

CITER: COLLABORATIVE INFERENCE FOR EFFICIENT LARGE LANGUAGE MODEL DECODING WITH TOKEN-LEVEL ROUTING

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ABSTRACT

Large language models (LLMs) have achieved remarkable success in natural language processing tasks but suffer from high computational costs during inference, limiting their deployment in latency-constrained applications. To address this issue, we propose a novel **Collaborative Inference with Token-LEvel Routing** (CITER) framework that introduces a token-level routing mechanism, enabling efficient collaboration between small and large language models (SLMs & LLMs). Specifically, CITER enables routing non-critical tokens to an SLM to reduce computational overhead, while critical tokens are processed by an LLM to maintain generation quality. We formulate the training of the router as a reinforcement learning task, where the router receives rewards based on both the quality of predictions and the inference cost of generation. This allows the router to learn to predict token-level routing scores and make routing decisions based on both the current token and the future impact of its decisions. To further accelerate the reward evaluation process, we introduce a shortcut for reward function estimation, significantly reducing the cost of the reward estimation and improving the practicality of our approach. Extensive experiments across four benchmark datasets demonstrate that CITER reduces inference cost while preserving high-quality generation, offering a promising solution for real-time and resource-constrained applications.

1 INTRODUCTION

Large language models (LLMs) have revolutionized a wide range of natural language processing tasks, from machine translation to context summarization and question answering (Coleman et al., 2024; Kamalloo et al., 2024; Eniser et al., 2024; He et al., 2024). However, their impressive performance comes with a substantial computational cost, particularly during inference. As these models grow in size, the cost of inference becomes a significant barrier to their practical deployment, especially in real-time applications. Thus, there is a growing need for accelerating the inference process without compromising the quality of the generated outputs.

Among the strategies (Dao et al., 2022; Sanh et al., 2020; Kou et al., 2024; Anagnostidis et al., 2024) to reduce inference costs, particularly the number of arithmetic operations, routing tasks to models of different sizes is a promising approach to accelerating LLM inference while maintaining output quality. In this approach, small language models (SLMs) handle simpler tasks with lower computational overhead, while more complex cases are routed to LLMs to ensure response accuracy. However, while promising, existing works largely focus on routing entire user queries to different models for generation (Ong et al., 2024; Mohammadshahi et al., 2024). This binary “either-or” approach limits routing flexibility and may reduce efficiency, as many responses are often routed to LLM to ensure the highest possible accuracy, even though SLM can often provide reasonably good answers.

To address this challenge, we present a novel framework, namely **Collaborative Inference with Token-LEvel Routing**(CITER). CITER introduces a token-level router that predicts whether a token is important by estimating the token-level routing score, and routes it to the appropriate model to balance the efficiency and accuracy of generation. Specifically, many tokens in the response that are not important to the final prediction, can be generated by SLM to reduce inference cost, while

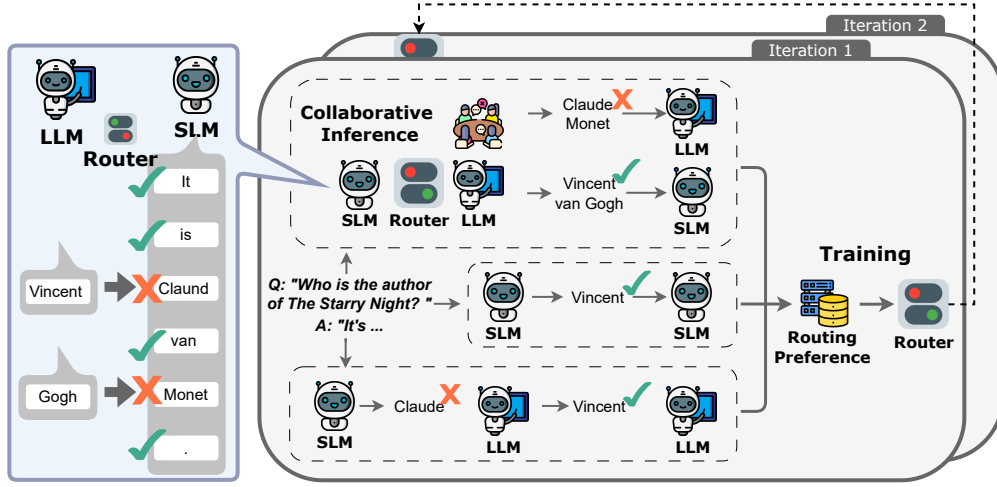


Figure 1: An overview of the CITER framework. A router is leveraged to perform collaborative inference between the SLM and LLM. The router is trained using routing preference collected through three cases. **Case 1:** The SLM generates the correct token, the routing preference is assigned to the SLM. **Case 2:** The SLM generates an incorrect token, while the LLM generates the correct token, the routing preference is assigned to the LLM. **Case 3:** None of the SLM or the LLM generates the correct token, then the collaborative inference is conducted to obtain the completed response for assigning the routing preference.

the LLM can be reserved to generate important tokens only. We formulate a reinforcement learning (RL) problem to train the router, with the objective of minimizing inference cost while preserving output quality. By employing this formulation, the router learns to predict token-level routing scores and make routing decisions not only based on the current token but also considering the impact of these decisions on future tokens. However, training the router using RL can be computationally expensive, as it requires estimating the reward function, which is defined as the accuracy of the responses generated via collaborative inference starting from each initial state. To make this formulation more practical, we present a shortcut to the reward function, where the prediction from the SLM and LLM is leveraged to estimate the final reward without completing the whole generation process, which significantly accelerates its training process. Through this RL framework, the router learns an optimal token-level decision-making strategy, enabling an SLM and an LLM to collaborate for effective and efficient autoregressive generation.

