Dual Decomposition of Weights and Singular Value Low Rank Adaptation

Anonymous ACL submission

Abstract

Parameter-Efficient Fine-Tuning (PEFT) has emerged as a critical paradigm for adapting Large Language Models (LLMs) to downstream tasks, among which Low-rank Adaptation (LoRA) represents one of the most widely adopted methodologies. However, existing LoRA-based approaches exhibit two fundamental limitations: unstable training dynamics and inefficient knowledge transfer from pre-trained models, both stemming from random initialization of adapter parameters. To overcome these challenges, we propose DuDe, a novel approach that decomposes weight matrices into magnitude and direction components, employing Singular Value Decomposition (SVD) for principled initialization. Our comprehensive evaluation demonstrates DuDe's superior performance and robustness, achieving up to 48.35% accuracy on MMLU and 62.53% (±1.59) accuracy on GSM8K. Our theoretical analysis and empirical validation collectively demonstrate that DuDe's decomposition strategy enhances optimization stability and better preserves pre-trained representations, particularly for domain-specific tasks requiring specialized knowledge. The combination of robust empirical performance and rigorous theoretical foundations establishes DuDe as a significant contribution to PEFT methodologies for LLMs.

1 Introduction

Pre-trained models have demonstrated exceptional capabilities across diverse applications from Natural Language Processing (NLP) tasks (Qin et al., 2023) to multi-modal scenarios (Li et al., 2023; Liu et al., 2023). However, fine-tuning these large models remains computationally expensive.

Parameter-Efficient Fine-Tuning (PEFT) methods have emerged as a promising solution to this challenge. In particular, Low-Rank Adaptation (LoRA) has gained significant attention due to its ability to maintain the model's original architecture while enabling efficient fine-tuning. LoRA achieves this by injecting trainable low-rank matrices into the pre-trained weights, significantly reducing the number of parameters that need to be updated.

Despite its widespread adoption, LoRA and its variants face two fundamental challenges: 1) Training instability caused by random initialization, and 2) Inefficient utilization of pre-trained knowledge. To address these limitations, we propose DuDe (Dual Decomposition of Weights and Singular Value Low Rank Adaptation), which employs dual decomposition and Singular Value Decomposition (SVD) based initialization. Our experimental results validate DuDe's effectiveness through: (1) More stable training across different random seeds with only ±1.59 standard deviation (Section 4.5), and (2) Superior performance on knowledge-intensive MMLU tasks achieving up to 48.35% average accuracy (Section 4.4).

DuDe combines two key technical innovations: magnitude-direction decomposition inspired by DoRA (Liu et al., 2024) and SVD-based initialization building on PiSSA (Meng et al., 2024). Our main contributions include:

- A novel dual decomposition strategy that separates weights into magnitude and direction components, enabling more stable optimization
- An SVD-based initialization method that effectively preserves and leverages pre-trained knowledge
- Theoretical analysis that demonstrates improved gradient properties and optimization stability
- Comprehensive experiments showing consistent performance improvements across diverse models and tasks

Our extensive evaluation demonstrates DuDe's strong empirical performance across multiple benchmarks. Notably, DuDe exhibits exceptional performance on complex tasks requiring domain expertise, indicating its superior ability to preserve and adapt pre-trained knowledge.

2 Related Work

Large Language Models (LLMs) containing billions of parameters pose substantial challenges in terms of complexity and computational resources when adapting them to new tasks. PEFT (Houlsby et al., 2019) offers an attractive approach by reducing the number of parameters to be fine-tuned and memory requirements, while maintaining performance comparable to full fine-tuning.

Existing PEFT methods can be broadly categorized into three main approaches: Adapter-based Methods (Houlsby et al., 2019; Lei, 2023; Edalati et al., 2022), Selective Tuning Methods (Ben Zaken et al., 2022; Liao et al., 2023), and Re-parameterization Methods.

Re-parameterization Methods transform the original parameters into a more efficient representation. The most prominent example is LoRA (Hu et al., 2022), which injects trainable adapters into the pre-trained weight through low-rank decomposition. Following LoRA, several improvements have been proposed. DoRA (Liu et al., 2024) decomposes the pre-trained weight into magnitude and direction components, enhancing both learning capacity and training stability. PiSSA (Meng et al., 2024) initializes the adaptor matrices with the principal components of the pre-trained weight, freezing the remaining components in a residual matrix. OFT (Li et al., 2024) exploits orthogonal factorization for model fine-tuning. LoRA-XS (Bałazy et al., 2024) and OLoRA (Büyükakyüz, 2024) further reduce the number of parameters while maintaining performance. VeRA (Kopiczko et al., 2024) introduces vector-based random matrix adaptation for more efficient parameterization. SVFT (Lingam et al., 2024) uses singular vectors for PEFT, sharing some conceptual similarities with our work.

Our work, DuDe, builds upon these advances by combining the strengths of DoRA's magnitude-direction decomposition with PiSSA's SVD-based initialization. Unlike previous methods that either focus on decomposition or initialization separately, DuDe integrates both aspects to achieve more stable training and better utilization of pre-trained knowledge. The key innovation lies in our dual decomposition approach, which not only separates magnitude and direction but also performs SVD to initialize the direction matrix, leading to more effective adaptation while maintaining parameter efficiency.

