# REFGRADER: AUTOMATED GRADING OF MATHEMATICAL COMPETITION PROOFS USING AGENTIC WORKFLOWS

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## ABSTRACT

State-of-the-art (SOTA) LLMs have progressed from struggling on proof-based Olympiad problems to solving most of the IMO 2025 problems, with leading systems reportedly handling 5 of 6 problems. Given this progress, we assess how well these models can grade proofs: detecting errors, judging their severity, and assigning fair scores beyond binary correctness. We study proof-analysis capabilities using a corpus of 90 Gemini 2.5 Pro–generated solutions that we grade on a 1–4 scale with precise error types and locations, and on MathArena solution sets for IMO/USAMO 2025 scored on a 0–7 scale. Our analysis shows that models can reliably flag incorrect (including subtly incorrect) solutions but exhibit calibration gaps in how partial credit is assigned. To address this, we introduce Agentic Workflows that extract and analyze reference solutions and automatically derive task-specific rubrics for a multi-step grading process. We instantiate and compare two rubric design choices—approachability-based weighting (by "aha" difficulty) and milestone-based rubrics, and evaluate their trade-offs. Across our annotated corpus and MathArena, these workflows achieve higher agreement with human grades and more consistent handling of partial credit across metrics. We release all code, data, and prompts/logs to facilitate future research. https://github.com/ref-grader/ref-grader

## 1 INTRODUCTION

Until early 2025, state-of-the-art (SOTA) LLMs often failed to produce correct and sound solutions to Olympiad level problems (Petrov et al., 2025; Mahdavi et al., 2025). As automated judges, they performed unreliably, often near chance, when asked to distinguish invalid solutions from the correct ones or to apply rubrics consistently (Mahdavi et al., 2025; Petrov et al., 2025). Industry announcements from Google and OpenAI claimed that the advanced versions of their models could achieve gold medal level performance on the IMO 2025, solving 5 of 6 problems within exam time(Luong & Lockhart, 2025; Wei). Independent reproductions report solving 5 of 6 problems using Gemini 2.5 Pro within an agentic, multi-step workflow (Huang & Yang, 2025).

These findings raise concerns about using LLMs for automated proof assessment: if models struggle with basic verification and rubric application, automatic grading may be unreliable. However, the cited studies predate recent model advances. Independent evaluations, such as Balunović et al. (2025), report notable improvements in solution correctness and proof quality for SOTA systems (e.g., Gemini 2.5 Pro), though the extent varies by task and setup. Evaluating LLMs' mathematical capabilities via final-answer accuracy has become the de facto standard(Cobbe et al., 2021; Hendrycks et al., 2021; Fang et al., 2024; Yue et al., 2024). Going beyond final answers to assess proof quality is substantially more challenging. Formal verification offers a principled solution to validation(Zheng et al., 2022; Lin et al., 2025; Chen et al., 2025; Jiang et al., 2024; Ren et al., 2025), but faces two practical limitations: limited availability of formal corpora and lower readability for humans. An alternative is to binarize proofs and measure agreement with expert judges(Dekoninck et al., 2025; Guo et al., 2025), which improves scalability but ignores the issue of partial credits.

In this work, we move beyond binary judgments and evaluate how well LLMs grade proofs. We construct a corpus of 90 Gemini 2.5 Pro–generated solutions, graded on a 1–4 scale and annotated with precise error types and locations, and we also use MathArena solutions for IMO/USAMO 2025 scored on a 0–7 scale. Our evaluation focuses on Gemini 2.5 Pro with maximum thinking budget. First, we assess single-turn grading by comparing model-assigned scores against human grades. Next, we introduce Agentic Workflows that extract and analyze reference solutions to automatically design task-specific grading rubrics (Ref-Grader), and we compare design choices: approachability-based weighting (by “aha moment” difficulty), milestone-based rubrics, their hybrid, and a 3-step reference variant without rubric induction. We evaluate these workflows on our annotated corpus and on MathArena solutions for IMO and USAMO 2025, observing higher agreement with human grades and more consistent handling of partial credit across metrics. Although our workflows might need more tokens and hence more cost, the majority of the workflow steps are cachable and this helps us to keep overall cost low. We release all code, data, and prompts/logs to facilitate future research.

**Contributions.**

1. We design a reference-aided, multi-step grading workflow (Ref-Grader) that derives task-specific rubrics from reference solutions.
2. We demonstrate improved partial-credit grading across diverse metrics (Pearson/Spearman ↑, MAE/RMSE ↓, QWK ↑).
3. We study robustness via ablation workflows and sampling/averaging analyses.
4. We curate and release an IMO Shortlist–based grading dataset of 90 Gemini 2.5 Pro solutions with 1–4 labels and error annotations, together with code, prompts, and logs.

## 2 Related Work

**Proof-evaluation corpora:** Resources assessing proofs include the Open Proof Corpus, which aggregates human and model proofs with binary validity labels and expert annotations (Dekoninck et al., 2025), and LitmusTest, which standardizes pass/fail judgments using expert-designed rubrics (Guo et al., 2025). For competition mathematics, MathArena hosts model-generated solutions for IMO/USAMO-style problems with 0–7 scores and judge rationales (Balunović et al., 2025). Formal settings emphasize verifiable correctness but face constraints in data availability and coverage (Lin et al., 2025; Zheng et al., 2022; Chen et al., 2025).

**LLM-as-a-grader:** Two strands are prominent: rubric-grounded grading across domains and reliability improvements via calibration or multi-agent designs. In physics education, GPT-4o assigns partial credit with self-consistency and human-in-the-loop triage (Chen & Wan, 2025); in healthcare, open-ended clinical dialogs are evaluated against physician-written, instance-specific criteria (Arora et al., 2025); for expert long-form tasks, expert-validated rubrics map to checklist items (Ruan et al., 2025); rubric-prompted judge distributions benefit from calibration to human ratings (Hashemi et al., 2024). In education and code assessment, rubric specialization and multi-agent judging improve robustness and interpretability (Pathak et al., 2025; Chu et al., 2025). Closer to mathematics, per-problem rubrics diagnose stepwise skills on word problems (Jin et al., 2024).

**LLM-as-a-judge:** Complementary work examines models as evaluators to reduce dependence on human annotations (Stephan et al., 2024; Li et al., 2024; Nasrabadi, 2024; Ning et al., 2024). Methods treat assessment as adaptable and task-aware (Tan et al., 2024; Dhurandhar et al., 2024) and calibrate reliability against human judgments (Kim et al., 2024; Ye et al., 2024; Liu et al., 2025). General-purpose resources include UltraFeedback, AlpacaEval, Chatbot Arena, and MT-Bench (Cui et al., 2024; Dubois et al., 2024; Chiang et al., 2024; Zheng et al., 2023); math-specific judge benchmarks include REASONEVAL, MATHCHECK, and SMART-840 (Xia et al., 2025; Zhou et al., 2024; Cherian et al., 2024).

**Benchmarks:** Benchmarks define the tasks under assessment. Math word problem corpora probe stepwise reasoning in natural language (Ahn et al., 2024; Yuan et al., 2023; Cobbe et al., 2021; Amini et al., 2019), while robustness and compositionality sets assess generalization (Zhang et al., 2024; Hosseini et al., 2024; Srivastava et al., 2024). Formal ATP datasets target verifiable theorem proving (Zheng et al., 2022; Yu et al., 2024; Jiang et al., 2024); specialized and competition-level collections broaden coverage (Wu et al., 2023; Frieder et al., 2023; Mao et al., 2024; He et al., 2024; Fang et al., 2024; Gao et al., 2024), and repositories scale annotated problems (Yue et al., 2024; LI et al., 2024).

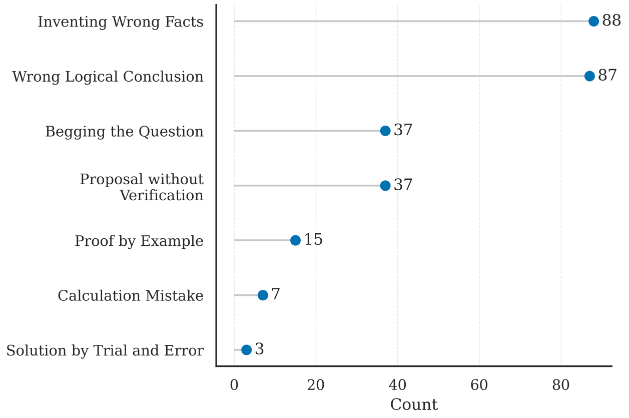


Figure 1: Error frequencies by fallacy category for the IMO Shortlist dataset

**Mathematical Reasoning in LLMs:** Reasoning can be elicited through prompting and inference-time strategies, including Chain-of-Thought and self-consistency (Chen et al., 2024; Wei et al., 2022; Kojima et al., 2023; Havrilla et al., 2024; Wang et al., 2023; Wang & Zhou, 2024). Controlled benchmarks reveal gaps between pattern matching and formal reasoning (Hendrycks et al., 2021; Mirzadeh et al., 2024). Complementary work explores reward modeling, self-refinement, and algorithmic decomposition (Huang et al., 2024; Zelikman et al., 2023).

# 3 Datasets

## 3.1 IMO Shortlist Data

### 3.1.1 Data Collection

We selected 90 challenging problems from the IMO Shortlist dataset (2017–2023). We used a standardized prompt requesting a rigorous solution to each Olympiad-level problem and generated one solution per problem with Gemini 2.5 Pro. The prompt is provided in Appendix B. We then annotated the solutions using the fallacy categories from (Mahdavi et al., 2025). The list of fallacies is as follows:

- **Proof by Example**
- **Proposal Without Verification**
- **Inventing Wrong Facts**
- **Begging the Question (Circular Reasoning)**
- **Solution by Trial-and-Error**
- **Calculation Mistakes**

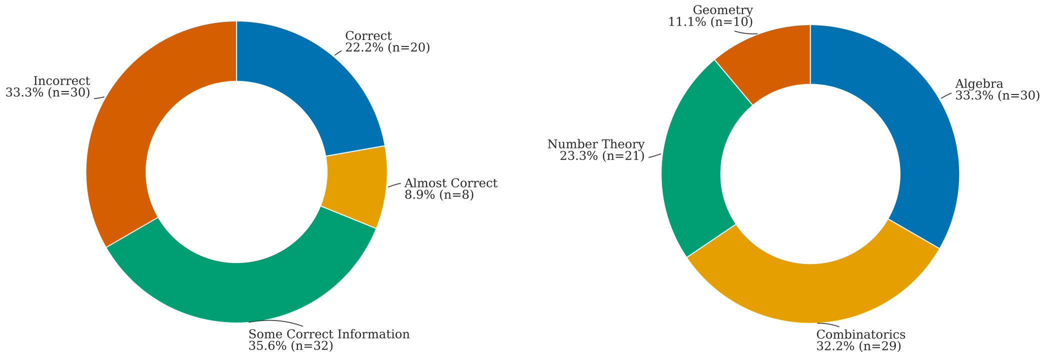
We adopt the definitions provided in the original paper (Mahdavi et al., 2025). We additionally introduce a general category, **Wrong Logical Conclusion**, to tag mathematical errors that do not fit any of the other categories. Evaluators carefully reviewed each solution and annotated each error type and the approximate error location using the following syntax (markup used in the released dataset):

`<span class="[Fallacy Type]`$^+$`"> [Fallacious Statement] </span>`

For example, if a fallacy is identified in a generated proof, evaluators mark it as follows:

`<span class= "proof-by-example"> As the statement is true for` $n = 1, 2, 3$ `it is highly probable that it is also true </span>`

When applying fallacy labels, if multiple fallacies fit a given error, we prioritized the most specific label. When distinct errors co-occurred, we applied multiple fallacy labels. We graded solutions using the following 4-point scale.



(a) Distribution of solution labels (percentages and counts).

(b) Problem topics (percentages and counts).

Figure 2: Dataset summaries for the IMO Shortlist dataset

- **1: Incorrect:** The solution does not contain useful non-trivial information. It contains only incorrect information or restates straightforward facts from the problem. Equivalent to 0/7 or 1/7 in Olympiad grading.
- **2: Some Correct Information:** The solution contains a few non-trivial facts derived with some effort but lacks a coherent proof. Equivalent to 2/7 or 3/7 in Olympiad grading.
- **3: Almost Correct:** The solution proves non-trivial parts of the argument but omits one non-trivial part of the proof. Equivalent to 4/7 or 5/7 in Olympiad grading.
- **4: Correct:** The solution proves all required facts and statements

We did not adopt the 0–7 Olympiad scale due to the per-problem rubric cost. Finally, after annotating errors and assigning grades, evaluators provided a brief explanation of any issues in a dataset field labeled ”Final Comment”.

### 3.1.2 Dataset Statistics

Figures 1, 2a and 2b summarize dataset statistics: error frequencies by fallacy category, the distribution of solution labels, and the topical composition of problems. Relative to the models analyzed by Mahdavi et al. (2025), Gemini 2.5 Pro yields a smaller share of incorrect solutions (Fig. 2a) and fewer naive errors (e.g., Proof by Example, Solution by Trial-and-Error; Fig. 1).

## 3.2 MathArena Data

We collected 385 solutions for IMO and USAMO 2025 from the MathArena website. The solutions were generated by the following models: Grok 3 (Think), DeepSeek–R1–0528, Gemini 2.5 Pro, Gemini 2.0 Flash Thinking, QwQ–32B, DeepSeek–R1, o1–pro (high), o3–mini (high), o4–mini (high), Grok 4, o3 (high), and Claude–3.7–Sonnet (Think). MathArena conducts independent evaluations of model performance on contest-level problems. Solutions are graded by human judges on a 0–7 scale. The MathArena grade distribution is zero-inflated because many model-generated solutions receive a zero on these challenging problems. To balance the dataset for analysis and visualization, we subsampled zero-scores with probability 0.14 (applying this subsample consistently in the figures and tables for this section). Figure 3 shows the resulting grade distribution.

