Outlier-Safe Pre-Training for Robust 4-Bit Quantization of Large Language Models

Anonymous ACL submission

Abstract

Large Language Models (LLMs) have experienced remarkable performance gains through increased parameter counts and training data, but this growth poses significant challenges for on-device deployment. Quantization has emerged as a critical technique to reduce compute and memory overhead in resource-constrained environments. Unfortunately, traditional quantization approaches are hampered by outliers–rare but extreme activation values that stretch quantization ranges and degrade performance. Recent work suggests that the Adam optimizer itself may contribute to outlier formation through its element-wise gradient normalization. In this paper, we introduce Muon as a practical alternative to Adam for large-scale LLM training. By employing efficient gradient orthogonalization via Newton-Schulz iterations, Muon avoids the heavy overhead common in second-order methods like Shampoo. We further propose an **Outlier-Safe Pre-Training (OSP)** framework that incorporates learnable embedding rotations and single-scale RMSNorm, suppressing outliers without architectural modifications at inference. Our ablation study on a 100 billion token corpus demonstrates that these components effectively mitigate outliers while maintaining model quality. We validate our approach by training a 1.4B-parameter LLM on 1 trillion tokens–to our knowledge, the first production-scale model trained without Adam. The resulting model exhibits distinct quantization behavior under 4-bit weight and activation (W4A4) quantization compared to existing open-source LLMs, suggesting new possibilities for robust low-bit pre-training in LLM development.

1 Introduction

Large Language Models (LLMs) have grown at an unprecedented pace, resulting in escalating compute, memory, and energy costs. Such growth poses significant challenges for on-device LLM deployment, especially on devices with limited resources.

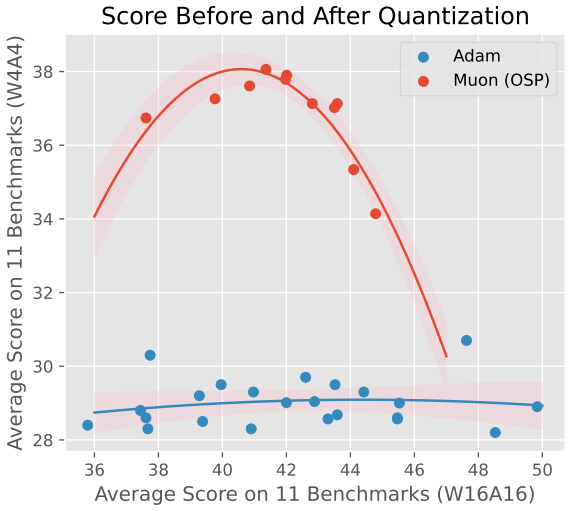


Figure 1: Comparison of Quantization-induced Degradation (QiD) patterns under 4-bit weight and activation (W4A4) quantization. In the plot, **Adam** refers to various open-source LLMs trained with the Adam optimizer, which exhibit one characteristic pattern of performance under quantization. **Muon (OSP)** represents checkpoints from our model trained with the Muon optimizer over **OSP** framework, revealing a distinctly different QiD pattern. Performance is evaluated across standard LLM benchmarks detailed in Section 4.1.

As a practical solution, various quantization methods (Frantar et al., 2023; Shao et al., 2024; Ashkboos et al., 2024b; Liu et al., 2024b) have been developed to reduce model size and computational overheads by representing weights and activations in lower-precision formats.

However, activation outliers in LLMs (Bondarenko et al., 2021; Wei et al., 2022; Liu et al., 2024a; Dettmers et al., 2022; Xiao et al., 2023; Ashkboos et al., 2024b; Liu et al., 2024b) present a persistent obstacle: these rare but extreme values expand the dynamic range, rendering simple methods like Round-To-Nearest ineffective. To accommodate these outliers, the quantization range must be widened, which results in a loss of pre-

cision for the majority of non-outlier values. To address this, prior work has explored strategies such as Quantization-Aware Training (QAT) (Liu et al., 2024a; Chen et al., 2024; Nrusimha et al., 2024), Post-Training Quantization (PTQ) that handles outliers in a mixed-precision manner (Zhao et al., 2024; Dettmers et al., 2022), shifting outliers between weights and activations (Xiao et al., 2023; Lin et al., 2024), or applying a random rotation matrix to mitigate outliers through computational invariance (Chee et al., 2023; Tseng et al., 2024; Ashkboos et al., 2024b; Liu et al., 2024b). Despite these efforts, existing methods often require additional training or calibration and have yet to fully explain why outliers arise.

Recent findings (Elhage et al., 2023; He et al., 2024; Caples and rrenaud, 2024) suspect that the Adam optimizer itself may be responsible for the occurrence of activation outliers. Adam's element-wise gradient normalization privileges the coordinate-wise basis of model parameters over arbitrary directions in parameter space (Elhage et al., 2023), leading to basis-aligned outliers. When models are trained with pure SGD, SOAP (Vyas et al., 2024), or other non-Adam optimizers, outliers appear less frequently.

In this work, we introduce Muon (Jordan et al., 2024; Bernstein and Newhouse, 2024a) as a practical, outlier-suppressing alternative to Adam for large-scale LLM training. Although Muon draws inspiration from second-order methods such as Shampoo (Gupta et al., 2018; Anil et al., 2020) and SOAP (Vyas et al., 2024), it circumvents the high overhead associated with full matrix preconditioning (as Table 2). By orthogonalizing gradients with a momentum buffer, essentially a simplified Shampoo (Bernstein and Newhouse, 2024b,a; Morwani et al., 2024), Muon achieves efficiency far superior to existing second-order methods. While Shampoo and SOAP often suffer from slow performance and large memory footprints, Muon matches the convergence speed of these second-order optimization methods while requiring less memory than Adam (Jordan et al., 2024). Our contributions can be summarized as follows:

- **Outlier-Safe Pre-Training (OSP).** We propose training LLMs without introducing outliers from the start by replacing Adam with Muon. We observe that Muon alone does not entirely eliminate outliers, contradicting some prior suggestions (He et al., 2024). Hence, we propose an **OSP** framework, adding an embedding rotation matrix and a single-scale RMSNorm during training. These modifications remove element-wise scaling while retaining architectural equivalence at inference (Ashkboos et al., 2024a,b; Liu et al., 2024b).

- **Scaling Up to 1 Trillion Tokens.** While most open-source LLMs still rely on Adam, we demonstrate that Muon maintains stable training and our framework can reduce outliers in a 1.4 billion parameter model trained on 1 trillion tokens. To our knowledge, this is the first production-level LLM of this scale trained without Adam. We open-source the trained model to facilitate further study of how different optimizers affect LLM behavior.

