Simple and Effective Masked Diffusion Language Models

Subham Sekhar Sahoo [1] **Marianne Arriola** [1] **Yair Schiff** [1] **Aaron Gokaslan** [1] **Edgar Marroquin** [1] **Justin T Chiu** [1] **Alexander Rush** [1] **Volodymyr Kuleshov** [1]

Abstract

While diffusion models excel at generating high-quality images, prior work reports a significant performance gap between diffusion and autoregressive (AR) methods in language modeling. In this work, we show that simple masked discrete diffusion is more performant than previously thought. We apply an effective training recipe that improves the performance of masked diffusion models and derive a simplified, Rao-Blackwellized objective that results in additional improvements. Our objective has a simple form—it is a mixture of classical masked language modeling losses—and can be used to train encoder-only language models that admit efficient samplers, including ones that can generate arbitrary lengths of text semi-autoregressively like a traditional language model. On language modeling benchmarks, a range of masked diffusion models trained with modern engineering practices achieves a new state-of-the-art among diffusion models, and approaches AR perplexity. We release our code at: https://github.com/kuleshov-group/mdlm

1. Introduction

Diffusion models excel at producing realistic, high-quality images and have received significant attention as potential tools for generating discrete data such as text (Austin et al., 2021; Li et al., 2021; Lou et al., 2023), biological sequences (Avdeyev et al., 2023), and graphs (Sun & Yang, 2023; Vignac et al., 2022). Unlike autoregressive (AR) approaches, diffusion-based methods are not constrained to generate data sequentially, and therefore have the potential to improve long-term planning, controllable generation, and sampling speed. However, discrete diffusion methods exhibit a performance gap relative to AR models (Austin et al., 2021; Gulrajani & Hashimoto, 2024; He et al., 2022; Lou et al., 2023), especially in language modeling. The standard measure of language modeling performance is log-likelihood: when controlling for parameter count, prior work reports a sizable log-likelihood gap between AR and diffusion models.

In this work, we show that a simple masked diffusion language modeling (MDLM) framework combined with effective training recipes significantly improves log-likelihood training of discrete diffusion models (Austin et al., 2021; He et al., 2022). We develop a well-engineered MDLM implementation based on a simple substitution-based parameterization of the reverse diffusion process that enables us to derive a Rao-Blackwellized continuous-time variational lower bound (ELBO) with improved tightness. Interestingly, our objective has a simple form: it is a weighted average of masked language modeling (MLM) losses (Devlin et al., 2018), and can be used to endow BERT-style, encoder-only models with principled generation capabilities. We complement this framework with efficient samplers—including ones that can generate semi-autoregressively like a typical language model.

Our masked diffusion models achieve a new state-of-the-art among diffusion models on language modeling benchmarks and approach the perplexity of AR models within 15-25%. Surprisingly, simple engineering choices significantly improve performance in both our models and simple baselines that were previously thought to perform poorly. Our framework also extends to non-language domains, including biological sequence modeling. We pre-train DNA sequence models and observe similar or higher downstream performance compared to classical BERT-style training, as well as adding generative capabilities that classical masked DNA language models lack.

Contributions We describe (1) a simple masked diffusion language modeling (MDLM) framework with a well-engineered implementation that outperforms all existing diffusion models across language modeling benchmarks (LM1B (Chelba et al., 2014), OWT (Gokaslan et al., 2019), DNA (Schiff et al., 2024)), and that significantly improves the performance of existing baselines (Austin et al., 2021;

[1]Department of Computer Science, Cornell Tech, NYC, USA. Correspondence to: Subham Sahoo <ssahoo@cs.cornell.edu>.

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He et al., 2022). Our MDLM framework implements (2a) a substitution-based parameterization (SUBS) of the reverse unmasking diffusion process; SUBS allows us to derive (2b) a simple, continuous-time, Rao-Blackwellized objective that improves tightness and variance of the ELBO, further increasing performance. We complement MDLM with (3) fast samplers that support semi-autoregressive (SAR) generation and outperform previous SAR models.

2. Background

2.1. Diffusion Models

Diffusion models are trained to iteratively undo a forward corruption process q that takes clean data $\mathbf{x}$ drawn from the data distribution $q(\mathbf{x})$ and defines latent variables $\mathbf{z}_t$ for $t \in [0,1]$ that represent progressively noisy versions of $\mathbf{x}$ (Ho et al., 2020; Sohl-Dickstein et al., 2015; Song et al., 2020). The standard forward process for continuous $\mathbf{x}$ is

$$\mathbf{z}_t = \sqrt{\alpha_t} \cdot \mathbf{x} + \sqrt{1-\alpha_t} \cdot \boldsymbol{\epsilon} \tag{1}$$

where $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0},\mathbf{I})$ and $(\alpha_t)_{t\in[0,1]}$ is a noise schedule, monotonically decreasing in t. The parameterized reverse diffusion model p_θ over $\mathbf{x}$ and $\mathbf{z}_t$ is trained to maximize a variational lower bound on log-likelihood (ELBO). Given a number of discretization steps T, defining $s(i) = (i-1)/T$ and $t(i) = i/T$, and using $D_{\mathrm{KL}}[\cdot]$ to denote the Kullback–Leibler divergence, the ELBO equals (Sohl-Dickstein et al., 2015):

$$\mathbb{E}_q \Bigg[\underbrace{\log p_\theta(\mathbf{x}|\mathbf{z}_{t(0)})}_{\mathcal{L}_{\text{recons}}} - \underbrace{\sum_{i=1}^{T} D_{\mathrm{KL}}[q(\mathbf{z}_{s(i)}|\mathbf{z}_{t(i)},\mathbf{x}) \| p_\theta(\mathbf{z}_{s(i)}|\mathbf{z}_{t(i)})]}_{\mathcal{L}_{\text{diffusion}}} \Bigg]$$
$$- \underbrace{D_{\mathrm{KL}}[q(\mathbf{z}_{t(T)}|\mathbf{x}) \| p_\theta(\mathbf{z}_{t(T)})]}_{\mathcal{L}_{\text{prior}}} \tag{2}$$

For brevity, we drop i from $t(i)$ and $s(i)$ below; in general, s will denote the time step before t.

2.2. Discrete Diffusion Models

Applications of diffusion modeling to discrete data can be broken into two broad categories. First are works that embed discrete structures in continuous space and then perform the Gaussian diffusion defined above on these continuous representations (Chen et al., 2022; Dieleman et al., 2022; Gulrajani & Hashimoto, 2024; Han et al., 2022; Li et al., 2022; Lovelace et al., 2024; Strudel et al., 2022). More related to our method are works that define a diffusion process directly on discrete structures. D3PM (Austin et al., 2021) introduces a framework with a Markov forward process $q(\mathbf{z}_t|\mathbf{z}_{t-1}) = \text{Cat}(\mathbf{z}_t; Q_t \mathbf{z}_{t-1})$ defined by the multiplication of matrices Q_t over T discrete time steps. This process induces marginals

$$q(\mathbf{z}_t|\mathbf{x}) = \text{Cat}(\mathbf{z}_t; \bar{Q}_t \mathbf{x}) = \text{Cat}(\mathbf{z}_t; Q_t \cdot Q_{t-1} \cdots Q_1 \mathbf{x}) \tag{3}$$

that represent the discrete-state form of (1). Extending this formalism to continuous time (as in (1)) relies on continuous time Markov chain (CTMC) theory (Campbell et al., 2022). The CTMC framework in turns leads to generalizations of the score matching perspective on diffusion modeling (Song & Ermon, 2019) to discrete data (Lou et al., 2023; Sun et al., 2022). Notably, SEDD (Lou et al., 2023) connects score-based approaches with ELBO maximization, enabling performant likelihood-based training of score-based models.

3. Simple Masked Diffusion Models

While previous work on discrete diffusion supports general forward processes (e.g., general Q_t in D3PM), absorbing state (i.e., masking) diffusion consistently achieves the best performance (Austin et al., 2021; Lou et al., 2023). In this work, instead of supporting general noise processes, we focus on masking and derive tight Rao-Blackwellized objectives that outperform general approaches and do not require CTMC theory. We denote our overall approach as masked diffusion (MDLM in the context of language models).

Notation. We denote scalar discrete random variables with K categories as 'one-hot' column vectors and define $\mathcal{V} \in \{\mathbf{x} \in \{0,1\}^K : \sum_{i=1}^K \mathbf{x}_i = 1\}$ as the set of all such vectors. Define $\text{Cat}(\cdot;\boldsymbol{\pi})$ as the categorical distribution over K classes with probabilities given by $\boldsymbol{\pi} \in \Delta^K$, where Δ^K denotes the K-simplex. We also assume that the K-th category corresponds to a special [MASK] token and let $\mathbf{m} \in \mathcal{V}$ be the one-hot vector for this mask, i.e., $\mathbf{m}_K = 1$. Additionally, let $\mathbf{1} = \{1\}^K$ and $\langle \mathbf{a}, \mathbf{b} \rangle$ and $\mathbf{a} \odot \mathbf{b}$ respectively denote the dot and Hadamard products between two vectors $\mathbf{a}$ and $\mathbf{b}$.

