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Bayesian spatiotemporal modelling of wildfire occurrences and sizes for projections under climate change

A step-by-step marked point process approach using INLA-SPDE

Juliette Legrand¹ Biostatistics and Spatial Processes, INRAE, Avignon, France
François Pimont Ecologie des Forêts Méditerranéennes (URFM), INRAE, Avignon, France
Jean-Luc Dupuy Ecologie des Forêts Méditerranéennes (URFM), INRAE, Avignon, France
Thomas Opitz Biostatistics and Spatial Processes, INRAE, Avignon, France

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Abstract

Appropriate spatiotemporal modelling of wildfire activity is crucial for its prediction and risk management. Here, we focus on wildfire risk in the Aquitaine region in the Southwest of France and its projection under climate change. We study whether wildfire risk could further increase under climate change in this specific region, which does not lie in the historical core area of wildfires in Southeastern France, corresponding to the Southeast. For this purpose, we consider a marked spatiotemporal point process, a flexible model for occurrences and magnitudes of such environmental risks, where the magnitudes are defined as the burnt areas. The model is first calibrated using 14 years of past observation data of wildfire occurrences and weather variables, and then applied for projection of climate-change impacts using simulations of numerical climate models until 2100 as new inputs. We work within the framework of a spatiotemporal Bayesian hierarchical model, and we present the workflow of its implementation for a large dataset at daily resolution for 8km-pixels using the INLA-SPDE approach. The assessment of the posterior distributions shows a satisfactory fit of the model for the observation period. We stochastically simulate projections of future wildfire activity by combining climate model output with posterior simulations of model parameters. Depending on climate models, spline-smoothed projections indicate low to moderate increase of wildfire activity under climate change. The increase is weaker than in the historical core area, which we attribute to different weather conditions (oceanic versus Mediterranean). Besides providing a relevant case study of environmental risk modelling, this paper is also intended to provide a full workflow for implementing the Bayesian estimation of marked log-Gaussian Cox processes using the R-INLA package of the R statistical software.

Keywords: Climate projection, Integrated Nested Laplace Approximation, Marked log-Gaussian Cox process, Spatiotemporal model, SPDE approach, Wildfire

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¹Corresponding author: juliette.legrand@univ-brest.fr

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22 **1 Introduction**

23 As recently experienced in France during summer 2022, wildfires constitute a major threat for the
24 environment and the society. Climate change leads to an increase in societal and economic risks
25 related to wildfires (Riviere et al. 2022), and moreover to an increase in fire-prone regions worldwide
26 (Abatzoglou, Williams, and Barbero 2019). In France, the Southeast historical region (where wildfire
27 occurrence data have been systematically collected in the “Prométhée” database since the 1970s) has
28 received a lot of attention in recent years due to the large amount of burnt areas (e.g. Pimont et al.
29 2021; Opitz, Bonneau, and Gabriel 2020). In this study, we consider another French region that draws
30 attention to its wildfire activity and its potential increase under climate change, the Aquitaine region
31 in Southwest France. To better understand the processes driving wildfire activity in this specific
32 region, in particular its sensitivity to weather conditions, and to measure its potential evolution under
33 climate-change, we seek to stochastically model and describe wildfire activities. Statistical modelling
34 of wildfire activity is challenging because occurrence intensities and sizes of wildfires (corresponding
35 to the area burnt by a wildfire) vary in space and time according to meteorological conditions, land
36 cover and human activities, and these predictors may act differently on the probabilities of the
37 ignition of fires (occurrence) and their propagation after ignition has taken place (size).

38 More specifically, each wildfire can be characterized in space and time, for instance by its location \mathbf{s}
39 and time t of ignition where $\{\mathbf{s} \in \mathcal{S}, t \in \mathcal{T}\}$ would correspond to the space-time study domain. Doing
40 so, wildfire occurrences can be seen as the realization of a spatiotemporal point process, a stochastic
41 model for the occurrences of space-time events. Moreover, each point of this spatiotemporal point
42 process can be associated with a numerical information, such as the burnt area of the corresponding
43 fire. Adding this random quantitative mark, the spatiotemporal pattern of wildfire occurrences and

44 sizes can be viewed as a marked spatiotemporal point process. That is, there exists a random measure
 45 N that counts the number of points in Borel sets $B \subset \mathcal{S} \times \mathcal{T}$ with intensity function $\Lambda(\mathbf{s}, t)$ determining
 46 the expected number of points in any set B , i.e.

$$\mathbb{E} [N(B) \mid \{\Lambda(\mathbf{s}, t)\}_{\mathbf{s} \in \mathcal{S}, t \in \mathcal{T}}] = \int_B \Lambda(\mathbf{s}, t) d(\mathbf{s}, t).$$

47 In this study, we will consider such marked spatiotemporal point processes where the occurrences of
 48 wildfires are the points of the process and the burnt area the associated marks. But note that other
 49 quantitative marks have been considered in the wildfire risk literature, such as the duration of the
 50 wildfire (e.g. Quinlan, Díaz-Avalos, and Mena 2021).

51 In addition, we will assume that N conditionally on Λ is a Poisson point process with intensity
 52 function $\Lambda(\mathbf{s}, t)$, meaning that

$$N(B) \mid \{\Lambda(\mathbf{s}, t)\}_{\mathbf{s} \in \mathcal{S}, t \in \mathcal{T}} \sim \text{Poisson} \left(\int_B \Lambda(\mathbf{s}, t) d(\mathbf{s}, t) \right)$$

53 and for any disjoint Borel sets $B_1, B_2 \subset \mathcal{S} \times \mathcal{T}$, $N(B_1)$ and $N(B_2)$ are independent conditionally on
 54 $\{\Lambda(\mathbf{s}, t)\}_{\mathbf{s} \in \mathcal{S}, t \in \mathcal{T}}$.

