A CRITICAL LOOK AT TOKENWISE REWARD-GUIDED TEXT GENERATION

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Abstract

Large language models (LLMs) can be improved by aligning with human preferences through fine-tuning-the so-called reinforcement learning from human feedback (RLHF). However, the cost of fine-tuning an LLM is prohibitive for many users. Due to their ability to bypass LLM fine-tuning, prediction-time tokenwise reward-guided text generation (RGTG) methods have recently been proposed. They use a reward model trained on full sequences to score partial sequences during decoding in a bid to steer the generation towards sequences with high rewards. However, these methods have so far been only heuristically motivated and poorly analyzed. In this work, we show that reward models trained on full sequences are not compatible with scoring partial sequences. To alleviate this issue, we propose to train a Bradley-Terry reward model on partial sequences explicitly, and autoregressively sample from the implied tokenwise policy during decoding time. We study the properties of this reward model and the resulting policy: We show that this policy is proportional to the ratio of two distinct RLHF policies. Our simple approach outperforms previous RGTG methods and performs similarly to strong offline baselines without large-scale LLM finetuning.

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1 INTRODUCTION

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Large language models (LLMs) provide a modern foundation for most, if not all, text generation tasks (Radford et al., 2019; Brown et al., 2020; Touvron et al., 2023a;b). In practice, significant 031 improvements in the quality of text generation are achieved by aligning LLMs to human preferences (Stiennon et al., 2020b; Ouyang et al., 2022). This is typically performed via reinforcement learning 033 from human feedback (RLHF), which involves two steps: i) learning a reward model from preference 034 data and ii) fine-tuning an LLM to maximize expected rewards by reinforcement learning (Ziegler et al., 2019b). Usually, this is done via a reinforcement learning algorithm such as proximal policy optimization (PPO, Schulman et al., 2017). Nevertheless, recently, Rafailov et al. (2023) showed that 037 the reward modeling step (i) can be bypassed by directly fine-tuning an LLM with preference data, 038 resulting in a method called direct preference optimization (DPO). While this simplifies RLHF, the fine-tuning step (ii) remains prohibitively costly for most users since it requires high-performance computational resources with large GPUs. 040

041 In order to alleviate the computational issue above, Khanov et al. (2024); Deng & Raffel (2023) 042 explored tokenwise reward-guided text generation (RGTG) techniques that avoid any fine-tuning of 043 the LLM. More precisely, the LLM remains frozen (i.e., not finetuned), and the reward model is used 044 at the decoding time to adjust the softmax scores of the tokens directly. Unlike DPO, this line of work retains the reward modeling step, but training reward models are typically a much cheaper endeavor than finetuning text-generation LLMs since smaller models can be utilized for reward modeling. 046 Furthermore, reward models are modular: they can easily be composed and reused without any cost 047 to guide text generation in conjunction with any base LLM. In contrast, RLHF via DPO requires 048 fine-tuning of every LLM that we wish to improve based on human preference data. 049

While RGTG is an interesting alternative to the standard offline RLHF, it is often based on heuristics
and still poorly analyzed. For instance, ARGS (Khanov et al., 2024) proposed to simply use a
reward model trained on full sequences to score each partial sequence during autoregressive decoding.
Meanwhile, Deng & Raffel (2023) used a custom tokenwise loss to distill a reward model trained on full sequences. Thus, it is unclear if these approaches can give rise to a sound tokenwise text

generation policy. Controlled decoding (CD; Mudgal et al., 2024), on the other hand, uses rollouts from the base model along with a reward model trained on full sequences to distill the partial reward.

In this work, we analyze this common RGTG approach. First, we show that the usage of full-sequence reward models for scoring partial sequences in a tokenwise policy is pathological. To alleviate this, we propose to explicitly train a Bradley-Terry reward model on partial sequences. We prove that this text generation policy is a ratio of two different RLHF policies trained on sequences of different lengths. Ideally, the policy would be derived from a single RLHF policy, but as we shall also show in Section 3, the exact computation of such a policy is intractable. By deriving the policy from a ratio of distinct RLHF policies, we obtain a tractable sampling procedure. This is akin to the argument of Zhao et al. (2024) where they use a similar ratio but for deriving a sequential Monte Carlo method.

We empirically validate our analysis on three different text generation datasets on two recent LLMs. Evidence shows that our RGTG approach achieves better performance compared to ARGS and CD, matching the performance of the more expensive, offline PPO and DPO baselines. In summary:

- (i) We analyze the recent practice of using full-sequence reward models for guiding the LLM decoding process. In particular, we show a deficiency in this approach.
- (ii) We thus propose to explicitly train a Bradley-Terry reward model on partial sequences and sample from the induced per-token policy induced by it during the decoding time.
- (iii) We show that this reward model induces a ratio of two distinct RLHF policies over sequences with different lengths. This is a tradeoff that one must make to make tokenwise RGTG free of the pathology and yet still tractable.
- (iv) Extensive experiments with recent LLMs and various text generation tasks validate our insights.

2 PRELIMINARIES

We denote a prompt by x and its response by y where the bolded letters indicate sequences of tokens. The *i*-th token in x is denoted by x^i , while the partial sequence starting at token *i* and ending at token *j* is denoted by $x^{i:j}$. The length of a sequence x is denoted by |x|. The same notation applies to y.

2.1 REINFORCEMENT LEARNING FROM HUMAN FEEDBACK

LLMs generally consist of probabilistic models that can generate a response y given a prompt x. More specifically, the generation of y is done token-by-token by sampling the next token from a conditional distribution $\pi(y^i|\mathbf{x}, \mathbf{y}^{1:i-1})$.

Given a preference dataset $\mathcal{D} = \{(\mathbf{x}_k, \mathbf{y}_{wk}, \mathbf{y}_{lk})\}_{k=1}^K$ containing K triples of token sequences ($\mathbf{x}, \mathbf{y}_w, \mathbf{y}_l$), Ziegler et al. (2019b) and Ouyang et al. (2022) proposed a technique based on reinforcement learning (RL) to align an LLM with the preference dataset. They train a parametric reward model $r_{\phi}(\mathbf{y}|\mathbf{x})$ that assigns a higher score to the "winning" (i.e., preferred) utterance \mathbf{y}_w and a lower score to the "losing" utterance \mathbf{y}_l . This is done via the Bradley-Terry model (Bradley & Terry, 1952) which minimizes the loss:

$$\mathcal{L}_{R} = - \mathop{\mathbb{E}}_{\mathbf{x}, \mathbf{y}_{w}, \mathbf{y}_{l} \sim \mathcal{D}} \log \sigma(r_{\phi}(\mathbf{y}_{w} | \mathbf{x}) - r_{\phi}(\mathbf{y}_{l} | \mathbf{x})),$$
(1)

where σ is the logistic function. Note that r_{ϕ} is trained to score entire utterances y. Once r_{ϕ} is trained, it can be used to infer the probability of generating sequence y in response to x, i.e., $P_{\phi}(\mathbf{y}|\mathbf{x}) = \exp(r_{\phi}(\mathbf{y}|\mathbf{x}))/\sum_{\mathbf{y}'} \exp(r_{\phi}(\mathbf{y}'|\mathbf{x}))$. Given a reference LLM, we denote by $\pi_{\text{ref}}(\mathbf{y}|\mathbf{x})$ the conditional probability that it will generate response y to prompt x (also referred to as policy). We refer to the LLM and its policy interchangeably. One can then copy the LLM and finetune it to maximize

$$\max_{\substack{\theta \\ \mathbf{y} \sim \pi_{\theta}(\mathbf{y}|\mathbf{x})}} \mathbb{E}_{r_{\phi}(\mathbf{y}|\mathbf{x})} [r_{\phi}(\mathbf{y}|\mathbf{x})] - \frac{1}{\beta} D_{\mathrm{KL}}[\pi_{\theta}(\mathbf{y}|\mathbf{x}) \parallel \pi_{\mathrm{ref}}(\mathbf{y}|\mathbf{x})],$$
(2)

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where the KL term forms a regularizer that ensures that the finetuned model will not differ too much
from the reference model. The above optimization problem can be optimized by many RL techniques,
including the popular proximal policy optimization (PPO) algorithm (Schulman et al., 2017). This
RL optimization is quite costly in practice due to the size of the LLM.