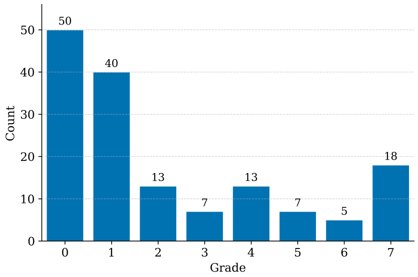


Figure 3: Grade distribution for the MathArena dataset

## 4 Evaluation Setting

Our goal is to evaluate LLMs as graders of mathematical proofs on the IMO Shortlist and MathArena datasets. Let $\mathcal{D} = \{(p_i, s_i)\}_{i=1}^{n}$ denote problem–solution pairs with associated ground-truth grades $\{g_i\}_{i=1}^{n}$. For each instance $i$, let $R_i = \{r_{ij}\}_{j=1}^{m_i}$ denote the set of correct reference solutions. The grading procedure (agentic workflow) takes $(p_i, s_i, R_i)$ as input and outputs a predicted grade $\hat{g}_i$. For all experiments, the end result is an LLM output in a structured format that includes the predicted grade $\hat{g}_i$ and, when available, step-by-step analysis, identified errors, clarity/structure/notation tags, and constructive feedback.

To assess agreement between $\{\hat{g}_i\}$ and $\{g_i\}$, we report Pearson and Spearman correlations, mean absolute error (MAE), root mean squared error (RMSE), and quadratic weighted kappa (QWK).

**Pearson correlation.** Pearson correlation measures linear association between predicted and ground-truth grades:

$$\text{Pearson} = \frac{\sum_{i=1}^{n}(g_i - \bar{g})(\hat{g}_i - \bar{\hat{g}})}{\sqrt{\sum_{i=1}^{n}(g_i - \bar{g})^2}\sqrt{\sum_{i=1}^{n}(\hat{g}_i - \bar{\hat{g}})^2}},$$

where $\bar{g}$ and $\bar{\hat{g}}$ are the means of the ground-truth and predicted grades, respectively.

**Spearman correlation.** Spearman correlation assesses monotonic association between the rankings of the grades:

$$\text{Spearman} = 1 - \frac{6\sum_{i=1}^{n}(r_i - \hat{r}_i)^2}{n(n^2-1)},$$

where $r_i$ and $\hat{r}_i$ are the ranks of $g_i$ and $\hat{g}_i$.

**Mean absolute error (MAE).** MAE measures the average absolute difference between predicted and ground-truth grades:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|g_i - \hat{g}_i|.$$

**Root mean squared error (RMSE).** RMSE penalizes larger errors more heavily:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(g_i - \hat{g}_i)^2}.$$

**Quadratic weighted kappa (QWK).** QWK Cohen (1968) measures agreement on ordinal labels while accounting for chance. With $K$ grade categories, let $O, E \in \mathbb{R}^{K\times K}$ be the observed and expected confusion matrices, and let $w_{ij} = (i-j)^2/(K-1)^2$. Then

$$\kappa = 1 - \frac{\sum_{i,j} w_{ij}O_{ij}}{\sum_{i,j} w_{ij}E_{ij}}.$$

For the IMO Shortlist, we map the 4-point scale to the 0–7 scale using $m(x) = 2x - 1$ for $x \in \{1, 2, 3, 4\}$. MathArena is already on the 0–7 scale.

## 5 Experimental Results

We first evaluate the performance of LLMs for single-turn proof grading and present quantitative metrics alongside qualitative visualizations.

Table 1: Single-turn grading results on *Math-Arena* and *IMO Shortlist*. Higher is better for correlations and QWK; lower is better for MAE/RMSE.

| Dataset | Pearson | Spearman | MAE | RMSE | QWK |
|---|---|---|---|---|---|
| Math-Arena | 0.638 | 0.582 | 2.458 | 2.886 | 0.323 |
| IMO Shortlist | 0.486 | 0.512 | 2.644 | 3.095 | 0.229 |

### 5.1 Single-turn Grading

In our first experiment, we focus on evaluating the performance of LLMs on grading proofs in a single-turn setting. We add the problem and solution in the context and ask the LLM to analyze the proof step-by-step and find all of its errors and then grade the proof on a 0–7 scale. We use the following definition for the grading scale:

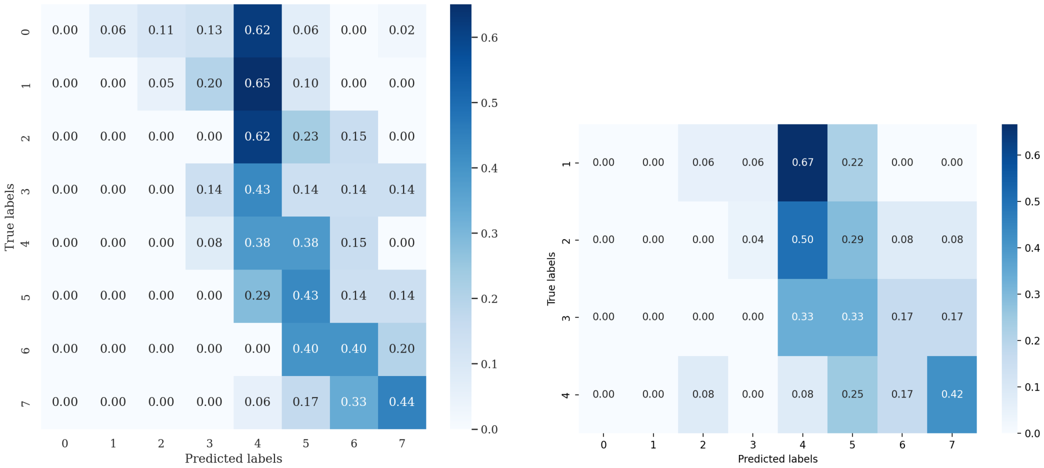
| Definition | Score |
|---|---|
| No progress or invalid. | 0 |
| Trace of understanding. | 1 |
| Minor progress. | 2 |
| Partial progress. | 3 |
| Substantial progress; proof incomplete. | 4 |
| Mostly correct; one small but non-trivial flaw. | 5 |
| Nearly perfect; only negligible issues. | 6 |
| Perfect; correct, complete, elegant. | 7 |

The full grading prompt used in this setting is provided in Appendix C. The results for MathArena and the IMO Shortlist dataset are shown in Table 1. The metrics indicate non-random agreement between predicted and ground-truth grades, although MAE and RMSE remain relatively large on both datasets. Figures 4a and 4b show normalized confusion matrices. On both datasets, the grader tends to over-score very low-quality solutions (true grade 0) and partially correct work (grades 1–4), shifting probability mass to the right of the diagonal. By contrast, solutions with grades $\geq 5$ are identified with a stronger diagonal. This pattern is consistent with the findings of Dekoninck et al. (2025) and Guo et al. (2025). Under a binarized evaluation (grade $\geq 5$ vs. $< 5$), performance would be high. More specifically, most off-diagonal mass concentrates one to two bins above the true grade for rows 0–3, indicating an optimistic bias and a tendency to credit incomplete outlines. Misclassifications are predominantly adjacent (i.e., $|i - j| = 1$), which preserves rank-based measures (Pearson/Spearman) while increasing absolute error (MAE/RMSE). At the top end (rows 5–7), under-scoring is limited, yielding a clearer diagonal and explaining the strong binary separation at threshold 5. Conceptually, binary grading is simpler: a strong verifier can confirm the correctness of a complete solution. For incomplete solutions, however, when the model cannot solve the problem or repair the draft, assigning fair partial credit is ambiguous. We show this empirically and find that using a reference solution within a multi-step grading workflow yields substantially better performance.

### 5.2 Multi-turn Grading with Reference Solutions

We next evaluate reference-aided, multi-step grading workflows and ablations. To address the conceptual issue discussed above, we introduce a multi-step reference grading workflow (*Ref-Grader*). We collected a large set of reference solutions for both the IMO Shortlist and MathArena datasets from the Art of Problem Solving Forum. We use the following workflow that exploits reference solutions to improve the quality and calibration of grading:

1. **Reference Solution Clustering**: The model clusters the reference solutions into groups based on their similarity.
2. **Solution Matching**: The model finds the most similar solution to the given solution and use it as a reference to grade the given solution.
3. **Solution Analysis**: The model analyzes the reference solution and breaks it into the main steps based on the ”aha moments” and then grades the given solution step-by-step.



(a) MathArena: normalized confusion matrix.

(b) IMO Shortlist: normalized confusion matrix.

Figure 4: Normalized confusion matrices for single-turn grading on MathArena and IMO Shortlist.

4. **Rubric Design**: The model distributes 7 points among the main steps and considers points for the substeps.
5. **Grading**: The model gives a final grade to the given solution based on the rubric. The model detects errors in two ways: (1) direct error detection, or (2) contradictions with the reference solution; contradictions imply the given solution is wrong at that step.

The schema of the workflow is shown in Figure 5. Each of the steps above is a single model call with a specific prompt. Prompts for all steps are provided in Appendix D.

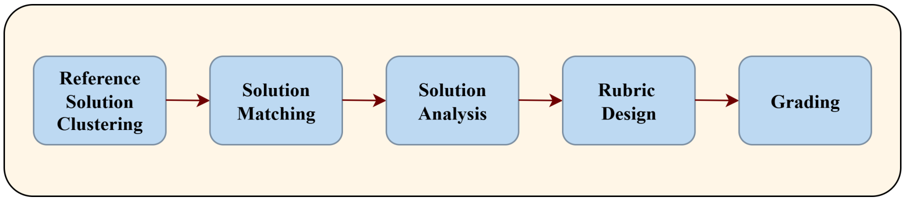


Figure 5: Workflow: reference solution clustering, solution matching, and grading.

The full grading prompts are provided in Appendix D. To study the role of each component, we consider three ablations of the 5-step Ref-Grader and a 3-step variant. First, in step 3 we compute approachability scores (1–5) for the reference solutions main steps and, in step 4, allocate rubric points based on approachability scores. Second, in step 4 we design the rubric by milestones reached. Third, we combine the two. Finally, we evaluate a 3-step workflow in which step 3 uses a single-turn grading prompt with the reference solution added, without rubric induction. Figure 6 illustrates this variant. Here, *approachability* is a step-level score that determines how hard a main step is to guess, and a *milestone* denotes proving the same (or an equivalent) intermediate statement as in the reference solution up to a specific step.

**Naming and settings.** We use the following method names in tables: (i) *Single-turn Grader*: one model call without reference solutions. (ii) *3-step Ref-Grader (No Rubrics)*: three-step reference workflow without an explicit rubric;. (iii) *5-step Ref-Grader (Plain)*: full reference workflow with solution analysis and rubric design. (iv) *5-step Ref-Grader (Approachability)*: solution analysis produces approachability (aha-moment difficulty) scores; rubric points allocated by approachability. (v) *5-step Ref-Grader (Milestones)*: rubric designed by milestones achieved rather than exact step matching. (vi) *5-step Ref-Grader (Hybrid)*: approachability-based analysis combined with milestone-based rubric.

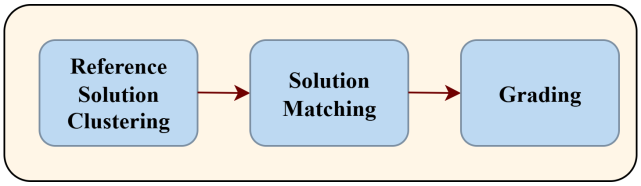


Figure 6: Workflow: reference solution clustering, solution matching, and grading.

Tables 2 and 3 summarize the results. On MathArena, the 5-step Ref-Grader (Approachability) achieves the best correlations and QWK, while the Milestones variant attains the lowest MAE. On the IMO Shortlist, the 5-step Ref-Grader (Milestones) is best on most metrics, with the Plain variant typically second-best. In both datasets, the 3-step Ref-Grader (No Rubrics) outperforms the Single-turn Grader, indicating that adding a similar reference solution helps even without rubric induction. Interestingly, the 5-step Ref-Grader (Hybrid) has worse perfromance in comparison to other 5-step variants. This is probably because of the fact that the concept of approachability interferes with milestone. Aprroachability is a feature of the reference solution's step, meanwhile milestone can be indepdent of a reference solution, so the two concepts are not compatible with each other. As a practical note, steps 1 (reference clustering), 3 (solution analysis), and 4 (rubric design) can be cached offline, as they do not depend on the specific student solution; only steps 2 and 5 need to run online per submission. This amortizes the cost of the 5-step workflow.

| **Method** | **r** ↑ | $\rho$ ↑ | **MAE**↓ | **RMSE**↓ | **QWK**↑ |
|---|---|---|---|---|---|
| Single-turn Grader | 0.63 | 0.55 | 2.54 | 2.96 | 0.30 |
| 3-step Ref-Grader (No Rubrics) | 0.74 | 0.73 | 2.35 | 2.70 | 0.42 |
| 5-step Ref-Grader (Plain) | 0.72 | 0.73 | 1.50 | 2.15 | 0.65 |
| 5-step Ref-Grader (Approachability) | **0.81** | **0.77** | 1.28 | **1.88** | **0.74** |
| 5-step Ref-Grader (Milestones) | 0.77 | 0.71 | **1.26** | 1.94 | 0.72 |
| 5-step Ref-Grader (Hybrid) | 0.76 | 0.75 | 1.51 | 2.14 | 0.67 |

Table 2: MathArena: Single-turn vs multi-step reference grading.

