chat (Touvron et al., 2023b) and Gemma2-2B-it (Team et al., 2024) for our JSON evaluation. For text-2-SQL generation experiments, we use Llama-2-7B-chat, Llama-3.2-1B, Llama-3.2-3B, and Gemma-2-2B-it. Furthermore, we chose models such as LLaMA-7B (Touvron et al., 2023a), WizardCoder-1B (Luo et al., 2023), and CodeGen-350M (Nijkamp et al., 2023) for code completion.

Datasets. We focus our evaluation on generating JSON, SQL, Python, and Go outputs. We choose JSON as it is supported by the baselines (Gerganov and et. al., 2024; Willard and Louf, 2023), which allows us to compare against them. We selected Python since it is extensively present in the training data employed for LLM training and fine-tuning. Conversely, we opted for Go due to its lower standard LLM accuracy and a relatively smaller presence in the training data. We consider JSON-Mode-Eval (NousResearch, 2024) dataset for text to JSON generation and HumanEval and MBXP (Athiwaratkun et al., 2023) dataset for evaluating Python and Go code generation. We display examples of prompts from these datasets in Appendix A.7.

- **JSON-Mode-Eval (NousResearch, 2024).** It consists of 100 zero-shot problems. Each problem prompt follows the chat format with a system prompt specifying a JSON schema and a user prompt requesting the LLM to generate a JSON object that contains specified contents.
- **Spider text-2-SQL.** Spider (Yu et al., 2018) text-to-SQL dataset consists of 1,034 problems of varying difficulty levels: *easy* (250), *medium* (440), *hard* (174), and *extra hard* (170).
- **Multilingual HumanEval (Athiwaratkun et al., 2023).** It is an extension of the original HumanEval collection (Chen et al., 2021), which comprises 164 Python programming problems, to include other languages like Go. Each problem in the dataset consists of a function definition, and text descriptions of the function as a part of the function docstring.
- **MBXP (Athiwaratkun et al., 2023).** It is extended from the MBPP (Austin et al., 2021) dataset for Python to support other languages such as Go. The dataset consists of 974 problems with the same format as HumanEval.

Grammars. For Python, we used the readily available grammar from the Lark repository. For Go, we converted an existing LL(*) grammar from (ANTLR,) implementation to LR(1) grammar for our use. We write the CFG for these languages using the Extended Backus-Naur Form (EBNF) syntax. We use a substantial subset of grammar for Python and Go syntactic generation with SynCode. The grammar has commonly used features of the language such as control flow, and loops, and excludes some features such as Python's support for lambda functions. Adding support for more features would require more engineering effort but it will not change the overall technique. The grammars we used are available in Appendix A.8. The JSON grammar consists of 19 rules and 12 terminals. The Python grammar we used contains 520 production rules and 94 terminals, whereas the Go grammar comprises 349 rules and 87 terminals.

Evaluating Syntax Errors. For evaluating the errors in the generated output in each of the languages, we use their respective standard compilers.

Experimental Setup. We run experiments on a 48-core Intel Xeon Silver 4214R CPU with 2 NVidia RTX A5000 GPUs. SynCode is implemented using PyTorch (Paszke et al., 2019a), HuggingFace transformers library (Wolf et al., 2020) and Lark library (Lark,).

Baselines. We evaluate three state-of-the-art baselines Outlines (Willard and Louf, 2023) v0.1.1, guidance (Lundberg et al., 2023) v0.1.16, and llama.cpp (Gerganov and et. al., 2024) v0.3.1 in our study. The algorithmic differences in the baselines and SynCode are discussed in Section 7. We perform a warmup run for each experiment where we measure inference time to ensure that one-time precomputation time is not included in the inference runtime. For a fair comparison with baselines, SynCode uses opportunistic masking (Beurer-Kellner et al., 2024), an optimization used in llama.cpp and guidance. Instead of computing the full logit vector mask upfront, the model generates a token and only computes the mask if the proposed token is incorrect.

6 Experimental Results

In this section, we evaluate SynCode on generating various formal languages. We compare SynCode with state-of-the-art baselines and perform various ablation studies.

SynCode allows the model to generate a special EOS token (indicating the end of generation) only when the output belongs to $L(G)$. In practice, however, LLM generation typically stopped after a fixed maximum number of tokens, n_{max}. Therefore, terminating with the EOS token within this limit is not always guaranteed potentially resulting in syntax errors.

Table 1: Effectiveness of SynCode in generating JSON with original and explicit prompts.

Model	Tool	Syntax Errors		Validation Accuracy (%)		Generation Time (s)	
		Original	Explicit	Original	Explicit	Original	Explicit
Llama-2-7B-chat	**SynCode**	**0**	**0**	**66%**	**84%**	**3.07**	**3.02**
	Standard	98	41	2%	58%	3.58	3.11
	llama.cpp	23	23	63%	68%	21.91	20.84
	guidance	13	11	57%	65%	5.14	4.14
	Outlines†	16	14	62%	56%	38.07	41.79
	GCD	2	0	62%	64%	6.08	4.01
Gemma2-2B-it	**SynCode**	**0**	**0**	**99%**	**100%**	4.82	4.74
	Standard	59	59	41%	41%	4.32	5.82
	llama.cpp	7	7	92%	91%	22.06	21.97
	guidance	1	1	96%	96%	6.09	5.47
	Outlines	2	0	67%	90%	**1.99**	**2.64**
	GCD	1	0	96%	95%	19.12	9.49

† We observed issues when using Llama-2-7B-chat with Outlines v0.1.1 and therefore, we use older version v0.0.46.

6.1 Effectiveness of SynCode for JSON Generation

We evaluate the effectiveness of SynCode in guiding LLMs with the JSON grammar to generate syntactically correct JSON. We run the inference with Llama-2-7B-chat and Gemma2-2B-it with SynCode, llama.cpp, Outlines, guidance, GCD, and standard generation on the 100 problems from the JSON-Mode-Eval dataset. We select these models for the JSON experiment as they are supported by all considered baselines.

We run llama.cpp on a CPU as it requires a specific CUDA version not compatible with our machine. We set max new tokens $n_{max} = 400$. We also report an evaluation of augmenting the prompts with an explicit request to output only JSON. We present an example of these explicit prompts in Appendix A.7. We evaluate the correctness of JSON generated by an LLM by first evaluating whether the JSON string can be parsed and converted to a valid JSON object. We further evaluate whether the generated JSON is valid against the schema specified in the prompt. Although the SynCode does not enforce the specific schema to the JSON output for each task, we believe it is an important research question to check whether the reduced syntax errors due to SynCode can also lead to improved schema validity.

Table 1 presents our evaluation results. We report results for both the prompts taken directly from the dataset (denoted as "Original") and after augmenting these prompts with an explicit request to output JSON (denoted as "Explicit"). In the "Validation Accuracy" column, we compute the percentage of valid completions against their respective schemas. In the "Generation Time (s)" column, we report the average time taken to generate a completion to a prompt from the dataset. Guiding Llama-2-7B-chat and Gemma2-2B-it with the JSON grammar via SynCode eliminates syntax errors in generated JSON. On the other hand, standard generation results in syntactically incorrect JSON for 98% and 59% of completions to the original prompts for the Llama-2-7B-chat and Gemma2-2B-it models respectively. A majority of these errors are due to the generation of natural language before and after the JSON. Explicit prompts somewhat mitigate this issue, but still results in syntactically invalid outputs to 41% and 59% of these prompts for standard Llama-2-7B-chat and Gemma2-2B-it generation respectively, primarily due to errors such as unmatched braces and unterminated string literals.llama.cpp, Outlines, guidance, and GCD face similar problems with closing braces and terminating strings.

Notably, SynCode significantly improves the JSON schema validation accuracy of Gemma2-2B-it completions over standard generation, from 41% to 99% and 41% to 100% for original and explicit prompts respectively. Furthermore, SynCode outperforms llama.cpp, Outlines,guidance, and GCD in validation accuracy of Llama-2-7B-chat completions by 3%, 4%, 9%, and 4% respectively for original prompts and 16%, 28%, 19%, and 20% for explicit prompts. The remaining schema validation errors with SynCode are semantic errors, including data type mismatch between the generation JSON and schema, missing fields required by the schema, and adding extra fields not allowed by the schema. SynCode is faster than all baseline grammar-guided generation methods for Llama-2-7B-chat and all but Outlines for Gemma2-2B-it. The low generation time with Outlines for Llama-2-7B-chat can largely be attributed to the fact that many of its completions to prompts are empty JSON (35% of original and 7% of explicit) which takes few tokens to generate, but often does not conform to the schema.

Interestingly, we observe that for Llama-2-7B-chat, SynCode also reduces the average generation time over standard generation. We attribute this finding to the fact that without grammar-guided generation, the model generates syntactically invalid output, such as natural language, in addition to JSON and thus generates more tokens in response to the same prompt than with SynCode. Thus, augmenting LLMs with SynCode can significantly improve syntactical correctness and runtime efficiency.

6.2 Effectiveness of SynCode for SQL Generation

This study demonstrates that SynCode improves text-to-SQL generation by enforcing grammar constraints, ensuring that generated SQL queries are syntactically accurate. We evaluate the following models for SQL generation: Llama-3.2-1B, Llama-3.2-3B (base models) and Llama-2-7B-chat, Gemma-2-2B-it (instruct-tuned models). We observe that despite explicitly prompting to only generate the SQL query, the instruct-tuned Gemma-2-2B-it model often enclosed generated SQL queries within markers, such as `` ``` `` or `` ```sql ``. Thus, we consider another baseline for Gemma-2-2B where we extract the SQL query substring within these markers, handling cases where the output format is either `` ```{SQL query} ``` `` or `` ```sql {SQL query} ``` ``.

For evaluation, we use the Spider (Yu et al., 2018) text-to-SQL dataset, which consists of 1,034 problems of varying difficulty levels: *easy* (250), *medium* (440), *hard* (174), and *extra hard* (170). We prompt models with schema information and text queries, instructing them to generate SQL queries only. Using greedy decoding and `\n\n` is used as an additional stopping condition for all experiments.

Table 2: Comparison of SynCode and unconstrained generation on SQL generation.