55 Within a Bayesian modelling framework, it then appears natural to consider log-Gaussian Cox
 56 processes (LGCPs, Møller, Syversveen, and Waagepetersen 1998) which are a particular case of
 57 Poisson point processes where $\{\log \Lambda(\mathbf{s}, t)\}_{\mathbf{s} \in \mathcal{S}, t \in \mathcal{T}}$ is assumed to be a Gaussian random field. A
 58 Bayesian interpretation of the LGCP is that we consider a log-Gaussian prior process for the intensity
 59 function of a Poisson process. LGCPs allow flexible inclusion of covariate information and are useful
 60 when points tend to occur in clusters, a behavior that is captured by the stochastic nature of the
 61 point process intensity function given by a log-Gaussian process. LGCPs have found numerous
 62 applications in risk modelling, especially for wildfires, but also for ecological data (e.g. Illian, Sørbye,
 63 and Rue 2012; Illian et al. 2013; Soriano-Redondo et al. 2019).

64 A fast, accurate and widely used Bayesian inference scheme when dealing with LGCPs is the integrated
 65 nested Laplace approximation framework (INLA, Rue, Martino, and Chopin 2009; Illian, Sørbye, and
 66 Rue 2012), which astutely exploits Laplace approximations (Tierney and Kadane 1986) and has proven
 67 to be computationally faster than simulation-based methods such as Markov chain Monte Carlo
 68 (MCMC) (e.g. Taylor and Diggle 2014). INLA allows estimating Bayesian hierarchical models and
 69 assumes that the latent structure of the model is a Gaussian Markov random field (GMRF), which
 70 means that the precision matrix (i.e., the inverse of the variance-covariance matrix) is sparse and
 71 therefore allows for fast computations even with high-dimensional Gaussian vectors with up to tens
 72 of thousands of latent variables. The stochastic partial differential equation (SPDE) approach is then
 73 often combined with INLA (providing the so-called INLA-SPDE approach) in order to approximate
 74 Gaussian random fields with Matérn covariance by GMRFs with sparse precision matrix (Lindgren,
 75 Rue, and Lindström 2011).

76 The INLA-SPDE approach has been intensively applied to wildfires. For instance, Pereira et al. (2013)
 77 have considered the occurrences of wildfires over Portugal taking into account specific topographic
 78 and land cover covariates, along with the average precipitation prior to the fire season. Serra et al.
 79 (2014) modeled wildfire occurrences in Catalonia given the potential causes of wildfires by considering
 80 a zero-inflated Poisson model. They also took into account topographic variables, the distance to
 81 anthropic areas and land uses. Another example is the work of Opitz, Bonneau, and Gabriel (2020),
 82 who modeled wildfire occurrences in the Southeastern core area in France by including predictors
 83 related to temperature and precipitation, as well as numerous covariates based on land use and land
 84 cover.

Table 1: First lines of the wildfire dataset.

PIX	XL2e	YL2e	DEP	YEAR	DOY	FWI	FA	NB0.1	BA
6373	476000	2081000	24	2019	227	2.7619048	2831.982	0	0
6373	476000	2081000	24	2019	225	0.8491777	2831.982	0	0
6373	476000	2081000	24	2013	158	6.7485680	2831.982	0	0
6373	476000	2081000	24	2013	155	3.4603725	2831.982	0	0
6373	476000	2081000	24	2019	229	8.2332123	2831.982	0	0
6373	476000	2081000	24	2013	154	1.8662924	2831.982	0	0

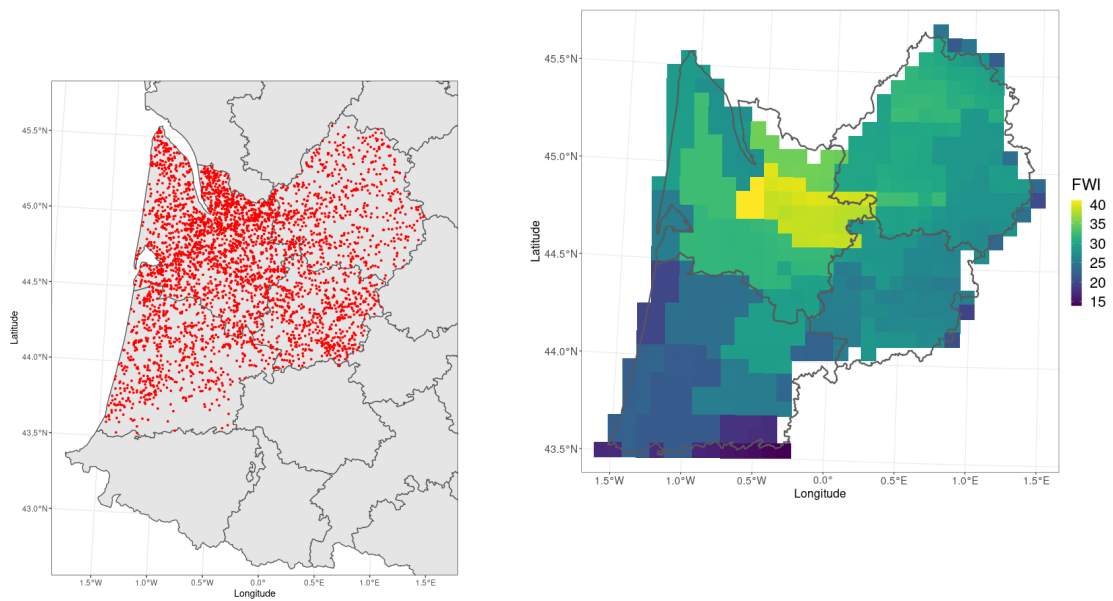
85 In the following, we build on the previous study of Pimont et al. (2021) but we implement several
86 extensions of their *Firelihood* model, a marked spatiotemporal log-Gaussian Cox process model
87 for wildfire activities. Their model was initially applied in the Southeast of France where climatic
88 conditions are sensibly different from the Aquitaine region. Note that to better account for burnt areas
89 of extreme wildfires, Koh et al. (2023) extended the *Firelihood* model by considering a two-component
90 mixture model for moderate and extreme wildfire sizes and other improvements that we include in
91 our model. Specifically, we will focus on using the Gamma distribution for wildfire sizes, which was
92 shown to provide a good fit in the historical Southeast region in Koh et al. (2023) without the need
93 to construct rather complex mixture models. In Section 2, we describe the data used in this study.
94 Section 3 presents the structure of our model and its inference using the INLA-SPDE approach. In
95 particular, we detail a subsampling approach for occurrence observations that are zero to keep fully
96 Bayesian inference with INLA-SPDE feasible. Then in Section 4, we generate wildfire activities over
97 the historical period in order to assess the validity of our methodology. Finally, in Section 5, we
98 derive future wildfire activities over the Aquitaine region under different climate scenarios.

