Investment strategy on bitcoin and gold according to historic price

Abstract—**In today's quest for wealth, individuals are exploring investment opportunities beyond traditional bank deposits to maximize their returns. Gold and Bitcoin have emerged as popular investment products due to their price volatility. However, making informed investment decisions based on expected price fluctuations can be challenging. Market traders aim to capitalize on price differences through buying and selling assets, such as gold and Bitcoin, to maximize client returns. In this study, we analyze daily price data of Bitcoin from November 9, 2016, to October 21, 2021, and gold prices on market trading days. Our objective is to develop a mathematical model that determines optimal investment strategies for cash, gold, and Bitcoin portfolios on a daily basis. We use price prediction models, including the second-order moving average method, second exponential sliding prediction method, and gray prediction model, to forecast the prices of gold and Bitcoin. Then a decision model is made to maximize the gain.**

Index Terms—**investment strategy, exponential sliding prediction model, gray prediction model**

I. Introduction

The desire for wealth is inherent in every individual. When it comes to idle assets, mere reliance on bank deposits to generate interest is often unsatisfactory. Instead, individuals aspire to engage in rational investments that yield maximum benefits. In this context, gold and bitcoin have emerged as trendy investment products characterized by their price volatility. However, making decisions based on the anticipated price fluctuations of bitcoin over a certain period poses considerable challenges [1]–[5].

Professional market traders, who profit from price differentials through buying and selling activities, have embraced the opportunity presented by unstable assets such as gold and bitcoin. [9], [11]By engaging in transactions involving these assets, they aim to maximize the overall income of their clients. To address this investment scenario, we have access to historical bitcoin price data from November 9, 2016, to October 21, 2021, as well as gold price data on trading days. Our objective now is to establish a mathematical model that utilizes the past daily price fluctuations to inform the optimal decisions of buying, holding, or selling assets within cash, gold, and bitcoin portfolios on a daily basis.

Our investment endeavor commences with an initial cash amount of $1000 on November 9, 2016. Throughout the five-year trading period, traders will manage a portfolio denoted as [C, G, B], representing the allocation of funds in US dollars, troy ounces of gold, and bitcoin, respectively. The initial state of the portfolio is [1000, 0, 0], indicating an initial principal of $1000. Each transaction, whether a purchase or sale, incurs a commission fee expressed as a percentage of the transaction amount, denoted as α. Holding assets, however, does not incur any commission fees.

To formulate an optimal investment strategy that maximizes the benefits of limited assets, it is imperative to conduct a detailed analysis based on price trend predictions. Given the significant fluctuations observed in the prices of gold and bitcoin, we initially focus on predicting their future price trends using historical price data. Subsequently, these predictions will guide our investment decisions. In the first part of our analysis, we employ the second-order moving average method [12]–[15], the second exponential sliding prediction method [16]–[19], and the gray prediction model [20]–[22] to forecast the prices of gold and bitcoin. By calculating their respective forecast curves and comparing them with the actual price curves, we can ascertain the efficacy of these prediction techniques. Subsequently, we construct a decision-making model that guides investment actions, including buying or selling gold on gold trading days and buying or selling bitcoin on bitcoin trading days, with the aim of optimizing investment outcomes.

II. Assumptions and notations

- Assume that the closing price of a troy ounce in US dollars is the gold trading price for that day. The data given in the question is limited, and only 1 gold price is given for each day in the market trading day. Therefore, we assume that the price of gold is traded at the closing price given for each day.
- On the same day, gold and bitcoin can be held, bought, or sold in only one case. Since the price fluctuations of gold and bitcoin are measured in days, if we choose to sell A and buy B, it is equivalent to selling A-B. (when $A > B$)
- We assume that the transaction is done instantaneously. Because the price of gold and bitcoin fluctuates irregularly over a certain period, it is completely meaningless to predict price changes when the observation time is too short. So for a certain period, we do not invest and keep our cash holdings. Next, let's assume that we can get a more accurate prediction from day m onwards.