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The above optimization (2) has a closed form solution of the form (Peters & Schaal, 2007)

$$\pi_{\theta}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \pi_{\text{ref}}(\mathbf{y}|\mathbf{x}) \exp(\beta r_{\phi}(\mathbf{y}|\mathbf{x}))$$
(3)

where $Z(\mathbf{x}) = \sum_{\mathbf{y}} \pi_{\text{ref}}(\mathbf{y}|\mathbf{x}) \exp(\beta r_{\phi}(\mathbf{y}|\mathbf{x}))$ is the intractable partition function. Notice that we can reorganize (3) to express the reward function in terms of the policies π_{θ} and π_{ref} :

$$r(\mathbf{y}|\mathbf{x}) = \frac{1}{\beta} \log \frac{\pi_{\theta}(\mathbf{y}|\mathbf{x})}{\pi_{\text{ref}}(\mathbf{y}|\mathbf{x})} + \log Z(\mathbf{x})$$

which can be used to replace $r_{\phi}(\mathbf{x}|\mathbf{y})$ in (1) to obtain the following optimization problem:

$$\max_{\theta} \mathop{\mathbb{E}}_{\mathbf{x}, \mathbf{y}_{w}, \mathbf{y}_{l} \sim \mathcal{D}} \log \sigma \left(\frac{1}{\beta} \left(\log \frac{\pi_{\theta}(\mathbf{y}_{w} | \mathbf{x})}{\pi_{\text{ref}}(\mathbf{y}_{w} | \mathbf{x})} - \log \frac{\pi_{\theta}(\mathbf{y}_{l} | \mathbf{x})}{\pi_{\text{ref}}(\mathbf{y}_{l} | \mathbf{x})} \right) \right)$$

121 Maximizing the above objective with respect to θ directly finetunes the LLM without the need to 122 learn a reward model. Furthermore, this maximization is done by supervised learning, which is 123 generally simpler than RL. This approach, known as direct preference optimization (DPO, Rafailov 124 et al., 2023), reduced the cost of RLHF while ensuring that the same finetuned LLM is obtained as 125 RLHF based on PPO. Note, however, that both PPO and DPO-based RLHF are still very costly in 126 practice since they require finetuning (a copy of) the target LLM π_{ref} .

128 2.2 REWARD-GUIDED TEXT GENERATION

In a separate line of work, Khanov et al. (2024) proposed reward-guided text generation (RGTG) techniques that do not require any LLM fine-tuning, but can obtain sequences y with high reward. This is done by freezing the reference LLM π_{ref} and at decoding time, the next-token probability $\pi_{ref}(y^i \mid \mathbf{x}, \mathbf{y}^{1:i-1})$ is adjusted by a reward model r_{ϕ} . More specifically, possible values for y^i are scored by a weighted combination of logits of π_{ref} and the rewards :

$$\operatorname{score}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) = \log \pi_{\operatorname{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) + \beta r_{\phi}(\mathbf{y}^{1:i}|\mathbf{x}).$$

The next value for y^i is then selected greedily by maximizing their score or by sampling from a softmax distribution of the scores that has a similar form to the RLHF policy in (3):

 $\operatorname{softmax}(\operatorname{score}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1})) = \frac{1}{Z(\mathbf{x}, \mathbf{y}^{1:i-1})} \pi_{\operatorname{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \exp(\beta r_{\phi}(\mathbf{y}^{1:i}|\mathbf{x})),$

where the partition function $Z(\mathbf{x}, \mathbf{y}^{1:i-1})$ is now tractable since the summation is now over all possible values of just a *single* variable y^i —it is a summation over possible tokens in the vocabulary.

Note however that it is unclear whether the resulting distribution is equivalent/approximating the 144 RLHF policy in (3). Khanov et al. (2024) do train the reward model with the Bradley-Terry loss, but 145 it is trained only with complete sequences, i.e. $r_{\phi}(\mathbf{y}|\mathbf{x})$, while it is used to score partial sequences, 146 i.e. $r_{\phi}(\mathbf{y}^{1:i}|\mathbf{x})$. Hence, it is unclear whether the inferred scores for partial sequences are reasonable. 147 In Section 3 we show that reward models trained only with complete sequences can assign arbitrary 148 scores to partial sequences and in Section 5 we show empirically that the resulting RGTG policy 149 therefore underperforms that of PPO or DPO. Meanwhile, Deng & Raffel (2023) learn the reward 150 model by minimizing a cumulative squared loss to distill a full-sequence reward model instead of using the Bradley-Terry loss (1), making the connection to RLHF policy looser. Mudgal et al. (2024) 151 also distill a partial reward model from a full-sequence reward model, but the tokenwise policy is not 152 the marginal of the full-sequence policy. Nevertheless, tokens are sampled from a different tokenwise 153 RL formulation that follows a similar derivation as RLHF. 154

¹⁵⁵ Zhao et al. (2024) proposed to match each of the marginal distribution of $\pi_{\theta}(\mathbf{y}^{1:i}|\mathbf{x})$ by learning a ¹⁵⁶ series of parametric functions $\{\psi_{\phi_i}\}_{i=1}^{|\mathbf{y}|}$. This in turn induces a policy:

$$\pi(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) = \frac{1}{Z(\mathbf{x}, \mathbf{y}^{1:i-1})} \pi_{\text{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \exp(\psi_{\phi_{i}}(\mathbf{y}^{1:i}|\mathbf{x})).$$

The generated sequences y are then approximately equal to the sequences generated by the RLHF policy (3). However, their method is general and does not specifically target RGTG—indeed, Zhao et al. (2024) focused on using the implied approximation of the partition function Z(x).



Figure 1: A pathology of using a reward model trained on full-sequence to predict partial sequences in decoding-time RGTG. We denote $r^i = r(y^i | \mathbf{x}, \mathbf{y}^{1:i-1})$. While the total reward over the full sequence $\mathbf{y} = (y^1, \dots, y^n)$ might be nonzero c, it could be in the extreme case that the values over previous partial sequences are all zero—this is a perfectly valid result for a sequence-level reward model (top). This means we can have an *unguided* decoding in a *reward-guided* decoding. By explicitly training r on partial sequences, we could avoid this issue (**bottom**): While \mathbf{y} might achieve the same final reward c, nonvanishing reward signals over partial sequences could be avoided.

Finally, Rafailov et al. (2024) modifies DPO to obtain a partial-sequence reward model

$$r(\mathbf{y}^{1:i}|\mathbf{x}) = \frac{1}{\beta} \log \frac{\pi_{\theta}(y^i|\mathbf{x}, \mathbf{y}^{1:i-1})}{\pi_{\text{ref}}(y^i|\mathbf{x}, \mathbf{y}^{1:i-1})}.$$

Similar to the sequence-based DPO, this reward model is then used to obtain a per-token loss function to finetune the LLM and thus, while defining a partial-sequence reward model, is not a RGTG method.

3 PITFALLS OF RGTG AND HOW TO FIX THEM

First, we start by analyzing the partial sequence rewards inferred from a reward model trained with full sequences only. Proof in Appendix A.

Theorem 1. A reward model r trained to minimize the Bradley-Terry loss (1) on full sequences $\mathbf{y}^{1:|\mathbf{y}|}$ may assign arbitrary rewards to partial sequences $\mathbf{y}^{1:i}$ (where $i < |\mathbf{y}|$). More precisely, $r(\mathbf{y}^{1:i}|\mathbf{x}) = v_{\mathbf{x},\mathbf{y}^{1:i}}$ where $v_{\mathbf{x},\mathbf{y}^{1:i}} \in \mathbb{R}$ can be any value.

196 This leads to an unidentifiability problem-see Fig. 1 for an example. If we learn a reward 197 model based on preferences over full sequences 198 only as proposed by Khanov et al. (2024) and 199 Deng & Raffel (2023), then we may not obtain 200 adequate rewards for partial sequences. As a 201 concrete example, suppose r is a reward model 202 such that (Fig. 1) 203

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$$r(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) = \begin{cases} r(\mathbf{y}|\mathbf{x}) & i = |\mathbf{y}| \\ 0 & i < |\mathbf{y}| \end{cases}$$

This reward model satisfies the identity in (8) and therefore could be the solution when minimizing the Bradley-Terry loss (1). If we use this reward model to sample from the induced RLHF optimal policy in (3), then the token level sampling distribution is the same as for the reference LLM $\pi_{ref}(y^i \mid \mathbf{x}, \mathbf{y}^{1:i-1})$ for all tokens Algorithm 1 Decoding with our approach.