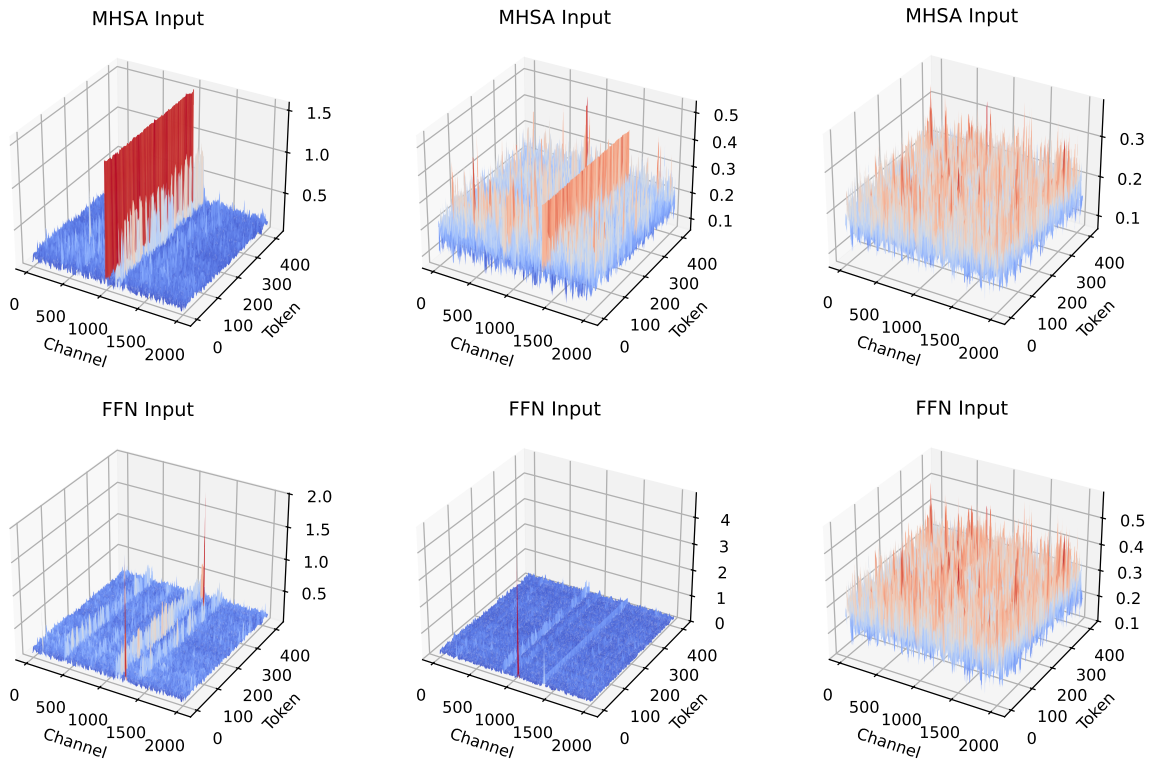
- **State-of-the-Art Low-Bit Robustness.** Combining Muon training with post-training quantization (PTQ) achieves the best 4-bit weight and activation over Round-To-Nearest (RTN) quantization performance among open-source LLMs, substantially improving *robustness to low-bit quantization*.

2 Preliminary

Several prior works have investigated activation outliers in LLMs. For instance, Nrusimha et al. (2024) explored how outliers emerge during pre-training and proposed a combination of Quantization-Aware Training (QAT) and kurtosis regularization to suppress them. While this approach reduces extreme activation values, it still does not eliminate them entirely and incurs a significant training slowdown (over 10%) due to QAT's overhead. Likewise, He et al. (2024) proposed an Outlier-Protected (OP) Transformer block, but this architectural modification complicates existing inference pipelines and requires entropy regularization to prevent issues like entropy collapse. These approaches highlight two main challenges: (1) modifications to the training pipeline (e.g., slow QAT or custom CUDA kernels) and (2) architectural changes that complicate deployment.

3 Method

Meanwhile, He et al. (2024) also explored to replace Adam optimizer with SOAP variants, suppressing outlier emergence while pre-training LLMs. This aligns with findings that Adam's



(a) Adam **(b) Muon** **(c) Muon (OSP)**

Figure 2: Activation histograms from the 20th layer input of Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) in 1.4B-parameter models trained on 100B tokens. Three optimization approaches are compared: (a) standard Adam optimizer, (b) Muon optimizer without modifications, and (c) Muon with learnable embedding rotations and single-scale RMSNorm (SSNorm). The histograms demonstrate that replacing Adam with Muon alone is insufficient to fully mitigate activation outliers.

element-wise gradient normalization may contribute to outlier emergence (Elhage et al., 2023; Caples and rrenaud, 2024).

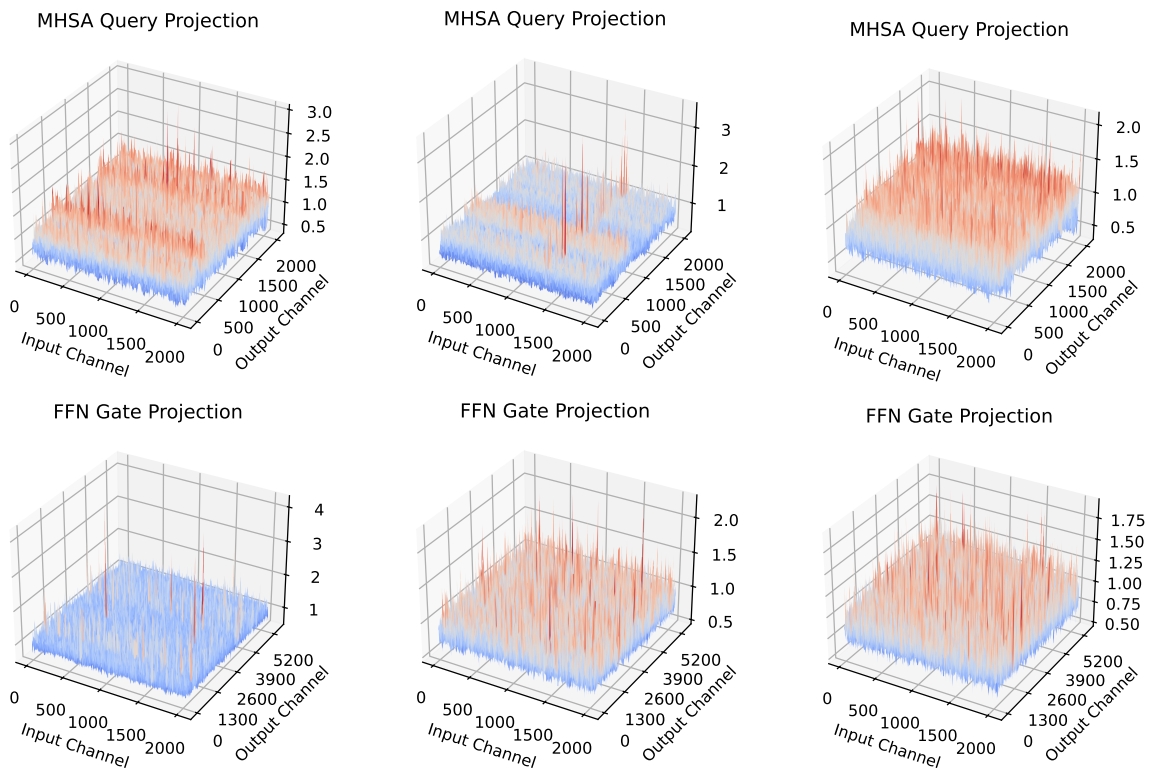
Building on these insights, We introduce a training pipeline for Large Language Models (LLMs) that inherently avoids the activation outliers impeding low-bit quantization. In contrast to approaches that require architectural modifications or additional training overhead like Quantization-Aware Training, our method employs (1) a simpler yet effective second-order-inspired optimizer called Muon, (2) learnable embedding rotations, and (3) a single-scale RMSNorm. Below, we detail the motivation behind these design choices and describe how each component helps mitigate outliers without incurring substantial computational or implementation costs.

