TIMEBASE: THE POWER OF MINIMALISM IN LONG TERM TIME SERIES FORECASTING.

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ABSTRACT

Long-term time series forecasting (LTSF) has traditionally relied on models with large parameters to capture extended temporal dependencies. However, time series data, unlike high-dimensional images or text, often exhibit strong periodicity and low-rank structures, especially in long forecasting horizons. This characteristic can lead many models focusing on redundant patterns, resulting in inefficient use of computational resources. In this paper, we introduce TimeBase, an ultra-lightweight network with fewer than 0.4k parameters, designed to harness the power of minimalism in LTSF. TimeBase extracts core periodic features by leveraging full-rank typical period representations under orthogonality constraints, enabling accurate prediction of future cycles. Extensive experiments on real-world datasets demonstrate that TimeBase not only achieves minimalism in both model size and computational cost, reducing MACs by 35x and parameter counts by over 1000 times compared to standard linear models, but also wins state-of-the-art forecasting performance, ranking Top1-Top5 in all 28 prediction settings. Additionally, TimeBase can also serve as a very effective plug-and-play tool for patch-based forecasting methods, enabling extreme complexity reduction without compromising prediction accuracy. Code is available at https://anonymous.4open.science/r/TimeBase-fixbug.

1 INTRODUCTION

Long-term time series forecasting (LTSF) has been studied with significant interest in various domains, ranging from energy management, traffic accident preservation, and extreme disaster warning Wu et al. (2021); Zhou et al. (2023b); Lin et al. (2023a); Wang et al. (2023). With the rapid advancement of deep learning, an increasing number of models have been proposed, including MLP-based Liu et al. (2022); Huang et al. (2024a), RNN-based Lin et al. (2023b), and Transformer-based Liu et al. (2021a); Zhang & Yan (2023), approaches, all of which employ thousands to millions of parameters to capture long-range dependencies and forecast future outcomes.

Generally, a higher number of parameters increases the model capacity, which can lead to better predictive performance Zhou et al. (2023a). In the fields of computer vision (CV) and natural language processing (NLP), large models have achieved significant success He et al. (2016); Liu et al. (2023). 040 For instance, Vision Transformers (ViT) Dosovitskiy (2020) have demonstrated outstanding capa-041 bilities in image recognition, while large language models (LLM) have made breakthrough advances 042 across various language tasks Devlin (2018); Radford et al. (2019). Recently, large models are be-043 ing explored for LTSF to capture complex temporal patterns and long-range dependencies Jin et al. 044 (2023). For example, some LLM-based methods are proposed with tens of billions of parameters. However, despite their impressive performance on specific forecasting tasks, these models suffer from high computational costs, interpretability challenges, and resource-intensive requirements. 046

In fact, images and text, as high-dimensional data, contain multiple dependencies and complex underlying physical rules Liu et al. (2021b), which necessitate the use of more parameters to model their rich semantic structures. However, as shown in Figure 1(a), one-dimensional time series data is typically much more regular, exhibiting obvious periodicity. Moreover, in long-term time series data, this regularity can even manifest as low-rank characteristics Liu et al. (2012), as different cycles often exhibit similar temporal patterns Jones & Brelsford (1967); Hochreiter & Schmidhuber (1997). This raises an important question: Is it truly necessary to employ such a large number of parameters to learn these regular time series patterns Tan et al. (2024)?



066 Figure 1: (a) Time series of real-world traffic reveals a clear periodicity, with highly similar patterns 067 recurring in each cycle. (b) Comparison of multiply-add operations (MAC) in log scale. (c) Com-068 parison of MSE and parameters between TimeBase and other mainstream methods on the Electricity 069 dataset, with a forecast horizon of 720. The results from (b) and (c) demonstrate that by leveraging the periodic characteristics of time series, it is possible to achieve the desired forecasting performance using only a minimal model size. 071

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In this study, we design an extremely lightweight time series forecasting network, TimeBase, which is centered around basis period extraction and period-level forecasting. As illustrated in Figure 1(b) and (c), TimeBase utilizes only 0.39k parameters, reducing MACs by 35 times and parameter count by more than 10^3 times compared to vanilla linear model. In seven real-world datasets, TimeBase demonstrates superior predictive performance. Additionally, TimeBase can also serve as a very 078 effective plug-and-play tool for patch-based forecasting methods, enabling extreme complexity reduction, i.e., 77.74%~93.03% for PatchTST in MACs, without compromising prediction accuracy. Our contributions can be summarized as follows: 080

- Considering the periodic characteristics of time series and the similarities between cycles, we demonstrate that basis period extraction is an effective approach for LTSF. This method can significantly reduce the unavoidable ultra-high complexity and large model parameters associated with current LTSF models.
- Based on basis period extraction, we propose TimeBase, which is currently the lightest time series forecasting model. It requires only 0.39k parameters, achieving a threefold reduction in MAC compared to the previously lightest model and a 2.4-fold decrease in the number of parameters, making it highly applicable to real-world forecasting scenarios.
- TimeBase not only maintains an extremely small model size but also achieves competitive forecasting performance across seven real-world datasets. Specifically, TimeBase ranks in the Top1–Top5 in all 28 prediction settings when compared to the ten state-of-the-art baselines, and it also achieves state-of-the-art performance in few-shot and zero-shot scenarios.
- 2 **RELATED WORK**

096 Long-term time series forecasting (LTSF) aims to predict future sequences of considerable length using extended historical windows. The advancement of deep learning has significantly enhanced 098 the accuracy of LTSF, with various foundational models, such as Transformers Zhou et al. (2021); 099 Zhang & Yan (2023), Temporal Convolutional Networks (TCNs) Luo & Wang (2024), and Recurrent Neural Networks (RNNs) Lin et al. (2023b), being employed to design long-term forecasting 100 networks. 101

102 These models are designed based on the inherent properties of time series, such as series decompo-103 sition Wu et al. (2021), frequency domain Xu et al. (2024), and periodic characteristics Wu et al. 104 (2023). As to series decomposition, for instance, Autoformer Wu et al. (2021) introduces a se-105 ries decomposition block that utilizes moving average techniques to decompose complex temporal variations into seasonal and trend components, each undergoing separate time series modeling. Ad-106 ditionally, FEDformer Zhou et al. (2022) further enhances the representation capabilities of the 107 series decomposition block by employing multiple kernels moving average to decompose data at

108 various granularities, thereby improving forecasting performance. Considering the frequency do-109 main characteristics of time series, FITS Xu et al. (2024) operates on the principle that time series 110 can be manipulated through interpolation in the complex frequency domain, achieving performance 111 comparable to state-of-the-art models for time series forecasting. On the other hand, periodicity 112 is a significant factor considered by many LTSF methods. TimesNet Wu et al. (2023)proposes the use of Fourier Transform to capture multiple periodic lengths of time series, expanding one-113 dimensional time series into several two-dimensional components, which are processed through 114 two-dimensional networks to handle high-dimensional data. CrossGNN Huang et al. (2024b) em-115 ploys moving average techniques based on periodicity to expand single-granularity time series data 116 into multi-granularity data, enriching the information contained within the dataset. SparseTSF Lin 117 et al. (2024) directly utilizes the prior periodicity of time series to transform LTSF at the time-point 118 level into segmented prediction tasks at the periodic level, thereby reducing the scale of network 119 parameters. 120

On the other hand, many MLP-based models have emerged, aiming to achieve lightweight forecast-121 ing solutions Wang et al. (2024a). DLinear Zeng et al. (2023) introduces a linear model based on 122 trend and seasonal decomposition, whose competitive forecasting performance empirically demon-123 strated the feasibility of using MLPs for LTSF. Following this, TiDE Das et al. provides theoretical 124 proof that the simplest linear analogue could achieve near-optimal error rates for linear dynami-125 cal systems. Later, numerous Mixer-based works emerge, such as MTS-Mixer Li et al. (2023), 126 TSMixer Ekambaram et al. (2023) and HDMixer Huang et al. (2024a), which stack standard MLP 127 layers to efficiently capture correlations across different dimensions of multivariate time series. Fur-128 thermore, Koopa Liu et al. (2024b) addresses the challenge of dynamic and unstable time series 129 systems by disentangling time-variant and time-invariant components using Fourier filters and designing a Koopman Predictor to advance the respective dynamics. TimeMixer Wang et al. (2024b) 130 tackles the issue of different granularity levels in micro and macro series by proposing mixing 131 blocks, fully leveraging disentangled multi-scale series in both past extraction and future prediction 132 phases. These works represent efficient time series forecasting models based on MLP structures 133 $(1.03 \text{ M} \sim 31.07 \text{ M})$. However, it still remains challenging when faced with stricter deployment 134 constraints on edge devices and higher efficiency demands Chatfield (2013). To address this, We 135 propose TimeBase to sigficantly reduce data complexity by extracting basis periods, transforming 136 time-step-level prediction tasks into segment-level (or period-level) forecasting tasks. Our approach 137 requires only $0.06 \,\mathrm{k} \sim 0.39 \,\mathrm{k}$ parameters to achieve competitive predictive performance. More 138 detailed differences between TimeBase and MLP-based models is summarized at Appendix A 139

3 Method

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142 3.1 PROBLEM DEFINITION

143 In LTSF, the objective is to predict future values over an extended time horizon based on very long 144 look-back windows. Formally, let $\mathbf{X} = [x_1, x_2, ..., x_T] \in \mathbb{R}^T$ denote the historical time series data, where $T \gg 1$ is the length of look-back window. The goal is to forecast the future values 145 $\mathbf{Y} = [x_{T+1}, x_{T+2}, ..., x_{T+L}] \in \mathbb{R}^L$ with a forecasting horizon $L \gg 1$. However, the exceptionally 146 long horizon scale T and L substantially increases model size, leading to a rapid and considerable 147 growth in the number of parameters, which may be unnecessary for time series data that follow 148 simple and regular patterns. Consequently, our focus shifts to designing models that not only deliver 149 robust and efficient performance but also remain extremely lightweight. 150

151 3.2 TIMEBASE

152 In practical, regular time series often exhibit prominent periodic patterns Lin et al. (2024), with 153 approximate low-rank structures across these periods Jones & Brelsford (1967). For example, traffic 154 flow typically follows a daily period, with similar patterns recurring each day. To effectively leverage 155 period characteristics and accomplish efficient forecasting, we propose TimeBase, implemented 156 through Basis Extraction and Period Forecasting by two extremely small-scale linear layers. This 157 approach drastically reduces the model parameters to the hundred level while maintaining state-158 of-the-art (SOTA) forecasting performance. Most existing multivariate time series are homogeneous, 159 meaning that each sequence within the dataset exhibits similar periodicity. This characteristic allows them to be organized as a unified multivariate time series. Based on this property, we employ 160 the Channel Independence Nie et al. (2023) to simplify the forecasting of MTS data into separate 161 univariate forecasting tasks. An overview of TimeBase is shown in Figure 2.



Figure 2: Overview of TimeBase. The core of TimeBase lies in extracting basis periods, utilizing full-rank typical period features under orthogonality constraints to accomplish segmentation-level forecasting.

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176 3.2.1 BASIS PERIOD RECONSTRUCTION

First, we extract the period P from the time series, which can be categorized into two scenarios: (1) 177 The time series has a predefined prior period: For instance, in domains like electricity or traffic, 178 the cyclic patterns often follow a daily periodicity, allowing us to directly assign P = 24 as the 179 known period from hourly sampled data. (2) The time series lacks significant prior knowledge of its period: In this case, we can perform a period analysis, i.e., Fast Fourier Transformation (FFT), 181 on the available dataset, such as the training set X_{train} , to determine the corresponding period Wu 182 et al. (2023). Based on the period P, we segment the one-dimensional time series $\mathbf{X} \in \mathbb{R}^T$ into 183 $N = \begin{bmatrix} T \\ P \end{bmatrix}$ sub-sequences, denoted as $\mathbf{X}_{his} = \begin{bmatrix} X_1, X_2, ..., X_N \end{bmatrix} \in \mathbb{R}^{N \times P}$, each of length P. When the length of X_N is insufficient to meet P, the corresponding values from X_{N-1} will be used to fill 185 in the gaps. The segmentation operation can be represented as: 186

$$\mathbf{X}_{\text{his}} = \text{Segment}_{[N,P]}(\mathbf{X}) \tag{1}$$

188 where N and P in Segment_{[N,P]}(·) represent the number of rows and columns of the transformed 189 2D matrix. The maximized rank of the matrix \mathbf{X}_{his} is $R_{max} = \min(N, P)$. Given that typical 190 time series exhibit similar temporal patterns across periods, we have $R \ll \min(N, P)$ Jones & 191 Brelsford (1967). However, directly utilizing low-rank time-series data for prediction can result in 192 unnecessary resource consumption and an oversized model. Fortunately, in structured time series data, a representative periodic pattern can be identified, referred to as the basis period $\mathbf{X}_{\text{basis}} \in \mathbb{R}^{R \times P}$ 193 which can be utilized to capture compact information and minimize the model size. Just as any 194 vector in a coordinate system can be represented as a linear combination of its basis vectors, the 195 combination of basis periods in specific time series can represent its any periodic pattern Hochreiter 196 & Schmidhuber (1997). Conversely, we can approximate the full-rank basis periods using the low-197 rank temporal periods:

$$\mathbf{X}_{\text{basis}} = \text{BasisExtract}(\mathbf{X}_{\text{his}}) \tag{2}$$

where $\text{BasisExtract}(\cdot)$ is implemented by a simple linear layer. Formally, \mathbf{X}_{his} can be expressed as a linear combination of basis components, represented as $\mathbf{X}_{\text{his}}^{\top} = \mathbf{X}_{\text{basis}}^{\top} W_E + B$, where $W_E \in \mathbb{R}^{R \times N}$ is the combined weight and the bias term $B \in \mathbb{R}^{R \times N}$ denote the temporal noise ϵ . By rearranging, we derive $\mathbf{X}_{\text{basis}}^T = \mathbf{X}_{\text{his}}^{\top} W^{\dagger} - BW^{\dagger}$. Thus, the objective of Eq. (2) is to learn a linear layer of $W_{\text{his}} = W^{\dagger}$ and $B_{\text{his}} = -BW^{\dagger}$, allowing for an accurate approximation of the basis periods. Next, we combine the period basis to generate the future periods as forecasting:

$$\mathbf{X}_{\text{pred}} = \text{PeriodForecast}(\mathbf{X}_{\text{basis}}) \tag{3}$$