99 2 Wildfire data

100 The wildfire data considered in this study are partially provided by the [BDIFF](#) database which collects
101 information on wildfires since 2006 on the whole French territory but for which no stochastic models
102 similar to *Firelihood* have been developed so far. Since BDIFF data are known to show some gaps,
103 they have been completed with those from a public-private network working on territorial risk
104 management (GIP ATGeRi). We extracted data from 2006 to 2020 in the former administrative
105 Aquitaine region, corresponding to four French “départements” Dordogne, Gironde, Landes and
106 Lot-et-Garonne (see the left display in Figure 1). Due to meteorological factors and specific human
107 activities (agriculture, tourism), winter and summer wildfires correspond to two different fire regimes.
108 Therefore, in this study, we focus on the summer wildfires, which are more numerous and on average
109 also much bigger in terms of size, and we keep only wildfires that have occurred between May to
110 October.

111 The meteorological data used are weather reanalysis data for variables such as temperature, pre-
112 cipitation, humidity and wind speed, provided by Météo France from the SAFRAN model (Vidal et
113 al. 2010). These data are then combined in order to obtain a Fire Weather Index (hereinafter FWI),
114 a unit-less indicator of fire danger. Since the SAFRAN model is defined on a regular grid of 8km
115 resolution, wildfire data are aggregated to the SAFRAN pixels; i.e., for each pixel-day B , we calculate
116 the number of wildfire occurrences in B ; however, we do not aggregate the marks (i.e., the burnt
117 areas) but continue working with the observed burnt area for each of the wildfire occurrences.

118 In the final dataset, we have for each pixel-day the corresponding daily FWI, the forest area FA
119 (i.e. the fuel surface), the number of fires that are greater than 0.1 hectares NB0.1 (for measurements
120 issues) and the burnt area BA in hectares (see Table 1).



(a) Locations of wildfire activities during 2006-2020 over (b) Spatial footprint of FWI observed on July 4, 2011 the region of interest.

Figure 1: Maps of wildfire activity during the observation period and spatial variability of the Fire Weather Index (FWI), which is commonly used to aggregate the meteorological fire drivers into a single variable, for a given day.

3 Model and inference

3.1 Subsampling

There are 1308930 observations in the wildfire dataset, but only 2331 actual wildfires. In order to keep the model manageable for INLA, we subsample observations with count 0. This approach is similar to Koh et al. (2023) and it does not bias the analysis since we reweight the Poisson intensities estimated in the model to take into account the factor by which we have subsampled. For instance, if we keep on average one in ten zeros when fitting the model, we attribute a ten times larger Poisson intensity to the zeros remaining in the observations used for estimation; this works since Poisson intensity parameters are additive under convolution of Poisson-distributed random variables.

We keep all observations with positive count (corresponding to at least one actual wildfire) since they are the most informative and account only for a small number of observations. For such observations, we put a weight equal to 1.

Then, among the n observations with 0 count, we only keep a number equal to n_{ss} and much smaller than n , such that the associated weights are then equal to n/n_{ss} . However, since wildfires tend to occur most often for large FWI, we further use a stratified approach by sampling with a larger probability weight p (i.e., the probability of keeping an observation) observations associated to large FWI values such that the model will be able to appropriately discriminate between wildfire presence (positive count) and absence (zero count) for relatively large FWI values. A large value of FWI is defined as exceeding a given high threshold u_q given by a specific q -quantile, $q \in (0, 1)$. If we denote by $(w_k)_{1 \leq k \leq n_{ss}}$ the subsampling weights of observation with count 0, then, for observations with FWI greater than u_q , necessarily $n\mathbb{P}(\text{FWI} \geq u_q) = n_{ss}pw_k$, i.e. $w_k = n(1 - q)/(n_{ss}p)$. Similarly, for FWI smaller than u_q , $n\mathbb{P}(\text{FWI} < u_q) = n_{ss}(1 - p)w_k$ leading to $w_k = nq/(n_{ss}(1 - p))$.