Model	Method	Accuracy (%)					Execute (%)	Tokens	Time (s)
		Easy	Medium	Hard	Extra	Overall			
Gemma2-2B-it	Standard	0.0	0.0	0.0	0.0	0.0	0.0	221.43	9.883
	Standard+	44.8	18.9	21.9	17.1	25.4	77.6	221.43	9.893
	SynCode	**45.8**	18.7	**23.3**	**20.6**	**26.3**	**78.2**	**135.17**	**5.876**
Llama-2-7b-chat	Standard	34.4	22.0	12.1	4.1	20.4	32.6	44.74	1.148
	SynCode	**40.0**	**27.3**	**13.8**	**5.9**	**24.6**	**41.6**	50.33	1.483
Llama-3.2-1B	Standard	40.8	24.8	20.7	10.6	25.6	51.1	48.00	0.509
	SynCode	**46.8**	**28.2**	**23.0**	**10.6**	**28.8**	**59.0**	56.36	0.916
Llama-3.2-3B	Standard	38.0	29.5	28.2	12.9	28.6	67.4	47.78	0.846
	SynCode	**47.2**	**34.8**	**32.8**	**19.4**	**34.9**	**81.4**	47.63	1.164

Table 2 presents a comparison of SynCode and unconstrained generation across key metrics. The Accuracy (%) column shows the percentage of correctly generated SQL queries across different difficulty levels. Execute (%) reflects the percentage of queries successfully executed without runtime errors in SQLite. The Tokens column reports the average number of tokens generated, and Time(s) shows the average generation time. Standard+ row for Gemma2 denotes the result for the additional baseline where we extract the SQL query from the full generation using regex matching.

We observe that SynCode achieves better performance over the baselines in terms of both execution percentage and execution accuracy. For example, with the Llama-3.2-3B model, SynCode achieves an execution success rate of 81.4%, compared to 67.4% for unconstrained generation. Further, the execution accuracy improves from 28.6% to 34.9%. In the case of the Gemma2-2B-it model, we observe that SynCode shows a moderate improvement over the Standard+ accuracy. However, it shows a significant gain in the speed (1.7x) of generation and a reduction in the number of tokens generated. Although the Gemma2-2B-it model has a good execution percentage without any runtime errors. The instruct-tuned models tends to use large number of tokens that are not part of the query. In applications where the goal is to use LLMs to generate SQL queries without additional explanations, the result with Gemma2-2B-it shows that SynCode is useful in improving the efficiency of LLM generation along with the improvements in accuracy.

6.3 Effectiveness of SynCode for GPL

Table 3: Number of programs with syntax errors for standard and SynCode generation (↓ shows how much SynCode reduces the occurrence of the syntax errors compared to Standard generation.

Dataset	Model	Python			Go		
		Standard	SynCode	↓	Standard	SynCode	↓
HumanEval	CodeGen-350M	271	**15**	95%	573	**49**	91%
	WizardCoder-1B	36	**3**	92%	1031	**50**	95%
	LLaMA-7B	291	**2**	99%	725	**10**	99%
MBXP	CodeGen-350M	78	**4**	95%	212	**2**	99%
	WizardCoder-1B	28	**2**	93%	243	**14**	94%
	LLaMA-7B	148	**5**	97%	414	**1**	99%

We run inference with CodeGen-350M, WizardCoder-1B, and LLaMA-7B with SynCode and with the standard no-masking approch. We do not compare SynCode with the other baselines as none of these works support general-purpose programming language grammars. We experiment with both Python and Go programming languages, evaluating performance on zero-shot problems from the HumanEval and MBXP datasets. For each dataset, we generate $n = 20$ and $n = 1$ samples per problem with the LLM, respectively. We run the LLM-generated code completion against a predefined set of unit tests. For each unit test, we record the error type when running the generated program against that test case. We use the hyperparameters temperature $= 0.2$ and top $p = 0.95$. Table 3 presents our results for Python and Go. The columns standard and SynCode represent the total number of generated programs with syntax errors for the respective approaches. The column ↓ designates the percentage reduction in syntax errors from the standard generation to the SynCode generation. In this evaluation, across both HumanEval and MBXP datasets, we generate a total of 4154 samples for each language. On average, of all standard generated samples, 6% and 25% have syntax errors for Python and Go, respectively.

Notably, our experiments reveal that SynCode reduces the number of syntax errors by over 90% over the baseline in most experiments. Moreover, SynCode reduces the number of syntax errors to less than 1% of the total samples. Interestingly, we observe significantly more Syntax errors in standard LLM-generated Go code than in Python code, likely because the LLMs are trained more extensively on Python code than Go. Thus, SynCode can be especially effective for Go and more underrepresented programming languages, where LLMs are more likely to generate syntax errors due to a limited understanding of the language. SynCode can bridge this gap by guiding the LLM to sample only the syntactically valid tokens during decoding.

We further analyze the errors in Python and Go code generated by the LLMs augmented with SynCode, an example of which is presented in Appendix A.6. All of the errors were because the LLM failed to generate a complete program within the maximum token limit. Recall, SynCode provides guarantees of completeness for syntactically correct partial programs. However, it does not guarantee convergence to a syntactically correct and complete program.

Functional Correctness for Code Generation. We investigate whether augmenting LLMs with SynCode improves the functional correctness of the generated code. We evaluate functional correctness using the pass@k metric, where k samples are generated per problem, and a problem is considered solved if any sample passes a set of unit tests, and the fraction of solved problems is calculated. Table 4 reports our results for pass@1 and pass@10 for generated code completions to problems from the HumanEval dataset.

Table 4: Functional correctness on HumanEval problems

Metric	Architecture	Python		Go	
		Standard	SynCode	Standard	SynCode
pass@1	CodeGen-350M	6.8%	**6.9%**	3.6%	3.6%
	WizardCoder-1B	20.0%	20.0%	9.3%	**9.5%**
	LLaMA-7B	11.2%	**11.5%**	3.8%	**4.25%**
pass@10	CodeGen-350M	10.6%	10.6%	5.6%	**6.1%**
	WizardCoder-1B	27.6%	**28.4%**	12.5%	**13.7%**
	LLaMA-7B	17.1%	**18.9%**	8.8%	8.8%

Table 5: DFA Mask store creation time and memory

Model	$\|V\|$	Python		Go	
		Time(s)	Memory	Time(s)	Memory
CodeGen-350M	51200	602.26	1.87GB	603.03	1.58GB
WizardCoder-1B	49153	588.28	1.83GB	588.84	1.54GB
LLaMA-7B	32000	382.26	1.17GB	380.49	1.06GB

We observe that augmenting LLMs with SynCode has a slight improvement in functional correctness over standard generation. This observation indicates that for these state-of-the-art models, syntactic correction can result in a small improvement in the logical correctness of the code.

6.4 Mask Store Overhead

We analyze the time and memory overhead involved in generating a DFA mask store using SynCode. The DFA mask store for Llama-2-7B-chat took 113.72 seconds to create and consumes 181 MB of memory. Additionally, we report the creation time and memory overhead of DFA mask stores for models used for Python and Go in Table 5. Each row shows the SynCode store generation time in seconds, and memory in GBs, for a particular LLM and grammar. The $|V|$ column represents the total vocabulary size of the tokenizer of the particular LLM. We see that generating the store requires less than 2GB of memory and several minutes across the evaluated models and grammars. This overhead is minimal for practical SynCode use cases, as the mask store is a one-time generation task. Thereafter, the mask store can be efficiently loaded into memory and used for repeated inference. We see smaller generation time and memory with Llama-2-7B-chat and JSON grammar as opposed to LLaMA-7B, WizardCoder-1B, and CodeGen-350M with Python and Go grammars since the size of the mask store is proportional to the number of terminals in the grammar.

7 Related Work

Our work focuses on enhancing the syntactical accuracy LLMs by using a constrained decoding algorithm. Prior research has explored two other primary directions to enhance LLMs' accuracy in generating formal language: 1) Fine-tuning or prompt engineering (Bassamzadeh and Methani, 2024; Weyssow et al., 2024), which demands substantial data, compute resources, and time, often without any formal guarantees. 2) Modifications to the LLM's architecture or tokenization (Murty et al., 2023; Dong et al., 2023; Zhu et al., 2024), although these techniques have not yet achieved performance comparable to the current state-of-the-art standard LLMs. However, both fine-tuning and architectural changes are complementary to the grammar-guided decoding approach that we focus on in our work, and any gains through those techniques will improve the overall quality of LLM generation.

There are several recent works on constrained LLM generation (Wei et al., 2023; Beurer-Kellner et al., 2023; Lundberg et al., 2023; Willard and Louf, 2023; Scholak et al., 2021; Poesia et al., 2022; Gerganov and et. al., 2024; Geng et al., 2023; Beurer-Kellner et al., 2024; Agrawal et al., 2023; Melcer et al., 2024). This includes recent works that have used language-server (tools built for communication between IDEs and programming language-specific tools like static analyzers and compilers) suggestions to enforce language-specific semantic constraints during decoding (Agrawal et al., 2023; Wei et al., 2023). These techniques do not guarantee syntactical accuracy and rely on the availability and efficiency of language servers.

Structured LLM Generation. We focus our further discussion on comparison to the techniques that constrain LLM for structured generation according to a formal language. We compare SynCode with prior works in terms of precision and efficiency of the algorithms and generality and scalability of frameworks. Table 6 presents the various recent techniques for structured LLM generation. The columns "Regex" and "CFG" indicate regular expression and CFG constraining features, respectively. The "Precomputed" column denotes techniques that precompute certain structures to enhance generation efficiency. The "GPL" column specifies if the tools support general-purpose PLs. "Max CFG" displays the number of production rules in the largest supported Grammar by these techniques. We obtained these numbers by examining the built-in grammars that were provided in the corresponding libraries. Finally, the "Input Format" column

Table 6: Overview of various constrained decoding methods

	Regex	CFG	Precomputed	GPL	Max CFG	Input format
LMQL (Beurer-Kellner et al., 2023)	✓	✗	✗	✗	50-100	LMQL DSL
GUIDANCE (Lundberg et al., 2023)	✓	✓	✗	✗	50-100	Python DSL
OUTLINES (Willard and Louf, 2023)	✓	✓	✓	✗	50-100	Lark EBNF
PICARD (Scholak et al., 2021)	✓	✓	✗	✗	50-100	Haskell
SYNCHROMESH (Poesia et al., 2022)	✓	✓	✗	✗	‡	ANTLR
LLAMA.CPP (Gerganov and et. al., 2024)	✓	✓	✗	✗	50-100	GBNF DSL
GCD (Geng et al., 2023)	✓	✓	✗	✗	50-100	GF
DOMINO (Beurer-Kellner et al., 2024)	✓	✓	✓	✗	50-100	GBNF DSL
SYNCODE (ours)	✓	✓	✓	✓	500+	Lark EBNF

† Implementation issues ‡ Synchromesh is closed-source and the information about DSL grammars is unavailable

GF: Grammatical Framework, GBNF is a DSL defined by LLAMA.CPP

indicates the format used to specify generation constraints. In addition to the improvement over the baselines presented in the evaluation, our work focuses on rigorously formalizing the correctness of our CFG-guided generation approach.