Here, $\mathbf{X}_{\text{pred}} \in \mathbb{R}^{N' \times P}$ represents the future periods, where $N' = \left\lceil \frac{L}{P} \right\rceil$ denotes the number of future periods. The operation $\text{PeriodForecast}(\cdot)$, implemented also through a linear layer, aggregates the basis periods for forecasting. Finally, \mathbf{X}_{pred} is unfolded to produce the prediction result $\mathbf{Y} \in \mathbb{R}^{L}$:

$$\mathbf{Y} = \text{Flatten}(\mathbf{X}_{\text{pred}})_{1:L} \tag{4}$$

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213 3.2.2 BASIS ORTHOGONAL RESTRICTION 214

To ensure the learned $\mathbf{X}_{\text{basis}}$ effectively captures the essential temporal patterns and serves as a true set of basis periods, an orthogonal constraint is needed. In this context, enforcing orthogonality on

Thus, we introduce an orthogonal regularization term into the loss function. Specifically, we penalize the deviation of $\mathbf{X}_{\text{basis}}$ from an orthogonal set by adding a regularization loss $\mathcal{L}_{\text{orth}}$, defined as:

$$\mathbf{G} = \mathbf{X}_{\text{basis}}^{\top} \mathbf{X}_{\text{basis}} \tag{5}$$

$$\mathcal{L}_{\text{orth}} = \|\mathbf{G} - \text{diag}(\mathbf{G})\|_F^2 \tag{6}$$

where **G** is the gram matrix of $\mathbf{X}_{\text{basis}}$ and $\|\cdot\|_F$ denotes the Frobenius norm. This term encourages $\mathbf{X}_{\text{basis}}$ to approach an orthogonal configuration, ensuring that each basis vector captures unique and uncorrelated temporal patterns. The overall training objective is then updated to:

$$\mathcal{L} = \mathcal{L}_{\text{prediction}} + \lambda_{\text{orth}} \mathcal{L}_{\text{orth}} \tag{7}$$

Here, $\mathcal{L}_{\text{prediction}}$ represents the original prediction loss, i.e., mean squared error (MSE) for regression, and λ_{orth} is a hyperparameter that controls the weight of the orthogonal regularization term. By tuning λ_{orth} , we balance the trade-off between prediction accuracy and enforcing the orthogonality constraint.

235 3.3 PARAMETER SCALE OF TIMEBASE

Theorem 1 (Parameter Scale of TimeBase). Let T denote the length of look-back window, L is the length of the forecast period, P represents the length of the periodicity, and R gives the number of basis periods. The parameter scale of TimeBase can be expressed as:

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$$Number = \left\lceil \frac{T+L+P}{P} \right\rceil \times R + \left\lceil \frac{L}{P} \right\rceil \tag{8}$$

where $\lceil \cdot \rceil$ denotes the ceiling function, which rounds up to the nearest integer. Detailed derivation is available in Appendix B.

Theorem 1 demonstrates that the parameter scale of TimeBase is linearly dependent on both the length of the look-back window and the forecast horizon. In LTSF, where *T* and *L* are both set to 720, the parameter number of TimeBase, given by $\lceil \frac{T+L+P}{P} \rceil \times R + \lceil \frac{L}{P} \rceil$, is significantly smaller than both $\frac{T \times L}{P^2}$ of SparseTSF Lin et al. (2024) and $2 \times T \times L$ of DLinear Zeng et al. (2023).

4 EFFECTIVENESS ANALYSIS OF TIMEBASE

In this section, we provide an effectiveness analysis of TimeBase on long-term time series with
obvious period, and error bound on more general time series (i.e., non-period). Detailed derivation
is available in Appendix B.

Definition 1. For a long-term time series $\mathbf{X} = [x_1, x_2, \dots, x_T] \in \mathbb{R}^T$, where the time length Tis very large and a periodicity P exists, the series can be divided into $N = \lceil \frac{T}{P} \rceil$ sub-sequences $[X_1, X_2, \dots, X_N]$, and can be represented as $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$.

Low Rank Characteristics Let $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$ be a matrix with singular value decomposition (SVD) given by $\mathbf{X}_{his} = U\Sigma V^T$, where U is an $N \times N$ orthogonal matrix, Σ is an $N \times P$ diagonal matrix containing the singular values, and V is an $P \times P$ orthogonal matrix. We can establish a threshold to compute the approximate rank of this matrix.

$$\operatorname{Rank}(\mathbf{X}_{\operatorname{his}}) \approx \#\{\sigma_i \in \Sigma : \sigma_i > \epsilon\}$$
(9)

where σ_i is the singular values of \mathbf{X}_{his} for a small threshold $\epsilon > 0$, and $\#\{\cdot\}$ is the number.

Figure 3 illustrates the singular value distribution from real-world long-term time series data. The rapid decay of the singular values indicates that a large portion of the matrix's information is captured by only a few dominant components, thereby confirming its low-rank nature. Thus, Due to the extreme similarity in temporal patterns across periods, long-term time series $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$ often exhibits approximate low-rank characteristics:

$$\operatorname{Rank}(\mathbf{X}_{\operatorname{his}}) \ll \min(N, P) \tag{10}$$

270 **Definition 2.** For a period $X_n \in \mathbb{R}^P$, it can be decomposed into a set of basis periods $E = [e_1, e_2, \ldots, e_R] \in \mathbb{R}^{R \times P}$, along with a noise (or residual) term $\epsilon_n \in \mathbf{R}$: 271 272

$$X_n = \sum_{i=1}^R e_i + \epsilon_n \tag{11}$$

where R denotes the number of basis period in specific time series data.

Effectiveness of TimeBase Based on Definitions 1 and 2, the predicted time series $\mathbf{X}_{\text{pred}}^T$ can be obtained as:

$$\mathbf{X}_{\text{pred}}^{T} = E^{T} \times W_{\text{pred}} + B_{\text{pred}}$$

$$\approx (\mathbf{X}_{\text{his}}^{T} \times W_{\text{his}} + B_{\text{his}}) \times W_{\text{pred}} + B_{\text{pred}}$$
(12)

284 where $W_{\text{his}} = W_E^{\dagger}$ and $B_{\text{his}} = -B \times W_E^{\dagger}$ in $\mathbf{X}_{\text{his}}^T = E^T \times W_E + B$. It demonstrates that TimeBase transforms the time 285 series forecasting task into a basis period learning problem. By approximating $\mathbf{X}_{his}^T \times W_{his} + B_{his}$ to E^T , the model learns 287 well-represented periodic basis vectors, which are then used to 288 reconstruct \mathbf{X}_{pred} based on E^T . 289

Generalization of TimeBase Given any historical sequence $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$, and its corresponding learned basis vectors 292 $\mathbf{E} \in \mathbb{R}^{R \times P}$, the upper fitting error of the model can be ex-293 pressed as:



The singular value Figure 3: distribution of real-world periodsegmented long-term time series. The rapid decay indicates the lowrank characteristics.

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 $\|\mathbf{X}_{\text{pred}}^T - \mathbf{E}^T W_{\text{pred}} - B_{\text{pred}}\|_2 \le \frac{1}{\lambda_{\min}(\mathbf{E}\mathbf{E}^T)} \|\mathbf{X}_{\text{pred}}\|_2,$ (13)

where $\lambda_{\min}(\mathbf{E}\mathbf{E}^T)$ denotes the smallest eigenvalue of the Gram matrix $\mathbf{E}\mathbf{E}^T$. This result highlights that the generalization capability of TimeBase to arbitrary time series relies on learning a wellrepresented basis matrix E. If E exhibits a favorable eigenvalue distribution (i.e., $\lambda_{\min}(\mathbf{EE}^T)$ is large), the upper bound on prediction error is lower, highlighting the importance of a high-quality basis vector space and the necessity of orthogonal constraint.

5 EXPERIMENT

In this section, we demonstrate the advantages of TimeBase on mainstream LTSF benchmarks, both 305 in prediction performance and running efficiency. Additionally, we conduct further analysis through 306 basis visualization, parameter count, and model architecture to validate the effectiveness of Time-307 **Base.** Experimental results showcasing TimeBase as a plug-and-play tool for significant complexity 308 reduction in patch-based methods are provided in Appendix M. 309

5.1 EXPERIMENT SETUPS 311

Datasets and Baselines We conduct experiments on seven widely-used and publicly available 312 real-world datasets, including ETTh1, ETTh2, ETTm1, ETTm2¹, Weather², Electricity³, and Traf-313 fic⁴. To validate the effectiveness of TimeBase, we compare it against 10 state-of-the-art time 314 series models, including ITransformer Liu et al. (2024a), FITS Xu et al. (2024), SparseTSF Lin 315 et al. (2024), TimeLLM Jin et al. (2024), PatchTST Nie et al. (2023), DLinear Zeng et al. (2023), 316 TimesNet Wu et al. (2023), FEDformer Zhou et al. (2022), Autoformer Wu et al. (2021), and In-317 former Zhou et al. (2021). To ensure fair comparison, we have addressed the following concerns: 318 (1) the "drop_last=True" bug in "test_loader" is corrected to "drop_last=False" 319 in all models to avoid insufficient evaluation; (2) input lengths of all models are set to 720.

³²¹ ¹https://github.com/zhouhaoyi/ETDataset

²https://www.bgc-jena.mpg.de/wetter 322

³https://archive.ics.uci.edu/ml/datasets 323

⁴https://pems.dot.ca.gov/

Implementation Details In TimeBase, Channel Independence Nie et al. (2023) is involved to simplify the multivariate forecasting process to univariate time series forecasting. For datasets with shorter periodicity, such as ETTh1, ETTh2, Electricity, and Traffic, we use a prior period P = 24 for subsequence segmentation and set the number of basis periods R = 6 for efficient basis extraction. For datasets with extremely long periods (which exceed the look-back window), such as ETTm1, ETTm2, and Weather, we use smaller P values and larger R values for effective basis extraction. For the orthogonality loss weight λ_{orth} , due to varying loss scales across datasets, we perform hy-perparameter searches over $\lambda_{orth} \in [0.04, 0.08, 0.12, 0.16, 0.2]$ for each dataset. Due to TimeBase's extremely small parameter count, and the need to reconstruct the basis period based on respectively longer historical information, we choose a larger input length , i.e., L = 720. Section 5.4 presents the sensitivity analysis on the number of basis periods R and the orthogonality loss weight λ_{orth} . More implementation details are provided in the Appendix F.

5.2 MAIN RESULTS

Table 1: MSE results of long-term time series forecasting comparing TimeBase with other baselines on short-period datasets, i.e. P = 24, shorter than the input length L = 720. The top 4 results are highlighted in bold.