3.1 Learnable Embedding Rotations

Contrary to He et al. (2024), we observe that non-diagonal preconditioning alone fails to remove all outliers, often leaving residual extremes in the embedding and unembedding layers. This is partially because large embedding matrices (e.g., vocab size $V\times$ hidden dimension m) are often handled differently by standard second-order routines. For instance, Shampoo or SOAP might skip the vocabulary dimension for the preconditioner factorization, and Muon may revert to Adam for embeddings.

Drawing on the concept of *computational invariance* (Section 5.3), we insert learnable rotation matrices around the embedding layers. Let $E = [e_1, e_2, \ldots, e_V]^T \in \mathbb{R}^{V\times m}$ be the embedding matrix and $Q \in \mathbb{R}^{m\times m}$ be a learnable matrix initialized to be orthogonal. Each embedding vector e_i is then mapped to $\hat{e}_i = e_i Q$, effectively rotating the embedding space. A similar rotation is applied before the unembedding layer.

Since those rotations are simply linear transformations, they can be merged into single effective matrices for inference, i.e. $\hat{V} = VQ$, preserving the original architecture and no additional overhead. Empirically, we observed that training additional matrices reduces outliers within the embedding



(a) Adam **(b) Muon** **(c) Muon (OSP)**

Figure 3: Weight histograms from the 20th layer input of Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) in 1.4B-parameter models trained on 100B tokens. Three optimization approaches are compared: (a) standard Adam optimizer, (b) Muon optimizer without modifications, and (c) Muon with learnable embedding rotations and single-scale RMSNorm (SSNorm). The histograms reveal that weight matrices, like activations, exhibit outliers that complicate quantization.

space and the resulting model size increases by only about 0.6%, making it a practical solution for mitigating outliers in large-scale LLMs.

3.2 Single-Scale RMSNorm

Normalization layers also play a critical role in outlier formation (Wei et al., 2022; He et al., 2024; Nrusimha et al., 2024). He et al. (2024) employ Simple RMSNorm (SRMSNorm) (Qin et al., 2023), which rescale the output vector by $\sqrt{m}$ without learnable parameters, where m is a dimensionality of input activation vector.

Inspired by SRMSNorm, we propose a single-scale RMSNorm (SSNorm), which use a single learnable scalar to control the scale of activation norm. SSNorm uses only a single scalar $\gamma \in \mathbb{R}$ to scale the normalized activations:

$$\text{SSNorm}(x) = \gamma \frac{x}{||x||_2}. \quad (1)$$

This design removes the channel-wise multiplicative degrees of freedom that can inflate specific coordinates and cause outliers. By controlling all dimensions uniformly with one parameter, SSNorm helps stabilize activations and reduces extreme values across the dimensions.

4 Experiments

4.1 Experimental Setup

We train 1.4B-parameter LLAMA (Touvron et al., 2023) models to evaluate quantization performance under different optimizers and architectural configurations. Training data is drawn from a mixture of FineWeb-Edu (Lozhkov et al., 2024), FineMath (Allal et al., 2025), Cosmopedia (Ben Allal et al., 2024), and Python code samples from StarCoder's training set (Li et al., 2023).

Training is conducted on TPU v4-512 Pod Slice, utilizing the Adam optimizer with a learning rate of 5×10^{-3} and Muon optimizer with a learning rate of 5×10^{-4}. We use a batch size of 4M tokens with sequence length of 2048 tokens. We use weight decay of 0.01 and trapezoidal learning rate scheduling (Hägele et al., 2024; Wen et al., 2024), of which learning rates are increased from 0 to maximum

Optimizer	EmbRot	SSNorm	Ex. Kurt.	Had.	16-16-16		4-8-16		4-8-8		4-4-16		4-4-4	
					Avg.	PPL	Avg.	PPL	Avg.	PPL	Avg.	PPL	Avg.	PPL
Adam	✗	✗	1818.56	✗	40.9	11.4	38.4	21.6	38.3	21.6	28.3	1e5	28.3	8e4
				✓	40.9	11.4	39.9	22.3	40.0	22.3	29.2	3e4	28.9	3e4
Muon† (w/o Adam)	✗	✗	361.35	✗	40.9	11.7	38.4	14.8	38.3	14.8	28.3	1e6	28.3	8e5
				✓	40.9	11.7	37.4	15.4	37.4	15.4	33.9	24.5	33.7	24.8
Muon	✗	✗	1575.12	✗	41.3	11.4	39.8	13.8	39.8	13.8	30.8	934.3	30.5	1e4
				✓	41.3	11.4	40.4	12.9	40.5	12.9	38.4	15.7	38.3	15.8
Muon	✓	✗	703.23	✗	39.9	12.3	38.4	14.8	38.4	14.8	32.2	99.7	31.9	114.6
				✓	39.9	12.3	39.2	13.9	39.3	13.9	36.6	22.1	36.7	22.3
Muon	✗	✓	66.69	✗	**41.4**	**11.2**	**40.8**	12.4	**40.8**	12.4	36.7	43.3	36.6	44.2
				✓	**41.4**	**11.2**	**40.6**	12.2	**40.6**	12.2	38.5	33.7	38.3	34.1
Muon	✓	✓	**0.04**	✗	41.2	**11.2**	40.4	**12.2**	40.5	**12.2**	**38.1**	**19.4**	**37.8**	**19.6**
				✓	41.2	**11.2**	40.4	**12.1**	40.4	**12.1**	**39.1**	**13.4**	**39.0**	**13.5**

Table 1: Ablation study of models trained on 100B tokens. **EmbRot** denotes models with learnable embedding rotations, and **SSNorm** indicates single-scale RMSNorm. **Ex. Kurt** represents excess kurtosis (Section 5.1), while **Had.** indicates online Hadamard transformation from QuaRot Stage 1. Bit-width configurations (e.g., 16-16-16, 4-8-16) specify quantization precision for weights, activations, and key-value cache respectively. **Avg.** shows mean performance across 11 LLM benchmarks, and **PPL** reports WikiText-2 perplexity. †Model trained with Muon without Adam on embedding layers, applying Newton-Schulz orthogonalization to embedding gradients.

value for 5 billion tokens, and decayed to zero at the last 20% of the training steps.

To achieve faster training throughput, we adopt Fully-Sharded Data-Parallel (Xu et al., 2020) with parameters sharded across 16 accelerator cores. We further implement a distributed variant of Muon, distributing the Newton–Schulz iterations across 8 *optimizer-parallel* ranks for orthogonalization. As shown in Table 2, our distributed Muon incurs only about 2% overhead relative to Adam, whereas Distributed Shampoo (Anil et al., 2020) degrades performance by over 30% even with an relaxed update frequency (every 32 steps).

