LEARNING BY CAUSALITY TO IMPROVE CHANNEL DE-PENDENCY MODELING IN MULTIVARIATE TIME SERIES FORECASTING

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ABSTRACT

Beyond the conventional long-term temporal dependency modeling, multivariate time series (MTS) forecasting has rapidly shifted toward channel dependency (CD) modeling. This shift significantly improves modeling quality by fully leveraging both multivariate relationships and temporal dependencies. Recent methods primarily model channel dependency through correlation learning (e.g., cross-attention) or non-trainable statistical techniques (e.g., cross-correlation), however, these approaches are insufficient to fully capture the intrinsic relationships within MTS, particularly those stemming from directed cause-effect (i.e., causality) and nonstationary variates originating from diverse sources. In addition, causality may arise in MTS with different temporal behaviors, such as varying periodicity or discrete event sequences, which remains underexplored. In this paper, we propose CALAS (Causality-enhanced Attention with Learnable and Adaptive Spacing), the first end-to-end learning method for MTS forecasting that uncovers causality among variates without relying on statistical measures or prior knowledge. To model underlying causality, which consists of causal strength and propagation delay, we newly design a hypernetworks-based 1D convolutions mechanism. Inspired by dilated convolution with learnable spacings (DCLS) and spiking neural networks (SNNs), we extend discrete time delay into a continuous Gaussian kernel. Combining the hypernetworks-generated Gaussian kernel and convolutional weights (i.e., attention or causal strength), we achieve the end-to-end dynamic causality modeling mechanism, improving the prediction accuracy, quality, and interpretability of both MTS forecasting results and models. For evaluation, we conduct extensive experiments with six real-world datasets and qualitative analysis to demonstrate CALAS's superiority in capturing varying causality in a data-agnostic manner. The experiment results indicate that CALAS has significantly improved MTS forecasting accuracy compared to state-of-the-art methods by dynamically modeling causality among variates.

1 Introduction

Multivariate time series (MTS) forecasting is one of the core problems in time series analysis, widespread in various domains such as traffic (Li & Shahabi, 2018; Lee et al., 2020; Wu et al., 2020), finance (Zeng et al., 2023; Zhu & Shasha, 2002), and weather (Matsubara et al., 2014; Wu et al., 2021). The fundamental idea of MTS forecasting methods is to build a mapping function from input (i.e., historical) to output (i.e., future) MTS by learning inherent dependencies of MTS. To model the intrinsic dependencies of MTS, the main flow of MTS forecasting is evolved into two distinguished categories – temporal dependency modeling and channel (or, variate) dependency modeling.

For temporal dependency modeling, researchers have proposed novel deep learning architectures, ranging from RNN-based models (Li et al., 2018; Jiang et al., 2023; Zonoozi et al., 2018) to Transformer-based architectures (Vaswani et al., 2017; Zhou et al., 2021; Liu et al., 2022), or have modified existing model for improved dependency modeling (Bai et al., 2020; Wu et al., 2021; Zhou et al., 2022). While these advances have significantly improved MTS forecasting quality, they often fail to capture important temporal characteristics, such as periodicity (Oreshkin et al., 2020; Wu et al., 2021; Zeng et al., 2023). To address this issue, some approaches incorporate prior knowledge

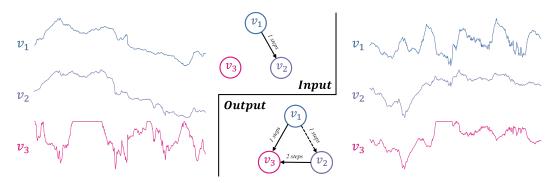


Figure 1: Distribution shift over lead-lag relationships in the Weather dataset (1,3, and 4-th variates in the dataset). (Left) In the historical data from time t-T+1 to t-1, two variates, v_1 and v_2 , exhibit a strong causality, while v_3 has no causal relation with v_1 and v_2 (top causal graph). (Right) In the consecutive output signals, the relationship between v_1 and v_2 weakens, while both v_1 and v_2 newly establish a strong causal relation with v_3 , invoking need for a layer-agnostic causality modeling.

from time series analysis, such as time series decomposition (Wu et al., 2021; Zeng et al., 2023; Zhou et al., 2022; Nie et al., 2023), or integrate statistical methods, including correlation and Fourier transformation (Zhou et al., 2022; Wu et al., 2023; Yi et al., 2023b), into their models. Nie et al. (2023) have recently introduced a patch-based input representation that enables models to learn semantic information directly without prior knowledge. Additionally, several methods have been developed to handle non-stationary behavior and distribution shifts frequently observed in real-world MTS data (Kim et al., 2022; Zhou et al., 2023).

On the other hand, a line of research has focused on modeling channel dependencies (CDs) in MTS, named channel dependency (CD) modeling. Their strategies can be categorized into two main approaches: correlation-based and causality-based. In correlation-based methods, CD are typically captured using advanced neural architectures, such as GNNs (Guo et al., 2019; Wu et al., 2019; Li et al., 2018), CNNs (Wu et al., 2023; Zeng et al., 2024; donghao & wang xue, 2024), and Attention mechanisms (Zhang & Yan, 2023; Liu et al., 2023; Zhou et al., 2021; Wang et al., 2024), often incorporating Fourier transformation and multi-periodicity decomposition (Wu et al., 2023; Yi et al., 2023a). Recently, some researchers have argued that CDs in MTS are inherently directional, meaning they involve cause-and-effect relationships (i.e., causality) and cannot be fully understood through correlation-based, non-directional models (Wu et al., 2020; Li & Shahabi, 2018; Zhao & Shen, 2024; Lee et al., 2022). Certain GNN-based methods attempt to learn cause-effect relationships using node embeddings; however, they struggle to fully capture the dynamic nature of causality (Wu et al., 2020). Recently, Zhao & Shen (2024) have proposed a novel method that leverages causality extracted from cross-correlation to align output signals with propagation delay along with corresponding leading indicators. While this approach addresses dynamic changes in causality, questions remain about the capacity of cross-correlation and sensitivity to hyperparameters (e.g., the maximum number of leading indicators). These issues highlights two ongoing challenges in adopting causality modeling in MTS: 1) lack of end-to-end, learning-based dynamic causality modeling methods; and 2) limitation of prior knowledge or statistical methods in fully leveraging intrinsic causality in MTS.

Figure 1 presents a motivating example from the Weather dataset, where local causality changes in consecutive signals, highlighting the need for dynamic causality modeling. It also emphasizes a need for layer-agnostic causality modeling methods that can be applied in any level of input, intermediate, or output layers, since causal structure may changed even within model forwarding. Additionally, it suggests a potential side-effect of statistic-based CD modeling approaches, such as cross-correlation (e.g., LIFT (Zhao & Shen, 2024)), which could mislead models to focus on less relevant variates.