Our primary contribution is CITER, which accelerates LLM inference by employing a token-level router to select the appropriate model for generating each token. Experiments on four benchmark datasets demonstrate the effectiveness of our approach, achieving a reduction in LLM inference cost while maintaining high output accuracy, with comparable accuracy while up to 30% fewer calls to LLM or delivering up to a 25% improvement in accuracy with the same ratio of calls to LLM compared to co-LLM (Shen et al., 2024). Additionally, our ablation study experiments also demonstrate that token-level routing offers more flexibility for achieving more promising results compared to query-level routing and that considering the long-term impact of routing decisions boosts the acceleration performance significantly.

2 COLLABORATIVE INFERENCE WITH TOKEN-LEVEL ROUTING(CITER)

In this section, we describe our **Collaborative Inference with Token-level Routing (CITER)** framework that leverages token-level routing to accelerate LLM inference. As illustrated in Figure 1, in CITER, we introduce a router to facilitate collaborative inference between a powerful but computationally expensive LLM and a fast but potentially inaccurate SLM. Specifically, the router is leveraged to predict the token-level routing score for each token, and a pre-defined threshold τ is used to determine which model should generate this token. The key challenge of our framework is the training process of the router. To feed the router with the knowledge on making the global optimal

routing decisions not only based on the accuracy of the current token but also the long-term impact of its decision, we formulate the training process of the router as a reinforcement learning problem, aiming to minimize the inference cost while maintaining the generation quality. To be more specific, we first formulate the RL problem and derive the reward function as token-wise routing preference, which should be computed to collect during the router training process. Subsequently, we introduce a shortcut for the reward function estimation, leveraging both the SLM and LLM’s prediction to estimate the reward, to significantly accelerate the collection process of the token-wise routing preference. Finally, we propose an iterative training process for the router to mitigate the potential inconsistencies of the routing decisions in the preference collection phase and deployment. In the rest of this section, we will outline the router training and collaborative inference processes in detail.

2.1 ROUTER TRAINING

2.1.1 PROBLEM FORMULATION

To equip the router with the ability to predict token-level routing scores and decide which model to use based not only on the accuracy of the current token but also on the impact of future token generation, we formulate the training process of the router as a reinforcement learning agent. Let the current state \mathcal{T}_h be the historical context input to the LLM before h -th token, i.e., $\mathcal{T}_h = \{\mathbf{x}, y_1, \dots, y_{h-1}\}$, where \mathbf{x} is the input prompts. At each step $h \in [H]$, the RL agent select either the LLM π_L and the SLM π_S with action $a = \{L, S\}$. Then the state \mathcal{T}_{h+1} is composed by first sampling the next token $y_h \sim \pi_a(\cdot|\mathcal{T}_h)$ and then concatenating it to previous state as $\mathcal{T}_{h+1} = \mathcal{T}_h + \{y_h\}$. In addition, we assume the reward is defined by the whole trajectory by $\mathcal{R}(\mathcal{T}_H, \mathcal{A}_H)$ where \mathcal{A}_H is the sequence of a routing policy, i.e. $\mathcal{A}_H = \{a_1, \dots, a_H\}$, $a_h = \{L, S\}$. Reward \mathcal{R} comprehensively encodes the performance of the routing strategy, including the quality of generated response \mathcal{T}_H and the computational cost required by performing \mathcal{A}_H . We further define the (meta) policy for the routing agent as $\Pi = \{\Pi_h(\cdot|\mathcal{T}_h)\}_h$ and the state-action value function as $Q_h^\Pi(\mathcal{T}_h, a) = \mathbb{E}[\mathcal{R}(\mathcal{T}_H, \mathcal{A}_{h:H})|\mathcal{T}_h, a, \Pi]$, where the expectation is taken over \mathcal{T}_H and $\mathcal{A}_{h:H}$ given history context \mathcal{T}_h , action $a \in \{L, S\}$ and (meta) policy Π . The objective of the routing agent is to optimize the policy by

$$\Pi_h = \arg \max_{\Pi_h} \mathbb{E}_{a \sim \Pi_h(\cdot|\mathcal{T}_h)} Q_h^\Pi(\mathcal{T}_h, a) - \text{KL}(\Pi_h \parallel \mu), \quad (1)$$

where μ is the pre-defined policy with $\mu(L) = \mu(S) = 0.5$, meaning that the SLM and LLM are randomly selected. The closed-form solution for equation 1 is therefore $\Pi_h(a|\mathcal{T}_h) \propto \mu(a) \exp(Q_h^\Pi(\mathcal{T}_h, a))$.

2.1.2 EFFICIENT COLLECTION OF TOKEN-WISE ROUTING PREFERENCES

Generally, it is hard to evaluate the quantity of $\mathcal{R}(\mathcal{T}_H, \mathcal{A}_H)$ because it contains both the quality of \mathcal{T}_H and the efficiency of \mathcal{A}_H . We inject the pairwise preference $\mathbb{1}_h[S \succ L]$ following the Bradley-Terry model (Bradley & Terry, 1952) as:

$$\Pr_h(S \succ L|\mathcal{T}_h, \Pi) = \frac{1}{1 + \exp(Q_h^\Pi(\mathcal{T}_h, L) - Q_h^\Pi(\mathcal{T}_h, S))} \quad (2)$$

Following (Rafailov et al., 2024), we have that

$$Q_h^\Pi(\mathcal{T}_h, L) - Q_h^\Pi(\mathcal{T}_h, S) = \log \frac{\Pi_h(L|\mathcal{T}_h)}{\mu(L)} - \log \frac{\Pi_h(S|\mathcal{T}_h)}{\mu(S)} = \log \frac{\Pi_h(L|\mathcal{T}_h)}{\Pi_h(S|\mathcal{T}_h)}, \quad (3)$$

where the equation 3 is due to $\log(\mu(L)/\mu(S)) = 0$. Plugging equation 3 into equation 2 yields

$$\Pr(S \succ L|\mathcal{T}_h, \Pi)_h = \frac{1}{1 + \Pi_h(L|\mathcal{T}_h)/\Pi_h(S|\mathcal{T}_h)} = \Pi_h(S|\mathcal{T}_h), \quad (4)$$

where the last equation is due to the fact that $\Pi_h(S|\mathcal{T}_h) + \Pi_h(L|\mathcal{T}_h) = 1$. Therefore, given a context \mathcal{T}_h , once we have labeled the preference $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi]$, $\Pi_h(S|\mathcal{T}_h)$, the routing agent Π can be learned by minimizing the cross-entropy loss

$$\mathcal{L} = \sum_{\mathcal{T}_h} \mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi] \log \Pi_h(S|\mathcal{T}_h) + \mathbb{1}_h[L \succ S|\mathcal{T}_h, \Pi] \log \Pi_h(L|\mathcal{T}_h), \quad (5)$$

where the $\mathbb{1}_h[L \succ S|\mathcal{T}_h, \Pi]$ is defined similarly to $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi]$, but it takes action L (selecting the LLM) instead of S (selecting the SLM) at step h , conditioned on the state \mathcal{T}_h .

