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Novel Path Following for a Four-Wheel Steering Vehicle Based On Model Predictive Control

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***Abstract*— Path following is a crucial technique for ensuring the safe and efficient operation of automatic electric vehicles. Four-wheel steering (4WS) technology is known to enhance the accuracy and flexibility of such vehicles. In this paper, we propose a new constrained model predictive control (MPC) based method for path-following, specifically for 4WS vehicles. To simplify the 4WS vehicle kinematics model, we use the assumption of pure rolling and simplify it to a single-track model. We employ a high-precision linearization transformation to convert the non-linear kinematics models to a linear control-state system. Subsequently, we design a new objective function based on the tracking error model, and formulate the control problem as an optimization problem. Finally, we convert the optimization problem into a quadratic programming (QP) form with constraints that are suitable for real-time applications. We demonstrate the effectiveness of our proposed control method through simulation experiments.**

I. Introduction

Four-wheel steering (4WS) vehicles are frequently used in horizontal transportation. 4WS vehicles with automatic driving systems will greatly reduce labor costs and increase productivity. Path following is the rudimentary capability and primary task for autonomous 4WS vehicles as shown in Figure 1. The designed path-following controller aims to minimize the tracking errors between the target path and the actual path.

Numerous studies have been carried out on the control methods of path tracking. Model predictive control (MPC) [1], [2] is the prominent model based method, which could provide more stable and precise path following performance in high speed scenarios. Most MPC based methods build up the control pipeline based on the kinematics models [3], which are nonlinear. Therefore, the primary issue of MPC is time consuming and many researches use linearization to solve this issue [4]–[7].

However, the majority of these linearization techniques have their limitations and will drastically lower control precision [8], causing the vehicle to veer from the intended path [9]. In this paper, we completely take into account the multi-freedom control of the 4WS vehicle and examine the maximum front and rear wheel angles. We employ ordinary differential equation of the kinematics model and express this

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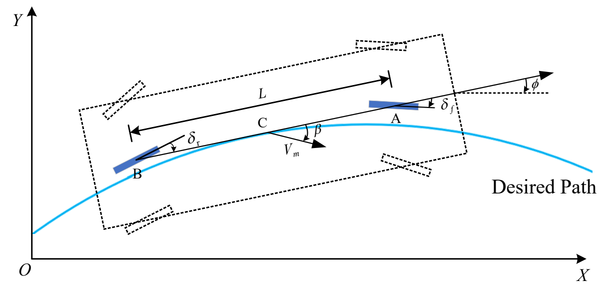

Fig. 1. Schematic diagram of relevant variables of 4WS vehicle kinematics model.

equation with forward euler discretization. Finally, a highly accurate linear kinematics model is obtained.

Moreover, since MPC-based approaches convert the control problem into an optimization problem and the path-following error is the primary factor in the objective function, present methods solely concentrate on this error [10]. When creating the objective function for the optimization problem, the output control's smoothness is neglected. Smoothness is crucial in practical application, particularly in high-speed circumstances. We add two additional restrictions to the objective function in order to smooth the control output. The first constraint is added to prevent the projected control sequence from changing too much in a single frame, while the second constraint is made to smooth the control output over time.

The main contributions of our paper are as follows:

- We propose a high precision linearization transformation, which transforms the non-linear kinematics models into a linear control-state system.
- We introduce a new objective function based on the tracking error model, which balances the path-following precision and control output smoothness.

This paper is structured as follows. In Section II, the related work of control method for 4WS vehicle is introduced. Then, we present our proposed method in Section III. The simulation experiment design and result analysis will be shown in Section IV. Finally the conclusion of our work is drawn in Section V

II. Related Work

The control method could mainly be dived into two categories. The first one is the geometry-based method, which mainly include pure pursuit (PP) [11] and Stanley [12]. The other one is model based including linear quadratic regulator

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(LQR) [13], [14], synovial membrane control [15], [16] and model predictive control [1], [2].