In the following, we choose $n_{ss} = 20000$, $q = 0.9$ and $p = 0.5$ (i.e. we keep as many observations with large FWI value as with lower FWI value) leading to a data set with dimension manageable by INLA on a personal computer and with computation times around the order of several minutes. Note that p and q are parameters that can be freely chosen based on the particularities of the data.

3.2 Model definition

In the following, the occurrences and wildfire sizes are considered as realizations of a spatiotemporal marked point process as explained in Section 1.

To account for the fact that the meteorological covariate FWI considered in this study is provided on a regular grid (see Section 2), we discretize our space-time study domain $\mathcal{S} \times \mathcal{T}$ and assume that the intensity of the LGCP $\Lambda(\mathbf{s}, t)$ is constant in each space-time cell $C_i \in \{1, \dots, M\}$, i.e.

$$\Lambda(\mathbf{s}, t) \equiv \Lambda_i \text{ if } (\mathbf{s}, t) \in C_i.$$

We then consider the occurrences. We model the number of fires, denoted N_i , in each space-time cell $C_i \in \{1, \dots, M\}$, corresponding to a SAFRAN pixel and a given day of the year. With these assumptions, the data discretized to the pixel grid are still coherent with the LGCP model introduced before, and we represent each pixel by its center coordinate.

In our model, we assume that for each space-time cell C_i , the number of fires N_i is distributed according to Poisson distribution with a log-link function and a random intensity parameter, described in the following hierarchical structure with the data layer (first line), the latent process layer (second line)

160 and the hyperparameter layer (third line):

$$\begin{aligned} N_i | \Lambda_i, \boldsymbol{\theta} &\sim \text{Poisson}(w_i \Lambda_i) \\ \log \Lambda_i &= \beta_0 + f_{\text{YEAR}}(\text{YEAR}_i) + f_{\text{DOY}}(\text{DOY}_i) + f_{\text{FWI}}(\text{FWI}_i) + f_{\text{FA}}(\text{FA}_i) + f_{\text{Spatial}}(A_i) \\ \boldsymbol{\theta} &= (\boldsymbol{\theta}^{\text{YEAR, size}}, \boldsymbol{\theta}^{\text{DOY}}, \boldsymbol{\theta}^{\text{FWI}}, \boldsymbol{\theta}^{\text{FA}}, \boldsymbol{\theta}^{\text{Spatial}}) \sim \text{Hyperpriors} \end{aligned} \quad (1)$$

161 The random 1-dimensional functional effects $f_{\{\text{FWI}, \text{DOY}, \text{FA}\}}$ are defined through quadratic splines,
162 for which SPDE prior models are available, and f_{Spatial} is a spatial field. Finally, to better capture the
163 yearly variability in the data, we add an iid random effect f_{YEAR} .

164 We work in a Bayesian framework, that is we put Gaussian prior distributions on all the random
165 effects. For instance, for the function representing the effect of FWI in the predictor, we set the
166 following prior structure:

$$\begin{cases} f_{\text{FWI}}(\text{FWI}_i) = \sum_{k=1}^n \beta_k^{\text{FWI}} b_k^{\text{FWI}}(\text{FWI}_i) \\ \boldsymbol{\beta}^{\text{FWI}} = (\beta_1^{\text{FWI}}, \dots, \beta_n^{\text{FWI}})^T \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{\text{FWI}}^{-1}) \end{cases}$$

167 where $(b_k^{\text{FWI}})_{k \in \{1, \dots, n\}}$ are spline basis functions. Denoting $\mathbf{B}^{\text{FWI}} = (b_k^{\text{FWI}}(\text{FWI}_i))_{(1 \leq k \leq n, 1 \leq i \leq M)}$ the
168 matrix containing the values of the spline basis function at the observed FWI values, and similarly
169 for the other covariates, the linear predictor in Equation 1 can be rewritten as follows

$$\log \Lambda = (\log \Lambda_1, \dots, \log \Lambda_M) = \mathbf{1}\beta_0 + \sum_{\text{eff} \in \text{Effects}} \mathbf{B}^{\text{eff}} \boldsymbol{\beta}^{\text{eff}} \quad (2)$$

170 where Effects contains all the effects considered in the model Equation 1, i.e. Effects =
171 {FWI, FA, YEAR, DOY, Spatial}. In the decomposition of Equation 2, the second term on the
172 right-hand side is decomposed for each effect by a product between the effect (i.e., the coefficients
173 $\boldsymbol{\beta}^{\text{eff}}$ to be estimated) and a projector matrix (i.e., the deterministic matrices \mathbf{B}^{eff} calculated from the
174 spline basis and the covariate values).

175 To fit this hierarchical model, we use the INLA framework (Rue, Martino, and Chopin 2009), which
176 leverages an astutely designed deterministic approximation of the posterior distributions, unlike
177 simulation-based methods such as MCMC.

178 For each component except the yearly effect, a latent Gaussian random field prior is approximated
179 using the stochastic partial differential equations (SPDE, Lindgren, Rue, and Lindström 2011) approach.
180 This approach allows us to approximate a continuous random field by a discrete random field with a
181 finite number of Gaussian variables used as priors for basis function coefficients, where interpolation
182 across continuous space provided is the deterministic basis functions. Shortly, recall that a Gaussian
183 random field $f(\mathbf{s})$ on \mathbb{R}^d can be obtained as the solution to the following SPDE

$$(\kappa^2 - \Delta)^{\alpha/2} \tau f(\mathbf{s}) = W(\mathbf{s}), \quad \alpha = \nu + d/2, \quad \mathbf{s} \in \mathbb{R}^d \quad (3)$$