Recent works such as GUIDANCE (Lundberg et al., 2023) and LMQL (Beurer-Kellner et al., 2023) mitigate the unpredictability of LLM responses by using template or constraint-based controlled generation techniques. These libraries feature a templating engine where prompts are expressed with holes for the generation to fill. LMQL (Beurer-Kellner et al., 2023) supports general regular expression constraints, but not CFG constraints. GUIDANCE (Lundberg et al., 2023) supports CFG-guided generation. It uses Earley parsing (Earley, 1970) for constrained decoding. Similar to other related works, it incurs high inference overhead as it checks the syntactical validity of the entire model vocabulary at each step. It uses a trie similar to (Poesia et al., 2022; Willard and Louf, 2023; Beurer-Kellner et al., 2024). As shown in our evaluation it incurs higher overhead for JSON generation than SYNCODE. It iterates over the vocabulary in order of the next token probability to efficiently compute the next token. However, this leads to a lack of generality and it cannot be directly combined with an arbitrary decoding strategy.

OUTLINES (Willard and Louf, 2023) is a library originally focused on regular expression-guided generation and recently extended to support grammar-guided generation. During LLM generation, OUTLINES employs an incremental Lark-based LALR parser to determine the next acceptable terminals based on the grammar. It constructs a larger regular expression by computing the union of regular expressions from all terminals, which is then converted into a DFA during inference. It then iterates over all LLM tokens and collects the set of tokens that lead to a valid path through this combined DFA. As shown in our evaluation, SYNCODE performs better than OUTLINES on generating with JSON grammar and it currently lacks support for large GPL grammars.

LLAMA.CPP (Gerganov and et. al., 2024), has also recently introduced support for grammar-guided generation. This approach models a nondeterministic pushdown automaton with N stacks to maintain possible parse states. LLAMA.CPP defines a new grammar syntax and implements a simplified basic parser in C++. While this implementation in C++ reduces some parsing overhead compared to heavier LR(1) parsers implemented in Python on top of Lark for SYNCODE, it is algorithmically inefficient. This inefficiency again is due to the requirement to iterate over the entire vocabulary and update stack states during inference. Moreover, the non-standard grammar syntax and limited support for general grammar features restrict its evaluation to simpler grammars such as JSON. We anticipate that LLAMA.CPP and OUTLINES would perform even slower on grammars with more rules, terminals, and complex regular expressions, such as those found in Python and Go. As shown in our evaluation, SYNCODE is more efficient and results in fewer syntax errors.

SYNCHROMESH (Poesia et al., 2022) is a proprietary a tool from Microsoft that supports CFG-guided syntactic decoding of LLMs. Similar to OUTLINES, it creates a union of regular expressions of terminals during LLM generation. Further, Synchromesh uses a non-incremental parser for parsing. Both of which lead to lower time complexity. Synchromesh uses techniques like Target Similarity Tuning for semantic example selection and Constrained Semantic Decoding to enforce user-defined semantic constraints and works on DSLs. In contrast, our work, SynCode focuses exclusively on syntactic generation.

Picard (Scholak et al., 2021) uses a specific decoding strategy that maintains a beam of multiple candidate outputs and promptly rejects the candidates that violate the syntax. It utilizes an incremental monadic parser and was developed specifically to support SQL generation. Introducing a new grammar into Picard necessitates considerable effort, as it lacks support for a grammar-defining language to provide grammar rules.

Recent work Domino (Beurer-Kellner et al., 2024) does CFG-guided LLM generation. It avoids traversing the whole vocabulary during inference by precomputing a prefix tree corresponding to each NFA state of the terminals of the grammar. The purpose of creating this structure is similar to SynCode's DFA mask store. We believe that SynCode's mask store is more efficient than Domino's prefix tree since on modern machines (especially with GPUs) the union of the boolean masks from mask store can be performed quite efficiently in practice (Paszke et al., 2019b). Domino defines the *minimally invasive* property which is equivalent to SynCode's soundness property. One key difference between SynCode and Domino is that Domino applies under-approximation, permitting only tokens that align with the lookahead of the parser, while SynCode adopts a conservative over-approximation approach, allowing tokens as long as their prefixes match the parser lookahead. Due to the under-approximation, they claim that it requires ∞ parser lookahead to get this soundness, whereas SynCode ensures soundness for any lookahead. Further, the largest grammar that Domino can support currently is highly simplified C grammar with 70 rules with roughly 25% overhead. Domino's code is not available yet to experimentally compare it with SynCode.

Fixed Schema Generation. Many recent works perform constrained LLM decoding to ensure that the generated output follows a fixed schema of JSON or XML (Zheng et al., 2023; Beurer-Kellner et al., 2024; Willard and Louf, 2023; Sengottuvelu and et. al., 2024). When employing a fixed schema, many intermediate points in the generation process offer either a single syntactical choice (e.g., key in the JSON schema) or present only a handful of distinct options. In cases where only one choice exists, the generation of the next token through the LLM can be entirely skipped. Alternatively, when there are multiple but limited choices, techniques like speculative decoding can be used to expedite the generation process (Chen et al., 2023; Leviathan et al., 2023). SynCode does not focus on generation problems with fixed schema, it solely focuses on CFG-guided generation. We made the same observation as in (Beurer-Kellner et al., 2024), techniques such as speculation are not useful for CFGs where the schema is not fixed.

8 Conclusion

Existing methods for guiding LLMs to produce syntactically correct output have been notably slow and restrictive. In this paper, we present SynCode, an efficient and general framework to enhance LLMs' ability to generate syntactical output for various formal languages. During decoding, SynCode incrementally parses the partially generated output, computes the unparsed remainder and acceptable terminal sequences, and then leverages the remainder, accept sequences, and pre-computed DFA mask store to compute a mask to constrain the LLM's vocabulary to only syntactically valid tokens. We evaluated SynCode on generating syntactically correct JSON, SQL, Python, and Go code with different combinations of datasets, models, and tasks. SynCode eliminates syntax errors in JSON completions and significantly improves JSON schema validation over the baselines. Furthermore, SynCode reduces the number of syntax errors in generated Python and Go code by 96.07% on average compared to standard generation. We believe that our approach will pave the way for more efficient and higher-quality structured LLM generation in real-world applications.

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A Appendix

A.1 List of Symbols

G	Formal Grammar
$L(G)$	Language of a grammar
$L_p(G)$	Prefix language of a grammar
l	lexical tokens
l_i	i-th lexical token in the parsed output
τ	A terminal in the grammar
τ_i	Terminal type of i-th lexical token
Γ	Set of all terminals in the grammar
$L^\Gamma(G)$	Language of terminals for grammar G
$L_p^\Gamma(G)$	Prefix language of terminals
P	Parser
Λ	Sequence of terminals
$\mathcal{T}$	Tokenizer in an LLM
V	Vocabulary of an LLM
V_k	Subset of vocabulary containing acceptable tokens at k-th LLM generation iteration
ρ_τ	Regular expression for a terminal τ
ρ_i	Regular expression corresponding to i-th lexical token
$\preccurlyeq$	Partial order over set of terminal sequences
r	Remainder from SynCode parsing the partial output
C_k	Partial output at k-th iteration of LLM generation
$C_k^\square$	Parsed prefix of partial output C_k at k-th iteration of LLM generation
$\mathcal{A}$	Set of accept sequences
$\mathcal{M}_\alpha$	DFA lookup store function for terminal sequences of length α
$dmatch$	Match with DFA walk as defined in Section 4
$pmatch$	Partial match with regular expression
$pparse$	Partial parsing function
m	Boolean mask
D	Deterministic finite automaton
Q	States in a DFA
Σ	Set of characters i.e. alphabet for DFA
δ	Transition function in a DFA
δ^*	Extended transition function in a DFA
q_0	Start state of a DFA
F	Set of final states in DFA
$live$	Live states of the DFA
Q_Ω	Set containing all DFA states for DFAs of all terminals in the grammar
A_0	Set of terminals acceptable for current lexical token
A_1	Set of terminals acceptable as for next lexical token
Lex	Lexer function
len	Length of a sequence
T_{cur}	Current set of tokens
$\mathcal{S}$	Map for storing parser state

A.2 Proofs for Theorems

Lemma 1. *Given* $\Lambda = \{\tau_{f+1}, \tau_{f+2} \ldots \tau_{f+d}\}$, $\Lambda^p = \{\tau_{f+2} \ldots \tau_{f+d}\}$ *and* $\rho_\Lambda = (\rho_{f+1}, \rho_{f+2}, \ldots, \rho_{f+d})$, $dmatch(w, q_0^{\tau_1}, \Lambda^p) \iff pmatch(w, \rho_\Lambda)$.

Proof. (a) First we prove $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p) \implies pmatch(w, \rho_\Lambda)$ We prove this using induction on the length i of w.
For $i = 0$, $pmatch(w, \rho_\Lambda)$ is trivially true.
Now, we assume that for w of length $i < k$, $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p) \implies pmatch(w, \rho_\Lambda)$.
We consider w of length k and $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$.
We consider 3 conditions from Definition 10.
If condition 1 is true, $\delta^*_{\tau_{f+1}}(w, q_0^{\tau_{f+1}}) \in live(Q_{\tau_{f+1}})$. Let $q_1 = \delta^*(w, q_0^{\tau_{f+1}})$. By Definition 9, $\exists w_1$ s.t. $\delta^*_{\tau_{f+1}}(w_1, q_1) \in F_{\tau_{f+1}}$. Hence,

$$\delta^*(w.w_1, q_0^{\tau_{f+1}}) \in F_{\tau_{f+1}} \implies w.w_1 \in L(\rho_{\tau_{f+1}})$$

We assume that each terminal $L(\tau_i)$ is non-empty. Hence,

$$\exists w_2 \in L(\rho_{\Lambda^p}) \implies w.w_1.w_2 \in L(\rho_\Lambda)$$

Hence, by condition 2 from Definition 8, $pmatch(w, \rho_\Lambda)$.