3 Method

3.1 Preliminaries

Building upon the hypothesis that fine-tuning updates exhibit a low "intrinsic rank" (Aghajanyan et al., 2021), LoRA (Hu et al., 2022) employs the product of two low-rank matrices to efficiently update pre-trained weights (Figure 1a). For a pre-trained weight matrix $W_0 \in \mathbb{R}^{d\times k}$, LoRA parameterizes the weight update $\Delta W \in \mathbb{R}^{d\times k}$ as a low-rank decomposition BA, where $B \in \mathbb{R}^{d\times r}$ and $A \in \mathbb{R}^{r\times k}$ are low-rank matrices with rank $r \ll \min(d, k)$. The fine-tuned weight W' is therefore formulated as:

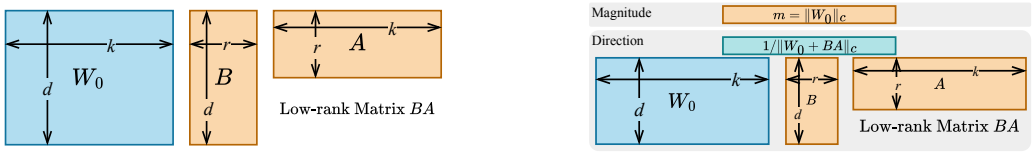
$$W' = W_0 + \Delta W = W_0 + \underline{BA} \tag{1}$$

During fine-tuning, W_0 remains frozen while only the low-rank matrices are trained. The matrices A are initialized using the Kaiming uniform distribution (He et al., 2015), while matrices B are initialized to zero, ensuring that $\Delta W = BA$ starts from zero at the beginning of training; thus the injection of adapters does not affect the model's output initially.

Inspired by Salimans and Kingma (2016), DoRA (Liu et al., 2024) decomposes the pre-trained weight into magnitude and direction components, and fine-tunes both components simultaneously (Figure 1b). To efficiently update the directional component with its large parameter space, DoRA adopts LoRA's low-rank decomposition approach. The formulation is expressed as:

$$W' = \underline{m}\frac{W_0 + \Delta W}{\|W_0 + \Delta W\|_c} = \underline{m}\frac{W_0 + \underline{BA}}{\|W_0 + \underline{BA}\|_c} \tag{2}$$

where $m \in \mathbb{R}^k$ represents the trainable magnitude vector, $\Delta W = BA$ is the directional update parameterized by two low-rank matrices $B \in \mathbb{R}^{d\times r}$ and $A \in \mathbb{R}^{r\times k}$ (with $r \ll \min(d, k)$), $\|\cdot\|_c$ denotes the column-wise vector norm, and underlined parameters are trainable during fine-tuning. Following LoRA, matrices B and A are initialized to ensure $\Delta W = 0$ at the beginning of training, maintaining the model's initial behavior while enabling effective adaptation.



(a) (b)

Figure 1: The blue parts in the figure represent frozen components, while the orange parts represent trainable components. (a) shows the diagrams of LoRA and PiSSA. The difference between them is that LoRA initializes matrix $B \in \mathbb{R}^{d\times r}$ to 0 and matrix $A \in \mathbb{R}^{r\times d}$ to Kaiming uniform distribution, while PiSSA first performs SVD on matrix W_0 to obtain $W_0 = U\Sigma V^\top$, then sets $B = U_r\sqrt{\Sigma_r}$, $A = \sqrt{\Sigma_r}V_r^\top$, and $W_0 = W_0 - BA$. (b) shows the diagrams of DoRA and DuDe. $m \in \mathbb{R}^k$ is the magnitude vector. For the direction matrix, DoRA initializes matrices B and A in the same way as LoRA, while DuDe initializes matrices B and A in the same way as PiSSA.

Model	Method	BoolQ	PIQA	SIQA	HellaSwag	WinoGrande	ARC-e	ARC-c	OBQA	Avg.
Qwen1.5-7B	LoRA	83.43	72.47	44.68	71.78	61.96	87.83	77.29	75.20	71.83
	DoRA	83.24	70.95	44.68	71.82	61.88	88.01	77.29	76.00	71.73
	PiSSA	84.04	74.32	44.73	71.53	61.64	87.65	78.31	74.20	72.05
	DuDe	84.04	75.57	44.98	71.37	62.98	87.65	78.31	75.40	**72.54**
Qwen2.5-32B	LoRA	89.85	90.75	46.16	92.11	79.16	97.53	93.90	89.40	84.86
	DoRA	90.03	90.59	46.26	92.06	79.08	97.35	93.56	89.80	84.84
	PiSSA	89.76	89.61	46.62	91.91	77.66	97.53	91.86	88.60	84.19
	DuDe	90.00	90.26	47.54	92.12	77.82	97.18	93.22	91.00	**84.89**
LLaMA2-13B	LoRA	65.84	73.78	53.68	48.63	51.78	79.01	59.32	60.00	61.51
	DoRA	61.07	73.94	54.25	49.98	51.46	79.19	61.02	60.40	61.41
	PiSSA	66.09	70.18	45.39	51.94	52.33	82.19	59.32	62.40	61.23
	DuDe	72.72	74.10	45.75	60.39	51.22	82.54	61.02	62.20	**63.74**

Table 1: Accuracy comparison of Qwen1.5-7B, Qwen2.5-32B, and LLaMA2-13B with different PEFT methods on eight commonsense reasoning tasks. The best results are highlighted in bold.

3.2 Dual Decomposition of Weights and Singular Value Low Rank Adaptation

In this section, we present our proposed method, Dual Decomposition of Weights and Singular Value Low Rank Adaptation (DuDe). As illustrated in Figure 1, DuDe performs SVD on the pre-trained weight matrix W_0 to derive optimal initialization parameters for low-rank adaptation. When applying SVD to a matrix $W_0 \in \mathbb{R}^{d\times k}$, we obtain the decomposition $W_0 = U\Sigma V^\top$, where $U \in \mathbb{R}^{d\times p}$ and $V \in \mathbb{R}^{k\times p}$ are orthogonal matrices containing the left and right singular vectors, and $\Sigma \in \mathbb{R}^{p\times p}$ is a diagonal matrix containing the singular values of W_0 in descending order, with $p = \min(d, k)$.