Moreover, correlation-based methods often struggle with long-term MTS, especially in signals with strong short-term periodicity. To further describe this phenomena, we present Figure 2 that shows synthetic signals with a periodic seasonal pattern, sinusoidal trends, and varying noise level. While all three signals shares the same seasonal pattern, v_2 and v_3 exhibit a stronger causal relationship than others. Nevertheless, statistical methods tend to emphasize the seasonal patterns, leading to errors in identifying the correct lead-lag relationship, as illustrated in the bottom graph of Figure 2.

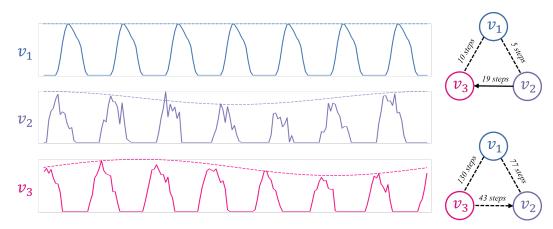


Figure 2: A illustration of long-term lead-lag relationships over the synthetic signal with seasonality and trend. Dashed line in time series shows trends. The top-right graph is the actual causality graph during data generation, while the bottom-right one is causality graph calculated with cross-correlation. In causality graph, dashed line means weak (or possibly undirected) relationship. There are large misalignment on the lead-lag steps.

In this work, we propose a novel end-to-end causality modeling method named CALAS¹ (Causalityenhanced Attention with Learnable and Adaptive Spacing) that converts correlation-based attention into causality by incorporating dilated convolution with learnable spacing (DCLS) (Hassani et al., 2023; Khalfaoui-Hassani et al., 2023) and hypernetworks (Ha et al., 2017) in a novel way. CALAS consists of three components: two hypernetworks and one convolutional neural network. To uncover the dynamic causal relations among MTS while considering its propagation delay, we extend the static delay modeling with the Gaussian kernel by introducing input-based dynamic delay modeling. Specifically, we adopt the hypernetworks–neural networks that generate the weights for another network—to model both the dynamic causal strength and propagation delay (i.e., causality map). We utilize two hypernetworks: one for estimating the Gaussian kernel, which serves as the time lag (or delay) of causality, and the other to measure the strength of the causality. After generating the weights, CALAS applies the resulting causality map directly through the convolutional architecture. In addition to enabling flexible causality learning beyond the methods relying on prior knowledge, our design enhances interpretability by separating causality map into causal strength and propagation delay. Furthermore, unlike the other causality-based CD models, CALAS can learn causality in input space, output space, or both, offering greater flexibility in integrating CALAS with other causality modeling methods, which will be discussed in the experimental results.

The main contributions of this paper can be summarized as follows.

- We introduce a new perspective on converting the correlation-based attention score into causality-based relationships by decomposing the causality map into learnable delay and causal strength.
- We design a novel architecture, CALAS, that enables the end-to-end learning of dynamic cause-and-effect relationships among variates for MTS forecasting. By design, CALAS can be applied in a model-agnostic way at any stage of the MTS forecasting models, including input and output space. To the best of our knowledge, CALAS is the first work to model causality for MTS forecasting in an end-to-end manner.
- Experimentally, we demonstrate that CALAS significantly improves the state-of-the-art methods. Specifically, our model improves the existing models by 7.61% on average, outperforming existing methods (e.g., LIFT) by 2.21 percentage points. We highlight that the improvements become larger for long-term forecasting, 9.45% on average. We have also revealed that CALAS can be integrated with other causality-based plug-and-play modules, such as LIFT, resulting in substantial performance gains.

¹Github link for source code will be provided.

2 RELATED WORK

MTS forecasting could be categorized into two research theme: temporal dependency modeling without inter-variate dependencies (i.e., channel-independent methods) and channel dependency (CD) modeling methods. Since our paper aims to solve CD modeling problem, we briefly introduce CI methods and move on to CD methods.

In MTS forecasting, CI methods are well-studied and are still on-going research directions, ranging from statistical models (e.g., ARIMA (Box et al., 2015), hidden Markov model (Pesaran et al., 2004)) to recent deep learning models (Liu et al., 2023; Zhao & Shen, 2024). Recent models are focusing on the long-term dependency modeling by leveraging the traditional studies, such as Fourier transformation (Zhou et al., 2022; Yi et al., 2023b; Xu et al., 2024) or by proposing novel input representation methods (Nie et al., 2023; Wu et al., 2021; Zeng et al., 2023; Hu et al., 2024; Jia et al., 2023). Those methods contribute to MTS forecasting domain by deeply understanding in-nature characteristics of time series (i.e., temporal dependency).

On the other hands, CD methods aim to uncover the intrinsic channel-wise dependency. Based on the methodology and assumption for channel dependency, CD methods can be divided into four categories: 1) model-based dynamic correlation learning (Learnable Correlation in Table 1), 2) model-based static causality learning (Learnable Static Causality in Table 1), 3) statistics-based dynamic causality learning (Statistics-based Causality in Table 1), and 4) model-based dynamic causality learning (Ours in Table 1). For CD modeling, there are primarily two directions—learnable correlation with attention mechanism (Zheng et al., 2020; Vaswani et al., 2017; Behrouz et al., 2024; Zhang & Yan, 2023) and learnable static causality with graph neural networks (GNNs) and learnable embedding (Cao et al., 2020; Guo et al., 2019; Wu et al., 2020; 2019; Lee et al., 2022). While attention mechanism aims to reconstruct correlation map by projecting inputs into the feature space, GNN-based methods try to approximate underlying, time-independent causal structure with learnable node embedding. However, those approaches merely discuss the lead-lag relationship and propagation delay, which is crucial for the CD modeling.

Recently, a few researches (Wu et al., 2020; Zhao & Shen, 2024) depict that when two signals have cause-effect relationship, there exist a time-lag for 'cause' signal to influence the 'effect' signal, which we call propagation delay. To fully leverage the benefits of CD methods, it is necessary to consider propagation delay, since ignorance of propagation delay making the model refer the outdated information (Zhao & Shen, 2024). To the best of our knowledge, LIFT (Zhao & Shen, 2024) is the only work that explicitly aims to solve the propagation delay by leveraging cross-correlation, which is statistics-based causality in our categorization. However, overrelying the statistical measure introduces a number of limitations, such as the information bottleneck from the excessively large number of recurring patterns (Lee et al., 2022), possible misalignment, and need of reference signals that limit the utilization of model in various situation. We describe those properties in Table 1.

Table 1: Comparison of existing CD methods. Δ means the method partially satisfies the condition.