To optimize equation 5, the routing preference $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi]$ is determined by whether the fully generated response, starting from state \mathcal{T}_{h+1} , is correct. The state \mathcal{T}_{h+1} is reached by taking action S (selecting the SLM) from state \mathcal{T}_h . Specifically, if the state \mathcal{T}_{h+1} is a completed state \mathcal{T}_H (e.g., ending with an $\langle \text{EOS} \rangle$ token), the routing preference is assigned based on whether the generated response \mathcal{T}_H is correct. If the state \mathcal{T}_{h+1} is an intermediate state, the routing agent Π will be leveraged to process the collaborative inference between the SLM and the LLM, obtaining the completed state \mathcal{T}_H . Subsequently, the routing preference $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi]$ will be determined as whether the generated response \mathcal{T}_H is correct. The whole process can be formulated as follows:

$$\begin{aligned} \mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi] &= \mathbb{1}(\text{Correct}(\mathcal{T}_H)), & \text{if } \mathcal{T}_{h+1} \text{ is completed,} \\ \mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi] &= \mathbb{1}(\Pi_{h+1}(S|\mathcal{T}_{h+1}) > \tau) \mathbb{1}_{h+1}[S \succ L|\mathcal{T}_{h+1}, \Pi] + & (6) \\ &\quad \mathbb{1}(\Pi_{h+1}(S|\mathcal{T}_{h+1}) \leq \tau) \mathbb{1}_{h+1}[L \succ S|\mathcal{T}_{h+1}, \Pi], & \text{otherwise,} \end{aligned}$$

where the $\text{Correct}(\cdot)$ is used to determine whether the generated response is correct. The threshold τ is a hyperparameter that determines the routing score required to select the SLM. If the routing score exceeds τ , the token is considered non-critical and the SLM's output will be committed to the final response. Otherwise, the token is marked as critical and routed to the LLM for re-generation.

However, the second case in equation 6 requires generating the full response \mathcal{T}_H and calculating the reward from each starting state \mathcal{T}_h , which is computationally expensive. To mitigate this, we introduce a shortcut for estimating the reward function, significantly reducing the cost of reward computation. We hypothesize that if the SLM cannot generate a token correctly when provided with the previous ground truth contextual history, then this token is particularly challenging to generate. Based on this hypothesis, we first provide both the SLM and LLM with the ground truth historical context and their output tokens are then compared with the ground truth. Then, we assign the routing preference $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi]$ directly without generating the full response \mathcal{T}_H if at least one of them generates the correct token. Specifically, we define the ground truth context before h -th token is $\mathcal{T}_h^* = \{\mathbf{x}, y_1^*, \dots, y_{h-1}^*\}$, and y_h^* is defined as the ground truth h -th token. Subsequently, we feed \mathcal{T}_h^* to the SLM to generate the next token. If the SLM can generate the correct token y_h^* based on the ground truth context, the SLM will be selected, and we will assign the routing preference $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi] = 1$. Otherwise, we then feed \mathcal{T}_h^* to the LLM, and if LLM can generate the correct token y_h^* , we will choose the LLM and assign $\mathbb{1}_h[S \succ L|\mathcal{T}_h, \Pi] = 0$. Only when both models fail to generate the correct token y_h^* based on ground truth context, the full response generation is required to compute the reward. This shortcut allows us to obtain routing preferences for most tokens without generating the full response. Empirically, we find that about 80% \sim 90% of tokens can be correctly predicted by either the SLM or LLM, making the shortcut significantly reduce the computational cost of the reward function estimation. After collecting the preference, it will be used to train the router using equation 5.

2.1.3 ITERATIVE TRAINING PROCESS

Ideally, we would expect the router Π used during preference data collection to make the same routing decision as the one used during deployment. However, as the router is updated throughout training using the collected data in Section 2.1.2, its behavior is likely to change, leading to potential inconsistencies. To address this issue, we propose a multi-iteration router training process. In each iteration k , the router Π_{k-1} from the previous iteration $k - 1$ is used to collect routing preferences. These newly collected routing preferences are then utilized to train a new router Π_k . With each iteration, the router's behavior becomes more consistent, eventually reaching convergence. This iterative process continues until the collected routing preferences match those from the previous iteration or until a predefined number of iterations K is reached. The only exception occurs in the first iteration, where no trained router exists. In this case, a simple routing policy is employed, routing all tokens to the SLM to collect the initial preference data. By following this approach, we can train the router in a practical and efficient manner. The full process is outlined in Algorithm 1.