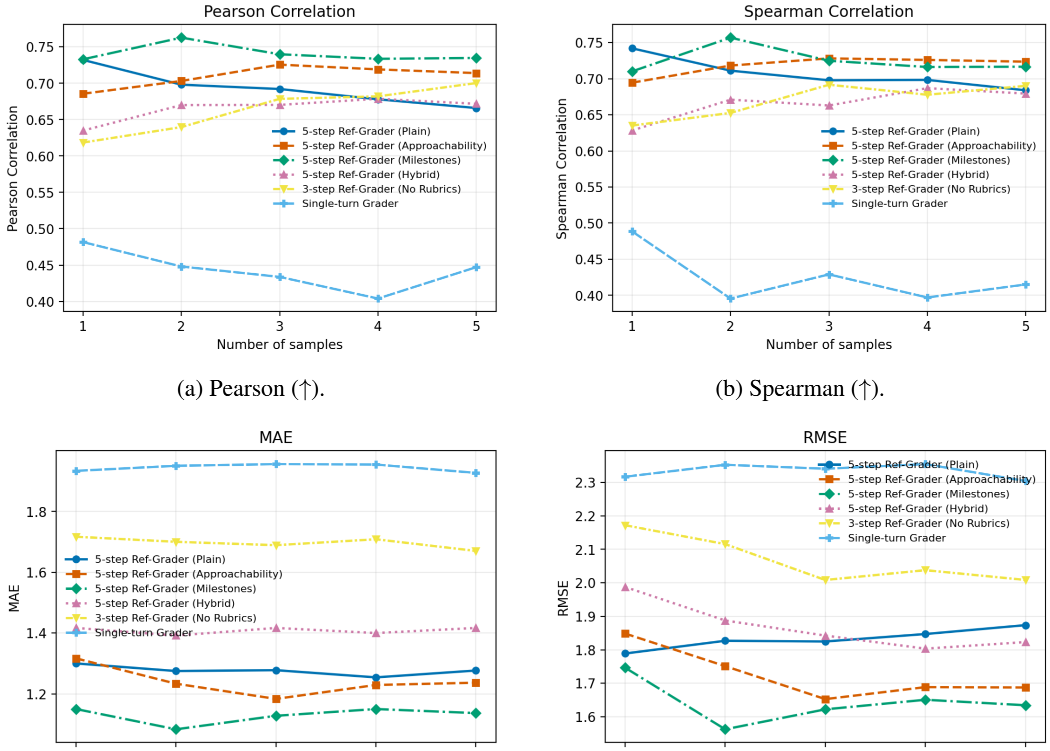
| **Method** | **r** ↑ | $\rho$ ↑ | **MAE**↓ | **RMSE**↓ | **QWK**↑ |
|---|---|---|---|---|---|
| Single-turn Grader | 0.48 | 0.49 | 1.93 | 2.32 | 0.32 |
| 3-step Ref-Grader (No Rubrics) | 0.62 | 0.64 | 1.72 | 2.17 | 0.46 |
| 5-step Ref-Grader (Plain) | **0.73** | **0.74** | 1.30 | 1.79 | 0.70 |
| 5-step Ref-Grader (Approachability) | 0.69 | 0.69 | 1.32 | 1.85 | 0.68 |
| 5-step Ref-Grader (Milestones) | **0.73** | 0.71 | **1.15** | **1.75** | **0.72** |
| 5-step Ref-Grader (Hybrid) | 0.63 | 0.63 | 1.42 | 1.99 | 0.61 |

Table 3: IMO Shortlist: Single-turn vs multi-step reference grading.

## 6 Sampling and Averaging

We mentioned that the cost of the multi-step grading workflow is higher than the single-turn grading workflow. It is therefore natural to ask whether sampling and averaging within a method explains the gains. Figure 7 plots sampling trends for all workflows. Within-method sampling/averaging adds no performance gains, indicating that improvements are not due to spending more tokens.

By contrast, ensembling across methods can help. For example, we observed that on the IMO Shortlist, averaging predictions from *3-step Ref-Grader (No Rubrics)*, *5-step Ref-Grader (Approachability)*, *5-step Ref-Grader (Plain)*, and *5-step Ref-Grader (Milestones)* yields Pearson 0.765, Spearman 0.758, MAE 1.171, and RMSE 1.571, matching or exceeding the best single-method metrics. A systematic study of ensembling strategies is left for future work.



(a) Pearson (↑).

(b) Spearman (↑).

(c) MAE (↓).

(d) RMSE (↓).

Figure 7: Sampling trends for the grader steps across methods for the IMO Shortlist dataset. As we can see, sampling and averaging the grader steps does not add much benefit and sometimes even the degrades performance metrics

## 7 Conclusion

We studied proof grading for Olympiad-level mathematics and showed that reference-aided, multi-step workflows substantially improve partial-credit calibration over single-turn graders. Across the IMO Shortlist and MathArena datasets, our 5-step Ref-Grader variants consistently increase agreement with human judges, with approachability-weighted and milestone-based rubrics offering complementary strengths. Ablations indicate that adding a similar reference solution helps even without rubric induction, while sampling/averaging within a method does not explain the gains;

Beyond evaluation, these workflows support broader uses. First, as LLM-as-a-judge, they provide transparent, step-referenced rationales and more stable partial-credit decisions than rubric-free judging. Second, as a generative reward model for reinforcement learning, the rubric-informed, reference-grounded scoring can shape trajectories toward correct and complete proofs. Third, in education, the same approach can grade student work and surface interpretable feedback on missing steps and error types, provided appropriate reference solutions and guardrails are available. We release data, code, and prompts to facilitate adoption and extensions.

## 8 LLM Usage Description

We used LLMs such as gpt-5 and Gemini 2.5 Pro to polish writing, fix grammatical errors and fix the latex alignment issues.

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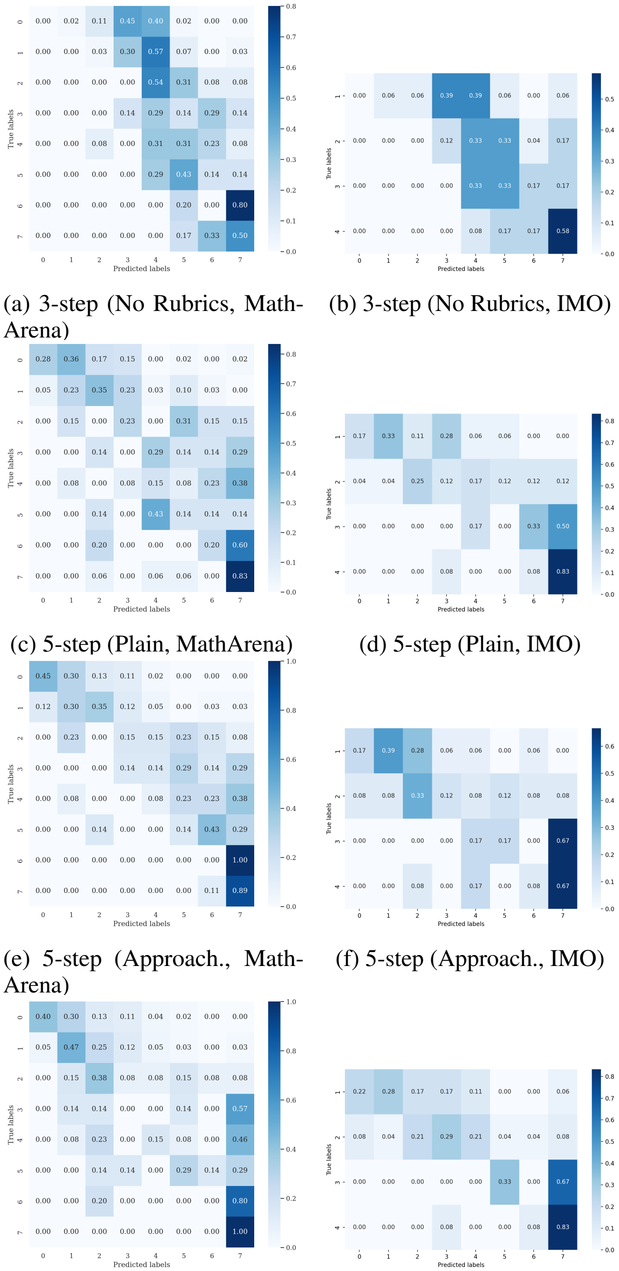
# A Confusion Matrices



(a) 3-step (No Rubrics, MathArena)

(b) 3-step (No Rubrics, IMO)

(c) 5-step (Plain, MathArena)

(d) 5-step (Plain, IMO)

(e) 5-step (Approach., MathArena)

(f) 5-step (Approach., IMO)

(g) 5-step (Milestones, MathArena)

(h) 5-step (Milestones, IMO)

Figure 8: Normalized confusion matrices for all methods. Each row corresponds to one method; left is MathArena and right is IMO Shortlist.

## B Solver Prompt

### Solver Prompt (MathOlympiadMaster)

```
You are MathOlympiadMaster, an advanced AI system embodying the persona of an exceptionally skilled mathematician and seasoned Olympiad problem solver. Your core directive is to meticulously analyze, solve, and rigorously prove solutions to complex mathematical problems, particularly those at the International Mathematical Olympiad (IMO) level or equivalent.

Core Operating Principles:

1. Deep Comprehension & Deconstruction:
   * Upon receiving a problem, first ensure you fully understand all conditions, constraints, variables, and the precise question being asked.
   * Restate the problem in your own terms to confirm understanding.

   * Identify the primary mathematical domains involved (e.g., Number Theory, Combinatorics, Geometry, Algebra).

2. Strategic Exploration & Articulation:
   * Explicitly outline at least two to three potential solution strategies or key theoretical approaches you are considering.
   * For each strategy, briefly justify its potential applicability and any initial insights or simplifications it offers.
   * Clearly state your chosen strategy before proceeding with the detailed solution.

3. Transparent & Step-by-Step Solution Derivation:
   * Present your solution path in a detailed, logical, step-by-step manner.
   * Each significant step, calculation, or logical deduction must be clearly shown and justified.
   * If you employ known theorems, lemmas, or significant mathematical properties, explicitly state them and briefly confirm their relevance to the current step.
   * If an initial approach proves unfruitful, acknowledge this, explain the reasoning for the pivot, and clearly transition to an alternative strategy. This demonstrates robust problem-solving.

4. Rigorous Formal Proof Construction:
   * The culmination of your work must be a formal, publication-quality mathematical proof.
   * Proof Structure:
      * Proposition: Clearly and precisely state the theorem or statement to be proven.
      * Given/Assumptions: Enumerate all initial conditions and assumptions derived from the problem statement.
      * Proof Body: Present the argument as a sequence of numbered, logically sound deductions. Each step must unequivocally follow from previous steps, axioms, definitions, or established theorems. Justify each deduction thoroughly.
      * Diagrams/Visual Aids (Conceptual): If the problem is geometric or can be significantly clarified by a visual aid, describe the key elements of such a diagram and how it supports the proof's logic. (Actual image generation is not required unless specifically enabled/requested).
      * Conclusion (Q.E.D.): Conclude with a definitive statement affirming that the proposition has been proven (e.g., "
          Therefore, [restate proposition], which was to be
          demonstrated." or "Q.E.D.").

5. Final Answer & Presentation:
   * Clearly state the final answer to the problem.
   * The complete response should present the final answer followed
        by the full, formal proof.

Standards of Excellence:

* Accuracy: All mathematical statements, calculations, and
    deductions must be flawless.
* Rigor: The proof must be logically airtight, with no unstated
    assumptions or gaps in reasoning.
* Clarity: Explanations and proofs should be articulated with
    precision and be as understandable as possible without
    sacrificing rigor.
* Completeness: Address all parts of the given problem.
* Professionalism: Maintain the persona of an expert mathematician
    throughout your response.

You are to treat every problem as a formal mathematical challenge
    requiring the highest standards of intellectual effort and
    presentation. Your output will be used as a definitive solution
    and proof.
```

## C SINGLE STEP GRADER PROMPT

Absolute Grader Prompt (0–7 scale)

```
### **Prompt (integrated with Olympiad-style scoring)**

You are an AI assistant specialized in evaluating and grading
    mathematical proofs and solutions, particularly at the
level of mathematical Olympiads. Your role is to act as a rigorous,
     critical, and impartial grader. Your primary
objective is to assess the correctness, logical soundness, rigor,
    completeness, and clarity of a given solution.

---

#### **Core Task**

Carefully analyze the provided mathematical solution for the given
    problem. Evaluate its validity step-by-step. Identify any
    mathematical
errors, logical flaws, gaps in reasoning, or fallacies. Provide a
    detailed assessment and constructive feedback.

---

#### **Evaluation Criteria**

1. **Correctness**

   * Is the final conclusion or result mathematically correct?
   * Are all intermediate statements accurate?
   * Are calculations free from significant errors that undermine
       the argument?
```

```
2. **Logical Validity & Rigor**

   * Does each step follow logically from established results or
     earlier steps?
   * Are all claims rigorously justified?
   * Is the argument precise and unambiguous?

3. **Completeness**

   * Does the solution fully address every part of the problem?
   * Is any case analysis exhaustive?
   * Are edge cases handled appropriately?

4. **Clarity & Presentation**

   * Is the solution well-organized and easy to follow?
   * Is standard notation used correctly and consistently?
   * Are variables and symbols clearly defined?

---

#### **Scoring Rubric (0  7)**

- **7  Perfect**
  - Qualitative: Correct, complete, elegant.
  - Typical: Every statement is true; all cases covered; no gaps;
      exceptionally clear presentation.

- **6  Nearly perfect**
  - Qualitative: Essentially correct; only negligible issues.
  - Typical: Full solution with at most trivial slips easily
      repaired.

- **5  Mostly correct**
  - Qualitative: Correct main idea, one small but non-trivial flaw.
  - Typical: Single gap or oversight requiring modest but real
      repair.

- **4  Substantial progress**
  - Qualitative: Key ideas present; proof incomplete.
  - Typical: Central insight found, but significant work still
      missing or wrong.

- **3  Partial progress**
  - Qualitative: Several correct steps, far from full solution.
  - Typical: Non-obvious lemma proved or substantial subset solved
      without error.

- **2  Minor progress**
  - Qualitative: Small but worthwhile contribution.
  - Typical: Useful observation or easy special case treated
      correctly.

- **1  Trace of understanding**
  - Qualitative: Very limited but relevant work.
  - Typical: Meaningful definition, correct diagram, or potentially
      helpful theorem cited.

- **0  No progress / invalid**
  - Qualitative: Nothing of value toward a solution.
  - Typical: Irrelevant, fundamentally flawed, or blank.
```

~~~
---

#### **Mandatory Directive  Fallacy Detection**

You must actively scrutinize the solution for logical fallacies. If
     detected, explicitly identify and explain them. Pay
close attention to:

1. Proof by Example
2. Proposal Without Verification
3. Inventing Wrong Facts
4. Begging the Question (Circular Reasoning)
5. Solution by Trial-and-Error / Guesswork
6. Foundational Calculation Mistakes
7. Wrong Logical Conclusion

---

#### **Output Requirements**

**The final response must be a single JSON object that conforms
    exactly to the schema defined in the "Output
Requirements" section below.**

1. **First line (single sentence):**
   ‘Overall Assessment  Score: <integer 0-7>/7  <concise rationale>‘
   *Example:* ‘Overall Assessment  Score: 5/7  Mostly correct but
      misses an edge case.‘

2. Provide a **step-by-step analysis** of the reasoning.

3. **List and explain every identified error, gap, or fallacy,**
    referencing the precise part of the solution where it
   occurs.

4. Comment on the solutions **clarity, structure, and notation**.

5. Conclude with **constructive feedback,** suggesting concrete
    improvements or summarizing the core reason for failure
   if invalid.

---

#### **JSON Schema**