Methods	Learnable Correlation	Learnable Static Causality	Statistics-based Causality	Ours (CALAS)
Delay-awareness	×	×	✓	√
Directed	×	✓	Δ	✓
Dynamic	✓	×	✓	✓
Learnable	✓	✓	×	✓
Example Methods	Attention (Vaswani et al., 2017)	GNNs (Wu et al., 2020)	LIFT (Zhao & Shen, 2024)	None (Ours)

3 Preliminaries

Let $\mathbf{x}_t \in \mathbb{R}^N$ denote a multivariate time series (MTS) with N variates (i.e., channels) at a time step t. Given T-length historical observation $\mathbf{X}_t = \{\mathbf{x}_{t-T+1}, \dots, \mathbf{x}_t\} \in \mathbb{R}^{T \times N}$ at time t, MTS forecasting aims to predict corresponding future L-length MTS $\mathbf{Y}_t = \{\mathbf{x}_t, \dots, \mathbf{x}_{t+L}\} \in \mathbb{R}^{L \times N}$. In the remaining sections, we denote the i-th variate of MTS as $\mathbf{X}^{(i)} \in \mathbb{R}^T$, and each point at time t as $\mathbf{x}_t^{(i)}$. Formally, MTS forecasting could be described as below:

Definition 3.1. Given MTS data \mathbf{X}_t , MTS forecasting aims to approximate the model $f(\cdot, \theta)$: $\mathbb{R}^{T \times N} \to \mathbb{R}^{L \times N}$ by minimizing $L(f(\mathbf{X}_t, \theta), \mathbf{Y}_t)$.

For the channel-independent (CI) models, $f(\cdot,\theta)$ shares the same parameter for all variates, enabling the universal representation learning for MTS, on the other hand, it merely models the channel dependency that is crucial in MTS forecasting. It therefore focuses more on the temporal dependency modeling.

Given MTS \mathbf{X}_t at time t, we assume that there is unknown dynamic causal graph \mathbf{A}_t , where each element (i.e., relationship from i-th variate to j-th variate) is defined as $a_{i,j} = \{w_{i,j}, \delta_{i,j}\} \in \mathbb{R} \times \mathbb{N}$. $w_{i,j}$ means the strength of causality and $\delta_{i,j}$ means the discrete delay. In addition to the temporal dependency, Channel dependency (CD) modeling methods aim to approximate the causal graph \mathbf{A}_t as below:

Definition 3.2. Channel dependency modeling aims to approximate $f(\cdot, \theta | \tilde{A}_t)$ that minimize $L(f(\mathbf{X}_t, \theta | \tilde{A}_t), \mathbf{Y})$, where \tilde{A}_t is approximation of causal graph \mathbf{A}_t with learnable parameter θ_A such that $\tilde{A}_t = g(\theta_A)$ or $\tilde{A}_t = g(\mathbf{X}, \theta_A)$.

3.1 Detailed Categorization for Channel Dependency Modeling

From the literature review, we have identified two distinctions in CD methods: the perspective on temporal variation and the type of relationship. Regarding temporal variation, CD modeling can be classified as either static or dynamic. Static CD modeling assumes that the underlying causal structure remains constant over time (i.e., $\mathbf{A}_t = \mathbf{A}_{t'} \forall t, t'$), thus avoiding instantaneous effects or potential misleading caused by anomalous values. This approach is widely used in early traffic forecasting (Wu et al., 2020; 2019; Li et al., 2018; Lee et al., 2022) and causal discovery (Cheng et al., 2023; 2024). In contrast, dynamic CD modeling assumes that CD can change over time. It is a dominant research trend in long-term MTS forecasting, including Crossformer (Zhang & Yan, 2023) and LIFT (Zhao & Shen, 2024). Dynamic CD modeling offers the advantage of adjusting the strength or nature of dependencies and also help mitigate distribution shifts in channel dependency (Zhao & Shen, 2024; Jiang et al., 2023).

Additionally, CD can be categorized into two types of relationships: undirected and directed. Correlation, an undirected relationship, is a traditional approach to modeling time series data. Transformer-based models are a representative example,utilizing self-attention mechanism to model channel dependencies. Recent patch-based models (Nie et al., 2023; Zhang & Yan, 2023) or inverted embedding methods (Liu et al., 2023) also fall under correlation-based model due to their input representations. On the other hand, there are a few researches focus on directed causal relation, using GNNs (Wu et al., 2020; Jiang et al., 2023) or statistical methods (Zhao & Shen, 2024). Directed CD methods explicitly target lead-lag, propagation, or causality as their objective, often relying on either static structure learning (Wu et al., 2020) or statistical methods, such as cross-correlation (Zhao & Shen, 2024). Moreover, there is no literature that explicitly learns the delay in MTS, limiting the interpretability.

4 METHODS

In this section, we introduce proposed methods, CALAS, a end-to-end dynamic causality modeling methods for MTS forecasting. Before we detail CALAS, we revisit the Granger causality and causal discovery in real-world MTS data to discuss the gap between the real-world MTS and causal discovery. We then explain decomposition of the causality graph into strength and propagation delay. It disentangles the causal strength and time-lag for propagation, enabling in-depth analysis over channel dependencies. Afterwards, we explain the propagation of causal effects in terms of 1D convolution (Section 4.2). Lastly, we describe how CALAS achieves dynamic causality modeling with two hypernetworks (Section 4.3). Our model first generate the convolutional weights (Section 4.3) and then feed them into Gaussian kernel (Equation 4 in Section 4.2). Then, it applies generated causal graph with 1D convolution (Equation 3) and forward to next layer (in case of Figure 3a).

Motivations Even though many researches discuss the importance of **propagation delay** (Wu et al., 2020; Zhao & Shen, 2024; Jin et al., 2023), which means the time taken for a cause signal to actually influence the effect signal, there are less effort to explicitly model the discrete delay in MTS forecasting. This ignorance eventually induces the misleading of causality by introducing additional complexity in CD modeling. Recent research utilizes cross-correlation to find the lead-lag relationship and its propagation delay, however, it also relies on the causal strength rather than actually model

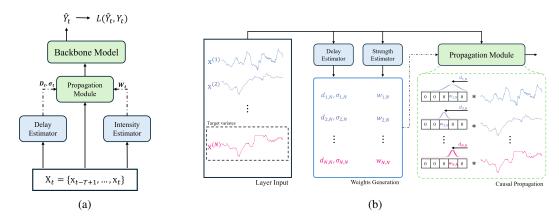


Figure 3: (a) CALAS when applied to input space and (b) an overview of CALAS forwarding path with respect to output channel N. Given input MTS (b, left), two hypernetworks of delay and causal strength estimators calculate propagation delay, standard deviation, and causal strength (b, middle). Standard deviation is utilized to convert discrete delay into continuous form. Finally, Gaussian 1D convolutional kernel is constructed and CALAS operates causal propagation with the propagation module (b, right). For the sake of simplicity, we assume that σ is sufficiently small, so we have an almost discrete kernel. The two blue modules are hypernetworks and dash-dotted line means supplying the generated parameters from the hypernetworks.

the discrete delay (Zhao & Shen, 2024). The importance of delay learning is also actively discussed and is demonstrated in spiking neural networks, which also solves the problem with propagation delays (Maass, 1997; Maass & Schmitt, 1999; Shrestha & Orchard, 2018; Hammouamri et al., 2024). Maass & Schmitt (1999) theoretically and experimentally demonstrated that k adjustable delays can compute a much richer class of functions than one with k adjustable weights. Motivated by those factors, we newly introduce hypernetworks-based 1D convolution mechanism that learns both dynamic causal strength and propagation delay in end-to-end manner.