If condition 2 is true, $\exists w_1, w_2$ such that $w_1.w_2 = w$, $\delta^*_{\tau_{f+1}}(w_1, q_0^{\tau_{f+1}}) \in F$ and $\Lambda^p = \{\}$. Here, $w_1 \in L(\rho_{\tau_{f+1}})$. Since $\Lambda^p = \{\}$, $\rho_\Lambda = \rho_1$, and hence, $w_1 \in L(\rho_\Lambda)$. Hence by condition 1 from Definition 8, $pmatch(w, \rho_\Lambda)$.

If condition 3 is true, $\exists w_1, w_2$ such that $w_1.w_2 = w$, $\delta^*_{\tau_{f+1}}(w_1, q_0^{\tau_{f+1}}) \in F_{\tau_{f+1}}$, and $dmatch(w_2, q_0^{\tau_{f+2}}, \{\tau_{f+3} \ldots \tau_{f+d}\}) = true$.

$$\delta^*_{\tau_{f+1}}(w_1, q_0^{\tau_{f+1}}) \in F_{\tau_{f+1}} \implies w_1 \in L(\rho_{\tau_{f+1}})$$

Since length of $w_2 < k$, by our induction hypothesis, $pmatch(w_2, \rho_{\Lambda^p}) = true$. By Definition 8, there are two possibilities. Suppose $\exists w_2 = w_3.w_4$ such that $w_3 \in L(\rho_{\Lambda^p})$.

$$w_1.w_3 \in L(\rho_\Lambda) \implies pmatch(w, \rho_\Lambda) = true$$

Alternatively, if $\exists w_3$ such that $w_2.w_3 \in L(\rho_{\Lambda^p})$

$$w_1.w_2.w_3 \in L(\rho_\Lambda) \implies pmatch(w, \rho_\Lambda) = true$$

Hence, our induction proof is complete and $pmatch(w, \rho_\Lambda) = true$

(b) Next we prove $pmatch(w, \rho_\Lambda) \implies dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$ We prove this using induction on the length i of w.
For $i = 0$, $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$ is trivially true.
Now, we assume that for w of length $i < k$, $pmatch(w, \rho_\Lambda) \implies dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$
Now we consider w of length k and $pmatch(w, \rho_\Lambda)$.
By Definition 8, there are two possible conditions

Case 1: $\exists w_1 \in \Sigma^*, w_2 \in \Sigma^+$ such that $w = w_1.w_2$ and $w_1 \in L(\rho_\Lambda)$
Hence, $\exists w_3, w_4$ such that $w_1 = w_3.w_4$ and $w_3 \in L(\rho_{\tau_{f+1}})$ and $w_4 \in L(\rho_{\Lambda^p})$. By induction hypothesis,

$$pmatch(w_4.w_2, \rho_{\Lambda^p}) \implies dmatch(w_4 w_2, \{\tau_{f+2}, \tau_{f+3} \ldots \tau_{f+d}\})$$

Since $w = w_3.w_4.w_2$ and

$$w_3 \in L(\rho_{\tau_{f+1}}) \implies \delta^*_{\tau_{f+1}}(w_3, q_0^{\tau_{f+1}}) \in F_{\tau_{f+1}}$$

Hence, by condition 3 in Definition 10, $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$

Case 2: $\exists w_1$ such that $w.w_1 \in L(\rho_\Lambda)$
Hence, $\exists w_2, w_3$ s.t $w.w_1 = w_2.w_3$ and $w_2 \in L(\rho_{\tau_{f+1}})$ and $w_3 \in L(\rho_\Lambda)$
Now there are two possibilities, either w is prefix of w_2 or w_2 is prefix of w_2
Supoose w is prefix of w_2, then $\delta^*_{\tau_{f+1}}(w, q_0^{\tau_{f+1}}) \in live(Q_{\tau_{f+1}})$ and hence by Definition 10, $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$ Alternatively, if w_2 is prefix of w then $\exists w_4$ s.t. $w = w_2 w_4$
Hence, $w_4.w_1 = w_3 \in L(\rho_{\tau_{f+1}})$ and thus $pmatch(w_4, \rho_{\Lambda^p})$
By induction hypothesis $dmatch(w_4, q_0^{\tau_{f+2}}, \{\tau_{f+3}, \tau_4 \dots \tau_{f+d}\})$
and since $w = w_2.w_4$ and $\delta^*_{\tau_{f+1}}(w_2, q_0^{\tau_{f+1}}) \in F_{\tau_{f+1}}$. We get $dmatch(w, q_0^{\tau_{f+1}}, \Lambda^p)$

□

Lemma 2. *If $q = \delta^*_\tau(r, q_0^\tau)$ and no prefix of r is in $L(\tau)$ i.e. $\nexists w_1 \in \Sigma^*, w_2 \in \Sigma^*$ such that $w_1.w_2 = r$ and $\delta^*_\tau(w_1, q_0^\tau) \in F_\tau$ then $dmatch(t, q, \Lambda) \iff dmatch(r.t, q_0^\tau, \Lambda)$.*

Proof. (a) First, we prove $dmatch(t, q, \Lambda) \implies dmatch(r.t, q_0^\tau, \Lambda)$.
From Definition 10, either of the 3 conditions hold true for $dmatch(t, q, \Lambda)$.

If condition 1 is true then

$$\delta^*_{\tau_1}(t, q) \in live(Q_\tau) \implies \delta^*_\tau(r.t, q_0^\tau) \in live(Q_\tau) \implies dmatch(r.t, q_0^\tau, \Lambda)$$

If condition 2 is true, $\exists w_1, w_2$ such that $w_1.w_2 = t$, $\delta^*_\tau(w_1, q) \in F$ and $\Lambda = \{\}$. Therefore,

$$\delta^*_\tau(r.w_1, q) \in F \implies dmatch(r.t, q_0^\tau, \Lambda)$$

If condition 3 is true, $\exists w_1, w_2$ such that $w_1.w_2 = t$, $\delta^*_\tau(w_1, q) \in F$
and $dmatch(w_2, q_0^{\tau_1}, \{\tau_2 \dots \tau_d\}) = true$. Therefore,

$$\delta^*_\tau(r.w_1, q) \in F \implies dmatch(r.t, q_0^\tau, \Lambda)$$

Therefore, in all cases, $dmatch(rt, q_0^\tau, \Lambda)$ must hold.

(b) Now, we prove $dmatch(rt, q_0^\tau, \Lambda) \implies dmatch(t, q, \Lambda)$.
From Definition 10, either of the 3 conditions hold true for $dmatch(r.t, q_0^\tau, \Lambda)$.

If condition 1 is true then

$$\delta^*_{\tau_1}(r.t, q_0^\tau) \in live(Q_\tau) \implies \delta^*_\tau(t, q) \in live(Q_\tau) \implies dmatch(t, q, \Lambda)$$

If condition 2 is true, $\exists w_1, w_2$ such that $w_1.w_2 = r.t$, $\delta^*_\tau(w_1, q_0^\tau) \in F$ and $\Lambda = \{\}$. Since no prefix of r is accepted by $L(\tau)$, $\exists w_3$ s.t. $w_3 w_4 = t$ and

$$\delta^*_\tau(w_3, q) \in F \implies dmatch(t, q, \Lambda)$$

If condition 3 is true, $\exists w_1, w_2$ such that $w_1.w_2 = r.t$, $\delta^*_\tau(w_1, q_0^\tau) \in F$
and $dmatch(w_2, q_0^{\tau_1}, \{\tau_2 \dots \tau_d\}) = true$. Since no prefix of r is accepted by $L(\tau)$, $\exists w_3$ s.t. $w_3 w_4 = t$ and

$$\delta^*_\tau(w_3, q) \in F \implies dmatch(t, q, \Lambda)$$

Therefore, in all cases, $dmatch(t, q, \Lambda)$ must hold.

□

Theorem 1. *Let $C_k \in L_p(G)$ be the partial output and any integer $d \geq 1$, let $\mathcal{A}_d \subseteq \Gamma^d$ contain all possible accept terminal sequences of length d and $r \in \Sigma^*$ denote the remainder. If $m = GrammarMask(\mathcal{A}, r)$ then for any $t \in V$, if $C_k.t \in L_p(G)$ then $t \in set(m)$*

Proof. Let $r, \Lambda^\square = pparse(C_k)$ where $\Lambda^\square = \tau_1, \tau_2 \ldots \tau_f$ and let $r_1, \Lambda_1 = pparse(C_k.t)$ where $\Lambda_1 = \tau_1, \tau_2 \ldots \tau_f \ldots \tau_{f+g}$
Hence, we can split $r.t$ such that for $w \in \Sigma^*$, $r.t = w.r_1$ and $w \in L(\tau_{f+1} \ldots \tau_{f+g})$
There are two possible cases:
Case 1: $g < d$

$$w \in L(\tau_{f+1} \ldots \tau_{f+g})$$

$$\implies w \in L_p(\tau_{f+1} \ldots \tau_{f+g})$$

By our assumption on $\mathcal{A}_d$ there must exist $\Lambda_2 = \tau_{f+1} \ldots \tau_{f+d}$ s.t. $\tau_{f+1} \ldots \tau_{f+g}$ is prefix of Λ_2. Hence,

$$\implies w \in L_p(\Lambda_2)$$

$$\implies pmatch(r.t, \Lambda_2)$$

Case 2: $g \geq d$
Since we assume that $\mathcal{A}_d$ contains all possible accept sequence of length d, $\Lambda_2 = \tau_{f+1} \ldots \tau_{f+d}$ must be contained in $\mathcal{A}_d$
Hence, $\exists w_1, w_2 \in \Sigma^*$ such that $w = w_1.w_2$ and

$$w_1 \in L(\Lambda_2)$$

$$\implies w \in L_p(\Lambda_2)$$

$$\implies pmatch(r.t, \Lambda_2)$$

In both cases, $pmatch(r.t, \Lambda_2)$. Using Lemma 1,

$$\implies dmatch(r.t, q_0^{\tau_{f+1}}, \{\tau_{f+2} \ldots \tau_{f+d}\})$$

Using Lemma 2 if $q = \delta^*_{\tau_{f+1}}(r, q_0^{\tau_{f+1}})$

$$dmatch(r.t, q_0^{\tau_{f+1}}, \{\tau_{f+2} \ldots \tau_{f+d}\}) \implies dmatch(t, q, \{\tau_{f+2} \ldots \tau_{f+d}\})$$

Here from Definition 12, if $\mathcal{M}_{d-1}(q, \{\tau_{f+2} \ldots \tau_{f+d}\}) = m_2$ then $t \in set(m_2)$.
Since $m_2 \subseteq m$, we have our result $t \in set(m)$.