3.1. Interpolating Discrete Diffusion

We restrict our attention to forward processes q that interpolate between clean data $\mathbf{x} \in \mathcal{V}$ and a target distribution $\text{Cat}(.;\boldsymbol{\pi})$, forming a direct extension of Gaussian diffusion in (1). The q define a sequence of increasingly noisy latent variables $\mathbf{z}_t \in \mathcal{V}$, where the time step t runs from $t=0$ (least noisy) to $t=1$ (most noisy). The marginal of $\mathbf{z}_t$ conditioned on $\mathbf{x}$ at time t is

$$q(\mathbf{z}_t|\mathbf{x}) = \text{Cat}(\mathbf{z}_t; \alpha_t \mathbf{x} + (1-\alpha_t)\boldsymbol{\pi}), \tag{4}$$

where $\alpha_t \in [0,1]$ is a strictly decreasing function in t, with $\alpha_0 = 1$ and $\alpha_1 = 0$. This implies transition probabilities $q(\mathbf{z}_t|\mathbf{z}_s) = \text{Cat}(\mathbf{z}_t; \alpha_{t|s}\mathbf{z}_t + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top \mathbf{z}_t)$ where $\alpha_{t|s} = \alpha_t/\alpha_s$ and

$$q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}) = \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{z}_t + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top \mathbf{z}_t] \odot [\alpha_s \mathbf{x} + (1-\alpha_s)\boldsymbol{\pi}]}{\alpha_t \mathbf{z}_t^\top \mathbf{x} + (1-\alpha_t)\mathbf{z}_t^\top \boldsymbol{\pi}}\right) \tag{5}$$

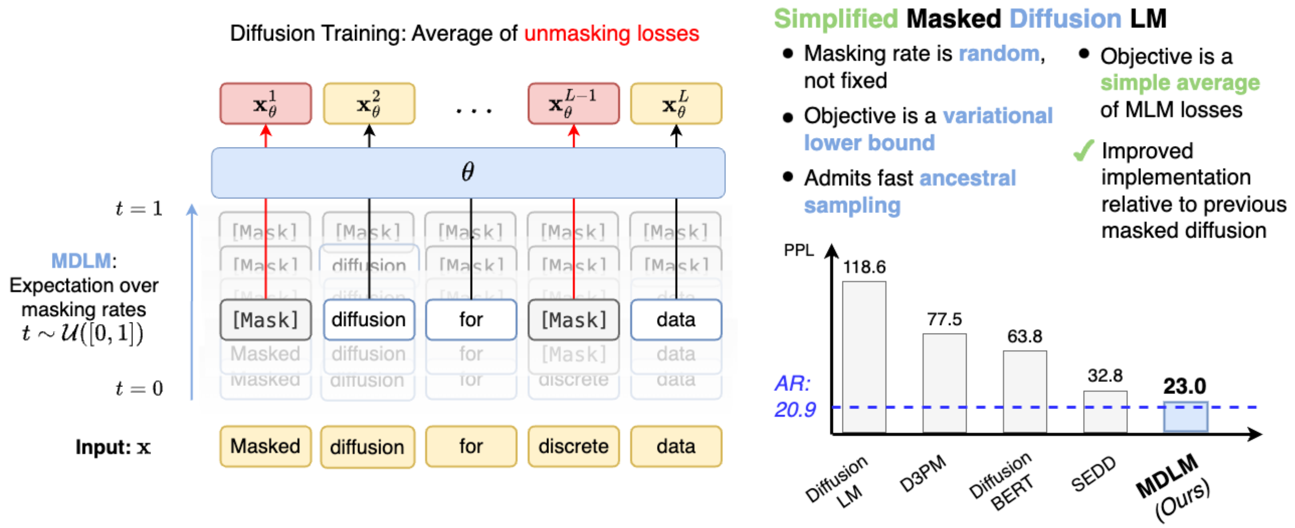

Figure 1: *(Left)* Our proposed masked diffusion language model (MDLM) is trained using a weighted average of masked cross entropy losses. (*Top Right*) In comparison to masked language models (MLM), MDLM's objective correspond to a principled variational lower bound, and supports generation via ancestral sampling. (*Bottom Right*) Perplexity (PPL) on One Billion Words benchmark.

See Suppl. 14 for details. While (4) and (5) represent a special case of the more general diffusion processes proposed in D3PM (Austin et al., 2021), we show below that they yield a simplified variational lower bound objective and admit straightforward continuous time extensions.

3.2. Masked Diffusion

Next, we focus on masking processes and derive a simple Rao-Blackwellized objective for this choice of q. This objective incurs lower variance during training and improves tightness.

3.2.1. Forward Masking Process

In masked (i.e., absorbing state) diffusion, we set $\boldsymbol{\pi} = \mathbf{m}$. At each noising step, the input $\mathbf{x}$ transitions to a 'masked' state $\mathbf{m}$ with a probability increasing in t. If an input transitions to $\mathbf{m}$ at any time t', it will remain in this state for all $t > t' : q(\mathbf{z}_t \,|\, \mathbf{z}_{t'} = \mathbf{m}) = \text{Cat}(\mathbf{z}_t; \mathbf{m})$. At time T, all inputs are masked with probability 1.

The marginal of the forward process (4) is given by $q(\mathbf{z}_t|\mathbf{x}) = \alpha_t \mathbf{x} + (1-\alpha_t)\mathbf{m}$. Using properties of the masking process, the posterior $q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x})$ simplifies (5); see Suppl. A:

$$q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}) = \begin{cases} \text{Cat}(\mathbf{z}_s;\mathbf{z}_t) & \mathbf{z}_t \neq \mathbf{m}, \\ \text{Cat}\left(\mathbf{z}_s; \frac{(1-\alpha_s)\mathbf{m} + (\alpha_s - \alpha_t)\mathbf{x}}{1-\alpha_t}\right) & \mathbf{z}_t = \mathbf{m}. \end{cases} \tag{6}$$

3.2.2. Reverse Unmasking Process

The reverse process inverts the noise process defined by q. We consider both a finite number of steps T, as well as a continuous time model corresponding to $T \to \infty$. We begin with the discrete-time case for which the generative model is expressed as $p_\theta(\mathbf{x}) = \int_{\mathbf{z}} p_\theta(\mathbf{z}_1) p_\theta(\mathbf{x}|\mathbf{z}_0) \prod_{i=1}^{T} p_\theta(\mathbf{z}_s|\mathbf{z}_t) \mathrm{d}\mathbf{z}_{0:T}$.

The optimal form for $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$ matches the true posterior in (6): this follows immediately from the definition of the diffusion objective in (2), which is a sum of terms of the form $\mathrm{D}_{\mathrm{KL}}(q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}) \| p_\theta(\mathbf{z}_s|\mathbf{z}_t))$. However, (6) is conditioned on $\mathbf{x}$, which we do not know. Therefore, we introduce a model $\mathbf{x}_\theta(\mathbf{z}_t, t) : \mathcal{V} \times [0,1] \to \Delta^K$ that approximates $\mathbf{x}$ with a neural network. We can also omit explicit dependence of $\mathbf{x}_\theta$ on time t, which simplifies sampling, yielding a 2x inference speed-up (see Suppl. H).

3.2.3. SUBS Parameterization

The specific parameterization for $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$ that we use is

$$p_\theta(\mathbf{z}_s|\mathbf{z}_t) = \begin{cases} \text{Cat}(\mathbf{z}_s;\mathbf{z}_t), & \mathbf{z}_t \neq \mathbf{m}, \\ \text{Cat}\left(\mathbf{z}_s; \frac{(1-\alpha_s)\mathbf{m} + (\alpha_s - \alpha_t)\mathbf{x}_\theta(\mathbf{z}_t,t)}{1-\alpha_t}\right). & \mathbf{z}_t = \mathbf{m}. \end{cases} \tag{7}$$

In order for $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$ to be a valid probability, $\mathbf{x}_\theta(\mathbf{z}_t, t)$ must satisfy two requirements. We implement these as substitutions to the output of $\mathbf{x}_\theta(\mathbf{z}_t, t)$, hence we call our parameterization SUBS.

Zero Masking Probabilities First, notice that by definition, $\langle \mathbf{x}, \mathbf{m} \rangle = 0$. For this reason, we design the denoising network such that $\langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle = 0$, i.e., we substitute the logit index corresponding to the [MASK] token with $-\infty$. This property enables the simplified expression of

(7) (Suppl. A.2.2) and ensures that case 2 in (7) is a valid probability.

Carry-Over Unmasking Second, if $\mathbf{z}_t$ is unmasked, then we desire $\mathbf{x}_\theta(\mathbf{z}_t, t) = \mathbf{z}_t$, i.e., unmasked latents are 'carried over'. We accomplish this by substituting the output of our network to simply copy unmasked inputs. This ensures that case 1 in (7) always holds, and furthermore reduces $\mathcal{L}_{\text{recons}}$ to 0.

3.3. Rao-Blackwellized Likelihood Bounds

Recall from (2) that the diffusion traning objective has the form $\mathcal{L}_{\text{recons}} + \mathcal{L}_{\text{diffusion}} + \mathcal{L}_{\text{prior}}$. For the simplified reverse process in (7), the discrete-time diffusion loss for finite T simplifies to (Suppl. B.1):

$$\mathcal{L}_{\text{diffusion}} = \sum_{i=1}^{T} \mathbb{E}_q [\mathrm{D}_{\mathrm{KL}}(q(\mathbf{z}_{s(i)} | \mathbf{z}_{t(i)}, \mathbf{x}) \| p_\theta(\mathbf{z}_{s(i)} | \mathbf{z}_{t(i)}))]$$
$$= \sum_{i=1}^{T} \mathbb{E}_q \left[\frac{\alpha_{t(i)} - \alpha_{s(i)}}{1 - \alpha_{t(i)}} \log \langle \mathbf{x}_\theta(\mathbf{z}_{t(i)}), \mathbf{x} \rangle \right]. \tag{8}$$

Note that this objective is simpler and more well-behaved than the expression one would obtain for $\mathrm{D}_{\mathrm{KL}}(q(\mathbf{z}_s | \mathbf{z}_t,\ \mathbf{x}) \| p_\theta(\mathbf{z}_s | \mathbf{z}_t))$ under the parameterization induced by using $p_\theta(\mathbf{z}_s | \mathbf{z}_t) = q(\mathbf{z}_s | \mathbf{z}_t, \mathbf{x} = \mathbf{x}_\theta(\mathbf{z}_t, t))$ from (5), which is similar to what is used by D3PM (Austin et al., 2021) (see Suppl. 27):

$$\left[\frac{\alpha_s - \alpha_t}{1 - \alpha_t} \log \frac{\alpha_t \langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle + (1 - \alpha_t)}{(1 - \alpha_t) \langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{x} \rangle} + \frac{1 - \alpha_s}{1 - \alpha_t} \log \frac{(1 - \alpha_s)(\alpha_t \langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle + (1 - \alpha_t))}{(1 - \alpha_t)(\alpha_s \langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle + (1 - \alpha_s))} \right] \langle \mathbf{z}_t, \mathbf{m} \rangle. \tag{9}$$

We refer to the process of obtaining (8) in lieu of (9) as a form of Rao-Blackwellization. Specifically, we analytically compute expectations such as $\langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle = 0$ in order to simplify objective (9) to obtain (8). Without analytical simplifications, a model must learn θ such that $\langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{m} \rangle = 0$ holds. Unlike in regular Rao-Blackwellization, simplifications are possible because of modeling choices for $\mathbf{x}_\theta(\mathbf{z}_t, t)$ (zero masking probabilities and carry-over unmasking). In that sense, our approach has similarities to graphical modeling, where incorporating conditional independencies into p_θ sets certain log-likelihood terms to zero. However, our approach also empirically helps reduce variance, hence we refer to it as Rao-Blackwellization, somewhat abusing the usual terminology.

