NAVIGATING CONCEPT DRIFT AND TEMPORAL SHIFT: DISTRIBUTION SHIFT GENERALIZED TIME-SERIES FORECASTING

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ABSTRACT

Time-series forecasting finds broad applications in real-world scenarios. Due to the dynamic nature of time series data, it is crucial for time-series forecasting models to produce robust predictions under potential distribution shifts. In this paper, we initially identify two types of distribution shifts in time series: concept drift and temporal shift. We acknowledge that while existing studies primarily focus on addressing temporal shift issues in time series, designing proper concept drift methods for time series data received comparatively less attention.

Motivated by the need to mitigate potential concept drift issues in time-series forecasting, this work proposes a novel soft attention mechanism that effectively leverages and ensemble information from the horizon time series. Furthermore, recognizing that both concept drift and temporal shift could occur concurrently in time-series forecasting scenarios while an integrated solution remains missing, this paper introduces ShifTS, a model-agnostic framework seamlessly addressing both concept drift and temporal shift issues in time-series forecasting. Extensive experiments demonstrate the efficacy of ShifTS in consistently enhancing the forecasting accuracy of agnostic models across multiple datasets, and consistently outperforming existing concept drift, temporal shift, and combined baselines.

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1 INTRODUCTION

Time-series forecasting finds applications in various real-world scenarios such as economics, urban computing, and epidemiology (Zhu & Shasha, 2002; Zheng et al., 2014; Deb et al., 2017; Mathis et al., 2024). These applications involve predicting future trends or events based on historical time-series data. For example, economists use forecasts to make financial and marketing plans, while sociologists use them to allocate resources and formulate policies for traffic or disease control.

The recent advent of deep learning has revolutionized time-series forecasting, resulting in series of advanced forecasting models (Lai et al., 2018; Torres et al., 2021; Salinas et al., 2020; Nie et al., 2023; Zhou et al., 2021). However, despite these success, time-series forecasting faces certain 040 challenges from distribution shifts due to the dynamic and complex nature of time series data. The 041 distribution shifts in time series can be categorized into two types (Granger, 2003). First, the data 042 distributions of the time series data themselves can change over time, including shifts in mean, 043 variance, and autocorrelation structure, which is referred to as non-stationarity or temporal drift 044 issues in time-series forecasting (Shimodaira, 2000; Du et al., 2021). For example, in influenza-like illness (ILI) forecasting, the distribution of influenza cases varies between summer and winter, with higher infection rates typically observed during the winter seasons. Second, time-series forecasting is 046 compounded by unforeseen exogenous factors, which shifts the distribution of target time series. A 047 prominent example is the COVID-19 pandemic, which led to an abnormal excess of influenza cases 048 than normal years. These types of phenomena, categorized as concept drift problems in time-series forecasting (Gama et al., 2014; Lu et al., 2018), make it even more challenging. 050

While prior research has investigated strategies to mitigate temporal shifts (Liu et al., 2022; Kim et al., 2021; Fan et al., 2023), addressing concept drift issues in time-series forecasting has been largely overlooked. Although concept drift is a well-studied problem in general machine learning (Sagawa et al., 2019; Arjovsky et al., 2019; Ahuja et al., 2021), adapting these solutions to time-series

forecasting is challenging. Many of these methods require environment labels, which are typically
unavailable in time-series datasets (Liu et al., 2024a). Indeed, the few concept drift approaches
developed for time-series data are designed exclusively for online settings (Guo et al., 2021), limiting
their generalizability to standard time-series forecasting tasks. Moreover, while both concept drift
and temporal shift can simultaneously impact time-series forecasting, as shown in the previous ILI
forecasting example, few existing researches or practical solutions address both issues together.

We aim to close this gap in the literature in this paper - this study aims to design an integrated framework that effectively addresses both concept drift, which has not been studied well by itself, and temporal shift. Our method involves ensembling time series across multiple horizon time steps to enhance generalization and mitigate concept drift, with seamless integration with normalization strategies to address temporal shift. The contributions of this paper are:

- 1. **Concept Drift for Time-Series:** We introduce soft attention masking (SAM) designed to mitigate concept drift issues by effectively using exogenous information from the horizon window. The soft attention allows the time-series forecasting models to weigh the ensemble of the time series at multiple horizon time steps to enhance the generalization ability.
- 2. Integrated Framework: We propose ShifTS, a practical and model-agnostic framework that tackles both concept drift and temporal shift in time-series forecasting tasks. ShifTS seamlessly integrates the proposed soft attention mechanism with established temporal shift mitigation techniques, facilitating enhanced forecasting accuracy.
- 3. **Comprehensive Evaluations:** We conduct extensive experiments on various time series datasets with multiple advanced time-series forecasting models. The proposed ShifTS demonstrates effectiveness by consistent performance improvements to agnostic forecasting models, as well as outperforming distribution shift baselines in better forecasting accuracy.

2 RELATED WORKS

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080 Time-Series Forecasting. Classical statistical time-series forecasting models, such as ARIMA (Hyn-081 dman & Athanasopoulos, 2018), often face limitations in capturing complicated patterns and depen-082 dencies due to inherent model constraints (Nadaraya, 1964; Williams & Rasmussen, 1995; Smola & 083 Schölkopf, 2004). Recent works in deep learning have achieved notable achievements in time-series 084 forecasting, such as RNNs, LSTNet, N-BEATS (Sherstinsky, 2020; Lai et al., 2018; Oreshkin et al., 085 2020). State-of-the-art models build upon the successes of self-attention mechanisms (Vaswani et al., 2017) with transformer-based architectures and significantly improve forecasting accuracy, such as 087 Informer, Autoformer, Fedformer, PatchTST, iTransformer, FRNet (Zhou et al., 2021; Wu et al., 2021; Zhou et al., 2022; Nie et al., 2023; Liu et al., 2024b; Zhang et al., 2024). However, these 088 advanced models primarily rely on empirical risk minimization (ERM) with IID assumptions, i.e., 089 train and test dataset follows the same data distribution, which exhibits limitations when potential 090 distribution shifts in time series. 091

Distribution Shift in Time-Series Forecasting. In recent decades, learning under non-stationary distributions, where the target distribution over instances changes with time, has attracted attention within learning theory (Kuh et al., 1990; Bartlett, 1992). In the context of time series, the distribution shift can be categorized into concept drift and temporal shifts.

General concept drift methods (Arjovsky et al., 2019; Ahuja et al., 2021; Krueger et al., 2021;
Pezeshki et al., 2021; Sagawa et al., 2019) assume instances sampled from various environments and
propose to identify and utilize invariant predictors across these environments. However, when applied
to time-series forecasting, these methods encounter limitations. Additional methods specifically
tailored for time series data also encounter certain constraints: DIVERSITY (Lu et al., 2023) is
designed for time series classification and detection only. OneNet (Wen et al., 2024) is tailored
solely for online forecasting scenarios using online ensembling. PeTS (Zhao et al., 2023) focuses on
distribution shifts induced by the specific phenomenon of performativity.

Other works specifically crafted for time-series forecasting aim to address temporal shift issues (Kim et al., 2021; Liu et al., 2022; Fan et al., 2023; Liu et al., 2023). These approaches implement carefully crafted normalization strategies to ensure that both the lookback and horizon of a univariate time series adhere to normalized distributions. This alignment helps alleviate potential temporal shifts, where the statistical properties of the lookback and horizon time series may differ, over time.

¹⁰⁸ 3 PROBLEM FORMULATION

110 3.1 TIME-SERIES FORECASTING

112 Time-series forecasting involves predicting future values of one or more dependent time series based on historical data, potentially augmented with exogenous covariate features. Let denote the 113 target time series as Y and its associated exogenous covariate features as X. At any time step t, 114 time-series forecasting aims to predict $\mathbf{Y}_t^H = [yt + 1, y_{t+2}, \dots, y_{t+H}] \in \mathbf{Y}$ using historical data 115 $(\mathbf{X}_t^L, \mathbf{Y}_t^L)$, where L represents the length of the historical data window, known as the *lookback* 116 window, and H denotes the forecasting time steps, known as the horizon window. Here, $\mathbf{X}_t^L =$ 117 $[x_{t-L+1}, x_{t-L+2}, \dots, x_t] \in \mathbf{X}$ and $\mathbf{Y}_t^L = [y_{t-L+1}, y_{t-L+2}, \dots, y_t] \in \mathbf{Y}$. For simplicity, we denote $\mathbf{Y}_t^H = \{\mathbf{Y}_t^H\}$ for $\forall t$ as the collection of horizon time-series of all time steps, and similar for \mathbf{Y}^L and 118 119 120 \mathbf{X}^{L} . Conventional approaches to time-series forecasting involve learning a model parameterized by θ 121 through empirical risk minimization (ERM) to obtain $f_{\theta} : (\mathbf{X}^L, \mathbf{Y}^L) \to \mathbf{Y}^H$ for all time steps t. 122

In this study, we focus on univariate time-series forecasting with exogenous features, where $d_{\mathbf{Y}} = 1$ and $d_{\mathbf{X}} \ge 1$. Our methodology and this setup can be extended to multivariate time-series forecasting by employing multiple univariate forecastings (Lim & Zohren, 2021; Gruver et al., 2024).