$\square$

Lemma 3. *Given $\mathcal{A}_1$ and $\mathcal{A}_2$ are set of accept sequences such that $\mathcal{A}_1 \preccurlyeq \mathcal{A}_2$ and $m_1 = GrammarMask(\mathcal{A}_1, r)$ and $m_2 = GrammarMask(\mathcal{A}_2, r)$ then $set(m_2) \subseteq set(m_1)$*

Proof. Since $\forall \Lambda_2 \in \mathcal{A}_2 \exists \Lambda_1 \in \mathcal{A}_1 \exists \Lambda_3 \in \Gamma^*$ s.t. $\Lambda_2 = \Lambda_1.\Lambda_3$, Hence

$$pmatch(w, \rho_{\Lambda_2}) \implies pmatch(w, \rho_{\Lambda_1})$$

Hence, for the mask $set(m_2) \subseteq set(m_1)$ $\square$

Theorem 2. *Let $C_k \in L_p(G)$ be the partial output, let $\mathcal{A}_d \subseteq \Gamma^d$ contain all possible accept terminal sequences of length d and $r \in \Sigma^*$ denote the remainder. Suppose for any $t \in V, d > len(t)$ and $m = GrammarMask(\mathcal{A}_d, r)$ such that $t \in set(m)$ then $C_k.t \in L_p(G)$*

For the simplicity of presenting the proof, we assume that $d > 2$.

Since $t \in set(m)$ for some $\Lambda_1 = \{\tau_{f+1}, \tau_{f+2} \dots \tau_{f+d}\} \in \mathcal{A}$

$$\implies dmatch(t, q, \{\tau_{f+2} \dots \tau_{f+d}\}) \implies dmatch(r.t, q_0^{\tau_{f+1}}, \{\tau_{f+2} \dots \tau_{f+d}\})$$

$$\implies pmatch(r.t, \{\rho_{\tau_{f+1}}.\rho_{\tau_{f+2}} \dots \rho_{\tau_{f+d}}\})$$

By Definition 8, there are two possible cases:

1. $\exists w_1 \in \Sigma^*, w_2 \in \Sigma^+$ such that $r.t = w_1.w_2$ and $w_1 \in L(\rho_{\tau_{f+1}}.\rho_{\tau_{f+2}} \dots \rho_{\tau_{f+d}})$
   We show that this case is not possible since our terminal sequence Λ_1 is long enough that no prefix of $r.t$ cannot be in $L(\rho_{\tau_{f+1}}.\rho_{\tau_{f+2}} \dots \rho_{\tau_{f+d}})$
   We can infer that $len(w_1) < len(r.t) \implies len(w_1) < len(r) + len(t)$
   Further, from the assumption $d > len(t)$, we have

   $$len(w_1) < d + len(r)$$

   Firstly, note that $r \notin L(\rho_{\tau_{f+1}}.\rho_{\tau_{f+2}})$ by the definition of remainder r
   Note that we assume no terminal contains empty string i.e. $\epsilon \notin L(\rho_{\tau_i})$
   Hence, every string in $L(\rho_{\tau_{f+2}} \dots \rho_{\tau_{f+d}})$ should have length at least $d - 1$

   Clearly, r is prefix of w_1. Let $w_3 \in \Sigma^*$, $r.w_3 = w_1$ and hence $len(w_3) > d - 1$
   Hence,

   $$len(r) + d - 1 < len(w_1)$$

   $$len(r) + d - 1 < len(w_1) < d + len(r)$$

   This is not possible and hence such w_1 cannot exist.

2. $\exists w_1 \in \Sigma^*$ such that $r.t.w_1 \in L(\rho_{\tau_{f+1}}.\rho_{\tau_{f+2}} \dots \rho_{\tau_{f+d}})$

   By Definition 5, we have $\Lambda^\square, r = pparse(C_k)$ s.t $C_k = C_k^\square.r$, $\Lambda^\square = \tau_1, \tau_2 \dots \tau_f$ $C_k^\square \in L(\rho_{\tau_1}.\rho_{\tau_2} \dots \rho_{\tau_f})$.
   Let $\Lambda_1 = \tau_{f+1}, \tau_{f+2} \dots \tau_{f+d}$
   Since, $C_k.t = C_k^\square.r.t$, $C_k^\square \in L(\Lambda^\square)$ and $r.t.w_1 \in L(\Lambda_1)$, we have

   $$C_k^\square.r.t.w_1 \in L(\Lambda^\square.\Lambda_1)$$

   $$C_k.t.w_1 \in L(\Lambda^\square.\Lambda_1)$$

   By Definition 7 of accept sequence, $\Lambda^\square.\Lambda_1 \in L_p^\Gamma(G)$, Hence

   $$C_k.t.w_1 \in L_p(G) \implies C_k.t \in L_p(G)$$

Thus, our proof is complete and $C_k.t \in L_p(G)$

A.3 Incremental Parsing Algorithm

Our parsing algorithm achieves incrementality in LLM generation by utilizing a map $\mathcal{S}$ to store the parser state. This map associates a list of lexical tokens with the corresponding parser state after parsing those tokens. Frequently, in subsequent LLM generation iterations, the count of lexical tokens remains the same—either the next vocabulary token is appended to the final lexical token, or it increases. Although uncommon, there are cases where the number of parsed lexical tokens may decrease during iterations. For example, in Python, an empty pair of double quotes, "", is recognized as a complete lexical token representing an empty string. On the other hand, """ serves as a prefix to a docstring, constituting an incomplete parser token. Consequently, the addition of a single double quote " reduces the overall count of lexical tokens in these iterations. We observe that while the total count of lexer tokens at the end may undergo slight changes during these iterations, the majority of prefixes of the parsed lexical tokens remain consistent. Hence, we establish a mapping between lists of prefixes of lexical tokens and the corresponding parser state after parsing those tokens. Subsequently, when parsing a new list of lexer tokens, we efficiently determine the maximum length prefix of the lexer token list that is already present in $\mathcal{S}$. This incremental approach significantly reduces the complexity of our parsing algorithm.

Algorithm 4 Incremental Parsing

Inputs: C_k: partial output, $\mathcal{S}$: state map

function PARSE(C_k)
    $l_1, l_2 \dots l_f \leftarrow Lex(C_k)$
    $\gamma, S_\gamma \leftarrow \text{RestoreState}(\mathcal{S}, L)$
    $P \leftarrow \text{Initialize}(S_\gamma)$
    $parsed \leftarrow l_1.l_2 \dots l_{\gamma-1}$
    for $l_i \in l_\gamma, l_{\gamma+1} \dots l_f$ **do**
        $Next(P, l_i)$
        if $P.state = Error$ **then**
            break
        $parsed \leftarrow parsed + l_i$
        $A_0 \leftarrow A_1$
        $A_1 \leftarrow Follow(P)$
        $S_i \leftarrow P.state$
        $\text{Store}(\mathcal{S}, parsed, S_i)$
    if $C_k = parsed$ **then**
        $r = l_f$
        $\mathcal{A} \leftarrow \{\tau_f, A_1[0]\}, \{\tau_f, A_1[1]\} \dots \}$
            $\cup \{A_0[0]\}, \{A_0[1]\} \dots \}$
    else
        $r = C_k - parsed$
        $\mathcal{A} \leftarrow \{A_1[0]\}, \{A_1[1]\} \dots$
    return $\mathcal{A}, r$

While it could be feasible to introduce incrementality in the lexing operation, our experiments revealed that lexing consumes insignificant time in comparison to parsing. As a result, we opted to focus only on performing parsing incrementally.

Our incremental parsing algorithm uses a standard non-incremental base parser P that maintains a parser state and supports two functions *Next* and *Follow*. The *Next* function accepts the next lexer token and then updates the parser state. The *Follow* function returns a list of acceptable terminals at the current parser state. These functions are present in common parser generator tools (Lark, ; ANTLR,).

The Algorithm 4 presents our incremental parsing algorithm. The algorithm utilizes a lexer to tokenize the partial output. The function RestoreState is used to restore the state of the parser to the maximal matching prefix of lexical tokens that exist in $\mathcal{S}$. The main loop iterates through the tokens and maintains a parser state map. For each token, it updates the parser state, stores the mapping in $\mathcal{S}$, and retrieves the next set of acceptable terminals. The process continues until the end of the partial output. The algorithm returns accept sequences $\mathcal{A}$ and remainder r.

A.4 Ablation Studies

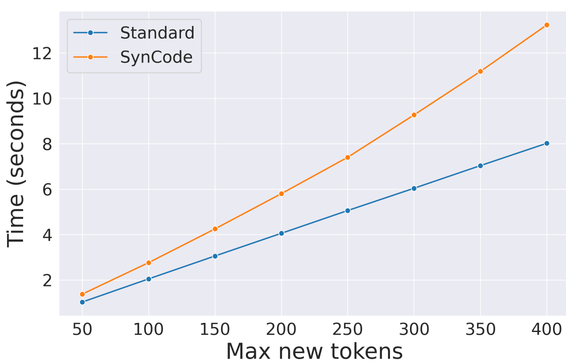
In this section, we perform an ablation study for incremental parsing and max new tokens.

Incremental Parsing. We compare the runtime efficiency of utilizing incremental parsing over re-running parsing from scratch in SYNCODE. We run inference on CodeGen-350M with SYNCODE using incremental parsing and parsing from scratch on Python prompts from the HumanEval dataset. We generate $n = 1$ samples and control the max new tokens in the code completion. Our results are presented in Figure 10b, where the x-axis represents the max new tokens and the y-axis represents the average generation time, in

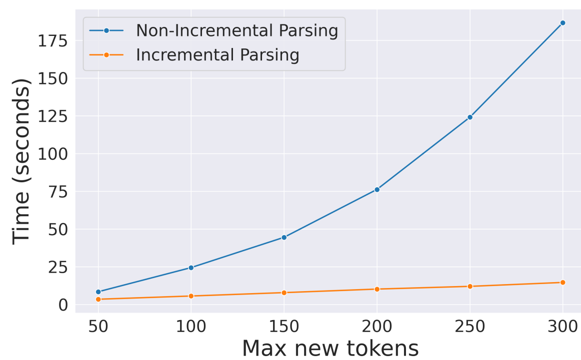
Table 7: SynCode on few-shot prompting

Architecture	Error Type	Standard	SynCode	↓
CodeGen-350M	Syntax	53	0	100%
	Indentation	15	3	80%
WizardCoder-1B	Syntax	40	2	95%
	Indentation	22	1	95%
Llama-7B	Syntax	110	0	100%
	Indentation	40	5	88%

seconds, with and without incremental parsing. As shown in the figure, the average generation time when re-parsing from scratch increases significantly as the maximum length of code that the LLM can generate increases. On the other hand, the average generation time increases slowly with incremental parsing. For max new tokens = 300, SynCode with incremental parsing achieves 9x speedup over running parsing from scratch. Our results collectively demonstrate that augmenting SynCode with incremental parsing dramatically improves generation time, especially when generating longer completions.