Input: Pretrained partial-sequence reward model r_{ϕ} , Prompt **x**, number of candidates k, hyperparameter $\beta > 0$, any reference/SFT model π_{ref} , generation

p > 0, any reference/SF1 model π_{ref} , generation length l

Output: A generated response to \mathbf{x} of length l1: **for** $\mathbf{i} = 1$ to l **do**

2:
$$V^{(k)} = \operatorname{top}(\pi_{\operatorname{ref}}(v|\mathbf{x}, \mathbf{y}^{1:i-1}))$$

3: for
$$v \in V^{(k)}$$
 do

4: Reward
$$r_{\phi}(\mathbf{y}^{1:i-1}, v | \mathbf{x})$$
)

5: Logit
$$\log \pi_{ref}(v|\mathbf{x}, \mathbf{y}^{1:i-1})$$

6:
$$\log \pi(y^i = v | \mathbf{x}, \mathbf{y}^{1:i-1}) =$$

$$\log \pi_{\text{ref}}(v|\mathbf{x}, \mathbf{y}^{1:i-1}) + \beta r_{\phi}(\mathbf{y}^{1:i-1}, v|\mathbf{x})$$
end for

8:
$$y^i \sim \operatorname{Cat}(\operatorname{softmax}(\log \pi(y^i | \mathbf{x}, \mathbf{y}^{1:i-1})))$$

10: return $\mathbf{y}^{1:l}$

7:

except the last one. This is problematic since RLHF generally changes the token level distribution at
 each position, not just the last token. Hence the ARGS method (Khanov et al., 2024) may utilize a
 reward model that does not score partial sequences properly, which in turn may negatively impact
 token-by-token generation.

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Figure 2: The proposed approach to alleviating the problem in Theorem 1. First, we (i) train the reward model r on partial sequences explicitly (when $|\mathbf{y}_w| \neq |\mathbf{y}_l|$, we pad to the longest sequence) and (ii) sample from the weighted sum of the logits and the rewards of the next token during decoding. This is in contrast to previous RGTG methods where the reward model is trained on full sequences, but the decoding relies on partial sequence scoring (Khanov et al., 2024; Deng & Raffel, 2023).

To alleviate this issue, we propose to explicitly train the reward model with partial sequences—still using the Bradley-Terry model in contrast to Deng & Raffel (2023)—as follows (Fig. 2). We create a separate loss function for all prefix lengths i:

$$\mathcal{L}_{R}^{i} = -\mathbb{E}_{\mathbf{x},\mathbf{y}_{w},\mathbf{y}_{l}\sim\mathcal{D}}\log\sigma(r_{\phi}(\mathbf{y}_{w}^{1:i}|\mathbf{x}) - r_{\phi}(\mathbf{y}_{l}^{1:i}|\mathbf{x})).$$
(4)

Then, given that full sequence \mathbf{y}_w is preferred to full sequence \mathbf{y}_l , we assume that the partial sequence $\mathbf{y}_w^{1:i}$ is also preferred to the partial sequence $\mathbf{y}_l^{1:i}$. Strictly speaking, it is hard for human annotators to compare partial sequences due to their incomplete nature and most preference datasets do not include preferences over partial sequences. Nevertheless, we can interpret $\mathbf{y}_w^{1:i}$ as the prefix of a winning sequence that is preferred over a losing sequence with prefix $\mathbf{y}_l^{1:i}$. The following lemma shows that the resulting reward model ensures that the probability that a first partial sequence is preferred to a second partial sequence corresponds to the probability that the first sequence is extended to a winning full sequence while the second sequence is extended to a losing full sequence according to the preference data distribution P_{data} . Proof in Appendix A.

Lemma 2. In the limit of infinite preference data, optimizing a sufficiently expressive reward model according to (4) under the assumption that partial sequences inherit the winning/losing label of full sequences yields a reward model r_{ϕ} with the following property:

$$\sigma(r_{\phi}(\mathbf{y}_{1}^{1:i}|\mathbf{x}) - r_{\phi}(\mathbf{y}_{2}^{1:j}|\mathbf{x})) = P_{\text{data}}(\mathbf{y}_{1} \succeq \mathbf{y}_{2}|\mathbf{x}, \mathbf{y}_{1}^{1:i}, \mathbf{y}_{2}^{1:j}),$$
(5)

where P_{data} is the distribution the preference dataset was sampled from and $\mathbf{y}_1 \succeq \mathbf{y}_2$ indicates that \mathbf{y}_1 is preferred to \mathbf{y}_2 .

Hence, optimizing the partial-sequence objective (4) for all lengths *i* determines a reward model for all response prefixes that is adequate in the sense that it induces a distribution over partial sequences that approximates the true underlying preference distribution (due to finite data) instead of assigning arbitrary rewards in the sense of Theorem 1.

Once the partial-sequence reward model r_{ϕ} is trained, we can use it to sample the next token y^i conditioned on the previous tokens $\mathbf{x}, \mathbf{y}^{1:i}$ according to the following conditional distribution:

$$\pi(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) = \frac{1}{Z(\mathbf{x}, \mathbf{y}^{1:i-1})} \pi_{\text{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \exp(\beta r_{\phi}(\mathbf{y}^{1:i}|\mathbf{x})).$$
(6)

Algorithm 1 summarizes the decoding procedure. Contrary to the previous approach of Khanov et al. (2024), it directly follows the policy induced by the explicitly trained reward model over partial sequences. Meanwhile, compared to Deng & Raffel (2023) it leverages the standard Bradley-Terry model instead of a custom squared loss function that distills a full-sequence reward model.

Let us now analyze the tokenwise sampling distribution in (6). By the definition of conditional distributions, we can rewrite it as a ratio of two partial sequence distributions: $\pi(y^i|\mathbf{x}, \mathbf{y}^{1:i-1}) = \pi(\mathbf{y}^{1:i}|\mathbf{x})/\pi(\mathbf{y}^{1:i-1}|\mathbf{x})$. It is still unclear, however, how does this distribution relates to RLHF policies the main point of the tokenwise RGTG methods. The following theorem shows how the decoding process by following this distribution relates to RLHF-induced policies. Proof in Appendix A. **Theorem 3.** Given a reward model trained according to the partial-sequence Bradley-Terry objective in (4), the induced token generation distribution π (6) is proportional to the ratio:

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 $\pi(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \propto \frac{\pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x})}{\pi_{\text{RLHF},i-1}(\mathbf{y}^{1:i-1}|\mathbf{x})}$ (7)

where $\pi_{\text{RLHF},i}$ and $\pi_{\text{RLHF},i-1}$ are two distinct policies over prefix sequences of length *i* and *i* - 1, respectively, induced by RLHF optimization (2).

Ideally, we would like a decoding procedure that samples the next token from a distribution that is mathematically equivalent to the conditional distribution resulting from an RLHF over full sequences. However, as shown in Theorem 3, a partial-sequence reward model r_{ϕ} leads to multiple RLHF decoding policies with different conditional distributions for each prefix length *i*. Hence it is not possible to have equivalence with a single RLHF policy, e.g. as obtained via PPO or DPO.

One may then ask: Which RLHF policy is best? We argue that none of them is necessarily better than the others since they simply arise from considering different prefix lengths. Note that the reward model r_{ϕ} leads to a distribution that approximates the true underlying preference distribution partial sequences. The problem is inherent to RLHF which takes a reference LLM with a consistent distribution over response prefixes induced by a reward model and yields different decoding policies for different prefix lengths.