Algorithm 1: The Iterative Training Process of our Router

Input: The training data $\mathcal{D} = \{(\mathcal{T}_h, y_h^*)\}_i$, the initial router Π_0 , the SLM \mathcal{M}_S and LLM \mathcal{M}_L , the pre-defined threshold τ and the number of iterations K

Output: The trained router Π

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1 Initialization: iter  $k \leftarrow 0$ , previous routing preference  $\mathcal{S}_0 \leftarrow \emptyset$ 
2 for  $k = 1$  to  $K$  do
3    $\mathcal{S}_k \leftarrow \emptyset$ 
4   for  $i = 1$  to  $|\mathcal{D}|$  do
5      $y_h^S \leftarrow \mathcal{M}_S(\mathcal{T}_h)$ 
6     if  $y_h^S == y_h^*$  then
7        $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \{1\}$ 
8       Continue
9      $y_h^L \leftarrow \mathcal{M}_L(\mathcal{T}_h)$ 
10    if  $y_h^L == y_h^*$  then
11       $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \{0\}$ 
12      Continue
13    Generate the full response  $\mathcal{T}_H$  starting from  $\mathcal{T}_h \cup y_h^S$  with router  $\Pi_{k-i}$  and threshold  $\tau$ 
14     $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \{\mathbb{1}(\text{Correct}(\mathcal{T}_H))\}$ 
15  if  $\mathcal{S}_k == \mathcal{S}_{k-1}$  then
16    break
17  Train the router  $\Pi_k$  with the routing preference  $\mathcal{S}_k$  and dataset  $\mathcal{D}$  as the loss
18    function equation 5
19   $\Pi \leftarrow \Pi_k$ 
20 Return  $\Pi$ 

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2.2 COLLABORATIVE INFERENCE

After router training, during inference, we aim to leverage the SLM to collaboratively generate tokens to reduce the number of tokens generated by the LLM, thereby improving the efficiency of the inference process. Specifically, we start by feeding both the input prompt and the previously generated tokens into the SLM, obtaining the output token and corresponding hidden states. Then, the router Π trained in Section 2.1 is leveraged to predict the token-level routing score based on the hidden states from the SLM. During this process, the router considers both the current token and the historical context to make routing decisions based not only on the accuracy of the current token but also on the long-term influence of its decision on future token generation. Subsequently, a pre-defined threshold τ is used to determine whether the SLM or the LLM should handle the current token. If the routing score exceeds the threshold τ , indicating that the SLM is confident with its output. The output token from the SLM will be committed to the final response and the generation process will go on. Otherwise, the token will be routed to the LLM for re-generation and the SLM's output will be discarded. During the preference data collection process for router training, most tokens are assigned to the SLM, with only a few routed to the LLM through our shortcut. As a result, the SLM efficiently generates the majority of tokens. This process continues until an $\langle \text{EOS} \rangle$ token is produced by either the SLM or LLM. In this way, the router dynamically routes each token between the SLM and LLM, offloading non-critical tokens to the SLM to reduce computational overhead while utilizing the LLM's capabilities to maintain response quality.

3 EXPERIMENTS

In this section, we evaluate the performance of CITER aiming to answer the following questions: (1) Compared with the previous works on speeding up the inference of LLM, how does our framework perform in terms of the computational cost and the quality of the generated response? (2) Does the components we proposed in our framework boost the performance of the router? (3) Does the iterative training process of the router improve the performance of our framework? (4) How does the performance of our framework change with the size of the LLM? (5) Can the router distinguish the critical and non-critical tokens correctly?

Table 1: The statistics of our evaluation datasets.

Dataset	Domain	Task	# choices	Train size	Test size
Commonsense QA	General	CoT + Multi-choice	5	9,741	1,221
ARC-Challenge	Reasoning	CoT + Multi-choice	4	1,119	299
GSM8k	Math	Question answering	N/A	7,473	1,319
MMLU-Professional Psychology	Psychology	CoT + Multi-choice	4	612	69

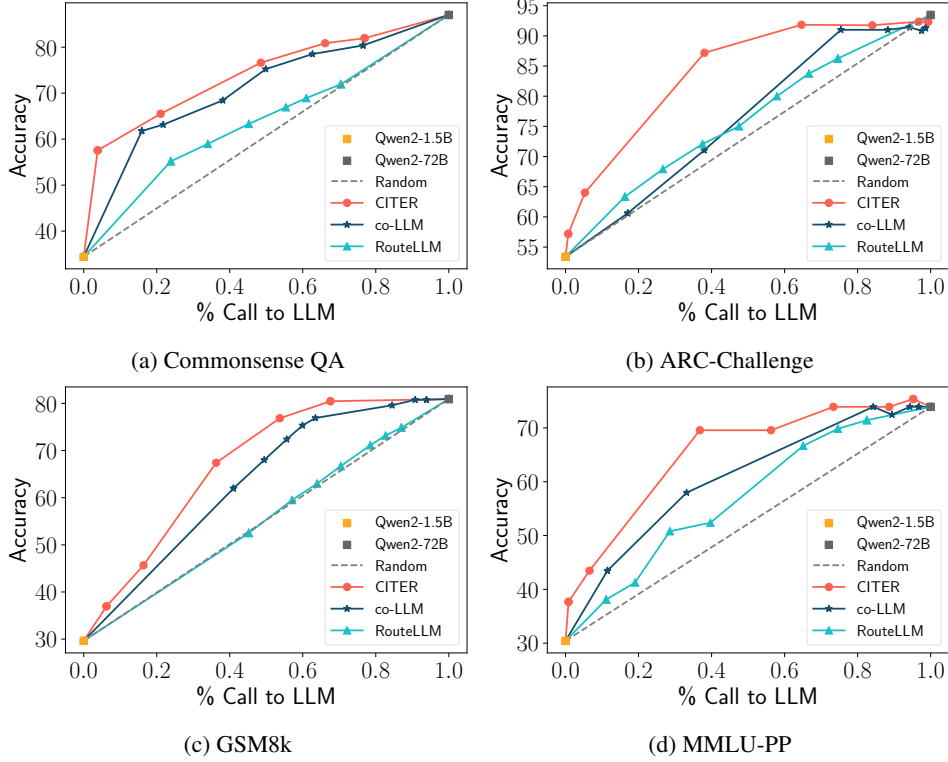


Figure 2: The accuracy vs % calls to LLM curve of CITER and the baselines. Points closer to the top-left corner indicate better acceleration performance.