(a) Average generation time for different max new tokens



(b) Average generation time with and without incremental parser

Figure 10: Ablation studies on CodeGen-350M model.

Max New Tokens. We conduct an ablation study into the relationship between the maximum length of code that the LLMs can generate and generation times. We used Python prompts from the HumanEval dataset and leveraged CodeGen-350M to generate the code completions, both with and without the augmentation of the SynCode. As shown in Figure 10a, as we increase the max new tokens, we observe a corresponding increase in generation time.

A.5 Few-Shot Prompting

Few-shot prompting (Ren et al., 2018) refers to the idea that language models do not need to be specifically trained for a downstream task such as classification or question answering. Rather, it is sufficient to train them on broad text-sequence prediction datasets and to provide context in the form of examples when invoking them. We study the performance of utilizing SynCode on few-shot prompting code generation tasks. We selected Python few-shot examples from the MBXP dataset and generated code completions with CodeGen-350M, LLaMA-7B, and WizardCoder-1B with SynCode and the standard no-masking generation. We present our results in Table 7. The columns standard and SynCode represent the total number of errors of a particular Error Type of LLM-generated code completions to problems in a particular dataset when utilizing that respective generation approach. The column ↓ represents the percentage reduction from the standard column to the SynCode column. As shown in the table, SynCode exhibits effectiveness not only in zero-shot but also in the context of few-shot prompting tasks. This signifies the versatility of SynCode in enhancing code generation across different prompt engineering techniques.

```python
def max_fill(grid, capacity):
    """You are given a rectangular grid of wells. Each row represents a single well,
       and each 1 in a row represents a single unit of water.
       Each well has a corresponding bucket that can be used to extract water from it,
       and all buckets have the same capacity. Your task is to use the buckets to empty the wells.
       Output the number of times you need to lower the buckets."""
    if len(grid) < 2:
        return 0
    if len(grid) == 1:
        return 1
    if len(grid) == 2:
        return grid[0][1] - grid[0][0]
    if len(grid) == 3:
        return grid[0][1] - grid[0][0] - grid[1][1]
    … 11 more lines
    if len(grid) == 9:
        return grid[0][1] - grid[0][0] - grid[1][1] - grid[2][1] - grid[
```

Figure 11: Syntactically Incorrect SynCode Program

A.6 SynCode Syntax Errors

Figure 11 presents an example of when the SynCode augmented LLM fails to generate a complete program within the maximum token limit for a problem from the HumanEval dataset. While the code is a syntactically correct partial program, it is not a syntactically correct complete program. Recall, that SynCode guarantees completeness for syntactically correct partial programs but does not guarantee termination with a syntactically correct complete program.

A.7 Prompts Used in the Evaluation

```
<s>[INST] <<SYS>>
You are a helpful assistant that answers in JSON. Here’s the json schema you must adhere to:
<schema>
{’title’: ’Person’, ’type’: ’object’, ’properties’: {’firstName’: {’type’: ’string’, ’description’: "The person’s first name."}, ’lastName’: {’type’: ’string’, ’description’: "The person’s last name."}, ’age’: {’description’: ’Age in years which must be equal to or greater than zero.’, ’type’: ’integer’, ’minimum’: 0}}, ’required’: [’firstName’, ’lastName’, ’age’]}
</schema>

<</SYS>>

Please generate a JSON output for a person’s profile that includes their first name, last name, and age. The first name should be ’Alice’, the last name ’Johnson’, and the age 35. [/INST]
```

Listing 1: Example original JSON Prompt from the JSON-Mode-Eval dataset (NousResearch, 2024). The prompt consists of a system message that specifies a schema and a user message requesting JSON output given certain parameters.

```
<s>[INST] <<SYS>>
You are a helpful assistant that answers in JSON. Here’s the json schema you must adhere to:
<schema>
{’title’: ’Person’, ’type’: ’object’, ’properties’: {’firstName’: {’type’: ’string’, ’description’: "The person’s first name."}, ’lastName’: {’type’: ’string’, ’description’: "The person’s last name."}, ’age’: {’description’: ’Age in years which must be equal to or greater than zero.’, ’type’: ’integer’, ’minimum’: 0}}, ’required’: [’firstName’, ’lastName’, ’age’]}
</schema>

<</SYS>>

Please generate a JSON output for a person’s profile that includes their first name, last name, and age. The first name should be ’Alice’, the last name ’Johnson’, and the age 35. Output only JSON. [/INST]
```

Listing 2: Example JSON prompt from the JSON-Mode-Eval dataset (NousResearch, 2024) after augmentation with an explicit request to only output JSON.

```

db_id: concert_singer
db_info: # stadium ( stadium_id , location , name , capacity , highest , lowest , average )
# singer ( singer_id , name , country , song_name , song_release_year , age , is_male )
# concert ( concert_id , concert_name , theme , stadium_id , year )
# singer_in_concert ( concert_id , singer_id )
# concert.stadium_id = stadium.stadium_id
# singer_in_concert.singer_id = singer.singer_id
# singer_in_concert.concert_id = concert.concert_id

question: How many singers do we have? Only output the SQL query.
SQL:
```

Listing 3: text-2-SQL prompt.

```
def has_close_elements(numbers: List[float], threshold: float) -> bool:
        """ Check if in given list of numbers, are any two numbers closer to each other than
        given threshold.
        >>> has_close_elements([1.0, 2.0, 3.0], 0.5)
        False
        >>> has_close_elements([1.0, 2.8, 3.0, 4.0, 5.0, 2.0], 0.3)
        True
        """
```

Listing 4: Example Python prompt from the HumanEval dataset (Athiwaratkun et al., 2023)

```
package main

import (
  "encoding/json"
  "reflect"
)
// You’re an expert Golang programmer
// Check if in given list of numbers, are any two numbers closer to each other than
```

```
// given threshold.
// >>> has_close_elements([1.0, 2.0, 3.0], 0.5)
// False
// >>> has_close_elements([1.0, 2.8, 3.0, 4.0, 5.0, 2.0], 0.3)
// True
//
func has_close_elements (numbers []float64, threshold float64) bool {
```

Listing 5: Example Go prompt from the HumanEval dataset (Athiwaratkun et al., 2023)

A.8 Grammars Used in the Evaluation

A.8.1 JSON Grammar

```
?start: value

?value: object
| array
| UNESCAPED_STRING
| SIGNED_NUMBER      -> number
| "true"             -> true
| "false"            -> false
| "null"             -> null

array  : "[" [value ("," value)*] "]"
object : "{" [pair ("," pair)*] "}"
pair   : UNESCAPED_STRING ":" value

UNESCAPED_STRING: /\"[^"]*\"/

DIGIT: "0".."9"
HEXDIGIT: "a".."f"|"A".."F"|DIGIT
INT: DIGIT+
SIGNED_INT: ["+"|"-"] INT
DECIMAL: INT "." INT? | "." INT


_EXP: ("e"|"E") SIGNED_INT
FLOAT: INT _EXP | DECIMAL _EXP?
NUMBER: FLOAT | INT
SIGNED_NUMBER: ["+"|"-"] NUMBER
WS: /[ \t\f\r\n]/+

%ignore WS
```

Listing 6: JSON Grammar

A.8.2 SQL Grammar

```

start: set_expr ";"? -> final

set_expr: query_expr
        | set_expr "UNION"i ["DISTINCT"i] set_expr -> union_distinct
        | set_expr "UNION"i "ALL"i set_expr -> union_all
        | set_expr "INTERSECT"i ["DISTINCT"i] set_expr -> intersect_distinct
        | set_expr "EXCEPT"i ["DISTINCT"i] set_expr -> except_distinct
        | set_expr "EXCEPT"i "ALL"i set_expr -> except_all

query_expr: select [ "ORDER"i "BY"i (order_by_expr ",")*  order_by_expr] [ "LIMIT"i
     limit_count [ "OFFSET"i skip_rows ] ]

select: "SELECT"i [SELECT_CONSTRAINT] [(select_expr ",")*] select_expr "FROM"i [(from_expr
     ",")*] from_expr [ "WHERE"i where_expr ] [ "GROUP"i "BY"i [(groupby_expr ",")*]
     groupby_expr ] [ "HAVING"i having_expr] [ "WINDOW"i window_expr ]

where_expr: bool_expression

select_expr.0: expression_math [ "AS"i alias ] -> select_expression

?from_expr: from_item -> from_expression

order_by_expr: order -> order_by_expression