126 127 3.2 DISTRIBUTION SHIFT IN TIME SERIES

Given the time-series forecasting setups, a time-series forecasting model aims to predict the target distribution $P(\mathbf{Y}^{H}) = P(\mathbf{Y}^{H}|\mathbf{Y}^{L})P(\mathbf{Y}^{L}) + P(\mathbf{Y}^{H}|\mathbf{X}^{L})P(\mathbf{X}^{L})$, which should be generalizable for both training and testing time steps. However, due to the dynamic nature of time-series data, forecasting faces challenges from distribution shifts, categorized into two types: temporal shift and concept drift. These two types of distribution shifts are defined as follows:

Definition 3.1 (Temporal Shift (Shimodaira, 2000; Du et al., 2021)) Temporal shift (also known as virtual shift (Tsymbal, 2004)) refers to the marginal probability distributions can change over time, and the conditional distributions are the same.

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Definition 3.2 (Concept Drift (Lu et al., 2018)) Concept drift (also known as real concept drift (Gama et al., 2014)) refers to the conditional distributions can change over time, and the marginal probability distributions are the same.

Intuitively, a temporal shift indicates unstable marginal distributions (e.g. $P(\mathbf{Y}^{H}) \neq P(\mathbf{Y}^{L})$), while a concept drift indicates unstable conditional distributions $(P(\mathbf{Y}_{i}^{H}|\mathbf{X}_{i}^{L}) \neq P(\mathbf{Y}_{j}^{H}|\mathbf{X}_{j}^{L})$ for some $i, j \in t$). Existing methods for distribution shifts in time-series forecasting typically focus on mitigating temporal shifts through normalization, ensuring $P(\mathbf{Y}^{H}) = P(\mathbf{Y}^{L})$ by both normalizing to standard 0-1 distributions (Kim et al., 2021; Liu et al., 2022; Fan et al., 2023).

147 Nevertheless, in addition to temporal shift, time-series forecasting also faces challenges from concept 148 drift: The correlations between **X** and **Y** can change over time, making the conditional distributions 149 $P(\mathbf{Y}^{H}|\mathbf{X}^{L})$ unstable and less predictable. Moreover, \mathbf{X}^{L} may not fully explain or determine \mathbf{Y}^{H} , 150 meaning that modeling the relationship solely through $P(\mathbf{Y}^{H}|\mathbf{X}^{L})$ may fail to capture the true 151 correlations between **X** and **Y**. A demonstration visualizing the differences and relationships 152 between temporal shift and concept drift is provided in Appendix A.

153 While the concept drift issue has received considerable attention in existing studies on general machine 154 learning, applying existing methods to time-series forecasting tasks presents certain challenges. 155 Firstly, these methods typically rely on explicit environment labels as input (e.g., labeled rotation or 156 noisy images in image classification), which are not readily available in time series datasets. Secondly, 157 existing concept drift methods often require leveraging all correlated exogenous features to the target variable (Liu et al., 2024a), which may not be adequately captured in time series datasets (e.g., 158 weather conditions affecting ILI forecasting, but not included in the current ILI dataset). Additionally, 159 while both temporal shift and concept drift can manifest simultaneously in time-series forecasting 160 (e.g., when both $P(\mathbf{Y}^H) \neq P(\mathbf{Y}^L)$ and $P(\mathbf{Y}^H | \mathbf{X}^L)$ are unstable), few existing solutions effectively 161 addresses both issues in the context of time-series forecasting.

¹⁶² 4 METHODOLOGY

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4.1 METHODOLOGY OVERVIEW

The high-level idea of our methodology lies in effectively harnessing information from the horizon window through soft attention masking SAM to mitigate concept drift in time-series forecasting. Moreover, acknowledging the absence of an integrated framework capable of addressing both temporal shift and concept drift within a single solution, we introduce a model-agnostic framework ShifTS tailored to tackle both challenges in time-series forecasting.

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4.2 MITIGATING CONCEPT DRIFT

As defined in Definition 3.2, concept drift in time-series refers to the changing correlations between X and Y over time $(P(\mathbf{Y}_i^H | \mathbf{X}_i^L) \neq P(\mathbf{Y}_j^H | \mathbf{X}_j^L)$ for $i, j \in t$), which introduces instability when when modeling conditional distribution $P(\mathbf{Y}^H | \mathbf{X}^L)$. This instability in time-series forecasting arises from the insufficient information in \mathbf{X}^L to fully determine \mathbf{Y}^H . Conventional concept drift methods necessarily assume that the inputs contain sufficient information to predict the output (Sagawa et al., 2019; Arjovsky et al., 2019), which may not always be valid in this context.

For example, an influenza-like illness (ILI) outbreak can be caused by multiple factors, including either extremely cold winter or hot summer weather (Nielsen et al., 2011; Jaakkola et al., 2014). In such cases, the stable conditional distribution to predict a winter ILI outbreak is $P(\mathbf{Y}^{H} = \text{outbreak} | \mathbf{X}^{L} =$ hot, or $\mathbf{X}^{H} = \text{cold}$). However, without considering \mathbf{X}^{H} , modeling $P(\mathbf{Y}^{H} | \mathbf{X}^{L})$ can become unstable, as \mathbf{X}^{L} alone may not sufficiently determine \mathbf{Y}^{H} . That is, both $P(\mathbf{Y}^{H} = \text{outbreak} | \mathbf{X}^{L} = \text{hot})$ and $P(\mathbf{Y}^{H} = \text{outbreak} | \mathbf{X}^{L} \neq \text{hot})$ are possible, causing unstable conditional distributions over years.

188 To address unstable conditional distributions 189 over time, we propose SAM, which mitigates 190 concept drift by employing a weighted ensemble 191 of multiple conditional distributions across the 192 horizon. The intuition behind SAM is twofold: (1) Given that \mathbf{X}^{L} alone cannot sufficiently determine 193 \mathbf{Y}^{H} , SAM incorporates both lookback and horizon 194 information from exogenous features to improve 195 target prediction. This enables modeling multiple 196 conditional distributions with inputs containing suf-197 ficient information to determine \mathbf{Y}^{H} , specifically 198 $[P(\mathbf{Y}_{t}^{H}|\mathbf{X}_{t}^{L}), P(\mathbf{Y}_{t}^{H}|\mathbf{X}_{t+1}^{L}), \cdots, P(\mathbf{Y}_{t}^{H}|\mathbf{X}_{t+H}^{L})]$ 199 at each time step t. (2) Once sufficient determi-200 nation is achieved through multiple conditional 201 distributions, SAM uses soft attention masking to 202

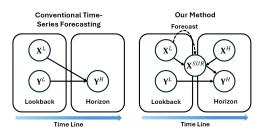


Figure 1: Comparison between conventional time-series forecasting and SAM. SAM aggregates both lookback and horizon information into \mathbf{X}^{SUR} to identify stable aggregated conditional distributions and mitigate concept drift.

identify and aggregate those distributions that remain stable over time. Conditional distributions
 exhibiting variant patterns are learned with lower attention weights during empirical risk minimization
 and can be filtered via sparsity regularization, while those with high attention weights are recognized
 as invariant patterns, which remain unchanged during test time steps. Figure 1 illustrates the
 difference between SAM and conventional time-series forecasting from a causal perspective.