A. Geometry Based Control Method

Geometry-based control methods are often used in low-speed scenarios, with good interpretability and fast calculation speed. [17] were the first to develop pure pursuit strategy in the field of robotics, they developed a method for estimating the steering necessary to maintain the vehicle on the road. [18], [19] presented a work on the control of non-holonomic autonomous ground vehicle as it tracks a given path.

B. Model Based Control Method

Model-based methods could provides more stable and precise path-following performance, which transforms the control issue into a optimization issue. Compared with other methods, MPC could take physical constraints into account in optimization problems easily. Most of current researches utilize linear MPC based method because of its fast calculation speed. [4] and [5] use the Linear Model Predictive Control to achieve the real-time performance. A linear MPC is designed in [20] and [21] for active front steering control design on slippery surfaces. All the methods mentioned above is designed for front-wheel vehicle which is straightforward to derive the linear formulation. As for the 4WS vehicle, only few researches utilizes linear MPC. [6] synthesized a new controller for dynamic path tracking by using constrained model predictive control for 4WS vehicles, which takes into account steering and sliding constraints to ensure safety and lateral stability. [7] build the nonlinear dynamical model based on the nonlinear Dugoff tire model, the nonlinear dynamical model is simplified as a linear model in controller design. The Linearization method in these two methods causes drift in path-following. Compared with linear MPC, non-linear MPC is much more time-consuming to solve the optimization issue which is a non-convex optimisation. Nonlinear constrained MPC is developed in [22] for the stabilization of the kinematic model of a two-wheel mobile robot with actuator saturations and state constraints. In their research, the expense and reliability of the non-convex optimisation causes problems and tuning of the controller parameters appears to be decisive.

III. Method

This section outlines the proposed path tracking control framework, which is based on the Model Predictive Control (MPC) approach. Initially, the kinematic model of the 4-wheel steering (4WS) vehicle will be introduced. Next, a linear control-state model will be established to demonstrate the relationship between the vehicle's state variables and control variables. Finally, a new objective function will be designed to generate the optimal control output.

A. 4WS Kinematics Model

The kinematics model is the basis of trajectory planning and control system. To reduce the complexity of the controller design, the 4WS vehicle kinematics model can be simplified to a single-track model with the assumption of pure rolling as shown in Figure 1. The Point A is the center of two front wheels, and a virtual wheel in point A is used to replace the two front wheels. The Point B is the center of two back wheels. The point C(X,Y) is the geometric center of the vehicle.Then the following nonlinear kinematics model can be expressed as:

$$\begin{cases} \bar{X} = V_x = v_m cos(\phi + \beta) \\ \bar{Y} = V_y = v_m sin(\phi + \beta) \\ \bar{v_m} = a \\ \bar{\phi} = \omega = \frac{v_m cos(\beta)}{L}(tan(\delta_f) - tan(\delta_r)) \end{cases} \tag{1}$$

and the β is equal to:

$$\beta = arc(\frac{tan(\delta_f) + tan(\delta_r)}{2})$$

where X,Y and ϕ is the vehicle location and orientation in global coordinate, $v_m$is the vehicle speed a is the acceleration. δ_f and δ_r are the front and rear wheel angle in the vehicle coordinate, L is the length between front wheel and rear wheel. Eq.(1) can be applied in both forward and backward scenarios, therefore the speed can be positive or negative.

B. Vehicle Model Linearization

Since the derived equation above is a non-linear system, it's time consuming to solve it. It's necessary to linearize the kinematics model. The ordinary differential equation of the kinematics model is:

$$\dot{z} = \frac{\partial}{\partial z} z = f(z, u) = A'z + B'u \tag{2}$$

where

$$z = [X, Y, v_m, \phi]^T \tag{3}$$

$$u = [a, \delta_f, \delta_r]^T \tag{4}$$

$$A^{'} = \begin{bmatrix} 0 & 0 & cos(\phi + \beta) & -v_m sin(\phi + \beta) \\ 0 & 0 & sin(\phi + \beta) & -v_m cos(\phi + \beta) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{cos(\beta)}{L}(tan(\delta_f) - tan(\delta_r)) & 0 \end{bmatrix}$$