4.1 CAUSALITY, GRANGER-CAUSES, AND CAUSAL DISCOVERY IN REAL-WORLD MTS DATA

Causality In the causal discovery domain, the causality is commonly based on several assumptions: Markovian condition, faithfulness, sufficiency, no instantaneous effect, and stationarity (Cheng et al., 2024). However, in the real-world MTS, it is hard to observe "all possible events and causes in the world," the sufficiency—all common causes of variables are observed—and stationarity—all the causal relationships remain constant throughout time— are hard to be hold. For the real-world MTS data, we need to assume that there exists latent variables or events that changes causality of observed variates. Therefore, in this paper, *causality* is based on the assumptions for Markovian condition, faithfulness, and sufficiency. We emphasize that *causality* in this paper is not necessarily be **true causality**, which have assumptions for no latent compounder and no unseen events.

Concepts of CALAS The basic idea of CALAS stems from the concepts of multivariate Granger causality (GC) analysis (Granger, 1969). Multivariate GC analysis is performed by fitting a VAR model with given maximum possible propagation delay k as below:

$$X_t = \sum_{\tau=1}^k W_\tau X_{t-\tau} + \epsilon(t),\tag{1}$$

where $W_{\tau} \in \mathbb{R}^{N \times N}$ is an estimated causal strength with propagation delay τ . Several approaches are proposed to resolve limitations on GC, such inability to capture non-linear causal relationship (Tank et al., 2022; Cheng et al., 2023). However, they stick to the static and individual causality measure for each time lag, disregarding the continuous property of MTS. To resolve these limitations, we set our design rationales as: 1) time lag (i.e., propagation delay) for two time series should be unique and be estimated with continuous function, 2) causal strength and propagation delay should be disentangled, and 3) the causality estimation should be time-dependent, more precisely, should handle latent compounder and unseen events.

4.2 Causal Propagation into Temporal Convolution

Given $\mathbf{X}^{(i)}$ and $\mathbf{X}^{(j)}$, let assume that they have $w_{i,j}$ lead-lag relationship with $d_{i,j}$ propagation delay. Let assume that we can derive $\mathbf{x}_t^{(i)}$ from a set of 'cause' signals. Then, we can formulate $\mathbf{x}_t^{(i)}$ as below:

$$\mathbf{x}_{t}^{(i)} = \sum_{j \in \mathcal{N}_{t}^{(i)}} w_{i,j} \mathbf{x}_{t-d_{i,j}}^{(j)},$$
(2)

where $\mathcal{N}_t^{(i)}$ is a set of 'cause' signals with respect to $\mathbf{x}_t^{(i)}$ at time t. In our paper, we assume that all variates could act as potential cause signals (i.e., $|\mathcal{N}_t^{(i)}| = N$) at model initialization. We can reformulate Equation 2 into one dimensional temporal convolution with Dirac delta function δ as follows:

$$\mathbf{x}_t^{(i)} = \sum_{j \in \mathcal{N}_t^{(i)}} w_{i,j} \delta(t - d_{i,j}) * \mathbf{x}_t^{(i)},$$
(3)

where * is 1D convolution and $\delta(t-a)*f(t)=f(t-a)$. We have detailed this process in right side of Figure 3b. To approximate Dirac delta function, we utilize Gaussian kernel with maximum delay (kernel size in trivial 1D convolution) (Hassani et al., 2023; Khalfaoui-Hassani et al., 2023). Let assume that we have k-size convolutional kernel for our model. Then, we approximate Gaussian kernel centered at $k-d_{i,j}-1$, where $d_{i,j}\in[0,k-1]$, with standard deviation $\sigma_{i,j}\in\mathbb{R}^*$. Then, $\forall n\in[0,\ldots,k-1]$ our k-size convolutional kernel will be:

$$k_{i,j}[n] = \frac{w_{i,j}}{A} exp\left(-\frac{1}{2} \left(\frac{n-k+d_{i,j}+1}{\sigma_{i,j}}\right)^2\right), \text{ where}$$
 (4)

$$A = \epsilon + \sum_{n=0}^{k-1} exp\left(-\frac{1}{2}\left(\frac{n-k+d_{i,j}+1}{\sigma_{i,j}}\right)^2\right)$$

with $\epsilon \in \mathbb{R}^*$ to avoid division by zero. In Equation 4, $w_{i,j} \in \mathbb{R}$, $d_{i,j} \in [0,k-1]$, and $\sigma \in \mathbb{R}^*$ are trainable parameter. Gaussian kernel converts the discrete Dirac delta function, which is not differentiable, into the continuous function, enabling the gradient calculation. To avoid underfitting of the delay, we penalize the σ with boundary value v, which continuously decreases up to 0.01 during the training phase. In the inference phase, we replace the Gaussian kernel with pulse function (i.e., hard voting). As a results, we can obtain almost discrete kernel representation as Figure 3b.

Despite the Gaussian kernel-based 1D convolution successfully parameterized both propagation delay and causal strength, it still has two limitations for dynamic causality modeling. First of all, trained causal map is fixed matrix and will not changed. It thus will be vulnerable to distribution shifts of causality. Secondly, it cannot directly observe and utilize the input information and lead-lag relationship, which make the model hardly understand intrinsic nature of MTS. Therefore, we additionally introduce hypernetworks to generate parameters in Equation 4.

4.3 CAUSALITY-ENHANCED ATTENTION WITH LEARNABLE AND ADAPTIVE SPACING

In this section, we describe the overall flow of CALAS with hypernetworks when it applied to input layer, as illustrated in Figure 3a. Specifically, we utilize two hypernetworks, one for the causal strength, and the other for delay learning. The generation process are shared for both hypernetworks, however, the causal strength is shared for k possible delays. Specifically, in the Equation 4, $w_{i,j} \in \mathbb{R}$, $\sigma_{i,j} \in \mathbb{R}$, $d_{i,j} \in \mathbb{R}^k$. By sharing same causal strength for the same pair of variates, CALAS achieves better generalization for causal strength.