SAM operates through the following steps: First, it concatenates $[\mathbf{X}^{L}, \mathbf{X}^{H}]$ to form an entire time series of length L + H. Second, it slices the entire time series using a sliding window of size H, resulting in L + 1 slices (candidates). Next, it applies a learnable soft attention mask \mathcal{M} to weigh and ensemble all slices, producing the ensembled time series \mathbf{X}^{SUR} , which is the surrogate exogenous time series that sufficiently supports and predicts the target series \mathbf{Y}^{H} . We denote this process as SAM ($[\mathbf{X}^{L}, \mathbf{X}^{H}]$), and can be mathematically described as:

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$$\mathbf{X}^{\text{SUR}} = \text{SAM}([\mathbf{X}^{L}, \mathbf{X}^{H}]) = \sum_{L+1} \mathcal{M}(\text{Slice}([\mathbf{X}^{L}, \mathbf{X}^{H}]))$$
(1)

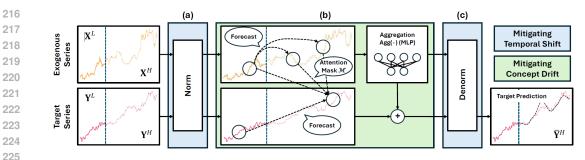


Figure 2: Diagram of ShifTS, consisting of three components: (a) normalization at the start (c) denormalization at the end to address temporal shifts, and (b) a two-stage forecasting process-The first stage predicts surrogate exogenous features, $\hat{\mathbf{X}}^{\text{SUR}}$, identified by the SAM, which capture invariant patterns essential for forecasting the target; The second stage uses both the predicted surrogate exogenous features and the original Y^L to predict Y^H .

where Slice(·) denotes the sliding window process (i.e., slicing the time series $[L + H, d_{\mathbf{X}}] \rightarrow [H, L+1, d_{\mathbf{X}}]$), and $\mathcal{M} \in \mathbb{R}^{L+1 \times d_{\mathbf{X}}}$ is the learnable soft attention mask with sparsity regularization:

Softmax :
$$\mathcal{M}_{j} = \text{Softmax}(\mathcal{M}_{j})$$

Sparsity : $\mathcal{M}_{ij} = \mathcal{M}_{ij} \cdot \mathbb{1}_{(\mathcal{M}_{ij} - \mu(\mathcal{M}_{j})) \ge 0}$
Normalize : $\mathcal{M}_{j} = \frac{\mathcal{M}_{j}}{|\mathcal{M}_{j}|}$
(2)

where *i*, *j* are the first and second dimensions of \mathcal{M} . The intuition behind sparsity regularization is to filter out variant conditional distributions with learned attention weights, leaving only invariant ones, which are to be unchanged during testing. In practice, \mathbf{X}^{SUR} may include horizon information that is unavailable during testing. Therefore, SAM estimates the surrogate features $\hat{\mathbf{X}}^{\text{SUR}}$ with agnostic forecasting models. The surrogate loss that aims to estimate $\hat{\mathbf{X}}^{\text{SUR}}$ is defined as:

$$\mathcal{L}_{SUR} = MSE(\mathbf{X}^{SUR}, \hat{\mathbf{X}}^{SUR})$$
(3)

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4.3 MITIGATING TEMPORAL SHIFT

Mitigating temporal shifts (referred to as 'non-stationary' problems in related literature) has received significant attention in the time-series forecasting community. The core intuition behind popular methods for addressing temporal shifts is to normalize data distributions before processing by the model and to denormalize the outputs afterward. This approach allows the normalized sequences to maintain more consistent mean and variance between the inputs and outputs of the forecasting model, i.e., $P(\mathbf{X}_{\text{Norm}}^L) \approx P(\mathbf{X}_{\text{Norm}}^H) \sim \text{Dist}(0, 1)$ and $P(\mathbf{Y}_{\text{Norm}}^L) \approx P(\mathbf{Y}_{\text{Norm}}^H) \sim \text{Dist}(0, 1)$, thus mitigating temporal shifts (i.e., marginal distribution shifts over time).

257 While the primary contribution of this work focuses on mitigating concept drift in time-series forecasting, addressing temporal shift is also crucial for effectively mitigating concept drift. The 258 underlying intuition is that SAM aims to learn invariant patterns that yield a stable conditional 259 distribution $P(\mathbf{Y}^H | \mathbf{X}^{SUR})$. However, achieving this stability becomes challenging without fixing a 260 stable marginal distribution (e.g., $P(\mathbf{Y}^H)$ or $P(\mathbf{X}^{SUR})$), as these marginal distributions may vary 261 over time. Therefore, a natural solution is to learn the conditional distribution under standardized 262 marginal distributions which is achieved by temporal shift methods through instance normalization 263 techniques. 264

Among the various approaches, Reversible Instance Normalization (RevIN) (Kim et al., 2021) is
particularly notable and is utilized in this work due to its simplicity and effectiveness. Advanced
techniques, such as SAN Liu et al. (2023) and N-S Transformer Liu et al. (2022), also show promise
in mitigating temporal shift but require modifications to forecasting models or pre-training strategies.
Exploring these advanced temporal shift methods remains promising but is beyond the scope of this study.

270 4.4 SHIFTS: THE INTEGRATED FRAMEWORK 271

272 By integrating SAM to mitigate concept drift and RevIN to address temporal shift, we propose 273 ShifTS, a comprehensive framework that addresses both challenges in time-series forecasting. ShifTS is also model-agnostic, as it processes to identify stable conditional distributions, which can 274 be learned by any time-series forecasting model. The workflow of ShifTS is illustrated in Figure 2 275 and consists of the following steps: (1) Normalize the input time series; (2) Forecast exogenous 276 features $\hat{\mathbf{X}}^{\text{SUR}}$ that sufficiently support the target series, as determined by SAM; (3) An aggregation 277 MLP that uses $\hat{\mathbf{X}}^{SUR}$ to forecast the target, denoted as $Agg(\cdot)$ in Figure 2 and Algorithm 1; (4) 278 Denormalize the output time series. Conceptually, steps 1 and 4 mitigate the temporal shift, step 2 279 addresses concept drift, and step 3 performs weighted aggregation of exogenous features to support 280 the target series. The optimization objective of ShifTS is described as follows: 281

$$\mathcal{L} = \mathcal{L}_{SUR}(\mathbf{X}^{SUR}, \hat{\mathbf{X}}^{SUR}) + \mathcal{L}_{TS}(\mathbf{Y}^{H}, \hat{\mathbf{Y}}^{H})$$
(4)

Here, \mathcal{L}_{SUR} is the surrogate loss that encourages learning to forecast exogenous features that sufficiently support the target series, and $\mathcal{L}TS$ is the MSE loss used in conventional time-series forecasting. The pseudo-code for training and testing ShifTS is provided in Algorithm 1.

Algorithm 1 ShifTS

1: Training: Require: Training data \mathbf{X}^L , \mathbf{X}^H , \mathbf{Y}^L , \mathbf{Y}^H ; Initial parameters f_0 , \mathcal{M}_0 , Agg_0 ; **Output:** Model parameter f, \mathcal{M}, Agg

2:	For	i	in	range	(E):
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292	Ζ.	For i in range (E).	
293	3:	Normalization:	$[\mathbf{X}_{\mathrm{Norm}}^{L}, \mathbf{Y}_{\mathrm{Norm}}^{L}] = \mathrm{Norm}([\mathbf{X}^{L}, \mathbf{Y}^{L}])$
294	4:	Time-series forecasting:	$[\mathbf{\hat{X}}_{\text{Norm}}^{\text{SUR}}, \mathbf{\hat{Y}}_{\text{Norm}}^{H}] = f_i([\mathbf{X}_{\text{Norm}}^{L}, \mathbf{Y}_{\text{Norm}}^{L}])$
295	5:	Exogenous feature aggregation:	$\mathbf{\hat{Y}}_{ ext{Norm}}^{H} = \mathbf{\hat{Y}}_{ ext{Norm}}^{H} + ext{Agg}_{i}(\mathbf{\hat{X}}_{ ext{Norm}}^{ ext{SUR}})$
296	6:	Denormalization:	$[\mathbf{\hat{X}}^{\mathrm{SUR}}, \mathbf{\hat{Y}}^{H}] = \mathrm{Denorm}([\mathbf{\hat{X}}^{\mathrm{SUR}}_{\mathrm{Norm}}, \mathbf{\hat{Y}}^{H}_{\mathrm{Norm}}])$
297	7:	Obtain sufficient ex-features:	$\mathbf{X}^{ ext{SUR}} = ext{SAM}([\mathbf{X}^L, \mathbf{X}^H])$
298 299	8:	Compute loss:	$\mathcal{L} = \mathcal{L}_{ ext{SUR}}(\mathbf{X}^{ ext{SUR}}, \mathbf{\hat{X}}^{ ext{SUR}}) + \mathcal{L}_{ ext{TS}}(\mathbf{Y}^{H}, \mathbf{\hat{Y}}^{H})$
299 300	9:	Update model parameter:	$f_{i+1} \leftarrow f_i, \mathcal{M}_{i+1} \leftarrow \mathcal{M}_i, \operatorname{Agg}_{i+1} \leftarrow \operatorname{Agg}_i$
301	10:	Final model parameters: $f \leftarrow f_E$, \mathcal{M}	$l \leftarrow \mathcal{M}_E, \mathrm{Agg} \leftarrow \mathrm{Agg}_E$
302	11.	Testing: Require: Test data \mathbf{X}^L , \mathbf{Y}^L	Output: Forecast target $\mathbf{\hat{v}}^{H}$
303	11:	Testing: Require: Test data A , I	, Output: Forecast target 1
304	12:	Normalization:	$[\mathbf{X}_{\text{Norm}}^{L}, \mathbf{Y}_{\text{Norm}}^{L}] = \text{Norm}([\mathbf{X}^{L}, \mathbf{Y}^{L}])$
305	13:	Time-series forecasting:	$[\mathbf{\hat{X}}_{\text{Norm}}^{\text{SUR}}, \mathbf{\hat{Y}}_{\text{Norm}}^{H}] = f([\mathbf{X}_{\text{Norm}}^{L}, \mathbf{Y}_{\text{Norm}}^{L}])$
306	14.	Exagencies feature aggregation:	$\mathbf{\hat{v}}^{H} = \mathbf{\hat{v}}^{H} + \Lambda_{gg}(\mathbf{\hat{v}}^{SUR})$

$$\begin{split} \hat{\mathbf{Y}}_{\text{Norm}}^{H} &= \hat{\mathbf{Y}}_{\text{Norm}}^{H} + \text{Agg}(\hat{\mathbf{X}}_{\text{Norm}}^{\text{SUR}}) \\ [\hat{\mathbf{X}}^{\text{SUR}}, \hat{\mathbf{Y}}^{H}] &= \text{Denorm}([\hat{\mathbf{X}}_{\text{Norm}}^{\text{SUR}}, \hat{\mathbf{Y}}_{\text{Norm}}^{H}]) \end{split}$$
14: Exogenous feature aggregation: 15: Denormalization:

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5 EXPERIMENTS

Datasets. We conduct experiments using six time-series datasets as leveraged in Liu et al. (2024a): 315 The daily reported currency exchange rates (Exchange) (Lai et al., 2018); The weekly reported 316 influenza-like illness patients (ILI) (Kamarthi et al., 2021); Two-hourly/minutely reported electricity 317 transformer temperature (ETTh1/ETTh2 and ETTm1/ETTm2, respectively) (Zhou et al., 2021). 318 We follow the established experimental setups and target variable selections in previous works(Wu 319 et al., 2021; 2022; Nie et al., 2023; Liu et al., 2024b). Datasets such as Traffic (PeMS) (Zhao et al., 320 2017) and Weather (Wu et al., 2021) are excluded from our evaluations, as their time series exhibit 321 near-stationary behavior, with only moderate distribution shift issues. Further details on the dataset 322 differences are discussed in Appendix B.1. 323

Baselines. We include two types of baselines for comprehensive evaluation on ShifTS:

^{5.1} Setup

324	Μ	lodel	C	rossform	er (ICLR'	23)	1	PatchTST	(ICLR'2	3)	iTransformer (ICLR'24)			
325	M	ethod	EF	ERM ShifTS		EF	RM	ShifTS		ERM		ShifTS		
326	Da	ataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
		24	3.409	1.604	0.674	0.590	0.772	0.634	0.656	0.618	0.824	0.653	0.799	0.642
327		36	4.001	1.772	0.687	0.617	0.763	0.649	0.694	0.602	0.917	0.738	0.690	0.640
328	ILI	48	3.720	1.724	0.652	0.611	0.753	0.692	0.654	0.630	0.772	0.699	0.680	0.665
329		60	3.689	1.715	0.658	0.633	0.761	0.724	0.680	0.656	0.729	0.710	0.672	0.667
		IMP.			81.9%	64.0%			12.0%	7.1%			13.8%	6.5%
330	6	96	0.338	0.475	0.102	0.237	0.130	0.265	0.102	0.236	0.135	0.272	0.115	0.255
331	Exchange	192	0.566	0.622	0.203	0.338	0.247	0.394	0.194	0.332	0.250	0.376	0.209	0.343
332	sha	336	1.078	0.867	0.407	0.484	0.522	0.557	0.388	0.477	0.450	0.503	0.426	0.495
333	XE	720	1.292	0.963	1.165	0.813	1.171	0.824	0.995	0.747	1.501	0.941	1.138	0.827
	щ	IMP.			53.5%	38.9%			20.9%	12.6%			15.2%	6.9%
334		96	0.145	0.312	0.055	0.180	0.064	0.193	0.056	0.181	0.061	0.190	0.056	0.181
335	11	192	0.240	0.420	0.072	0.206	0.085	0.222	0.073	0.209	0.076	0.219	0.072	0.205
336	ETTh1	336	0.240	0.424	0.084	0.228	0.096	0.244	0.089	0.235	0.086	0.227	0.083	0.225
	Ы	720	0.391	0.553	0.095	0.244	0.128	0.282	0.097	0.245	0.085	0.232	0.082	0.230
337		IMP.			68.2%	48.8%			14.5%	7.2%			5.1%	3.3%
338		96	0.255	0.408	0.137	0.286	0.154	0.309	0.139	0.287	0.141	0.292	0.137	0.288
339	5	192	1.257	1.034	0.182	0.338	0.204	0.374	0.191	0.345	0.194	0.347	0.184	0.339
	ETTh2	336	0.783	0.771	0.234	0.388	0.252	0.406	0.222	0.381	0.229	0.383	0.225	0.381
340	Ы	720	1.455	1.100	0.234	0.389	0.259	0.411	0.236	0.390	0.266	0.413	0.235	0.390
341		IMP.			71.4%	52.9%			9.2%	6.5%			5.4%	2.5%
342		96	0.050	0.174	0.028	0.126	0.031	0.135	0.029	0.128	0.030	0.131	0.030	0.131
	nl	192	0.271	0.454	0.043	0.158	0.048	0.166	0.044	0.161	0.049	0.171	0.046	0.165
343	ETTm1	336	0.731	0.805	0.057	0.184	0.058	0.190	0.058	0.186	0.066	0.199	0.059	0.188
344	풘	720	0.829	0.849	0.083	0.219	0.083	0.223	0.080	0.219	0.082	0.219	0.079	0.217
345		IMP.			77.3%	61.0%			4.6%	3.0%			5.1%	2.5%
		96	0.153	0.315	0.069	0.190	0.078	0.206	0.067	0.188	0.073	0.200	0.073	0.195
346	n2	192	0.408	0.526	0.105	0.242	0.113	0.246	0.101	0.237	0.119	0.251	0.108	0.248
347	ETTm2	336	0.428	0.504	0.146	0.289	0.176	0.320	0.134	0.278	0.157	0.302	0.144	0.291
348	Б	720	1.965	1.205	0.191	0.342	0.220	0.368	0.185	0.334	0.196	0.347	0.193	0.344
		IMP.			71.3%	52.0%			15.9%	8.6%			4.8%	2.1%
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Table 1: Performance comparison on forecasting errors without (ERM) and with ShifTS. Employing ShifTS shows consistent performance gains agnostic to forecasting models. The top-performing method is in bold. 'IMP.' denotes the average improvements over all horizons of ShifTS vs ERM.

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356 Forecasting Model Baselines: ShifTS is model-agnostic, we include six time-series forecast-357 ing models (referred to as 'Model' in Table 1 and 4), including: Informer (Zhou et al., 2021), Pyraformer (Liu et al., 2021), Crossformer (Zhang & Yan, 2022), PatchTST (Nie et al., 2023), 358 TimeMixer (Wang et al., 2024) and iTransformer (Liu et al., 2024b), which of the last two are 359 the state-of-the-art (SOTA) forecasting model. These models are used to demonstrate that ShifTS 360 consistently enhances forecasting accuracy across various models, including SOTA. 361

362 Distribution Shift Baselines: We compare ShifTS with various distribution shift methods (referred 363 to as 'Method' in Table 2): (1) Three non-stationary methods for addressing temporal distribution shifts in time-series forecasting N-S Trans. (Liu et al., 2022), RevIN (Kim et al., 2021), and SAN (Liu 364 et al., 2023). We omit **Dish-TS** (Fan et al., 2023) and **SIN** (Han et al., 2024) from the main text due to their instability on univariate targets. (2) Four concept drift methods, including GroupDRO (Sagawa 366 et al., 2019), IRM (Arjovsky et al., 2019), VREx (Krueger et al., 2021), and EIIL (Creager et al., 367 2021), which are primarily designed for general applications. (3) Three combined methods for both 368 temporal distribution shifts and concept drift: IRM+RevIN, EIIL+RevIN, and SOTA time-series 369 distribution shift method FOIL (Liu et al., 2024a). These comparisons aim to highlight the advantages 370 of ShifTS in distribution shift generalization over existing distribution shift approaches. 371

Evaluation. We measure the forecasting errors using mean squared error (MSE) and mean absolute error (MAE). The formula of the metrics are: $MSE = \frac{1}{n} \sum_{i=1}^{n} (y - \hat{y})^2$ and $MSE = \frac{1}{n} \sum_{i=1}^{n} |y - \hat{y}|$. 372 373 The proposed ShifTS does not introduce any additional hyperparameter beyond those inherent in 374 the forecasting models. Therefore, we omit the hyperparameter sensitivity study in our experiments. 375

Reproducibility. All models are trained on NVIDIA Tesla V100 32GB GPUs. All training data and 376 code are anonymously available at: https://anonymous.4open.science/r/shifts_ 377 iclr-56A0. More experiment details are presented in Appendix B.2.