We evaluate model performance using perplexity on WikiText-2 (Merity et al., 2016) and accuracy on 11 standard LLM benchmarks: ARC (Clark et al., 2018), CommonsenseQA (Talmor et al., 2019), GSM8k (Cobbe et al., 2021) (8-shot), HellaSwag (Zellers et al., 2019), MMLU (Hendrycks et al., 2021), OpenBookQA (Mihaylov et al., 2018), PIQA (Bisk et al., 2020), SIQA (Sap et al., 2019), TriviaQA (Joshi et al., 2017) (5-shot), TruthfulQA (Lin et al., 2022), and WinoGrande (Sakaguchi et al., 2020).

4.2 Ablation Study

To analyze each component of our proposed method, we conduct an ablation study on a 100B-token subset of our training corpus. By default, we optimize embedding matrices using Adam for two reasons: orthogonalizing gradients for large vocabularies is computationally expensive, and Adam achieves better final performance than Muon alone (Jordan et al., 2024). This aligns with other second-order optimizers, which typically exclude the vocabulary dimension from their preconditioners due to the computational constraints of handling such large matrices. While we use Adam for embeddings by default, we also evaluate full Muon optimization as part of our ablation study.

Optimizer	TPS	Relative Speed
Adam	4.07M	–
Muon	3.99M	97.9%
Shampoo†	3.07M	75.5%

Table 2: Performance comparison of different optimizers on TPU-v4 512 Pod Slice. **TPS** denotes tokens processed per second during training, and **Relative Speed** indicates training speed relative to Adam. Muon demonstrates significantly better throughput than Distributed Shampoo. †Distributed Shampoo results shown with update frequency of 32 steps.

Quantization robustness is first evaluated through a simple Round-To-Nearest (RTN) approach. We also test QuaRot (Ashkboos et al., 2024b) Stage 1 by applying an online Hadamard transform to rotate hidden states within the FFN layers. Each experimental setting involves four distinct quantization scenarios summarized in Table 1.

As shown in Table 1, using *both* learnable embedding rotations and single-scale RMSNorm (SS-

Model	Params.	Tokens	ARC	CSQA	GSM8K	HS	MMLU	OBQA	PIQA	SIQA	TQA	TFQA	WG	Avg.
Pythia	1.4B	0.3T	27.2	21.5	0.0	25.8	26.2	24.8	53.2	37.2	0.0	50.1	49.0	28.6
TinyLlama	1.1B	2T	28.3	22.9	0.0	26.6	26.2	21.2	48.7	**40.7**	0.0	49.1	49.0	28.4
OPT	1.3B	0.3T	25.0	21.6	0.0	26.5	25.6	28.2	49.6	36.9	0.0	48.0	49.5	28.3
OLMo	1.2B	3T	27.7	25.8	0.0	27.0	26.1	25.8	54.1	37.4	0.0	49.0	**51.9**	29.5
MobileLLaMA	1.4B	1.3T	27.4	23.5	0.0	26.7	26.0	22.4	49.6	38.3	0.0	49.4	49.6	28.5
Qwen 1.5	1.8B	2.4T	27.2	25.4	0.0	28.3	25.7	25.0	54.1	39.1	0.0	50.1	49.3	29.5
Qwen 2	1.5B	7T	30.9	27.7	0.4	35.7	26.2	28.4	56.5	38.3	0.8	44.0	48.6	30.7
Qwen 2.5	1.5B	–	27.7	25.0	0.0	26.9	25.7	24.0	52.2	38.4	0.0	49.9	47.5	28.9
LLaMA 3.2	1.2B	–	29.3	24.7	**0.5**	30.1	25.8	27.4	53.3	39.5	0.1	45.3	50.3	29.7
Stable LM 2	1.6B	2T	26.2	24.0	0.0	27.0	24.6	27.0	51.1	37.8	0.0	**50.2**	51.1	29.0
SmolLM	1.7B	1T	28.4	25.7	0.0	27.0	26.1	28.0	51.0	38.8	0.0	48.9	48.4	29.3
SmolLM 2	1.7B	11T	25.8	22.4	0.0	25.9	24.2	26.6	51.5	36.0	0.0	48.0	50.0	28.2
Trained from scratch														
Adam	1.4B	1T	25.7	25.3	0.0	26.8	25.4	26.0	49.0	37.2	0.0	49.1	49.9	28.6
Muon (OSP)	1.4B	1T	**42.1**	**32.4**	0.0	**41.5**	**28.5**	**32.0**	**61.2**	38.8	**2.8**	45.5	50.8	**34.1**

Table 3: W4A4 quantization performance comparison across 12 open-source LLMs and our implementations. **Params.** indicates model size in parameters, and **Tokens** shows training dataset size. Benchmarks include CSQA (CommonsenseQA), HS (HellaSwag), OBQA (OpenBookQA), TQA (TriviaQA), TFQA (TruthfulQA), and WG (WinoGrande). Models trained with **Outlier-Safe Pre-Training** exhibit distinct quantization behavior across the benchmark suite.

Norm) yields minimal excess kurtosis (Section 5.1) and preserves performance under both RTN and Hadamard rotation. Histograms of activations and weights in Figures 2 and 3 further confirm that outliers are substantially mitigated.

Notably, simply switching from Adam to a non-diagonal optimizer for all parameters is insufficient to remove outliers; skipping the vocabulary dimension or partially applying Muon can still leave outlier issues. Moreover, applying Newton–Schulz updates to the embedding matrices raises training costs by about 2.2% overall, yet still underperforms our final approach on the 11 LLM benchmarks and WikiText-2 perplexity.

Figure 4 visualizes perplexities on WikiText-2 for weight and activation quantization ranging from 8 bits down to 4 bits (via RTN). Our method consistently preserves performance more effectively across all bit configurations.

4.3 Scaling Up to 1 Trillion Tokens

We scale our training to 1 trillion tokens, a dataset size commonly used for commercial LLMs but rarely explored in academic research on outlier mitigation (He et al., 2024; Nrusimha et al., 2024). While previous works limited their experiments to hundreds of billions of tokens, we demonstrate that our **OSP** framework maintains its effectiveness at trillion-token scale.

Table 3 compares W4A4 quantization performance for 12 open-source small LMs, each trained on a large corpus. Most baseline models suffer heavy accuracy drops under W4A4, and scores on

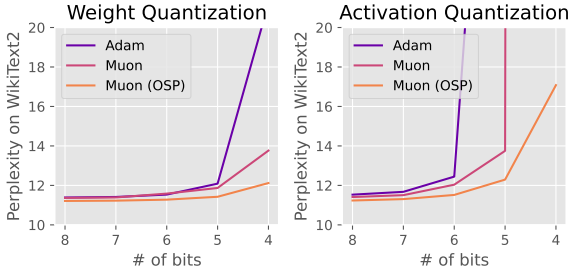


Figure 4: WikiText-2 perplexity under varying weight and activation quantization bit-widths for models trained on 100B tokens. Three configurations are compared: standard Adam, Muon, and Muon with **Outlier-Safe Pre-Training (OSP)**. The results demonstrate different quantization robustness patterns, particularly in low-bit scenarios.

multiple-choice benchmarks like ARC and CommonsenseQA degrade to near-random baselines (around 25%). By contrast, our approach exhibits notably stronger retention of performance, suggesting that both Muon and our architectural modifications better maintain quantization resilience.