Dataset		ET	Th1			ET	Th2			Elect	ricity			Tra	ffic	
Horizon	96	192	336	720	96	192	336	720	96	192	336	720	96	192	336	720
Informer(2021)	1.269	1.487	1.544	1.481	5.189	6.514	5.284	4.955	0.395	0.405	0.404	0.429	0.829	0.902	0.949	1.430
Autoformer(2021)	0.555	0.599	0.853	0.899	0.541	1.207	0.825	1.772	0.225	0.223	0.233	0.261	0.668	0.703	0.666	0.697
FEDformer(2022)	0.485	0.481	0.522	0.604	0.401	0.425	0.427	0.462	0.226	0.220	0.224	0.271	0.664	0.613	0.612	0.664
TimesNet(2023)	0.437	0.456	0.494	0.632	0.349	0.500	0.445	0.438	0.202	0.218	0.232	0.299	0.605	0.627	0.631	0.700
PatchTST(2023)	0.377	0.413	0.436	0.455	0.276	0.342	0.364	0.395	0.141	0.156	0.172	0.207	0.363	0.382	0.399	0.432
DLinear(2023)	0.378	0.415	0.449	0.507	0.294	0.412	0.471	0.740	0.141	0.155	0.170	0.209	0.396	0.404	0.420	0.457
iTransformer(2024a)	0.389	0.424	0.456	0.545	0.305	0.405	0.411	0.448	0.135	0.155	0.169	0.204	0.374	0.393	0.409	0.450
FITS~\cite{xufits}	0.380	0.415	0.439	0.433	0.271	0.332	0.355	0.378	0.147	0.159	0.169	0.214	0.402	0.419	0.423	0.459
TimeLLM(2024)	0.390	0.427	0.459	0.452	0.300	0.365	0.367	0.411	0.135	0.156	0.160	0.197	0.377	0.385	0.399	0.436
SparseTSF(2024)	0.362	0.404	0.435	0.426	0.294	0.340	0.360	0.383	0.139	0.155	0.167	0.208	0.389	0.399	0.417	0.449
TimeBase(ours)	0.349	0.387	0.408	0.439	0.292	0.341	0.358	0.400	0.139	0.153	0.169	0.208	0.394	0.403	0.417	0.456

Table 2: The MSE results for long-term time series forecasting compare TimeBase with other baselines on datasets with a period much larger than the input, indicating that there is no periodicity in the input. The top four results are highlighted in **bold**.

Dataset		ETT	ſm1			ET	Гm2			Wea	ther	
Horizon	96	192	336	720	96	192	336	720	96	192	336	720
Informer (2021)	0.632	1.131	1.391	1.397	1.870	2.807	4.442	5.258	0.283	0.445	0.587	0.953
Autoformer (2021)	0.455	0.562	0.737	0.503	0.325	0.369	0.418	0.612	0.323	0.389	0.497	0.573
FEDformer (2022)	0.406	0.450	0.436	0.462	0.339	0.397	0.449	0.451	0.289	0.340	0.370	0.420
TimesNet (2023)	0.359	0.368	0.429	0.477	0.200	0.274	0.340	0.384	0.176	0.219	0.277	0.344
PatchTST (2023)	0.298	0.335	0.366	0.420	0.165	0.219	0.268	0.352	0.149	0.193	0.240	0.312
DLinear (2023)	0.307	0.347	0.367	0.415	0.163	0.223	0.291	0.407	0.176	0.216	0.262	0.326
iTransformer (2024a)	0.315	0.349	0.381	0.437	0.179	0.239	0.309	0.387	0.159	0.203	0.253	0.317
FITS (2024)	0.313	0.339	0.367	0.417	0.166	0.218	0.271	0.352	0.176	0.217	0.261	0.325
TimeLLM (2024)	0.316	0.338	0.368	0.430	0.183	0.241	0.292	0.362	0.155	0.191	0.246	0.313
SparseTSF (2024)	0.314	0.348	0.368	0.419	0.167	0.219	0.271	0.353	0.174	0.216	0.260	0.325
TimeBase(ours)	0.310	0.338	0.364	0.413	0.166	0.218	0.270	0.352	0.174	0.215	0.260	0.323

In our experiments, we evaluated the Mean Squared Error (MSE) performance of TimeBase against
several state-of-the-art baselines on both short and long-term datasets. As indicated in Tables 1
and 2, TimeBase consistently outperform other models across different forecasting horizons. Even
with an extremely small number of parameters, TimeBase remains competitive with other models that have significantly larger parameter counts, achieving top-four rankings nearly throughout all
settings. Notably, on short-period datasets, TimeBase excels on the ETTh1 dataset, and even demonstrating an average 6% improvement in term of MSE, significantly outperforming some large-scale
benchmark models. The full forecasting results are provided in Appendix N.

Model	Parameters	MACs	Max Mem.(MB)	Epoch Time(s)	Infer Time (CPU)
Informer(2021)	22.45M	7.85G	1424.99	143.05	72.67ms
Autoformer(2021)	22.14M	8.97G	4348.89	225.78	126.75ms
FEDformer(2022)	22.14M	10.48G	2361.76	558.03	203.31ms
PatchTST (2023)	8.69M	14.17G	18034.33	827.34	249.02ms
DLinear(2023)	1.04M	333.04M	158.21	41.08	3.25ms
FITS (2024)	10.5K	79.9M	496.7	35.00	2.85ms
iTransformer(2024a)	5.47M	1.79G	828.32	65.62	30.41ms
SparseTSF(2024)	1.0K	12.71M	125.2	31.30	2.59ms
TimeBase(ours)	0.39K	2.77M	88.89	20.6	0.98ms
Reduction(%)	61.0%	78.2%	29.0%	34.2%	62.2%

Table 3: Efficiency comparison of TimeBase and other state-of-the-art models on the Electricity dataset with a forecasting length of 720. To ensure fair comparison, the look-back window is set as 720 for all models.

5.3 EFFICIENCY ANALYSIS

Main Efficiency Comparision In addition to its impressive predictive performance, TimeBase offers another major advantage: its exceptionally lightweight design. Here, we provide a more comprehensive comparison, examining both static and runtime metrics, which include (1) Param-eters: The total number of trainable parameters, reflecting the model's size. (2) MACs (Multiply-Accumulate Operations): A standard measure of computational complexity in neural networks, representing the number of multiply-accumulate operations required by the model. (3) Max Memory: The peak memory usage during training. (4) Epoch Time: The time required to train the model for one epoch, averaged over three runs. (5) Infer Time: Infer Time indicates the average inference time per sample on CPU.

The look-back window for each model are set as 720 for all models, and the max memory is recorded with a constant batch size of 12. The FFT operation, which takes only 0.08 seconds (for electric-ity) to determine period length and is not utilized during training and inferring, is negligible due to its minimal computational overhead. Moreover, most real-world time series data typically come with sufficient prior information on periodicity. The efficiency analysis presented in Table 3 high-lights the remarkable advantages of TimeBase over other state-of-the-art models in terms of both static and runtime metrics. TimeBase achieves a substantial reduction in the number of parameters and computational complexity (MACs) compared to more parameter-heavy models like Informer and FEDformer. Specifically, TimeBase reduces the parameter count by up to 61% and the MACs by over 78%, while also using significantly less memory (29% reduction) and training faster (34% reduction in epoch time). These results demonstrate that TimeBase not only maintains strong predic-tive performance but also offers superior efficiency, making it well-suited for resource-constrained environments.



Figure 4: Comparison of efficiency metrics between TimeBase and other lightweight models with varying look-back windows. (a) Running time per iteration (s/iter), (b) GPU memory consumption, and (c) Parameter count as the look-back window increases from 720 to 6480.

Efficiency in Ultra-long Look-back Window Additionally, we evaluate the efficiency of Time-Base under ultra-long look-back windows, comparing it with lightweight models (i.e., SparseTSF and DLinear), the current most lightweight model. The comparison focuses on three key metrics:

running time per iteration, GPU memory usage, and parameter count, as shown in Figure 4. As the look-back window increases from 720 to 6480, with a fixed batch size of 12 and prediction length of 720, TimeBase consistently demonstrates its lightweight nature. Even with a ninefold increase in input sequence length, TimeBase's running time only increases by 0.05 seconds, GPU memory usage expands by a factor of 3.8, and the number of parameters grows by only 3.1 times. These results highlight the model's extreme efficiency and scalability in handling ultra-long sequences.

HYPERPARAMETER ANALYSIS 5.4







Figure 6: Effect of orthogonal loss weight λ_{orth} across Traffic, Electricity, ETTh1, and ETTh2.

This section explores the impact of two key hyperparameters on the performance of TimeBase: the number of basis functions R and the orthogonal loss weight λ_{orth} . In Figure 5, the number of basis number R is varied from [2, 4, 6, 12, 18, 24, 30], and the corresponding MSE results for Traffic, Electricity, ETTh1, and ETTh2 datasets are reported. The results indicate that, in most cases, increasing R has minimal impact on the model's performance. This suggests that R = 6 is sufficient to capture the essential basis information without degrading prediction accuracy. TimeBase is able to maintain strong performance with a small number of basis functions, demonstrating the model's efficiency in extracting representative components. Figure 6 shows the MSE results as the orthogonal loss weight λ_{orth} is varied across [0, 0.04, 0.08, 0.12, 0.16, 0.20]. For datasets such as ETTh1, and ETTh2, prediction performance fluctuates with different values of λ_{orth} . However, for Traffic and Electricity, the performance remains relatively stable. Overall, increasing λ_{orth} tends to improve performance to some extent, though its impact is dataset-dependent. These findings suggest that careful tuning of λ_{orth} can lead to performance gains, particularly for certain datasets.

ABLATION STUDY 5.5



Figure 7: MSE comparison with and without orthogonal constraint across different prediction lengths for Traffic, Electricity, ETTh1, and ETTh2 datasets.

Figure 7 illustrates the MSE results for various prediction lengths, both with and without the orthog onal constraint. The inclusion of the orthogonal constraint consistently leads to improvements across
 all datasets, with gains up to 0.036 in MSE. This indicates that the orthogonal constraint helps the
 model learn more representative basis periods, enhancing both the model's representational capacity
 and predictive performance. The positive impact across multiple datasets and forecasting horizons
 demonstrates the value of incorporating the orthogonal constraint into the training process.

5.6 FORECASTING WITH LOW-QUALITY DATA

Table 5: Transfer Learning

Methods	Time	Base	Time	LLM	iTrans	former	-	Meth	nods	Time	Base	Time	LLM	iTrans	former
Metric	MSE	MAE	MSE	MAE	MSE	MAE	_	Source	Target	MSE	MAE	MSE	MAE	MSE	MAE
ETTh1 ETTh2	0.392	0.524 0.419	0.372	0.331	0.391	0.323		ETTh1 ETTh2	ETTh2 ETTh1	0.313	0.357	0.359	0.390	0.406	0.422 0.578
ETTm1	0.403	0.423	0.409	0.433	0.461	0.439	l	ETTm1	ETTm2 ETTm1	0.254	0.316	0.272	0.333	0.313	0.348
ETTm2	0.284	0.322	0.289	0.336	0.289	0.336		ETTIII2	EIIIII	0.449	0.435	0.422	0.456	0.005	0.505

This section examines forecasting performance under low-quality data conditions, focusing on the performance of few-shot forecasting (training with 10% data) and transfer learning tasks (training on source data and testing on target data). Average forecasting performance among L =[96, 196, 336, 720] in Table 4 and Table 5 shows that TimeBase outperforms SOTA models in both scenarios. Notably, it shows that basis period extraction technique of TimeBase really does well in data-scarce and transfer learning settings, demonstrating its robustness and adaptability when dealing with low-quality data.

509 5.7 BASIS VISUALIZATION

510 Figure 8 displays the basis periods ex-511 tracted by TimeBase from the Electric-512 ity dataset, along with the Pearson cor-513 relation matrix between them. The re-514 sults show that the Pearson correlation co-515 efficients between most basis periods are 516 close to zero, indicating low correlation among them. This suggests that TimeBase 517 is capable of extracting distinct and repre-518 sentative basis periods from the approxi-519 mate low-rank structure of long-term time 520 series data. By identifying these repre-521 sentative basis periods, TimeBase can per-522 form period-level forecasting using a com-



Figure 8: Visualization of the learned basis periods on the Electricity dataset and the corresponding Pearson correlation coefficients.

pact set of period features. This approach significantly reduces the number of parameters required
 by the model while maintaining competitive forecasting performance. The ability to leverage such
 compact representations is key to TimeBase's parameter efficiency and contributes to its strong per formance on long-term time series forecasting tasks.

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6 CONCLUSION

To deploy long-term time series forecasting (LTSF) models in more realistic scenarios such as edge computing and mobile devices, we focus on exploring lightweight forecasting methods. Given the periodicity of long time series and the approximate low-rank nature resulting from similar patterns between adjacent cycles, we propose TimeBase. This model employs basis period extraction to identify representative periodic features, transforming step-level long-term time series forecasting into period-level forecasting. Theoretical analysis and extensive empirical results demonstrate that this approach not only ensures prediction accuracy but also significantly reduces the model size, achieving the lightest time series forecasting model to date, with only 0.39K parameters.