3.4. Continuous-Time Likelihood Bounds

Previous works have shown empirically and mathematically that increasing the number of steps T yields a tighter approximation to the ELBO (Kingma et al., 2021). Following a similar argument, we form an continuous extension of (8) by taking $T \to \infty$ (see Suppl. B.2), which yields

$$\mathcal{L}_{\text{diffusion}}^{\infty} = \mathbb{E}_q \int_{t=0}^{t=1} \frac{\alpha_t'}{1 - \alpha_t} \log \langle \mathbf{x}_\theta(\mathbf{z}_t, t), \mathbf{x} \rangle \mathrm{d}t \tag{10}$$

Invariance to the noise schedule The function α_t is invertible due to the monotonicity assumption in Sec. 3.1, and so we can perform the following change of variables in (10): $\boldsymbol{\gamma} \equiv \log(1 - \alpha_t)$. Thus, the diffusion loss can be equivalently expressed as $\mathcal{L}_{\text{diffusion}}^{\infty} = -\mathbb{E}_q \int_{\gamma=-\infty}^{\gamma=0} \log \langle \mathbf{x}_\theta^i(\mathbf{x}_{\boldsymbol{\gamma}}), \mathbf{x}^i \rangle \mathrm{d}\boldsymbol{\gamma}$; see Suppl. 10 for details. This new formulation demonstrates that the diffusion loss is invariant to the functional form of α_t, which we verify empirically in Suppl. D.

3.5. Masked Diffusion Language Models

Next, we apply masked diffusion to language modeling over sequences $\mathbf{x}^{1:L}$ of L tokens, with $\mathbf{x}^\ell$ denoting the ℓ-th token. We make the assumption that the forward noising process is applied independently across a sequence and that, conditioned on a sequence of latents $\mathbf{z}_t^{1:L}$, the denoising process factorizes independently across tokens, i.e., $p_\theta(\mathbf{z}_s^{1:L} | \mathbf{z}_t^{1:L}) = \prod_{\ell=1}^{L} p_\theta(\mathbf{z}_s^\ell | \mathbf{z}_t^{1:L})$. Thus, we use a single model to compute $\mathbf{x}_\theta^\ell(\mathbf{z}_t^{1:L}, t)$ for each ℓ from a masked sequence $\mathbf{z}_t$, optimizing:

$$\mathcal{L}_{\text{diffusion}}^{\infty} = \mathbb{E}_q \int_{t=0}^{t=1} \frac{\alpha_t'}{1 - \alpha_t} \sum_{\ell} \log \langle \mathbf{x}_\theta^\ell(\mathbf{z}_t), \mathbf{x}^\ell \rangle \mathrm{d}t \tag{11}$$

Interestingly, our objective has a simple form: it is the weighted average of masked language modeling (MLM) losses (Devlin et al., 2018). Thus our work establishes a connection between generative diffusion models and encoder-only BERT models. Our objective enables principled selection of a (randomized) masking rate, and also endows BERT-style models with principled generation capabilities, see Sec. 6.

3.5.1. Efficient Training for Masked Diffusion

One of the key contributions of our work is a well-engineered implementation of masked diffusion models. Our experiments demonstrate that these improvements greatly boost performance even for methods previously thought to perform poorly, e.g., Austin et al. (2021). Below we briefly summarize these implementation details. First, we find that tokenization is critical to performance. Small vocabularies, such as the 8k vocabulary in Austin et al. (2021), result in longer-range dependencies that decrease the performance of both diffusion and AR models. Additionally, by focusing on masked diffusion, we are able to provide a numerically stable implementation of the objective function. Namely, since previous formulations of discrete diffusion were constructed to accommodate a wide range of limiting distributions (Austin et al., 2021), the objective was implemented by materializing the full transition matrices $\bar{Q}_t$ and posterior probabilities. In contrast, we evaluate $D_{\mathrm{KL}}[q(\mathbf{z}_s \mid \mathbf{z}_t, \mathbf{x}) || p_\theta(\mathbf{z}_s \mid \mathbf{z}_t)]$ by examining only the masked token indices rather than comparing the full true and approximate posterior distributions.

Furthermore, we modernize the architecture for the denoising network relative to D3PM (Austin et al., 2021). In lieu of the T5 architecture used in D3PM, we use the diffusion transformer (DiT) introduced in Peebles & Xie (2023), which integrates time step conditioning into a standard encoder-only transformer (Vaswani et al., 2017) and uses rotary positional embeddings (Su et al., 2021). In addition, we implement a low-discrepancy sampler that reduces the variance of the ELBO, similar to Kingma et al. (2021) and draws correlated samples t_i rather than performing i.i.d. sampling.

4. Inference and Sampling in Masked Diffusion Language Models

4.1. Efficient Ancestral Sampling

To generate a sequence of length L, the reverse diffusion process starts with the sequence $\mathbf{z}_{t=1}^{1:L}$ where $\mathbf{z}_{t=1}^{\ell} = \mathbf{m}, \forall \ell \in \{1, \ldots, L\}$. Then the subsequent latents, $\mathbf{z}_t^{1:L}$ are generated by discretizing the reverse diffusion process with some finite T. Given $\mathbf{z}_t^{1:L}$, we construct $\mathbf{z}_s^{1:L}$ by sampling each token $\mathbf{z}_s^{\ell}$ independently from the distribution $p_\theta(\mathbf{z}_s^{\ell} | \mathbf{z}_t^{1:L})$ given in (7).

4.2. Semi-Autoregressive Masked Diffusion Language Models

Our method also admits an effective semi-autoregressive (SAR) decoding method that allows the model to generate sequences of arbitrary length. Let $\tilde{\mathbf{x}}^{1:L}$ represent the output from sampling a sequence of L tokens using the reverse diffusion process described above. To generate additional $L' < L$ tokens, we propose a generation algorithm in which the latter $L - L'$ tokens $\tilde{\mathbf{x}}^{L':L-L'}$ are used as a prefix for an additional round of generation. Given the carry-over unmasking described in Sec. 3.2.3, these prefix tokens will simply be copied over at each decoding step. The remaining tokens are generated as above with $\mathbf{z}_s^{\ell} \sim p_\theta(\mathbf{z}_s^{\ell} \mid \mathbf{z}_t^{L':L+L'})$ for all $\ell \in \{L+1, \ldots L+L'\}$, with $\mathbf{z}_1^{L':L-L'}$ initialized to $\tilde{\mathbf{x}}^{L':L-L'}$ as opposed to being initialized as masked tokens $\mathbf{m}$. At the end of this process, we have produced $L + L'$ tokens $\mathrm{concat}[\tilde{\mathbf{x}}^{1:L}, \tilde{\mathbf{x}}^{L+1:L+L'}]$, where $\mathrm{concat}[\cdot]$ denotes concatenation along the sequence length dimension. This process can repeat indefinitely, with the prefix shifted for every new round of generation.

5. Experiments

5.1. Masked Diffusion Language Models

Experimental Setup We evaluate MDLM as a generative model of language and as a representation model via fine-tuning on downstream tasks.

For language modeling likelihood evaluation, we conduct experiments on two datasets: The One Billion Words Dataset (LM1B; (Chelba et al., 2014)) and OpenWebText (OWT; (Gokaslan et al., 2019)). We use the `bert-base-uncased` tokenizer for One Billion Words, and report perplexities on the test split. Models have a context size of 128. For OWT, which does not have a pre-defined split, we reserve the last 100K documents as a held-out validation set and report perplexities on this set. We use the `GPT2` tokenizer (Radford et al., 2019) for OWT. Models have a context size of 1,024. We utilize the transformer architecture from Lou et al. (2023), which augments the diffusion transformer (Peebles & Xie, 2023) with rotary embeddings (Su et al., 2021). MDLM was trained for 1M or 10M steps (corresponding to 33B, 330B tokens, respectively) on LM1B and 1M steps on OWT (which corresponds to 262B tokens). The corresponding AR baseline was trained for half the number of steps to ensure similar number of tokens seen (details in Suppl. F). Full hyperparameters are given in Suppl. I.1. On OWT, we train with and without time step conditioning.

For representation learning, we pre-train models on the C4 dataset (Raffel et al., 2020), then fine-tune and evaluate models on the GLUE benchmark (Wang et al., 2019). Models have a context size of 128. We use the `bert-base-uncased` tokenizer for the representation learning experiments. We utilize the MosaicBERT architecture from Portes et al. (2024), an extension of the original BERT architecture (Devlin et al., 2018). We pre-train a bidirectional MosaicBERT using an MLM objective for 37B tokens of C4, as well as a causal variant on the same data. We further fine-tune MosaicBERT model using the MDLM for 327M tokens, less than 1% of the pre-training data. We provide the full hyperparameters in Suppl. I.3.

Likelihood Evaluation On LM1B, MDLM outperforms all previous diffusion methods (Table 1). Compared to the SEDD baseline reported by Lou et al. (2023), trained for 66B tokens, MDLM, which we train for the same amount, achieves a 17% improvement on the perplexity bound. Finally, MDLM gets within 14% of an AR baseline and continues to improve with more training. We see the same trend for models trained on OWT, a larger dataset, shown in Table 2 – MDLM outperforms prior diffusion methods, closing the gap towards AR models. Results on OWT time step conditioning are in Table 10, Suppl. C.3 where we find that models trained with and without time conditioning attain similar perplexities. Additionally, Figure 2 demonstrates the reduced variance

Table 1: Test perplexities (PPL; ↓) on LM1B. †Reported in He et al. (2022). Best diffusion value is bolded.