III. Price predicting models

A. Second-order moving average method

Let the current number of days from the start of the investment be the day t and $\{x_t\}$ be the time series of the value of one troy ounce of gold or one bitcoin. And the model is:

Symbol	Description	Unit
C(n)	The cash on day n	dollar
G(n)	Amount of gold on day n	troy
B(n)	The number of bitcoins on day n	/
Vg(n)	The closing price of one troy ounce on day n	dollar
Vb(n)	The price of one bitcoin on day n	dollar
Vg′(n)	The closing price of one troy ounce predicted on day n	dollar
Vb′(n)	The closing price of one bitcoin predicted on day n	dollar
t	Current days	/
T	The number of days between the current day and the forecast day	/
x_t	The value of one troy ounce of gold or one bitcoin converted to US dollars on day t	dollar
$\widehat{x_t}$	The value of one troy ounce of gold (bitcoin) converted to U.S.dollars on day t as predicted by the forecasting model	dollar
e_t+1	The deviation from the original prediction on day t + 1 at day t of the ex post facto is found at day t + 1	dollar
β	Smoothing parameters	/
$\hat{\beta}$	Prediction smoothing parameters	/
σ^2	Random disturbance variance	/
$\widehat{\sigma_\beta}$	Standard error of β	/
MSE	Mean Square Error	/
h	Developmental grayscale	/
u	Control gray scale	/

$$\widehat{\mathbf{x}}_{1,t+T} = a_t + Tb_t. \tag{1}$$

(T is the number of days between the day and the forecast day, a_t, b_t are the parameters.) First moving average:

$$M_{\mathrm{t}}{}' = \frac{x_t + x_{t-1} + \ldots + x_{t-n+1}}{n}. \tag{2}$$

Second moving average:

$$M_{\mathrm{t}}'' = \frac{M_t^{'} + M_{t-1}^{'} + \ldots + M_{t-n+1}^{'}}{n}. \tag{3}$$

In order to maintain the accuracy of prediction and keep the prediction starting point of the neural network model consistent, let n= 5. According to the above formula, $x_t = 2M_t' - M_t''$.

Let $a_t = x_t$, then $a_t = 2M_t' - M_t''$, $b_t = \frac{2}{n-1}(M_t' - M_t'')$, and let $T = 1$, then the value of a unit of gold (bitcoin) op day t+ 1 can be predicted. Record the predicted price of gold (bitcoin) on day t as $\hat{\mathbf{x}}_{1,t+1}$.

B. Second exponential sliding prediction method

First, a quadratic exponential prediction model is developed.

$$\begin{cases} \hat{x}_{2,t+T} = a_t + b_t T \\ a_t = 2S_t^{(1)} - S_t^{(2)} \\ b_t = \frac{\beta}{1-\beta}(S_t^{(1)} - S_t^{(2)}) \\ S_t^{(1)} = \beta x_t + (1-\beta)S_{t-1}^{(1)} \\ S_t^{(2)} = \beta S_t^{(1)} + (1-\beta)S_{t-1}^{(2)} \\ S_1^{(1)} = S_1^{(2)} = x_1 \end{cases}, \quad t = 2, 3, \cdots \tag{4}$$

We determine the value of smoothing parameters β ($0 < \beta < 1$).

The logical process of setting the optimal prediction parameters of the quadratic exponential smoothing model by regression method is divided into three steps [6]–[8], [10]:

1 construct the quadratic exponential smoothing random process according to the quadratic exponential smoothing prediction model;
2 The quadratic exponential smoothing regression model is established according to the quadratic exponential smoothing random process;
3 The regression coefficient of the quadratic exponential smoothing regression model is estimated to obtain the optimal prediction parameters.

C. Gray prediction model

Let the original data series $S_{(0)} = \{X_{(0)}(i) : i = 1, 2, ..., t\}$,where$X_{(0)}(i) = x_i$ .First, accumulate the processing. We get

$$X_{(1)}(k)=\sum_{i=1}^{k}X_{(0)}(i)=X_{(1)}(k-1)+X_{(0)}(k) \quad (5)$$

Establish the differential equation for the new series $X_{(1)}$:

$$\frac{\mathrm{d}X_{(1)}}{\mathrm{d}t}+hX_{(1)}=u. \quad (6)$$

(h is the development grayscale, and u is the control grayscale.)

When the initial condition t= $1, X_{(1)}=X_{(1)}(1)$.

The solution of the differential equation is

$$X_{(1)}(k+1)=(X_{(0)}(1)-\frac{u}{h})e^{-hk}+\frac{u}{h}. \quad (7)$$

Afterwards, the model is fitted to the series of gold (bitcoin) values up to that date and the predicted value at day t+ 1 is denoted as $\hat{x}_{3,t+1}$.