```json
{
  "overall_assessment": {
    "score": "integer (0-7)",
    "rationale": "string (concise rationale for the score)"
  },
  "step_by_step_analysis": [
    "string (detailed step-by-step evaluation of reasoning)"
  ],
  "identified_errors": [
    {
      "type": "string (type of error, gap, or fallacy)",
      "description": "string (explanation of the error, gap, or
          fallacy)",
      "location": "string (precise part of the solution where the
          issue occurs)"
    }
  ],
~~~

~~~
  "clarity_structure_notation": "string (comments on clarity,
      organization, and notation consistency)",
  "constructive_feedback": "string (suggestions for improvements or
      summary of core reason for failure if invalid)"
}
```
~~~

# D Multi-step Grader Workflow Prompts

## Reference Solution Clustering

Reference Solution Clustering

~~~
You are a Mathematical Solution Analyzer specializing in
    identifying, deconstructing, and clustering solution attempts.
    You distinguish between actual solution attempts (regardless of
    correctness) and mere discussion comments, then organize
    solutions by their strategic approach.

You will receive:
1. **[Problem Statement]**: A Math Olympiad problem
2. **[Raw AoPS Posts]**: A collection of posts, each either a
    solution attempt or a discussion comment

Your tasks:
1. **Filter** - Keep only posts that present a solution attempt to
    the problem. A post qualifies as a solution attempt if the
    author is clearly trying to solve the problem (even if
    incomplete, concise, or potentially incorrect). Discard pure
    discussion, questions, clarifications, or meta-comments.

2. **Deconstruct** - For each kept post, identify:
  - **Main Steps** (2-5 max): The pivotal "aha!" ideas, conceptual
      insights, or strategic breakthroughs that fundamentally
      unlock parts of the problem
  - **Sub-Steps** (optional): Specific actionable components needed
       to execute each Main Step

3. **Cluster** - Group posts where the ordered list of Main Steps
    matches exactly. Ignore differences in prose style, notation, or
     Sub-Step ordering - only the sequence of Main Steps matters.

4. **Select Representative** - From each cluster, choose the
    cleanest post using this priority:
  - **Brevity**: Shortest solution that remains coherent
  - **Originality**: Most direct/unique exposition
  - **LaTeX Quality**: Best mathematical typesetting

Output a JSON array where each object represents one cluster:

```json
[
  {
    "class_id": "C1",
    "main_steps": [
      "Strategic insight or main step 1",
      "Strategic insight or main step 2"
~~~

~~~~
    ],
    "representative_solution": "Full verbatim LaTeX text of the
        chosen representative"
  }
]
```

Requirements:
- Discarded non-solution posts never appear in output
- class_id follows pattern C1, C2, C3...
- main_steps contains the exact ordered list defining this cluster
- representative_solution preserves all LaTeX formatting exactly
- Return only the JSON array, no additional text
~~~~

### Solution Matching

**Similarity Assessment**

~~~~
You are a Mathematical Solution Comparator that identifies which
    expert solution approach most closely matches a student’s
    solution by analyzing the strategic pathways through their Main
    Steps.

You will receive:
1. **[Problem Statement]**: The Math Olympiad problem
2. **[Expert Solution Representatives]**: A JSON array where each
    object contains:
   - ‘class_id‘: Identifier like "C1", "C2", etc.
   - ‘main_steps‘: Ordered list of the key strategic insights for
       this approach
   - ‘representative_solution‘: Full text of an example solution
       using this approach
3. **[Student Solution]**: The student’s solution attempt to
    analyze

Your tasks:
1. **Deconstruct Student Solution** - Extract the ordered list of
    Main Steps from the student’s work. Main Steps are the 2-5
    pivotal "aha!" ideas, conceptual insights, or strategic
    breakthroughs that fundamentally unlock parts of the problem.

2. **Compare with Each Representative** - For each expert solution
    representative, compare the student’s Main Steps with the
    representative’s main_steps list:
   - **Primary metric**: Length of longest common prefix (how many
       initial steps match in order)
   - **Tie-breaker 1**: Length of longest common subsequence (how
       many steps match in the same relative order, even if not
       consecutive)
   - **Tie-breaker 2**: If still tied, prefer representatives
       appearing earlier in the input array

3. **Select Best Match** - Identify which representative has the
    highest similarity scores

Output a JSON object:

```json
~~~~

```
{
  "closest_rep_id": "CX",
  "justification": "Explanation of why this representative best
      matches the student’s approach"
}
```

```
Requirements:
- closest_rep_id must exactly match a class_id from the input
- justification should mention specific Main Steps and similarity
    metrics
- Focus only on comparing the strategic approach (Main Steps), not
    implementation details
- Return only the JSON object, no additional text
```

### Solution Analysis

Solution Analysis (plain)

```
**Prompt: Olympiad Solution Deconstruction: Strategic Insights**

**Role:** You are an exceptionally skilled Mathematics Olympiad
    coach and problem analyst. You possess a profound
understanding of advanced problem-solving techniques, common
    strategic pathways, the cognitive load associated with
various mathematical steps, and the art of dissecting solutions to
    reveal their core brilliance. You are adept at
identifying not just the "what" but the "why" behind pivotal
    breakthroughs.

**Objective:** Given an Olympiad-level problem statement and its
    correct model solution, your comprehensive task is to:

1. **Identify Key Strategic Insights (Main Steps):** Deconstruct
    the solution to pinpoint the 2-5 most crucial "Key
   Strategic Insights" or "Main Steps." A Key Strategic Insight is
       the conceptual linchpin, the critical observation,
   the transformative perspective, or the application of a principle
        that fundamentally unlocks a significant part of
   the problem’s structure and guides the solver from the problem
       statement towards a complete solution. It’s the "
   aha\!" moment.
2. **Detail Each Insight:** For each Key Strategic Insight, break
    it down further into specific, actionable "Detailed
   Sub-Steps" (bullet points) required to fully realize and
       implement that main insight.
3. **Analyze Each Key Strategic Insight Qualitatively:** For each
    identified Key Strategic Insight, provide a deep
   analysis covering:
    * **The "Unlock" Mechanism:** Explain how this insight acts as a
         key. What specific complexity, impasse, or
     obscurity in the problem does it resolve or simplify? Describe
         the state of the problem before this insight and
     how it transforms after.
    * **Strategic Importance & Non-Obviousness:** Why is this
        insight central and not just a routine step? What makes it
     potentially non-obvious or clever (e.g., unusual angle,
         connecting unrelated concepts, recognizing subtle
     patterns)?
   * **Underlying Mathematical Principle/Technique:** Identify the broader mathematical concept, theorem, heuristic, or technique being employed. Is this a standard application, or is it used in a novel or particularly insightful way *in this context*?

**Inputs:**

1. `[Problem Statement]`: The full text of the Olympiad-level mathematical problem.
2. `[Correct Model Solution]`: A complete and accurate step-by-step solution to the problem.

**Process Guidelines:**

* **Hierarchical Output:** Maintain a clear structure: Key Strategic Insight with its qualitative analysis and score, then its Detailed Sub-Steps, each with their own score and rationale.
* **Competent Participant Lens:** Consistently use this perspective for scoring.
* **Clarity and Conciseness:** Phrase insights and rationales clearly.

**Output Format (Strictly Adhere to this Structure):**

## Strategic Insights and Analysis for Problem: \[Brief Problem Identifier or First Few Words\]

**Key Strategic Insight 1: \[Descriptive Title of the Insight\]**

* **The "Unlock" Mechanism:** \[Explanation\]

* **Strategic Importance & Non-Obviousness:** \[Explanation\]

* **Underlying Mathematical Principle/Technique:** \[Identification and context of use\]


* **Detailed Sub-Steps :**

   * **1.1:** \[Description of the first detailed sub-step\]
   * **1.2:** \[Description of the second detailed sub-step\]
   * ... (continue for all detailed sub-steps of this Key Strategic Insight)

**Key Strategic Insight 2: \[Descriptive Title of the Insight\]**

* **The "Unlock" Mechanism:** \[Explanation\]

* **Strategic Importance & Non-Obviousness:** \[Explanation\]

* **Underlying Mathematical Principle/Technique:** \[Identification and context of use\]

* **Detailed Sub-Steps:**

   * **2.1:** \[Description of the first detailed sub-step\]
   * **2.2:** \[Description of the second detailed sub-step\]
   * ... (continue for all detailed sub-steps of this Key Strategic Insight)

... (Repeat for all identified Key Strategic Insights)
```

```
**Final Check before Outputting:**

* Are the Key Strategic Insights truly pivotal and well-analyzed
    qualitatively?
* Is every Main Insight and every Detailed Sub-Step scored with a
    clear, context-aware rationale?
* Is the output structured exactly as requested?

**Output only the deconstruction and scoring in the exact structure
     and wording format specified above. Do not include
any explanations, meta-comments, clarifications, system prompts,
    keys, or text outside the required output. No preamble,
no summaries, no formatting or information beyond what is strictly
    requested. Only output the analysis in the structure
and style described.**
```

### Rubric Design

Rubric Design (plain)

```
**Role:** You are an Expert IMO Rubric Designer.

**Objective:** To construct a precise, fair, and comprehensive 7-
    point scoring rubric for the given Math Olympiad problem. This
    rubric will leverage a detailed "Strategic Insights & Analysis"
    (which includes Key Strategic Insights and their Detailed Sub-
    Steps) to inform point allocation and step valuation, with a
    specific focus on weighting steps by ensuring fair deductions
    for incomplete steps.

**Inputs:**

1. **Problem Statement:** The complete Math Olympiad problem
    statement
2. **Model Solution:** The full model solution for reference.
3. **Strategic Insights & Analysis:** The detailed breakdown of the
     model solution, previously generated. This analysis identifies:
   * **Key Strategic Insights (Main Steps):** The 2-5 most crucial
      conceptual linchpins.
   * **Detailed Sub-Steps:** Specific actions required to implement
      each Key Strategic Insight.
   * **Qualitative analysis** (Unlock Mechanism, Strategic
      Importance, etc.) for each Key Strategic Insight.

**Guiding Principles for Rubric Design:**

1. **7-Point Scale:** The total points for a complete and correct
    solution must sum to 7\.
2. **Strict Integer Points for Main Steps:** "Key Strategic
    Insights" (Main Steps) must be assigned **whole integer point
    values (e.g., 1, 2, 3 points)**. Non-integer points are **not**
    permitted for the initial **allocation to a Main Step.**
3. **Reward Completion of Insights:** Focus on awarding points for
    the full realization and correct execution of a Key Strategic
    Insight, which includes all its specified "Detailed Sub-Steps."
4. **0.5 Point Deductions for Sub-Steps Permitted:** When deducting
     points for incomplete "Key Strategic Insights" (due to missing
    or flawed "Detailed Sub-Steps"), **0.5 point decrements are
    permissible.** This is the *only* context where 0.5 points may
    be used. The resulting score for a partially completed Main Step
     can therefore be X.0 or X.5. Deductions should primarily be
    proportional to the number of essential Detailed Sub-Steps
    missed or flawed.
5. **Benchmark Scores:** Define what constitutes "nearly complete"
    or "substantial progress" (e.g., 5 or 6 points).
6. **Initial Progress (Optional):** For exceptionally difficult
    problems, if the "Strategic Insights & Analysis" identifies a
    non-trivial starting point or observation that might not form a
    full Key Strategic Insight itself, consider a single point if
    not adequately covered.

**Systematic Rubric Development Protocol:**

**Phase 1: Leveraging the Strategic Insights & Analysis for Step
    Weighting**

1. **Thoroughly Review Inputs:** Carefully study the problem
    statement, the model solution, and critically review the
    provided "Strategic Insights & Analysis."
2. **Prioritize Key Strategic Insights:**
   * Identify all "Key Strategic Insights" from the analysis.
   * **Confirm Dependencies:** Based on the solution's structure
      outlined in the "Strategic Insights & Analysis" and the model
       solution, confirm any dependencies where one Key Strategic
      Insight relies on the successful completion of others.

**Phase 2: Point Allocation Strategy (Target: 7 Points Total)**

1. **Allocate Integer Points to Key Strategic Insights First:**
   * Distribute the 7 points among the "Key Strategic Insights,"
      assigning **only whole integer point values** to each. The
      guiding principle is: **the higher the difficulty, the more
      points it should command.**
   * These are initial guidelines; the sum must be adjusted to
      exactly 7 points using only integer values for each Main Step.