I	Dataset	ILI		Exchange		ETTh1		ETTh2	
Ν	Method			MSE	MAE	MSE	MAE	MSE	MAE
Base	ERM	3.705	1.704	0.819	0.732	0.254	0.427	0.937	0.828
Concept	GroupDRO	2.285	1.287	0.821	0.751	0.278	0.453	1.150	0.936
Concept Drift Method	IRM	2.248	1.237	0.846	0.754	0.201	0.367	0.878	0.792
	VREx	2.285	1.286	0.821	0.742	0.314	0.486	1.142	0.938
Method	EIIL	2.036	1.159	0.822	0.749	0.212	0.433	1.122	0.930
Temporal	RevIN	0.815	0.708	0.475	0.476	0.085	0.224	0.205	0.358
Shift	N-S Trans.	0.781	0.688	0.484	0.481	0.086	0.226	0.203	0.355
Method	SAN	0.757	0.715	0.415	0.453	0.088	0.225	<u>0.199</u>	<u>0.348</u>
	IRM+RevIN	0.809	0.711	0.481	0.476	0.089	0.231	0.202	0.362
Combined	EIIL+RevIN	0.799	0.706	0.483	0.485	0.085	0.225	0.218	0.380
Method	FOIL	<u>0.735</u>	<u>0.651</u>	0.497	0.481	<u>0.081</u>	<u>0.219</u>	0.206	0.357
	ShifTS (Ours)	0.668	0.613	<u>0.470</u>	<u>0.468</u>	0.076	0.214	0.194	0.348

Table 2: Averaged performance comparison between ShifTS and distribution shift baselines with Crossformer. ShifTS achieves the best and second-best performance in 6 and 2 out of 8 evaluations. The best results are highlighted in bold and the second-best results are underlined.

5.2 Performance Improvement across Base Forecasting Models

To showcase the effectiveness of ShifTS in reducing forecasting errors, we conduct experiments to compare performance with and without the inclusion of ShifTS across various time series datasets and forecasting horizons, utilizing five transformer-based forecasting models. Evaluation results for Crossformer, PatchTST, and iTransformer are presented in Table 1. Additional evaluations for older models, including Informer, Pyraformer, and TimeMixer, are provided in Table 4 in Appendix C.1.

401 The results highlight the effectiveness of ShifTS in consistent performance improvements over 402 agnostic forecasting models. articularly remarkable is its ability to consistently enhance performance, 403 even when incorporated with advanced models like iTransformer, yielding reductions of up to 15% in 404 forecasting errors. Moreover, ShifTS demonstrates heightened effectiveness when applied to other 405 non-state-of-the-art forecasting models, such as Informer and PatchTST.

⁴⁰⁶ In addition to the observed performance improvements, our results reveal two further insights:

407 The effectiveness of ShifTS relies on the insights provided by the horizon data. The performance 408 enhancements exhibit variations across different datasets. For instance, the application of ShifTS 409 on ILI and Exchange datasets yields greater performance improvements compared to ETT datasets 410 overall. To interpret the phenomenon and determine the conditions under which ShifTS could 411 be most effective in practical scenarios, we quantify the mutual information $I(\mathbf{X}^{H}; \mathbf{Y}^{H})$ shared 412 between \mathbf{X}^{H} and \mathbf{Y}^{H} (detailed setup provided in Appendix B.2). We plot the relationship between 413 $I(\mathbf{X}^{H}; \mathbf{Y}^{H})$ and performance gains in Figure 3(a). The scatter plot illustrates a positive linear 414 correlation between $I(\mathbf{X}^{H}; \mathbf{Y}^{H})$ and performance gains, supported by a p-value $p = 0.012 \le 0.05$. 415 This observation suggests that the greater the amount of useful information from exogenous features 416 within the horizon window, the more substantial the performance gains achieved by ShifTS. This 417 insight aligns with the innovation of ShifTS, which is to comprehensively exploit and leverage 418 information from the horizon window, which has been overlooked by existing methodologies. 419

The extent of quantitative performance gains achieved by ShifTS depends on the underlying 420 forecasting model. Notably, the extent of performance enhancements achieved by ShifTS varies 421 across different forecasting models. For example, the performance gains on the simpler Informer 422 model by ShifTS is more significant than the SOTA iTransformer model. Importantly, we emphasize 423 two key observations: Firstly, even when applied to the iTransformer model, ShifTS demonstrates 424 a notable performance boost of approximately 15% on both ILI and Exchange datasets, consistent 425 with the aforehead intuition. Secondly, integrating ShifTS into forecasting processes should, at the 426 very least, maintain or improve the performance of standalone forecasting models, as evidenced by 427 consistent performance enhancements observed across all datasets with iTransformer model.

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5.3 COMPARISON WITH DISTRIBUTION SHIFT METHODS

To illustrate the advantages of ShifTS over other model-agnostic methods for addressing distribution shifts, we conduct experiments to compare performance across distribution shift baselines

432 following Liu et al. (2024a), where the evaluations on minutely ETT datasets were omitted, as their 433 data characteristics and forecasting quality generally align with those of hourly ETT datasets. We use 434 Crossformer as the forecasting model. The averaged results are summarized in Table 2.

435 The results highlight the advantages of 436 ShifTS over existing distribution shift 437 methods, achieving the highest average 438 forecasting accuracy in 6 out of 8 evalua-439 tions, with the remaining 2 evaluations 440 ranking second. Notably, as discussed 441 in Section 4.3, ShifTS is flexible in in-442 tegrating other advanced temporal shift methods to enhance performance. For in-443 stance, in the Exchange dataset, where 444 SAN outperforms ShifTS, ShifTS can 445

further improve its accuracy by incorpo-

Horizon	ShifTS	SAN	ShifTS+SAN
96	0.102	0.091	0.089
192	0.207	0.195	0.187
336	0.407	0.373	0.372
720	1.165	1.001	0.981
Avg.	0.470	0.415	0.407

Table 3: MSE comparison between ShifTS, SAN, and ShifTS+SAN on Exchange dataset. ShifTS+SAN achieves the best performance on all evaluations.

446 rating SAN in place of RevIN. Detailed MSE values are provided in Table 3. Additionally, the results 447 illustrate the further benefits of addressing concept drift using SAM when temporal shift is effectively 448 managed. 449

5.4 ABLATION STUDY

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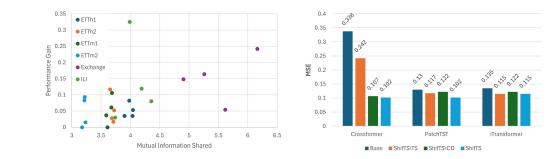
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To demonstrate the effectiveness of each module in ShifTS, we conducted an ablation study using 453 two modified versions: ShifTS\TS and ShifTS\CD. ShifTS\TS excludes the temporal shift adjustment via RevIN, while ShifTS\CD excludes the concept drift handling via SAM. Additionally, 455 conventional forecasting models that do not address either concept drift or temporal shift are denoted 456 as 'Base'. We performed experiments on the Exchange datasets using previous three baseline forecasting models, with a fixed forecasting horizon of 96. The results are visualized in Figure 3(b). The visualization reveals the following observations: 458



469 Figure 3: Left (a): The performance gains of ShifTS versus the mutual information shared between 470 \mathbf{X}^{H} and \mathbf{Y}^{H} . Greater mutual information in \mathbf{X}^{H} compared to \mathbf{Y}^{H} correlates with more significant 471 performance gains achieved by ShifTS. Right (b): Ablation Study. Addressing either concept drift 472 or temporal shift individually provides certain benefits in reducing forecasting error, but ShifTS, 473 which tackles both, achieves the lowest forecasting error.

475 First, addressing both temporal shift and concept drift together, as implemented in ShifTS, yields 476 lower forecasting errors than addressing only one type of distribution shift (ShifTS\TS and ShifTS/CD) or not considering any distribution shift adjustments (Base). This suggests that 477 temporal shift and concept drift are likely interrelated and co-existed in time series data, and address-478 ing both provides significant benefits. 479

480 Second, for forecasting models that inherently address temporal shift, such as PatchTST and iTrans-481 former that incorporate norm/denorm, the performance gains from mitigating concept drift are more 482 significant than those from additionally mitigating temporal shift using RevIN. In contrast, for models 483 without any temporal shift mitigation, such as Crossformer, tackling temporal shift leads to a greater performance improvement than addressing concept drift. This distinction highlights the coexistence of 484 both concept drift and temporal shift in time-series forecasting tasks. While handling temporal shifts 485 is a fundamental challenge that has already received considerable attention, once resolved, mitigating

concept drift—an issue largely overlooked in current research and a unique key contribution of our work—can lead to promising improvements in forecasting accuracy.