		Parameters	PPL (↓)
Autoregressive	Transformer-X Base (Dai et al., 2019)	0.46B	23.5
	OmniNet$_T$ (Tay et al., 2021)	100M	21.5
Diffusion	BERT-Mouth (Wang & Cho, 2019)	110M	≤142.89
	D3PM (absorb) (Austin et al., 2021)	70M	≤77.50
	Diffusion-LM (Li et al., 2022)†	80M	≤118.62
	DiffusionBert (He et al., 2022)	110M	≤63.78
	SEDD (Lou et al., 2023) (33B tokens)	110M	≤ 32.79
Autoregressive (Retrained)	Transformer (33B tokens)	110M	22.32
	Transformer (330B tokens)		20.86
Diffusion (Ours)	MDLM (33B tokens)	110M	≤27.04
	MDLM (330B tokens)		≤**23.00**

Table 2: Test perplexities (PPL; ↓) on OWT for models trained for 262B tokens. † denotes retrained models.

	PPL (↓)
AR†	17.54
SEDD†	≤24.10
MDLM (Ours)	≤**23.21**

we achieve from our objective, when compared to previous masked diffusion models, such as SEDD (Lou et al., 2023).

Zero-Shot Likelihood Evaluation We also explore models' ability to generalize by taking models trained on OWT and evaluating how well they model unseen datasets. We compare the perplexities of our MDLM with a SEDD parameterization and an AR Transformer language model. Our zero-shot datasets include the validation splits of Penn Tree Bank (PTB; (Marcus et al., 1993)), Wikitext (Merity et al., 2016), LM1B, Lambada (Paperno et al., 2016), AG News (Zhang et al., 2015), and Scientific Papers (Pubmed and Arxiv subsets; (Cohan et al., 2018)). Full experimental details are available in Suppl. I.1.

MDLM consistently outperforms the SEDD diffusion parameterization. In some cases, e.g., for Lambada and Scientific Papers, MDLM attains better perplexity than AR. We hypothesize that these datasets are farther from OWT, and that diffusion models may be more robust to out-of-domain evaluation due to the unmasking-based objective.

Downstream Task Evaluation We find that BERT fine-tuned with MDLM to be a generative model results in strong perplexities while preserving performance on downstream tasks. On the C4 validation set, the AR model attains perplexity (PPL) of 22, the pre-trained BERT attains a PPL upper bound of 78 (evaluated using the MDLM variational bound), and BERT + MDLM-FT attains a PPL upper bound of 35. In Table 4, we further find that BERT + MDLM fine-tuning has no degradation in downstream GLUE performance compared to the BERT initialization. While the perplexity of our method is higher than the AR baseline, the downstream task performance is significantly better.

Semi-Autoregressive Modeling To test the SAR decoding algorithm presented in Sec. 4.2, we compare to SSD-LM (Han et al., 2022) a diffusion model that was designed to generate blocks of text autoregressively. We generate 200 sequences of length 2048 tokens on a single `3090` GPU and evaluate generative perplexity under a pre-trained GPT-2 (Radford et al., 2019) model. The SSD-LM sequences are generated using blocks of 25 tokens (as implemented in their pre-trained model) and the MDLM sequences are generated using $L' = 512$. In Table 5, we find that in addition to achieving better generative perplexity, MDLM enables ∼25-30x faster SAR decoding relative to SSD-LM.

5.2. Masked Diffusion DNA Models

We also explore the use of our generative formulation in conjunction with Structured State Space models (Gu et al., 2021). Namely, we build on the recently proposed Caduceus (Schiff et al., 2024) model, which uses as a backbone the data-dependent SSM Mamba block (Gu & Dao, 2023).

Experimental Setup We pre-train the encoder-only Caduceus (Schiff et al., 2024), which is an MLM, on the HG38 human reference genome (Consortium, 2009) and perform fine-tuning using our diffusion parameterization. We use a context length of 1024 tokens and follow Schiff et al. (2024) for the experimental setup, other than learning rate which was reduced to 1e-3. See Suppl. I.4 for full experimental details. We assess both generative performance

Table 3: Zero-shot validation perplexities (↓) of models trained for 524B tokens on OWT. All perplexities for diffusion models are upper bounds.

	PTB	Wikitext	LM1B	Lambada	AG News	Pubmed	Arxiv
AR (Retrained)	**82.05**	**25.75**	**51.25**	51.28	**52.09**	49.01	41.73
SEDD (Retrained)	100.09	34.28	68.20	49.86	62.09	44.53	38.48
MDLM (Ours)	95.26	32.83	67.01	**47.52**	61.15	**41.89**	**37.37**

Table 4: GLUE evaluation results. Evaluation measures (↑) are F1 score for QQP and MRPC, Spearman correlations for STS-B, and accuracy for the rest. For MNLI, we report match/mismatch accuracies.

	MNLI (m/mm)	QQP	QNLI	SST-2	COLA	STS-B	MRPC	RTE	Avg
AR	80.94/80.78	86.98	86.16	90.14	33.43	84.32	83.88	47.29	74.88
BERT	84.43/85.35	88.41	**90.46**	**92.20**	54.81	**88.41**	89.16	61.37	81.62
+MDLM-FT	**84.76/85.07**	**88.49**	90.30	**92.20**	**57.69**	87.48	**90.53**	**62.09**	**82.06**

Table 5: Semi-AR generative perplexity (Gen. PPL; ↓) for sequences of 2048 tokens.

	Gen. PPL (↓)	Sec/Seq (↓)
SSD-LM	35.43	2473.9
MDLM (Ours)	**27.18**	**89.3**

Table 6: Test perplexities (PPL; ↓) of generative fine-tuning of the Caduceus MLM (Schiff et al., 2024) on the HG38 reference genome. Best diffusion model values are bolded. Error bars indicate the difference between the maximum and minimum values across 5 random seeds used for fine-tuning. † denotes retrained models.

		Params	PPL (↓)
AR†	Mamba	465K	$3.067 \pm .0104$
	HyenaDNA	433K	$3.153 \pm .001$
Dif†	Plaid	507K	$\leq 3.240 \pm .005$
	SEDD	467K	$\leq 3.216 \pm .003$
Dif (Ours)	MDLM	467K	$\leq \mathbf{3.199} \pm .010$

using perplexity and downstream performance on Genomics Benchmarks (Grešová et al., 2023) across language diffusion paradigms and AR models.

Generative Performance We fine-tune the Caduceus MLM across diffusion parameterizations and compare perplexities against AR models. We report perplexity values in Table 6. MDLM outperforms all other diffusion language modeling schemes.

Downstream Task Fine-tuning We perform downstream evaluation with the Genomics Benchmarks (Grešová et al., 2023), a recently proposed benchmark with eight regulatory element classification tasks. As shown in Table 7, our generative fine-tuning paradigm preserves or improves upon downstream performance from MLM pre-training. Absorbing-state diffusion methods outperform Plaid across tasks except for the simplest task Human vs. Worm, where all methods have roughly the same performance. For tasks where the input is a biased subsample of the full genome, we observe that the correlation between perplexity and downstream performance is weaker; see Suppl. I.4.

5.3. Ablation Analysis

In Table 8, we can see the effect of our streamlined masked diffusion implementation. The improvements described in Sec. 3.5.1 allow us to greatly reduce perplexity of previously discounted models, such as D3PM (see the bottom row of this table, which is mathematically equivalent to the D3PM formulation). While most works assumed that D3PM achieves mediocre log-likelihoods, we show that is is incorrect: our re-implementation almost matches state-of-the-art score-based methods. This introduces a new strong baseline that opens new research opportunities. Additionally, in Table 8, we ablate different components of MDLM. We observe that the perplexity for MDLM trained with a discrete $T = 1000$ marginally worsens by 0.1 compared to MDLM trained in continuous time. Additionally, removing the “carry over” operation from the SUBS parameterization increases the perplexity by 2 points. However, further removing the “zero masking” operation does not lead to any meaningful change in perplexity.

We provide further ablations for the continuous time formulation in the Appendix, showing in Table 9 that for a pre-trained

Table 7: Genomic Benchmarks. Top-1 accuracy (↑) across 5-fold cross-validation (CV) for a pre-trained AR Mamba, and a pre-trained Caduceus model fine-tuned with different diffusion parameterizations. The best values per task are bolded and the second best are italicized. Error bars indicate the difference between the maximum and minimum values across 5 random seeds used for CV.

Model Fine-Tuning Objective (Parameter Count)	Mamba AR (465K)	Caduceus MLM (467K)	Caduceus Plaid (507k)	Caduceus SEDD (467k)	Caduceus MDLM (ours) (467k)
Mouse Enhancers	0.763 {±0.008}	**0.810** {±0.016}	0.745 {±0.079}	0.784 {±0.058}	*0.795* {±0.029}
Coding vs. Intergenomic	0.897 {±0.004}	**0.913** {±0.003}	*0.908* {±0.003}	**0.913** {±0.005}	**0.913** {±0.003}
Human vs. Worm	0.967 {±0.002}	*0.970* {±0.002}	**0.971** {±0.001}	*0.970* {±0.003}	*0.970* {±0.003}
Human Enhancers Cohn	0.734 {±0.027}	0.737 {±0.001}	*0.743* {±0.010}	**0.746** {±0.015}	*0.743* {±0.016}
Human Enhancer Ensembl	0.856 {±0.003}	**0.907** {±0.000}	0.885 {±0.003}	*0.905* {±0.006}	0.899 {±0.004}
Human Regulatory	0.861 {±0.008}	**0.874** {±0.003}	*0.868* {±0.010}	0.828 {±0.037}	*0.868* {±0.004}
Human OCR Ensembl	0.806 {±0.005}	*0.821* {±0.000}	0.820 {±0.004}	0.816 {±0.008}	**0.823** {±0.008}
Human NonTATA Promoters	0.926 {±0.008}	*0.935* {±0.014}	*0.935* {±0l007}	*0.935* {±0.014}	**0.940** {±0.007}

Table 8: Test perplexities (PPL; ↓) for MDLM ablations on LM1B. All the models were trained for 200K steps. Standard deviation is measured over 5 seeds during evaluation.

	PPL
MDLM	$\mathbf{33.59{\pm}.11}$
w/o Continuous time	$33.70{\pm}.07$
& carry-over	$35.57{\pm}.15$
& zero masking	$35.31{\pm}.16$

model, at inference, increasing T yields better likelihoods.