We design our hypernetworks with one multi-layer perceptron (MLP) and feature embedding. MLP layer takes the input vector $z \in \mathbb{R}^{T \times N_i}$ (in Figure 3a, \mathbf{X}_t), where N_i is input channels, and project it into hidden size of H. To map the projected input to the N_o channels, we introduce embedding vectors $\mathbf{M} = \{\mathbf{m}^{(1)}, \dots, \mathbf{m}^{(N_o)}\}$, where $\mathbf{m}^{(i)} \in \mathbb{R}^{h \times N_o}$. \mathbf{M} is initially random vectors and will be trained to represent the static features of each output variate. For the output channels, we can

interchangeably use MLP or embedding vectors with users' intention. Thus, we generate weights $w_{i,j}$ as follows:

$$w_i^{in} = w_{MLP}z^{(i)} + b_{MLP}; w_{i,j} = g(w_i^{in}, \mathbf{m}^{(j)})$$

where $w_{MLP} \in \mathbb{R}^{T \times H}$ and $b_{MLP} \in \mathbb{R}^{H}$ are learnable parameters. Function $g(\cdot, \cdot)$ could be any function or operation to generate $\mathbb{R}^{N_i \times N_o}$ map, including matrix multiplication and self-attention. To generate causal strength, we utilize matrix multiplication with LeakyReLU activation function. For the propagation delay, we mimic multi-head attention (Vaswani et al., 2017), which generates $\mathbb{R}^{N_i \times N_o \times k}$ output. Generated weights will be directly feed into Gaussian kernel (Equation 4) and then forward to propagation module (Figure 3b). CALAS will be served as the plug-in module injected in input-, output-, or intermediate-layers. In case of Figure 3a, CALAS has plugged into the input layer and its output will be served as the input for backbone model.

5 EXPERIMENTS

5.1 EXPERIMENTAL SETTINGS

Datasets Following previous researches (Zhou et al., 2021; 2022; Nie et al., 2023), we conduct experiments on six MTS datasets, including Weather, Electricity, Traffic, Solar, Wind, and PeMSD8. In this paper, we have excluded ETT* datasets because they are recorded from the same sources, which means they simultaneously changes and have no causal relationship (Zhou et al., 2021). We also exclude Exchange-rate dataset since it is well known that x_{t-1} is the best predictor for x_t if the market is efficient (Fama, 1970; Rossi, 2013) and Zeng et al. (2023) have experimentally demonstrated that simply repeating the last value can outperform or be comparable to the best results.

Baselines To demonstrate performance of CALAS as a plug-in module, we conduct the experiments with CALAS and four MTS forecasting backbones, PatchTST (Nie et al., 2023), DLinear (Zeng et al., 2023), Crossformer (Zhang & Yan, 2023), and MTGNN (Wu et al., 2020). PatchTST and DLinear is channel-independent modeling methods and others are channel-dependent methods. We further compared our results with statistics-based plug-in module, LIFT (Zhao & Shen, 2024), to prove effectiveness of end-to-end dynamic causality modeling. We also provide comparative experiments with Linear model-based CALAS with baseline methods, additionally including FEDformer (Zhou et al., 2022), Autoformer (Wu et al., 2021), and LightMTS (Zhao & Shen, 2024).

Setups We follow the same experimental settings as previous MTS forecasting works (Zhou et al., 2022; Nie et al., 2023), where the forecasting horizon $L \in \{96, 192, 336, 720\}$ and lookback window T=336. All the baselines follow the experimental settings, including hyperparemeter settings, in official code. For CALAS, we adopt the Adam optimizer and search the optimal learning rate in $\{0.1, 0.05, 0.01, 0.005, 0.001, 0.0005\}$. We set hidden size H=64 and batch size to 32. We have run each experiment five times and reported average performance. We also provide quantitative and qualitative results for causal discovery in Appendix B.

5.2 Main Results

Table 2 indicates the experimental results among four state-of-art methods, LIFT, and CALAS on six MTS datasets. CALAS significantly improves the backbone models by 6.65% on average, outperforming LIFT by 2.01 percentage points. This results indicates both importance of end-to-end dynamic causality modeling and limitation of statistical methods, especially in long-term forecasting. We have omitted the error bars for the experimental results since LIFT and CALAS are stable and has less than 0.001. We provide comparative analysis and depict important findings in following paragraphs.

Improvements over backbone models CALAS significantly improve the performance of backbone models, implying the importance of both channel dependency and propagation delay. For channel-independent (CI) methods, such as PatchTST and DLinear, CALAS achieves an average improvements of 9.88%. We particularly highlight the Weather, Solar, and PeMSD8 datasets, which have shown the largest gains with CALAS. These results are attributed to the complex spatio-temporal dependencies

Table 2: Performance comparison in terms of forecasting errors. The **bold text** means the best results among backbone, with LIFT, and with CALAS and <u>underlined text</u> means the best results among all the methods for each dataset. We also provide improvements over 1) baseline and 2) LIFT in the second-rightmost and rightmost columns, respectively. Impr. over LIFT displays percent point of improvements.

_																											
Me	thod		hTST	+L			LAS							Crossi					LAS				IFT		LAS	Impr.	Impr.
		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	(baseline)	(LIFT)										
H			0.199																							6.76	1.06
- ři			0.243																							5.59	0.82
Ve ₈			0.283																							4.56	1.56
_	720	0.320	0.335	0.315	0.333	0.301	0.325	0.323	0.362	0.317	0.349	0.289	0.337	0.323	0.364	0.321	0.360	0.317	0.358	0.335	0.369	0.333	0.365	0.322	0.355	5.02	4.01
≥			0.222																							4.23	1.15
:5			0.240																							3.45	0.79
5			0.261																							5.95	2.20
面	720	0.202	0.291	0.195	0.289	0.190	0.282	0.203	0.301	0.198	0.295	0.193	0.294	0.264	0.353	0.224	0.312	0.213	0.311	0.242	0.327	0.216	0.305	0.213	0.295	9.87	2.76
Hic	96	0.367	0.251	0.352	0.242	0.350	0.243	0.410	0.282	0.394	0.273	0.388	0.269	0.519	0.293	0.462	0.284	0.457	0.277	0.479	0.289	0.464	0.286	0.461	0.282	5.55	1.28
			0.259																							4.36	1.04
屋			0.265																							4.05	2.33
	720	0.434	0.287	0.429	0.286	0.425	0.283	0.466	0.315	0.454	0.307	0.443	0.297	0.584	0.369	0.543	0.322	0.536	0.312	0.616	0.352	0.532	0.340	0.520	0.335	8.19	2.40
	96	0.176	0.227	0.174	0.224	0.174	0.225	0.222	0.291	0.185	0.238	0.178	0.233	0.179	0.245	0.174	0.224	0.176	0.221	0.167	0.224	0.166	0.223	0.160	0.214	9.16	2.11
	192	0.205	0.260	0.190	0.245	0.188	0.243	0.249	0.309	0.194	0.253	0.190	0.252	0.204	0.254	0.197	0.250	0.181	0.248	0.180	0.243	0.179	0.239	0.169	0.228	12.2	3.61
			0.252																							11.6	2.46
	720	0.229	0.282	0.203	0.261	0.200	0.256	0.271	0.327	0.207	0.260	0.204	0.265	0.211	0.250	0.202	0.255	0.198	0.254	0.197	0.256	0.195	0.251	0.192	0.245	11.6	1.38
Pu	96	0.186	0.216	0.175	0.213	0.170	0.207	0.197	0.230	0.177	0.214	0.174	0.212	0.172	0.218	0.169	0.208	0.168	0.202	0.170	0.211	0.169	0.208	0.167	0.210	6.01	1.80
	192	0.204	0.229	0.191	0.224	0.187	0.219	0.218	0.245	0.193	0.226	0.193	0.225	0.189	0.230	0.187	0.222	0.187	0.213	0.186	0.223	0.184	0.220	0.181	0.219	6.14	1.61
			0.239																							5.90	1.03
	720	0.231	0.253	0.215	0.247	0.212	0.244	0.254	0.278	0.225	0.256	0.218	0.261	0.237	0.286	0.224	0.254	0.221	0.251	0.200	0.236	0.200	0.232	0.197	0.229	7.68	1.38
	96	0.445	0.316	0.410	0.305	0.397	0.300	0.562	0.421	0.449	0.336	0.431	0.328	0.373	0.290	0.368	0.286	0.366	0.288	0.393	0.304	0.386	0.297	0.385	0.293	10.6	2.09
S	192	0.519	0.354	0.471	0.337	0.466	0.334	0.611	0.443	0.502	0.364	0.505	0.370	0.409	0.312	0.399	0.303	0.397	0.302	0.440	0.333	0.429	0.324	0.420	0.317	9.28	0.66
			0.366																							10.6	1.54
P	720	0.653	0.403	0.563	0.378	0.542	0.369	0.748	0.519	0.597	0.414	0.595	0.424	0.488	0.356	0.468	0.338	0.451	0.322	0.511	0.379	0.484	0.342	0.481	0.338	14.4	2.18