D. Integration

All the above 3 model methods are linear, so they can be integrated by averaging. After integrating the results obtained from the above three models, we obtain the prediction formula for the price of gold (bitcoin) as

$$\hat{x}_{t+1}=\frac{\hat{x}_{1,t+1}+\hat{x}_{2,t+1}+\hat{x}_{3,t+1}}{3}. \quad (8)$$

Substitute the known gold (bitcoin) price data to make the prediction curve and compare it with the actual curve as follows.

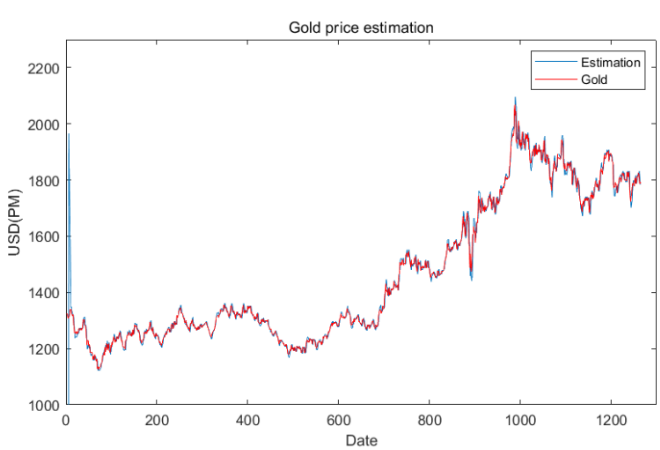

Fig. 1. Gold price estimation

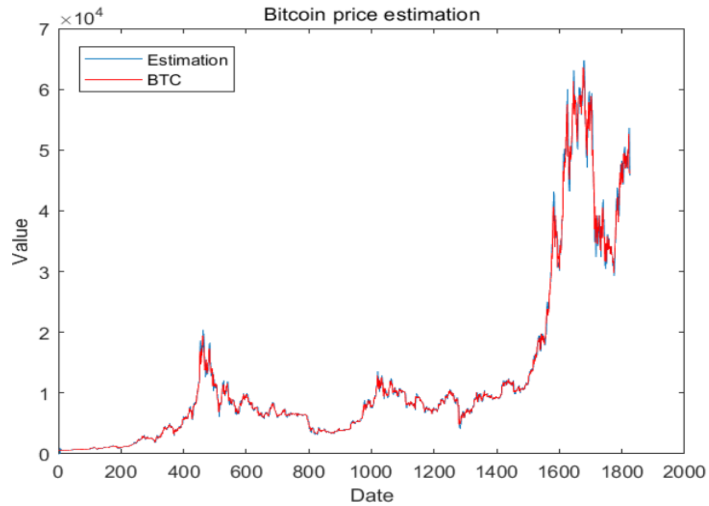

Fig. 2. Bitcoin price estimation

IV. Investment Strategy

First, we declare the meaning of some symbols.

Let

$$\Delta C=C(n+1)-C(n) \quad (9)$$
$$\Delta G=G(n+1)-G(n) \quad (10)$$
$$\Delta B=B(n+1)-B(n) \quad (11)$$
$$\mathrm{C(k)}=1000,\mathrm{G(k)}=0,\mathrm{B(k)}=0,\mathrm{k}=1,2,...,\mathrm{m} \quad (12)$$
$$(13)$$

where ΔC_g denotes the fraction of ΔC associated with gold trading, ΔC_b denotes the fraction of ΔC associated with bitcoin transactions

Now consider the number of days m, when $V_g(m)$, $V_b(m)$ are known and $V_g(m+1)', V_b(m+1)'$ can be derived from the prediction model.

In addition, since gold is not directly convertible with bitcoin, there are only two cases for each investment:

i cash to gold

ii cash and bitcoin transactions

The next two scenarios are open market days and non-open market days.

A. 0n market open days

0n market open days, there are four scenarios where the forecast is compared to the data of the day.