4.4 Analysis of Quantization Robustness

Finally, we investigate how models trained with our method respond to standard Post-Training Quantization (PTQ) techniques. Since our approach focuses on pre-training, it can be combined with any PTQ method. As shown in Table 4, Adam-trained models suffer significant degradation under 4-bit weight and activation quantization when using minimal PTQ methods like QuaRot and GPTQ, which perform limited calibration. In contrast, models

Quantization	Adam	Muon (OSP)
RTN	14475.51	162.81
+ QuaRot†	4794.00	88.40
+ GPTQ	3723.46	18.72
+ SpinQuant	14.94	17.44

Table 4: WikiText-2 perplexity after applying various Post-Training Quantization (PTQ) methods to models trained with Adam versus **Outlier-Safe Pre-Training (OSP)**. The minimal PTQ methods (QuaRot and GPTQ) show limited effectiveness in mitigating quantization error for Adam-trained models. Models trained with **OSP** maintain consistent performance when combined with SpinQuant. †Only applies online Hadamard transform to Feed-Forward Network (FFN) hidden states.

trained with Muon and OSP show remarkable robustness to quantization, particularly on WikiText-2 where perplexity remains nearly unchanged after applying the more advanced SpinQuant method. This demonstrates that SpinQuant's learnable rotations serve a similar function to non-diagonal preconditioner optimizers in preserving model quality under quantization, which aligns to the previous observation (He et al., 2024).

Recent work (Ouyang et al., 2024) indicates that more extensively trained models are prone to severe Quantization-induced Degradation (QiD) in the low-bit setting. In Table 3, for instance, Qwen 2.5 and SmolLM 2 are trained on significantly more data than earlier versions, thereby achieving higher benchmark scores but suffering worse drops under W4A4 quantization.

To analyze quantization error patterns, we collect 11 checkpoints from our OSP model during its training and compare their performance before and after W4A4 RTN quantization against various Adam-trained open-source LLMs. As shown in Figure 1, models trained with our OSP approach follow a distinctly different Quality-in-Distribution (QiD) pattern than Adam-trained models, achieving higher absolute maximum performance. This discovery of a different quantization behavior pattern highlights the importance of exploring and analyzing non-Adam-trained LLMs for robust low-bit compression.

5 Related Works

5.1 Quantization

Quantization reduces the precision of weights and activations by mapping continuous floating-point values to discrete integers. While standard floating-point formats typically require 32 or 16 bits, quantization can reduce each value to 8 bits or even fewer than 4 bits. Let N be the target number of bits. A common baseline, Round-To-Nearest (RTN), is expressed as:

$$\hat{X} = \alpha \left\lfloor \frac{X - \beta}{\alpha} \right\rceil + \beta, \tag{2}$$

where $\alpha = \frac{\max(|X|)}{2^{N-1}-1}, \beta = 0$ for symmetric quantization, or $\alpha = \frac{\max(X)-\min(X)}{2^{N-1}-1}, \beta = \min(X)$ for asymmetric quantization. In both cases, the quantization scale α is heavily influenced by the extreme values in X, so outliers can severely degrade the reconstruction error $||X - \hat{X}||$. As a workaround, some prior works keep outliers in higher precision while compressing the remainder (Xiao et al., 2023; Lin et al., 2024), or they mitigate outliers by applying additional training (Liu et al., 2024a; Chen et al., 2024; Nrusimha et al., 2024).

To quantify those activation outliers, researchers commonly use *kurtosis*, specifically *excess kurtosis* (shkolnik et al., 2020; He et al., 2024; Caples and rrenaud, 2024; **?**). Excess kurtosis is defined as:

$$\text{Kurt}[X] - 3 = \mathbb{E}\left[\left(\frac{X - \mu}{\sigma}\right)^4\right] - 3, \tag{3}$$

where μ and σ are the mean and standard deviation of activation X, respectively.

5.2 Second-Order Optimization

Early second-order methods in deep learning were hindered by large computation and memory overhead, relying on relying on quasi-Newton or Hessian-based approximations (Nocedal and Wright, 1999). Kronecker-Factored Approximate Curvature (K-FAC) (Martens and Grosse, 2015) alleviated these costs by approximating the Fisher information matrix with Kronecker-factored statistics. Shampoo (Gupta et al., 2018) leverages the multi-dimensional (tensor) structure of parameters, maintaining and updating factorized preconditioners along each dimension of the gradient tensor, thereby improving scalability; its distributed version (Anil et al., 2020) demonstrated strong wall-clock performance.

Formally, let $W, G \in \mathbb{R}^{m \times n}$ denote a weight matrix and its gradient. Let $L \in \mathbb{R}^{m \times m}$ and $R \in \mathbb{R}^{n \times n}$ accumulate the statistics GG^T and G^TG,

respectively. Shampoo updates W as follows:

$$W \leftarrow W - \eta L^{-1/4} G R^{-1/4}, \tag{4}$$

where $\eta \in \mathbb{R}$ is the learning rate. Vyas et al. (2024) showed that Shampoo is equivalent to Adafactor (Shazeer and Stern, 2018) in the eigenbasis of its preconditioner, and introduced SOAP to adapt that eigenbasis to Adam.

More recently, Muon (Jordan et al., 2024) was proposed to orthogonalize gradients via the Newton–Schulz algorithm (Higham, 2008; Schulz, 1933), effectively approximating the Singular Value Decomposition (SVD) of gradients. Specifically, the Newton–Schulz iteration transforms

$$G = U\Sigma V^T \mapsto UV^T, \tag{5}$$

where $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$ are the singular vectors, and $\Sigma \in \mathbb{R}^{m\times n}$ is the diagonal matrix of singular values. Unlike Shampoo and SOAP, which require momentum buffers for both the preconditioner and its inverse, Muon maintains only a momentum buffer for the gradients. Subsequent works (Bernstein and Newhouse, 2024b,a; Duvvuri et al., 2024) has shown that Muon coincides with Shampoo in the absence of preconditioner accumulation.

5.3 Computational Invariance

Ashkboos et al. (2024a) introduced *computational invariance* in Transformer architectures, whereby inserting an orthogonal rotation into the residual stream and removing it later yields identical outputs. Modern large language models usually employ RMSNorm (Zhang and Sennrich, 2019) as a pre-normalization layer. Let $X \in \mathbb{R}^{m\times n}$ be an activation matrix, and $Q \in \mathbb{R}^{n\times n}$ be orthogonal. By definition of RMSNorm, the following holds:

$$\text{RMSNorm}(XQ)Q^T = \text{RMSNorm}(X). \tag{6}$$

Additionally, when linear layers follow RMSNorm, multiplication by Q^T can be merged with these layers via the associative property of matrix multiplication. By carefully placing Q and Q^T across normalization, projection, and embedding layers, one can preserve network-level invariance.

Building on this idea, QuaRot (Ashkboos et al., 2024b) applies a random Hadamard rotation to the Transformer's embedding space, while SpinQuant (Liu et al., 2024b) employs a learnable rotation matrix which is optimized via *Cayley SGD* (Li et al., 2020).