6. Discussion, Prior Work, and Conclusion

Comparison to D3PM Masked diffusion is a strict subset of D3PM (Austin et al., 2021); setting $Q_t = \alpha_{t|s} I + (1 - \alpha_{t|s}) \mathbf{1}\mathbf{m}^\top$ in their framework yields our forward diffusion. We improve over D3PM in three ways: (1) we adopt the SUBS parameterization for $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$; (2) this allows us to derive a simplified objective that analytically simplifies certain expectations to zero; (3) we adopt well-engineered training recipes that improve performance. Both (1) and (2) are possible because we focus on masking instead of developing a general discrete diffusion framework. Surprisingly, (3) has the largest contribution to performance.

Comparison to CTMC Most implementations of diffusion work best in continuous time. However, extending D3PM in this way requires computing the limit of the product of an infinite number of matrices $Q_T \cdot Q_{T-1} \cdots Q_t$ as $T \to \infty$, which requires advanced CTMC theory (Campbell et al., 2022). Our work describes simple continuous-time formulations for the most common noise processes (e.g., masking and uniform $\boldsymbol{\pi}$), thus helping make an important part of the literature more accessible. Our results remain compatible with CTMC: we effectively use rate matrices $R_t = \alpha'_t(\mathbf{1}\mathbf{m}^\top - I)$.

Comparison to Score Estimation Score-based approaches to diffusion (Song & Ermon, 2019) extend to discrete states, although they typically further build upon advanced CTMC theory. In particular, SEDD (Lou et al., 2023) derives and optimizes an ELBO that is a function of the score model, obtaining state-of-the-art log-likelihoods among diffusion models. We use a much simpler approach that requires no advanced theory.

Comparison to BERT Our work provides a principled way of making BERT generative when trained with randomized masking rates. Previous work on generating from BERT used Gibbs sampling or ad-hoc methods (Ghazvininejad et al., 2019; Liao et al., 2020; Wang & Cho, 2019). The connection between BERT and diffusion was first made by Austin et al. (2021): their objective effectively involves unmasking. He et al. (2022) additionally starts training from a pretrained BERT. However, both works use an objective that is similar to (9), which is less numerically stable than our objective (see Section 3.5.1). Austin et al. (2021) describe in their appendix how their ELBO can simplify to a weighted masking (MLM) loss similar to (8), but using a more complex formula for the weights. However, they do not train with that objective. Our work derives a simpler expression for the average of MLM losses, implements it, and obtains better likelihoods.

Conclusion In this work, we explore masked diffusion. With a well-engineered implementation that supports a simple variational objective, we attain state-of-the-art diffusion perplexities on language benchmarks and demonstrate how to efficiently convert BERT-style encoders into generative models. Given we are working on language modeling, we carry any of the inherent risks and opportunities that come with this line of research.

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A. Discrete time ELBO

This section is organized as follows: First, we derive the expressions for the true posterior and the approximate posterior as outlined in Suppl. A.1. We then simplify these expressions specifically for the case of absorbing state diffusion in Suppl. A.2. Finally, we derive the expression for the ELBO for absorbing state diffusion in Suppl. A.2.3.

A.1. Generic case

A.1.1. $q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x})$

Given the state transition matrix Q_t, prior $\boldsymbol{\pi}$, and the latent variables $\mathbf{z}_s$ and $\mathbf{z}_t$, where $s<t$, the forward process defined in (3) has the following posterior (Austin et al., 2021):

$$q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}) = \text{Cat}\left(\mathbf{z}_s; \frac{Q_{t|s}\mathbf{z}_t \odot Q_s^\top \mathbf{x}}{\mathbf{z}_t^\top Q_t^\top \mathbf{x}}\right) \tag{12}$$

$$Q_{t|s} = \alpha_{t|s}\mathbf{I}_n + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top \tag{13}$$

which we simplify to the following:

$$\begin{aligned}
&q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x})\\
&= \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{I}_n + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top]\mathbf{z}_t \odot [\alpha_s\mathbf{I}_n + (1-\alpha_s)\mathbf{1}\boldsymbol{\pi}^\top]^\top \mathbf{x}}{\mathbf{z}_t^\top [\alpha_t\mathbf{I}_n + (1-\alpha_t)\mathbf{1}\boldsymbol{\pi}^\top]^\top \mathbf{x}}\right)\\
&= \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{z}_t + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top \mathbf{z}_t] \odot [\alpha_s\mathbf{x} + (1-\alpha_s)\boldsymbol{\pi}]}{\mathbf{z}_t^\top [\alpha_t\mathbf{x} + (1-\alpha_t)\boldsymbol{\pi}\mathbf{1}^\top \mathbf{x}]}\right)\\
&\text{Using the property } \mathbf{1}^\top\mathbf{x}=1 \text{ we get,}\\
&= \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{z}_t + (1-\alpha_{t|s})\mathbf{1}\boldsymbol{\pi}^\top \mathbf{z}_t] \odot [\alpha_s\mathbf{x} + (1-\alpha_s)\boldsymbol{\pi}]}{\alpha_t\mathbf{z}_t^\top\mathbf{x} + (1-\alpha_t)\mathbf{z}_t^\top\boldsymbol{\pi}}\right).
\end{aligned} \tag{14}$$

A.1.2. $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$

Austin et al. (2021) approximate the reverse process in the following manner:

$$p_\theta(\mathbf{x}_s|\mathbf{x}_t) = q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}=\mathbf{x}_\theta(\mathbf{z}_t,t)) = \text{Cat}\left(\mathbf{x}_s; \frac{Q_{t|s}\mathbf{x}_t \odot Q_s^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}{\mathbf{x}_t^\top Q_t^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right). \tag{15}$$

where $\mathbf{x}_\theta(\mathbf{z}_t,t): \mathcal{V}\times[0,1] \to \Delta^K$ is an approximation for $\mathbf{x}$.

A.2. Absorbing state

For the absorbing state diffusion process we have $\boldsymbol{\pi}=\mathbf{m}$.

A.2.1. $q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x})$

Since, $\mathbf{z}_t \in \{\mathbf{x},\mathbf{m}\}$, takes only 2 values we consider the separate cases: $\mathbf{z}_t=\mathbf{x}$ and $\mathbf{z}_t=\mathbf{m}$.

Case 1. Consider the case $\mathbf{z}_t = \mathbf{x}$ i.e. $\mathbf{z}_t$ is unmasked. From (14), we have the following:

$$\begin{aligned}
&q(\mathbf{z}_s|\mathbf{z}_t = \mathbf{x}, \mathbf{x}) \\
&= \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{x} + (1-\alpha_{t|s})\mathbf{1}\mathbf{m}^\top \mathbf{x}] \odot [\alpha_s \mathbf{x} + (1-\alpha_s)\mathbf{m}]}{\alpha_t \mathbf{x}^\top \mathbf{x} + (1-\alpha_t)\mathbf{x}^\top \mathbf{m}}\right) \\
&= \text{Cat}\left(\mathbf{z}_s; \frac{[\alpha_{t|s}\mathbf{x}] \odot [\alpha_s \mathbf{x} + (1-\alpha_s)\mathbf{m}]}{\alpha_t}\right) && \text{since } \mathbf{x}^\top \mathbf{m} = 0 \\
&= \text{Cat}\left(\mathbf{z}_s; \frac{\alpha_t \mathbf{x}}{\alpha_t}\right) && \text{since } \mathbf{x}^\top \mathbf{m} = 0 \text{ and } \alpha_t = \alpha_{t|s}\alpha_s \\
&= \text{Cat}(\mathbf{z}_s; \mathbf{x}) && \text{since } \alpha_t = \alpha_{t|s}\alpha_s
\end{aligned} \tag{16}$$

Thus, we have the following:

$$q(\mathbf{z}_s|\mathbf{z}_t = \mathbf{x}, \mathbf{x}) = \text{Cat}(\mathbf{z}_s; \mathbf{x}). \tag{17}$$

Case 2. Consider the case $\mathbf{z}_t = \mathbf{m}$. By substituting $\mathbf{z}_t = \mathbf{m}$ and $\boldsymbol{\pi} = \mathbf{m}$ in (14), $q(\mathbf{z}_s|\mathbf{z}_t, \mathbf{x})$ simplifies to the following:

$$\begin{aligned}
q(\mathbf{z}_s|\mathbf{z}_t = \mathbf{m}, \mathbf{x}) &= \text{Cat}\left(\frac{(\alpha_{t|s}\mathbf{m} + (1-\alpha_{t|s})\mathbf{1}) \odot (\alpha_s \mathbf{x} + (1-\alpha_s)\mathbf{m})}{(1-\alpha_t)}\right) \\
&= \text{Cat}\left(\frac{(\alpha_{t|s}(1-\alpha_s)\mathbf{m} + (1-\alpha_{t|s})(1-\alpha_s)\mathbf{m} + (\alpha_s - \alpha_t)\mathbf{x})}{(1-\alpha_t)}\right) \\
&= \text{Cat}\left(\mathbf{z}_s; \frac{(1-\alpha_s)\mathbf{m} + (\alpha_s - \alpha_t)\mathbf{x}}{1-\alpha_t}\right)
\end{aligned} \tag{18}$$

Note that the above categorical distribution is non-zero for $\mathbf{z}_s \in \{\mathbf{x}, \mathbf{m}\}$ and zero for every other value. The non-zero values are specified as follows:

$$q(\mathbf{z}_s = \mathbf{x}|\mathbf{z}_t = \mathbf{m}, \mathbf{x}) = \frac{\alpha_s - \alpha_t}{1-\alpha_t} \tag{19}$$

$$q(\mathbf{z}_s = \mathbf{m}|\mathbf{z}_t = \mathbf{m}, \mathbf{x}) = \frac{1-\alpha_s}{1-\alpha_t} \tag{20}$$

A.2.2. $p_\theta(\mathbf{z}_s|\mathbf{z}_t)$

For the absorbing state diffusion process with $\boldsymbol{\pi} = \mathbf{m}$, we want to simplify the (15). For this reason, we consider 2 cases: first, when $\mathbf{z}_t \neq \mathbf{m}$ (**case 1**), second, when $\mathbf{z}_t \neq \mathbf{m}$ (**case 2**).