To effectively capture the most important features, the top r singular values and their corresponding singular vectors are extracted from Σ, U, and V, which are denoted as $\Sigma_r \in \mathbb{R}^{r\times r}$, $U_r \in \mathbb{R}^{d\times r}$, and $V_r \in \mathbb{R}^{k\times r}$. These components form the update matrix:

$$\Delta W = U_r\Sigma_r V_r^\top \tag{3}$$

The remaining components of the original weight matrix are preserved as:

$$W_f = W_0 - \Delta W \tag{4}$$

where W_f remains frozen during fine-tuning.

The low-rank matrices are initialized using the SVD components for efficient parameterization:

$$A = \sqrt{\Sigma_r}V_r^\top \in \mathbb{R}^{r\times k} \tag{5}$$

$$B = U_r\sqrt{\Sigma_r} \in \mathbb{R}^{d\times r} \tag{6}$$

where B and A are low-rank matrices with rank $r \ll p$.

The final fine-tuned weight W' integrates the frozen component W_f with the trainable low-rank update, scaled by a trainable magnitude vector m:

$$W' = \underline{m}\frac{W_f + \Delta W}{\|W_f + \Delta W\|_c} = \underline{m}\frac{W_f + \underline{BA}}{\|W_f + \underline{BA}\|_c} \tag{7}$$

At initialization, since $\Delta W = BA$, the fine-tuned weight W' is equivalent to the original weight W_0, ensuring that the model's initial behavior is preserved while enabling effective adaptation during training.

3.3 Gradient Analysis

In this section, we analyze the gradient of DuDe and demonstrate how our proposed decomposition enables more stable and efficient fine-tuning.

From Eq. (7), the gradient of loss $\mathcal{L}$ with respect to m and $W_0 = W_f + \Delta W$ can be derived as:

$$\frac{\partial \mathcal{L}}{\partial m} = \frac{\partial \mathcal{L}}{\partial W'} \frac{W_0}{\|W_0\|_c} \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial W_0} = \frac{m}{\|W_0\|_c} \left(\mathbb{I} - \frac{W_0 W_0^\top}{\|W_0\|_c^2} \right) \frac{\partial \mathcal{L}}{\partial W'} \tag{9}$$

Eq. (9) reveals that the gradient of W_0 undergoes two key transformations: scaling by $\frac{m}{\|W_0\|_c}$ and projection onto the orthogonal complement of W_0. These transformations help align the gradient's covariance matrix more closely with the identity matrix $\mathbb{I}$, promoting optimization stability.

Since $W_0 = W_f + \Delta W$, the gradient $\frac{\partial \mathcal{L}}{\partial W_0}$ is equivalent to $\frac{\partial \mathcal{L}}{\partial \Delta W}$. Consequently, all optimization benefits from this decomposition directly transfer to ΔW, enhancing DuDe's learning stability.

Furthermore, because the top r singular values and their corresponding singular vectors capture the most significant features of W_0, the gradient $\frac{\partial \mathcal{L}}{\partial \Delta W}$ contains more stable and informative signals compared to LoRA's gradient, leading to improved convergence properties.

Our experiments, as illustrated in Figure 2a, show that DuDe's loss and gradient norm curves closely resemble those of full fine-tuning, confirming that our dual decomposition effectively transfers the benefits of full fine-tuning while maintaining parameter efficiency.

4 Experiments

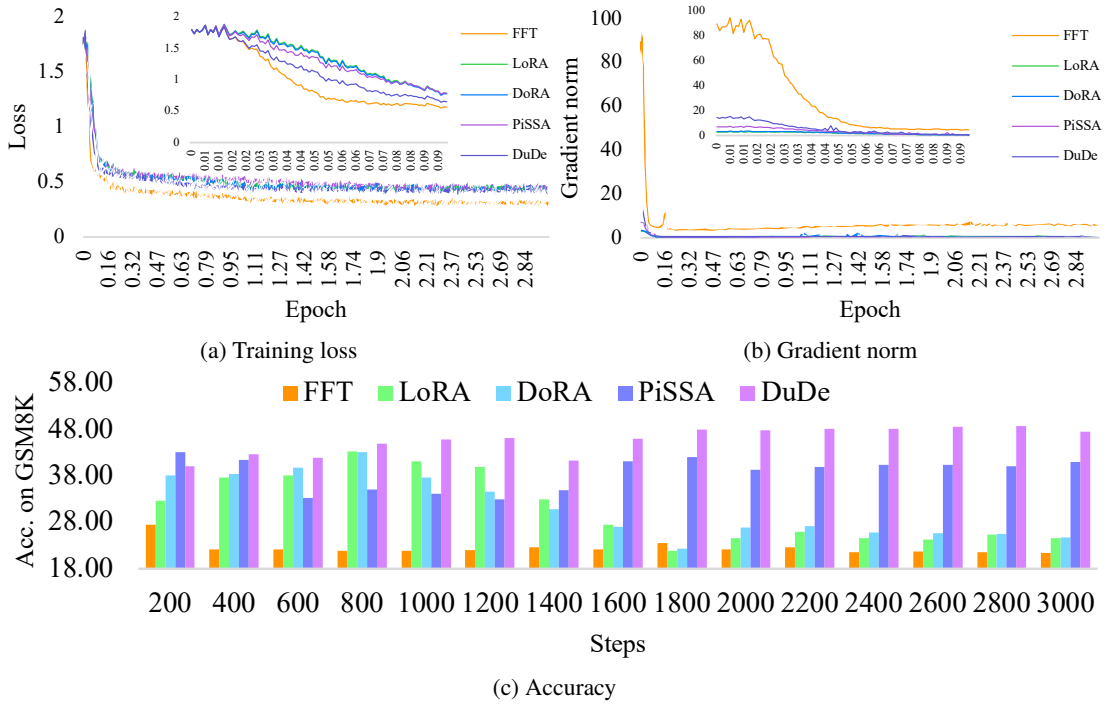
4.1 Commonsense Reasoning

DuDe is comprehensively evaluated against established PEFT methods (LoRA, DoRA, and PiSSA) on commonsense reasoning tasks across three different models: Qwen1.5-7B (Team, 2024), Qwen2.5-32B (Qwen et al., 2025), and LLaMA2-13B (Touvron et al., 2023). The evaluation suite is comprised of eight diverse commonsense reasoning benchmarks: BoolQ (Clark et al., 2019), PIQA (Bisk et al., 2019), SIQA (Sap et al., 2019), HellaSwag (Zellers et al., 2019), Winogrande (Sakaguchi et al., 2021), ARC-e/ARC-c (Clark et al., 2018), and OpenBookQA (Mihaylov et al., 2018). For all experiments, CommonsenseQA (Talmor et al., 2019) is used for fine-tuning and evaluations are performed on the respective test sets using the OpenCompass (Contributors, 2023) framework. For fair comparison, identical hyperparameters including rank r, learning rate, batch size, and training epochs are shared across all methods, with details being provided in Table 5.

