NetSciX2026: International School and Conference on Network Science
February 17^{th}-20^{th}, 2026 - Auckland, New Zealand

First return processes of non-backtracking random walks on random networks

Keywords: first return time, random walk, non-backtracking random walk, random network, Kac's lemma

Extended Abstract

The concept of first return time—the random time for a stochastic walker to return to its starting point is a fundamental property in the study of diffusion and search processes on discrete structures. On simple lattices such as the one-dimensional (1D) line classical results yield explicit distributions and scaling behaviors for these return statistics, reflecting the underlying recurrence or transience. Extending to general graphs, including random or regular networks, first return time distributions reveal deep connections between structural features and dynamical behavior. Their relevance to search processes stems from the fact that return events determine both the efficiency of exploration and the likelihood of redundant visits, thus providing key insights into how quickly and thoroughly a walker can cover a given structure.

In this talk I will present analytical results for the distribution of first return (FR) times of non-backtracking random walks (NBWs) on random networks consisting of N nodes with degree distribution $P(k)$. Starting from a random initial node i at time $t = 0$, an NBW hops into a random neighbor of i at time $t = 1$ and at each subsequent step it continues to hop into a random neighbor of its current node, excluding the previous node. For the first return process, we calculate the tail distribution $P(T_{\rm FR} > t)$ of first return times from a random initial node to itself. It is found that $P(T_{\rm FR} > t)$ is given by a discrete Laplace transform of the degree distribution $P(k)$. This result exemplifies the relation between structural properties of a network, captured by the degree distribution, and properties of dynamical processes taking place on the network.

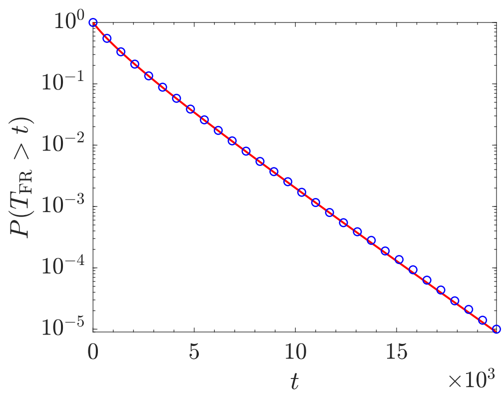

Figure 1: Analytical results for the tail distribution $P(T_{\rm FR} > t)$ (solid lines) of first return times of an NBW on a configuration model network of size $N = 1000$ which exhibits a power-law distribution with $k_{\rm min} = 3$, $k_{\rm max} = 30$ and $\gamma = 2.5$. The analytical results are in very good agreement with the results obtained from computer simulations (circles).

In Fig. 1 we present analytical results for the tail distribution $P(T_{\rm FR} > t)$ (solid line) of first return times of an NBW on a configuration model network of size $N = 1000$ that exhibits a power-law degree distribution with $k_{\min} = 3$, $k_{\max} = 30$ and $\gamma = 2.5$, The analytical results are in very good agreement with the results obtained from computer simulations (circles).

Using the tail-sum formula, we calculate the mean first return time $E[T_{\mathrm{FR}}]$. Surprisingly, $E[T_{\mathrm{FR}}]$ coincides with the result obtained from the Kac's lemma [1] that applies to classical random walks (RWs). The results obtained from this approach will be presented for Erdős-Rényi networks and configuration model networks with exponential and power-law degree distributions and will be generalized to the broader context of first-passage processes.

These results provide useful insight on the advantages of NBWs which scan the network faster and more efficiently than classical RWs. As a result, they are exceptionally useful for network exploration, sampling procedures and search processes on networks.

References

[1] R. Fitzner and R. van der Hofstad, Non-backtracking random walks, *J. Stat. Phys.* **150** 264 (2013).

[2] M. Kac, On the notion of recurrence in discrete stochastic processes, *Bull. Amer. Math. Soc.* **53** 1002 (1947).

[3] N. Alon, I. Benjamini, E. Lubetzky and S. Sodin, Non-backtracking random walks mix faster, *Commun. Contemp. Math.* **9** 585 (2007).

[4] D. Fasino, A. Tonetto and F. Tudisco, Hitting times for second-order random walks, *Euro. Jnl of Applied Mathematics* **34** 642 (2023).