recorded at 5- or 10-minute intervals, requiring more sophisticated dependency modeling. For instance, although both Traffic and PeMSD8 datasets consists of traffic recordings from a large number of sensors (862 and 510 sensors, respectively), the PeMSD8 dataset exhibits much greater fluctuation and recording errors, making it more difficult for CI models to accurately predict future states by relying solely on temporal dependencies. For channel-dependent (CD) methods, such as Crossformer and MTGNN, CALAS makes an average improvements of 5.34% across six datasets. Notably, Electricity and Traffic datasets show the largest improvements, with gains of up to 18.46%. This is due to the presence of multiple short-term periodicities in the data, which require an understanding of frequency or propagation delay. By injecting an awareness for propagation delay, CALAS significantly enhances the modeling qualities on highly periodic datasets.

Improvements over plug-in method Compared to the recently developed plug-in method, LIFT, CALAS achieves an additional improvement of 2.21 percentage points. This improvement is even more pronounced in datasets containing signals from a variety of sensors (e.g., Weather) and in long-term forecasting tasks. We hypothesize that this performance gain is caused by several factors: **F1**: the presence of short-term distribution shifts in channel dependency, **F2**: the limited diversity of statistically extracted 'cause' signals, and **F3**: limitation of LIFT's layer-specific design.

Across all datasets, we observe that CALAS tends to outperform LIFT, particularly in long-term forecasting tasks. We will showcase three cases: Weather, Electricity, and Traffic datasets. Weather dataset consists of signals exhibiting diverse patterns caused by various sources. Moreover, their causality can shift in the short-term period–for example, humidity and temperatures indicate no causality during certain periods but exhibit strong negative causality during the onset of the rainy season. These short-term changes, even between the input and output signals, degrade LIFT's performance. This degradation experimentally proves **F1**: distribution shifts in channel dependency, and **F3**: LIFT's reliance on input to generate output channel dependencies, highlighting the need of layer-agnostic usage of the causality modeling as in CALAS.

Electricity and Traffic datasets are recorded at hourly intervals and have strong daily periodicity, generating multiple short-term periodic patterns. As a result, LIFT, particularly through cross-correlation, tends to focus on capturing the short-term periodicities while overlooking trend changes. Additionally, its top-k selection process with redundant patterns creates an information bottleneck, limiting access to diverse information (**F2**). In contrast, CALAS learns the causal relationships in an end-to-end manner, allowing the model to avoid such bottlenecks and ultimately produce richer forecasting results. Furthermore, we have found that CALAS can support statistics-based model, LightMTS (Zhao & Shen, 2024), by modeling input causality, improving additional 1.2 percentage points over CALAS, 3.41 percentage points with respect to original LightMTS, as Figure 4c.

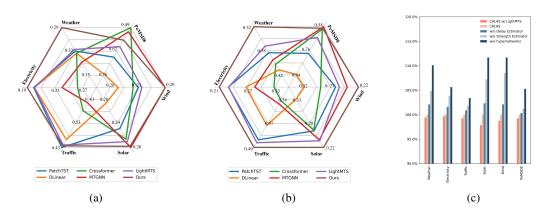


Figure 4: Performance of Linear model-based CALAS (Ours, Brown line) and five baseline models on (a) all variates and (b) top-10% variates with strong causal relationship (i.e., highly affected from the other variates). (c) Performance of CALAS with LightMTS and ablation study on CALAS with hypernetworks, introducing partially (or fully) static causality modeling.

CALAS with Linear Backbone To showcase the importance of dynamic causality modeling, we conduct additional experiments with simple Linear backbone with RevIN (Kim et al., 2022). As shown in Figure 4a, CALAS with one linear projection still shows comparable performance to state-of-the-art models. We also indicate that if certain variates have strong causal relationship, which is extracted with CALAS, our model outperform the existing models by utilizing strong causal information and propagation delay. For example, PeMSD8, Solar, and Wind datasets require more sophisticate modeling, degrading the linear models. In contrast, CALAS gains comparable performance with complicate CD modeling methods and even outperform them for the variates with strong causal relationship, as illustrated in Figure 4b.

5.3 ABLATION STUDY

In Figure 4c, we provide the ablation study results with our hypernetworks, reporting relative MSE amplification with respect to CALAS. In this study, we compared CALAS with three variants – 1) without delay estimator, 2) without strength estimator, and 3) without both estimators. We utilize the one layer linear function as backbone model and the performance is reported on average over the prediction length 96, 192, 336, and 720. By ablating hypernetworks, CALAS lost its ability to dynamically modeling causality, as a results, degrading the forecasting quality. We can depict that loosing dynamic causal strength modeling (i.e., w/o strength estimator) induces inability to distribution shifts on CD, showing inferior performance. The delay estimator, in contrast, often shows less performance drop than strength estimator, since CALAS still can dynamically estimate causality. Furthermore, the ablation results of CALAS w/o hypernetworks imply the need of transition from static causality modeling to dynamic causality modeling.