3.1 EXPERIMENTAL SETUP

Dataset Description. We evaluate CITER and our baselines on four widely-used academic benchmark datasets: the commonsense QA dataset (Talmor et al., 2019) contains 12,102 questions requiring different types of commonsense knowledge to answer; the ARC-Challenge dataset (Clark et al., 2018), including 1,418 genuine grade-school level, multiple-choice science questions; the GSM8k dataset (Cobbe et al., 2021) with 8.5K high quality linguistically diverse grade school math word problems; and the MMLU-Professional Psychology dataset (Hendrycks et al., 2021), consisting of 874 multiple-choice questions on psychology, respectively. The statistics of the datasets are in Table 1.

Evaluation. We evaluate the performance of CITER and the baseline methods using the test sets and corresponding evaluation metrics for each dataset. Specifically, a threshold τ is applied in each method to balance the trade-off between response accuracy and the % calls to LLM, which is defined as the ratio of calls routed to the LLM. We then plot the accuracy vs. the % calls to LLM curve to illustrate the acceleration performance of both CITER and the baselines. The optimal point is located in the top-left corner of the curve, corresponding to the highest accuracy with the lowest inference cost.

Baselines. We compare CITER with both a representative query-level routing method (RouteLLM (Ong et al., 2024)) and a token-level routing method (co-LLM (Shen et al., 2024)).

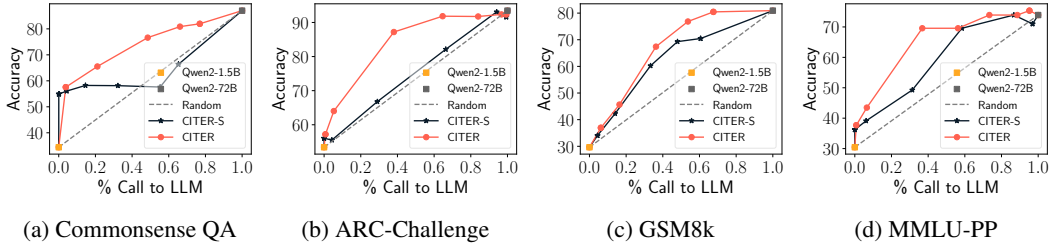


Figure 3: The accuracy vs % calls to LLM curve of CITER and the variant CITER-S. Points closer to the top-left corner indicate better acceleration performance.

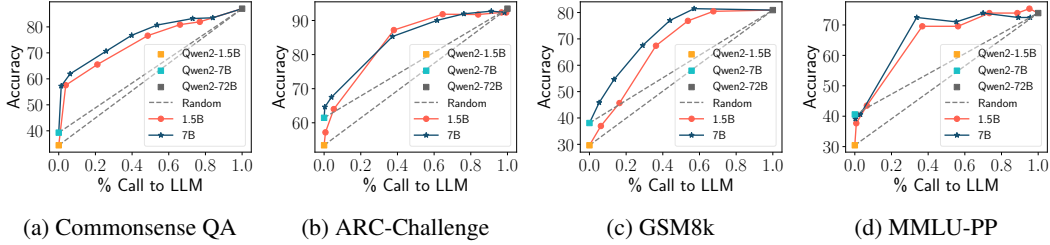


Figure 4: The accuracy vs % calls to LLM curve of CITER with 1.5B SLM and CITER with 7B SLM. Points closer to the top-left corner indicate better acceleration performance.

RouteLLM makes routing decisions for entire queries, directing them to different models for generation, while co-LLM breaks down the generation process, routing each token to different models.

Implementation Details We implement our framework using the Hugging Face Transformers library (Wolf et al., 2020). For the SLM and LLM, we utilize Qwen2-1.5b and Qwen2-72b, respectively. The router is implemented as a multilayer perceptron (MLP) network with three hidden layers, ReLU activation (Agarap, 2019), BatchNorm normalization (Ioffe & Szegedy, 2015), and a 0.1 dropout rate. It is trained using the Adam optimizer (Kingma & Ba, 2017) with a learning rate of 1×10^{-7} , betas of (0.9, 0.99), and no weight decay. Training is performed on a single NVIDIA H100 GPU with a batch size of 80. The iterative training process runs for 2 rounds.

3.2 OVERALL PERFORMANCE

We conduct extensive experiments to assess the performance of CITER across all benchmark datasets, comparing it against baseline methods. The results are presented in Figure 2. Notably, all token-level routing methods, including CITER and co-LLM, significantly outperform the query-level routing method, RouteLLM, across all datasets, particularly on the Commonsense QA and GSM8k datasets, reducing up to 30% calls to LLM while maintaining the same accuracy or achieving up to 12% higher accuracy with the same ratio of calls to LLM. This emphasizes the effectiveness of token-level routing, which provides enhanced flexibility in reducing computational costs while preserving response quality. Furthermore, CITER consistently surpasses co-LLM, achieving comparable accuracy with up to 27% fewer calls to LLM or delivering up to a 17% improvement in accuracy with the same ratio of calls to LLM. These findings demonstrate the success of our framework in accelerating LLM inference. This outcome is expected, as co-LLM does not consider long-term information during the router training phase, which is crucial for token-level routing. In the following section, we present experiments to further demonstrate the importance of incorporating long-term information in router training.

3.3 ANALYSIS OF LONG-TERM INFLUENCE

In this section, we conduct an ablation study on a key component of our framework: the long-term influence of routing decisions, to evaluate its effectiveness. For this purpose, we design an ablation variant, CITER-S, where the SLM is selected if both the SLM and LLM provide incorrect predictions during the routing preference collection, disregarding the long-term impact of routing

decisions. The results are shown in Figure 3. Clearly, CITER significantly outperforms the ablation variant CITER-S across all datasets, reducing calls to LLM by up to 42% while maintaining the same accuracy, or achieving up to a 23% accuracy improvement with the same ratio of calls to LLM. These findings highlight the critical role of accounting for the long-term influence of routing decisions.