Model	Method	Score
Qwen1.5-7B	LoRA	20.20
	DoRA	22.22
	PiSSA	19.19
	DuDe	**24.75**
Qwen2.5-14B	LoRA	39.39
	DoRA	40.40
	PiSSA	40.91
	DuDe	**41.41**
Mistral-7B v0.1	LoRA	15.66
	DoRA	20.20
	PiSSA	20.71
	DuDe	**23.74**
Phi4 small	LoRA	30.81
	DoRA	33.33
	PiSSA	35.35
	DuDe	**39.90**

Table 2: Score comparison of Qwen1.5-7B, Qwen2.5-14B, Mistral-7B v0.1, and Phi4 small with different PEFT methods on GPQA task. The best results are highlighted in bold.

As shown in Table 1, DuDe consistently outperforms all baseline methods across all three models. For Qwen1.5-7B, DuDe achieves an average accuracy of 72.54%, surpassing LoRA (71.83%), DoRA (71.73%), and PiSSA (72.05%), with particularly strong improvements on PIQA (+3.10% over LoRA) and Winogrande (+1.02% over LoRA). On Qwen2.5-32B, DuDe maintains its advantage with 84.89% average accuracy, showing notable gains on SIQA (+1.38% over LoRA). The most substantial improvements appear with LLaMA2-13B, where DuDe achieves 63.74% average accuracy, significantly outperforming LoRA (61.51%) by 2.23%. In this case, DuDe demonstrates remarkable gains on HellaSwag (+11.76% over LoRA) and BoolQ (+6.88% over LoRA), highlighting its



(a) Training loss

(b) Gradient norm

(c) Accuracy

Figure 2: Comparison of Full finetuning, DuDe and other PEFT methods on Mistral 7B v0.2 model: (a) Training loss, (b) Gradient norm during training on MetaMathQA-395K dataset for 3 epochs, and (c) Evaluation accuracy on GSM8K dataset measured every 200 steps over 3000 total training steps.

effectiveness in adapting different models to commonsense reasoning tasks.

4.2 GPQA Task

In this section, DuDe is evaluated on the GPQA (Rein et al., 2024) dataset, a challenging benchmark of graduate-level questions in biology, physics, and chemistry that cannot be easily answered through online searches. Deep domain knowledge and sophisticated reasoning capabilities are required by these questions.

Four different models (Qwen1.5-7B, Qwen2.5-14B, Mistral-7B v0.1 (Jiang et al., 2023), and Phi4 small (Abdin et al., 2024)) are fine-tuned on both the Main and Extended splits of GPQA, and their performance is evaluated on the Diamond split using the OpenCompass framework. Similar to the commonsense reasoning experiments, identical hyperparameters are maintained across all PEFT methods (LoRA, DoRA, PiSSA, and DuDe), including rank r, learning rate, and training epochs. However, due to the complexity of GPQA, a smaller batch size (set to 4) is used compared to the commonsense tasks. For Phi4 small model, W_{qkv} is used as the target module due to its different architecture, while the same target modules as in commonsense experiments are maintained for the other models.

Table 2 presents our findings. DuDe consistently outperforms all baseline methods across all models tested. With Qwen1.5-7B, DuDe achieves 24.75%, significantly surpassing LoRA (20.20%), DoRA (22.22%), and PiSSA (19.19%). On Qwen2.5-14B, DuDe reaches 41.41%, maintaining a consistent advantage over the baselines. For Mistral-7B v0.1, DuDe scores 23.74%, outperforming LoRA by a substantial 8.08 percentage points. The most dramatic improvement appears with Phi4 small, where DuDe achieves 39.90%, exceeding LoRA (30.81%) by 9.09 percentage points.

These results demonstrate DuDe's effectiveness in adapting various model architectures to complex, knowledge-intensive tasks requiring specialized expertise. The consistent performance improvements across different models highlight DuDe's versatility and robustness as a PEFT method.

4.3 Robustness to Different Epochs Settings

In this section, Mistral-7B v0.2 model is finetuned on MetaMathQA-395K (Yu et al., 2024) dataset. The detailed configuration is shown in Table 6. The training loss and gradient norms are visualized and evaluated on the GSM8K (Cobbe et al., 2021) dataset every 200 steps, by which quicker conver-