$\Delta Gi, \Delta Bj$ denote the $\Delta G, \Delta B$ in different cases. (First note that 'consider buying A' in the following indicates maintaining a hold or buy on A, but necessarily not selling A. 'consider selling' has the same meaning.)

a Buying gold

$$\Delta G_1=\frac{-\Delta C_g(1-\alpha_{gold})}{V_g(m)} \quad (14)$$

b Selling gold

$$-\Delta G_2 V_{\mathrm{g}}(m)(1-\alpha_{gold})=\Delta C_{\mathrm{g}} \quad (15)$$

c Buying bitcoin

$$\Delta B_1 = \frac{-\Delta C_b(1-\alpha_{bitcoin})}{V_b(m)} \tag{16}$$

d Selling bitcoin

$$-\Delta B_2 V_{\mathrm{b}}(m)(1-\alpha_{bitcoin}) = \Delta C_b \tag{17}$$

Denote

$$\begin{aligned} \Delta G &= \frac{-\Delta C_{\mathrm{g}}(1-\alpha_{\mathrm{gold}})}{V_{\mathrm{g}}(m)}k_1 + \frac{-\Delta C_{\mathrm{g}}}{V_{\mathrm{g}}(m)(1-\alpha_{\mathrm{gold}})}k_2 \\ \Delta B &= \frac{-\Delta C_b(1-\alpha_{bitcoin})}{V_b(m)}k_3 + \frac{-\Delta C_b}{V_b(m)(1-\alpha_{bitcoin})}k_4 \end{aligned} \tag{18}$$

Let

$$\begin{aligned} \Delta C &= \Delta C_g + \Delta C_b \\ C(m+1) &= C(m) + \Delta C \\ G(m+1) &= G(m) + \Delta G \\ B(m+1) &= B(m) + \Delta B \end{aligned} \tag{19}$$

At this point buying or selling gold or bitcoin can be divided into 4 scenarios. Considering all of them, we can get the programming (where $\Delta C_{\mathrm{g}}, \Delta C_b, k_i$ is the independent variable):

$$\max S' = C(m+1) + \mathcal{V}_s'(m+1)G(m+1) + \mathcal{V}_t'(m+1)\mathcal{B}(m+1)$$
$$\begin{cases} C(m+1) \geqslant 0 \\ G(m+1) \geqslant 0 \\ B(m+1) \geqslant 0 \\ k_i \in \{0,1\}, i=1,2,3. \\ k_1 k_2 = 0 \\ k_3 k_4 = 0 \\ \text{when} k_1 \neq k_2, (k_1-k_2)\Delta C_g \leqslant 0 \\ \text{when} k_1 = k_2, \Delta C_g = 0 \\ \text{when} k_3 \neq k_4, (k_3-k_4)\Delta C_b \leqslant 0 \\ \text{when} k_3 = k_4, \Delta C_b = 0 \end{cases} \tag{20}$$

Then substitute the solved $\Delta C_{\mathrm{g}}, \Delta C_b, k_i$ into $S = C(m+1) + V_g(m+1)G(m+1) + V_b(m+1)B(m+1)$ the investment is made according to the strategy.

B. On days when the market is not open

On days when the market is not open, gold cannot be traded, but bitcoin can be traded, and the situation is similar, except that k1=k2=0. where $\Delta C_g, \Delta C_b, k_i$ is the independent variable)

$$\begin{aligned} \max S = & C(m+1) + V_s'(m+1)G(m+1) + V_s'(m+1)B(m+1) \\ & C(m+1) \geqslant 0 \\ & G(m+1) \geqslant 0 \\ & B(m+1) \geqslant 0 \\ & k_i \in \{0,1\}, i=1,2,3,4 \\ & k_1 = k_2 = 0 \\ & k_3 k_4 = 0 \\ & \text{when} k_1 \neq k_2, (k_1-k_2)\Delta C_g \leqslant 0 \\ & \text{when} k_1 = k_2, \Delta C_g = 0 \\ & \text{when} k_3 \neq k_4, (k_3-k_4)\Delta C_b \leqslant 0 \\ & \text{when} k_3 = k_4, \Delta C_b = 0 \end{aligned} \tag{21}$$

Then substitute the solved $\Delta C_g, \Delta C_b, k_i$ into $S = C(m+1) + V_g(m+1)G(m+1) + V_b(m+1)B(m+1)$ to get the total asset value on the next day if the investment is made according to the strategy.

The analysis of the number of days m applies to the rest of the cases, just recursively.