184 where Δ is the Laplacian operator and $W(\mathbf{s})$ is a standard Gaussian white noise process. Then the
185 only stationary solution to Equation 3 is a Gaussian random field with Matérn covariance function

$$\text{Cov}(f(\mathbf{0}), f(\mathbf{s})) = \sigma^2 2^{1-\nu} (\kappa \|\mathbf{s}\|)^\nu K_\nu(\kappa \|\mathbf{s}\|) / \Gamma(\nu)$$

186 with Euclidean distance $\|\cdot\|$, Gamma function Γ , modified Bessel function of the second kind K_ν , and
187 $\sigma, \nu > 0$. In practice, we often fix $\nu = 1$ (as we do here), and then approximate solutions of Equation 3,
188 having a Markov structure, are obtained using a finite element method with a triangulation of the
189 space if $d = 2$, or splines if $d = 1$. Such solutions are Gaussian Markov random fields with sparse

190 precision matrices, allowing for fast numerical calculations even in high dimension (in terms of the
 191 number of basis functions, equal to the dimension of the corresponding latent Gaussian vector).

192 We then model the associated size components $(S_{i,1}, \dots, S_{i,N_i})$ given that $N_i > 0$, considered as the
 193 marks of the point process, through a Gamma distribution. As for the occurrence model, we consider
 194 the SPDE approach for the physical predictors FA and FWI, and an iid random effect for the year. We
 195 also consider an iid random effect for each “département”, which leads to the following model:

$$\begin{aligned} (S_{i,1}, \dots, S_{i,N_i}) \mid \alpha_i, \boldsymbol{\theta}^{\text{size}}, (N_i > 0) &\stackrel{\text{iid}}{\sim} \text{Gamma}(\alpha_i, \phi) \\ \log \alpha_i = \beta_0 + f_{\text{YEAR}}^{\text{size}}(\text{YEAR}_i) + f_{\text{FWI}}^{\text{size}}(\text{FWI}_i) + f_{\text{FA}}^{\text{size}}(\text{FA}_i) + f_{\text{Spatial}}^{\text{size}}(\text{DEP}_i) & \quad (4) \\ \boldsymbol{\theta}^{\text{size}} = (\boldsymbol{\theta}^{\text{YEAR,size}}, \boldsymbol{\theta}^{\text{FWI,size}}, \boldsymbol{\theta}^{\text{FA,size}}) &\sim \text{Hyperpriors} \end{aligned}$$

196 Note that with INLA, the parametrization of the Gamma likelihood is given by $E(S_{i,k}) = \alpha_i$ and
 197 $\text{Var}(S_{i,k}) = \alpha_i^2 / \phi$, for all $k \in \{1, \dots, N_i\}$ where ϕ is a hyperparameter included in the vector $\boldsymbol{\theta}^{\text{FWI,size}}$.
 198 The Gamma distribution behaves similarly to a Gaussian distribution when ϕ is large, whereas its
 199 tails become more and more heavy when ϕ approaches 0.

200 3.3 INLA settings

201 For the SPDE-based effects, namely FWI, FA, DOY and Spatial, we construct a Matérn SPDE model
 202 with penalized complexity (PC) priors for the hyperparameters of the Matérn field (Fuglstad et al.
 203 2019). The construction of the SPDE is achieved using the function `inla.spde2.pcmatern()`. We
 204 also define the associated projector matrix B^{eff} with `inla.spde.make.A()`, mapping the projections
 205 of the SPDE to the observation points. Then, the function `inla.spde.make.index()` is used to define
 206 the indexes of the latent variable for the SPDE model (i.e., an identifier that runs from 1 to the number
 207 of basis functions).

208 The 1-dimensional effects $f_{\text{FWI,FA,DOY}}$ in Equation 1 are defined through quadratic splines with 5
 209 knots for FWI and DOY, and 6 knots for FA. For FWI, we want to extrapolate values as a constant in
 210 cases where new covariate values used for prediction are larger than the observed covariate values
 211 used for fitting the model, so we impose a Neumann upper-bound condition corresponding to a zero
 212 first derivative.

213 Regarding the spatial effect f_{Spatial} , the triangulation mesh depicted in Figure 2 has 1051 nodes.
 214 In order to avoid non-stationary effects of the SPDE solution near the boundaries, we define two
 215 regions with a lower density of triangulation nodes in the outer region. Based on previous studies
 216 (e.g. Pimont et al. 2021), the parameters for the PC priors for the SPDE model (corresponding to
 217 exponential prior distributions for the standard deviation and the range parameter) are set using the
 218 following conditions: we set a probability of 50% to have a standard deviation larger than 1 and a
 219 probability of 5% to have a spatial range smaller than 50km.

220 3.4 Estimation of the occurrence model

221 To perform the model estimation, we gather all the information in a stack, a data format used in the
 222 R-INLA package that is appropriate for INLA and contains the data, the projection matrices and the
 223 different effects. In the model, we only have one fixed effect, which is the intercept, and five random
 224 effects. For the SPDE-based effects we include the indices of their associated SPDE. Then, we define
 225 the model formula as in Equation 1.

226 We then fit the model calling `inla()` with a number of user-specific settings to control how Laplace
 227 approximations are carried out and which posterior quantities are calculated.