3.4 ANALYSIS OF ITERATIVE TRAINING PROCESS

To highlight the importance of the iterative training process, we present the performance curve of CITER with the router over the first three iterations on the Commonsense QA dataset. As shown in Figure 5, the results demonstrate a clear improvement in performance from the first to the second iteration. In the second iteration, CITER reduces $\sim 5\%$ calls to LLM while maintaining the same accuracy or achieves 2 \sim 3% higher accuracy with the same ratio of calls to LLM compared to the first. This improvement underscores the effectiveness of our proposed iterative training process. Moreover, the performance curve of the third iteration closely follows that of the second, indicating that the router has already converged by the second iteration. The rapid convergence of the router emphasizes the robustness of our training strategy, suggesting that optimal performance can be achieved without excessive computational costs or extended training periods.

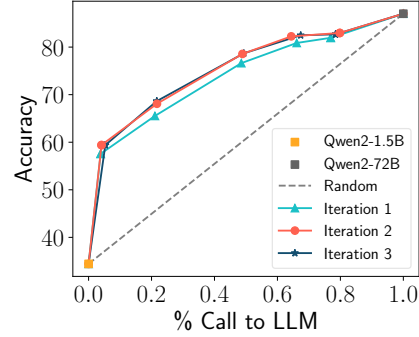


Figure 5: Accuracy vs. % calls to LLM curve of CITER with router over the first three iterations on the commonsense QA datasets. Points closer to the top-left corner indicate better acceleration performance.

3.5 ANALYSIS OF THE IMPACT OF SLM MODEL SIZE

We further scale up the SLM size from Qwen2-1.5B to Qwen2-7B, while keeping the LLM fixed to Qwen2-72B, to understand the scalability of our framework. As shown in Figure 4, the results clearly demonstrate that CITER reduces calls to LLM by up to 10% while maintaining the same level of accuracy or achieves up to 11% higher accuracy with the same ratio of calls to LLM when using Qwen2-7B as the SLM compared to Qwen2-1.5B, particularly on the commonsense QA and GSM8k datasets, underscoring our framework’s scalability with larger SLMs. However, the performance gap is most noticeable when only a few calls to LLM are made, and it gradually diminishes or even disappears as the ratio of calls to LLM increases. This is expected, as the SLM’s capacity limits its performance, and the quality of responses increasingly depends on the LLM as more calls are routed to it.

3.6 CASE STUDY ANALYSIS ON THE ROUTER

Finally, we perform a case study to further analyze the decision-making process of the router in our framework. A selection of examples, along with their corresponding routing decisions, is shown in Figure 6. In the left example, it is clear that our router accurately identifies the critical tokens, including the first occurrence of the answer “Midwest” and the word “fertile,” which describes the farmland in the Midwest, both crucial to the final answer. Moreover, most non-critical tokens are efficiently offloaded to the SLM, effectively reducing computational costs.

In the right example, we compare CITER with the token-level routing method co-LLM. It is evident that our router outperforms co-LLM by correctly identifying potential critical tokens, particularly time-related words. In co-LLM’s response, at the first red-marked word “morning,” co-LLM incorrectly routes the word “the” to the LLM while assigning the contextually important word “morning” to the SLM, leading to an initial error in the response. Similarly, co-LLM routes the critical phrase “afternoon meal” to the SLM, which results in the final incorrect prediction. In contrast, our router correctly identifies the critical word “day” and routes it to the LLM, followed by routing the phrase “has already” to accurately capture the reasoning process, ultimately leading to the correct prediction.

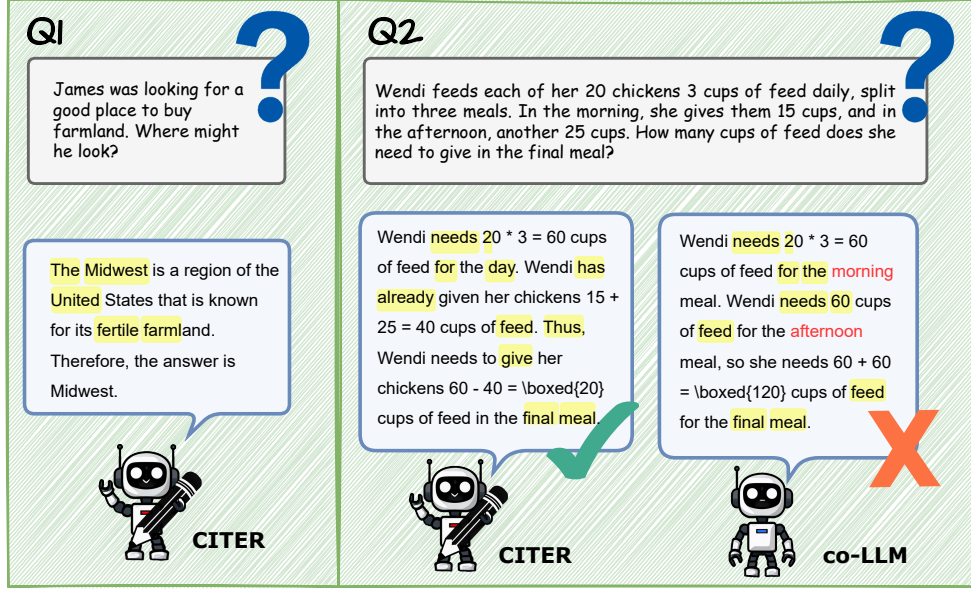


Figure 6: The case study analysis of our router. The words highlighted with yellow background are generated by the LLM, while other words are generated by the SLM. The red-marked words are the mistakes in co-LLM’s response.

These examples illustrate that the router in CITER effectively distinguishes between critical and non-critical tokens, offloading non-critical tokens to the SLM to minimize computational overhead, while leveraging the LLM to ensure the quality of the generated response.

4 RELATED WORK

In this section, we conduct a literature review that mainly focuses on prior LLM inference acceleration methods, especially those that involve using routing mechanisms and collaborative inference between LLMs for inference acceleration.