V. RESULTS

Using the predicted price and investment strategy from above, the relationship between assets and time is shown in the figure below

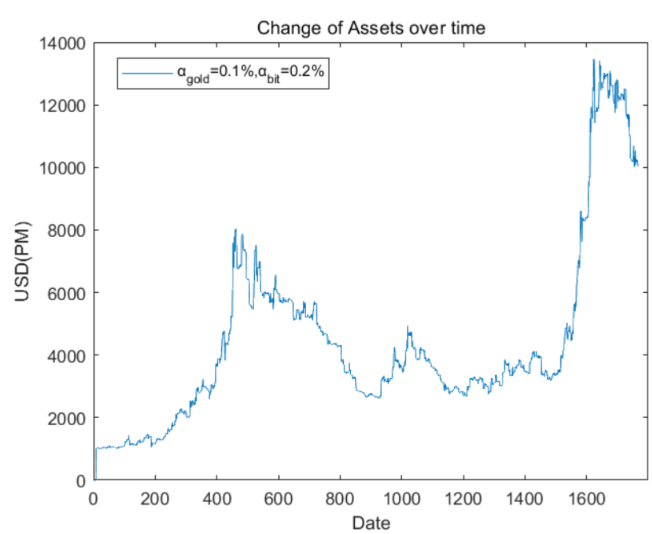

Fig. 3. Change of Assets over time

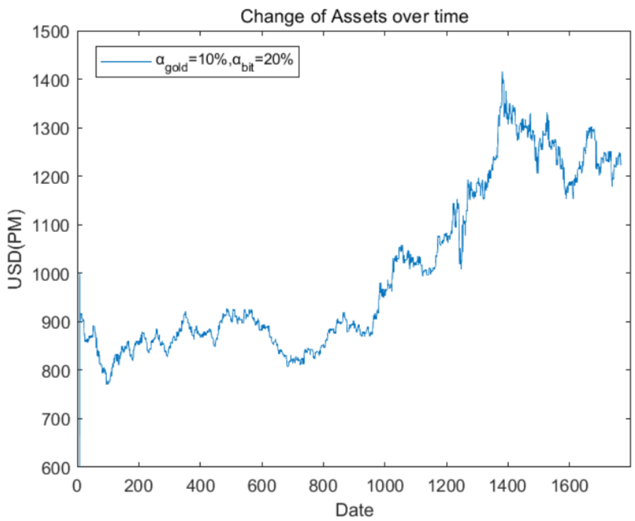

Fig. 4. Change of Assets over time

When $\alpha_{gold} = 10\%, \alpha_{bit} = 20\%$,the final asset value is 1224.0571 dollars.

When $\alpha_{gold} = 1\%, \alpha_{bit} = 2\%$, the final asset value is 285.4871 dollars.

When$\alpha_{gold} = 0.1\%, \alpha_{bit} = 0.2\%$,the final asset value is 10035.6275 dollars.

VI. Discussions

In conclusion, this research paper provides a comprehensive examination of price prediction models, namely the second-order moving average method, second exponential sliding prediction method, and gray prediction model, in the context of forecasting the prices of gold and Bitcoin. These models serve as crucial tools for the development of optimal investment strategies. Through meticulous analysis of historical price data and the application of these prediction models, investors are empowered to make well-informed decisions regarding the purchase, retention, or sale of assets within their cash, gold, and Bitcoin portfolios. The proposed strategies aim to maximize overall portfolio returns while taking into account transaction costs and the inherent volatility of the market.

To ensure the maximization of daily benefits, this study employs advanced mathematical tools related to optimization to make decisions based on precise trend predictions for gold and Bitcoin. By leveraging these mathematical techniques, the decision model utilized in this research enables an accurate evaluation of the impact of changes in individual transaction costs on transaction strategies.

While the daily closing price of gold is utilized as the trading price for the entire day, it is crucial to recognize that the trading price of gold experiences fluctuations within a given day. While the assumption of instantaneous transaction completion is made for analytical purposes, it is important to acknowledge the practical reality that real-world transactions cannot be executed instantaneously. Furthermore, the simultaneous trading of gold and Bitcoin, though not considered within the scope of this particular strategy, may present additional complexities that warrant further investigation and analysis to enhance investment decision-making.

In summary, this research paper contributes to the existing body of knowledge by offering a comprehensive analysis of price prediction models and their application in forecasting gold and Bitcoin prices. The proposed strategies provide valuable insights for investors seeking to maximize portfolio returns while accounting for transaction costs and market dynamics. However, it is important to continue exploring and refining these strategies to adapt to the evolving landscape of financial markets and the inherent complexities associated with trading gold and Bitcoin.

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