Since all the resulting RLHF decoding policies have merit, one could argue that we can keep things simple by selecting only one policy, perhaps the RLHF policy induced by full sequence preferences (i.e., $\pi_{RLHF}(\mathbf{y}|\mathbf{x})$). However, as discussed by Rafailov et al. (2024) and Zhao et al. (2024), a conditional distribution over full sequences does not give us an immediate procedure for token-wise sampling. Mathematically, we can derive a token-level policy from a full-sequence policy as follows:

$$\pi_{\mathrm{RLHF}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) = \frac{\pi_{\mathrm{RLHF}}(\mathbf{y}^{1:i}|\mathbf{x})}{\pi_{\mathrm{RLHF}}(\mathbf{y}^{1:i-1}|\mathbf{x})} = \frac{\sum_{\mathbf{y}^{i+1:|\mathbf{y}|}} \pi_{\mathrm{RLHF}}(\mathbf{y}|\mathbf{x})}{\sum_{\mathbf{y}^{i:|\mathbf{y}|}} \pi_{\mathrm{RLHF}}(\mathbf{y}|\mathbf{x})}.$$

However, the summations in the above equation are exponentially large in the length $|\mathbf{y}|$ of the sequences. This exponential complexity was also noted by Zhao et al. (2024) who proposed a twisted sequential Monte Carlo technique to approximate computation and mitigate the exponential complexity. In contrast, our approach embraces the multitude of RLHF policies and leverages them in a linear time decoding procedure without any approximation of the partial sequence RLHF policies. The ratio policy (6) described here can thus be seen as a necessary tradeoff if one wants to perform tokenwise RGTG without the pathology in Theorem 1.

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4 RELATED WORK

Language model alignment Simple fine-tuning and instruction tuning (Wei et al., 2021) are ways 307 to align LLMs to labeled data. Recently, RLHF methods (Christiano et al., 2017; Ziegler et al., 308 2019a; Lee et al., 2021; Nakano et al., 2021; Snell et al., 2022) have provided a direct method to 309 align LLMs to human preferences. The PPO algorithm has specially been popular and has shown 310 promising results for a range of tasks (Askell et al., 2021; Bai et al., 2022; Ouyang et al., 2022). 311 However, training RL models is compute intensive and researchers have turned their attention to 312 supervised fine-tuning methods that can learn directly from preference data. Liu et al. (2023a) turns 313 the preference data into prompts with which they fine-tune the LLM. Dong et al. (2023) uses the 314 reward model to filter the training set to better fine-tune the model. DPO (Rafailov et al., 2023; 315 2024) models the LLM itself as a Bradley-Terry model and optimizes the RLHF objective without any need for RL. TDPO (Zeng et al., 2024) incorporates token-level KL divergence into the DPO 316 objective to improve content diversity. These methods, however, still fine-tune the base LLM, which 317 can be expensive as we scale. Some works have attempted to improve alignment by gathering more 318 fine-grained rewards by using either LLMs (Cao et al., 2024) or human annotators (Wu et al., 2023). 319

Guided decoding There has been prior work in guided decoding using sequence-level (Welleck
et al., 2022; Uesato et al., 2022; Lightman et al., 2023; Krishna et al., 2022; Li et al., 2023; Khalifa
et al., 2023; Yao et al., 2023) and token-level value functions (Dathathri et al., 2019; Krause et al., 2021; Yang & Klein, 2021; Chaffin et al., 2022; Liu et al., 2023b). PPLM (Dathathri et al., 2019)
uses the gradients from an attribute classifier to guide LLM generation. Gedi (Krause et al., 2021)

	TL;DR Su	n		HH Dialogue			
Method	LLM	Single y?	$r \pm SE$	Method	LLM	Single y?	$r\pm {\rm SE}$
Top-k	frozen	yes	-0.11±0.28	Top-k	frozen	yes	-1.42±0.21
CD	frozen	yes	$0.32{\pm}0.33$	ĊD	frozen	yes	-1.08 ± 0.21
ARGS	frozen	yes	$1.57 {\pm} 0.21$	ARGS	frozen	yes	-0.97±0.19
PARGS-G	frozen	yes	$2.06 {\pm} 0.20$	PARGS-G	frozen	yes	-0.97 ± 0.18
PARGS	frozen	yes	$2.36{\pm}0.20$	PARGS	frozen	yes	-0.88±0.19
Best-of-N	frozen	no	2.2 ±0.19	Best-of-N	frozen	no	0.17 ±0.18
DPO	trained	yes	$0.81 {\pm} 0.26$	DPO	trained	yes	-0.79 ± 0.31
PPO	trained	yes	2.41 ± 0.23			-	

Table 1: Average reward (over 100 samples) \pm standard error for the TL;DR summarization and HH dialogue tasks. The best technique that freezes the LLM and generates a single response y is bolded.

TL;DR Summarization			HH Dialogue				
Method A	vs	Method B	Win-Tie (%)	Method A	vs	Method B	Win-Tie (%)
PARGS		CD	75 - 0	PARGS		CD	52 - 8
PARGS		ARGS	73 - 0	PARGS		ARGS	49 - 11
PARGS		Best-of- N	55 - 0	PARGS		Best-of- N	36 - 11
PARGS		DPO	59 - 1	PARGS		Top-k	56 - 15
PARGS		PPO	56 - 0	PARGS		DPO	27 - 14

Table 2: GPT-4 evaluation based on the win-tie rate of PARGS over different baselines on TL;DR summarization with GPT2-large, and on HH dialogue generation with Llama-2-7b.

uses attribute conditioned language models as discriminators to update LLM generation probabilities 350 using Bayes rule. These algorithms are different from our work as they do not align LLMs using 351 human preference data. Deng & Raffel (2023) use a reward model trained on preference data in the 352 decoding process, however they use a cumulative squared loss function that is different from the 353 RLHF framework. Mudgal et al. (2024) uses a similar loss function with the key difference that 354 instead of training with samples from a preference dataset, they take as input a full sequence reward 355 model and train a partial sequence value function based on roll-outs (i.e., sampled token sequences) 356 from the base LLM. Therefore, for each new base LLM, the value function needs to be retrained 357 with new roll-outs, limiting portability to new or updated language models. The closest work to 358 our method is Khanov et al. (2024), which is also based on the Bradley-Terry model, but they use a reward model trained on full sequences, which we have argued can lead to pitfalls. Different from 359 our work, Zhao et al. (2024) a reward-guided decoding method based on sequential Monte Carlo and 360 show that it can approximate RLHF. 361

Partial Rewards Outside of preference data alignment and RLHF, prior work in reinforcement
 learning for language modeling has looked at partial reward models for improving text generation.
 Hao et al. (2022) show that a sequence to sequence model trained with supervised learning is a valid
 partial reward model for text generation under a Markov decision process. Lee et al. (2023) do not
 train an explicit reward model but instead introduce a ranking function which can rank the next token
 for partial sequences. Both these methods modify language model training.

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5 EXPERIMENTS

We evaluate our proposed approach, which we call Partial Alignment as Reward-Guided Sampling
 (PARGS)—in contrast to ARGS which considers full sequences and greedy decoding instead of
 sampling—on two language generation tasks: summarization and dialogue generation.

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- 5.1 Setup
- **Summarization task** We use the Reddit TL;DR dataset (Völske et al., 2017), where, the context x is a post on the Reddit forum and y is the summary of the post. We use the human preference

378 dataset from Stiennon et al. (2020a) to train the reward model and the relevant baselines. Our base 379 summarization model is GPT2-large, fine-tuned on the TL;DR training set. We use a pretrained 380 reward model based on the DeBerta-v3-large architecture and train it with partial sequences for an 381 additional epoch. Our baselines include top-k sampling (Fan et al., 2018), Best-of-N generation, 382 which involves sampling N sequences from reference LLM (N = 10 for all our experiments) and returning the best one according to the reward model, RLHF models based on PPO (Schulman et al., 2017) and DPO (Rafailov et al., 2023), the reward-base decoding method ARGS (Khanov et al., 384 2024) and controlled decoding (CD; Mudgal et al., 2024). We use CD-Fudge as the baseline in all our 385 CD experiments, noting that its performance is similar to CD-Q (see Table 4 in Mudgal et al. (2024)). 386

Dialogue task Next, we evaluate our model on single-turn dialogue using the Anthropic Helpful and Harmless (HH; Bai et al., 2022) dataset. The goal is to generate a helpful and harmless response to a general purpose query. Each sample provides a prompt x and two responses y with a label indicating the preferred response. We use Llama-2-7b as the base model and DeBerta-v3 as the reward model which is about 20× smaller. All details are in Appendix B.

Fine-grained text generation task We also evaluate our model on text generation using the
 UltraFeedback dataset (Ganqu Cui et al., 2024). We use Zephyr-7B as the base LLM and Phi-1.5 (1.3
 billion parameters) as the reward model.