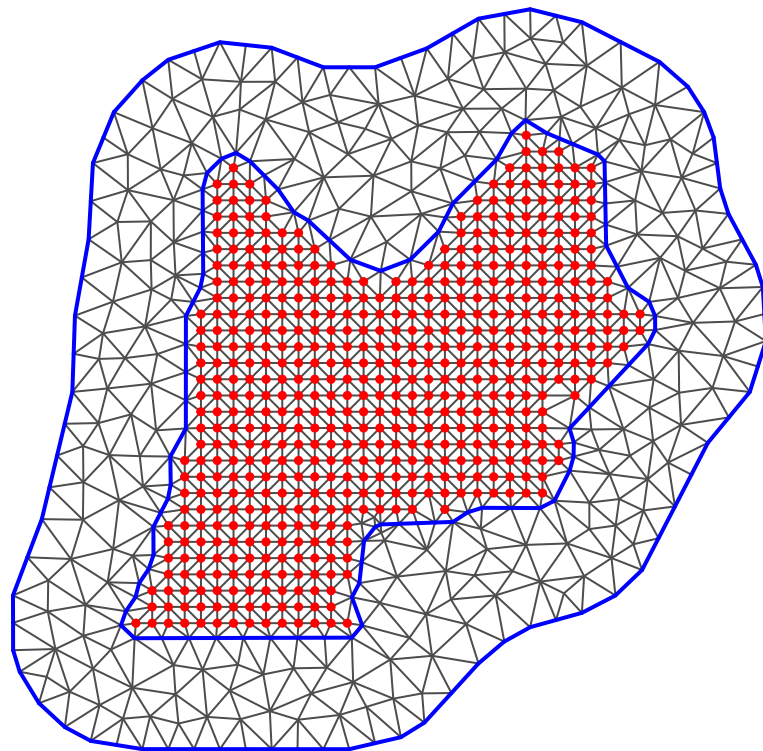


Figure 2: Constructed spatial mesh with 1051 nodes. Red dots represent the centers of the SAFRAN pixels for the study area.

228 We can visualize summaries of the posterior distributions of the random effects. For the SPDE effects,
 229 we have to project the effects on a 1-d (or 2-d for the spatial effect) grid that contains the initial mesh.
 230 The new projector matrix (i.e., the matrix B containing the new values of covariates and evaluated
 231 spline bases) is obtained with the function `inla.mesh.projector()`. Results are depicted in Figure 3.

232 From Figure 3, we can conclude that the yearly effect captures an inter-annual variability that cannot
 233 be described by the physical parameters FWI and FA. Regarding the seasonal effect DOY, we see that
 234 it decreases in mid-September, after the high summer heat. Both FWI and Forest area (FA) effects
 235 increase almost linearly, which is something that we expect since wildfire activity increases with
 236 FWI and the amount of fuel material. Looking at the spatial effect, we can observe a high spatial
 237 correlation between the different locations and clear separated clusters highlighting different fire
 238 regimes. An interpretation of these results is that FWI and FA are able to explain a substantial part of
 239 the spatiotemporal variability in wildfire activity, but that there also remains strong spatiotemporal
 240 residual correlation that is captured by the other random effects. Lastly, the magnitude of the effects
 241 appears to be smaller for the yearly effect than for the other effects.