Query-Level Routing Mechanisms. Previous routing methods (Jang et al., 2023; Chronopoulou et al., 2023; Diao et al., 2023; Lu et al., 2023; Cheng et al., 2024; Lu et al., 2024; Chen et al., 2023b; Wang et al., 2024b) for efficient inference mainly focus on routing entire user queries to different models for generation. For example, Routoo (Mohammadshahi et al., 2024) proposes a performance predictor and a cost-aware decoder to route between LLMs, considering both performance and resource constraints; RouteLLM (Ong et al., 2024) formulates the routing problem as a classification problem and employs a data augmentation framework to significantly expand the dataset used for training the router. FrugalGPT Chen et al. (2023b) formulates the routing problem as a constrained optimization problem, where the final generated quality is maximized under a budget or inference cost constraint. However, as highlighted in Section 1, routing at the query-level granularity may lead to suboptimal performance, as non-critical tokens in complex queries may be generated inefficiently, while critical tokens in simple queries may suffer from inaccuracy. In contrast, token-level routing methods offer more fine-grained control over the routing process, improving both inference costs and the quality of the generated response.

Token-Level Routing Mechanisms. Unlike query-level routing methods, previous token-level routing methods (Pfeiffer et al., 2021; Belofsky, 2023; Muqeeth et al., 2024; Wang et al., 2024a; Wu et al., 2024; Xu et al., 2024) mainly focus on routing input tokens to different specialized experts to enhance performance without considering the computational cost. For example, Arrow (Ostapenko et al., 2024) reuses a library of expert LoRAs to build a mixture-of-experts (MoE)

architecture, dynamically routing inputs to different LoRAs during inference. Similarly, Branch-Train-MiX (Sukhbaatar et al., 2024) fine-tunes LLMs on different domains from a seed LLM, creating specialized experts to form an MoE framework. Among these methods, co-LLM (Shen et al., 2024) is the most relevant to our framework CITER, introducing a router to route tokens to models of different sizes. However, co-LLM only considers the current outputs from SLM and LLM when generating ground truth labels to train the router. This may lead to suboptimal performance since the influence of current decisions on future tokens is not considered. Moreover, similar to other token-level routing methods, co-LLM focuses on enhanced response quality without taking the computational cost of the inference process into account. In contrast, our CITER framework considers both the current token and the future impact of each decision, enabling more accurate and efficient routing.

Other Methods for LLM Inference Acceleration. In addition to routing methods, several approaches ranging from algorithmic to system optimizations (Leviathan et al., 2023; Miao et al., 2023; Kwon et al., 2023; Bhendawade et al., 2024; Cai et al., 2024; Chen et al., 2024b;a) have been proposed to accelerate LLM inference. Speculative Decoding (Leviathan et al., 2023; Chen et al., 2023a) employs a small draft model to generate potential next tokens, which are concatenated with previously generated tokens. These guesses are then processed by the target LLM in parallel to verify their correctness. Tokens are only committed to the final output if confirmed by the target LLM. Although this approach reduces inference time by generating multiple tokens in a single forward pass, it does not lower the overall computational complexity (e.g., the total amount of FLOPs). Speculative Streaming (Bhendawade et al., 2024) addresses the computational overhead of Speculative Decoding by predicting n-grams instead of individual tokens in each forward pass. However, it requires redesigning the LLM architecture, necessitating re-pretraining, which is computationally prohibitive for many use cases. Medusa (Cai et al., 2024) mitigates the re-pretraining issue by adding auxiliary heads to the original LLM, allowing n-gram predictions without modifying the core model. These heads can be trained while keeping the original LLM frozen, thereby avoiding the need for re-pretraining. SpecInfer and Sequoia (Miao et al., 2023; Chen et al., 2024b) leverage tree-based parallelism for decoding and verification to further accelerate inference.

5 CONCLUSION AND FUTURE WORK

In this paper, we propose a novel Collaborative Inference with Token-Level Routing (CITER) framework to address the challenge of high computational costs associated with large language models. By introducing a token-level routing mechanism, CITER enables efficient collaboration between smaller and larger models, achieving a balance between computational efficiency and generation quality. The key challenge lies in training the router, which we formulate as a reinforcement learning task. The router receives rewards based on both prediction quality and inference cost, allowing it to learn to predict token-level routing scores and make routing decisions by considering both the current token and the future impact of its decision. Additionally, we introduce an iterative training process to mitigate inconsistencies during the dataset collection phase. To further accelerate the learning process, we propose a shortcut where the reward is estimated based on the predictions from both the small and large models at each step. Extensive experiments demonstrate that CITER significantly reduces inference costs while maintaining high-quality generation, offering a promising solution for real-time and resource-constrained applications.

Future work can explore several directions to further enhance the CITER framework. One potential avenue is the application of CITER to other modalities, such as vision or multimodal tasks, to assess its generalizability and effectiveness beyond language models. Additionally, investigating the use of self-supervised learning to pretrain the router and reduce reliance on costly iterative training could also be a promising direction. To be more specific, other estimation methods could be introduced to further accelerate the routing preference collection for the case where both the SLM and the LLM make the incorrect next token prediction. Future research could also focus on improving the interpretability of the routing mechanism, providing more transparency on how the model makes token-level routing decisions. Finally, exploring the integration of CITER with model compression techniques or dynamic architectures may yield further reductions in computational costs, making it even more suitable for deployment in real-time, edge-based applications.

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A DATASET DESCRIPTION

In this section, we describe our benchmark datasets with more details.

A.1 COMMONSENSE QA

CommonsenseQA is a large-scale, multiple-choice question-answering dataset designed to challenge and evaluate systems on their ability to leverage commonsense knowledge. The dataset consists of 12,102 questions, each accompanied by one correct answer and four distractor (incorrect) options, requiring models to distinguish the correct answer by understanding various types of commonsense reasoning. What sets CommonsenseQA apart is its emphasis on requiring a broader array of everyday knowledge, involving not only basic facts but also causal, temporal, and conceptual reasoning.

A.2 ARC-CHALLENGE

The AI2 ARC dataset is a comprehensive collection of 7,787 grade-school-level multiple-choice science questions, meticulously curated to stimulate advancements in question-answering systems. The dataset is strategically divided into two subsets: the ARC-Easy Set and the ARC-Challenge Set. The ARC-Challenge Set, which is the subset we utilized in our work, comprises a selection of particularly difficult questions. These questions were specifically included because they were misclassified by both a traditional retrieval-based algorithm and a word co-occurrence algorithm, making them a true test of a model’s ability to understand and reason through complex scientific concepts. The ARC-Challenge subset serves as an ideal benchmark for testing sophisticated models, as it presents questions that require more than surface-level understanding or simple pattern matching.