6 CONCLUSION

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491 In this paper, we identify that both concept drift and temporal shift issues can coexist in time series 492 forecasting. While mitigating temporal shifts has received significant attention from the time-series 493 forecasting community, concept drift issues have been largely neglected. To address this gap, we first 494 propose a soft attention mechanism, SAM, which effectively mitigates concept drift in time-series 495 forecasting by incorporating horizon information of exogenous features to enhance generalization 496 ability. We then introduce ShifTS, a model-agnostic framework that tackles both concept drift and 497 temporal shift issues. Our comprehensive evaluations demonstrate the effectiveness of ShifTS, and 498 the benefit of SAM is further illustrated through an ablation study.

References

- Kartik Ahuja, Ethan Caballero, Dinghuai Zhang, Jean-Christophe Gagnon-Audet, Yoshua Bengio,
 Ioannis Mitliagkas, and Irina Rish. Invariance principle meets information bottleneck for out-of distribution generalization. *Advances in Neural Information Processing Systems*, 34:3438–3450,
 2021.
- Martin Arjovsky, Léon Bottou, Ishaan Gulrajani, and David Lopez-Paz. Invariant risk minimization.
 arXiv preprint arXiv:1907.02893, 2019.
- Peter L Bartlett. Learning with a slowly changing distribution. In *Proceedings of the fifth annual workshop on Computational learning theory*, pp. 243–252, 1992.
- Elliot Creager, Jörn-Henrik Jacobsen, and Richard Zemel. Environment inference for invariant
 learning. In *International Conference on Machine Learning*, pp. 2189–2200. PMLR, 2021.
- Chirag Deb, Fan Zhang, Junjing Yang, Siew Eang Lee, and Kwok Wei Shah. A review on time series forecasting techniques for building energy consumption. *Renewable and Sustainable Energy Reviews*, 74:902–924, 2017.
- Yuntao Du, Jindong Wang, Wenjie Feng, Sinno Pan, Tao Qin, Renjun Xu, and Chongjun Wang.
 Adarnn: Adaptive learning and forecasting of time series. In *Proceedings of the 30th ACM international conference on information & knowledge management*, pp. 402–411, 2021.
- Wei Fan, Pengyang Wang, Dongkun Wang, Dongjie Wang, Yuanchun Zhou, and Yanjie Fu. Dish-ts:
 a general paradigm for alleviating distribution shift in time series forecasting. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 37, pp. 7522–7529, 2023.
 - João Gama, Indrė Žliobaitė, Albert Bifet, Mykola Pechenizkiy, and Abdelhamid Bouchachia. A survey on concept drift adaptation. *ACM computing surveys (CSUR)*, 46(4):1–37, 2014.
- 526 Clive WJ Granger. Time series concepts for conditional distributions. *Oxford Bulletin of Economics and Statistics*, 65:689–701, 2003.
- Nate Gruver, Marc Finzi, Shikai Qiu, and Andrew G Wilson. Large language models are zero-shot time series forecasters. *Advances in Neural Information Processing Systems*, 36, 2024.
- Husheng Guo, Shuai Zhang, and Wenjian Wang. Selective ensemble-based online adaptive deep neural networks for streaming data with concept drift. *Neural Networks*, 142:437–456, 2021.
- Lu Han, Han-Jia Ye, and De-Chuan Zhan. Sin: Selective and interpretable normalization for long-term
 time series forecasting. In *Forty-first International Conference on Machine Learning*, 2024.
- Rob J Hyndman and George Athanasopoulos. *Forecasting: principles and practice*. OTexts, 2018.
- Kari Jaakkola, Annika Saukkoriipi, Jari Jokelainen, Raija Juvonen, Jaana Kauppila, Olli Vainio, Thedi Ziegler, Esa Rönkkö, Jouni JK Jaakkola, Tiina M Ikäheimo, et al. Decline in temperature and humidity increases the occurrence of influenza in cold climate. *Environmental Health*, 13:1–8, 2014.

540 541 542	Harshavardhan Kamarthi, Lingkai Kong, Alexander Rodriguez, Chao Zhang, and B Aditya Prakash. When in doubt: Neural non-parametric uncertainty quantification for epidemic forecasting. <i>Advances in Neural Information Processing Systems</i> , 34:19796–19807, 2021.
543 544 545 546	Taesung Kim, Jinhee Kim, Yunwon Tae, Cheonbok Park, Jang-Ho Choi, and Jaegul Choo. Reversible instance normalization for accurate time-series forecasting against distribution shift. In <i>International Conference on Learning Representations</i> , 2021.
547 548 549	David Krueger, Ethan Caballero, Joern-Henrik Jacobsen, Amy Zhang, Jonathan Binas, Dinghuai Zhang, Remi Le Priol, and Aaron Courville. Out-of-distribution generalization via risk extrapolation (rex). In <i>International Conference on Machine Learning</i> , pp. 5815–5826. PMLR, 2021.
550 551 552	Anthony Kuh, Thomas Petsche, and Ronald Rivest. Learning time-varying concepts. Advances in neural information processing systems, 3, 1990.
553 554 555	Guokun Lai, Wei-Cheng Chang, Yiming Yang, and Hanxiao Liu. Modeling long-and short-term temporal patterns with deep neural networks. In <i>The 41st international ACM SIGIR conference on research & development in information retrieval</i> , pp. 95–104, 2018.
556 557 558	Bryan Lim and Stefan Zohren. Time-series forecasting with deep learning: a survey. <i>Philosophical Transactions of the Royal Society A</i> , 379(2194):20200209, 2021.
559 560 561	Haoxin Liu, Harshavardhan Kamarthi, Lingkai Kong, Zhiyuan Zhao, Chao Zhang, and B Aditya Prakash. Time-series forecasting for out-of-distribution generalization using invariant learning. <i>Forty-first International Conference on Machine Learning</i> , 2024a.
562 563 564	Shizhan Liu, Hang Yu, Cong Liao, Jianguo Li, Weiyao Lin, Alex X Liu, and Schahram Dust- dar. Pyraformer: Low-complexity pyramidal attention for long-range time series modeling and forecasting. In <i>International conference on learning representations</i> , 2021.
565 566 567 568	Yong Liu, Haixu Wu, Jianmin Wang, and Mingsheng Long. Non-stationary transformers: Exploring the stationarity in time series forecasting. <i>Advances in Neural Information Processing Systems</i> , 35: 9881–9893, 2022.
569 570 571	Yong Liu, Tengge Hu, Haoran Zhang, Haixu Wu, Shiyu Wang, Lintao Ma, and Mingsheng Long. itransformer: Inverted transformers are effective for time series forecasting. In <i>International Conference on Learning Representations (ICLR)</i> , 2024b.
572 573 574 575	Zhiding Liu, Mingyue Cheng, Zhi Li, Zhenya Huang, Qi Liu, Yanhu Xie, and Enhong Chen. Adaptive normalization for non-stationary time series forecasting: A temporal slice perspective. In <i>Thirty-seventh Conference on Neural Information Processing Systems</i> , 2023.
576 577	Jie Lu, Anjin Liu, Fan Dong, Feng Gu, Joao Gama, and Guangquan Zhang. Learning under concept drift: A review. <i>IEEE transactions on knowledge and data engineering</i> , 31(12):2346–2363, 2018.
578 579 580 581	Wang Lu, Jindong Wang, Xinwei Sun, Yiqiang Chen, and Xing Xie. Out-of-distribution representation learning for time series classification. In <i>International Conference on Learning Representations</i> , 2023.
582 583 584 585	Sarabeth M Mathis, Alexander E Webber, Tomás M León, Erin L Murray, Monica Sun, Lauren A White, Logan C Brooks, Alden Green, Addison J Hu, Roni Rosenfeld, et al. Title evaluation of flusight influenza forecasting in the 2021–22 and 2022–23 seasons with a new target laboratory-confirmed influenza hospitalizations. <i>Nature Communications</i> , 15(1):6289, 2024.
586 587	Elizbar A Nadaraya. On estimating regression. <i>Theory of Probability & Its Applications</i> , 9(1): 141–142, 1964.
588 589 590 591	Yuqi Nie, Nam H. Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. A time series is worth 64 words: Long-term forecasting with transformers. In <i>International Conference on Learning Representations</i> , 2023.
592 593	Jens Nielsen, Anne Mazick, Steffen Glismann, and Kåre Mølbak. Excess mortality related to seasonal influenza and extreme temperatures in denmark, 1994-2010. <i>BMC infectious diseases</i> , 11:1–13, 2011.