6 Conclusion

In this work, we emphasize the importance of **delay learning** in channel dependency modeling and propose new perspective for understanding channel dependencies in MTS forecasting. We introduce a novel end-to-end learning method, CALAS, which learns underlying lead-lag relationship in MTS while considering both causal strength and propagation delay. By integrating Gaussian kernel-based 1D convolution with two hypernetworks, CALAS reformulate undirected channel dependency modeling into directed, delay-aware causality modeling. We experimentally demonstrate the superiority of CALAS over state-of-the-art MTS forecasting models and statistical causality modeling methods. We believe that delay learning and causality-aware modeling are promising research directions for future developments of MTS forecasting models. As a next step, we plan to investigate the better model architecture for delay learning, including multi-periodicity decomposition by adjusting stride or introducing the uncertainty concepts into the CD modeling.

REFERENCES

- Lei Bai, Lina Yao, Can Li, Xinazhi Wang, and Can Wang. Adaptive graph convolutional recurrent network for traffic forecasting. In *Advances in Neural Information Processing Systems*, volume 33, 2020.
- Ali Behrouz, Michele Santacatterina, and Ramin Zabih. Chimera: Effectively modeling multivariate time series with 2-dimensional state space models. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL https://openreview.net/forum?id=ncYGjx2vnE.
- George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, and Greta M. Ljung. *Time series analysis: forecasting and control.* John Wiley, 2015.
- Defu Cao, Yujing Wang, Juanyong Duan, Ce Zhang, Xia Zhu, Congrui Huang, Yunhai Tong, Bixiong Xu, Jing Bai, Jie Tong, and Qi Zhang. Spectral temporal graph neural network for multivariate time-series forecasting. In *Advances in Neural Information Processing Systems*, volume 33, 2020.
- Yuxiao Cheng, Runzhao Yang, Tingxiong Xiao, Zongren Li, Jinli Suo, Kunlun He, and Qionghai Dai. CUTS: Neural causal discovery from irregular time-series data. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=UG8bQcD3Emv.
- Yuxiao Cheng, Ziqian Wang, Tingxiong Xiao, Qin Zhong, Jinli Suo, and Kunlun He. Causaltime: Realistically generated time-series for benchmarking of causal discovery. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=iadlyyyGme.
- Luo donghao and wang xue. ModernTCN: A modern pure convolution structure for general time series analysis. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=vpJMJerXHU.
- Eugene F. Fama. Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2):383–417, 1970. ISSN 00221082, 15406261. URL http://www.jstor.org/stable/2325486.
- C. W. J. Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, 37(3):424–438, 1969.
- Shengnan Guo, Youfang Lin, Ning Feng, Chao Song, and Huaiyu Wan. Attention based spatial-temporal graph convolutional networks for traffic flow forecasting. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pp. 922–929, 2019.
- David Ha, Andrew M. Dai, and Quoc V. Le. Hypernetworks. In *International Conference on Learning Representations*, 2017.
- Ilyass Hammouamri, Ismail Khalfaoui-Hassani, and Timothée Masquelier. Learning delays in spiking neural networks using dilated convolutions with learnable spacings. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=4r2ybzJnmN.
- Lu Han, Han-Jia Ye, and De-Chuan Zhan. The capacity and robustness trade-off: Revisiting the channel independent strategy for multivariate time series forecasting, 2023. URL https://arxiv.org/abs/2304.05206.
- Ismail Khalfaoui Hassani, Thomas Pellegrini, and Timothée Masquelier. Dilated convolution with learnable spacings. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=Q3-1vRh3HOA.
- Jiaxi Hu, Disen Lan, Ziyu Zhou, Qingsong Wen, and Yuxuan Liang. Time-ssm: Simplifying and unifying state space models for time series forecasting, 2024. URL https://arxiv.org/abs/2405.16312.

- Yuxin Jia, Youfang Lin, Xinyan Hao, Yan Lin, Shengnan Guo, and Huaiyu Wan. WITRAN: Waterwave information transmission and recurrent acceleration network for long-range time series forecasting. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023. URL https://openreview.net/forum?id=y08bkEtNBK.
 - Renhe Jiang, Zhaonan Wang, Jiawei Yong, Puneet Jeph, Quanjun Chen, Yasumasa Kobayashi, Xuan Song, Shintaro Fukushima, and Toyotaro Suzumura. Spatio-temporal meta-graph learning for traffic forecasting. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 37, pp. 8078–8086, 2023.
 - Seungmin Jin, Hyunwook Lee, Cheonbok Park, Hyeshin Chu, Yunwon Tae, Jaegul Choo, and Sungahn Ko. A visual analytics system for improving attention-based traffic forecasting models. *IEE Transaction on Visualization and Computer Graphics*, 29(1):1102–1112, 2023.
 - Ismail Khalfaoui-Hassani, Thomas Pellegrini, and Timothée Masquelier. Dilated convolution with learnable spacings: beyond bilinear interpolation. In *ICML 2023 Workshop on Differentiable Almost Everything: Differentiable Relaxations, Algorithms, Operators, and Simulators*, 2023. URL https://openreview.net/forum?id=j8FPBCltB9.
 - Taesung Kim, Jinhee Kim, Yunwon Tae, Cheonbok Park, Jang-Ho Choi, and Jaegul Choo. Reversible instance normalization for accurate time-series forecasting against distribution shift. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=cGDAkQo1C0p.
 - Chunggi Lee, Yeonjun Kim, Seungmin Jin, Dongmin Kim, Ross Maciejewski, David Ebert, and Sungahn Ko. A visual analytics system for exploring, monitoring, and forecasting road traffic congestion. *IEEE Transactions on Visualization and Computer Graphics*, 26(11):3133–3146, 2020. doi: 10.1109/TVCG.2019.2922597.
 - Hyunwook Lee, Seungmin Jin, Hyeshin Chu, Hongkyu Lim, and Sungahn Ko. Learning to remember patterns: Pattern matching memory networks for traffic forecasting. In *International Conference on Learning Representations*, 2022.
 - Hongming Li, Shujian Yu, and Jose Principe. Causal recurrent variational autoencoder for medical time series generation. In *Proceedings of the Thirty-Seventh AAAI Conference on Artificial Intelligence and Thirty-Fifth Conference on Innovative Applications of Artificial Intelligence and Thirteenth Symposium on Educational Advances in Artificial Intelligence*, AAAI'23/IAAI'23/EAAI'23, 2023.
 - Yaguang Li and Cyrus Shahabi. A brief overview of machine learning methods for short-term traffic forecasting and future directions. *SIGSPATIAL Special*, 10(1):3–9, 2018.
 - Yaguang Li, Rose Yu, Cyrus Shahabi, and Yan Liu. Diffusion convolutional recurrent neural network: Data-driven traffic forecasting. In *International Conference on Learning Representations*, 2018.
 - Yong Liu, Haixu Wu, Jianmin Wang, and Mingsheng Long. Non-stationary transformers: Exploring the stationarity in time series forecasting. In *Advances in Neural Information Processing Systems*, 2022.
 - Yong Liu, Tengge Hu, Haoran Zhang, Haixu Wu, Shiyu Wang, Lintao Ma, and Mingsheng Long. itransformer: Inverted transformers are effective for time series forecasting. *arXiv preprint arXiv:2310.06625*, 2023.
 - Wolfgang Maass. Networks of spiking neurons: The third generation of neural network models. *Neural Networks*, 10(9):1659–1671, 1997. ISSN 0893-6080. doi: https://doi.org/10.1016/S0893-6080(97)00011-7.
 - Wolfgang Maass and Michael Schmitt. On the complexity of learning for spiking neurons with temporal coding. *Information and Computation*, 153(1):26–46, 1999. ISSN 0890-5401. doi: https://doi.org/10.1006/inco.1999.2806.
 - Yasuko Matsubara, Yasushi Sakurai, Willem G. van Panhuis, and Christos Faloutsos. FUNNEL: automatic mining of spatially coevolving epidemics. In *The ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pp. 105–114. ACM, 2014.