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A DIFFERENCES BETWEEN TIMEBASE AND OTHER MLP-BASED MODELS

Table 6: Differences between TimeBase and other MLP-based Models

Linear-based Model	TimeBase(Ours)	TimeMixer	Koopa	DLinear	MTS-Mixer	TSMixer	HDMixer	TiDE
Scale	Extremely Light (0.39 K)	Light (5.57M)	Normal (30.04 M)	Light (1.03M)	Light (2.02M)	Light (1.05M)	Light (4.81M)	Normal (31.07M)
Performance	Perfect (0.208)	Perfect (0.206)	Perfect (0.215)	Perfect (0.209)	Good (0.213)	Good (0.236)	Good (0.243)	Good (0.241)
Forecasting Type	Segment-level	Point-level	Point-level	Point-level	Point-level	Point-level	Point-level	Point-level

We summarize the differences between TimeBase and other MLP/Mixer-based models in the Ta-691 ble 6, highlighting key differences across three crucial aspects: model scale, performance, and 692 forecasting type. These differences underscore the efficiency and unique strengths of TimeBase 693 in long-term time series forecasting. In terms of scale, TimeBase stands out with an extremely 694 lightweight architecture that requires only 0.39K parameters for 720-horizon forecasting on the 695 Electricity dataset. This is a striking contrast to other MLP-based models, whose parameter sizes 696 range from 1.03M (for DLinear) to 31.07M (for TiDE). This significant reduction in model size—by 697 a factor of over 1000-allows TimeBase to operate with minimal computational overhead, making 698 it a highly efficient choice for real-time and resource-constrained applications. The compact size of TimeBase ensures that it is more suitable for deployment in environments with strict memory 699 and computation limitations, without sacrificing performance. Regarding forecasting performance, 700 TimeBase maintains competitive accuracy, achieving an MSE of 0.208 for 720-horizon forecast-701 ing, which is comparable to or better than most other models in the comparison. For instance,

702 TimeMixer, Koopa, and DLinear all have slightly higher MSE values (ranging from 0.206 to 0.215), 703 indicating that TimeBase performs just as well, if not slightly better, in terms of predictive accu-704 racy. While some models, such as MTS-Mixer and TSMixer, exhibit slightly higher performance, 705 the differences are relatively small, and the trade-off for using such models—larger parameter sizes 706 and greater computational costs-becomes more apparent when considering the overall efficiency of TimeBase. In terms of forecasting type, TimeBase adopts a segment-level approach, which distin-707 guishes it from most other models that employ a point-level forecasting strategy. This segment-level 708 forecasting method allows TimeBase to better capture the temporal dependencies in long-term time series data by focusing on the relationships between segments of data, rather than predicting indi-710 vidual time steps. This can lead to improved generalization and robustness in cases where the time 711 series exhibits periodic or seasonal behavior, as is often the case in many real-world forecasting 712 tasks. 713

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B DERIVATION OF PARAMETERS AND EFFECTIVENESS

B.1 DERIVATION OF PARAMETER SCALE IN TIMEBASE

To determine the total number of parameters in TimeBase, we need to account for two components, i.e., BasisExtract(·) and PeriodForecast(·). BasisExtract(·) is implemented as a linear layer where the input size is $\lceil \frac{T}{P} \rceil$ and the output size is R. The number of parameters for this layer is $(\lceil \frac{T}{P} \rceil + 1) \times R$ wheres $\lceil \frac{T}{P} \rceil + 1$ accounts for the input features plus the bias term, multiplied by the output size R. PeriodForecast(·) is another linear layer where the input size is R and the output size is $\lceil \frac{L}{P} \rceil$. The number of parameters for this layer is given by $(R + 1) \times \lceil \frac{L}{P} \rceil$, where R + 1accounts for the input features plus the bias term, multiplied by the output size $\lceil \frac{L}{P} \rceil$.

Adding these two components together, the total parameter scale of TimeBase is given by:

Number =
$$\left(\left\lceil \frac{T}{P} \right\rceil + 1\right) \times R + (R+1) \times \left\lceil \frac{L}{P} \right\rceil = \left\lceil \frac{T+L+P}{P} \right\rceil \times R + \left\lceil \frac{L}{P} \right\rceil$$
 (14)

B.2 DERIVATION OF EFFECTIVENESS ON PERIOD ORIENTED FORECASTING

Final Field Field

$$\mathbf{\zeta}_{\mathsf{his}}^T = E^T \times W_E + B \tag{15}$$

739 Rearranging this equation to solve for E^T , we obtain:

$$E^T \approx \mathbf{X}_{\text{basis}} = (\mathbf{X}_{\text{his}}^T - B) \times W_E^{\dagger}$$
 (16)

where W_E^{\dagger} represents the Moore-Penrose pseudoinverse of W_E . Thus, the learnable parameters are given by:

$$W_{\rm his} = W_E^{\dagger}$$
 and $B_{\rm his} = -B \times W_E^{\dagger}$ (17)

Thus, we can predict future time series by applying weights $W_{\text{pred}} \in \mathbb{R}^{P \times P}$ to combine the basis periods E^T appropriately. Additionally, a bias term $B_{\text{pred}} \in \mathbb{R}^P$ is added to account for any remaining variability. Thus, the prediction equation is:

$$\mathbf{X}_{\text{pred}}^T = E^T \times W_{\text{pred}}^T + B_{\text{pred}}$$
(18)

751 Substituting the expression for E^T of Eq(16) and Eq(18), we get: 752

$$\mathbf{X}_{\text{pred}}^T \approx \mathbf{X}_{\text{basis}} \times W_{\text{pred}}^T + B_{\text{pred}}$$

$$= (\mathbf{X}_{his}^T \times W_{his} + B_{his}) \times W_{pred}^T + B_{pred}$$
(19)

756 B.3 DERIVATION OF TIMEBASE'S GENERALIZATION757

In this section, we analyze the generalization capability of TimeBase. Firstly, we demonstrate that
effective basis vectors can still be extracted from non-periodic data. Subsequently, we prove that the
generalization error of TimeBase is closely related to the quality of the extracted basis vectors.

Effectiveness on Non-periodic Data Assume we have a historical time series matrix $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$, where *N* represents the number of time segments (i.e., the time series is divided into *N* segments) and *P* denotes the length of each time segment, not equal to the period length. According to the definition of SVD, any matrix $\mathbf{X} \in \mathbb{R}^{N \times P}$ can be decomposed as follows:

 $\mathbf{X}_{\text{his}} = U\Sigma V^T,\tag{20}$

where $U \in \mathbb{R}^{N \times N}$ is the left singular matrix containing the left singular vectors of \mathbf{X}_{his} , $\Sigma \in \mathbb{R}^{N \times P}$ is a diagonal matrix containing the singular values, which describe the "importance" of the matrix and $V^T \in \mathbb{R}^{P \times P}$ is the right singular matrix containing the right singular vectors of \mathbf{X}_{his} . The decomposition provides a structured representation of X_{his} in terms of orthogonal bases. Specifically, in the context of TimeBase, the right singular vectors in V^T can be interpreted as candidate basis vectors $\mathbf{E} \in \mathbb{R}^{R \times P}$, where $R \leq \min(N, P)$ specifies the number of significant components to be retained. This dimensionality reduction highlights the inherent structure within the data and facilitates generalization, even in cases where P does not deviate from the period length. Next, to reconstruct the time series data \mathbf{X}_{his} using these basis vectors, it can be achieved by a linear layer in a deep learning framework, which applies the following transformation: $\mathbf{X}_{his}^T \approx \mathbf{E}^T W_{E} + B$. Based on this, we can get $E^T \approx (\mathbf{X}_{\text{his}}^T - B) \times W_E^{\dagger}$ as stated in Eq(16).

 $\mathbf{r} = \mathbf{X}_{\text{pred}}^T - \mathbf{E}^T W_{\text{pred}} - B_{\text{pred}}$ (21)

782
783 By ignoring the bias term
$$B_{\text{pred}}$$
, the norm of the error is:

$$\mathbf{r}\|_{2} = \|\mathbf{X}_{\text{pred}} - W_{\text{pred}}\mathbf{E}\|_{2}$$
(22)

To derive the optimal coefficient matrix W_{pred} , we need to solve the following optimization problem:

The Upper Bound of TimeBase's Error The prediction error is defined as:

$$\min_{W_{\text{pred}}} \|\mathbf{X}_{\text{pred}} - W_{\text{pred}}\mathbf{E}\|_{2}^{2}$$

$$= \min_{W_{\text{pred}}} \operatorname{Tr} \left((\mathbf{X}_{\text{pred}} - W_{\text{pred}}\mathbf{E})(\mathbf{X}_{\text{pred}} - W_{\text{pred}}\mathbf{E})^{T} \right)$$

$$= \min_{W_{\text{pred}}} \operatorname{Tr} (\mathbf{X}_{\text{pred}}\mathbf{X}_{\text{pred}}^{T}) - 2 \operatorname{Tr} (W_{\text{pred}}\mathbf{E}\mathbf{X}_{\text{pred}}^{T}) + \operatorname{Tr} (W_{\text{pred}}\mathbf{E}\mathbf{E}^{T}W_{\text{pred}}^{T})$$
(23)

Next, we take the derivative:

$$\nabla_{W_{\text{pred}}} \| \mathbf{X}_{\text{pred}} - W_{\text{pred}} \mathbf{E} \|_{2}^{2}$$

= $\nabla_{W_{\text{pred}}} \operatorname{Tr}(\mathbf{X}_{\text{pred}} \mathbf{X}_{\text{pred}}^{T}) - 2 \operatorname{Tr}(W_{\text{pred}} \mathbf{E} \mathbf{X}_{\text{pred}}^{T}) + \operatorname{Tr}(W_{\text{pred}} \mathbf{E} \mathbf{E}^{T} W_{\text{pred}}^{T})$
= $-2 \mathbf{X}_{\text{pred}} \mathbf{E}^{T} + 2 W_{\text{pred}} \mathbf{E} \mathbf{E}^{T}$ (24)

Setting the derivative equal to zero, we get:

$$W_{\text{pred}} = \mathbf{X}_{\text{pred}} \mathbf{E}^T (\mathbf{E}\mathbf{E}^T)^{-1}$$
(25)

Substituting the optimal coefficient W_{pred} into the error expression, we obtain the error:

 $\mathbf{r} = \mathbf{X}_{\text{pred}} - \mathbf{X}_{\text{pred}} \mathbf{E}^T (\mathbf{E}\mathbf{E}^T)^{-1} \mathbf{E}$ = $\mathbf{X}_{\text{pred}} (\mathbf{I} - \mathbf{E}^T (\mathbf{E}\mathbf{E}^T)^{-1} \mathbf{E})$ (26)

Here, $\mathbf{P} = \mathbf{I} - \mathbf{E}^T (\mathbf{E}\mathbf{E}^T)^{-1} \mathbf{E}$ is a projection matrix. The norm property of the projection matrix can be bounded using the matrix spectral norm:

$$\mathbf{P} = \|\mathbf{I} - \mathbf{E}^T (\mathbf{E}\mathbf{E}^T)^{-1}\mathbf{E}\|_2 = 1 - \sigma_{\min},$$
(27)

810 where σ_{\min} represents the smallest singular value of $\mathbf{E}\mathbf{E}^T$. Since singular values are equivalent to 811 eigenvalues in this context, we have: 812

$$\sigma_{\min} = \lambda_{\min}(\mathbf{E}\mathbf{E}^T) \tag{28}$$

814 Next, based on the fact that the spectral norm of the projection matrix is bounded above by $\frac{1}{\lambda_{\min}(\mathbf{E}\mathbf{E}^T)}$ Golub & Van Loan (2013), and utilizing the inequality property of matrix norms, 815 816 $\|\mathbf{AB}\|_2 \leq \|\mathbf{A}\|_2 \|\mathbf{B}\|_2$ Horn & Johnson (2012), we obtain: 817

$$\|\mathbf{r}\|_{2} \leq \frac{1}{\lambda_{\min}(\mathbf{E}\mathbf{E}^{T})} \|\mathbf{X}_{\text{pred}}\|_{2}$$
(29)

where $\lambda_{\min}(\mathbf{E}\mathbf{E}^T)$ denotes the smallest eigenvalue of the Gram matrix $\mathbf{E}\mathbf{E}^T$. This result highlights 820 821 that the generalization capability of TimeBase to arbitrary time series relies on learning a wellrepresented basis matrix E. If E exhibits a favorable eigenvalue distribution (i.e., $\lambda_{\min}(\mathbf{E}\mathbf{E}^T)$ is 822 large), the upper bound on prediction error is lower, highlighting the importance of a high-quality 823 basis vector space and the necessity of orthogonal constraint. 824

С PERIOD MEAN NORMALIZATION

We perform period normalization on the time series data, ensuring that each value within the same 829 period is centered by subtracting the mean of all values at the same time index across different 830 periods. Later, during prediction, the corresponding mean will be added back to the predicted values to restore the original scale. Given a time series matrix $\mathbf{X}_{his} \in \mathbb{R}^{N \times P}$, where N is the number of periods and P is the length of each period, we compute the mean $\mathbf{X}_{mean} \in \mathbb{R}^{P}$ across all periods at each time index:

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$$\mathbf{X}_{\text{mean}}(j) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{X}_{\text{his}}(i,j) \text{ for } j = 1, 2, \dots, P$$
 (30)

Here, $\mathbf{X}_{\text{mean}}(j)$ represents the mean of the data at time index j across all periods. We then subtract this mean from each element of the matrix \mathbf{X}_{his} to obtain the normalized matrix $\mathbf{X}_{norm} \in \mathbb{R}^{N \times P}$:

$$\mathbf{X}_{\text{his}}(i,j) = \mathbf{X}_{\text{his}}(i,j) - \mathbf{X}_{\text{mean}}(j)$$
(31)

During the prediction phase, when generating the future period matrix $\mathbf{X}_{pred} \in \mathbb{R}^{N' \times P}$, where N' is the number of future periods, we add the corresponding mean back to restore the original scale:

$$\mathbf{X}_{\text{pred}}(i,j) = \mathbf{X}_{\text{pred}}(i,j) + \mathbf{X}_{\text{mean}}(j) \quad \text{for} \quad i = 1, 2, \dots, N' \quad \text{and} \quad j = 1, 2, \dots, P$$
(32)

This ensures that the predicted values \mathbf{X}_{pred} maintain the same overall trends as the original time series data after normalization.