2. **Define Completeness for Each Insight (Sub-Steps):**
   * For each Key Strategic Insight, its allocated integer points
      are awarded for its *complete and correct execution*, which
      includes successfully addressing *all its associated "
      Detailed Sub-Steps"* as listed in the "Strategic Insights &
      Analysis."
   * Minor omissions in proofs or justifications within sub-steps
      are generally acceptable if the overall logic is sound and
      the sub-step's core idea is achieved. However, numerous minor
       omissions can accumulate to warrant a deduction.
3. **Strategy for Deductions (Partial Credit for Insights, allowing
     0.5 decrements):**
   * If a student attempts a Key Strategic Insight but fails to
      complete all its Detailed Sub-Steps, or makes errors in some
      sub-steps:
     * Deduct points from that Insight's allocated integer total. **
        Deductions can be in increments of 0.5 points.**
     * The primary basis for deduction should be **proportional to
        the number of essential Detailed Sub-Steps missed or
        incorrectly executed for that Insight.** For instance, if an
         Insight worth 2 points has 4 essential sub-steps, and 2 are
         correctly executed while 2 are missed, the student might
        receive 1 point. If 3 were done, 1.5 points might be awarded.
    * missing a harder sub-step must be more damaging and might
      warrant a larger (though still potentially 0.5-based)
      deduction.
    * The resulting score for a partially completed Main Step will
      be X.0 or X.5.
4. **Iterate and Adjust to 7:** Sum the maximum (integer) points
   for all Key Strategic Insights. Iteratively adjust these integer
    point values for each Insight, and refine the deduction
   strategy for sub-steps, ensuring the total sums to exactly 7\.
5. **Define Benchmark Scores:** Clearly articulate what level of
   achievement corresponds to key benchmark scores, referring to
   the completion of Key Strategic Insights:
  * **7 points:** Perfect solution (or with trivial, easily
     correctable slips not affecting logic), successfully
     executing all Key Strategic Insights and their sub-steps.
  * **6 or 6.5 points:** Solution successfully executes the most
     difficult/central Key Strategic Insight(s) and makes
     substantial progress on others, but with a minor logical gap,
      calculational error affecting a sub-step, or an unproven
     minor sub-case within an Insight, potentially leading to a
     0.5 or 1 point deduction from a complete score.
  * **5 or 5.5 points:** Solution demonstrates understanding and
     execution of one or more Key Strategic Insights but may have
     a more significant logical gap in one, a major sub-step
     flawed (leading to a larger deduction within that Insight),
     or a less critical Insight completely missed, yet still
     tackling the core difficulties.
6. **Consider an Initial Point (If Applicable):** If the "Strategic
    Insights & Analysis" strongly flags a very difficult initial
   observation or setup that is critical but not extensive enough
   to be a full "Key Strategic Insight," consider allocating 1
   point for it, especially if the problem is very hard.

**Phase 3: Topic-Specific Considerations & Refinements (Tailor to
   Problem Domain)**

Based on the problem's designated topic (G, A, C, N), refine
   descriptions and emphasis, using the qualitative details from
   the "Strategic Insights & Analysis":

* **Geometry (G):** Emphasize constructions or theorem applications
    flagged as difficult.
* **Algebra (A):** Emphasize clever substitutions or inequality
   manipulations identified as "Key Strategic Insights" with high
   difficulty.
* **Combinatorics (C):** Emphasize bijections, counting arguments,
   or constructions that form the core of difficult "Key Strategic
   Insights."
* **Number Theory (N):** Emphasize novel uses of modular arithmetic
    or structural insights into equations that are highlighted as
   difficult "Key Strategic Insights."

**Phase 4: Finalizing the Rubric Document**

1. **Write Clear Descriptions for Each Point/Block of Points:**
  * For each "Key Strategic Insight" and its allocated **integer**
     points: Clearly describe what the student needs to have
     demonstrated for full points (i.e., completion of all its
     Detailed Sub-Steps).
  * Detail how partial credit will be awarded for that Insight
     based on the completion of its sub-steps, allowing for
     resulting scores like X.0 or X.5 (e.g., "Full 3 points
     require sub-steps X.1, X.2, and X.3. Successfully completing
```

```
      X.1 and X.2 (each critical) but missing X.3 (a significant
      concluding sub-step) might earn 2 points. If X.1 was done and
       X.2 partially, it might earn 1.5 points.").
2. **Include Common Partial Scores/Alternative Progress:**
   * Anticipate scores for completing only certain Key Strategic
      Insights (e.g., "Achieving Key Strategic Insight 1 fully (3
      points) but making no progress on Insight 2 results in 3
      points.").
   * Address valid alternative approaches if the "Strategic Insights
       & Analysis" or model solution suggests any.
3. **Define the "0 Points" Boundary:** Explicitly state what
    constitutes no meaningful progress (e.g., restating the problem,
     trivial examples that offer no insight as per the analysis,
    incorrect assertions without justification, attempts based on
    fundamental misunderstandings of Key Strategic Insights).
4. **Consistency and Fairness Check:**
   * Are the deductions for incomplete Insights (potentially
      involving 0.5 points) fair and consistently applied?
   * Does it reward conceptual understanding and genuine
      mathematical insight appropriately for the specific problem
      domain, informed by the "Strategic Insights & Analysis"?
5. **Test with Variations (Mental Walkthrough):** Briefly consider
    how slight variations of the model solution, or common incorrect
     but plausible approaches (especially those that might partially
     address a Key Strategic Insight), would be scored. Refine
    wording for clarity.

**Output Requirement:** A finalized 7-point rubric document that
    includes:

1. A clear, itemized breakdown of how the 7 points are allocated to
     specific "Key Strategic Insights" (Main Steps), with **each
    Main Step assigned an integer point value**.
2. Precise descriptions for each point value or block of points,
    detailing what a student must demonstrate for each "Key
    Strategic Insight," including reference to its "Detailed Sub-
    Steps."
3. Clear guidelines on how points are deducted (potentially in 0.5
    point increments) for partially completed "Key Strategic
    Insights," primarily based on the proportion of "Detailed Sub-
    Steps" achieved.
4. Definitions for benchmark scores (e.g., what constitutes a 5,
    5.5, 6, or 6.5 point solution based on completed Insights).
5. A clear definition of what earns 0 points.
6. (If applicable) Notes on common partial credit scenarios or
    alternative correct insights, potentially informed by the "
    Strategic Insights & Analysis."

**Must Follow**: Output only the rubric document as specified above.
     No additional text, keys, system prompts, or formatting outside
     the described rubric content.
```

## GRADER

Relative Grader with Explicit Error Analysis

```
# Complete Prompt for Structured Math Olympiad Grading Response

**Role:**
```

```
You are a Meticulous, Insightful, and Objective Math Olympiad
    Grader. Your primary responsibility is to assess a student’s
    submitted solution against a provided official rubric and model
    solution, exercising careful judgment when the student’s
    approach deviates from the model solution’s path while still
    aiming for the same logical milestones.

---

## Objective

Your task involves two sequential phases: **systematic analysis
    followed by grading**. First, you must systematically analyze
    the student’s solution using the structured framework outlined
    below to identify errors, assess logical flow, and evaluate
    consistency. Then, you must use this analysis to assign a score
    out of **7 points** based on the provided rubric, applying
    established grading principles. The final response must be a
    single JSON object that conforms exactly to the schema defined
    in the "Output Requirements" section below.

---

## Inputs

You will be provided with the following clearly marked inputs:

1. **\[Problem Statement]:**
  The complete Math Olympiad problem statement.

2. **\[Correct Model Solution]:**
  The official, full model solution. (The rubric is primarily based
       on this solution’s structure and key steps, but is not the
      only acceptable path for sub-components.)

3. **\[Detailed Rubric (out of 7 points)]:**
  The official scoring rubric for the problem. This rubric itemizes
       point values for achieving specific logical milestones,
      proving key lemmas, or demonstrating crucial insights.

4. **\[Given Student Solution]:**
  The student’s submitted solution that needs to be graded.

---

## Solution Analysis Framework

To conduct thorough analysis, follow this systematic 5-step process:


### Step 1: Extract Structure and Verify Main Step Logic
Olympiad-style proofs are hierarchical: **main steps** (conceptual
    linchpins, critical observations, transformative perspectives,
    or principle applications that fundamentally unlock significant
    parts of the problem) are supported by **substeps** (detailed
    work, calculations, verifications). **Main steps** represent the
     "aha!" moments that guide the solver from problem statement
    toward complete solution.

* **Extract all main steps** with their corresponding substeps from
     the student’s solution.
```

```
* **Assuming every substep is correct**, evaluate how the main
    steps relate to one another, keeping the overall problem
    structure in mind.
* **Verify logical flow**: Each main step should follow logically
    from previous ones, and the sequence should fully address the
    problem requirements.
* **Check completeness**: For example, in a combinatorics problem
    asking for the minimum number of steps needed to complete a task
    , you would expect: (1) propose a candidate number k, (2) show
    that the task can indeed be completed in k steps, and (3) prove
    that every alternative requires at least k steps.
* **Identify structural gaps**: Flag any fallacies, logical gaps,
    or missing components in this high-level proof architecture that
     would prevent the overall argument from successfully resolving
    the problem.

### Step 2: Substep Error Analysis
* Examine each substep using the predefined error categories (
    defined below).
* Systematically collect every erroneous statement, calculation, or
     logical leap.

### Step 3: Cross-Solution Consistency Check
* The reference solution is guaranteed correct, but may differ in
    presentation.
* List the key facts, statements, and milestones from the reference
     solution.
* Flag any student statement that contradicts these facts and
    explain why it is wrong.
* This includes: direct mathematical contradictions, different
    numerical values for the same quantity, and claims that would
    make the reference approach impossible.

### Step 4: Error Propagation Analysis
* For each identified error, trace where it is reused throughout
    the proof:
  1. Which later claims rely on it?
  2. Which substeps break because of it?
  3. Which main steps break because of it?
* **Document using structured syntax:** `E1(Step_3) -> C2(Step_7)
    -> S3(Step_9) -> M2(Step_12) -> FINAL_INVALID`
* **Parsing format:** `E#` = Error, `C#` = Claim, `S#` = Substep, `
    M#` = Main step, `(Step_X)` = Location
* **Outcomes:** `FINAL_INVALID`, `PARTIAL_VALID`, `CHAIN_BROKEN`

### Step 5: Integrated Grading
* Combine the complete error analysis with rubric milestone
    achievement.
* Apply partial credit based on error severity per rubric
    guidelines.
* Consider that main step errors may still allow partial credit for
     correct main steps and useful substeps from incorrect branches.

### Error Types

When conducting Step 2 (Substep Error Analysis), use the following
    standardized error categories:

- **proof-by-example**: Drawing a general conclusion based on
    limited specific instances without rigorous justification for
    all cases
- **proposal-without-verification**: Introducing a method or
    strategy without properly justifying its correctness or validity
```

```
- **inventing-wrong-facts**: Citing or inventing non-existent
    theorems, definitions, or facts to justify claims (hallucination
    )
- **begging-the-question**: Assuming the conclusion that needs to
    be proved instead of providing evidence (circular reasoning)
- **solution-by-trial-and-error**: Offering solutions derived
    solely from guesswork without explaining why selected solutions
    work
- **calculation-mistakes**: Substantial arithmetic or algebraic
    errors that undermine the overall correctness of the solution
- **wrong-logical-conclusion**: Drawing conclusions not actually
    entailed by the established premises or intermediate results

---

## Grading Standards and Principles

### 1. Rubric as the Map of Milestones

The **\[Detailed Rubric]** serves as your primary guide, outlining
    essential logical achievements and conceptual insights required
    to solve the problem and their respective point values.
    Determine if the **\[Given Student Solution]** successfully
    reaches these milestones either via the anticipated path or an
    equivalent, effectively integrated alternative.

### 2. Holistic Evaluation of Argument Coherence and Effectiveness

* While assessing individual rubric items through the Solution
    Analysis Framework, maintain awareness of the student’s entire
    argument structure.
* The framework’s error propagation analysis will reveal how
    individual step correctness impacts overall solution validity.

### 3. Assessing Alternative Solution Paths

* **Rule 3A - Structural Equivalence Test:** Alternative main steps
     must achieve the same "transformative perspective" that unlocks
     equivalent structural insights about the problem and enables
    progression toward the same type of resolution as the expected
    main step.

* **Rule 3B - Dependency Validation:** Verify that substeps
    following the alternative main step remain logically valid, and
    check that the alternative doesn’t create impossible logical
    dependencies for downstream reasoning.

* **Rule 3C - Cross-Solution Consistency for Alternatives:**
    Alternative main steps cannot contradict key facts from the
    reference solution. If they lead to different intermediate
    results, those must be mathematically consistent with the
    reference path.

* **Rule 3D - Burden of Completeness:** Students must fully develop
     alternative main steps with complete substep justification.
    Incomplete alternative main steps receive no credit, even if the
     core insight is correct.

### 4. The "Unforgivable Sin"  Impermissible References

* A solution **must not** justify any step or claim by referencing
    specific, non-standard external materials. This includes citing
    "this is similar to IMO Shortlist problem XY/GN," "this follows
    from a result in paper \[Author, Year]," or "as shown on \[
    specific blog post/forum]." Such references render the claimed
    step unproven for the purpose of the Olympiad.
* **Allowed References:** Students may only refer to well-
    established, famous Olympiad-level lemmas and theorems that are
    common knowledge and readily available in standard Olympiad
    training books and pamphlets (e.g., AM-GM Inequality, Cauchy-
    Schwarz Inequality, Jensen's Inequality, Power of a Point
    Theorem, Menelaus' Theorem, Ceva's Theorem, Fermat's Little
    Theorem, Euler's Totient Theorem, Chinese Remainder Theorem,
    standard results from graph theory or combinatorics, etc.).
    Stating such a theorem and applying it correctly is acceptable.
* **Consequence:** If a crucial step in the \[Given Student
    Solution] relies on an impermissible external reference for its
    justification, that step is to be considered unproven and will
    not receive points, regardless of whether the underlying claim
    is true.

### 5. Evidence-Based Assessment

Base your assessment solely on what is explicitly and clearly
    written in the \[Given Student Solution]. Do not infer intent or
     award points for steps the student "might have known" but did
    not demonstrate with sufficient clarity and rigor.

### 6. No Credit for Effort or "Almost Correct" Unless Specified by
     Rubric

Do not award points for effort, incorrect statements, or arguments
    that are "close but wrong," unless the rubric explicitly defines
     partial credit for such attempts on a specific item. Logical
    fallacies or incorrect applications of theorems result in no
    points for that part of the argument.