rank r	PEFT Method	Humanities	Social Science	STEM	Other	Avg.	Weighted Avg.
2	LoRA	49.09	51.87	40.54	47.74	46.51	45.11
	DoRA	48.88	51.95	40.17	47.39	46.28	44.83
	PiSSA	49.16	52.24	41.86	48.60	**47.24**	45.53
	DuDe	49.06	52.48	41.57	48.37	47.13	**45.56**
4	LoRA	49.28	51.38	40.05	47.11	46.15	44.93
	DoRA	48.87	52.62	40.19	47.62	46.48	45.05
	PiSSA	49.28	51.77	40.62	47.24	46.45	44.98
	DuDe	49.94	52.34	40.57	47.46	**46.75**	**45.34**
8	LoRA	47.91	51.02	38.30	47.66	45.30	43.68
	DoRA	47.65	50.79	36.47	46.87	44.40	43.09
	PiSSA	48.65	52.02	39.22	47.24	45.90	44.21
	DuDe	49.07	52.08	40.01	47.12	**46.24**	**44.72**
16	LoRA	48.64	51.17	41.48	47.32	46.48	44.99
	DoRA	49.44	53.12	39.86	48.37	46.78	45.41
	PiSSA	50.00	52.78	41.35	47.50	47.13	45.57
	DuDe	50.11	53.04	41.50	47.84	**47.34**	**45.88**
32	LoRA	49.71	52.63	39.97	48.52	46.81	45.48
	DoRA	49.78	52.82	40.49	47.83	46.88	45.48
	PiSSA	50.12	51.90	40.96	47.86	46.92	45.45
	DuDe	50.44	53.84	42.85	49.26	**48.35**	**46.52**

Table 3: Comparison of the average accuracy between LoRA and DuDe method across various rank settings for MMLU tasks. DuDe consistently outperforms LoRA at all rank settings. We also compare DuDe with DoRA and PiSSA, and find that DuDe achieves better performance than DoRA and PiSSA at all rank settings. The best results are highlighted in bold.

gence and superior performance of DuDe compared to other PEFT methods are demonstrated.

As shown in Figure 2, DuDe demonstrates superior performance compared to other PEFT methods across multiple metrics. From the training loss curve in Figure 2a, we observe that DuDe converges more quickly compared to LoRA, DoRA, and PiSSA. This faster convergence can be attributed to DuDe's dual decomposition approach and SVD-based initialization, which provides a better starting point for optimization.

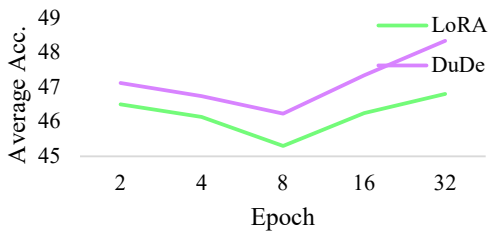
Most notably, the accuracy plot in Figure 2c demonstrates DuDe's consistent performance advantage. Starting from early training steps, DuDe achieves higher accuracy on the GSM8K evaluation set and maintains this lead throughout the training process. By the end of training, DuDe reaches a significantly higher final accuracy compared to baseline methods, indicating better generalization capabilities.

These empirical results validate our theoretical analysis that DuDe's decomposition strategy leads to more stable optimization dynamics and better utilization of the pre-trained model's knowledge. The combination of magnitude-direction decomposition and SVD-based initialization appears to create a more favorable optimization landscape, resulting in both faster convergence and superior final performance.

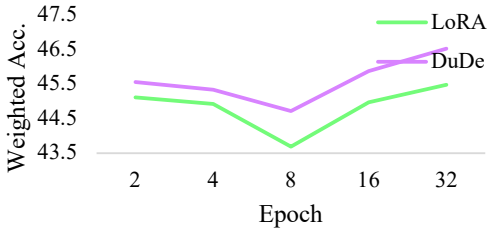
4.4 Robustness to Different Rank Settings

In this section, how different rank settings affect model performance is investigated by comparing DuDe with other PEFT methods. Experiments are conducted on Qwen2.5-0.5B using MMLU tasks (Hendrycks et al., 2021), where the rank r is varied among $\{2, 4, 8, 16, 32\}$. The detailed configuration is presented in Table 7. The results are presented in Figure 3 and Table 3.

As illustrated in Figure 3, DuDe demonstrates consistently superior performance across all rank configurations. The performance advantage becomes more pronounced as rank increases, with DuDe achieving the best results at $r = 32$ (48.35%



(a) Accuracy



(b) Weighted Average Accuracy

Figure 3: Performance comparison between LoRA and DuDe on MMLU tasks with varying rank settings. (a) Average accuracy across all MMLU categories shows DuDe consistently outperforming LoRA, especially at larger ranks. (b) Weighted average accuracy demonstrates similar trends, with DuDe maintaining superior performance across all rank configurations.

average accuracy and 46.52% weighted average accuracy). This represents improvements of 1.54% and 1.04% over LoRA respectively.

A detailed analysis of Table 3 reveals several key findings: 1) Performance Scaling: DuDe shows better scaling with increased rank compared to baseline methods. At $r = 32$, DuDe achieves the highest scores across all categories, with particularly strong performance in STEM (42.85%) and humanities (50.44%) subjects. 2) Low-Rank Efficiency: At lower ranks ($r = 2, 4$), while all methods perform similarly due to limited parameter capacity, DuDe maintains a slight advantage in weighted average accuracy (45.56% at $r = 2$, 45.34% at $r = 4$).

These results show that DuDe's dual decomposition and initialization strategies enable better model capacity utilization and achieve more robust performance across different ranks.

4.5 Robustness to Different Seed Settings

In this section, a comprehensive analysis of DuDe's robustness across different random seed settings is conducted. Qwen1.5-7B is finetuned on GSM8K tasks using five different random seeds (42, 78, 512, 1234, 3407).

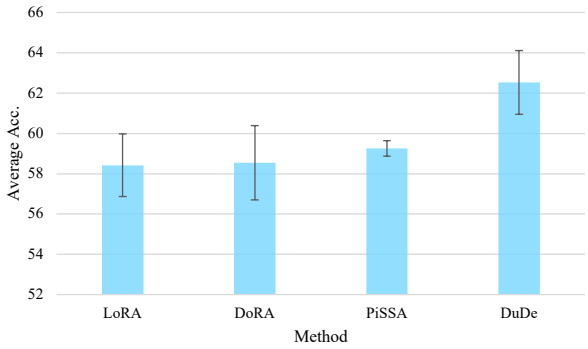


Figure 4: Average accuracy of DuDe and LoRA on MMLU tasks with different seeds.