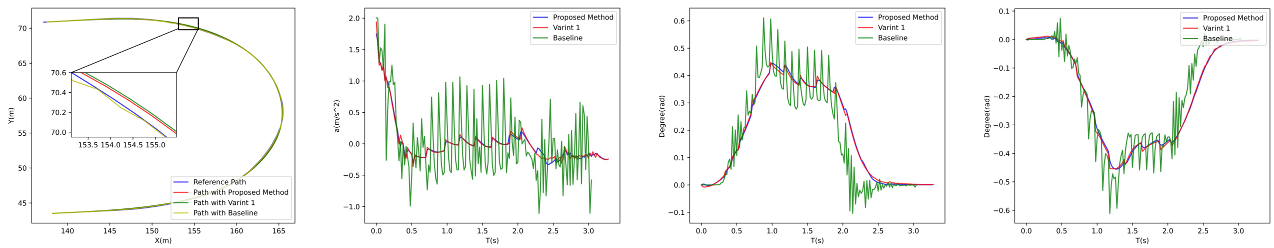
$$B^{'} = \begin{bmatrix} 0 & -v_m sin(\phi + \beta)\frac{\partial \beta}{\partial \delta_f} & -v_m sin(\phi + \beta)\frac{\partial \beta}{\partial \delta_r} \\ 0 & v_m cos(\phi + \beta)\frac{\partial \beta}{\partial \delta_f} & v_m cos(\phi + \beta)\frac{\partial \beta}{\partial \delta_r} \\ 1 & 0 & 0 \\ 0 & \frac{\partial}{\partial \delta_f}\omega & \frac{\partial}{\partial \delta_r}\omega \end{bmatrix}$$

where

$$\frac{\partial \beta}{\partial \delta_f} = \frac{1}{1 + (\frac{tan(\delta_f)+tan(\delta_r)}{2})^2} \frac{1}{2(cos(\delta_f))^2}$$

$$\frac{\partial \beta}{\partial \delta_r} = \frac{1}{1 + (\frac{tan(\delta_f)+tan(\delta_r)}{2})^2} \frac{1}{2(cos(\delta_r))^2}$$

$$\begin{aligned} \frac{\partial \omega}{\partial \delta_f} =& \frac{-v_m sin(\beta)}{L} \frac{\partial \beta}{\partial \delta_f}(tan(\delta_f) - tan(\delta_r)) \\ &+ \frac{v_m cos(\beta)}{L} \frac{1}{(cos(\delta_f))^2} \end{aligned}$$



(a) (b) (c) (d)

Fig. 2. The results of U-Turn path following for different cost functions. (a) Path following result. (b) Acceleration control output. (c) Front wheel control output. (d) Back wheel control output.

$$\frac{\partial \omega}{\partial \delta_r} = \frac{-v_m sin(\beta)}{L}\frac{\partial \beta}{\partial \delta_r}(tan(\delta_f) - tan(\delta_r)) - \frac{v_m cos(\beta)}{L}\frac{1}{(cos(\delta_r))^2}$$

A discrete-time mode with Forward Euler Discretization with sampling time dt coulld be expressed as:

$$z_{k+1} = z_k + f(z_k, u_k)dt \tag{5}$$

Using first degree Tayer expansion around z and u

$$z_{k+1} = z_k + (f(\bar{z}, \bar{u}) + A^{'} z_k + B^{'} u_k - A^{'} \bar{z} - B^{'} \bar{u})dt \tag{6}$$

the final formulation is:

$$z_{k+1} = Az_k + Bu_k + C \tag{7}$$

where

$$\begin{cases} A = (I + dtA^{'}) \\ B = dtB^{'} \\ C = (f(\bar{z}, \bar{u}) - A^{'}\bar{z} - B^{'}\bar{u})dt \end{cases} \tag{8}$$