Case 1. Consider the case when $\mathbf{z}_t \neq \mathbf{m}$. (15) simplifies to the following:

$$\begin{aligned}
p_\theta(\mathbf{z}_s|\mathbf{z}_t \neq \mathbf{m}) &= \mathrm{Cat}\left(\mathbf{x}_s; \frac{Q_{t|s}\mathbf{z}_t \odot Q_s^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}{\mathbf{z}_t^\top Q_t^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right) && (21)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{Q_{t|s}\mathbf{z}_t \odot Q_s^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}{[Q_t\mathbf{z}_t]^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{z}_t] \odot [\alpha_s \mathbf{I}_n + (1-\alpha_s)\mathbf{m}\mathbf{1}^\top]\mathbf{x}_\theta(\mathbf{z}_t,t)}{[\alpha_t\mathbf{z}_t]^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{z}_t] \odot [\alpha_s \mathbf{x}_\theta(\mathbf{z}_t,t) + (1-\alpha_s)\mathbf{m}\langle \mathbf{1},\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle]}{\alpha_t\langle \mathbf{z}_t,\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle}\right)\\
&\text{since } \langle \mathbf{1},\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle = 1, \text{ we have the following:}\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{z}_t] \odot [\alpha_s \mathbf{x}_\theta(\mathbf{z}_t,t) + (1-\alpha_s)\mathbf{m}]}{\alpha_t\langle \mathbf{z}_t,\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle}\right)\\
&\text{since } \mathbf{z}_t \odot \mathbf{m} = \mathbf{0}, \text{ we have the following:}\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{\alpha_t\mathbf{z}_t \odot \mathbf{x}_\theta(\mathbf{z}_t,t)}{\alpha_t\langle \mathbf{z}_t,\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle}\right) && (22)
\end{aligned}$$

Case 2. Consider the case when $\mathbf{z}_t = \mathbf{m}$. (15) simplifies to the following:

$$\begin{aligned}
p_\theta(\mathbf{x}_s|\mathbf{z}_t = \mathbf{m}) &= \mathrm{Cat}\left(\mathbf{x}_s; \frac{Q_{t|s}\mathbf{m} \odot Q_s^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}{\mathbf{m}^\top Q_t \mathbf{x}_\theta(\mathbf{z}_t,t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{Q_{t|s}\mathbf{m} \odot Q_s^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}{[Q_t^\top\mathbf{m}]^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{m} + (1-\alpha_{t|s})\mathbf{1}] \odot [\alpha_s \mathbf{I}_n + (1-\alpha_s)\mathbf{m}\mathbf{1}^\top]\mathbf{x}_\theta(\mathbf{z}_t,t)}{[\alpha_t\mathbf{m} + (1-\alpha_t)\mathbf{1}]^\top \mathbf{x}_\theta(\mathbf{z}_t,t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{m} + (1-\alpha_{t|s})\mathbf{1}] \odot [\alpha_s \mathbf{x}_\theta(\mathbf{z}_t,t) + (1-\alpha_s)\mathbf{m}\langle \mathbf{1},\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle]}{\alpha_t\langle \mathbf{m},\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle + (1-\alpha_t)\langle \mathbf{1},\mathbf{x}_\theta(\mathbf{z}_t,t)\rangle}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{[\alpha_{t|s}\mathbf{m} + (1-\alpha_{t|s})\mathbf{1}] \odot [\alpha_s \mathbf{x}_\theta(\mathbf{z}_t,t) + (1-\alpha_s)\mathbf{m}]}{\alpha_t\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle + (1-\alpha_t)}\right)\\
&= \mathrm{Cat}\left(\mathbf{x}_s; \frac{\alpha_t\mathbf{m} \odot \mathbf{x}_\theta(\mathbf{z}_t,t) + (\alpha_s-\alpha_t)\mathbf{x}_\theta(\mathbf{z}_t,t) + (1-\alpha_s)\mathbf{m}}{\alpha_t\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle + (1-\alpha_t)}\right) && (23)
\end{aligned}$$

Note that the above categorical distribution, we can obtain the values for $p_\theta(\mathbf{x}_s = \mathbf{x}|\mathbf{x}_t = \mathbf{m})$ and $p_\theta(\mathbf{x}_s = \mathbf{m}|\mathbf{x}_t = \mathbf{m})$ which are as follows:

$$p_\theta(\mathbf{x}_s = \mathbf{x}|\mathbf{x}_t = \mathbf{m}) = \frac{(\alpha_s-\alpha_t)\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle}{\alpha_t\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle + (1-\alpha_t)} \tag{24}$$

$$p_\theta(\mathbf{x}_s = \mathbf{m}|\mathbf{x}_t = \mathbf{m}) = \frac{\alpha_s\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle + (1-\alpha_s)}{\alpha_t\langle \mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle + (1-\alpha_t)} \tag{25}$$

As a sanity check, we can verify that (24) reduces to (19), and (25) reduces to (20) if our denoising network can reconstruct $\mathbf{x}$ perfectly, i.e., $\mathbf{x}_\theta(\mathbf{z}_t,t) = \mathbf{x}$.

A.2.3. Diffusion Loss

For a given T, Let $\mathcal{L}_T = \mathbb{E}_{t\in\{1,\dots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})} T \mathrm{D}_{\mathrm{KL}}(q(\mathbf{x}_s|\mathbf{x}_t,\mathbf{x}) \| p_\theta(\mathbf{x}_s|\mathbf{x}_t))$ denote the diffusion loss. We break down the computation of $\mathrm{D}_{\mathrm{KL}}(q(\mathbf{x}_s|\mathbf{x}_t,\mathbf{x}) \| p_\theta(\mathbf{x}_s|\mathbf{x}_t))$ into 2 cases: $\mathbf{z}_t = \mathbf{x}$ (**case 1**) and $\mathbf{z}_t = \mathbf{m}$ (**case 2**).

Case 1. consider the case $\mathbf{z}_t = \mathbf{x}$. Let's simplify $\mathrm{D}_{\mathrm{KL}}(q(\mathbf{z}_s|\mathbf{z}_t = \mathbf{x},\mathbf{x}) \| p_\theta(\mathbf{z}_s|\mathbf{z}_t = \mathbf{x}))$.

$$
\begin{aligned}
&\mathrm{D_{KL}}(q(\mathbf{z}_s|\mathbf{z}_t=\mathbf{x},\mathbf{x})\|p_\theta(\mathbf{z}_s|\mathbf{z}_t=\mathbf{x}))\\
&=\sum_{\mathbf{z}_s} q(\mathbf{z}_s|\mathbf{z}_t=\mathbf{x},\mathbf{x})\log\frac{q(\mathbf{z}_s|\mathbf{z}_t=\mathbf{x},\mathbf{x})}{p_\theta(\mathbf{z}_s|\mathbf{z}_t=\mathbf{x})}\\
&\text{Since } q(\mathbf{z}_s|\mathbf{z}_t,\mathbf{x}) \text{ is 1 only for } \mathbf{z}_s=\mathbf{x} \text{ we get,}\\
&=\log\frac{1}{p_\theta(\mathbf{z}_s=\mathbf{x}|\mathbf{z}_t=\mathbf{x})}\\
&=\log 1 && \text{From (21)}\\
&=0
\end{aligned}
\tag{26}
$$

Case 2. Consider the case $\mathbf{z}_t=\mathbf{m}$. Let's simplify $\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))$.

$$
\begin{aligned}
&\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))\\
&=\sum_{\mathbf{x}_s} q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\log\frac{q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})}{p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m})}\\
&=\sum_{\mathbf{x}_s\in\{\mathbf{x},\mathbf{m}\}} q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\log\frac{q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})}{p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m})}\\
&=\underbrace{q(\mathbf{x}_s=\mathbf{x}|\mathbf{x}_t=\mathbf{m},\mathbf{x})\log\frac{q(\mathbf{x}_s=\mathbf{x}|\mathbf{x}_t=\mathbf{m},\mathbf{x})}{p_\theta(\mathbf{x}_s=\mathbf{x}|\mathbf{x}_t=\mathbf{m})}}_{\text{Simplify using (19) and (24)}}\\
&\quad+\underbrace{q(\mathbf{x}_s=\mathbf{m}|\mathbf{x}_t=\mathbf{m},\mathbf{x})\log\frac{q(\mathbf{x}_s=\mathbf{m}|\mathbf{x}_t=\mathbf{m},\mathbf{x})}{p_\theta(\mathbf{x}_s=\mathbf{m}|\mathbf{x}_t=\mathbf{m})}}_{\text{Simplify using (20) and (25)}}\\
&=\frac{\alpha_s-\alpha_t}{1-\alpha_t}\log\frac{\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t)}{(1-\alpha_t)\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle}\\
&\quad+\frac{1-\alpha_s}{1-\alpha_t}\log\frac{(1-\alpha_s)(\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t))}{(1-\alpha_t)(\alpha_s\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_s))}
\end{aligned}
\tag{27}
$$

Thus, $\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t,\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t))$ can be written in the following manner where $\langle\mathbf{z}_t,\mathbf{x}\rangle$ evaluates to 1 if $\mathbf{z}_t=\mathbf{x}$ and $\langle\mathbf{z}_t,\mathbf{m}\rangle$ evaluates to 1 if $\mathbf{z}_t=\mathbf{m}$:

$$
\begin{aligned}
&\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t,\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t))\\
&=\underbrace{\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{x},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{x}))}_{=0\text{, from (26)}}\langle\mathbf{z}_t,\mathbf{x}\rangle+\underbrace{\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))}_{\text{Given by (27)}}\langle\mathbf{z}_t,\mathbf{m}\rangle
\end{aligned}
\tag{28}
$$

Thus, we derive the diffusion loss, $\mathcal{L}_T$, in the following manner:

$$
\begin{aligned}
\mathcal{L}_T&=\mathbb{E}_{t\in\{1,\ldots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}T\mathrm{D_{KL}}(q(\mathbf{x}_s|\mathbf{x}_t,\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t))\\
&=\mathbb{E}_{t\in\{1,\ldots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}T\Bigg[\frac{\alpha_s-\alpha_t}{1-\alpha_t}\log\frac{\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t)}{(1-\alpha_t)\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle}\\
&\qquad+\frac{1-\alpha_s}{1-\alpha_t}\log\frac{(1-\alpha_s)(\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t))}{(1-\alpha_t)(\alpha_s\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_s))}\Bigg]\langle\mathbf{z}_t,\mathbf{m}\rangle
\end{aligned}
\tag{29}
$$

Note that $\mathcal{L}_T$ is 0 if $\mathbf{z}_t$ is an unmasked token i.e. $\mathbf{z}_t=\mathbf{x}$.

B. MDLM: Rao-Blackwelization using SUBS parameterization

In this section we show how SUBS parameterization can simplify the functional form of the ELBO as defined in (29).