6 Conclusion

In this paper, we presented a pre-training pipeline that substantially mitigates activation outliers by replacing the Adam optimizer with Muon, introducing learnable embedding rotations, and adopting a single-scale RMSNorm. Unlike many previously proposed solutions, our method achieves almost outlier-free training without expensive quantization-aware fine-tuning or specialized architectural blocks that complicate model deployment. The transition away from Adam alleviates the basis-alignment issue that underpins outlier formation, while our framework eliminates residual extremes without negatively impacting convergence.

Extensive experiments on a 1.4B-parameter LLM trained over 1 trillion tokens confirm the reliability and efficiency of our approach. In particular, the resulting model remains robust under 4-bit quantization, outperforming comparable open-source models that rely on Adam. These findings motivate further exploration of how optimizer choices influence model behavior and quantization readiness. We believe our work on **Outlier-Safe Pre-Training** will encourage broader adoption of outlier-aware training practices as the field continues to develop larger, more efficiently deployable language models.

Limitations

Our study focused primarily on Muon without extensive comparisons to other second-order methods like Shampoo or SOAP. This limitation stems from practical constraints: TPU compilation times for training pipelines often exceed one hour, making comprehensive optimizer ablation studies prohibitively time-consuming given our available computational resources.

Additionally, while our experiments demonstrate effectiveness on a 1.4B-parameter model, we have not yet explored the impact across a range of model sizes, particularly the 3B and 7B parameter scales commonly targeted for mobile deployment. Looking ahead, we plan to extend our analysis to these larger models. Our distributed implementation of Muon in JAX achieves comparable efficiency to Adam, making such broader experiments computationally feasible.

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A Appendix

A.1 Comprehensive Benchmark Results for Open-Source LLMs

Table 5 presents the performance of various open-source LLMs across 11 benchmarks using W16A16 precision (16-bit weights and activations, without quantization). Notably, our Muon-trained model achieves comparable performance to Adam-trained models, marking the first successful application of Muon to trillion-token scale training.

A.2 Detailed Distribution Analysis

For a comprehensive view of activation and weight distributions, we provide detailed histograms in Figures 5, 6, 7, and 8. These visualizations allow direct comparison between Adam and Muon training approaches.

Model	Params.	Tokens	ARC	CSQA	GSM8K	HS	MMLU	OBQA	PIQA	SIQA	TQA	TFQA	WG	Avg.
Pythia	1.4B	0.3T	41.27	35.38	2.43	50.81	31.33	34.6	71.06	43.45	9.22	38.99	55.17	37.61
TinyLlama	1.1B	2T	36.51	25.39	1.74	53.98	32.64	23.0	70.29	41.30	23.48	35.44	49.96	35.79
OPT	1.3B	0.3T	39.33	39.97	0.91	52.22	29.58	35.8	71.00	42.27	11.14	38.86	53.28	37.67
OLMo	1.2B	3T	44.21	40.38	1.67	60.36	31.93	37.8	75.19	44.11	17.57	32.90	53.43	39.96
MobileLLAMA	1.4B	1.3T	42.65	37.02	1.97	54.18	31.78	34.4	73.29	43.04	24.48	34.95	55.41	39.38
Qwen 1.5	1.8B	2.4T	46.85	32.92	34.19	59.52	33.14	37.2	74.32	44.47	18.76	39.37	57.93	43.52
Qwen 2	1.5B	7T	48.18	30.96	58.07	63.89	37.42	36.8	75.35	44.22	23.99	45.85	59.19	47.63
Qwen 2.5	1.5B	–	58.77	34.32	61.56	66.47	40.26	39.6	75.68	44.88	20.59	46.67	59.43	49.84
LLAMA 3.2	1.2B	–	49.24	41.11	5.99	61.31	36.26	39.0	74.92	43.45	20.72	38.51	58.09	42.60
Stable LM 2	1.6B	2T	53.46	34.56	19.26	66.67	35.98	37.0	76.82	43.50	35.59	38.76	59.19	45.53
SmolLM	1.7B	1T	59.69	38.00	6.75	63.02	39.36	42.8	75.95	44.11	25.84	38.55	54.54	44.42
SmolLM 2	1.7B	11T	60.38	43.57	32.60	68.70	41.30	42.4	77.58	43.40	27.08	36.70	60.14	48.53
Trained from scratch														
Adam	1.4B	1T	59.50	40.62	14.48	63.97	39.52	41.0	76.06	43.60	23.91	40.88	56.59	45.47
Muon (OSP)	1.4B	1T	57.68	38.74	14.25	62.43	38.91	41.0	75.35	44.73	24.46	39.54	55.56	44.79

Table 5: W16A16 quantization performance comparison across 12 open-source LLMs and our implementations. **Params.** indicates model size in parameters, and **Tokens** shows training dataset size. Benchmarks include CSQA (CommonsenseQA), HS (HellaSwag), OBQA (OpenBookQA), TQA (TriviaQA), TFQA (TruthfulQA), and WG (WinoGrande).