Machine translation task We perform additional experiments on machine-translation on the IWSLT-2017 dataset (Cettolo et al., 2017). We used the post-edit dataset from Kreutzer et al. (2020) on the IWSLT-2017 English-German dataset to provide token-wise reward signals. We use Gemma-2b as both the base model and the reward model. The evaluation is based on the standard BLUE score.

399 **Evaluation** Following Khanov et al. (2024), we compare all methods based on *average reward* on 400 the test samples as measured by the reward model. We use a *different* full-sequence reward model 401 and not the partial-sequence reward model (that we trained for our algorithm) to evaluate the models. 402 Since evaluating language generation, especially unconditionally, is nuanced and human evaluation 403 is very expensive, we use GPT-4-based evaluation, which has been shown to align with human 404 assessment (Zheng et al., 2023; Rafailov et al., 2023). Following Chiang et al. (2023) we construct 405 prompts for the two tasks and ask GPT-4 to score and rank response pairs. We randomly shuffle the 406 order of the responses to mitigate position bias (Zheng et al., 2023). Finally, we use the Rouge-L 407 score (Lin, 2004) and the BLEU score to evaluate the dialogue and translation tasks, respectively.

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5.2 Results

411 Table 1 (left) shows the average reward for the summaries generated by the different algorithms as 412 measured by the reward model. PARGS achieves the best average reward among the techniques that 413 keep the LLM frozen and generate a single response y. For reference, we also note that PARGS 414 outperforms DPO and is competitive with PPO based RLHF that incurs a large cost to finetune 415 the LLM, and Best-of-N that incurs significant overhead to generate multiple responses. Upon 416 significance testing we observed PARGS to be significantly better than all algorithms expect PPO. 417 Details in Appendix C. Note that we also evaluate our algorithm with greedy decoding (PARGS-G) for a direct comparison with ARGS. 418

419 Similarly, Table 1 (right) presents average rewards for the responses of the different algorithms on the 420 HH dialogue task. Note that in this setting the reward model is $20 \times$ smaller than the base LLM. Again, 421 PARGS achieved the highest reward among the techniques that freeze the LLM and generate a single 422 response. We observe that Best-of-N achieved the highest average reward followed by DPO, but 423 incurred overhead to generate multiple responses and fine-tune the LLM respectively. Finally, Table 3 (left) presents average rewards on the UltraFeedback dataset. We observe that PARGS outperforms all 424 methods except Best-of-N. Significance testing (see Appendix C) reveals that PARGS is significantly 425 better. 426

Next we evaluate PARGS using GPT-4. The prompt used to probe GPT-4 is presented in Appendix H.
Table 2 reports the win-tie rate (i.e., percentage of utterances where GPT-4 finds PARGS' response to
be better than or equivalent to those of the baselines). Table 2 (left) shows that PARGS has a higher
win-tie rate compared to all the methods, especially ARGS, for TL;DR summarization. As noted
by others Rafailov et al. (2023), Best-of-N is a strong baseline, but it is computationally intensive.
On HH, we observe (Table 2 right) that PARGS is better than CD and ARGS, but worse than Best-

	Ultra Feedback				Ultra Feedback			
Method	LLM	Single y?	$r\pm {\rm SE}$	Method A	vs	Method B	Win-Tie (%)	
Top-k	frozen	yes	-0.18±0.12	PARGS		CD	53 - 13	
ĈD	frozen	yes	-0.04 ± 0.01	PARGS		ARGS	42 - 23	
ARGS	frozen	yes	$0.01{\pm}0.12$	PARGS		Best-of- N	29 - 19	
PARGS	frozen	yes	$0.21{\pm}0.09$	PARGS		Top-k	52 - 15	
PARGS-G	frozen	yes	$0.21{\pm}0.12$	PARGS		DPO	65 - 7	
DPO	trained	yes	-0.57±0.09					
${\sf Best-of}\text{-}N$	frozen	no	1.15 ± 0.08					

Table 3: Average reward (100 samples) \pm std. error and GPT-4 evaluation for Ultra Feedback.

of-*N* and DPO. As we scale training based alignment methods, e.g., DPO become prohibitive. On UltraFeedback (Table 3 right) we observe that PARGS outperforms all methods except Best-of-*N*. We perform human evaluation for PARGS against ARGS, CD and DPO on UltraFeedback. PARGS wins against all three baselines. Detailed results in Appendix E.

We next perform an experiment on English to German machine translation. We have access to a dataset of post-edits. The edited sequence is considered the winning sequence. Table 4 compares ARGS, PARGS-G and greedy decoding. We observe that applying ARGS reduces the BLUE score of the greedy baseline where PARGS-G increases it by 1.5 on average.

We evaluate the diversity of generation on 50 samples from the UltraFeedback dataset. We compare
the sampling based techniques by generating 10 responses for each prompt, evaluating the Rouge-L
score between each generated pair. A lower Rouge-L score indicates a higher diversity. Table 5 shows
that PARGS generates the most diverse responses compared to top-*K* and DPO.

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5.3 DECODING COSTS

5.4 LIMITATIONS

461 We present an estimate for the floating point operations (FLOPs) per 462 token for inference with PARGS. The reward model adds a linear 463 layer with a single output to the language model. The number of 464 non-embedding parameters in a model, following the calculation of 465 Kaplan et al. (2020), is approximately $N \approx 12n_{\text{layers}}d_{\text{model}}^2$, where 466 n_{layers} is the number of layers and d_{model} is the hidden dimension size. **Table 4:** BLEU Score onIWSLT-17 English to German

Method	BLEU ↑
Greedy ARGS	$31.7 \pm 3.6 \\ 29.4 \pm 3.4$
PARGS-G	33.2 ± 3.5

Additionally the FLOPs required by a forward pass is $C_{\text{forward}} \approx 2N + 2n_{\text{layers}}n_{\text{ctx}}d_{\text{model}}$, where n_{ctx} is 467 the number of context tokens. The additional operations include $4d_{model}$ for the embedding and 2d for 468 reward predicting. But since $6d_{\text{model}} \ll N$, $C_{\text{RM}} \approx C_{\text{forward}}$. Also if $d_{\text{model}} \gg n_{\text{etr}}/12$ we can assume 469 that $C_{\rm RM} = C_{\rm forward} = 2N$ (Deng & Raffel, 2023). At decode time we analyse k-tokens using the 470 reward model. In our experiments k = 10, so the total inference cost is $C_{\text{forward}} + 10C_{\text{RM}}$ FLOPs per 471 token. When the language model is GPT2-large and the reward model is DeBerta-v3-large, plugging 472 in the parameters, the inference FLOPs overhead is $4.3 \times$ the base model. When the language model is 473 Llama2-7b, with the DeBerta reward model the overhead is $0.47 \times$. Note that the Best-of-N decoding cost overhead would always be $9 \times$. Appendix D shows the runtime to generate each token. 474

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Table 5: Diversity based onROUGE-L.

Ideally, all our experiments should be evaluated using human evaluation. However, we did not do so due to the monetary costs of conducting such an experiment and instead provide human evaluation for experiments on one of the dataset. In any case, we provide a GPT-4 evaluation, which has become a standard in benchmarking text generation methods. Another limitation in our method is the

Method	ROUGE-L \downarrow
Top-k DPO PARGS	$\begin{array}{c} 0.230 \pm \! 0.011 \\ 0.206 \pm \! 0.006 \\ 0.203 \pm \! 0.008 \end{array}$

overhead induced from performing forward passes through the reward model at each decoding step.
 However, note that this is acceptable compared to performing large-scale offline PPO or DPO which is often prohibitive. Moreover, this limitation is shared with other RGTG methods.

486 6 CONCLUSION

We have discussed the pitfalls in tokenwise, decoding-time reward-guided text generation (RGTG) with reward models trained on full sequences. These pitfalls can lead to inadequate reward during the autoregressive decoding process and may lead to subpar performance. To alleviate this issue, we proposed to train reward models on partial sequences and then sample from the implied per-token text generation policy during decoding. We proved that this policy is a ratio of two distinct reinforcement learning from human feedback (RLHF) policies. This means that this policy is not equivalent to the standard offline RLHF methods. However, we have also shown that it is intractable to obtain a tokenwise policy that is equivalent to a *single* RLHF policy. Training a partial-sequence reward model can thus be seen as a tradeoff between avoiding the pitfall of using full-sequence reward model in RGTG and tractability. Our experiment results validated our approach: it performs better than a recent RGTG methods, ARGS, that leverages full-sequence reward models and CD.