B.1. ELBO

The SUBS parameterization, as described in Sec. 3.2.3, simplifies $\mathrm{D}_{\mathrm{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))$ ((27)) to the following:

$$\begin{aligned}
&\mathrm{D}_{\mathrm{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))\\
&=\frac{\alpha_s-\alpha_t}{1-\alpha_t}\log\frac{\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t)}{(1-\alpha_t)\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle}\\
&+\frac{1-\alpha_s}{1-\alpha_t}\log\frac{(1-\alpha_s)(\alpha_t\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_t))}{(1-\alpha_t)(\alpha_s\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle+(1-\alpha_s))}\\
&\text{Since SUBS sets } \langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{m}\rangle=0\text{, the above equation simplifies to the following:}\\
&=\frac{\alpha_s-\alpha_t}{1-\alpha_t}\log\frac{(1-\alpha_t)}{(1-\alpha_t)\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle}\\
&=\frac{\alpha_t-\alpha_s}{1-\alpha_t}\log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle
\end{aligned}\tag{30}$$

Using this, we obtain the following expression for the diffusion loss, $\mathcal{L}_T$:

$$\begin{aligned}
\mathcal{L}_T&=T\mathbb{E}_{t\in\{1,\dots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}\mathrm{D}_{\mathrm{KL}}(q(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m},\mathbf{x})\|p_\theta(\mathbf{x}_s|\mathbf{x}_t=\mathbf{m}))\langle\mathbf{z}_t,\mathbf{m}\rangle\\
&=T\mathbb{E}_{t\in\{1,\dots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}\frac{\alpha_t-\alpha_s}{1-\alpha_t}\log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle\langle\mathbf{z}_t,\mathbf{m}\rangle\\
&\text{When } \mathbf{z}_t=\mathbf{m},\ \log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle=0\text{; hence, the term } \langle\mathbf{z}_t,\mathbf{m}\rangle \text{ can be safely dropped to obtain:}\\
&=T\mathbb{E}_{t\in\{1,\dots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}\frac{\alpha_t-\alpha_s}{1-\alpha_t}\log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle
\end{aligned}\tag{31}$$

B.2. Continous Time ELBO

To derive the continuous-time diffusion loss, $\mathcal{L}^\infty_{\text{diffusion}}$, we consider the limiting case $\lim_{T\to\infty}\mathcal{L}_T$:

$$\begin{aligned}
\mathcal{L}^\infty_{\text{diffusion}}&=\lim_{T\to\infty}\mathcal{L}_T\\
&=\mathbb{E}_{t\in\{1,\dots,T\}}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}\left[\lim_{T\to\infty}T\frac{\alpha_t-\alpha_s}{1-\alpha_t}\log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle\right]\\
&\text{Using } \lim_{T\to\infty}T(\alpha_s-\alpha_t)=\alpha'_t\text{, we obtain:}\\
&=\mathbb{E}_{t\sim[0,1]}\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x})}\left[\frac{\alpha'_t}{1-\alpha_t}\log\langle\mathbf{x}_\theta(\mathbf{z}_t,t),\mathbf{x}\rangle\right]
\end{aligned}\tag{32}$$

C. Additional Experiments

C.1. LM1B ablations

We assess the importance of our continuous-time framework by performing ablation on diffusion steps T. In Table 9, we compare NLL and PPL under continuous and discrete T in MDLM. We find that NLL consistently decreases as $T\to\infty$.

C.2. Train NLL curves on OWT

In Figure 2, we show that MDLM achieves lower variance loss during training compared to a previous diffusion language model, SEDD. Training is performed over 1M steps on OWT (which corresponds to 524B tokens).

C.3. Time-conditioning ablation on OWT

In Table 10, we assess the importance of time conditioning in MDLM on OWT. We observe that time-conditioning has minimal impact on perplexity. Training is performed over 1M steps on OWT (which corresponds to 524B tokens).

Table 9: Discrete vs continuous time evaluation for MDLM on LM1B. MDLM was trained with $T=\infty$ and a smaller model containing 70M non-embedding parameters for 200K steps. We report test perplexity for a discrete T.

Method	NLL	PPL
$\text{MDLM}_{T=\infty}$	$\leq \mathbf{3.61 \pm 0.001}$	$\leq$**37.25**
$\text{MDLM}_{T=10}$	$\leq 4.14 \pm 0.003$	≤ 62.83
$\text{MDLM}_{T=100}$	$\leq 3.66 \pm 0.002$	≤ 39.04
$\text{MDLM}_{T=1000}$	$\leq 3.62 \pm 0.000$	≤ 37.38

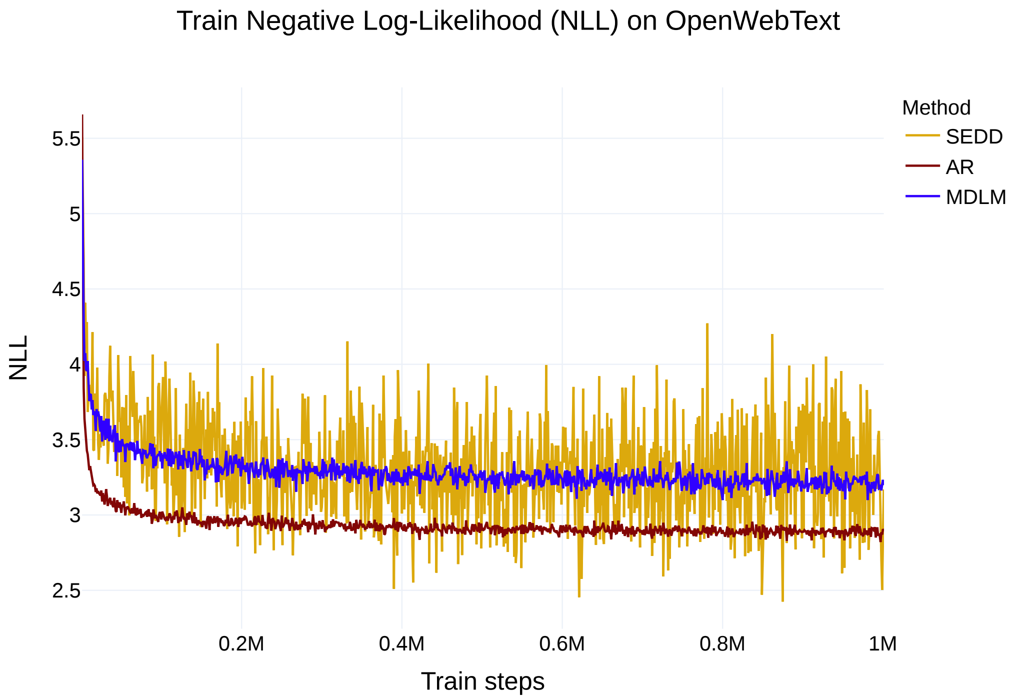

Figure 2: Train negative log-likelihood (NLL) curves across 1M gradient steps (524B tokens) on OpenWebText (Gokaslan et al., 2019). NLL is logged every 1K steps without value smoothing.

Table 10: Ablation on time-conditioning in MDLM on OWT.

Method	PPL
MDLM w/ time-conditioning	23.21
MDLM w/o time-conditioning	**23.05**

D. Noise schedule parameterization

As described in Sec. 3.4, the ELBO is invariant to the functional form of α_t. To demonstrate this, we evaluate MDLM, initially trained using a log-linear schedule on OWT, by replacing the noise schedule with various other noise schedules as mentioned below. Following prior works (Austin et al., 2021; Lou et al., 2023; Sohl-Dickstein et al., 2015), we parameterize $\alpha_t = e^{-\sigma(t)}$, where $\sigma(t): [0,1] \to \mathbb{R}^+$. Various functional forms of $\sigma(t)$ are listed below:

Log Linear (Austin et al., 2021; Lou et al., 2023; Sohl-Dickstein et al., 2015) The log linear schedule is given as:

$$\sigma(t) = -\log t \tag{33}$$

Cosine Squared schedule (Han et al., 2022) The Cosine Squared schedule is given as:

$$\sigma(t) = -\log\cos^2\left(\frac{\pi}{2}(1-t)\right) \tag{34}$$

Cosine schedule The Cosine schedule is given as:

$$\sigma(t) = -\log\cos^2\left(\frac{\pi}{2}(1-t)\right) \tag{35}$$

Linear The Linear schedule is given as:

$$\sigma(t) = \sigma_{\max}(1-t) \tag{36}$$

where $\sigma_{\max}$ is a very large number. In our experiments we set it to 10^8.

In Table 11 we demonstrate empirically that noise schedules with different functional forms evaluate to the same Likelihood which is consistent with our theory in Sec. 3.4. However, different schedules lead to different per data point variance.

Table 11: Likelihood in bits per dimension (BPD) for different noise schedules on OWT dataset, is reported along with the mean and variance associated with each noise schedule per data point. We empirically observe that noise schedules with different functional forms yield the same likelihood, consistent with our theory in Sec. 3.4; however, different schedules result in different variances. Notably, the log-linear schedule exhibits the lowest variance among all the noise schedules considered.

$\sigma(t)$	Mean	Variance per datapoint
Log Linear (33)	3.30	1.81
Cosine (35)	3.30	3.30
Cosine Squared (34)	3.30	3.30
Linear (36)	3.30	7.57

E. Likelihood Evaluation

How you do it Say that it incurs lower variance by referencing to the Ablattions table The variance is low because of the low discrepancy sampler

F. Avg. Number of Tokens seen

Given `training_steps`, `batch_size`, `context_length`, the number of tokens seen by the AR model is given as:

$$\texttt{training\_steps} \times \texttt{batch\_size} \times \texttt{context\_length}.$$

However, this expression doesn't hold true for a diffusion model, since at each training step, the model sees masked input. Let p_m be the probability of a token being masked at a timestep t. Then the diffusion model sees the following number of tokens in expection:

$$\begin{aligned}
&\mathbb{E}_t[\texttt{training_steps} \times \texttt{batch_size} \times \texttt{context_length} \times p_m] \\
&= \texttt{training_steps} \times \texttt{batch_size} \times \texttt{context_length} \times \mathbb{E}_t[p_m] \\
&\text{For log-linear schedule used in our experiments } p_m = t\text{; thus,} \\
&= \texttt{training_steps} \times \texttt{batch_size} \times \texttt{context_length} \times 0.5
\end{aligned} \tag{37}$$

G. Low discrepancy sampler

To reduce variance during training we use a low-discrepancy sampler, similar to that proposed in Kingma et al. (2021). Specifically, when processing a minibatch of N samples, instead of independently sampling N from a uniform distribution, we partition the unit interval and sample the time step for each sequence $i \in \{1,...,N\}$ from a different portion of the interval $t_i \sim U[\frac{i-1}{N}, \frac{i}{N}]$. This ensures that our sampled timesteps are more evenly spaced across the interval $[0,1]$, reducing the variance of the ELBO.