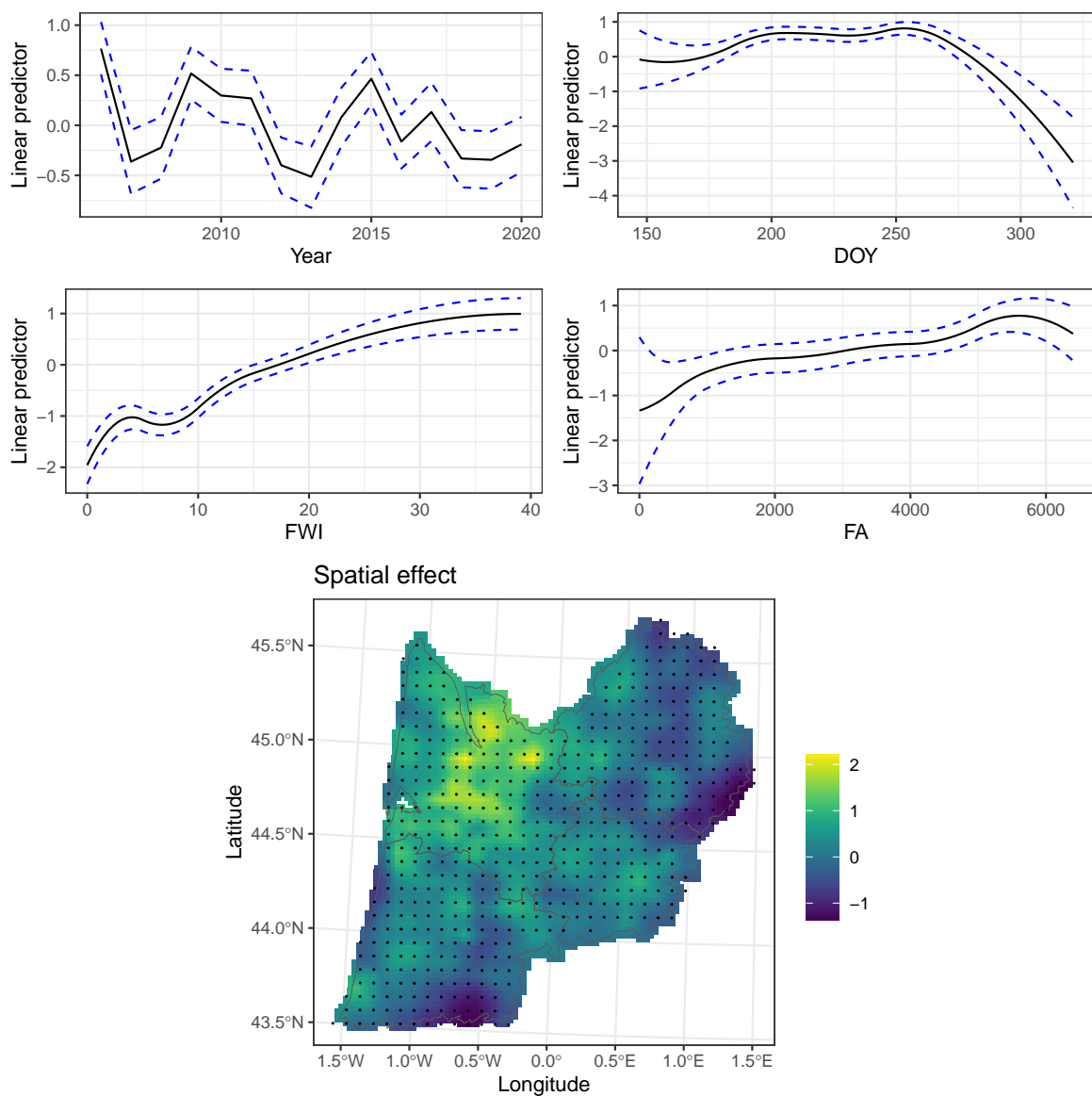


Figure 3: Partial effects of the occurrence model with the simulated data.

242 **3.5 Estimation of the size model**

243 Similarly to the occurrence model, we use INLA to perform the size model estimation. But prior to
244 that, we need to build the new projection matrices for the SPDE effects defined in Equation 4 since
245 we will consider hereafter only data such that the burnt area is greater than 0.1ha. We here perform
246 the estimation of the size model separately from the estimation of the occurrence model, which is
247 possible as long as we do not construct a model where some of the random effects are used in both
248 the occurrence model and the size model, i.e., where there are shared random effects, such as in Koh
249 et al. (2023). Separate estimation of the two models strongly reduces the overall computational cost
250 for running INLA.

251 Again, after running the INLA estimation, we can plot the estimated random effects, see Figure 4. It
252 can be seen that large wildfires tend to be associated with large values of FWI and FA. The spatial
253 effect is almost null except for the north-east region for which we have a negative effect. Finally,
254 looking at the amplitude of the effects, the FWI has the strongest influence on wildfire sizes.

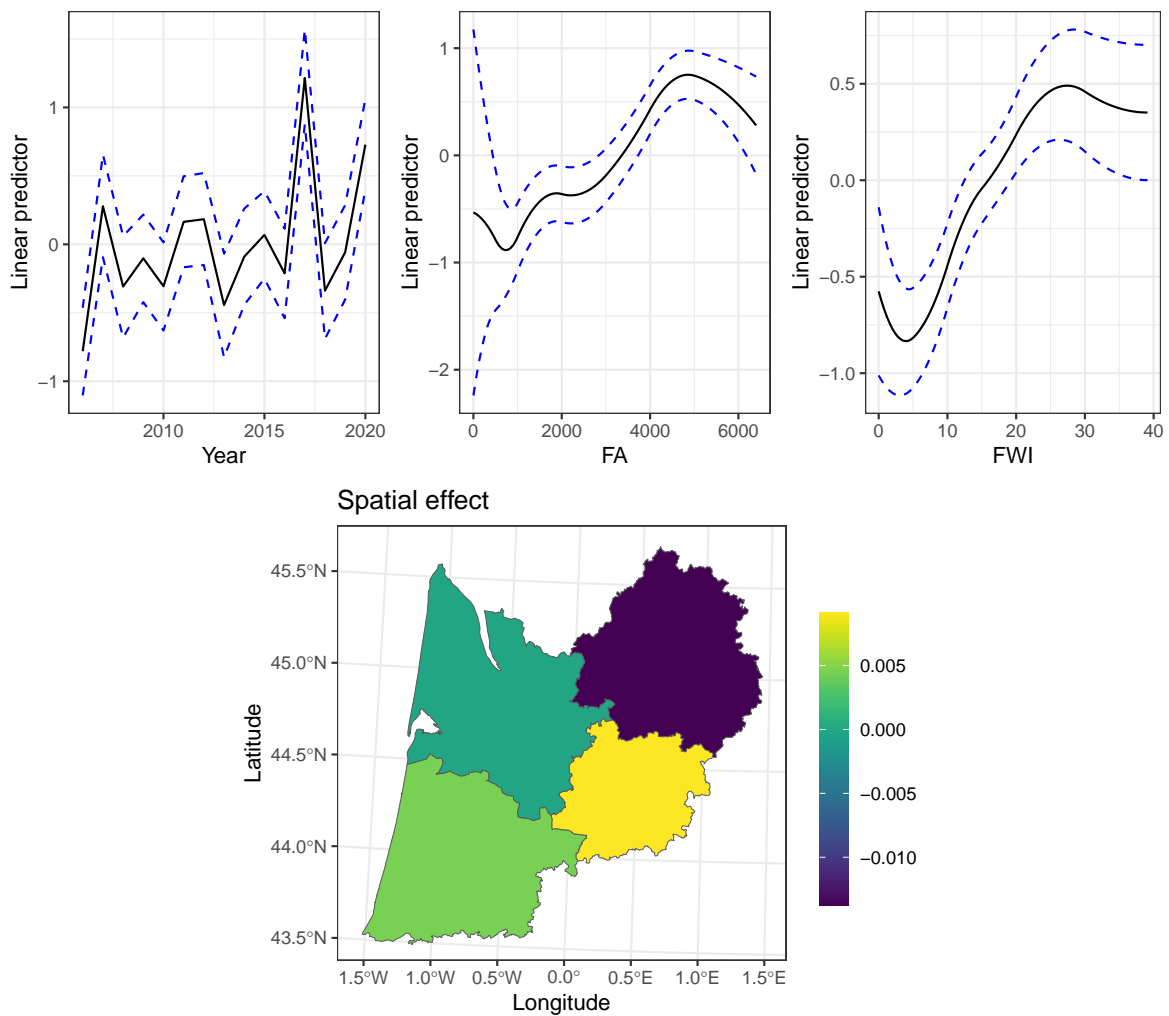


Figure 4: Partial effects of the size model.