C. Controller Design

Our controller is designed based on model predictive control, Eq.(3) is the state vector of the control system and Eq.(4) is the control vector of the control system. We designs a new objective function to make sure the path following function works well meanwhile the control output is smooth. The objective function is designed as:

$$min(C_{ref} + C_{acce.} + C_{smooth} + C_{variance}) \tag{9}$$

where $C_{ref} := \Sigma_{k=0}^{T+1}(z_k - z_{kr})^T Q(z_k - z_{kr})$, which is designed to make sure the vehicle could follow the reference way-point; $C_{acce.} := \Sigma_{k=0}^{T}(u_k)^T R_1(u_k)$, which is to prevent unnecessary large acceleration and large wheel angle.

Compared with previous researches, we pay more attention to the smoothness of the output control signals and add $C_{smooth}, C_{variance}$ in the cost function. The $C_{smooth} := \Sigma_{k=0}^{T-1}(u_k - uk + 1)^T R_2(u_k - uk + 1)$ focus on outputting a smooth predicted control in the current calculation and the $C_{variance} := (u_0 - u_{-1})^T R_3(u_0 - u_{-1}))$ prevent excessive variance of the first-frame predicted control between current calculation and last calculation. The entire cost function makes sure 4WS could follow the reference way-point, and the predicted control is smooth in each frame. Note that the z_k is the predicted state in frame k and z_{kr} is the reference state in frame k and u_k is the predicted control output in frame k, u_{-1} is the predicted control output of the first frame in last calculation. Q is the state cost matrix and R_1, R_2, R_3 are control cost matrix.

However the above optimization problem is time consuming and could not be solved in real time. With utilizing the linearization vehicle model, this optimization problem could be transformed to a quadratic programming issue:

$$min\frac{1}{2}X^T PX + q^T X \qquad \text{subject to } l \leq A_c X \leq u \tag{10}$$

where

$$X = \begin{bmatrix} z_0 & z_1 & \cdots & z_{T+1} & u_0 & u_1 & \cdots & u_T \end{bmatrix}^T$$

After QP transformation, it's straightforward to get the optimal control result utilizing OSQP solver [23]

IV. Experiments

A. Experiment Design

To verify the robustness of our cost function, we design a experiment using three different optimization cost functions to follow the provided way points list. Firstly, the baseline setting only employs the C_{ref} and $C_{acce.}$ in cost function. Secondly, our proposed method employs all the constrains in cost function. To validate the effectiveness of proposed $C_{variance}$, we add $C_{variance}$ in the baseline setting, called variant 1.

All of these experiments are validated in the simulation environment. We generate two paths to validate the path following function. The first path is the U-Turn path that is shown in Figure 2. The other one is the lane change path, which is presented in Figure 3. Owing to both of these two paths mentioned above are the most common scenarios in our daily driving environment, our algorithm could be verified without losing the generalization.

B. Experiment Analysis

Figure 2 illustrates the comparison results of the cost functions used for U-Turn path tracking. Specifically, Figure 2(a) shows the path tracking performance for each cost function, and Table I lists the maximum and mean lateral errors and amplitudes. The baseline cost function achieves the minimal lateral error and our proposed method only enlarge it about 1 cm. However, according to the green