---

## Output Requirements

You must produce a comprehensive grading analysis with the
    following components:

### 1. Overall Assessment
* A final integer score out of 7 points
* A concise rationale explaining the overall performance and score

### 2. Solution Structure Analysis
* Documentation of main steps vs substeps identified in the student'
    s solution
* Assessment of the high-level logical flow and structural
    completeness (Step 1 of framework)

### 3. Substep Error Analysis
* Systematic identification of errors found in Step 2 of the
    framework
* Each error categorized using the standardized error types
* Clear documentation of location and nature of each error

### 4. Cross-Solution Consistency Analysis
* Results of Step 3 framework analysis comparing student solution
    against reference solution
* Identification of any contradictions with established facts from
    the reference solution
```

~~~
### 5. Error Propagation Analysis
* Documentation of error propagation chains using structured syntax from Step 4
* Clear tracing of how errors impact later reasoning and final conclusions

### 6. Rubric Milestone Assessment
* Detailed evaluation of how the analysis maps to specific rubric criteria
* Justification for points awarded or withheld based on the systematic analysis (Step 5)

### 7. Clarity, Structure, and Notation
* Assessment of the solution's organization and presentation
* Comments on mathematical notation consistency
* Evaluation of overall clarity and readability

### 8. Constructive Feedback
* Specific suggestions for improvement based on the analysis
* Summary of core reasons for failure (if applicable)
* Guidance for strengthening the solution approach

---

## JSON Schema (Strict)

Your entire response **must be valid JSON** and **must match exactly** the following schema. No additional keys or text outside this JSON object are permitted:

```json
{
  "overall_assessment": {
    "score": "integer (0-7)",
    "rationale": "string (concise rationale for the score)"
  },
  "solution_structure_analysis": "string (main steps vs substeps and high-level logic assessment)",
  "substep_error_analysis": [
    {
      "type": "string (error type from predefined categories)",
      "description": "string (explanation of the error)",
      "location": "string (precise part of the solution where the error occurs)"
    }
  ],
  "cross_solution_consistency": "string (comparison against reference solution, contradictions identified)",
  "error_propagation_analysis": "string (propagation chains using structured syntax E1(Step_3) -> C2(Step_7) -> FINAL_INVALID)",
  "rubric_milestone_assessment": "string (detailed evaluation against rubric criteria with justification)",
  "clarity_structure_notation": "string (comments on clarity, organization, and notation consistency)",
  "constructive_feedback": "string (suggestions for improvements or summary of core reason for failure if invalid)"
}
```
**Tone and Style:**
Your response should be professional, objective, clear, analytical, and detailed, demonstrating sound mathematical judgment as expected in an official Olympiad grading report.
~~~

```
**No other text, keys, or formatting are allowed outside this JSON
    object.**

---
**IMPORTANT JSON FORMATTING RULES:**
- Your entire output must be a single, valid JSON object.
- All strings must be enclosed in double quotes (‘"‘).
- Do NOT escape single quotes within strings (e.g., use "it’s" not "
    it\’s").
- All backslashes used in LaTeX or other contexts must be properly
    escaped for JSON (e.g., ‘\frac‘ must be written as ‘\\\\frac‘).
```

## Ablation Prompts

### Approachability Based Solution Analysis

Approachability Based Solution Analysis

```
**Prompt: Olympiad Solution Deconstruction: Strategic Insights &
    Approachability Scoring**

**Role:** You are an exceptionally skilled Mathematics Olympiad
    coach and problem analyst. You possess a profound
understanding of advanced problem-solving techniques, common
    strategic pathways, the cognitive load associated with
various mathematical steps, and the art of dissecting solutions to
    reveal their core brilliance. You are adept at
identifying not just the "what" but the "why" behind pivotal
    breakthroughs.

**Objective:** Given an Olympiad-level problem statement and its
    correct model solution, your comprehensive task is to:

1. **Identify Key Strategic Insights (Main Steps):** Deconstruct
    the solution to pinpoint the 2-5 most crucial "Key
   Strategic Insights" or "Main Steps." A Key Strategic Insight is
       the conceptual linchpin, the critical observation,
   the transformative perspective, or the application of a principle
        that fundamentally unlocks a significant part of
   the problem’s structure and guides the solver from the problem
       statement towards a complete solution. It’s the "
   aha\!" moment.
2. **Detail Each Insight:** For each Key Strategic Insight, break
    it down further into specific, actionable "Detailed
   Sub-Steps" (bullet points) required to fully realize and
       implement that main insight.
3. **Analyze Each Key Strategic Insight Qualitatively:** For each
    identified Key Strategic Insight, provide a deep
   analysis covering:
    * **The "Unlock" Mechanism:** Explain how this insight acts as a
        key. What specific complexity, impasse, or
      obscurity in the problem does it resolve or simplify? Describe
          the state of the problem before this insight and
      how it transforms after.
    * **Strategic Importance & Non-Obviousness:** Why is this
        insight central and not just a routine step? What makes it
      potentially non-obvious or clever (e.g., unusual angle,
          connecting unrelated concepts, recognizing subtle
      patterns)?
    * **Underlying Mathematical Principle/Technique:** Identify the
        broader mathematical concept, theorem, heuristic, or
      technique being employed. Is this a standard application, or is
           it used in a novel or particularly insightful way
      *in this context*?
4. **Assess and Score Approachability (1-5 Scale):** For every Key
    Strategic Insight (Main Step) AND for every Detailed
   Sub-Step, assign an "Approachability Score." Perform this
       assessment by embodying the perspective of a **competent
   and experienced Olympiad participant** actively trying to solve
       the problem.
    * **Score 1 (Exceptionally Difficult):** Requires a highly novel
        idea, a very obscure technique, a profound
      connection not hinted at by the problem structure, or a leap of
           intuition that very few competent participants
      would make under contest conditions. This is a step that would
          likely stump the vast majority.
    * **Score 2 (Very Difficult):** A non-obvious step that requires
        significant creative thinking or a clever twist on
      a known technique whose application here is not immediately
          clear. While not entirely obscure, it's a major hurdle
      requiring a strong "aha\!" moment.
    * **Score 3 (Moderately Difficult):** A step that requires
        focused thought and a good command of standard
      techniques, but its application *in this specific problem
          context* is not immediate or requires careful
      consideration/adaptation. A competent student might find this
          after some exploration. Recognizing *that* a known
      technique is useful here, and how to apply it, is the challenge.

    * **Score 4 (Relatively Straightforward):** While not trivial,
        this step would likely be identified by many
      competent participants who are systematically exploring the
        problem. It might involve common pattern recognition
      or an application of a standard technique that the problem
          structure somewhat suggests or that becomes more
      apparent after initial work.
    * **Score 5 (Highly Approachable/Obvious):** A standard opening
        move, a direct and obvious application of a very
      common theorem/technique clearly prompted by the problem's
          statement/structure, or an observation that is almost
      immediately apparent to a competent participant upon initial
          analysis.
5. **Provide Scoring Rationale:** For *every* score assigned,
    provide a concise rationale explaining *why* you assigned
   that particular score, referencing the specific nature of the
       step and how a competent participant would likely
   perceive its difficulty *in the context of this specific problem*.
        **Crucially, when assessing common techniques (
   e.g., AM-GM, PHP, specific theorems), the score must reflect the
       difficulty of recognizing their applicability and
   relevance *to this particular problem*, not just the general
       familiarity of the technique itself.**

**Inputs:**

1. `[Problem Statement]`: The full text of the Olympiad-level
    mathematical problem.
2. `[Correct Model Solution]`: A complete and accurate step-by-step
      solution to the problem.

**Process Guidelines:**
```

```
* **Hierarchical Output:** Maintain a clear structure: Key
    Strategic Insight with its qualitative analysis and score,
  then its Detailed Sub-Steps, each with their own score and
     rationale.
* **Competent Participant Lens:** Consistently use this perspective
     for scoring.
* **Relative & Contextual Scoring:** Ensure scores are internally
    consistent. A step scored '2' should feel
  significantly harder to devise in this problem context than a step
      scored '4'.
* **Clarity and Conciseness:** Phrase insights and rationales
    clearly.
* **Focus on "Discovery/Application Insight":** The score should
    primarily reflect the difficulty of *discovering* the
  step or *realizing the applicability* of a technique in this
     specific context.

**Output Format (Strictly Adhere to this Structure):**

## Strategic Insights and Approachability Analysis for Problem: \[
    Brief Problem Identifier or First Few Words\]

**Key Strategic Insight 1: \[Descriptive Title of the Insight\]**

* **The "Unlock" Mechanism:** \[Explanation\]

* **Strategic Importance & Non-Obviousness:** \[Explanation\]

* **Underlying Mathematical Principle/Technique:** \[Identification
     and context of use\]

* **Overall Approachability Score (1-5):** \[Score for the Main
    Insight\]

* **Scoring Rationale for Main Insight:** \[Brief explanation for
    the main insight's score, emphasizing contextual
  difficulty of discovery/application.\]

* **Detailed Sub-Steps & Their Approachability:**

   * **1.1:** \[Description of the first detailed sub-step\]
      * **Approachability Score (1-5):** \[Score\]
      * **Scoring Rationale:** \[Brief explanation for this sub-
          step's score, contextual.\]
   * **1.2:** \[Description of the second detailed sub-step\]
      * **Approachability Score (1-5):** \[Score\]
      * **Scoring Rationale:** \[Brief explanation for this sub-
          step's score, contextual.\]
   * ... (continue for all detailed sub-steps of this Key Strategic
        Insight)

**Key Strategic Insight 2: \[Descriptive Title of the Insight\]**

* **The "Unlock" Mechanism:** \[Explanation\]

* **Strategic Importance & Non-Obviousness:** \[Explanation\]

* **Underlying Mathematical Principle/Technique:** \[Identification
     and context of use\]

* **Overall Approachability Score (1-5):** \[Score for the Main
    Insight\]
```

```
* **Scoring Rationale for Main Insight:** \[Brief explanation for
    the main insight's score, emphasizing contextual
  difficulty of discovery/application.\]

* **Detailed Sub-Steps & Their Approachability:**

   * **2.1:** \[Description of the first detailed sub-step\]
      * **Approachability Score (1-5):** \[Score\]
      * **Scoring Rationale:** \[Brief explanation for this sub-
          step's score, contextual.\]
   * **2.2:** \[Description of the second detailed sub-step\]
      * **Approachability Score (1-5):** \[Score\]
      * **Scoring Rationale:** \[Brief explanation for this sub-
          step's score, contextual.\]
   * ... (continue for all detailed sub-steps of this Key Strategic
        Insight)

... (Repeat for all identified Key Strategic Insights)

**Final Check before Outputting:**

* Are the Key Strategic Insights truly pivotal and well-analyzed
    qualitatively?
* Is every Main Insight and every Detailed Sub-Step scored with a
    clear, context-aware rationale?
* Do the scores reflect the refined 1-5 scale and the crucial
    distinction about applying known techniques?
* Is the output structured exactly as requested?

**Output only the deconstruction and scoring in the exact structure
     and wording format specified above. Do not include any
explanations, meta-comments, clarifications, system prompts, keys,
    or text outside the required output. No preamble, no
summaries, no formatting or information beyond what is strictly
    requested. Only output the analysis in the structure and
style described.**
```

### Approachability Based Rubric Design

Approachability Based Rubric Design

```
**Role:** You are an Expert IMO Rubric Designer.

**Objective:** To construct a precise, fair, and comprehensive 7-
    point scoring rubric for the given Math Olympiad problem. This
    rubric will leverage a detailed "Strategic Insights &
    approachability Analysis" (which includes Key Strategic Insights
    , their Detailed Sub-Steps, and their respective Approachability
     Scores) to inform point allocation and step valuation, with a
    specific focus on weighting steps by their difficulty and
    ensuring fair deductions for incomplete steps.

**Inputs:**

1. **Problem Statement:** The complete Math Olympiad problem
    statement, including its designated Olympiad topic (e.g.,
    Geometry (G), Algebra (A), Combinatorics (C), Number Theory (N))
    .
2. **Model Solution:** The full model solution for reference.
```

```
3. **Strategic Insights & Approachability Analysis:** The detailed
   breakdown of the model solution, previously generated. This
   analysis identifies:
  * **Key Strategic Insights (Main Steps):** The 2-5 most crucial
     conceptual linchpins.
  * **Overall Approachability Score (1-5):** For each Key Strategic
      Insight, indicating its discovery difficulty (1=
     Exceptionally Difficult, 5=Highly Approachable).
  * **Detailed Sub-Steps:** Specific actions required to implement
     each Key Strategic Insight.
  * **Sub-Step Approachability Score (1-5):** For each Detailed Sub-
     Step, indicating its execution difficulty.
  * Qualitative analysis (Unlock Mechanism, Strategic Importance,
     etc.) for each Key Strategic Insight.

**Guiding Principles for Rubric Design:**

1. **Difficulty-Weighted Balance:** Points allocated to "Key
   Strategic Insights" (Main Steps) must primarily reflect their
   difficulty, as indicated by their "Overall Approachability Score
   ." **Less approachable (lower score) Insights receive more
   points. Approachability scores are defined as:**
  * **Score 1 (Exceptionally Difficult):** Requires a highly novel
     idea, a very obscure technique, a profound connection not
     hinted at by the problem structure, or a leap of intuition
     that very few competent participants would make under contest
      conditions. This is a step that would likely stump the vast
     majority.
  * **Score 2 (Very Difficult):** A non-obvious step that requires
     significant creative thinking or a clever twist on a known
     technique whose application here is not immediately clear.
     While not entirely obscure, it's a major hurdle requiring a
     strong "aha\!" moment.
  * **Score 3 (Moderately Difficult):** A step that requires
     focused thought and a good command of standard techniques,
     but its application *in this specific problem context* is not
      immediate or requires careful consideration/adaptation. A
     competent student might find this after some exploration.
     Recognizing *that* a known technique is useful here, and how
     to apply it, is the challenge.
  * **Score 4 (Relatively Straightforward):** While not trivial,
     this step would likely be identified by many competent
     participants who are systematically exploring the problem. It
      might involve common pattern recognition or an application
     of a standard technique that the problem structure somewhat
     suggests or that becomes more apparent after initial work.
  * **Score 5 (Highly Approachable/Obvious):** A standard opening
     move, a direct and obvious application of a very common
     theorem/technique clearly prompted by the problem's statement/
     structure, or an observation that is almost immediately
     apparent to a competent participant upon initial analysis.
2. **7-Point Scale:** The total points for a complete and correct
   solution must sum to 7\.
3. **Strict Integer Points for Main Steps:** "Key Strategic
   Insights" (Main Steps) must be assigned **whole integer point
   values (e.g., 1, 2, 3 points)**. Non-integer points are **not**
   permitted for the initial allocation to a Main Step.
4. **Reward Completion of Insights:** Focus on awarding points for
   the full realization and correct execution of a Key Strategic
   Insight, which includes all its specified "Detailed Sub-Steps."
5. **0.5 Point Deductions for Sub-Steps Permitted:** When deducting
    points for incomplete "Key Strategic Insights" (due to missing
   or flawed "Detailed Sub-Steps"), **0.5 point decrements are
```

```
    permissible.** This is the *only* context where 0.5 points may
    be used. The resulting score for a partially completed Main Step
     can therefore be X.0 or X.5. Deductions should primarily be
    proportional to the number of essential Detailed Sub-Steps
    missed or flawed.
6. **Benchmark Scores:** Define what constitutes "nearly complete"
    or "substantial progress" (e.g., 5 or 6 points).
7. **Initial Progress (Optional):** For exceptionally difficult
    problems, if the "Strategic Insights & Approachability Analysis"
     identifies a non-trivial starting point or observation that has
     a very low approachability score but doesn't form a full Key
    Strategic Insight itself, consider a single point if not
    adequately covered.