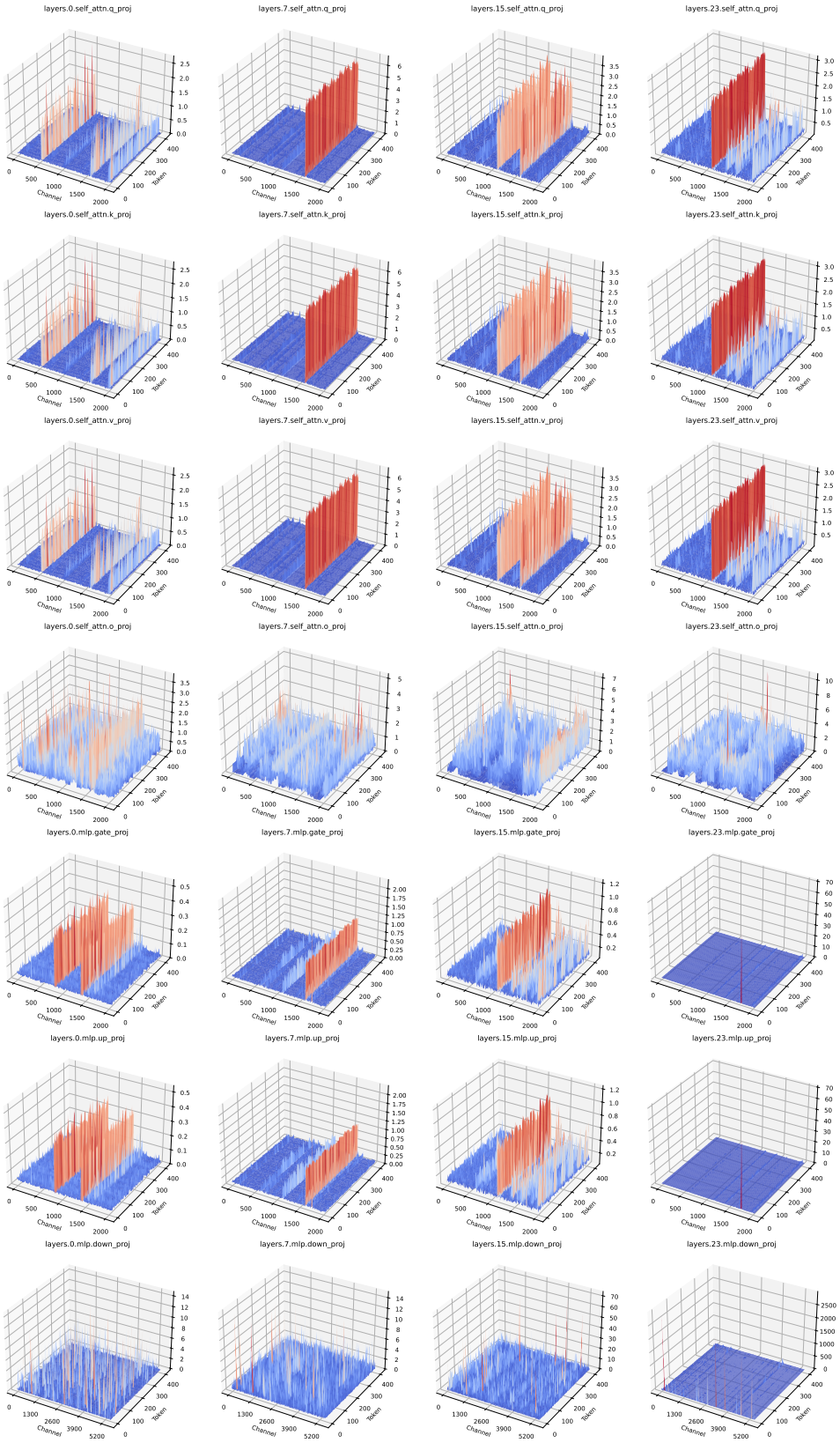

Figure 5: Activation histograms from Adam-trained models on 1 trillion tokens. The plots show input distributions to Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) layers at four different depths: 0th, 7th, 15th, and 23rd transformer blocks.

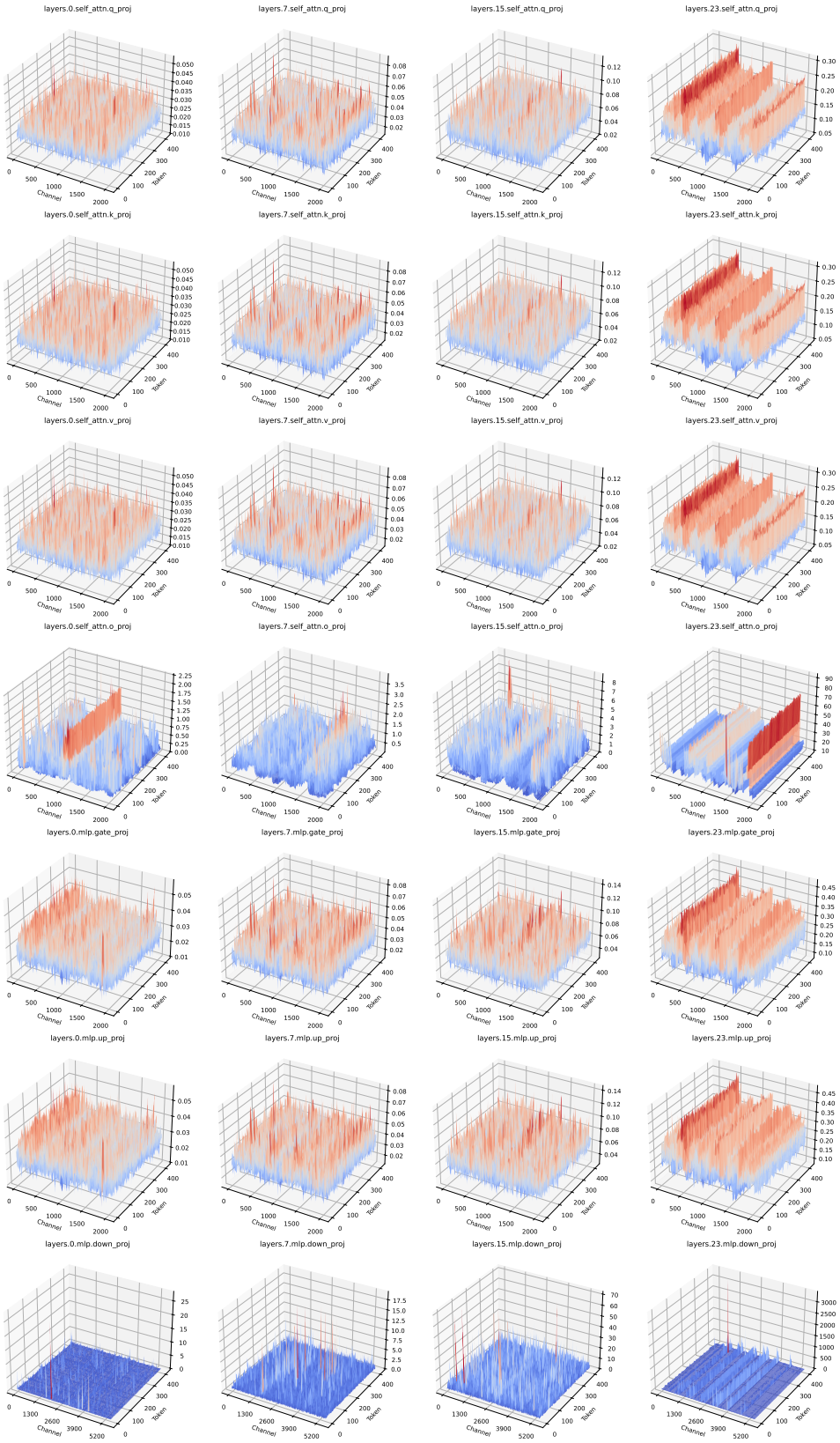

Figure 6: Activation histograms from Muon-trained models with **OSP** on 1 trillion tokens. The plots show input distributions to Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) layers at four different depths: 0th, 7th, 15th, and 23rd transformer blocks.