A.3 MMLU-PROFESSIONAL PSYCHOLOGY

The MMLU dataset is a comprehensive multitask benchmark that comprises multiple-choice questions across a vast range of knowledge domains, including subjects in the humanities, social sciences, hard sciences, and other fields. It covers 57 distinct tasks such as elementary mathematics, U.S. history, computer science, law, and more, aimed at evaluating a model’s general world knowledge and problem-solving capabilities.

In our work, we focused specifically on the “Professional Psychology” subset of MMLU. This subset contains questions rich in domain-specific terminology, including specialized terms related to psychology and, occasionally, biological concepts tied to psychological phenomena. It provides a robust test for assessing a model’s proficiency in understanding and reasoning within a specialized academic field, thus offering insights into the model’s capability to handle complex, domain-specific content.

A.4 GSM8K

GSM8k (Grade School Math 8k) is a dataset consisting of 8.5K high-quality, linguistically diverse grade school math word problems. Designed to evaluate and improve question-answering capabilities in basic mathematical problem-solving, this dataset emphasizes multi-step reasoning, requiring between 2 and 8 steps to arrive at the correct solution.

The problems involve a sequence of elementary calculations using basic arithmetic operations—addition, subtraction, multiplication, and division—along with some early Algebra concepts. However, the dataset ensures that all problems are approachable for a bright middle school student, avoiding the need for advanced mathematical tools like variable definitions in most cases.

One of the distinctive features of GSM8K is that the solutions are presented in natural language rather than purely in mathematical expressions. This design decision aligns with the dataset’s goal to illuminate the reasoning capabilities of large language models (LLMs), specifically how they simulate an “internal monologue” when reasoning through problems. The dataset’s natural language solutions provide a more interpretable and instructive resource for evaluating the logical progression of LLMs in real-world tasks.

B PROMPT

In this section, we illustrate the prompt we used for each dataset.

B.1 MULTIPLE-CHOICE QUESTION PROMPT

For multiple-choice question datasets, including the Commonsense QA dataset, the ARC-Challenge dataset, and the MMLU-Professional Psychology dataset, we leverage the following prompt to require LLMs to provide both an explanation and a final answer in a specific format.

Example:

Question: \$question

Choices:

A. \$choice_A

B. \$choice_B

C. \$choice_C

D. \$choice_D

E. \$choice_E

Please format your response in the following way:

[Explanation]. Therefore, the answer is answer (label).

Ensure the final sentence includes the answer followed by the label in parentheses.

Answer and Reasoning: \$response

B.2 GSM8K MATH PROBLEM PROMPT

For math problems in the GSM8k dataset, the following prompt is employed to require LLMs to provide a step-by-step solution with clear reasoning in LaTeX format, with the final answer enclosed in a box using `\boxed{}`.

Example:

Solve the following math problem step by step. Ensure all reasoning is shown clearly in LaTeX format, and the final answer should be enclosed using the LaTeX command `\boxed{}` to display it in a box.

Question: \$question

Solution: \$solution

C EXAMPLES

In this section, we list examples of each dataset, including the questions, prompts, and responses from LLMs to clearly demonstrate the prompt we use and the task of each dataset.

C.1 COMMONSENSE QA

Question: What do people aim to do at work?

Choices:

A. complete job

B. learn from each other

C. kill animals

D. wear hats

E. talk to each other

Please format your response in the following way:

[Explanation]. Therefore, the answer is answer (label).

Ensure the final sentence includes the answer followed by the label in parentheses.

Answer and Reasoning:

The answer should be something that people aim to do at work. People generally aim to complete jobs. Therefore, the answer is complete job (A).

C.2 ARC-CHALLENGE

Question: Juan and LaKeisha roll a few objects down a ramp. They want to see which

object rolls the farthest. What should they do so they can repeat their investigation?
Choices:

- A. Put the objects in groups.
- B. Change the height of the ramp.
- C. Choose different objects to roll.
- D. Record the details of the investigation.

Please format your response in the following way:

[Explanation]. Therefore, the answer is answer (label).

Ensure the final sentence includes the answer followed by the label in parentheses.

Answer and Reasoning:

To repeat their investigation, Juan and LaKeisha should record the details of the investigation. This will involve noting down the objects they rolled, the height of the ramp, and the distance each object rolled. This way, they can replicate the exact conditions and compare the results of their repeated investigation. This is the crucial step in the scientific method, as it ensures that the results are reliable and can be trusted. Therefore, the final answer is record the details of the investigation (D).

C.3 MMLU-PROFESSIONAL PSYCHOLOGY

Question: Justice primarily involves _____.

Choices:

- A. Recognizing fairness for all persons
- B. Promoting accuracy and truthfulness
- C. Benefitting others and do no harm
- D. Respecting cultural and individual difference

Please format your response in the following way:

[Explanation]. Therefore, the answer is answer (label).

Ensure the final sentence includes the answer followed by the label in parentheses.

Answer and Reasoning:

Justice primarily involves recognizing fairness for all individuals. It is about ensuring that everyone is treated equally and fairly, without any discrimination or bias. Therefore, the answer is Recognizing fairness for all persons (A).

C.4 GSM8K

Solve the following math problem step by step. Ensure all reasoning is shown clearly in LaTeX format, and the final answer should be enclosed using the LaTeX command `\boxed{}` to display it in a box.

Question: Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She sells the remainder at the farmers' market daily for \$2 per fresh duck egg. How much in dollars does she make every day at the farmers' market?

Solution: $16 - 3 = 13$ 13 eggs are left after breakfast. $13 - 4 = 9$ 9 eggs are left after baking. $9 * 2 = 18$ 18 dollars she makes at the farmers' market. The final answer is: $\boxed{18}$.