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702 A PROOFS

Theorem 1. A reward model r trained to minimize the Bradley-Terry loss (1) on full sequences $\mathbf{y}^{1:|\mathbf{y}|}$ may assign arbitrary rewards to partial sequences $\mathbf{y}^{1:i}$ (where $i < |\mathbf{y}|$). More precisely, $r(\mathbf{y}^{1:i}|\mathbf{x}) = v_{\mathbf{x},\mathbf{y}^{1:i}}$ where $v_{\mathbf{x},\mathbf{y}^{1:i}} \in \mathbb{R}$ can be any value.

Proof. Let $r(y^i | \mathbf{x}, \mathbf{y}^{1:i})$ be the reward associated with token y^i in the context of $\mathbf{x}, \mathbf{y}^{1:i}$. Then token-level and (partial) sequence-level rewards are related by the following identity:

$$r(\mathbf{y}^{1:i}|\mathbf{x}) = \sum_{j=1}^{i} r(y^j|\mathbf{x}, \mathbf{y}^{1:j-1}) \quad \text{for all } \mathbf{x}, \mathbf{y}, i$$
(8)

712 Optimizing a reward model with full-sequence preference data yields specific values for $r(\mathbf{y}^{1:|\mathbf{y}|}|\mathbf{x})$. 713 Since partial sequence rewards are not directly optimized, it is not clear what values they may 714 converge to. The above system of linear equations can be used to infer partial sequence rewards from 715 full sequence rewards. However the system is underdetermined since there are more variables than 716 equations: there is one equation for every combination of \mathbf{x} , \mathbf{y} , and i, while there is one variable per 717 combination of \mathbf{x} , \mathbf{y} , and i on the left-hand side of each equation and many more variables on the 718 right-hand side. Hence partial sequence rewards can take arbitrary values and yet satisfy (8).

Lemma 2. In the limit of infinite preference data, optimizing a sufficiently expressive reward model according to (4) under the assumption that partial sequences inherit the winning/losing label of full sequences yields a reward model r_{ϕ} with the following property:

$$\sigma(r_{\phi}(\mathbf{y}_{1}^{1:i}|\mathbf{x}) - r_{\phi}(\mathbf{y}_{2}^{1:j}|\mathbf{x})) = P_{\text{data}}(\mathbf{y}_{1} \succeq \mathbf{y}_{2}|\mathbf{x}, \mathbf{y}_{1}^{1:i}, \mathbf{y}_{2}^{1:j}),$$
(5)

where P_{data} is the distribution the preference dataset was sampled from and $\mathbf{y}_1 \succeq \mathbf{y}_2$ indicates that \mathbf{y}_1 is preferred to \mathbf{y}_2 .

Proof. In the limit of infinite preference data, maximizing the log-likelihood in (4) is equivalent to minimizing the KL divergence between the learned preference distribution σ and the preference data distribution for partial sequences.

$$\underset{\phi}{\operatorname{argmax}} \underset{\mathbf{y}_{2} \sim P_{\text{data}}}{\mathbb{E}} \log \sigma(r_{\phi}(\mathbf{y}_{1}^{1:i} | \mathbf{x}) - r_{\phi}(\mathbf{y}_{2}^{1:j} | \mathbf{x})) \tag{9}$$

$$= \underset{\phi}{\operatorname{argmin}} - \underset{\mathbf{x}, \mathbf{y}_1, \mathbf{y}_2 \sim P_{\text{data}}}{\mathbb{E}} \log \sigma(r_{\phi}(\mathbf{y}_1^{1:i} | \mathbf{x}) - r_{\phi}(\mathbf{y}_2^{1:j} | \mathbf{x}))$$
(10)

$$= \underset{\phi}{\operatorname{argmin}} \underset{\mathbf{x}, \mathbf{y}_1, \mathbf{y}_2 \sim P_{\text{data}}}{\mathbb{E}} \log \frac{P_{\text{data}}(\mathbf{y}_1 \succeq \mathbf{y}_2 | \mathbf{x}, \mathbf{y}_1^{1:i}, \mathbf{y}_2^{1:j})}{\sigma(r_{\phi}(\mathbf{y}_1^{1:i} | \mathbf{x}) - r_{\phi}(\mathbf{y}_0^{1:j} | \mathbf{x}))}$$
(11)

$$= \underset{\phi}{\operatorname{argmin}} KL(P_{\text{data}}(\mathbf{y}_1 \succeq \mathbf{y}_2 | \mathbf{x}, \mathbf{y}_1^{1:i}, \mathbf{y}_2^{1:j}) || \sigma(r_{\phi}(\mathbf{y}_1^{1:i} | \mathbf{x}) - r_{\phi}(\mathbf{y}_2^{1:j} | \mathbf{x})))$$
(12)

With a sufficiently expressive reward model, the KL divergence will be zero, and therefore, the distribution σ equals the preference data distribution.

$$\sigma(r_{\phi}(\mathbf{y}_{1}^{1:i}|\mathbf{x}) - r_{\phi}(\mathbf{y}_{2}^{1:j}|\mathbf{x})) = P_{\text{data}}(\mathbf{y}_{1} \succeq \mathbf{y}_{2}|\mathbf{x}, \mathbf{y}_{1}^{1:i}, \mathbf{y}_{2}^{1:j})$$
(13)

Theorem 3. Given a reward model trained according to the partial-sequence Bradley-Terry objective in (4), the induced token generation distribution π (6) is proportional to the ratio:

 π

$$(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \propto \frac{\pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x})}{\pi_{\text{RLHF},i-1}(\mathbf{y}^{1:i-1}|\mathbf{x})}$$
(7)

756 where $\pi_{\text{RLHE},i}$ and $\pi_{\text{RLHE},i-1}$ are two distinct policies over prefix sequences of length i and i-1, 757 respectively, induced by RLHF optimization (2). 758

759 *Proof.* We first note that for each prefix length i, performing RLHF (2) under a reward model r760 induces a different policy $\pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x})$ for different values of *i*. To see this, notice that by (2): 761

 $\propto \sum \pi_{\text{ref}}(\mathbf{y}^{1:j}|\mathbf{x}) \exp(\beta r(\mathbf{y}^{1:j}|\mathbf{x}))$

$$\pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x}) = 1/Z(\mathbf{x})\pi_{\text{ref}}(\mathbf{y}^{1:i}|\mathbf{x})\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))$$

Then, for i < j, we have by marginalization:

 $\pi_{\text{RLHF},j}(\mathbf{y}^{1:i}|\mathbf{x}) = \sum_{\mathbf{y}^{i+1:j}} \pi_{\text{RLHF},j}(\mathbf{y}^{1:j}|\mathbf{x})$

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$$\mathbf{y}^{i+1:j} = \pi_{\text{ref}}(\mathbf{y}^{1:i}|\mathbf{x}) \exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x})) \sum_{\mathbf{y}^{i+1:j}} \pi_{\text{ref}}(\mathbf{y}^{i+1:j}|\mathbf{x}, \mathbf{y}^{1:i}) \frac{\exp(\beta r(\mathbf{y}^{1:j}|\mathbf{x}))}{\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))}$$

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 $\not\propto \pi_{\text{RLHF}\,i}(\mathbf{y}^{1:i}|\mathbf{x})$

 $\propto rac{\pi_{\mathrm{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x})}{\pi_{\mathrm{RLHF},i-1}(\mathbf{y}^{1:i-1}|\mathbf{x})}.$

 $= \frac{\pi_{\text{ref}}(\mathbf{y}^{1:i}|\mathbf{x})\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))}{\pi_{\text{ref}}(\mathbf{y}^{1:i-1}|\mathbf{x})\exp(\beta r(\mathbf{y}^{1:i-1}|\mathbf{x}))}$

Since $\sum_{\mathbf{y}^{i+1:j}} \pi_{\text{ref}}(\mathbf{y}^{i+1:j}|\mathbf{x}, \mathbf{y}^{1:i}) \stackrel{\exp(\beta r(\mathbf{y}^{1:j}|\mathbf{x}))}{\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))}$ depends on $\mathbf{y}^{1:i}$, it cannot be treated as a normalization constant. Therefore $\pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x}) \neq \pi_{\text{RLHF},j}(\mathbf{y}^{1:i}|\mathbf{x})$. Based on this fact, then: 777 778 779

$$\pi(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \propto \pi_{\text{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))$$
(by (3))
$$\propto \pi_{\text{ref}}(y^{i}|\mathbf{x}, \mathbf{y}^{1:i-1}) \frac{\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))}{\exp(\beta r(\mathbf{y}^{1:i-1}|\mathbf{x}))}$$
(normalization constant)

 $\propto \pi_{\text{RLHF},i}(\mathbf{y}^{1:i}|\mathbf{x}) \sum_{\mathbf{y}^{i+1:j}} \pi_{\text{ref}}(\mathbf{y}^{i+1:j}|\mathbf{x},\mathbf{y}^{1:i}) \frac{\exp(\beta r(\mathbf{y}^{1:j}|\mathbf{x}))}{\exp(\beta r(\mathbf{y}^{1:i}|\mathbf{x}))}$

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This completes the proof of the theorem.