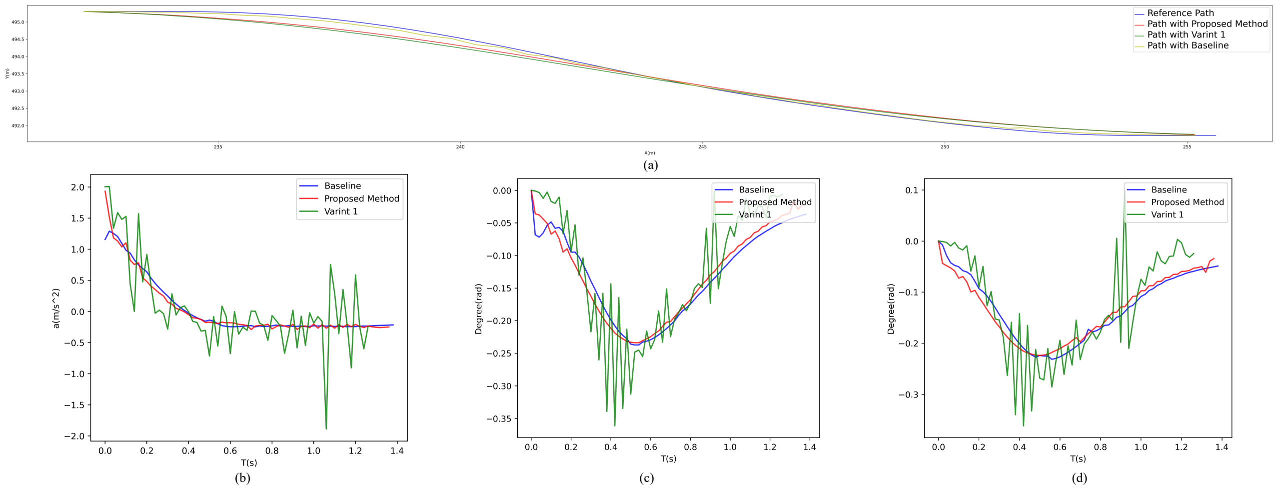


Fig. 3. The results of lane-change path following for different cost functions. (a) Path following result. (b) Acceleration control output. (c) Front wheel control output. (d) Back wheel control output.

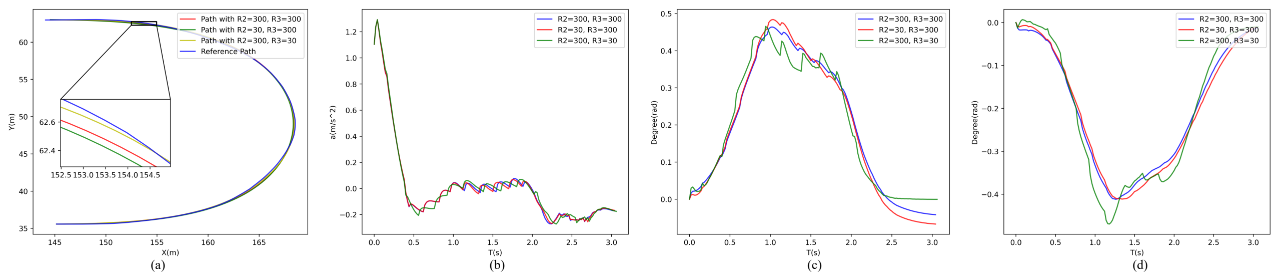


Fig. 4. The results of U-Turn path following for different R_2, R_3 settings. (a) Path following result. (b) Acceleration control output. (c) Front wheel control output. (d) Back wheel control output.

line in Figure 2(b)-(d), the control output of the baseline cost function changes dramatically, which will cause the wheels to vibrate violently. In contrast, our proposed method offers a better trade-off between path-following precision and control output smoothness. Moreover, the comparison with variant 1 cost function confirms that introducing $C_{variance}$ can improve path-following precision while also enhancing the smoothness of the results. These findings hold for lane-change path tracking as well, as demonstrated in Figure 3 and Table I.

To further evaluate the impact of different variables, we conducted experiments with various setting combinations. We used the same cost factor for acceleration in all experiments, as well as the same cost factor for both front and back wheels. The results of path following for the U-Turn path are shown in Figure 4, and the maximum and mean lateral errors are listed in Table II. The combination of (R_2=300, R_3=30) achieved the minimal lateral error, while the combination of (R_2=30, R_3=300) had the maximal lateral error. Moreover, Figure 4(b)-(d) indicate that the combination of (R_2=300, R_3=300) generated smoother control output than the combination of (R_2=300, R_3=30). Consistent with the results presented in Figure 5 and Table II, we concluded that increasing R_2 enhances path-following precision and generates smoother results, while the effect of R_3 is a trade-off between precision and smoothness. These findings apply to lane-change path verification as well.