The detailed performance trajectory across different seeds is visualized in Figure 4, which clearly illustrates DuDe's robust advantage over baseline methods. The experimental results demonstrate DuDe's superior stability and performance. Across all five seed settings, DuDe achieves the highest average accuracy of 62.53% with a standard deviation of 1.59. This represents a significant improvement over existing methods:

LoRA 58.42% average accuracy (±1.55 std)

DoRA 58.55% average accuracy (±1.85 std)

PiSSA 59.26% average accuracy (±0.38 std)

Most remarkably, even DuDe's worst performance (61.11% with seed 3407) surpasses the best results achieved by all baseline methods (LoRA's best: 60.35% with seed 42, DoRA's best: 60.96% with seed 42, PiSSA's best: 59.67% with seed 521). This demonstrates that DuDe not only achieves higher average performance but also maintains consistently superior results regardless of random initialization.

4.6 Differentiable Initialization

In this section, how different initialization strategies affect DuDe's performance is investigated. Specifically, two initialization variants are explored:

$$A = \Sigma_r V_r^\top, \quad B = U_r \tag{10}$$

and

$$A = V_r^\top, \quad B = U_r \Sigma_r \tag{11}$$

which we denote as DuDe$_\mathcal{A}$ and DuDe$_\mathcal{B}$ respectively. These variants differ in how they distribute the singular values between matrices A and B.

Qwen1.5-7B is finetuned on the MetaMathQA-395K dataset and Qwen2.5-0.5B is finetuned on

Model	Dataset	Metric	LoRA	DoRA	PiSSA	DuDe	DuDe$_{\mathcal{A}}$	DuDe$_{\mathcal{B}}$
Qwen1.5-7B	GSM8K	Acc.	60.35	60.96	59.44	64.22	67.48	66.72
Qwen2.5-0.5B	MMLU	Humanities	49.71	49.78	50.12	50.44	50.54	50.31
		Social Science	52.63	52.82	51.90	53.84	53.22	53.28
		STEM	39.97	40.49	40.96	42.85	42.40	42.27
		Other	48.52	47.83	47.86	49.26	49.57	48.61
		Avg.	46.81	46.88	46.92	48.35	48.17	47.87
		Weighted Avg.	45.48	45.48	45.45	46.52	46.62	46.10

Table 4: Performance comparison of different PEFT methods and DuDe variants on GSM8K and MMLU benchmarks. DuDe$_{\mathcal{A}}$ and DuDe$_{\mathcal{B}}$ represent different initialization strategies for the dual decomposition matrices.

MMLU tasks using different initialization methods. The experimental results are reported in Table 4.

The experimental results reveal several interesting patterns. For the GSM8K dataset using Qwen1.5-7B, both DuDe$_{\mathcal{A}}$ and DuDe$_{\mathcal{B}}$ significantly outperform the baseline DuDe implementation, achieving accuracies of 67.48% and 66.72% respectively, compared to DuDe's 64.22%. This suggests that carefully distributing singular values between matrices A and B during initialization can lead to better optimization dynamics.

For the MMLU benchmark using Qwen2.5-0.5B, the performance differences between initialization variants are more nuanced. DuDe$_{\mathcal{A}}$ shows slight improvements in Humanities (50.54% vs 50.44%) and Other categories (49.57% vs 49.26%), while performing marginally lower in Social Science (53.22% vs 53.84%) and STEM (42.40% vs 42.85%) compared to standard DuDe. DuDe$_{\mathcal{B}}$ generally performs slightly below both DuDe and DuDe$_{\mathcal{A}}$ across most categories, though the differences are relatively small.

Overall, while both initialization variants demonstrate competitive performance, DuDe$_{\mathcal{A}}$ appears to be the most promising, achieving the highest weighted average accuracy (46.62%) on MMLU and the best performance (67.48%) on GSM8K. This suggests that allocating singular values to matrix A during initialization may provide better optimization properties for PEFT.

5 Conclusion

In this paper, we introduced DuDe, a novel PEFT approach that combines dual decomposition of weights with singular value low-rank adaptation. Our method addresses two key limitations of existing PEFT approaches: training instability and under-utilization of pre-trained knowledge. Through the decomposition of weight matrices into magnitude and direction components, along with SVD-based initialization, DuDe achieves more stable optimization while better preserving the knowledge encoded in pre-trained models.

Our extensive experimental evaluation demonstrates DuDe's superior performance across multiple dimensions:

- Consistent improvements over baseline methods across different rank settings on the MMLU benchmark, achieving up to 48.35% average accuracy
- Robust performance across different random seeds on the GSM8K dataset, with an average accuracy of 62.53% (±1.59)
- Strong performance on complex tasks requiring deep domain expertise, suggesting better preservation of pre-trained knowledge

The theoretical analysis and empirical results validate our key design choices, showing how the dual decomposition strategy leads to more stable gradients and better optimization properties. These findings suggest that DuDe represents a meaningful step forward in PEFT, offering a more principled approach to adapting LLMs.

Future work could explore extending DuDe to other model architectures, investigating its effectiveness in multi-task scenarios, and further analyzing the theoretical foundations of its improved stability. Additionally, combining DuDe with other PEFT innovations could potentially yield even more efficient and effective adaptation methods.

6 Limitations

Despite DuDe's promising results, several key limitations need to be acknowledged. The SVD-based

initialization, while effective, introduces additional computational overhead during setup compared to simpler methods. This one-time cost can become significant when working with extremely large models or when rapid deployment is needed. Memory usage is also slightly higher than basic LoRA due to storing both magnitude and direction components, which may be problematic in resource-constrained environments.

Our current implementation focuses mainly on transformer architectures, particularly attention layers. The method's effectiveness on other architectures or different transformer components, remains to be thoroughly explored. The optimal application to emerging architectures such as mixture-of-experts models is also unclear.