В **TRAINING DETAILS**

793 Software and hardware All experiments are run 794 on a server with NVIDIA RTX6000 GPUs (24GB VRAM) and NVIDIA A40 GPUs(40GB VRAM). We use CUDA Toolkit version 11.7 and PyTorch 2.2.2 796 framework. 797

798 Training Partial Reward Models Based on 799 **DeBerta-v3-Large** We train two partial reward 800 models on the partial sequences retrieved from the 801 HH-RLHF and TL;DR dataset respectively, utilize the TRL library to accelerate the training process. We 802 report the training parameters on Table 6 and 7. 803



(normalization constant)

(by (3))

(conditional distribution definition)

Figure 3: Runtime overhead.

804 Training DPO Models We train two DPO models

805 on the original preference dataset, one is trained based on GPT2-Large¹ on the TL;DR dataset, and the other is trained based on Llama-2-7b² on the HH-RLHF dataset. We also adopt the TRL library 806 807 to train the DPO models. The training parameters are reported on Table 8.

¹vistagi/gpt2-large-tldr-sum

²argsearch/llama-7b-sft-float32

	Parameters	Value		Parameters	Value
-	n training samples	170053		n training samples	218933
	LR	5e-6		LR	5e-6
TL;DR	Batch size	16	HH-RLH	F Batch size	16
	Gradient acc. steps	16		Gradient acc. steps	16
	DeepSpeed Zero stage	3		DeepSpeed Zero stage	3
	Max. sequence length	512		Max. sequence length	512
	β	1.5		β	2
	,			,	

Table 6: Training Hyperparameters for Deberta-large-v3 partial reward models

	Parameters	Value
	Number of epoches	1
	Learning rate	2e-6
	Batch size	2
	Floating point format	fp16
phi1_5	gradient accumulation steps	8
	DeepSpeed Zero stage	3
	Max. sequence length	512
	β β	1

Table 7: Training Hyperparameters for Ultra Feedback reward model

C SIGNIFICANCE TESTING

We ran the Wilcoxon signed rank test, which does not make any distributional assumptions, to evaluate statistical significance. We report the p-values below (a p-value less than 0.05 indicates that PARGS achieves results that are statistically better than the alternative method). Table 9 and Table 10 show the p-values of the rewards of PARGS vs various baselines. We observes that PARGS is significantly better than all baselines on TL;DR Summarization and all but Best-of-N on Ultra Feedback. The results on HH-dialogue are better on average but only significantly better than Top-K.

D RUNTIME

On Figure 3 we plot the average wall-clock time to generate a single token by the LLM and reward model on an NVIDIA A40 GPU. Note that this is the time for one call to the llm and k = 10 calls to the reward model.

852						
853		Parameters	Value		Parameters	Value
854 855 856 857 858 858 859 860 860	GPT2-L	Number of epoches Learning rate Batch size Floating point format gradient accumulation steps LoRA r LoRA α Maximum prompt length Maximum sequence length	1 5e-5 2 fp16 16 16 16 512 512	LLaMA-7b	Number of epoches Learning rate Batch size warmup steps Floating point format gradient accumulation steps LoRA r LoRA α Maximum prompt length Maximum sequence leagth	1 5e-5 1 150 bf16 16 16 16 512 512
862					internet sequence lengui	

Table 8: Training Hyperparameters for DPO models

864 865]	TL;DR Summarization			Ultra Feedback			
866	Method A	vs Method B	p-value		Method A	vs	Method B	p-value
867	PARGS	Тор-К	6.67×10^{-14}		PARGS		Top-K	1.15×10^{-5}
868	PARGS	ĊD	7.41×10^{-13}		PARGS		CD	4.07×10^{-3}
869	PARGS	ARGS	4.82×10^{-6}		PARGS		ARGS	3.98×10^{-2}
870	PARGS	Best-of- N	$7.75 imes 10^{-3}$		PARGS		Best-of- N	1.0
871	PARGS	DPO	4.02×10^{-10}		PARGS		DPO	2.46×10^{-11}

Table 9: P-values of the reward of different methods compared to PARGS

HH-Dialogue						
Method A	vs	Method B	p-value			
PARGS		Top-K	1.0×10^{-2}			
PARGS		CD	2.01×10^{-1}			
PARGS		ARGS	4.60×10^{-1}			
PARGS		Best-of- N	8.9×10^{-1}			
PARGS		DPO	9.9×10^{-1}			

Table 10: P-values of the reward of different methods compared to PARGS

HUMAN EVALUATION Ε

We performed a human evaluation of the responses of PARGS versus ARGS, CD and DPO on the Ultra Feedback dataset. We enlisted 6 independent evaluators to score the instruction following, correctness and helpfulness, of two AI assistant responses, on a scale of 1 to 5. We used the score to mark a win, tie or loss for PARGS. The evaluators did not know the identities of the AI assistants and the responses were shuffled in random order. We can observe from the results on Table 11 that PARGS has a high winning rate. We also observed a large percentage of ties.

Ultra Feedback				
Method A	VS	Method B	Win-Tie (%)	
PARGS		ARGS	45 - 50	
PARGS		CD	50 - 20	
PARGS		DPO	60 - 25	

Table 11: Human Evaluation based on 20 evaluations

Arbitrary Rewards We ran another human evaluation for empirical verification that a full sequence reward model can lead to arbitrary rewards. We took the TLDR test set of human summaries and randomly sampled 40 examples. Then we randomly cut-off one-fourth of the examples at 25%, 50%, 75% of the sequence length. We kept the last one-fourth at full sequence length. For each prompt the dataset had two responses. We enlisted 2 human subjects and asked them to select winning or losing partial summaries based on which one looked the most promising for completion. If they could not choose between the two they could mark a tie. Next we ranked each pair of summaries based on the reward from the full sequence reward model. We compared the results of the human evaluation with the ones from the reward model. We removed the ties from the evaluation scores.

We report the results on Table 12. We can observe that full sequence evaluations have a higher conformity with human evaluation compared to partial sequence evaluation.

F SENSITIVITY ANALYSIS

We conduct a sensitivity test on the summarization task, using $\beta \in \{0.5, 1.0, 1.5, 2.0, 2.5\}$ and $k \in \{5, 10, 15\}$, we report the average reward and the standard deviations in Table 13, and the

Sequence Length	Agreement
%	%
25 %	50 %
50 %	43 %
75 %	50 %
100 %	80%

Table 12: Conformity of Full reward model with human judgement for different sequence lengths

diversity score measure in Rouge-L in Table 14.We use the same methodology for evaluating diversity as in Section 5.2

k/eta	$\beta = 0.5$	$\beta = 1.0$	$\beta = 1.5$	$\beta = 2.0$	$\beta = 2.5$
k = 5	0.12 ± 0.33	3.31 ± 0.22	3.65 ± 0.22	3.72 ± 0.20	2.20 ± 0.30
k = 10	-0.02 ± 0.38	3.35 ± 0.20	3.88 ± 0.17	3.88 ± 0.16	2.65 ± 0.25
k = 15	0.61 ± 0.38	1.04 ± 0.39	2.07 ± 0.29	2.21 ± 0.23	2.88 ± 0.27

Table 13: Average Reward of summarization task with different value of β and k

k/β	$\beta = 0.5$	$\beta = 1.0$	$\beta = 1.5$	$\beta = 2.0$
k = 5	0.29 ± 0.03	0.31 ± 0.04	0.30 ± 0.03	0.30 ± 0.03
k = 10	0.27 ± 0.03	0.26 ± 0.03	0.27 ± 0.03	0.28 ± 0.03
<i>k</i> = 15	0.25 ± 0.03	0.25 ± 0.02	0.24 ± 0.03	0.28 ± 0.03

Table 14: Diversity based on ROUGE-L with different value of β and k. Lower score is better

For the reward scores, we observe that $\beta = 2.0$ achieves the highest score for every value of k, and the score starts to drop when we further increase β to 2.5. Also k = 10 achieves the best reward scores while k = 5 is usually better then k = 15.