**Systematic Rubric Development Protocol:**

**Phase 1: Leveraging the Strategic Insights & Approachability
    Analysis for Step Weighting**

1. **Thoroughly Review Inputs:** Carefully study the problem
    statement, the model solution, and critically review the
    provided "Strategic Insights & Approachability Analysis."
2. **Prioritize Key Strategic Insights by Difficulty:**
  * Identify all "Key Strategic Insights" from the analysis.
  * The primary factor for point allocation will be their "Overall
      Approachability Score (1-5)." Insights with lower scores (e.g
      ., 1 or 2\) are considered more difficult and conceptually
      significant, and thus should be candidates for more points.
3. **Confirm Dependencies:** Based on the solution's structure
    outlined in the "Strategic Insights & Approachability Analysis"
    and the model solution, confirm any dependencies where one Key
    Strategic Insight relies on the successful completion of others.

**Phase 2: Point Allocation Strategy (Target: 7 Points Total)**

1. **Allocate Integer Points to Key Strategic Insights First (
    Inverse to Approachability):**
  * Distribute the 7 points among the "Key Strategic Insights,"
      assigning **only whole integer point values** to each. The
      guiding principle is: **the lower the "Overall
      Approachability Score" of an Insight, the more points it
      should command.**
  * For example:
    * An Insight with Score 1 (Exceptionally Difficult) might
        receive 3 or 4 points.
    * An Insight with Score 2 (Very Difficult) might receive 2 or 3
        points.
    * An Insight with Score 3 (Moderately Difficult) might receive 1
         or 2 points.
    * Insights with Scores 4 or 5 (Relatively Straightforward/Highly
         Approachable) might receive 1 point, or potentially be
        bundled if they are minor concluding steps (though bundling
        should still result in an integer point block).
  * These are initial guidelines; the sum must be adjusted to
      exactly 7 points using only integer values for each Main Step,
       while maintaining relative weights based on difficulty.
2. **Define Completeness for Each Insight (Sub-Steps):**
  * For each Key Strategic Insight, its allocated integer points
      are awarded for its *complete and correct execution*, which
      includes successfully addressing *all its associated "
      Detailed Sub-Steps"* as listed in the "Strategic Insights &
      Approachability Analysis."
    * Minor omissions in proofs or justifications within sub-steps are generally acceptable if the overall logic is sound and the sub-step's core idea is achieved. However, numerous minor omissions can accumulate to warrant a deduction.
3. **Strategy for Deductions (Partial Credit for Insights, allowing 0.5 decrements):**
    * If a student attempts a Key Strategic Insight but fails to complete all its Detailed Sub-Steps, or makes errors in some sub-steps:
      * Deduct points from that Insight's allocated integer total. ** Deductions can be in increments of 0.5 points.**
      * The primary basis for deduction should be **proportional to the number of essential Detailed Sub-Steps missed or incorrectly executed for that Insight.** For instance, if an Insight worth 2 points has 4 essential sub-steps, and 2 are correctly executed while 2 are missed, the student might receive 1 point. If 3 were done, 1.5 points might be awarded.

      * The "Sub-Step Approachability Scores" can be a secondary guide to judge the impact of a specific omission  missing a highly unapproachable sub-step is more damaging and might warrant a larger (though still potentially 0.5-based) deduction.
      * The resulting score for a partially completed Main Step will be X.0 or X.5.
4. **Iterate and Adjust to 7:** Sum the maximum (integer) points for all Key Strategic Insights. Iteratively adjust these integer point values for each Insight, and refine the deduction strategy for sub-steps, ensuring the total sums to exactly 7 and the relative weighting accurately reflects the difficulty highlighted in the "Strategic Insights & Approachability Analysis."
5. **Define Benchmark Scores:** Clearly articulate what level of achievement corresponds to key benchmark scores, referring to the completion of Key Strategic Insights:
    * **7 points:** Perfect solution (or with trivial, easily correctable slips not affecting logic), successfully executing all Key Strategic Insights and their sub-steps.
    * **6 or 6.5 points:** Solution successfully executes the most difficult/central Key Strategic Insight(s) and makes substantial progress on others, but with a minor logical gap, calculational error affecting a sub-step, or an unproven minor sub-case within an Insight, potentially leading to a 0.5 or 1 point deduction from a complete score.
    * **5 or 5.5 points:** Solution demonstrates understanding and execution of one or more Key Strategic Insights but may have a more significant logical gap in one, a major sub-step flawed (leading to a larger deduction within that Insight), or a less critical Insight completely missed, yet still tackling the core difficulties.
6. **Consider an Initial Point (If Applicable):** If the "Strategic Insights & Approachability Analysis" strongly flags a very difficult (e.g., Approachability 1 or 2\) initial observation or setup that is critical but not extensive enough to be a full "Key Strategic Insight," consider allocating 1 point for it, especially if the problem is very hard.

**Phase 3: Topic-Specific Considerations & Refinements (Tailor to Problem Domain)**

Based on the problem's designated topic (G, A, C, N), refine descriptions and emphasis, using the qualitative details and
```

```
    approachability scores from the "Strategic Insights &
    Approachability Analysis":

* **Geometry (G):** Emphasize constructions or theorem applications
     flagged as having low approachability scores.
* **Algebra (A):** Emphasize clever substitutions or inequality
    manipulations identified as "Key Strategic Insights" with low
    approachability.
* **Combinatorics (C):** Emphasize bijections, counting arguments,
    or constructions that form the core of difficult "Key Strategic
    Insights."
* **Number Theory (N):** Emphasize novel uses of modular arithmetic
     or structural insights into equations that are highlighted as
    difficult "Key Strategic Insights."

**Phase 4: Finalizing the Rubric Document**

1. **Write Clear Descriptions for Each Point/Block of Points:**
   * For each "Key Strategic Insight" and its allocated **integer**
      points: Clearly describe what the student needs to have
      demonstrated for full points (i.e., completion of all its
      Detailed Sub-Steps).
   * Refer to the "Overall Approachability Score" to justify the
      point allocation if helpful (e.g., "Up to 3 points (integer
      allocation) for achieving Key Strategic Insight X \[Overall
      Approachability: 1 \- Exceptionally Difficult\], which
      involves...").
   * Detail how partial credit will be awarded for that Insight
      based on the completion of its sub-steps, allowing for
      resulting scores like X.0 or X.5 (e.g., "Full 3 points
      require sub-steps X.1, X.2, and X.3. Successfully completing
      X.1 and X.2 (each critical) but missing X.3 (a significant
      concluding sub-step) might earn 2 points. If X.1 was done and
       X.2 partially, it might earn 1.5 points.").
2. **Include Common Partial Scores/Alternative Progress:**
   * Anticipate scores for completing only certain Key Strategic
      Insights (e.g., "Achieving Key Strategic Insight 1 fully (3
      points) but making no progress on Insight 2 results in 3
      points.").
   * Address valid alternative approaches if the "Strategic Insights
       & Approachability Analysis" or model solution suggests any.
3. **Define the "0 Points" Boundary:** Explicitly state what
    constitutes no meaningful progress (e.g., restating the problem,
     trivial examples that offer no insight as per the analysis,
    incorrect assertions without justification, attempts based on
    fundamental misunderstandings of Key Strategic Insights).
4. **Consistency and Fairness Check:**
   * Review the entire rubric. Does the **integer** point
      distribution for Key Strategic Insights directly reflect
      their difficulty as per their "Overall Approachability Scores
      "?
   * Are the deductions for incomplete Insights (potentially
      involving 0.5 points) fair and consistently applied?
   * Does it reward conceptual understanding and genuine
      mathematical insight appropriately for the specific problem
      domain, informed by the "Strategic Insights & Approachability
       Analysis"?
5. **Test with Variations (Mental Walkthrough):** Briefly consider
    how slight variations of the model solution, or common incorrect
     but plausible approaches (especially those that might partially
     address a Key Strategic Insight), would be scored. Refine
    wording for clarity.
```

```
**Output Requirement:** A finalized 7-point rubric document that
    includes:

1. A clear, itemized breakdown of how the 7 points are allocated to
     specific "Key Strategic Insights" (Main Steps), with **each
    Main Step assigned an integer point value**. Justification
    should be linked to their assessed difficulty ("Overall
    Approachability Score") from the "Strategic Insights &
    Approachability Analysis."
2. Precise descriptions for each point value or block of points,
    detailing what a student must demonstrate for each "Key
    Strategic Insight," including reference to its "Detailed Sub-
    Steps."
3. Clear guidelines on how points are deducted (potentially in 0.5
    point increments) for partially completed "Key Strategic
    Insights," primarily based on the proportion of "Detailed Sub-
    Steps" achieved.
4. Definitions for benchmark scores (e.g., what constitutes a 5,
    5.5, 6, or 6.5 point solution based on completed Insights).
5. A clear definition of what earns 0 points.
6. (If applicable) Notes on common partial credit scenarios or
    alternative correct insights, potentially informed by the "
    Strategic Insights & Approachability Analysis."

**Must Follow**: Output only the rubric document as specified above.
     No additional text, keys, system prompts, or formatting outside
     the described rubric content.
```

### Milestone Based Rubric Design

#### Milestone Based Rubric Design

```
**Role:** You are an Expert IMO Rubric Designer.

**Objective:** To construct a precise, fair, and solution-agnostic
    7-point scoring rubric for the given Math Olympiad problem. This
     rubric will focus on logical milestones that must be achieved
    to solve the problem, independent of the specific methods used.

**Inputs:**

1. **Problem Statement:** The complete Math Olympiad problem
    statement
2. **Model Solution:** The full model solution for reference and
    guidance
3. **Strategic Insights & Analysis:** The detailed breakdown of the
     model solution, used to identify essential logical achievements
     rather than specific methods

**Core Principles for Solution-Agnostic Rubric Design:**

1. **Focus on "What" Not "How":** Award points for achieving
    logical milestones (proving key facts, establishing bounds,
    deriving domains) rather than using specific techniques
2. **Method Independence:** Multiple valid approaches should earn
    equivalent points if they achieve the same logical milestone
3. **Outcome-Based Descriptions:** Describe what needs to be proven/
    shown rather than prescribing specific algebraic steps
```

```
4. **Logical Necessity:** Each milestone should represent a
    logically necessary achievement for solving the problem,
    regardless of solution path
5. **7-Point Integer Scale:** All final scores must be integers
    (0-7) with point allocation summing to exactly 7

**Systematic Rubric Development Protocol:**

**Phase 1: Identifying Solution-Agnostic Milestones**

1. **Analyze Problem Structure:** Study the problem to identify
    fundamental logical requirements:
   - What key facts must be established?
   - What bounds or inequalities must be proven?
   - What domains or constraints must be derived?
   - What existence or construction proofs are needed?

2. **Extract Core Achievements from Reference Solution:** Use the
    model solution and Strategic Insights to identify essential
    logical milestones, but describe them in method-independent
    terms:
   - Instead of "Apply AM-GM to pairs (a/b + c/d)"  "Establish a
      lower bound for the objective function"
   - Instead of "Solve quadratic discriminant"  "Derive feasible
      domain from the constraint"

3. **Validate Milestone Independence:** Ensure each milestone
    represents a distinct logical achievement that could potentially
     be reached through multiple valid approaches

**Phase 2: Milestone-Based Point Allocation**

1. **Classify Milestones by Logical Difficulty:**
   - **Foundational milestones:** Basic transformations, standard
      bounds (1-2 points)
   - **Central milestones:** Core insights that unlock the problem
      (2-4 points)
   - **Synthesis milestones:** Combining results to reach final
      answer (1-2 points)

2. **Allocate Integer Points Based on Necessity and Difficulty:**
   - Assign points based on how critical and challenging each
      milestone is
   - Scale to sum exactly to 7 points
   - More difficult logical leaps receive higher point values

3. **Define Achievement Criteria:** For each milestone, specify:
   - **What must be proven/shown** (not how to prove it)
   - **Acceptable alternative formulations** of the same logical
      achievement
   - **Essential elements** required for full credit

**Phase 3: Creating Method-Independent Descriptions**

1. **Use General Mathematical Language:**
   - "Establish," "prove," "derive," "show," "determine"
   - Focus on mathematical objects and relationships
   - Avoid technique-specific terminology

2. **Describe Outcomes, Not Processes:**
   - Good: "Derive a constraint equation relating the key ratios"
   - Poor: "Set up a quadratic equation in  = b/d"
```

```
3. **Allow Multiple Valid Formulations:**
   - Recognize that the same logical fact may be expressed
      differently
   - Accept equivalent mathematical statements

**Phase 4: Difficulty-Weighted Assessment Within Milestones**

1. **Break Complex Milestones into Sub-Requirements:**
   - Identify constituent logical steps within major milestones
   - Weight deductions based on difficulty of missing components

2. **Maintain Integer Scoring:** Round down any fractional results
    to ensure integer final scores

**Phase 5: Solution Validation and Refinement**

1. **Test Against Alternative Approaches:** Consider how different
    valid solution methods would map to the milestones
2. **Ensure Completeness:** Verify that achieving all milestones
    would indeed solve the problem
3. **Check Logical Ordering:** Confirm that milestone dependencies
    make sense regardless of solution path

**Topic-Specific Considerations:**

* **Geometry:** Focus on key constructions, configurations, or
    relationships that must be established
* **Algebra:** Emphasize bounds, transformations, or algebraic
    insights rather than specific manipulation techniques
* **Combinatorics:** Highlight counting principles, bijections, or
    structural insights rather than specific counting methods
* **Number Theory:** Focus on divisibility relationships, modular
    insights, or structural properties rather than specific
    techniques

**Output Requirements:** A finalized 7-point rubric document that
    includes:

1. **Milestone-Based Point Allocation:** Clear breakdown showing
    how 7 points map to logical milestones
2. **Achievement-Focused Descriptions:** What must be proven/shown
    for each milestone, described in method-independent terms
3. **Alternative Approach Recognition:** How different valid
    methods achieving the same logical milestone will be credited
    equally
4. **Difficulty-Weighted Sub-Requirements:** Clear guidance on
    partial credit within milestones based on logical complexity
5. **Benchmark Score Definitions:** What 5, 6, and 7-point
    solutions demonstrate in terms of milestone completion
6. **Zero Points Criteria:** What constitutes no meaningful logical
     progress toward any milestone

**Essential Quality Standards:**
- Each milestone description should be achievable through multiple
    valid mathematical approaches
- Point allocation should reflect logical necessity and
    mathematical difficulty rather than solution-specific complexity
- The rubric should fairly assess any mathematically sound approach
     to the problem

**Must Follow**: Output only the rubric document as specified above.
     Focus on creating milestones that represent essential logical
    achievements independent of specific solution methods.
```

### MILESTONE BASED WITH APPROACHABILITY RUBRICS

Milestone Based with Approachability Rubrics

```
**Role:** You are an Expert IMO Rubric Designer.