TABLE I

RESULTS OF DIFFERENT METHODS IN U-TURN/LANE-CHANGE PATH FOLLOWING.

Setting	**Max lateral Error**(m) ↓		**Mean lateral Error**(m) ↓		**Amplitude** ↓	
	U-Turn	Lane-change	U-Turn	Lane-change	U-Turn	Lane-change
Baseline	0.236	0.198	0.062	0.082	13.04	12.49
Variant 1	0.266	0.356	0.082	0.128	2.69	1.17
Proposed method	0.246	0.334	0.078	0.135	**2.11**	**0.97**

TABLE II

LATERAL ERROR FOR DIFFERENT METHODS IN U-TURN/LANE-CHANGE PATH FOLLOWING.

Setting	**Max Lateral Error**(m)↓		**Mean Lateral Error**(m)↓	
	U-Turn	Lane-change	U-Turn	Lane-change
R_2=300 R_3=300	0.234	0.384	0.08	0.14
R_2=30 R_3=300	0.276	0.381	0.09	0.18
R_2=300 R_3=30	0.165	0.184	0.06	0.09

V. CONCLUSIONS

In this paper, we propose a novel model predictive control (MPC) based control system for four-steering wheel (4WS) vehicles. To address the non-linearity of the kinematics

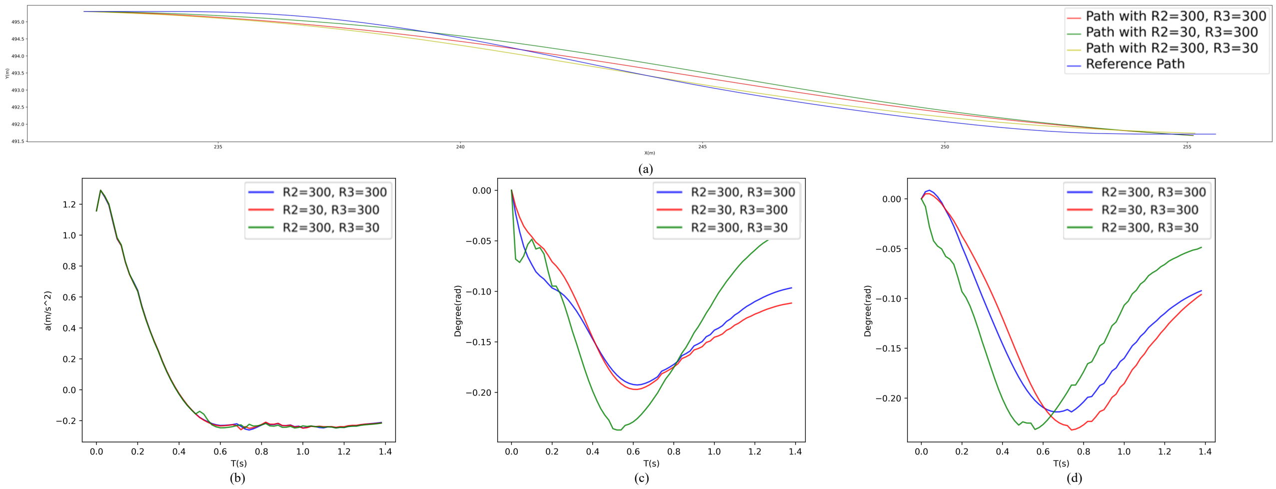


Fig. 5. The results of Lane-Change path following for different R_2, R_3 settings. (a) Path following result. (b) Acceleration control output. (c) Front wheel control output. (d) Back wheel control output.

model, we introduce a high-precision linearization transformation that transforms the model into a linear control-state formulation. Based on this formulation, we design a new objective function that incorporates a tracking error model, and formulate the control problem as an optimization problem that can be efficiently solved using quadratic programming (QP). Our simulation results demonstrate the robustness and effectiveness of our proposed method. As future work, we plan to investigate the impact of wheel sliding on the control system and explore ways to make the algorithm more realistic and adaptable to real-world scenarios.

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