While DuDe excels at complex tasks requiring domain expertise, its advantages may be less pronounced for simpler tasks where standard PEFT methods already perform well. This task-dependent variation makes it challenging to provide universal recommendations for its use. Additionally, while we offer some theoretical analysis, a complete understanding of why certain initialization strategies outperform others remains incomplete. The interaction between magnitude-direction decomposition and SVD-based initialization warrants deeper theoretical investigation.

Our experiments, though comprehensive, primarily focus on models up to 32B parameters. Further research is needed to understand DuDe's scaling behavior on larger models (70B+ parameters) and its interaction with other scaling laws. Future work should focus on developing more efficient initialization methods, extending architecture support, deepening theoretical understanding, and studying scaling properties in extremely large models.

References

Marah Abdin, Jyoti Aneja, Harkirat Behl, Sébastien Bubeck, Ronen Eldan, Suriya Gunasekar, Michael Harrison, Russell J. Hewett, Mojan Javaheripi, and Piero Kauffmann. 2024. Phi-4 Technical Report. *arXiv preprint arXiv:2412.08905*.

Armen Aghajanyan, Sonal Gupta, and Luke Zettlemoyer. 2021. Intrinsic Dimensionality Explains the Effectiveness of Language Model Fine-Tuning. In *Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing (Volume 1: Long Papers)*, pages 7319–7328, Online. Association for Computational Linguistics.

Klaudia Bałazy, Mohammadreza Banaei, Karl Aberer, and Jacek Tabor. 2024. LoRA-XS: Low-Rank Adaptation with Extremely Small Number of Parameters. *arXiv preprint arXiv:2405.17604*.

Elad Ben Zaken, Yoav Goldberg, and Shauli Ravfogel. 2022. BitFit: Simple parameter-efficient fine-tuning for transformer-based masked language-models. In *Proceedings of the 60th Annual Meeting of the Association for Computational Linguistics (Volume 2: Short Papers)*, pages 1–9, Dublin, Ireland. Association for Computational Linguistics.

Yonatan Bisk, Rowan Zellers, Ronan Le Bras, Jianfeng Gao, and Yejin Choi. 2019. PIQA: Reasoning about Physical Commonsense in Natural Language. *arXiv preprint arXiv:1911.11641*.

Kerim Büyükakyüz. 2024. OLoRA: Orthonormal Low-Rank Adaptation of Large Language Models. *arXiv preprint arXiv:2406.01775*.

Christopher Clark, Kenton Lee, Ming-Wei Chang, Tom Kwiatkowski, Michael Collins, and Kristina Toutanova. 2019. BoolQ: Exploring the Surprising Difficulty of Natural Yes/No Questions. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 2924–2936, Minneapolis, Minnesota. Association for Computational Linguistics.

Peter Clark, Isaac Cowhey, Oren Etzioni, Tushar Khot, Ashish Sabharwal, Carissa Schoenick, and Oyvind Tafjord. 2018. Think you have Solved Question Answering? Try ARC, the AI2 Reasoning Challenge. *arXiv preprint arXiv:1803.05457*.

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, and 1 others. 2021. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*.

OpenCompass Contributors. 2023. OpenCompass: A Universal Evaluation Platform for Foundation Models. https://github.com/open-compass/opencompass.

Ali Edalati, Marzieh Tahaei, Ivan Kobyzev, Vahid Partovi Nia, James J. Clark, and Mehdi Rezagholizadeh. 2022. KronA: Parameter Efficient Tuning with Kronecker Adapter. *arXiv preprint arXiv:2212.10650*.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. 2015. Delving Deep into Rectifiers: Surpassing Human-Level Performance on Imagenet Classification. In *2015 IEEE International Conference on Computer Vision (ICCV)*, pages 1026–1034, Santiago, Chile. IEEE.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. 2021. Measuring Mathematical

Problem Solving With the MATH Dataset. In *Thirty-Fifth Conference on Neural Information Processing Systems Datasets and Benchmarks Track (Round 2)*.

Neil Houlsby, Andrei Giurgiu, Stanislaw Jastrzebski, Bruna Morrone, Quentin De Laroussilhe, Andrea Gesmundo, Mona Attariyan, and Sylvain Gelly. 2019. Parameter-Efficient Transfer Learning for NLP. In *Proceedings of the 36th International Conference on Machine Learning*, pages 2790–2799. PMLR.

Edward Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. 2022. LoRA: Low-rank Adaptation of Large Language Models. In *International Conference on Learning Representations*.

Albert Q. Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, and Lucile Saulnier. 2023. Mistral 7B. *arXiv preprint arXiv:2310.06825*.

Dawid J Kopiczko, Tijmen Blankevoort, and Yuki M Asano. 2024. VeRA: Vector-Based Random Matrix Adaptation. In *International Conference on Learning Representations*.

Tao Lei. 2023. Conditional Adapters: Parameter-efficient Transfer Learning with Fast Inference. In *Advances in Neural Information Processing Systems*, volume 36, pages 8152–8172. Curran Associates, Inc.

Yingzhou Li, Haizhao Yang, Eileen R. Martin, Kenneth L. Ho, and Lexing Ying. 2024. Parameter-Efficient Orthogonal Finetuning Via Butterfly Factorization. In *The Twelfth International Conference on Learning Representations*.

Zhuang Li, Yuyang Chai, Terry Yue Zhuo, Lizhen Qu, Gholamreza Haffari, Fei Li, Donghong Ji, and Quan Hung Tran. 2023. FACTUAL: A benchmark for faithful and consistent textual scene graph parsing. In *Findings of the Association for Computational Linguistics: ACL 2023*, pages 6377–6390, Toronto, Canada. Association for Computational Linguistics.

Baohao Liao, Yan Meng, and Christof Monz. 2023. Parameter-efficient fine-tuning without introducing new latency. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 4242–4260, Toronto, Canada. Association for Computational Linguistics.