594 595 596 597	Boris N. Oreshkin, Dmitri Carpov, Nicolas Chapados, and Yoshua Bengio. N-BEATS: Neural basis expansion analysis for interpretable time series forecasting. In <i>International Conference on Learning Representations</i> , 2020. URL https://openreview.net/forum?id=rlecqn4YwB.
598 599 600	Mohammad Pezeshki, Oumar Kaba, Yoshua Bengio, Aaron C Courville, Doina Precup, and Guil- laume Lajoie. Gradient starvation: A learning proclivity in neural networks. <i>Advances in Neural</i> <i>Information Processing Systems</i> , 34:1256–1272, 2021.
601 602 603 604	Shiori Sagawa, Pang Wei Koh, Tatsunori B Hashimoto, and Percy Liang. Distributionally robust neural networks for group shifts: On the importance of regularization for worst-case generalization. <i>arXiv preprint arXiv:1911.08731</i> , 2019.
605 606 607	David Salinas, Valentin Flunkert, Jan Gasthaus, and Tim Januschowski. Deepar: Probabilistic forecasting with autoregressive recurrent networks. <i>International Journal of Forecasting</i> , 36(3): 1181–1191, 2020.
608 609 610	Alex Sherstinsky. Fundamentals of recurrent neural network (rnn) and long short-term memory (lstm) network. <i>Physica D: Nonlinear Phenomena</i> , 404:132306, 2020.
611 612	Hidetoshi Shimodaira. Improving predictive inference under covariate shift by weighting the log- likelihood function. <i>Journal of statistical planning and inference</i> , 90(2):227–244, 2000.
613 614 615	Alex J Smola and Bernhard Schölkopf. A tutorial on support vector regression. <i>Statistics and computing</i> , 14(3):199–222, 2004.
616 617 618	José F Torres, Dalil Hadjout, Abderrazak Sebaa, Francisco Martínez-Álvarez, and Alicia Troncoso. Deep learning for time series forecasting: a survey. <i>Big Data</i> , 9(1):3–21, 2021.
619 620	Alexey Tsymbal. The problem of concept drift: definitions and related work. <i>Computer Science Department, Trinity College Dublin,</i> 106(2):58, 2004.
621 622 623 624	Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. <i>Advances in neural information processing systems</i> , 30, 2017.
625 626 627	Shiyu Wang, Haixu Wu, Xiaoming Shi, Tengge Hu, Huakun Luo, Lintao Ma, James Y Zhang, and JUN ZHOU. Timemixer: Decomposable multiscale mixing for time series forecasting. In <i>International Conference on Learning Representations (ICLR)</i> , 2024.
628 629 630	Qingsong Wen, Weiqi Chen, Liang Sun, Zhang Zhang, Liang Wang, Rong Jin, Tieniu Tan, et al. Onenet: Enhancing time series forecasting models under concept drift by online ensembling. <i>Advances in Neural Information Processing Systems</i> , 36, 2024.
631 632 633	Christopher Williams and Carl Rasmussen. Gaussian processes for regression. Advances in neural information processing systems, 8, 1995.
634 635 636 637	Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with auto-correlation for long-term series forecasting. <i>Advances in neural information processing systems</i> , 34:22419–22430, 2021.
638 639 640	Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. In <i>The eleventh international</i> <i>conference on learning representations</i> , 2022.
641 642 643 644	Xinyu Zhang, Shanshan Feng, Jianghong Ma, Huiwei Lin, Xutao Li, Yunming Ye, Fan Li, and Yew Soon Ong. Frnet: Frequency-based rotation network for long-term time series forecasting. In <i>Proceedings of the 30th ACM SIGKDD Conference on Knowledge Discovery and Data Mining</i> , pp. 3586–3597, 2024.
645 646 647	Yunhao Zhang and Junchi Yan. Crossformer: Transformer utilizing cross-dimension dependency for multivariate time series forecasting. In <i>The eleventh international conference on learning</i>

representations, 2022.

- 648 Zheng Zhao, Weihai Chen, Xingming Wu, Peter CY Chen, and Jingmeng Liu. Lstm network: a deep 649 learning approach for short-term traffic forecast. IET intelligent transport systems, 11(2):68–75, 650 2017.
 - Zhiyuan Zhao, Alexander Rodriguez, and B Aditya Prakash. Performative time-series forecasting. arXiv preprint arXiv:2310.06077, 2023.
 - Yu Zheng, Licia Capra, Ouri Wolfson, and Hai Yang. Urban computing: concepts, methodologies, and applications. ACM Transactions on Intelligent Systems and Technology (TIST), 5(3):1-55, 2014.
- Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. 659 Informer: Beyond efficient transformer for long sequence time-series forecasting. In Proceedings of the AAAI conference on artificial intelligence, volume 35, pp. 11106–11115, 2021.
 - Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. FEDformer: Frequency enhanced decomposed transformer for long-term series forecasting. In Proc. 39th International Conference on Machine Learning (ICML 2022), 2022.
 - Yunyue Zhu and Dennis Shasha. Statstream: Statistical monitoring of thousands of data streams in real time. In VLDB'02: Proceedings of the 28th International Conference on Very Large Databases, pp. 358-369. Elsevier, 2002.
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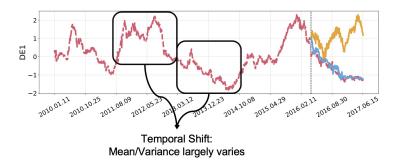
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APPENDIX A: TEMPORAL SHIFT AND CONCEPT DRIFT А

To highlight the differences between concept drift and temporal shift, we provide visualizations of 674 both phenomena. Figure 4 illustrates temporal shift, while Figure 5 demonstrates concept drift¹. 675

676 Temporal shift refers to changes in the statistical properties of a univariate time series data, such as mean, variance, and autocorrelation structures, over time. For instance, the mean and variance of the 677 given time series shift between lookback window and horizon window, as depicted in Figure 4. This 678 issue is inherent in time series forecasting and can occur on any given time series data, regardless of 679 whether the data pertains to the target series or exogenous features. 680

681 In contrast, concept drift describes to changes in the correlations between exogenous features and the 682 target series over time. Figure 5 illustrates this phenomenon, where increases in exogenous features at earlier time steps lead to increases in the target series, while increases at later time steps result in 683 decreases. Unlike temporal shift, concept drift involves multiple correlated time series and is not an 684 inherent issue in univariate time series analysis. 685



697 Figure 4: Demonstration of temporal shift phenomenon within time series data, showcasing the variations in statistical properties, including mean and variance, over time as the emergence of temporal shift (Red: ground truth; Yellow: N-BEATS prediction; Blue: N-BEATS+RevIN prediction). 699 700

¹Figures adapted from: https://github.com/ts-kim/RevIN

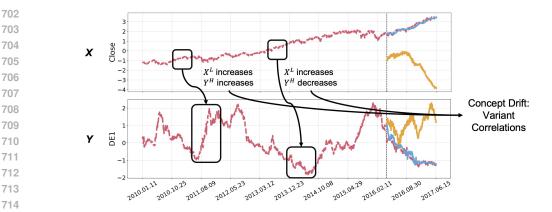


Figure 5: Demonstration of concept drift phenomenon within time series data, showcasing the variations in correlation structures between arget series **Y** and exogenous feature **X** over time as the emergence of concept drift (**Red:** ground truth; **Yellow:** N-BEATS prediction; **Blue:** N-BEATS+RevIN prediction).

B APPENDIX **B**: ADDITIONAL EXPERIMENT DETAILS

B.1 DATASETS

We conduct experiments on six real-world datasets, which are commonly used as benchmark datasets:

- ILI. The ILI dataset collects data on influenza-like illness patients weekly, with eight variables.
- Exchange. The Exchange dataset records the daily exchange rate of eight currencies.
- ETT. The ETT dataset contains four sub-datasets: ETTh1, ETTh2, ETTm1, ETTm2. The datasets record electricity transformer temperatures from two separate counties in China (distinguished by '1' and '2'), with two granularities: minutely and hourly (distinguished by 'm' and 'h'). All sub-datasets have seven variables/features.

We follow Wu et al. (2022); Nie et al. (2023); Liu et al. (2024b) to preprocess data, which guides
 splitting datasets into train/validation/test sets and selecting the target variables. All datasets are
 preprocessed using the zero-mean normalization method.

Additional popular time-series datasets, such as Traffic (which records road occupancy rates from various sensors on San Francisco freeways), Electricity (which tracks hourly electricity consumption for 321 customers), and Weather (which collects 21 meteorological indicators in Germany, such as humidity and air temperature), are omitted from our evaluations. These datasets exhibit strong periodic signals and display near-stationary properties, making distribution shift issues less prevalent. A visualization comparison between the ETTh1 and Traffic datasets, shown in Figure 6, further supports this observation.

B.2 BASELINE IMPLEMENTATION

We follow the commonly adopted setup for defining the forecasting horizon window length, as
outlined in prior works Wu et al. (2022); Nie et al. (2023); Liu et al. (2024b). Specifically, for datasets
such as ETT and Exchange, the forecasting horizon windows are chosen from the set [96, 192, 336,
720], with a fixed lookback window size of 96 and a consistent label window size of 48 for the
decoder (if required). Similarly, for the weekly reported ILI dataset, we employ forecasting horizon
windows from [24, 36, 48, 60], with a fixed lookback window size of 36 and a constant label window
size of 18 for the decoder (if required).