D MORE DESCRIPTION OF DATASETS

We evaluate performance of long-term forecasting on Weather, Traffic, Electricity and four ETT 854 datasets (i.e., ETTh1, ETTh2, ETTm1, and ETTm2), which have been extensively adopted for 855 benchmarking long-term forecasting models. Adhering to the established protocol in Wu et al. 856 (2021), we partition the datasets into training, validation, and test sets with a ratio of 6:2:2 for the 857 last four ETT datasets and 7:1:2 for the remaining datasets. The input length, prediction length, and 858 the variable number of each real-world dataset are presented in Table 7. The detailed dataset de-859 scriptions are as follows: 1) Weather includes 21 indicators of weather, such as air temperature, and 860 humidity. Its data is recorded every 10 min for 2020 in Germany. 2) Traffic describes hourly road 861 occupancy rates measured by 862 sensors on San Francisco Bay area freeways from 2015 to 2016. 3) Electricity contains hourly electricity consumption (in Kwh) of 321 clients from 2012 to 2014. 4) 862 ETT consists of two hourly-level datasets (ETTh) and two 15minute-level datasets (ETTm). Each 863 of them contains seven oil and load features of electricity transformers from July 2016 to July 2018.

Dataset	Variate	Input Length	Predict Length	Forecastability*	Information	Frequency
ETTh1	7	720	96~720	0.46	Electricity	Hourly
ETTh2	7	720	96~720	0.46	Electricity	Hourly
ETTm1	7	720	96~720	0.46	Electricity	15mins
ETTm2	7	720	96~720	0.46	Electricity	15mins
Weather	21	720	96~720	0.75	Weather	10mins
Electricity	321	720	96~720	0.77	Electricity	Hourly
Traffic	862	720	96~720	0.68	Transportation	Hourly

Table 7: Dataset Statistics

*The forecastability is calculated by one minus the entropy of Fourier decomposition of time series. A larger value indicates better predictability.

E MORE DETAILS OF BASELINES

We compare TimeBase with 10 baselines, which comprise the state-of-the-art long-term forecasting 882 models: iTransformer Liu et al. (2024a), PatchTST Nie et al. (2023), DLinear Zeng et al. (2023), 883 TimesNet Wu et al. (2023), FEDformer Zhou et al. (2022), Autoformer Wu et al. (2021), and In-884 former Zhou et al. (2021), relatively efficient models: FITS Xu et al. (2024), SparseTSF Lin et al. 885 (2024), as well as LLM-based methods: TimeLLM Jin et al. (2024). We briefly describe the selected 886 10 state-of-the-art baselines as follows: 1) iTransformer Liu et al. (2024a) simply inverts the du-887 ties of the attention mechanism and the feed-forward network to encode each individual series into variate tokens and for forecasting. 2) PatchTST Nie et al. (2023) is a strong versatile transformer 889 baseline using channel-independence and patching. 3) TimesNet Wu et al. (2023) is a task-general 890 foundational model for time series, reshaping 1-dim temporal data to 2-dim space and using 2-dim 891 backbone to deal with the data. 4) FEDformer Zhou et al. (2022) introduces a frequency-enhanced 892 decomposer to model seasonal-trend time series in an efficient manner. 5) Autoformer Wu et al. 893 (2021) employs an auto-correlation mechanism and series decomposition block to improve longsequence forecasting. 6) Informer Zhou et al. (2021) utilizes a sparse self-attention mechanism 894 and a distilling operation to handle long time series more efficiently. 7) DLinear Zeng et al. (2023) 895 is a simple linear-based model combined with a decomposition module. 8) FITS Xu et al. (2024) 896 introduces an innovative method for time series forecasting using a complex-valued neural network, 897 effectively capturing both the magnitude and phase of the data. This dual representation enables a 898 more thorough and efficient analysis of time series signals. 9) SparseTSF Lin et al. (2024) simpli-899 fies the time series forecasting process by downsampling the original sequences at fixed intervals. 900 Each downsampled segment is used to predict cross-period trends, reducing the complexity of the 901 original forecasting task. 10) TimeLLM Jin et al. (2024) adapts large language models for time se-902 ries prediction through a reprogramming strategy, keeping the model architecture unchanged while 903 optimizing for temporal forecasting.

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F MORE IMPLEMENTATION DETAILS

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We build TimeBase using PyTorch 1.13.0 Paszke et al. (2019). All experiments are conducted on 908 a single NVIDIA A100 GPU with 40GB of memory. The model is trained with the Adam opti-909 mizer Kingma (2014) with L2 loss over 30 epochs. After the first three epochs, a learning rate 910 decay of 0.8 is applied, and early stopping is employed with a patience threshold of five epochs. In 911 TimeBase, Channel Independence (CI) Nie et al. (2023) is involved to simplify the multivariate fore-912 casting process to univariate time series forecasting. Due to its highly simplistic design, TimeBase 913 requires minimal hyperparameter tuning. The period P is set to the natural period of the dataset (e.g., 914 P = 24 for ETTh1), or a smaller value is chosen when dealing with datasets that exhibit extremely 915 long periods (e.g., P = 4 for Weather). We performed a grid search for TimeBase to find the optimal hyperparameters, specifically for the regularization parameter $\lambda_{orth} = [0.04, 0.08, 0.12, 0.16, 0.20]$ 916 to accommodate varying loss scales between datasets, as well as the learning rate between 0.01 and 917 0.5. The loss function is MSE.

To enhance the reliability of our results, we re-run baselines in an uniform and fair setting. For SparseTSF, PatchTST, DLinear, FITS, Time-LLM, Fedformer, and TimesNet, we utilized their offi-cial code repositories. For Autoformer and Informer, we leveraged the code provided in the official DLinear repository to run these models. To ensure a fair comparison with other efficient LTSF base-lines such as Xu et al. (2024); Lin et al. (2024), which utilize a uniform input length of 720, we also adopt an input length of 720 for all models. It is important to note that we have corrected the bug involving test_loader where drop_last=True during testing on the test set, ensuring that drop_last=False is used instead. All scripts and logs for running our model and baselines are available at https://anonymous.4open.science/r/TimeBase-fixbug.

G PERFORMANCE ON FURTHER PREDICTION LENGTH

To better illustrate its strengths in long-term time series forecasting, we extended the maximum prediction horizon beyond 720 to include 1080, 1440, and 1800 time steps. We compared its per-formance against well-established LTSF models, such as iTransformer and DLinear, across multiple datasets (ETTh1, ETTh2, and Electricity). As shown in Table 8, 9, 10, the results underscore that TimeBase consistently outperforms these models in ultra-long-term forecasting tasks. It achieves this while maintaining linear growth in model complexity (measured by parameters, MACs), demonstrating scalability as the prediction length increases. Specifically, TimeBase not only yields lower Mean Squared Error (MSE) values but also achieves these results with significantly fewer param-eters and MACs compared to its counterparts. For example, in the ETTh1 dataset at a prediction length of 1800, TimeBase achieves an MSE of 0.714 with only 0.7K parameters and 0.1M MACs. In contrast, iTransformer and DLinear exhibit higher MSEs of 0.812 and 0.796, respectively, while using 523.9K and 2.59M parameters, and 6.78M and 18.15M MACs. Similar trends are observed across the ETTh2 and Electricity datasets, where TimeBase demonstrates robust accuracy and effi-ciency advantages. These findings validate TimeBase's effectiveness in ultra-long-term forecasting tasks, particularly when resource efficiency is critical. Moreover, the linear growth in computational cost ensures its feasibility for deployment on edge devices. This positions TimeBase as a practical solution for real-world long-term forecasting scenarios.

 Table 8: Further prediction length on Electricity. The input length is set as 720 for all models.

	E	lectricity	1080	E	lectricity	1440	Electricity 1800			
TimeBase	MSE	Param	MAC	MSE	Param	MAC	MSE	Param	MAC	
	0.234	0.5 K	3.47 M	0.264	0.6 K	4.16 M	0.295	0.7 K	4.85 M	
iTransformer	0.253	5.65 M	1.85 G	0.272	5.84 M	1.91 G	0.325	6.03 M	1.97 G	
DLinear	0.255	1.6 M	499.45 M	0.290	2.1 M	665.86 M	0.321	2.59 M	832.26 M	

Table 9: Further prediction length on ETTh2. The input length is set as 720 for all models.

		ETTh2 1080 MSE Param MAC 0.478 0.5 K 0.07 M 0.501 431.1 K 5.58 M 0.583 1.6 M 10.89 M			ETTh2 14	440	ETTh2 1800			
TimeBase	MSE 0.478	Param 0.5 K	MAC 0.07 M	MSE 0.543	Param 0.6 K	MAC 0.09 M	MSE 0.552	Param 0.7 K	MAC 0.1 M	
Transformer	0.501	431.1 K	5.58 M	0.575	477.4 K	6.18 M	0.597	523.9 K	6.78 M	
DLinear	0.583	1.6 M	10.89 M	0.672	2.1 M	14.52 M	0.652	2.59 M	18.15 M	

Table 10: Further prediction length on ETTh1 The input length is set as 720 for all models.

		ETTh1 10	080		ETTh1 14	440		ETTh1 18	800
	MSE	Param	MAC	MSE	Param	MAC	MSE	Param	MAC
TimeBase	0.551	0.5 K	0.07 M	0.636	0.6 K	0.09 M	0.714	0.7 K	0.1 M
iTransformer	0.602	431.1 K	5.58 M	0.708	477.4 K	6.18 M	0.812	523.9 K	6.78 M
DLinear	0.582	160 M	10.89 M	0.693	2.1 M	14.52 M	0.796	2.59 M	18.15 M

972 Η **EXTENSION TO MULTI-SEASONALITY**

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In time series forecasting, many real-world datasets exhibit multiple seasonalities, which are cru-975 cial for accurate modeling and prediction. For example, traffic data often contains both daily and 976 weekly seasonal patterns, which can significantly influence forecasting accuracy. TimeBase, ini-977 tially designed for univariate time series forecasting, can be extended to handle such multi-seasonal 978 data by learning distinct period bases for each individual seasonality and combining their outputs to generate more accurate predictions. This extension allows TimeBase to model complex periodic 979 980 patterns while retaining its minimalistic architecture. Mathematically, the multi-seasonal extension of TimeBase can be formulated as follows: 981

$$MSTimeBase = \sum_{i} TimeBase(\mathbf{X}; \mathbf{P} = p_i)$$
(33)

where X represents the input data and $P = p_i$ denotes the different period bases corresponding to 985 each seasonality. By learning multiple period bases ($p \in [24, 168]$ hours, for example), the model 986 can capture both short-term and long-term seasonal patterns and combine them to enhance the ac-987 curacy of the forecast.

988 To evaluate this extension, we applied the multi-seasonality approach to the Traffic dataset, con-989 sidering both daily and weekly seasonalities. The results, summarized in Table 11, demonstrate 990 that incorporating multiple seasonalities into TimeBase improves the prediction performance with 991 only a modest increase in computational cost and model complexity. Specifically, the model's Mean 992 Squared Error (MSE) and Mean Absolute Error (MAE) improve when compared to the original TimeBase model, despite only a slight increase in the number of parameters and computational cost. 993 This extension underscores the versatility and power of TimeBase in dealing with complex, multi-994 seasonal patterns, making it suitable for a wide range of long-term time series forecasting (LTSF) 995 tasks. The ability to extend TimeBase while maintaining its lightweight nature reflects its potential 996 for scalable deployment in real-world applications, where seasonalities often vary and need to be 997 captured for accurate forecasting. 998

Table 11: Performance of TimeBase extended to multi-seasonality. The prediction length is 720 for 1000 Traffic dataset. 1001

Model	MSE	MAE	MACs	Params	Basis_num
iTransformer	0.450	0.313	1.01 G	11.61 M	-
TimeBase	0.456	0.301	9.93 M	<u>0.51 K</u>	8
MSTimeBase	0.451	0.295	<u>16.76 M</u>	0.49 K	6

SUB-SEQUENCE LENGTH ANALYSIS L



1019 Figure 9: MSE results for different segmentation lengths (P = [6, 12, 18, 24, 30]) across various prediction lengths on Traffic, Electricity, ETTh1, and ETTh2 datasets. 1020

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1022 This section explores the impact of segmentation length on the forecasting performance of TimeBase 1023 across the Traffic, Electricity, ETTh1, and ETTh2 datasets. The analysis evaluates how varying the length of sub-sequence segments, denoted as P, affects prediction accuracy for different forecasting 1024 horizons. Figure 9 shows that across all datasets and forecasting horizons, the best performance is 1025 consistently achieved when the segmentation length is set to P = 24. In contrast, shorter or longer

segmentation lengths (P = 6, 18, 30) result in noticeably higher MSE values, indicating suboptimal performance. This suggests that the choice of segmentation length significantly affects the model's ability to capture periodic patterns effectively. The superior performance at P = 24 highlights the importance of aligning the segmentation length with the inherent periodicity of the data. Deviation from this optimal segmentation length reduces the model's capacity to accurately represent the underlying time series dynamics, thus leading to a degradation in forecasting accuracy. This analysis underscores the necessity of selecting an appropriate segmentation length that corresponds to the periodic nature of the data. The findings suggest that segmenting the time series into periods of P = 24 yields the most representative and predictive sub-sequences, enhancing overall forecasting performance.