Vijay Lingam, Atula Tejaswi, Aditya Vavre, Aneesh Shetty, Gautham Krishna Gudur, Joydeep Ghosh, Alex Dimakis, Eunsol Choi, Aleksandar Bojchevski, and Sujay Sanghavi. 2024. SVFT: Parameter-Efficient Fine-Tuning with Singular Vectors. In *Advances in Neural Information Processing Systems*.

Fangyu Liu, Guy Emerson, and Nigel Collier. 2023. Visual spatial reasoning. *Transactions of the Association for Computational Linguistics*, 11:635–651.

Shih-Yang Liu, Chien-Yi Wang, Hongxu Yin, Pavlo Molchanov, Yu-Chiang Frank Wang, Kwang-Ting Cheng, and Min-Hung Chen. 2024. DoRA: Weight-Decomposed Low-Rank Adaptation. In *Forty-First International Conference on Machine Learning*, pages 32100–32121. PMLR.

Fanxu Meng, Zhaohui Wang, and Muhan Zhang. 2024. PiSSA: Principal Singular Values and Singular Vectors Adaptation of Large Language Models. In *Advances in Neural Information Processing Systems*, volume 37, pages 121038–121072. Curran Associates, Inc.

Todor Mihaylov, Peter Clark, Tushar Khot, and Ashish Sabharwal. 2018. Can a Suit of Armor Conduct Electricity? A New Dataset for Open Book Question Answering. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 2381–2391, Brussels, Belgium. Association for Computational Linguistics.

Libo Qin, Qiguang Chen, Fuxuan Wei, Shijue Huang, and Wanxiang Che. 2023. Cross-lingual prompting: Improving zero-shot chain-of-thought reasoning across languages. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pages 2695–2709, Singapore. Association for Computational Linguistics.

Qwen, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, and Fei Huang. 2025. Qwen2.5 Technical Report. *arXiv preprint arXiv:2412.15115*.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R Bowman. 2024. GPQA: A Graduate-Level Google-Proof Q&A Benchmark. In *First Conference on Language Modeling*.

Keisuke Sakaguchi, Ronan Le Bras, Chandra Bhagavatula, and Yejin Choi. 2021. WinoGrande: An adversarial winograd schema challenge at scale. *Communications of the ACM*, 64(9):99–106.

Tim Salimans and Durk P Kingma. 2016. Weight Normalization: A Simple Reparameterization to Accelerate Training of Deep Neural Networks. In *Advances in Neural Information Processing Systems*, volume 29. Curran Associates, Inc.

Maarten Sap, Hannah Rashkin, Derek Chen, Ronan Le Bras, and Yejin Choi. 2019. Social IQa: Commonsense reasoning about social interactions. In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP)*, pages 4463–4473, Hong Kong, China. Association for Computational Linguistics.

Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. 2019. CommonsenseQA: A question answering challenge targeting commonsense knowledge. In *Proceedings of the 2019 Conference*

of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4149–4158, Minneapolis, Minnesota. Association for Computational Linguistics.

Qwen Team. 2024. Introducing Qwen1.5.

Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajjwal Bhargava, and Shruti Bhosale. 2023. Llama 2: Open Foundation and Fine-Tuned Chat Models. *arXiv preprint arXiv:2307.09288*.

Longhui Yu, Weisen Jiang, Han Shi, Jincheng Yu, Zhengying Liu, Yu Zhang, James T Kwok, Zhenguo Li, Adrian Weller, and Weiyang Liu. 2024. Metamath: Bootstrap Your Own Mathematical Questions for Large Language Models. In *The Twelfth International Conference on Learning Representations*.

Rowan Zellers, Ari Holtzman, Yonatan Bisk, Ali Farhadi, and Yejin Choi. 2019. HellaSwag: Can a Machine Really Finish Your Sentence? In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 4791–4800, Florence, Italy. Association for Computational Linguistics.

A Experiment Details

A.1 Commonsense Reasoning

In this section, we provide the details of the commonsense reasoning experiments. We fine-tune the models for one epoch using a batch size of 1 and gradient accumulation steps of 20. The learning rate is set to 2e-5 with cosine decay scheduling and 0.03% warmup rate. We apply our method to query and value matrices (W_q, W_v) in the attention layers with rank $r = 16$. The detailed hyperparameter settings are shown in Table 5.

A.2 Settings for Robustness Experiments to Different Epochs

In this section, we provide the details of the robustness experiments conducted across different training epochs. We randomly sampled 128,000 examples from the MetaMathQA-395K dataset using a fixed random seed of 42 to ensure reproducibility. For both PEFT methods and full fine-tuning experiments, we used identical learning rate settings to enable fair comparisons. Specifically, we trained each model configuration for 3 epochs to analyze the impact of training duration on model performance. The learning rate was set to 2e-5 with cosine decay scheduling and 0.03% warmup rate, consistent across all experimental conditions. The detailed configuration is shown in Table 6.

A.3 Settings for Robustness Experiments to Different Rank Settings

In this section, we provide the details of the robustness experiments to different rank settings. The detailed configuration is shown in Table 7.

rank r	**learning rate**	**epochs**	**warmup %**	**scheduler**	**packing**	**target module**
16	2e-5	1	0.03	cosine	false	W_q, W_v

Table 5: Configuration for commonsense reasoning experiments. Note that the batch size is set to 1 and the gradient accumulation steps is set to 20.

rank r	**learning rate**	**epochs**	**warmup %**	**scheduler**	**packing**	**target module**
16	2e-5	3	0.03	cosine	false	W_q, W_v

Table 6: Configuration for robustness experiments to different epochs. Note that the batch size is set to 8 and the gradient accumulation steps is set to 16.

learning rate	**epochs**	**warmup %**	**scheduler**	**packing**	**target module**
2e-5	1	0.03	cosine	false	W_q, W_v

Table 7: Configuration for robustness experiments to different rank settings. Note that the batch size is set to 1 and the gradient accumulation steps is set to 100.