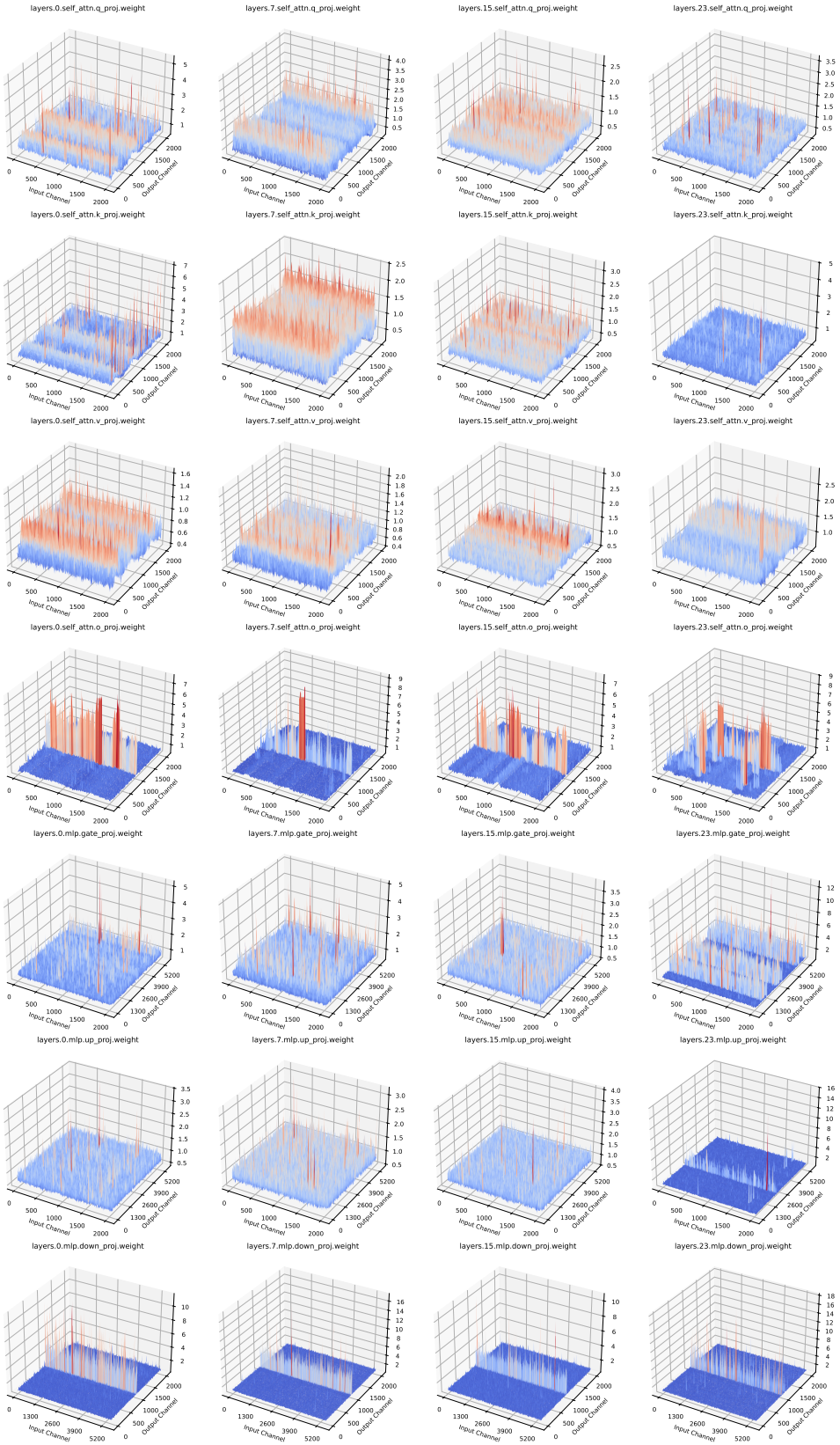

Figure 7: Weight histograms from Adam-trained models on 1 trillion tokens. The plots show weight distributions in Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) layers across four different depths: 0th, 7th, 15th, and 23rd transformer blocks.

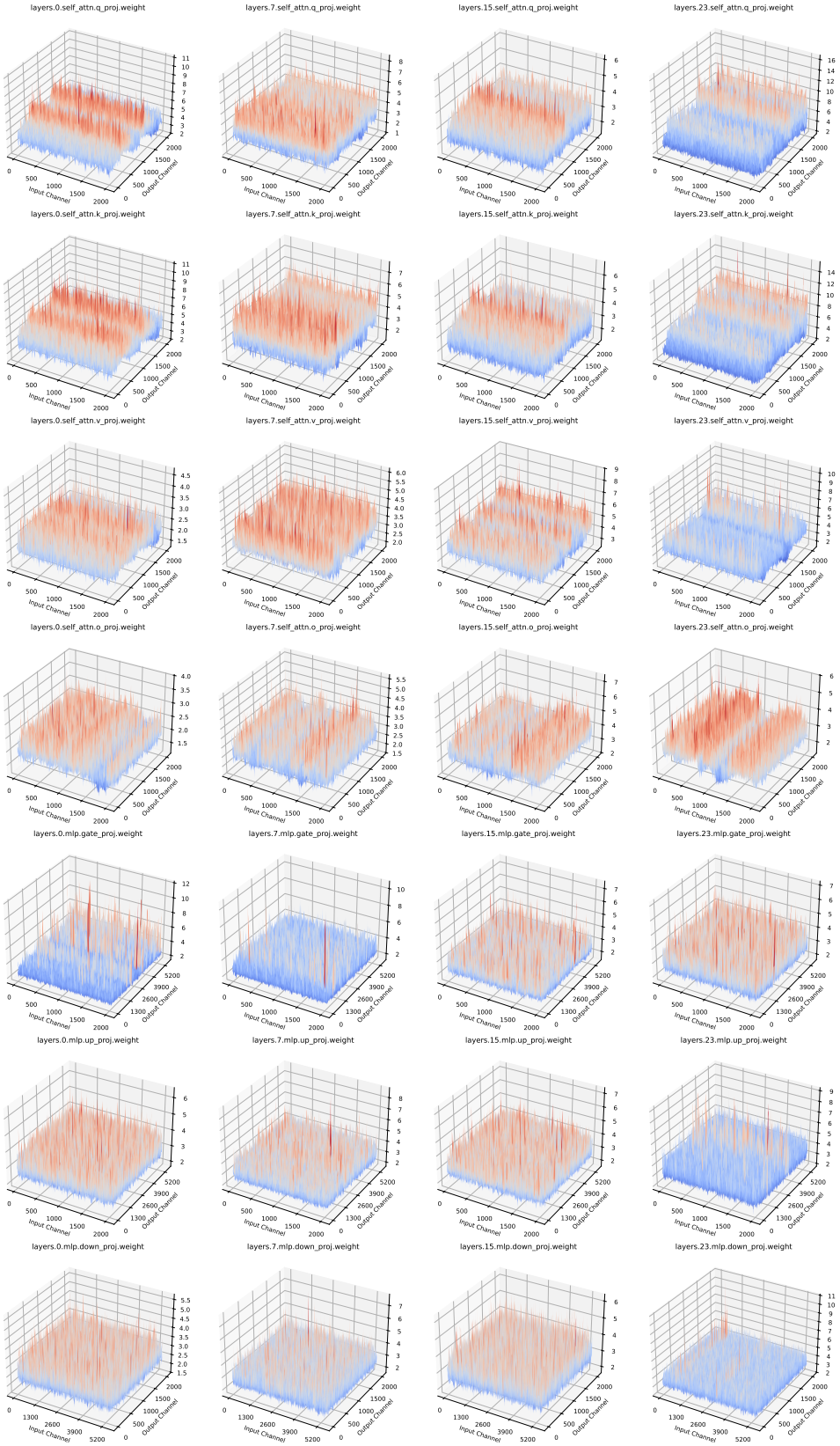

Figure 8: Weight histograms from Muon-trained models with **OSP** on 1 trillion tokens. The plots show weight distributions in Multi-Head Self-Attention (MHSA) and Feed-Forward Network (FFN) layers across four different depths: 0th, 7th, 15th, and 23rd transformer blocks.