- Meike Nauta, Doina Bucur, and Christin Seifert. Causal discovery with attention-based convolutional neural networks. *Machine Learning and Knowledge Extraction*, 1(1):312–340, 2019.
 - Yuqi Nie, Nam H. Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. A time series is worth 64 words: Long-term forecasting with transformers. In *International Conference on Learning Representations*, 2023.
 - Boris N. Oreshkin, Dmitri Carpov, Nicolas Chapados, and Yoshua Bengio. N-beats: Neural basis expansion analysis for interpretable time series forecasting. In *International Conference on Learning Representations*, 2020.
 - M.H. Pesaran, D. Pettenuzzo, and A. Timmermann. Forecasting time series subject to multiple structural breaks. Cambridge Working Papers in Economics 0433, Faculty of Economics, University of Cambridge, 2004.
 - Barbara Rossi. Exchange rate predictability. *Journal of Economic Literature*, 51(4):1063–1119, 2013. doi: 10.1257/jel.51.4.1063. URL https://www.aeaweb.org/articles?id=10.1257/jel.51.4.1063.
 - Sumit Bam Shrestha and Garrick Orchard. Slayer: spike layer error reassignment in time. In *Proceedings of the 32nd International Conference on Neural Information Processing Systems*, pp. 1419–1428, 2018.
 - Koichi Sameshima Suiz A. Baccalá. Partial directed coherence: a new concept in neural structure determination. *Biological Cybernetics*, 84:463–474, 2001.
 - Alex Tank, Ian Covert, Nicholas Foti, Ali Shojaie, and Emily B. Fox. Neural granger causality. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(8):4267–4279, 2022. ISSN 0162-8828.
 - Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Ł ukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *Advances in Neural Information Processing Systems*, volume 30, 2017.
 - Xue Wang, Tian Zhou, Qingsong Wen, Jinyang Gao, Bolin Ding, and Rong Jin. CARD: Channel aligned robust blend transformer for time series forecasting. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=MJksrOhurE.
 - Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with auto-correlation for long-term series forecasting. In *Advances in Neural Information Processing Systems*, volume 34, pp. 22419–22430, 2021.
 - Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. In *International Conference on Learning Representations*, 2023.
 - Zonghan Wu, Shirui Pan, Guodong Long, Jing Jiang, and Chengqi Zhang. Graph wavenet for deep spatial-temporal graph modeling. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 1907–1913, 2019.
 - Zonghan Wu, Shirui Pan, Guodong Long, Jing Jiang, Xiaojun Chang, and Chengqi Zhang. Connecting the dots: Multivariate time series forecasting with graph neural networks. In *Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, 2020.
 - Zhijian Xu, Ailing Zeng, and Qiang Xu. FITS: Modeling time series with \$10k\$ parameters. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=bWcnvZ3qMb.
 - Kun Yi, Qi Zhang, Wei Fan, Hui He, Liang Hu, Pengyang Wang, Ning An, Longbing Cao, and Zhendong Niu. FourierGNN: Rethinking multivariate time series forecasting from a pure graph perspective. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023a. URL https://openreview.net/forum?id=bGs1qWQ1Fx.

- Kun Yi, Qi Zhang, Wei Fan, Shoujin Wang, Pengyang Wang, Hui He, Ning An, Defu Lian, Longbing Cao, and Zhendong Niu. Frequency-domain mlps are more effective learners in time series forecasting. In *Advances in Neural Information Processing Systems*, volume 36, pp. 76656–76679, 2023b.
- Ailing Zeng, Muxi Chen, Lei Zhang, and Qiang Xu. Are transformers effective for time series forecasting? In *Proceedings of the AAAI Conference on Artificial Intelligence*, 2023.
- Chaolv Zeng, Zhanyu Liu, Guanjie Zheng, and Linghe Kong. Cmamba: Channel correlation enhanced state space models for multivariate time series forecasting, 2024. URL https://arxiv.org/abs/2406.05316.
- Yunhao Zhang and Junchi Yan. Crossformer: Transformer utilizing cross-dimension dependency for multivariate time series forecasting. In *The Eleventh International Conference on Learning Representations*, 2023.
- Lifan Zhao and Yanyan Shen. Rethinking channel dependence for multivariate time series fore-casting: Learning from leading indicators. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=JiTVtCUOpS.
- Chuanpan Zheng, Xiaoliang Fan, Cheng Wang, and Jianzhong Qi. GMAN: A graph multi-attention network for traffic prediction. In *Proceedings of the AAAI Conference on Artificial Intelligence*, pp. 1234–1241, 2020.
- Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. *Proceedings of the AAAI Conference on Artificial Intelligence*, 35(12):11106–11115, 2021.
- Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. FEDformer: Frequency enhanced decomposed transformer for long-term series forecasting. In *Proceedings of the 39th International Conference on Machine Learning*, volume 162 of *Proceedings of Machine Learning Research*, pp. 27268–27286, 2022.
- Tian Zhou, Peisong Niu, Xue Wang, Liang Sun, and Rong Jin. One fits all: Power general time series analysis by pretrained LM. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023. URL https://openreview.net/forum?id=qMS6FVZvmF.
- Yunyue Zhu and Dennis Shasha. Statstream: Statistical monitoring of thousands of data streams in real time. In *Proceedings of the International Conference on Very Large Databases*, pp. 358–369. Morgan Kaufmann, 2002.
- Ali Zonoozi, Jung-jae Kim, Xiao-Li Li, and Gao Cong. Periodic-crn: A convolutional recurrent model for crowd density prediction with recurring periodic patterns. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 3732–3738, 2018.