```

```
having_expr: bool_expression

groupby_expr: expression -> group_by

window_expr: [window_expr ","] _window_name "AS"i ( window_definition )

from_item: table_name [ "AS"i alias ] -> table
            | join -> join
            | cross_join -> cross_join_expression
            | subquery
table_name: name

subquery: ( "(" (query_expr | join | cross_join) ")" ) [ "AS"i alias ]

cross_join: from_item "CROSS"i "JOIN"i from_item
join: from_item JOIN_EXPR from_item [ "ON"i bool_expression ] -> join_expression

JOIN_EXPR.5: (JOIN_TYPE WS)? "JOIN"i
JOIN_TYPE: "INNER"i | "OUTER"i? | JOIN_DIRECTION (WS "OUTER"i)? | JOIN_DIRECTION
JOIN_DIRECTION: "FULL"i | "LEFT"i | "RIGHT"i

?expression_math: expression_product
                | expression_math "+" expression_product -> expression_add
                | expression_math "-" expression_product -> expression_sub
                | "CASE"i (when_then)+ "ELSE"i expression_math "END"i -> case_expression
                | "CAST"i "(" expression_math "AS"i TYPENAME ")" -> as_type
                | "CAST"i "(" literal "AS"i TYPENAME ")" -> literal_cast
                | AGGREGATION expression_math ")" [window_form] -> sql_aggregation
                | "RANK"i "(" ")" window_form -> rank_expression
                | "DENSE_RANK"i "(" ")" window_form -> dense_rank_expression
                | "COALESCE"i "(" [(expression_math ",")*] expression_math ")" -> coalesce_expression
                | subquery -> subquery_expression

window_form: "OVER"i "(" ["PARTITION"i "BY"i (partition_by ",")* partition_by] ["ORDER"i "BY"i (order ",")* order [ row_range_clause ] ] ")"

partition_by: expression_math

row_range_clause: ( ROWS | RANGE ) frame_extent
frame_extent: frame_between | frame_preceding
frame_between: "BETWEEN"i frame_bound "AND"i frame_bound
frame_bound: frame_preceding | frame_following | "CURRENT"i "ROW"i
frame_preceding: UNBOUNDED PRECEDING | INT_NUMBER PRECEDING
frame_following: UNBOUNDED FOLLOWING | INT_NUMBER FOLLOWING
RANGE: "RANGE"i
ROWS: "ROWS"i
UNBOUNDED: "UNBOUNDED"i
PRECEDING: "PRECEDING"i
FOLLOWING: "FOLLOWING"i

when_then: "WHEN"i bool_expression "THEN"i expression_math
order: expression_math ["ASC"i] -> order_asc
          | expression_math "DESC"i -> order_desc


?expression_product: expression_parens
                   | expression_product "*" expression_parens -> expression_mul
                   | expression_product "/" expression_parens -> expression_div

?expression_parens: expression
                  | "(" expression_parens "*" expression ")" -> expression_mul
                  | "(" expression_parens "/" expression ")" -> expression_div
                  | "(" expression_parens "+" expression ")" -> expression_add
                  | "(" expression_parens "-" expression ")" -> expression_sub

column_name: [name "."] (name | STAR)
?expression: column_name -> column_name
            | literal


SELECT_CONSTRAINT.9: "ALL"i | "DISTINCT"i
TYPENAME:  "object"i
         | "varchar"i
         | "integer"i
         | "int16"i
         | "smallint"i
         | "int32"i
         | "int64"i
         | "int"i
         | "bigint"i
```

```
          | "float16"i
          | "float32"i
          | "float64"i
          | "float"i
          | "bool"i
          | "datetime64"i
          | "timestamp"i
          | "time"i
          | "date"i
          | "cateSQLry"i
          | "string"i
AGGREGATION.8: ("SUM("i | "AVG("i | "MIN("i | "MAX("i | "COUNT("i "DISTINCT"i | "COUNT("i)
alias: name -> alias_string
_window_name: name
limit_count: INT_NUMBER -> limit_count
skip_rows: INT_NUMBER
bool_expression: bool_parentheses
                 | bool_expression "AND"i bool_parentheses -> bool_and
                 | bool_expression "OR"i bool_parentheses -> bool_or
bool_parentheses: comparison_type
                 | "(" bool_expression "AND"i comparison_type ")" -> bool_and
                 | "(" bool_expression "OR"i comparison_type ")" -> bool_or
                 | "EXISTS"i subquery -> exists
comparison_type: equals | not_equals | greater_than | less_than | greater_than_or_equal
| less_than_or_equal | between | in_expr | not_in_expr | subquery_in | subquery_not_in | is_null | is_not_null | like_expr | not_like_expr

equals: expression_math "=" expression_math
is_null: expression_math "IS"i "NULL"i
is_not_null: expression_math "IS"i "NOT"i "NULL"i
not_equals: expression_math ("<>" | "!=") expression_math
greater_than: expression_math ">" expression_math
less_than: expression_math "<" expression_math
greater_than_or_equal: expression_math ">=" expression_math
less_than_or_equal: expression_math "<=" expression_math
between: expression_math "BETWEEN"i expression_math "AND"i expression_math

// `LIKE` and `NOT LIKE`
like_expr: expression_math "LIKE"i expression_math
not_like_expr: expression_math "NOT"i "LIKE"i expression_math

// `IN` and `NOT IN`
in_expr: expression_math "IN"i "(" [expression_math ","]* expression_math ")"
subquery_in: expression_math "IN"i subquery
not_in_expr: expression_math "NOT"i "IN"i "(" [expression_math ","]* expression_math ")"
subquery_not_in: expression_math "NOT"i "IN"i subquery

?literal: boolean -> bool
       | number_expr -> number
       | /'([^']+)'|''/ -> string
       | timestamp_expression -> timestamp_expression
boolean: "TRUE"i -> true
       | "FALSE"i -> false
?number_expr: product

?product: INT_NUMBER -> integer
       | FLOAT -> float

INT_NUMBER: /[1-9][0-9]*/

STAR: "*"
window_definition:
timestamp_expression: "NOW"i "(" ")" -> datetime_now
                    | "TODAY"i "(" ")" -> date_today

date: YEAR "-" MONTH "-" DAY
YEAR: /[0-9]{4}/
MONTH: /[0-9]{2}/
DAY: /[0-9]{2}/
time: HOURS ":" MINUTES ":" SECONDS
HOURS: /[0-9]{2}/
MINUTES: /[0-9]{2}/
SECONDS: /[0-9]{2}/
name: CNAME | ESCAPED_STRING

_STRING_INNER: /(?:[^"\\]|\\.)*?/
ESCAPED_STRING: "\"" _STRING_INNER "\""

%import common.CNAME
%import common.WS
%import common.SQL_COMMENT
```

```
%import common.WS_INLINE
%import common.FLOAT

%ignore WS
%ignore SQL_COMMENT
```

Listing 7: SQL Grammar

A.8.3 Python Grammar

```
single_input: _NL | simple_stmt | compound_stmt _NL
start: (_NL | stmt)*
eval_input: testlist _NL*

!decorator: "@" dotted_name [ "(" [arguments] ")" ] _NL
decorators: decorator+
decorated: decorators (classdef | funcdef | async_funcdef)

async_funcdef: "async" funcdef
funcdef: "def" NAME "(" parameters? ")" ["->" test] ":" ( suite | _NL)

!parameters: paramvalue ("," paramvalue)* ["," [ starparams | kwparams]]
           | starparams
           | kwparams
starparams: "*" typedparam? ("," paramvalue)* ["," kwparams]
kwparams: "**" typedparam

?paramvalue: typedparam ["=" test]
?typedparam: NAME [":" test]

!varargslist: (vfpdef ["=" test] ("," vfpdef ["=" test])* ["," [ "*" [vfpdef] ("," vfpdef
     ["=" test])* ["," ["**" vfpdef [","]]] | "**" vfpdef [","]]]
  | "*" [vfpdef] ("," vfpdef ["=" test])* ["," ["**" vfpdef [","]]]
  | "**" vfpdef [","])

vfpdef: NAME

?stmt: (simple_stmt | compound_stmt ) ["eof"]
!?simple_stmt: small_stmt (";" small_stmt)* [";"] _NL
?small_stmt: (expr_stmt | del_stmt | pass_stmt | flow_stmt | import_stmt | global_stmt |
     nonlocal_stmt | assert_stmt)
?expr_stmt: testlist_star_expr (annassign | augassign (yield_expr|testlist)
         | ("=" (yield_expr|testlist_star_expr))*)
annassign: ":" test ["=" test]
!?testlist_star_expr: (test|star_expr) ("," (test|star_expr))* [","]
!augassign: ("+=" | "-=" | "*=" | "@=" | "/=" | "%=" | "&=" | "|=" | "^=" | "<<=" | ">>=" |
     "**=" | "//=")
// For normal and annotated assignments, additional restrictions enforced by the interpreter
del_stmt: "del" exprlist
pass_stmt: "pass"
flow_stmt: break_stmt | continue_stmt | return_stmt | raise_stmt | yield_stmt
break_stmt: "break"
continue_stmt: "continue"
return_stmt: "return" [testlist]
yield_stmt: yield_expr
raise_stmt: "raise" [test ["from" test]]
import_stmt: import_name | import_from
import_name: "import" dotted_as_names
// note below: the ("." | "...") is necessary because "..." is tokenized as ELLIPSIS
import_from: "from" (dots? dotted_name | dots) "import" ("*" | "(" import_as_names ")" |
     import_as_names)
!dots: "."+
import_as_name: NAME ["as" NAME]
dotted_as_name: dotted_name ["as" NAME]
!import_as_names: import_as_name ("," import_as_name)* [","]
dotted_as_names: dotted_as_name ("," dotted_as_name)*
dotted_name: NAME ("." NAME)*
global_stmt: "global" NAME ("," NAME)*
nonlocal_stmt: "nonlocal" NAME ("," NAME)*
assert_stmt: "assert" test ["," test]

compound_stmt: if_stmt | while_stmt | for_stmt | try_stmt | with_stmt | funcdef | classdef |
      decorated | async_stmt
async_stmt: "async" (funcdef | with_stmt | for_stmt)
if_stmt: "if" test ":" suite ("elif" test ":" suite)* ["else" ":" suite]
while_stmt: "while" test ":" suite ["else" ":" suite]
for_stmt: "for" exprlist "in" testlist ":" suite ["else" ":" suite]
```

```
try_stmt: ("try" ":" suite ((except_clause ":" suite)+ ["else" ":" suite] ["finally" ":"
    suite] | "finally" ":" suite))
with_stmt: "with" with_item ("," with_item)*  ":" suite
with_item: test ["as" expr]
// NB compile.c makes sure that the default except clause is last
except_clause: "except" [test ["as" NAME]]
suite: simple_stmt | _NL _INDENT stmt+ _DEDENT

?test: or_test ["if" or_test "else" test] | lambdef
?test_nocond: or_test | lambdef_nocond
lambdef: "lambda" [varargslist] ":" test
lambdef_nocond: "lambda" [varargslist] ":" test_nocond
?or_test: and_test ("or" and_test)*
?and_test: not_test ("and" not_test)*

?not_test: "not" not_test -> not
         | comparison
?comparison: expr (_comp_op expr)*
star_expr: "*" expr
?expr: xor_expr ("|" xor_expr)*
?xor_expr: and_expr ("^" and_expr)*
?and_expr: shift_expr ("&" shift_expr)*
?shift_expr: arith_expr (_shift_op arith_expr)*
?arith_expr: term (_add_op term)*
?term: factor (_mul_op factor)*
?factor: _factor_op factor | power

!_factor_op: "+"|"-"|"~"
!_add_op: "+"|"-"
!_shift_op: "<<"|">>"
!_mul_op: "*"|"@"|"/"|"%"|"//"
// <> isn't actually a valid comparison operator in Python. It's here for the
// sake of a __future__ import described in PEP 401 (which really works :-)
!_comp_op: "<"|">"|"=="|">="|"<="|"<>"|"!="|"in"|"not" "in"|"is"|"is" "not"

?power: await_expr ["**" factor]
!await_expr: ["await"] atom_expr

?atom_expr: atom_expr "(" [arguments] ")"      -> funccall
          | atom_expr "[" subscriptlist "]"  -> getitem
          | atom_expr "." NAME               -> getattr
          | atom

?atom: "(" [yield_expr|testlist_comp] ")" -> tuple
     | "[" [testlist_comp] "]"  -> list
     | "{" [dictorsetmaker] "}" -> dict
     | NAME -> var
     | number | string+
     | "(" test ")"
     | "..." -> ellipsis
     | "None"    -> const_none
     | "True"    -> const_true
     | "False"   -> const_false

!?testlist_comp: (test|star_expr) [comp_for | ("," (test|star_expr))+ [","] | ","]
!subscriptlist: subscript ("," subscript)* [","]
subscript: test | [test] ":" [test] [sliceop]
sliceop: ":" [test]
!exprlist: (expr|star_expr) ("," (expr|star_expr))* [","]
!testlist: test ("," test)* [","]
!dictorsetmaker: ( ((test ":" test | "**" expr) (comp_for | ("," (test ":" test | "**" expr)
    )* [","])) | ((test | star_expr) (comp_for | ("," (test | star_expr))* [","])) )

classdef: "class" NAME ["(" [arguments] ")"] ":" suite
!arguments: argvalue ("," argvalue)*  ["," [ starargs | kwargs]]
         | starargs
         | kwargs
         | test comp_for

!starargs: "*" test ("," "*" test)* ("," argvalue)* ["," kwargs]
kwargs: "**" test

?argvalue: test ["=" test]

comp_iter: comp_for | comp_if | async_for
async_for: "async" "for" exprlist "in" or_test [comp_iter]
comp_for: "for" exprlist "in" or_test [comp_iter]
comp_if: "if" test_nocond [comp_iter]

// not used in grammar, but may appear in "node" passed from Parser to Compiler
encoding_decl: NAME
```

```

yield_expr: "yield" [yield_arg]
yield_arg: "from" test | testlist


number: DEC_NUMBER | HEX_NUMBER | OCT_NUMBER | FLOAT_NUMBER

string: STRING | LONG_STRING

// Tokens
NAME: /[a-zA-Z_]\w*/
COMMENT: /#.*(\n[\t ]*)+/ | LONG_STRING
_NL: ( /(\r?\n[\t ]*)+/ | COMMENT)+

LONG_STRING: /[ubf]?r?("""(?<!\\).*?"""|'''(?<!\\).*?''')/is

DEC_NUMBER: /0|[1-9]\d*/i
HEX_NUMBER.2: /0x[\da-f]*/i
OCT_NUMBER.2: /0o[0-7]*/i
BIN_NUMBER.2 : /0b[0-1]*/i
FLOAT_NUMBER.2: /((\d+\.\d*|\.\d+)(e[-+]?\d+)?|\d+(e[-+]?\d+))/i

%import common.WS_INLINE

%declare _INDENT _DEDENT
%ignore WS_INLINE
%ignore /\\[\t \f]*\r?\n/   // LINE_CONT
%ignore COMMENT
```