H. Faster sampling with caching

In Figure 12 we compare the wall clock times of variaous methods: AR, SEDD, MDLM with caching, and MDLM without caching for generating 64 samples on a single GPU. We observe that MDLM without caching yields samples that consistently get better generative perplexity than SEDD. For $T=\{5k, 10k\}$, both SEDD and MDLM get better generative perplexity than the AR model.

Table 12: Wall clock time reported in seconds.

	$T=5k$	$T=10k$
MDLM	4215.9	7675.4
+ caching	2407.3	3626.6
Speedup	1.75x	2.12x

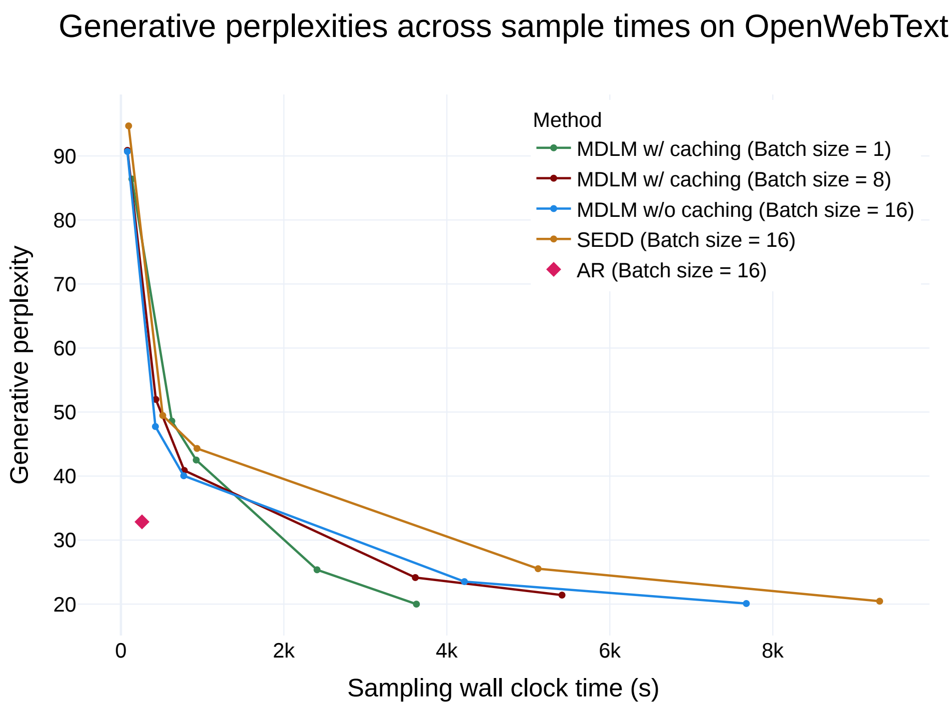

Figure 3: Generative perplexities across wall clock time for generating 64 samples on OWT using a single 32GB A5000 GPU are compared by varying $T \in \{100, 500, 1000, 5000, 10000\}$ in the reverse diffusion process. The samples are generated in mini-batches with a batch size of 16 for AR, SEDD, and MDLM without caching, as it is the largest batch size that fits on this GPU. For MDLM with caching, we vary the batch size.

I. Experimental details

I.1. Language Modeling

For our forward noise process, we use a log-linear noise schedule similar to Lou et al. (2023).

We detokenize the One Billion Words dataset following Lou et al. (2023), whose code can be found here. We tokenize the One Billion Words dataset with the `bert-base-uncased` tokenizer, following He et al. (2022). We pad and truncate sequences to a length of 128.

We tokenize OpenWebText with the `GPT2` tokenizer. We do not pad or truncate sequences – we concatenate and wrap them to a length of 1,024. When wrapping, we add the `eos` token in-between concatenated. We additionally set the first and last token of every batch to be `eos`. Since OpenWebText does not have a validation split, we leave the last 100k docs as validation.

We parameterize our autoregressive baselines, SEDD, and MDLM with the transformer architecture from Lou et al. (2023). We use 12 layers, a hidden dimension of 768, 12 attention heads, and a timestep embedding of 128 when applicable. Word embeddings are not tied between the input and output.

We use the AdamW optimizer with a batch size of 512, constant learning rate warmup from 0 to a learning rate of 3e-4 for 2,500 steps. We use a constant learning rate for 1M, 5M, or 10M steps on One Billion Words, and 1M steps for OpenWebText. We use a dropout rate of 0.1.

I.2. Zeroshot Likelihood

We evaluate zeroshot likelihoods by taking the models trained on OpenWebText and evaluating likelihoods on the validation splits of 7 datasets: Penn Tree Bank (PTB; Marcus et al. (1993)), Wikitext (Merity et al., 2016), One Billion Word Language Model Benchmark (LM1B; Chelba et al. (2014)), Lambada (Paperno et al., 2016), AG News (Zhang et al., 2015), and Scientific Papers (Pubmed and Arxiv subsets; Cohan et al. (2018)). We detokenize the datasets following Lou et al. (2023). For the AG News and Scientific Papers (Pubmed and Arxiv), we apply both the Wikitext and One Billion Words detokenizers. Since the zeroshot datasets have different conventions for sequence segmentation, we wrap sequences to 1024 and do not add `eos` tokens in between sequences.

I.3. Representation Learning

Following Devlin et al. (2018), we evaluate on all GLUE tasks (Wang et al., 2019), but exclude WNLI.

We pre-train a MosaicBERT model on C4 (Raffel et al., 2020) for 70k steps, corresponding to 36B tokens. We pad and truncate the data to 128 tokens using the `bert-base-uncased` tokenizer.

MosaicBERT (Portes et al., 2024) has a similar architecture to `bert-base-uncased` and has 137M parameters, 12 layers, 12 attention heads, a hidden dimension of 768, an intermediate size of 3072, and ALiBi attention bias (Press et al., 2022).

For pre-training, we use the following hyperparameters: A global batch size of 4096 with gradient accumulation, a learning rate of 5e-4, linear decay to 0.02x of the learning rate with a warmup of 0.06x of the full training duration, and the decoupled AdamW optimizer with 1e-5 weight decay and betas 0.9 and 0.98.

For diffusion fine-tuning we use AdamW with a warmup of 2,500 steps from a learning rate of 0 to 5e-5, betas 0.95 and 0.999, and batch size 512. We train for 5k steps total, corresponding to 32M tokens.

For GLUE evaluation, we use the HuggingFace script found here. We use the default parameters for all datasets, except for a batch size of 16, which we found helped with smaller datasets. This includes the default of 3 epochs for all datasets and learning rate of 2e-5.

I.4. Diffusion DNA Models

Dataset We pre-train the Caduceus MLM (Schiff et al., 2024) on the HG38 human reference genome (Consortium, 2009). Following Schiff et al. (2024), we use character- / base pair-level tokenization. The dataset is based on the splits used in Avsec et al. (2021): the training split comprises of 35 billion tokens covering the human genome. This consists of 34,021 segments extended to a maximum length of 1,048,576 (220 segments). We maintain a constant 2^{20} tokens per batch. For the Genomics Benchmark tasks, we use 5-fold cross-validation where we split the training set into 90/10 train/validation splits.

Architecture The Caduceus MLM uses as a backbone a bi-directional variant of the data-dependent SSM Mamba block proposed in Gu et al. (2021). This architecture is ideal as it contains inductive biases that preserve reverse complement (RC) equviariance, respecting the inherent symmetry of double-stranded DNA molecules (Mallet & Vert, 2021; Schiff et al., 2024; Zhou et al., 2022).

Training details All models are pre-trained on 10B tokens (10K steps) and fine-tuned on a generative objective for an additional 50B tokens (50K steps). We use a global batch size of 1024 for a context length of 1024 tokens. Downstream task fine-tuning is performed for 16K steps (1B tokens).

For performing Caduceus MLM pre-training, we follow Schiff et al. (2024) for the model size configuration, and hyperparameter selection. For pre-training, we use a fixed 15% mask rate as done in Devlin et al. (2018). Of the 'masked' tokens, 80% are replaced with [MASK] , 10% are replaced with a random token from the vocabulary, and 10% are left unchanged.

For fine-tuning all Mamba-based models (including Caduceus) on diffusion objectives, we lower the learning rate from 8e-3 to 1e-3. For fine-tuning HyenaDNA (Nguyen et al., 2024), we lower the learning rate from 6e-4 to 5e-5. Similar to Gu et al. (2021); Schiff et al. (2024), we found that Mamba-based models were robust to higher learning rates. We exclude timestep embeddings for all Diffusion DNA experiments, as we show it has minimal impact on generative performance (see Table 10, Suppl. C.3).

We perform downstream task fine-tuning on the final hidden state embedding from pre-training. We perform mean pooling across the sequence length, which may vary from 200 to approximately 2,000 bps. We report the mean and $\pm$ on max/min classification accuracy over 5-fold cross-validation (CV) using different random seeds, with early stopping on validation accuracy. For each task, we do a hyperparameter sweep over batch size and learning rate and report the values of the 5-fold CV for the best configuration.

Genomic Benchmark Task Distributions We use a subset of the Genomic Benchmark tasks with an emphasis on tasks from Human data. The positive samples for each dataset were generated by selecting samples that were annotated, either computationally or experimentally, in previous work (e.g enhancers, promoters, open chromatin regions (OCR)) (Grešová et al., 2023). These annotations each correspond to subsets of the genome of varying sizes that may exhibit different distributions of DNA than those observed globally over the reference genome. Due to this, the observed dataset may have a different distribution than the data used for pre-training and calculating perplexity. This might in turn lead to a case where perplexity and downstream performance may not necessarily correlate.