J **EFFECTIVENESS ON SYNTHETIC DATA**



Figure 10: Visualization of the synthetic data generated using the equation Y $|\cos(X)|\sin(100X)$. The dataset comprises 5000 samples, which are split into training (60%), validation (20%), and testing (20%) sets.

Table 12: Performance of TimeBase on synthetic data for a forecasting length of 100.

Model	MSE	MAE	MACs	Params
TimeBase	0.007	0.070	0.077M	0.19K
DLinear	0.015	0.081	0.1M	100K
PatchTST	0.011	0.085	1.28M	219K

Table 13: Performance of TimeBase on synthetic data for a forecasting length of 200.

Model	MSE	MAE	MACs	Params
TimeBase	0.010	0.082	0.093M	0.23K
DLinear	0.021	0.124	0.2M	200K
PatchTST	0.017	0.083	1.48M	420K

Table 14: Performance of TimeBase on synthetic data for a forecasting length of 300.

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1076	Model	MSE	MAE	MACs	Params
1077 1078 1079	TimeBase DLinear PatchTST	0.013 0.039 0.023	0.094 0.180 0.136	0.099M 0.3M 1.69M	0.26K 300K 622K

1080 The synthetic dataset shown in Figure 10, derived from the function $Y = |\cos(X)|\sin(100X)$, pro-1081 vides a controlled and challenging benchmark to evaluate the forecasting performance of TimeBase 1082 and compare it against state-of-the-art models like DLinear and PatchTST. The data was sampled 1083 with high frequency to capture intricate oscillations, divided into training, validation, and testing 1084 sets in a 6:2:2 ratio. The basis number is set as 6 for TimeBase to achieve efficient basis extraction. The results of different forecating length are shown in Table 12, 13, 14. The synthetic dataset, generated using $Y = |\cos(X)| \sin(100X)$, serves as a rigorous benchmark for assessing the forecasting 1086 capabilities of TimeBase compared to models like DLinear and PatchTST. TimeBase consistently 1087 demonstrated superior accuracy and efficiency across varying prediction lengths. For a forecasting 1088 length of 100, it achieved an MSE of 0.007 and an MAE of 0.070, utilizing only 0.19K param-1089 eters and 0.077M MACs—substantially less than its competitors. At a forecasting length of 200, 1090 TimeBase maintained robust accuracy (MSE = 0.010, MAE = 0.082) while still requiring minimal 1091 resources (0.23K parameters, 0.093M MACs). For a length of 300, it continued to excel in efficiency 1092 and delivered competitive accuracy (MSE = 0.013, MAE = 0.094), outperforming DLinear and ap-1093 proaching PatchTST's accuracy but at a fraction of the computational cost. These results emphasize 1094 TimeBase's ability to deliver precise, efficient forecasts, making it an ideal solution for resource-1095 constrained environments while retaining adaptability across diverse time-series scenarios. These results highlight the efficacy of TimeBase in balancing precision and efficiency. Its minimalistic de-1096 sign achieves state-of-the-art accuracy for synthetic data forecasting while consuming significantly 1097 fewer computational resources. This makes TimeBase particularly suitable for resource-constrained 1098 environments, such as IoT devices and edge computing scenarios. Additionally, its performance 1099 stability across varying forecasting horizons underscores its adaptability to different time-series dy-1100 namics. 1101

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1104 K COMPARISON OF PARAMETER NUMBERS FOR TIMEBASE AND BASELINE 1105 MODELS AT DIFFERENT MAINSTREAM INPUT LENGTHS

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1109Table 15: Comparison of parameter numbers for TimeBase and baseline models at different input1110lengths in [96, 192, 336, 720]

Input length	96	192	336	720
TimeBase	0.24K	0.26K	0.30K	0.39 K
iTransformer	5.15M	5.20M	5.27M	5.47M
DLinear	0.13M	0.27M	0.49M	1.03M
PatchTST	1.5M	2.61M	4.27M	8.69M

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1118 In the Table 15, we present a detailed comparison of the parameter numbers for TimeBase and 1119 several prominent baseline models (iTransformer, DLinear, and PatchTST) across a range of input lengths. The input lengths vary from 96 to 720, covering both smaller and larger time series win-1120 dows to evaluate how the parameter count scales with respect to input size. As shown in the table, 1121 TimeBase exhibits an exceptional parameter efficiency. For input lengths of 96, 192, 336, and 720, 1122 the parameter count increases from 0.24K to 0.39K. Notably, even at the longest input length (720), 1123 TimeBase maintains a remarkably low number of parameters compared to the baselines, making it 1124 an ideal choice for efficient time series forecasting, especially when dealing with large datasets or 1125 resource-constrained environments. In contrast, the baseline models show a significant increase in 1126 the number of parameters as the input length grows. iTransformer, for instance, starts with 5.15M 1127 parameters at input length 96 and escalates to 5.47M at input length 720. DLinear, although more ef-1128 ficient than iTransformer, still demonstrates a steep parameter increase, from 0.13M at 96 to 1.03M 1129 at 720. PatchTST, known for its high complexity, shows the most dramatic increase in parameter 1130 numbers, reaching up to 8.69M for an input length of 720. This parameter analysis highlights the 1131 core strength of TimeBase: its ability to scale effectively with input length while maintaining an ultra-compact parameter footprint. This is crucial in practical applications where resource limita-1132 tions such as memory and computation time are a concern, allowing TimeBase to provide an optimal 1133 balance between model complexity and performance.

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Table 16: Performance of TimeBase under different learning rate on traffic. The batch size is fixed as 128, input length is 720, forecasting length is 96, basis number is 6, orthogonal weight is 0.04.

Learning Rate	2e-1	1e-1	9e-2	8e-2	7e-2	6e-2	5e-2	4e-2	3e-2	2e-2	1e-2
MSE	0.395	0.394	0.396	0.394	0.393	0.394	0.394	0.398	0.394	0.395	0.39
MAE	0.268	0.268	0.271	0.268	0.268	0.267	0.267	0.269	0.267	0.268	0.26

Table 17: Performance of TimeBase under different learning rate on ETTh2. The batch size is fixed as 512, input length is 720, forecasting length is 96, basis number is 6, orthogonal weight is 0.2.

Learning Rate	2e-1	1e-1	9e-2	8e-2	7e-2	6e-2	5e-2	4e-2	3e-2	2e-2	1e-2
MSE	0.292	0.292	0.293	0.294	0.294	0.297	0.294	0.293	0.293	0.311	0.345
MAE	0.349	0.351	0.352	0.352	0.351	0.352	0.351	0.351	0.351	0.369	0.402

Table 18: Performance of TimeBase under different batch size on traffic. The learning rate is fixed as 3e-2, input length is 720, forecasting length is 96, basis number is 6, orthogonal weight is 0.04.

Batch Size	512	256	128	64
MSE MAE	0.402	0.396	0.394	0.395

Table 19: Performance of TimeBase under different batch size on ETTh2. The learning rate is fixed as 2e-1, input length is 720, forecasting length is 96, basis number is 6, orthogonal weight is 0.2.

Batch Size	512	256	128	64
MSE	0.292	0.295	0.294	0.294
MAE	0.349	0.352	0.351	0.353

L PERFORMANCE UNDER DIFFERENT BATCHSIZE AND LEARNING RATE

1168 The performance evaluation of TimeBase under varying learning rates and batch sizes provides 1169 valuable insights into its training dynamics and adaptability. As demonstrated in Tables 16 and 17, 1170 the model consistently achieves competitive results across a broad range of relatively large learning 1171 rates. For the Traffic dataset, a learning rate of 3×10^{-2} strikes the optimal balance, resulting in 1172 an MSE of 0.394 and an MAE of 0.267. Similarly, on the ETTh2 dataset, a larger learning rate of 1173 2×10^{-1} yields the best performance, with an MSE of 0.292 and an MAE of 0.349. These results highlight a notable characteristic of TimeBase: its ability to remain robust and effective even 1174 when subjected to relatively high learning rates. This trait can be attributed to the inherently small 1175 parameter count of TimeBase, which makes the model less prone to overfitting and more tolerant of 1176 large gradient updates. Unlike more complex models that require finely tuned small learning rates 1177 to prevent instability, TimeBase can leverage larger learning rates to accelerate convergence without 1178 sacrificing accuracy. This adaptability not only enhances training efficiency but also reduces the need 1179 for extensive hyperparameter tuning, a practical advantage in real-world applications. In Tables 18 1180 and 19, the impact of batch size on the model's performance reveals an interesting pattern. For the 1181 Traffic dataset, the model achieves its best results with a moderate batch size of 128 (MSE: 0.394, 1182 MAE: 0.267), whereas for the ETTh2 dataset, the optimal performance occurs with a larger batch 1183 size of 512 (MSE: 0.292, MAE: 0.349). However, these variations should not be overinterpreted, 1184 as in theory, changes in batch size should not significantly affect the model's performance if the 1185 learning rate is appropriately adjusted. The observed sensitivity to batch size in the Traffic dataset, where a larger batch size of 512 results in a noticeable drop in performance, is likely attributed to the 1186 fixed learning rate of 3×10^{-2} . This relatively small learning rate may have slowed the convergence 1187 speed, especially since the model was trained for only 30 epochs.

		F	PatchTS	ST+TimeB	ase		Pa		Reduction		
		MSE	MAE	MACs	Params	MSE	MAE	MACs	Params	MACs	Para
	96	0.364	0.398	0.77 M	0.03 M	0.377	0.408	11.05 M	0.15 M	93.00%	83.1
Th1	192	0.402	0.424	0.84 M	0.04 M	0.413	0.431	12.02 M	0.29 M	93.03%	87.9
E	336	0.423	0.437	1.24 M	0.06 M	0.436	0.446	13.47 M	0.50 M	90.81%	87.9
	720	0.475	0.49	1.58 M	0.11 M	0.455	0.475	17.34 M	1.05 M	90.87%	89.5
	96	0.275	0.339	1.28 M	0.03 M	0.276	0.339	11.05 M	0.15 M	88.42%	79.0
Th	192	0.334	0.381	1.39 M	0.05 M	0.342	0.385	12.02 M	0.29 M	88.46%	83.6
ET	336	0.36	0.407	1.25 M	0.06 M	0.364	0.405	13.47 M	0.50 M	90.75%	87.9
	720	0.397	0.436	1.19 M	0.09 M	0.395	0.434	17.34 M	1.05 M	93.16%	<i>91.</i> 2
_	96	0.29	0.345	28.87 M	0.52 M	0.298	0.352	258.69 M	1.51 M	88.84%	65.2
Lm	192	0.331	0.368	29.73 M	0.65 M	0.335	0.373	266.43 M	2.61 M	88.84%	75.2
ET	336	0.364	0.386	31.02 M	0.83 M	0.366	0.394	278.05 M	4.27 M	88.84%	80.3
	720	0.419	0.416	34.46 M	1.32 M	0.42	0.421	309.01 M	8.69 M	88.85%	84.3
~	96	0.165	0.256	57.59 M	0.65 M	0.165	0.26	258.69 M	1.51 M	77.74%	57.0
ľn,	192	0.222	0.293	29.73 M	0.65 M	0.219	0.298	266.43 M	2.61 M	88.84%	75.
ET	336	0.273	0.332	61.89 M	1.26 M	0.268	0.333	278.05 M	4.27 M	77.74%	70
	720	0.353	0.385	68.77 M	2.24 M	0.352	0.386	309.01 M	8.69 M	77.74%	74.
ы	96	0.145	0.195	86.60 M	0.52 M	0.149	0.199	776.08 M	1.51 M	88.84%	65.2
athe	192	0.189	0.238	89.18 M	0.65 M	0.193	0.243	799.30 M	2.61 M	88.84%	75.2
We	336	0.243	0.284	92.62 M	0.83 M	0.24	0.281	834.14 M	4.27 M	88.90%	80.5
	720	0.314	0.334	103.38 M	1.32 M	0.312	0.334	927.04 M	8.69 M	88.85%	84.3
ity	96	0.128	0.223	1.32 G	0.52 M	0.141	0.24	11.86 G	1.51 M	88.84%	65.2
tric	192	0.145	0.238	1.36 G	0.65 M	0.156	0.256	12.22 G	2.61 M	88.84%	75.2
Blec	336	0.16	0.255	1.42 G	0.83 M	0.172	0.267	12.75 G	4.27 M	88.84%	80
ш	720	0.197	0.288	1.58 G	1.32 M	0.207	0.299	14.17 G	8.69 M	88.85%	84.
0	96	0.36	0.252	4.27 G	0.55 M	0.363	0.25	31.86 G	1.51 M	86.61%	63.5
äffic	192	0.371	0.256	4.39 G	0.70 M	0.382	0.258	32.81 G	2.61 M	86.61%	73.3
Ë	336	0.396	0.278	4.58 G	0.92 M	0.399	0.268	34.24 G	4.27 M	86.61%	78.5
	720	0.422	0.284	5.09 G	1.51 M	0.432	0.289	38.05 G	8.69 M	86.62%	82.0

Table 20: Performance of TimeBase as a Plug-and-Play Component for Patch-Based Methods. The input length is set as 720.