Listing 8: Python Grammar

A.8.4 Go Grammar

```

start: package_clause eos (import_decl eos)* ((function_decl | method_decl | declaration) eos "eoc"?)*

package_clause: "package" NAME

import_decl: "import"  (import_spec | "(" (import_spec eos)* ")")

import_spec: ("." | NAME)? import_path

import_path: string_

declaration: const_decl | type_decl | var_decl

const_decl: "const"  (const_spec | "(" (const_spec eos)* ")")

const_spec: identifier_list (type_? "=" expression_list)?

identifier_list: NAME ("," NAME)*

expression_list: expression ("," expression)*

type_decl: "type" (type_spec | "(" (type_spec eos)* ")")

type_spec: alias_decl | type_def

alias_decl : NAME "=" type_

type_def : NAME type_parameters? type_

type_parameters : "[" type_parameter_decl ("," type_parameter_decl)* "]"

type_parameter_decl : identifier_list type_element

type_element : type_term ("|" type_term)*

type_term : "~"? type_

// Function declarations

function_decl: "func" NAME type_parameters? signature ("{" statement_list? ("}" | "eof"))?

method_decl: "func" receiver NAME signature block?

receiver: parameters

var_decl: "var" (var_spec | "(" (var_spec eos)* ")")

var_spec: identifier_list (type_ ("=" expression_list)? | "=" expression_list)

block: "{" statement_list? "}"

statement_list: ((";"? | EOS?) statement eos)+

statement: declaration | labeled_stmt | simple_stmt | go_stmt | return_stmt | break_stmt | continue_stmt | goto_stmt | fallthrough_stmt | block | if_stmt | switch_stmt | select_stmt | for_stmt | defer_stmt

simple_stmt: send_stmt | inc_dec_stmt | assignment | expression | short_var_decl

send_stmt: expression  "<-" expression

inc_dec_stmt: expression ("++" | "--")

assignment: expression assign_op expression | expression_list "=" expression_list

assign_op: "+=" | "-=" | "|=" | "^=" | "*=" | "/=" | "%=" | "<<=" | ">>=" | "&=" | "&^="

short_var_decl: expression_list ":=" expression_list

labeled_stmt: NAME ":" statement?

return_stmt: "return" expression_list?

break_stmt: "break" NAME?

continue_stmt: "continue" NAME?

```

```
goto_stmt: "goto"  NAME

fallthrough_stmt: "fallthrough"

defer_stmt: "defer" expression

if_stmt: "if"  ( expression | eos expression | simple_stmt eos expression) block ("else" (
     if_stmt | block))?

switch_stmt: expr_switch_stmt | type_switch_stmt

expr_switch_stmt: "switch"  (expression? | simple_stmt? eos expression?) "{"
     expr_case_clause* "}"

expr_case_clause: expr_switch_case ":" statement_list?

expr_switch_case: "case" expression_list | "default"

type_switch_stmt: "switch"  ( type_switch_guard | eos type_switch_guard | simple_stmt eos
     type_switch_guard) "{" type_case_clause* "}"

type_switch_guard: (NAME ":=")? NAME "." "(" "type"  ")"

type_case_clause: type_switch_case ":" statement_list?

type_switch_case: "case" type_list | "default"

type_list: (type_ | "nil" ) ("," (type_ | "nil"  ))*

select_stmt: "select" "{" comm_clause* "}"

comm_clause: comm_case ":" statement_list?

comm_case: "case" (send_stmt | recv_stmt) | "default"

recv_stmt: (expression_list "=" | identifier_list ":=")? expression

for_stmt: "for" [for_clause] block

for_clause: simple_stmt (eos expression eos simple_stmt)? | range_clause

range_clause: (expression_list "=" | expression_list ":=") "range"  expression

go_stmt: "go"expression

type_: literal_type | var_or_type_name type_args? | "(" type_ ")"

channel_type

type_args : "--"

var_or_type_name: NAME "." NAME | NAME | NAME "." "(" type_ ")"

array_type: "[" array_length "]" element_type

array_length: expression

element_type: type_

pointer_type: "*" type_

interface_type: "interface" "{" ((method_spec | type_element ) eos)* "}"

slice_type: "[" "]" element_type

// It's possible to replace 'type' with more restricted type_lit list and also pay attention
      to nil maps
map_type: "map" "[" type_ "]" element_type

channel_type: ("'chan"  | "chan"   "<-" |  "<-" "chan" ) element_type

method_spec: NAME parameters result | NAME parameters

function_type: "func" signature

signature: parameters result?

result: parameters | type_

parameters: "(" parameter_decl ("," parameter_decl)* ","? ")" | "(" ")"

```

```
// a comma-separated list of either (a) name, (b) type, or (c) name and type
// parameter_decl: identifier_list? "..."? type_


// Although following is overapproximate it's an easy way to avoid reduce/reduce conflicts
parameter_decl: (type_ | "..."? type_ | NAME type_)


expression: primary_expr
            | ("+" | "-" | "!" | "^" | "*" | "&" | "<-") expression
            | expression ("*" | "/" | "%" | "<<" | ">>" | "&" | "&^") expression
            | expression ("+" | "-" | "|" | "^") expression
            | expression ("==" | "!=" | "<" | "<=" | ">" | ">=") expression
            | expression "&&" expression
            | expression "||" expression

primary_expr: operand | primary_expr ("." (NAME | "(" type_ ")") | index | slice_ | arguments) | type_

// Giving operand higher precedence than type_ is a hack to avoid reduce/reduce conflicts
operand.3: literal | NAME | "(" expression ")" // removed NAME type_args?

literal: basic_lit | composite_lit | function_lit

basic_lit: "nil" | integer | string_ | FLOAT_LIT | CHAR_LIT

integer: DECIMAL_LIT | BINARY_LIT | OCTAL_LIT | HEX_LIT

DECIMAL_LIT: /0|[1-9]\d*/i
HEX_LIT.2: /0x[\da-f]*/i
OCTAL_LIT.2: /0o[0-7]*/i
BINARY_LIT.2 : /0b[0-1]*/i
FLOAT_LIT.2: /((\d+\.\d*|\.\d+)(e[-+]?\d+)?|\d+(e[-+]?\d+))/i
CHAR_LIT: /'.'/i

composite_lit: literal_type literal_value

literal_type: struct_type | array_type | "[" "..." "]" element_type | slice_type | map_type | "interface" "{" "}"

literal_value: "{" (element_list ","?)? "}"

element_list: keyed_element ("," keyed_element)*

keyed_element: (key ":")? element

key: expression | literal_value

element: expression | literal_value

struct_type: "struct" "{" (field_decl eos)* "}"

field_decl: (identifier_list type_ | embedded_field) string_?

string_: RAW_STRING_LIT | INTERPRETED_STRING_LIT

RAW_STRING_LIT: /`.*?`/
INTERPRETED_STRING_LIT: /".*?"/i

embedded_field: "*"? (NAME "." NAME | NAME)  type_args?

function_lit: "func" signature block // function

index: "[" expression "]"

slice_: "[" ( expression? ":" expression? | expression? ":" expression ":" expression) "]"

type_assertion: "." "(" type_ ")"

arguments: "(" ( expression_list? "..."? ","?)? ")"

eos: ";" | EOS // | {this.closingBracket()}?

NAME : /[a-zA-Z_]\w*/
EOS: _NL | ";" | "/*' .*? '*/"

COMMENT : /\/\/[^\n]*/
_NL: ( /(\r?\n[\t ]*)+/ | COMMENT)+

%ignore /[\t ]/
%ignore /\\[\t \f]*\r?\n/   // LINE_CONT
```

Listing 9: Go Grammar