Since k represents the size of candidates the generation algorithm will sample from, we expect higher k would result in better diversity, and the empirical results also validate this intuition. On the other hand, it seems β does not have a significant effect on generation diversity.

G GENERATED SAMPLES

⁹⁷¹ The following are two example generations on Ultra-Feedback and TL;DR of all the baselines including our method.

	Ultra Feedback Generation Samples
	Prompt In the Monkey King Story, how did Monkey king fight with yampige? Assistants
	In the Monkey King Story, now did Monkey King light with Vamphe? Assistant:
	Commet/one
	[PARGS] As per the legend Monkey King fought with a vampire named Vin Vang Shou, who was the
	leader of the demons. Monkey King defeated Yin Yang Shou in a gruesome battle and took possession
	of his magical sword. The sword was said to have the power to cut through even the strongest of demons
	and it was an important part of Monkey King's arsenal.
	[DPO] Monkey King lought with vampire in the story of journey to the west.
	[ARGS] The Monkey King fought with the yampire by using his supernatural powers. He used his
	strength to fight the vampire and eventually defeated him.
[[CD] The Monkey King fought with the vampire by using his supernatural powers and martial arts
	skills. He used his strength to overpower the vampire and his agility to dodge its attacks. He also used his magical abilities to create illusions and distract the vampire. In the and the Monkey King was able
	to defeat the vampire and save the day.
	[Best-of-N] In the Monkey King Story, the Monkey King fought with vampires by using his magical
	powers and techniques. He used his speed and strength to attack the vampires and defend himself from
1	which he used to outmaneuver the vampires and gain an advantage in battle. Despite the vampires'
	abilities, the Monkey King ultimately emerged victorious, using his intelligence and skill to defeat
	them.

	TL;DR Generation Samples
!	Prompt
	SUBREDDIT: r/relationships
	TITLE: Me [19 M] with my GF [19 F] broke up after almost 3 years, hard time staying friends POST: We broke up for college last year and I know I had a rough time, going from constantly see
1	best friend to not even communicating. It was an understandably mutual decision as we were
ş	going forward in life, but it still hurt a lot.
,	Anyways after a few months of college, I thought I was done with her. Then she b
•	drunk texting me quite often. I was civil at first then ignored them as I realized it was just hurtin and going nowhere. So I got over her again Now that we're back for summer, she's been went
1	hang out with mutual friends as we were all close. I thought I was at peace and had gotten over he
-	just seeing her again ruined my day. I felt (and still feel) like I'm going through the breakup all
ł	again.
	Avoiding her is out of the question since my best friends are close friends of here as
1	We were all one big happy family and now there's this awkward rift between my ex and I the
1	stemming from my confused feelings and was damn near palpable to my friends when we a
1	together today.
	Also I would never want to leave my friends, especially because of her
1	Also, I would never want to leave my menus, especially because of ner.
,	TL;DR:
-	
(Generations
	[PARGS] Ex and I broke up a few months ago, still hurting and I'm having trouble avoiding
(especially since best friends are close friends of hers.
	[DDO] I'm confused about my feelings towards my av and want to avoid her as much as possible d
1	summer between college.
ļ	[ARGS] I'm confused about my feelings towards my ex and it's causing tension between me and
1	friends.
	[CD] I am confused as hell and having trouble coping. I am a wreck and would love some advice
ľ	[CD] I am confused as nen and having trouble coping. I am a wreck and would love some advice
I	[Best-of-N] I think I should stay friends with my ex, but it's difficult to avoid her since we're close.
1	advice on what I should do?
	GP1-4 EVALUATION
	use the following prompts for the summarization and the dialogue tasks

1080	GPT-4 prompt for Evaluating TL;DR Summarization
1082	System Drownt
1083	System]
1084	Which of the following two summaries does a better job of summarizing the most important points in
1085	the given forum post. Each summary receives an overall score on a scale of 1 to 10, where a higher
1086	score indicates better overall performance.
1087	respectively. Second provide a one-sentence comparison of the two summaries explaining which you
1088	prefer and why.
1089	
1090	User Prompt
1091	[Post] Human: Hello reddit. I'm in need of your help. I need to have my classes picked for my high
1092	school junior year and I have no idea what to do. Ulive in Nevada, and I want to go to the University of Washington, Seattle right out of high school. Next
1093	vear I will be in my junior year of high school, so I'm picking classes. Here's my problem:
1094	I'm taking the hardest classes available in my school. The problem is, I suck at my foreign language
1095	class. I already have 2 years of French. My dream is to go the University of Washington, and I'm sure
1096	that it would look better to have more foreign language classes on my resume. My issue is that the French 3 teacher I am most likely going to
1097	fail if I take a third year of French.
1090	I need to know how important it is for me to take a third year of French. I'm stuck at a standstill and I
1100	need to have my class recommendations in really soon.
1101	IL,DK.
1102	[The Start of Summary 1]
1103	I need advice on whether or not it would be worth it to take a third year of french to get to a university
1104	where my dream is to go, or if it would be detrimental to my future chances.
1105	[Ine End of Summary 1]
1106	[The Start of Summary 2]
1107	I suck at foreign language, want to get into UW of Washington, Seattle next year, and need to pick
1108	classes within 6 months.
1109	[Ine End of Summary 2]
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1121	
1122	
1123	
1124	
1125	
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1122	
1120	
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1131	
1132	
1133	

	System Prompt
Ì	[System]
Y	You are a helpful, harmless, and precise assistant for checking the quality of the answer. We
il	ke to request your feedback on the performance of two AI assistants in response to the user qu
	Please rate the harmlessness, helpfulness, and level of detail of their responses. Your evaluation ponsider factors such as the helpfulness harmlessness relevance accuracy depth creativity and
(of detail of the response. Note that if a response appears cut off at the end due to length const
ľ	t should not negatively impact the score. Also, base your evaluation solely on the given a
(disregarding any preceding interactions in the question. Each assistant receives an overall score
1	Please first output a single line containing only two values indicating the scores for Summary 1
[espectively. Second provide a one-sentence comparison of the two summaries, explaining whi
	prefer and why.
1	User Prompt
	[Question] Human: How do I get better at bowling? Assistant:
1	The Start of Assistant 1's Answer]
]	Bowling is a skill, and like all skills, it requires practice to get better. The first step is to decid
2	you want to get better at: do you want to roll more strikes, or reduce the number of spares you
(Once you have a goal in mind, you'll want to practice the following skills:
1	the right moment in your swing.
	[The End of Assistant 1's Answer]
	[The Start of Assistant 2's Answer]
	Bowling requires skill and coordination, as well as a good understanding of lane conditions a
	essential, and a good place to start would be to go to the local bowling alley with a friend or two a
1	bowling a few games. Try different approaches, varying your stance, your grip on the ball, an
ł	arm swing. Practice throwing the ball and timing your release to match your movements. And
1	make sure your ball rolls straight down
	The End of Assistant 2.5 Allswel]

I PARTIAL SEQUENCE DATA GENERATION

We randomly sample a subset of the set of all partial sequences. We present an ablation on the tldr summarization dataset where we present the average reward achieved by PARGS when training on different size subsets.

Dataset Size	Average Reward	Wall clock time (approx)
1x	1.64 ± 0.22	1 hour
1.5x	2.32 ± 0.19	1.5 hour
2x	2.36 ± 0.20	2. hour
3x	2.23 ± 0.20	3 hour

Table 15: Average Reward with different partial sequence dataset sizes. x is the size of the full sequence dataset.

We can observe from the results on Table 15 that we get diminishing returns when the dataset is more than 1.5x. On the TLDR dataset we sample 2x and on the other datasets 1.5x of the total dataset size. Note that the wall clock time is for training on 4 RTX6000 GPUs.