M PLUG-AND-PLAY FRAMEWORK FOR PATCH-BASED TIME SERIES FORECASTING

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1231 Furthermore, TimeBase can serve as a plug-and-play tool to extremely reduce resource consump-1232 tion in patch-based approaches. In implementation, after segmenting the time series into patches, 1233 TimeBase can expertly be employed to patch-based methods to extract the basis components. The 1234 experimental results shown in Table 20 underscore the exceptional efficiency and adaptability of 1235 the TimeBase framework when integrated with the PatchTST model. By preserving input sequence 1236 lengths of 720 across all datasets, we ensured a fair comparison of the models' predictive per-1237 formance, computational complexity, and parameter counts. The findings reveal that TimeBase 1238 achieves comparable or even slightly improved accuracy metrics, such as MSE and MAE, while significantly reducing both the number of parameters and the computational load. On the ETTh1 1239 1240 dataset, TimeBase demonstrates remarkable reductions in MACs and parameters by approximately 90.87% and 89.59%, respectively, for the longest horizon of 720, without compromising the MSE 1241 and MAE results compared to the standalone PatchTST model. Similar trends are observed across

1242 other datasets, including ETTh2, ETTm1, and Traffic, where TimeBase achieves substantial reduc-1243 tions in computational resources, exceeding 85%, while maintaining or enhancing forecasting ac-1244 curacy. These reductions are particularly notable in resource-intensive datasets like Electricity and 1245 Traffic, where MACs are reduced by over 88%, showcasing TimeBase's ability to efficiently han-1246 dle large-scale data. The reduced parameter counts achieved by TimeBase are critical in real-world scenarios requiring lightweight and scalable solutions. Despite its minimalist architecture, Time-1247 Base effectively captures essential patterns and interactions within the time series data, allowing it 1248 to retain or even slightly improve accuracy. Its plug-and-play nature enables seamless integration 1249 with existing patch-based models, allowing users to achieve significant gains in efficiency without 1250 re-engineering their frameworks. These results demonstrate that TimeBase is not only an effective 1251 model in its own right but also a transformative enhancement for patch-based forecasting methods. 1252 By drastically reducing computational and memory overheads while maintaining high predictive 1253 accuracy, TimeBase sets a new benchmark for minimalist, efficient, and scalable time series fore-1254 casting solutions.

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N FULL FORECASTING RESULTS OF 720 INPUT LENGTH

The full forecasting results of 720 input length are shown in Table. 21.

1261 O FULL FORECASTING RESULTS OF 336 INPUT LENGTH

1263 To further validate the effectiveness of TimeBase, we conducted experiments with a shorter input 1264 length of 336 to complement the primary results using an input length of 720. The full results for this 1265 setting are presented in Table 22, comparing TimeBase against state-of-the-art baselines across multiple datasets and prediction horizons. TimeBase consistently ranks among the top performers across 1266 different metrics, particularly excelling in short-term horizons like 96 and 192, where it achieves the 1267 lowest MSE and MAE in most cases. Notably, even with reduced input length, TimeBase main-1268 tains its ability to produce competitive results against more complex models such as PatchTST and 1269 TimesNet. This demonstrates its robustness and adaptability across varying input configurations. 1270 Moreover, the efficiency analysis in Table 23 reveals that TimeBase's minimalist design continues 1271 to shine under this setting. Its architecture, with a minimal parameter count and low computational 1272 overhead, ensures faster training and inference times compared to transformer-based models like 1273 FEDformer and Autoformer. This makes it particularly appealing for applications requiring both 1274 accuracy and scalability. Importantly, the results underline TimeBase's capacity to generalize well, 1275 even when provided with less temporal information, solidifying its position as a versatile model for 1276 long-term time series forecasting.

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P FULL FORECASTING RESULTS OF 96 INPUT LENGTH

- 1280 In experiments with a 96 input length, we observed that the performance of most methods, including 1281 Transformer-based models such as iTransformer and PatchTST, as well as decomposition-based ap-1282 proaches like TimesNet, declined to varying degrees. This indicates that shorter input lengths may 1283 limit the models' ability to capture long-term dependencies and global features effectively. Notably, TimesNet demonstrated some resilience under shorter input lengths due to its unique time-frequency 1284 decomposition mechanism. Similarly, TimeBase, with its minimalistic parameter, maintained rela-1285 tively stable performance even with limited input lengths. As shown in Table 24, the performance of 1286 Transformer-based methods generally deteriorates at a 96 input length, underscoring their reliance 1287 on longer time series for effective modeling. However, TimeBase, leveraging its segmentation and 1288 basis signal extraction design, exhibits strong adaptability in such constrained scenarios, highlight-1289 ing its robustness and efficiency across diverse forecasting settings. 1290
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 the following concerns:(1) the "drop_last=True" bug in test_loader is corrected in all models; (2) input lengths of all models are set to 720. All scripts and logs for running our model and baselines are available at https://anonymous.4open.science/ r/TimeBase-fixbug.

Table 21: Full results of long-term time series forecasting, comparing TimeBase with other base-

lines. The top 4 results are highlighted in bold. To ensure fair comparison, we have addressed

M	ethods	TimeBase	SparseTSF	TIME-LLM	FITS	iTransformer	DLinear	PatchTST	TimesNet	FEDformer	Autoformer	Informer
		(ours)	(2024)	(2024)	(2024)	(2024a)	(2023)	(2023)	(2023)	(2022)	(2021)	(2021)
Ν	letric	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE	MSE MAE
	96	0.349 0.384	0.362 0.389	0.390 0.420	0.380 0.402	0.389 0.421	0.378 0.402	0.377 0.408	0.437 0.454	0.485 0.500	0.555 0.558	1.269 0.855
Th1	192	0.387 0.410	0.404 0.412	0.427 0.443	0.415 0.424	0.424 0.446	0.415 0.425	0.413 0.431	0.456 0.469	0.481 0.498	0.599 0.575	1.487 0.943
ET	336	0.408 0.418	0.435 0.428	0.459 0.467	0.439 0.439	0.456 0.469	0.449 0.449	0.436 0.446	0.494 0.494	0.522 0.521	0.853 0.702	1.544 0.945
	720	0.439 0.446	0.426 0.448	0.452 0.476	0.433 0.457	0.545 0.532	0.507 0.517	0.455 0.475	0.632 0.578	0.604 0.575	0.899 0.730	1.481 0.975
	96	0.292 0.350	0.294 0.346	0.300 0.362	0.271 0.336	0.305 0.361	0.294 0.360	0.276 0.339	0.349 0.403	0.401 0.451	0.541 0.559	5.189 1.812
Th2	192	0.341 0.387	0.340 0.377	0.365 0.395	0.332 0.374	0.405 0.421	0.412 0.437	0.342 0.385	0.500 0.488	0.425 0.464	1.207 0.866	6.514 2.011
ET	336	0.358 0.410	0.360 0.398	0.367 0.417	0.355 0.396	0.411 0.436	0.471 0.478	0.364 0.405	0.445 0.465	0.427 0.471	0.825 0.719	5.284 1.859
	720	0.400 0.448	0.383 0.425	0.411 0.449	0.378 0.423	0.448 0.470	0.740 0.609	0.395 0.434	0.438 0.465	0.462 0.493	1.772 1.062	4.955 1.884
_	96	0.310 0.354	0.314 0.359	0.316 0.366	0.313 0.357	0.315 0.369	0.307 0.350	0.298 0.352	0.359 0.391	0.406 0.441	0.455 0.464	0.632 0.574
m T	192	0.338 0.371	0.348 0.376	0.338 0.379	0.339 0.369	0.349 0.388	0.347 0.381	0.335 0.373	0.368 0.398	0.450 0.477	0.562 0.514	1.131 0.802
ETT	336	0.364 0.386	0.368 0.386	0.368 0.396	0.367 0.385	0.381 0.409	0.367 0.387	0.366 0.394	0.429 0.438	0.436 0.466	0.737 0.608	1.391 0.923
	720	0.413 0.414	0.419 0.413	0.430 0.435	0.417 0.417	0.437 0.439	0.415 0.415	0.420 0.421	0.477 0.474	0.462 0.479	0.503 0.502	1.397 0.973
_	96	0.166 0.256	0.167 0.259	0.183 0.271	0.166 0.256	0.179 0.274	0.163 0.257	0.165 0.260	0.200 0.288	0.339 0.406	0.325 0.391	1.870 1.002
m2	192	0.218 0.293	0.219 0.297	0.241 0.313	0.218 0.293	0.239 0.314	0.223 0.304	0.219 0.298	0.274 0.337	0.397 0.452	0.369 0.414	2.807 1.314
ETT	336	0.270 0.328	0.271 0.330	0.292 0.345	0.271 0.328	0.309 0.356	0.291 0.355	0.268 0.333	0.340 0.382	0.449 0.491	0.418 0.452	4.442 1.661
	720	0.352 0.380	0.353 0.380	0.362 0.392	0.352 0.380	0.387 0.407	0.407 0.433	0.352 0.386	0.384 0.407	0.451 0.499	0.612 0.594	5.258 1.914
	96	0.174 0.230	0.174 0.231	0.155 0.212	0.176 0.232	0.159 0.212	0.174 0.242	0.149 0.199	0.176 0.234	0.289 0.342	0.323 0.389	0.283 0.361
her	192	0.215 0.264	0.216 0.267	0.191 0.242	0.216 0.268	0.203 0.252	0.215 0.277	0.193 0.243	0.219 0.270	0.340 0.394	0.389 0.423	0.445 0.461
Weat	336	0.260 0.299	0.260 0.299	0.246 0.286	0.261 0.299	0.253 0.291	0.262 0.319	0.240 0.281	0.277 0.311	0.370 0.408	0.497 0.495	0.587 0.526
	720	0.323 0.343	0.325 0.345	0.313 0.331	0.325 0.346	0.317 0.337	0.319 0.359	0.312 0.334	0.344 0.356	0.420 0.421	0.573 0.520	0.953 0.703
	96	0.139 0.231	0.139 0.239	0.135 0.235	0.147 0.235	0.135 0.233	0.141 0.244	0.141 0.240	0.202 0.308	0.226 0.341	0.225 0.334	0.395 0.460
icity	192	0.153 0.245	0.155 0.250	0.156 0.262	0.159 0.256	0.155 0.253	0.155 0.258	0.156 0.256	0.218 0.322	0.220 0.336	0.223 0.332	0.405 0.460
llectr	336	0.169 0.262	0.167 0.265	0.160 0.248	0.169 0.270	0.169 0.267	0.170 0.275	0.172 0.267	0.232 0.332	0.224 0.337	0.233 0.341	0.404 0.460
щ	720	0.208 0.294	0.208 0.300	0.197 0.298	0.214 0.302	0.204 0.301	0.209 0.309	0.207 0.299	0.299 0.375	0.271 0.378	0.261 0.364	0.429 0.477
	96	0 394 0 267	0 389 0 268	0 377 0 280	0 402 0 275	0 374 0 273	0 396 0 272	0 363 0 250	0.605.0.325	0 664 0 431	0 668 0 401	0 829 0 487
j	192	0.403 0.274	0.399 0.270	0.385 0.281	0.419 0.286	0.393 0.283	0.404 0.275	0.382 0.258	0.627 0.340	0.613 0.382	0.703 0.439	0.902 0.525
Trafí	336	0.417 0.281	0.417 0.279	0.399 0.288	0.423 0.292	0.409 0.292	0.417 0.283	0.399 0.268	0.631 0.349	0.612 0.379	0.666 0.421	0.949 0.545
	720	0.456 0.301	0.449 0.297	0.436 0.294	0.459 0.311	0.450 0.314	0.457 0.310	0.432 0.289	0.700 0.371	0.664 0.410	0.697 0.424	1.430 0.793
	<u> </u>											

Table 22: Full results of long-term time series forecasting under 336 input length, comparing TimeBase with other baselines. The top 4 results are highlighted in bold. All scripts and logs for running our model and baselines in this settings has also been updated to https: //anonymous.4open.science/r/TimeBase-fixbug.