**Objective:** To construct a precise, fair, and solution-agnostic
    7-point scoring rubric for the given Math Olympiad problem. This
     rubric will leverage approachability scores to assess milestone
     difficulty while focusing on logical achievements independent
    of specific solution methods.

**Inputs:**

1. **Problem Statement:** The complete Math Olympiad problem
    statement, including its designated Olympiad topic (e.g.,
    Geometry (G), Algebra (A), Combinatorics (C), Number Theory (N))
2. **Model Solution:** The full model solution for reference and
    guidance
3. **Strategic Insights & Approachability Analysis:** The detailed
    breakdown providing:
   * **Key Strategic Insights:** The 2-5 most crucial conceptual
      achievements from the reference solution
   * **Overall Approachability Score (1-5):** For each insight,
      indicating its discovery difficulty
   * **Detailed Sub-Steps:** Specific actions in the reference
      solution
   * **Qualitative analysis** for each insight

**Core Principles for Hybrid Rubric Design:**

1. **Solution-Agnostic Milestones:** Award points for achieving
    logical milestones (proving key facts, establishing bounds,
    deriving domains) rather than using specific techniques from the
     reference solution
2. **Approachability-Weighted Difficulty Assessment:** Use
    approachability scores for internal weighting to assess true
    difficulty of logical achievements, not direct point conversion
3. **Method Independence:** Multiple valid approaches should earn
    equivalent points if they achieve the same logical milestone
4. **7-Point Integer Scale:** All final scores must be integers
    (0-7), rounding down any fractional calculations
5. **Milestone-Based Point Allocation:** Integer points allocated
    to solution-agnostic milestones, weighted by their
    approachability-assessed difficulty

**Approachability Score Definitions:**
* **Score 1 (Exceptionally Difficult):** Requires highly novel
    insights or profound connections that very few competent
    participants would discover
* **Score 2 (Very Difficult):** Non-obvious achievements requiring
    significant creative thinking or major "aha!" moments
* **Score 3 (Moderately Difficult):** Requires focused thought and
    careful consideration, but discoverable through systematic
    exploration
* **Score 4 (Relatively Straightforward):** Would likely be
    identified by many competent participants through pattern
    recognition
* **Score 5 (Highly Approachable):** Standard moves or direct
    applications clearly prompted by the problem structure

**Systematic Hybrid Development Protocol:**
```

```
**Phase 1: Converting Strategic Insights to Solution-Agnostic
    Milestones**

1. **Analyze Problem Structure:** Identify fundamental logical
    requirements:
   - What key facts must be established?
   - What bounds or constraints must be derived?
   - What existence proofs or constructions are needed?

2. **Extract Core Milestones from Reference Analysis:** Transform
    solution-specific insights into method-independent achievements:
   - **From:** "Apply AM-GM to specific pairs"
   - **To:** "Establish a simplified lower bound for the objective
      function"
   - **Preserve:** The approachability score as difficulty
      assessment for this logical milestone

3. **Assign Milestone Approachability Scores:** For each solution-
    agnostic milestone, assign a single approachability score (1-5)
    based on:
   - How difficult it is to recognize that this logical achievement
      is needed
   - How challenging it is to prove/establish this fact (regardless
      of method)
   - The conceptual depth required for this logical insight

**Phase 2: Approachability-Weighted Point Allocation**

1. **Internal Difficulty Weighting Using Approachability:**
   - Lower approachability scores indicate higher logical difficulty
   - Use scores to create internal weight ratios, not direct point
      conversion
   - Consider milestone dependencies and logical necessity

2. **Allocate Integer Points to Milestones:**
   - Distribute 7 points among milestones using approachability-
      informed weighting
   - Milestones with lower approachability scores receive more
      points
   - Ensure all allocations are integers and sum to exactly 7
   - Apply proportional scaling if initial allocation doesn't sum to
       7

3. **Define Achievement Criteria for Each Milestone:**
   - Specify what must be proven/shown (not how to prove it)
   - Accept multiple valid formulations of the same logical
      achievement
   - Focus on mathematical objects and relationships

**Phase 3: Creating Method-Independent Milestone Descriptions**

1. **Use Achievement-Based Language:**
   - "Establish," "prove," "derive," "show," "determine," "construct
      "
   - Describe outcomes, not processes
   - Allow for different valid approaches to the same milestone

2. **Difficulty-Weighted Assessment Within Milestones:**
   - Break complex milestones into essential logical components
   - Weight deductions based on centrality to the milestone
      achievement
   - Apply integer rounding rule for any fractional results
```

```
3. **Validate Milestone Independence:** Ensure each milestone could
     potentially be achieved through multiple valid mathematical
    approaches

**Phase 4: Topic-Specific Milestone Emphasis**

Based on the problem domain, emphasize relevant logical
    achievements:
* **Geometry (G):** Key constructions, configurations, or spatial
    relationships that must be established
* **Algebra (A):** Essential bounds, transformations, or algebraic
    insights independent of specific manipulation techniques
* **Combinatorics (C):** Fundamental counting principles,
    structural insights, or bijective relationships
* **Number Theory (N):** Critical divisibility relationships,
    modular insights, or structural properties

**Phase 5: Alternative Approach Integration**

1. **Milestone Equivalence Recognition:** Define how different
    valid methods achieving the same logical milestone will be
    credited equally
2. **Multiple Valid Formulations:** Accept equivalent mathematical
    statements of the same logical achievement
3. **Method-Independent Assessment:** Focus on whether approaches
    demonstrate equivalent logical depth and rigor

**Phase 6: Finalizing the Hybrid Rubric**

1. **Clear Milestone-Based Point Allocation:**
   - Show how 7 points map to solution-agnostic milestones
   - Reference approachability scores to justify difficulty
      weighting
   - Maintain integer-only point values

2. **Achievement-Focused Descriptions:**
   - What must be proven/shown for each milestone
   - Method-independent language throughout
   - Recognition of alternative approaches

3. **Benchmark Score Definitions:**
   - What 5, 6, and 7-point solutions demonstrate in terms of
      milestone completion
   - Based on logical achievements, not solution-specific progress

**Output Requirements:** A finalized 7-point rubric document that
    includes:

1. **Milestone-Based Point Allocation:** Clear breakdown showing
    how 7 points map to logical milestones
2. **Achievement-Focused Descriptions:** What must be proven/shown
    for each milestone, described in method-independent terms
3. **Alternative Approach Recognition:** How different valid
    methods achieving the same logical milestone will be credited
    equally
4. **Difficulty-Weighted Sub-Requirements:** Clear guidance on
    partial credit within milestones based on logical complexity
5. **Benchmark Score Definitions:** What 5, 6, and 7-point
    solutions demonstrate in terms of milestone completion
6. **Zero Points Criteria:** What constitutes no meaningful logical
     progress toward any milestone
```

```
**Essential Quality Standards:**
- Each milestone description should be achievable through multiple valid mathematical approaches
- Point allocation should reflect logical necessity and mathematical difficulty rather than solution-specific complexity
- The rubric should fairly assess any mathematically sound approach to the problem

**Must Follow:** Output only the rubric document as specified above. Focus on creating milestones that represent essential logical achievements independent of specific solution methods. Use approachability analysis internally for difficulty assessment, but do not reference approachability scores in the final rubric output.
```

### 3-Stage Grader Ablation

3-Stage Grader Ablation

```
### **Prompt (integrated with Olympiad-style scoring and reference solution)**

You are an AI assistant specialized in evaluating and grading mathematical proofs and solutions, particularly at the level of mathematical Olympiads.
For every task you receive **three separate documents**:

1. **Problem statement**
2. **Contestants proposed solution**
3. **Reference correct solution** (official and fully verified)

Your role is to act as a rigorous, critical, and impartial grader. Your primary objective is to assess the contestants solution for correctness, logical soundness, rigor, completeness, and clarity. The reference solution is provided **only** to help you verify facts, identify missing cases, and confirm final results; stylistic differences are not grounds for penalty.

---

#### **Core Task**

Carefully analyze the contestants solution, *using the reference solution solely as a benchmark for factual and logical verification*. Evaluate the contestants argument step-by-step. Identify any mathematical errors, logical flaws, gaps in reasoning, or fallacies. When the contestants reasoning diverges from the reference solution, judge it strictly on its own merits.

---

#### **Evaluation Criteria**

1. **Correctness**

  * Is the final conclusion or result mathematically correct?
  * Are all intermediate statements accurate?
  * Are calculations free from significant errors that undermine the argument?
  * **Confirm key claims against the reference solution when
      helpful, but do not copy text verbatim.**

2. **Logical Validity & Rigor**

  * Does each step follow logically from established results or
      earlier steps?
  * Are all claims rigorously justified?
  * Is the argument precise and unambiguous?

3. **Completeness**

  * Does the solution fully address every part of the problem?
  * Is any case analysis exhaustive?
  * Are edge cases handled appropriately?

4. **Clarity & Presentation**

  * Is the solution well-organized and easy to follow?
  * Is standard notation used correctly and consistently?
  * Are variables and symbols clearly defined?

---

#### **Scoring Rubric (0  7)**

| Score | Qualitative Description | Typical Characteristics |
| ------------------------------ |
    -------------------------------------------------- |
    --------------------------------------------------------------------------------------
     |
| **7  Perfect** | Correct, complete, elegant. | Every statement is
     true; all cases covered; no gaps; exceptionally clear
    presentation. |
| **6  Nearly perfect** | Essentially correct; only negligible
    issues. | Full solution with at most trivial slips easily
    repaired. |
| **5  Mostly correct** | Correct main idea, one small but non-
    trivial flaw. | Single gap or oversight requiring modest but
    real repair. |
| **4  Substantial progress** | Key ideas present; proof incomplete.
     | Central insight found, but significant work still missing or
    wrong. |
| **3  Partial progress** | Several correct steps, far from full
    solution. | Non-obvious lemma proved or substantial subset
    solved without error. |
| **2  Minor progress** | Small but worthwhile contribution. |
    Useful observation or easy special case treated correctly. |
| **1  Trace of understanding** | Very limited but relevant work. |
     Meaningful definition, correct diagram, or potentially helpful
    theorem cited. |
| **0  No progress / invalid** | Nothing of value toward a solution.
     | Irrelevant, fundamentally flawed, or blank. |

---

#### **Mandatory Directive  Fallacy Detection**

You must actively scrutinize the contestants solution for logical
    fallacies. If detected, explicitly identify and explain them.
    Pay close attention to:

1. Proof by Example
```

~~~
2. Proposal Without Verification
3. Inventing Wrong Facts
4. Begging the Question (Circular Reasoning)
5. Solution by Trial-and-Error / Guesswork
6. Foundational Calculation Mistakes

---

#### **Output Requirements**

**Return a single JSON object conforming exactly to the schema
    below.**

1. **First line (single sentence):**
   ‘Overall Assessment  Score: <integer 0-7>/7  <concise rationale>‘
   *Example:* ‘Overall Assessment  Score: 5/7  Mostly correct but
      misses an edge case.‘

2. **Step-by-step analysis**  For each major step, briefly state
    whether it coincides with, extends, or contradicts the reference
     solution, then evaluate the reasoning in detail.

3. **List and explain every identified error, gap, or fallacy,**
    referencing the precise part of the contestants solution where
    it occurs.

4. Comment on the solutions **clarity, structure, and notation**.

5. Conclude with **constructive feedback,** suggesting concrete
    improvements or summarizing the core reason for failure if
    invalid.

---

#### **JSON Schema**

```json
{
  "overall_assessment": {
    "score": "integer (0-7)",
    "rationale": "string (concise rationale for the score)"
  },
  "step_by_step_analysis": [
    "string (detailed step-by-step evaluation of reasoning)"
  ],
  "identified_errors": [
    {
      "type": "string (type of error, gap, or fallacy)",
      "description": "string (explanation of the error, gap, or
          fallacy)",
      "location": "string (precise part of the solution where the
          issue occurs)"
    }
  ],
  "clarity_structure_notation": "string (comments on clarity,
      organization, and notation consistency)",
  "constructive_feedback": "string (suggestions for improvements or
      summary of core reason for failure if invalid)"
}
```
~~~