In the context of concept drift baselines, several baselines like GroupDRO, IRM, and VREx require
 environment labels, which are typically absent in time series datasets. To address this, we partition
 the training set into k equal-length time segments to serve as predefined environment labels.

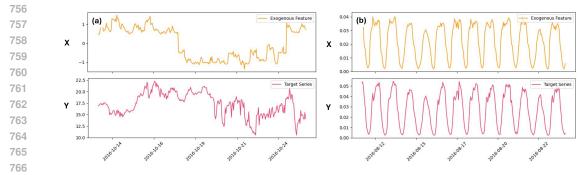


Figure 6: Distribution shift issues across datasets: Left (a): ETT. Both temporal shift and concept drift are present. The target series shows varying statistics over time (e.g., lower variance in earlier periods and higher variance later), causing temporal shift. The correlation between X and Y is unclear and unstable, causing concept drift. Right (b): Traffic. Both temporal shift and concept drift are moderate. The target series exhibits near-periodicity, making the temporal shift moderate. Moreover, the correlation between X and Y remains stable (e.g., both increase or decrease simultaneously), making concept drift moderate.

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For baseline time-series forecasting models, we follow implementations and suggested hyperparameters (with additional tuning) sourced from the Time Series Library². For concept drift baselines, we utilize implementations and hyperparameter tuning strategies recommended by DomainBed³.
For temporal shift baselines, we adopt implementations and hyperparameter configurations outlined in their respective papers. Additionally, we add an additional MLP layer to the end PatchTST to effectively utilize exogenous features, following Liu et al. (2024a).

In the ablation study, for the implementation of PatchTST and iTransformer, we follow the original approach by applying norm and denorm operations to the 'Base' model. To clarify our notation, ShifTS\TS refers to the model with standard norm/denorm operations and SAM, while ShifTS\CD denotes the version where the regular norm/denorm is replaced with RevIN.

786 B.3 MUTUAL INFORMATION VISUALIZATION

For a given time series dataset, we compute the mutual information $I(\mathbf{X}^{H}; \mathbf{Y}^{H})$ for each training time step and each exogenous feature dimension individually, following:

$$I(\mathbf{X}^{H}; \mathbf{Y}^{H}) = \sum_{x \in \mathbf{X}^{H}} \sum_{y \in \mathbf{Y}^{H}} P(x, y) \log \frac{P(x, y)}{P(x)P(y)}$$
(5)

We then average the mutual information across all time steps for each exogenous feature dimension
and identify the maximum averaged mutual information over all feature dimensions. This process
allows us to assess the information content of each feature dimension in relation to the target series.

We visualize the maximum averaged mutual information plotted against the corresponding performance gain in Figure 3(a). This visualization provides insights into how the information content of different feature dimensions relates to the performance improvement achieved in the forecasting model.

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C APPENDIX C: ADDITIONAL RESULTS

C.1 EVALUATIONS ON AGNOSTIC PERFORMANCE GAINS

To further demonstrate the benefit of ShifTS in improving the forecasting accuracy over agnostic forecasting models, we additionally evaluate the performance differences without and with ShifTS on Informer, Pyraformer, and TimeMixer. The detailed results are presented in Table 4. The additional

²https://github.com/thuml/Time-Series-Library

³https://github.com/facebookresearch/DomainBed

810	М	lodel]	Informer	(AAAI'2	1)	I	Pyraforme	r (ICLR'2	.1)	TimeMixer (ICLR'24)			
811	M	ethod		RM		fTS		ERM Shif		fTS	IS ERM			fTS
812	Da	ataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
		24	5.032	1.935	1.030	0.812	4.692	1.898	0.979	0.749	0.853	0.733	0.789	0.702
813		36	4.475	1.876	1.046	0.850	4.814	1.950	0.866	0.740	0.721	0.676	0.697	0.665
814	ILI	48	4.506	1.879	0.918	0.818	4.109	1.801	0.789	0.732	0.737	0.692	0.741	0.711
815		60	4.313	1.850	0.957	0.839	4.483	1.850	0.723	0.698	0.788	0.723	0.670	0.659
		IMP.			78.4%	56.0%			81.5%	61.1%			6.3%	3.0%
816	a	96	0.839	0.746	0.137	0.277	0.410	0.525	0.145	0.275	0.127	0.268	0.098	0.234
817	Exchange	192	0.862	0.773	0.210	0.346	0.529	0.610	0.300	0.404	0.229	0.355	0.214	0.352
	sha	336	1.597	1.063	0.378	0.485	0.851	0.778	0.440	0.506	0.553	0.560	0.440	0.491
818	EXC	720	4.358	1.935	0.760	0.655	1.558	1.067	1.509	0.963	1.173	0.834	0.962	0.747
819		IMP.			79.5%	59.7%			39.8%	31.5%			16.9%	9.1%
820		96	0.891	0.863	0.095	0.231	0.653	0.748	0.065	0.197	0.059	0.184	0.059	0.187
	hl	192	1.027	0.958	0.096	0.237	0.853	0.828	0.075	0.210	0.099	0.247	0.077	0.211
821	ETTh1	336	1.055	0.961	0.092	0.237	0.705	0.797	0.092	0.238	0.121	0.279	0.098	0.246
822	Ы	720	1.077	0.969	0.100	0.252	0.562	0.695	0.126	0.279	0.139	0.299	0.099	0.252
823		IMP.			90.7%	74.5%			86.4%	69.6%			23.3%	10.1%
		96	3.195	1.651	0.232	0.381	1.598	1.127	0.156	0.307	0.152	0.303	0.146	0.299
824	ETTh2	192	3.569	1.778	0.334	0.464	3.314	1.599	0.217	0.367	0.195	0.349	0.185	0.343
825	E	336	2.556	1.468	0.400	0.512	2.571	1.489	0.245	0.398	0.238	0.392	0.230	0.381
	Ц	720	2.723	1.532	0.489	0.579	2.294	1.409	0.261	0.410	0.273	0.421	0.249	0.397
826		IMP.			82.0%	69.5%			90.6%	73.5%			5.3%	2.9%
827		96	0.320	0.433	0.055	0.175	0.130	0.298	0.028	0.125	0.030	0.128	0.029	0.126
828	m1	192	0.459	0.582	0.079	0.211	0.240	0.4112	0.045	0.162	0.047	0.165	0.047	0.164
	ETTm1	336	0.457	0.556	0.104	0.243	0.359	0.512	0.062	0.192	0.063	0.191	0.060	0.189
829	E	720	0.735	0.760	0.148	0.294	0.657	0.750	0.091	0.231	0.083	0.223	0.081	0.220
830		IMP.			80.7%	60.3%			82.2%	62.6%			2.3%	1.1%
831		96	0.191	0.345	0.154	0.298	0.275	0.422	0.075	0.200	0.079	0.205	0.075	0.201
	m2	192	0.458	0.556	0.243	0.378	0.484	0.552	0.107	0.248	0.121	0.259	0.111	0.250
832	ETTm2	336	0.606	0.624	0.515	0.539	1.138	0.909	0.146	0.293	0.150	0.295	0.148	0.294
833	Ы	720	1.175	0.879	0.564	0.592	2.920	1.537	0.196	0.347	0.246	0.387	0.198	0.346
834		IMP.			33.4%	23.0%			82.8%	63.2%			8.5%	4.1%
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Table 4: Performance comparison on forecasting errors without (ERM) and with ShifTS on Informer, Pyraformer, and TimeMixer. Employing ShifTS again shows near-consistent performance gains agnostic to forecasting models. The top-performing method is in bold. 'IMP.' denotes the average improvements over all horizons of ShifTS vs ERM.

evaluations again show consistent performance improvements on these models. Moreover, compared to the results in Table 1, the performance gains on these older models are even more significant. This observation highlight the needs of mitigating both concept drift and temporal shift in timeseries forecasting, as such problem are rarely considered in these models, but the later models (e.g., PatchTST and iTransformer are compounded with normalizaiton/denormalizaiton processes).

D LIMITATION DISCUSSION

This work introduces SAM to address concept drift and proposes an integrated framework, ShifTS, which combines SAM with temporal shift mitigation techniques to enhance the accuracy of time-series forecasting. Extensive empirical evaluations support the effectiveness of these methods. However, the limitations of this study lie in two aspects: First, the distribution shift methods in time-series forecasting, including ShifTS, lack a theoretical guarantee. For example, no analysis quantifies how much the error bound can be tightened by addressing concept drift or temporal shift compared to vanilla time-series forecasting methods. Second, while this paper defines concept drift and temporal shift issues within the context of time-series forecasting, SAM and ShifTS are not the only possible solutions. Exploring alternative approaches remains an avenue for future research beyond the scope of this work. These two limitations highlight opportunities for future investigation.