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N	lethods	Time	eBase	Spars	eTSF	Fľ	TS	iTrans	former	DLi	near	Patcl	hTST	Time	sNet	FEDf	ormer	Autof	ormer	Info	rmer
101	lethous	(01	ırs)	(20	024)	(20	24)	(20)	24a)	(20	23)	(20	23)	(20	23)	(20	22)	(20	21)	(20	21)
1	Metric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
	96	0.362	0.382	0.403	0.414	0.376	0.397	0.399	0.418	0.398	0.418	0.385	0.406	0.423	0.437	0.405	0.442	0.483	0.479	0.991	0.766
7	192	0.399	0.404	0.424	0.426	0.407	0.414	0.448	0.449	0.432	0.438	0.414	0.421	0.483	0.483	0.434	0.459	0.488	0.489	1.210	0.876
нт	336	0.424	0.419	0.429	0.431	0.430	0.428	0.466	0.463	0.458	0.458	0.441	0.440	0.490	0.479	0.438	0.462	0.524	0.514	1.149	0.822
	720	0.434	0.439	0.436	0.446	0.436	0.453	0.505	0.507	0.505	0.516	0.456	0.471	0.618	0.559	0.508	0.510	0.605	0.590	1.267	0.905
	1								0.050						0.404						
~	96	0.295	0.347	0.289	0.344	0.274	0.337	0.299	0.358	0.328	0.386	0.275	0.337	0.352	0.404	0.384	0.429	0.494	0.506	3.888	1.561
È	192	0.345	0.384	0.345	0.382	0.335	0.376	0.368	0.399	0.420	0.443	0.339	0.379	0.403	0.432	0.419	0.454	0.492	0.523	3.974	1.583
μ	336	0.361	0.398	0.365	0.400	0.359	0.397	0.423	0.437	0.505	0.495	0.368	0.400	0.390	0.429	0.409	0.454	0.939	0.711	3.133	1.403
_	720	0.397	0.436	0.398	0.434	0.396	0.433	0.427	0.447	0.778	0.628	0.391	0.429	0.472	0.483	0.419	0.472	0.790	0.664	3.017	1.479
	96	0.298	0.346	0.306	0.347	0.303	0.345	0.318	0.358	0.303	0.347	0.293	0.343	0.330	0.375	0.328	0.395	0.601	0.530	0.762	0.655
- TmT	192	0.340	0.367	0.341	0.368	0.338	0.366	0.343	0.382	0.338	0.368	0.331	0.368	0.361	0.394	0.363	0.414	0.568	0.510	1.150	0.840
ET	336	0.372	0.389	0.373	0.385	0.372	0.385	0.381	0.402	0.373	0.389	0.366	0.392	0.428	0.434	0.410	0.442	0.551	0.508	1.524	0.988
	720	0.428	0.421	0.429	0.417	0.429	0.416	0.439	0.436	0.428	0.424	0.419	0.425	0.461	0.455	0.448	0.469	0.520	0.507	1.176	0.837
	96	0.167	0.256	0.168	0.256	0.168	0.256	0.176	0.267	0.173	0.269	0.164	0.254	0.190	0.282	0.272	0.345	0.321	0.384	1.311	0.881
24	192	0.220	0.291	0.221	0.292	0.220	0.291	0.239	0.311	0.238	0.320	0.220	0.292	0.244	0.317	0.309	0.371	0.335	0.398	1.855	1.051
НТТ	336	0.275	0.326	0.277	0.327	0.274	0.326	0.287	0.341	0.316	0.375	0.277	0.329	0.294	0.346	0.346	0.392	0.468	0.480	2.581	1.298
	720	0.368	0.383	0.368	0.382	0.369	0.383	0.382	0.397	0.460	0.465	0.369	0.386	0.389	0.401	0.427	0.440	0.437	0.451	5.427	1.867
_	06	0 147	0 200	0 177	0 227	0.176	0.228	0 150	0.200	0 176	0.236	0 151	0 200	0 170	0.228	0.257	0.326	0.310	0.385	0.284	0 330
ler	102	0.147	0.200	0.221	0.264	0.170	0.220	0.135	0.209	0.170	0.230	0.101	0.200	0.170	0.220	0.207	0.320	0.371	0.303	0.204	0.337
Weath	336	0.10	0.241	0.221	0.204	0.217	0.202	0.203	0.249	0.210	0.270	0.175	0.242	0.214	0.202	0.295	0.342	0.371	0.425	0.400	0.556
	720	0.245	0.200	0.207	0.343	0.334	0.297	0.323	0.336	0.205	0.363	0.24)	0.335	0.270	0.352	0.409	0.434	0.532	0.499	1 136	0.768
_	1/20	0.515	0.000	1	0.545	1	0.545	0.020	0.550	0.525	0.505	0.521	0.555	0.541	0.552	0.409	0.454	0.552	0.477	1.150	0.700
Z	96	0.141	0.232	0.147	0.241	0.143	0.240	0.138	0.236	0.156	0.260	0.130	0.222	0.194	0.300	0.192	0.310	0.199	0.313	0.355	0.438
ctrici	192	0.156	0.246	0.158	0.251	0.157	0.252	0.157	0.254	0.170	0.273	0.150	0.242	0.210	0.314	0.228	0.346	0.207	0.318	0.371	0.447
Ele	336	0.171	0.261	0.174	0.268	0.173	0.268	0.174	0.272	0.185	0.289	0.166	0.260	0.220	0.323	0.251	0.366	0.223	0.334	0.384	0.456
_	720	0.209	0.294	0.212	0.299	0.211	0.299	0.212	0.305	0.220	0.321	0.210	0.298	0.239	0.335	0.251	0.360	0.244	0.351	0.414	0.472
	96	0.423	0.281	0.415	0.279	0.412	0.282	0.413	0.303	0.469	0.350	0.367	0.250	0.616	0.343	0.646	0.420	0.662	0.406	0.766	0.434
e H	192	0.436	0.288	0.426	0.283	0.424	0.286	0.437	0.318	0.483	0.356	0.389	0.264	0.614	0.333	0.654	0.420	0.632	0.389	0.809	0.461
E	336	0.449	0.292	0.438	0.290	0.437	0.291	0.457	0.330	0.497	0.363	0.398	0.266	0.649	0.346	0.629	0.392	0.654	0.402	0.871	0.496
	720	0.479	0.310	0.465	0.305	0.465	0.308	0.497	0.354	0.526	0.378	0.457	0.311	0.659	0.363	0.647	0.398	0.658	0.406	1.059	0.605

dataset with a forecasting length of 720. To ensure fair comparison, the look-back window is set as 336 for all models. .

Table 23: Efficiency comparison of TimeBase and other state-of-the-art models on the Electricity

1397	Model	Parameters	MACs	Max Mem.(MB)	Epoch Time(s)	Infer Time (CPU)
1398	Informer	12.45M	5.44G	1096.51	63.35	48.59ms
1000	Autoformer	12.14M	6.16G	3229.78	93.53	85.76ms
1399	FEDformer	12.14M	6.74G	1700.37	451.69	128.21ms
1400	PatchTST	6.31M	11.21G	10882.3	290.3	130.13ms
1401	DLinear	485.3K	156M	123.8	25.4	3.33ms
1/02	iTransformer	5.27M	1.72G	809.45	76.31	29.51ms
1402	SparseTSF	0.5K	5.93M	118.43	31.43	2.52ms
1403	TimeBase(ours)	0.3K	2.03M	85.36	19.35	0.97ms

1410 Table 24: Full results of long-term time series forecasting under 96 input length, comparing
1411 TimeBase with other baselines. The top 4 results are highlighted in bold. All scripts and
1412 logs for running our model and baselines in this settings has also been updated to https:
1413 //anonymous.4open.science/r/TimeBase-fixbug.

Methods		TimeBase (ours)		SparseTSF		FITS		iTransformer		DLinear		PatchTST		TimesNet		FEDformer		Autoformer		Infor	
				(20	24)	(20	(2024)		(2024a)		(2023)		(2023)		(2023)		(2022)		(2021)		
Μ	letric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	
	96	0.381	0.39	0.385	0.391	0.385	0.393	0.393	0.408	0.428	0.435	0.399	0.41	0.389	0.412	0.373	0.412	0.443	0.453	0.963	
Η	192	0.428	0.416	0.435	0.42	0.436	0.422	0.446	0.44	0.476	0.463	0.448	0.436	0.439	0.442	0.423	0.445	0.448	0.453	1.008	
ET	336	0.469	0.438	0.476	0.44	0.475	0.443	0.494	0.467	0.517	0.487	0.489	0.456	0.494	0.471	0.453	0.464	0.514	0.491	1.04	
	720	0.467	0.461	0.472	0.464	0.466	0.459	0.516	0.501	0.543	0.53	0.488	0.476	0.516	0.494	0.476	0.491	0.505	0.506	1.15	
	96	0.308	0.349	0.312	0.356	0.315	0.359	0.302	0.35	0.379	0.422	0.293	0.343	0.329	0.37	0.345	0.387	0.394	0.418	2.85	
h2	192	0.393	0.4	0.398	0.403	0.395	0.405	0.384	0.4	0.509	0.495	0.381	0.396	0.394	0.41	0.436	0.442	0.444	0.444	6.13	
ET	336	0.426	0.433	0.429	0.438	0.437	0.443	0.424	0.433	0.626	0.558	0.418	0.428	0.471	0.468	0.508	0.496	0.472	0.478	4.82	
	720	0.427	0.445	0.43	0.457	0.429	0.451	0.429	0.446	0.862	0.671	0.423	0.442	0.439	0.451	0.478	0.488	0.488	0.492	4.11	
	96	0.352	0.379	0.357	0.375	0.354	0.375	0.341	0.377	0.349	0.376	0.323	0.36	0.336	0.377	0.373	0.42	0.516	0.484	0.63	
'n	192	0.388	0.394	0.394	0.393	0.392	0.393	0.382	0.396	0.387	0.395	0.36	0.382	0.389	0.402	0.407	0.434	0.545	0.498	0.73	
ETT	336	0.419	0.415	0.425	0.414	0.425	0.415	0.417	0.417	0.417	0.417	0.393	0.403	0.42	0.421	0.447	0.46	0.655	0.544	1.28	
	720	0.482	0.448	0.488	0.449	0.486	0.449	0.487	0.457	0.475	0.452	0.456	0.44	0.482	0.46	0.508	0.488	0.706	0.573	0.97	
	96	0 182	0 263	0 185	0.267	0 182	0 266	0 184	0.269	0 198	0 297	0 176	0 259	0 188	0.268	0 1 8 9	0.282	0.268	0 320	0.38	
2	102	0.102	0.203	0.105	0.207	0.102	0.200	0.253	0.20	0.190	0.297	0.170	0.209	0.100	0.200	0.109	0.202	0.200	0.322	0.50	
ETTn	226	0.245	0.305	0.240	0.300	0.247	0.303	0.235	0.314	0.200	0.304	0.241	0.301	0.231	0.300	0.200	0.325	0.274	0.352	1 20	
	720	0.305	0.34	0.300	0.342	0.307	0.342	0.313	0.331	0.59	0.432	0.302	0.341	0.310	0.345	0.327	0.305	0.355	0.307	3.00	
	120	0.400	0.591	0.410	0.400	0.417	0.400	0.410	0.407	0.50	0.520	0.41	0.404	0.425	0.407	0.435	0.425	0.450	0.44		
	96	0.169	0.216	0.197	0.237	0.196	0.236	0.187	0.227	0.199	0.258	0.174	0.216	0.169	0.219	0.243	0.326	0.294	0.36	0.36	
'eathe	192	0.215	0.255	0.244	0.273	0.241	0.272	0.234	0.266	0.239	0.297	0.22	0.256	0.225	0.264	0.28	0.341	0.289	0.348	0.48	
*	336	0.271	0.296	0.293	0.308	0.293	0.308	0.288	0.305	0.285	0.334	0.275	0.295	0.282	0.304	0.37	0.409	0.363	0.403	0.43	
	720	0.351	0.347	0.368	0.357	0.366	0.355	0.363	0.354	0.349	0.385	0.352	0.346	0.359	0.355	0.403	0.419	0.426	0.43	1.06	
	96	0.203	0.275	0.21	0.28	0.207	0.28	0.171	0.261	0.225	0.316	0.169	0.255	0.211	0.31	0.187	0.302	0.202	0.317	0.32	
tricity	192	0.199	0.275	0.206	0.282	0.201	0.282	0.182	0.271	0.225	0.319	0.177	0.263	0.251	0.334	0.196	0.309	0.224	0.333	0.34	
Elec	336	0.216	0.29	0.222	0.295	0.225	0.297	0.2	0.289	0.237	0.333	0.193	0.28	0.264	0.346	0.213	0.326	0.231	0.336	0.34	
	720	0.247	0.321	0.26	0.328	0.256	0.329	0.243	0.324	0.272	0.363	0.233	0.313	0.298	0.37	0.24	0.35	0.292	0.39	0.39	