4 Wildfire simulations from the posterior distribution for the observation period (2006-2020)

Before applying the fitted model to climate projections, we wish to assess the validity of our model. We here focus on the capacity of the model to reproduce the observed wildfires during the study period (2006-2020) with appropriate posterior uncertainty. R-INLA implements a method to obtain independent samples from the posterior distributions of hyperparameters and latent Gaussian variables, which can then be combined with new covariate data to calculate Monte-Carlo estimations of any posterior quantity of interest.

Note that the following results correspond to simulations obtained from the original data, which are not made available to reproduce the study. See Appendix B for more details.

4.1 Occurrence component

For the occurrence model, we first have to sample from the posterior distribution of all the coefficients in the occurrence model and then combine them with new effect values, using the additivity of Equation 2.

Hereinafter, we perform $n = 100$ simulations and results are depicted in Figure 5. The top line panels depict the spatial patterns of observed and simulated wildfire occurrences. In the bottom line, the yearly aggregated occurrences are shown on the left. Then, we looked for a specific year the daily and weekly aggregation (middle and right panels) of simulated wildfire occurrences. We chose to examine the year 2010, but any other year could have been considered.

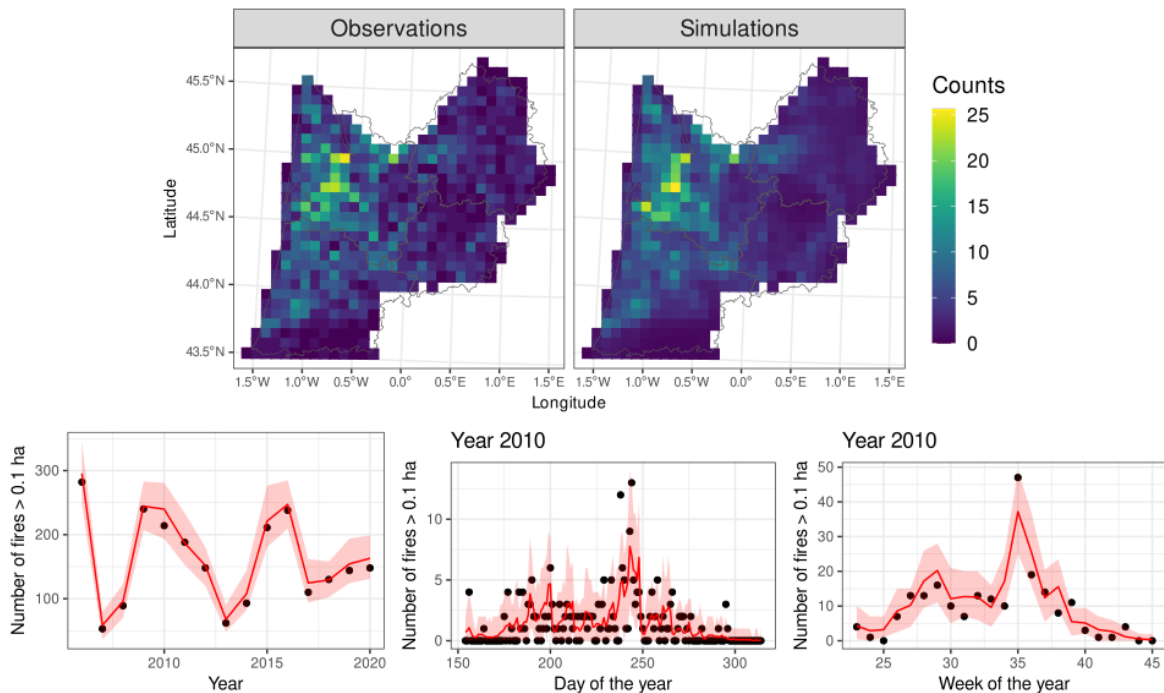


Figure 5: top: Spatial patterns of observed wildfire occurrences (left) and simulated occurrences (right). bottom: Simulated wildfire occurrences with 95% pointwise confidence intervals (in red) and observations (black dots).

Figure 5 highlights that the model successfully recovers both the spatial and temporal pattern of wildfire occurrences. Almost all observations are within the uncertainty bands of the posterior model.

276 The temporal aggregations also highlights the temporal trends and stochasticity of wildfire regimes,
 277 with for instance a stronger wildfire activity at the end of August.

278 4.2 Size component

279 As for the occurrence model, we illustrate the applicability of our model through simulations and
 280 compare the simulated sizes with the historical data (see Figure 6). The simulation scheme is as
 281 follows: for each of the 100 previously simulated samples of wildfire occurrences, we generate the
 282 associated sizes by sampling the posterior distributions of the fitted size model effects and use the
 283 additivity of the linear predictor as defined in Equation 4.

284 Simulations are presented in Figure 6. Again, the spatial aggregation is shown in the top line panels.
 285 The yearly aggregated burnt areas are depicted in the bottom-left panel. Finally, the middle and right
 286 panels in the bottom line depict the weekly aggregated occurrences of fires greater than 1 ha and 10
 287 ha respectively.

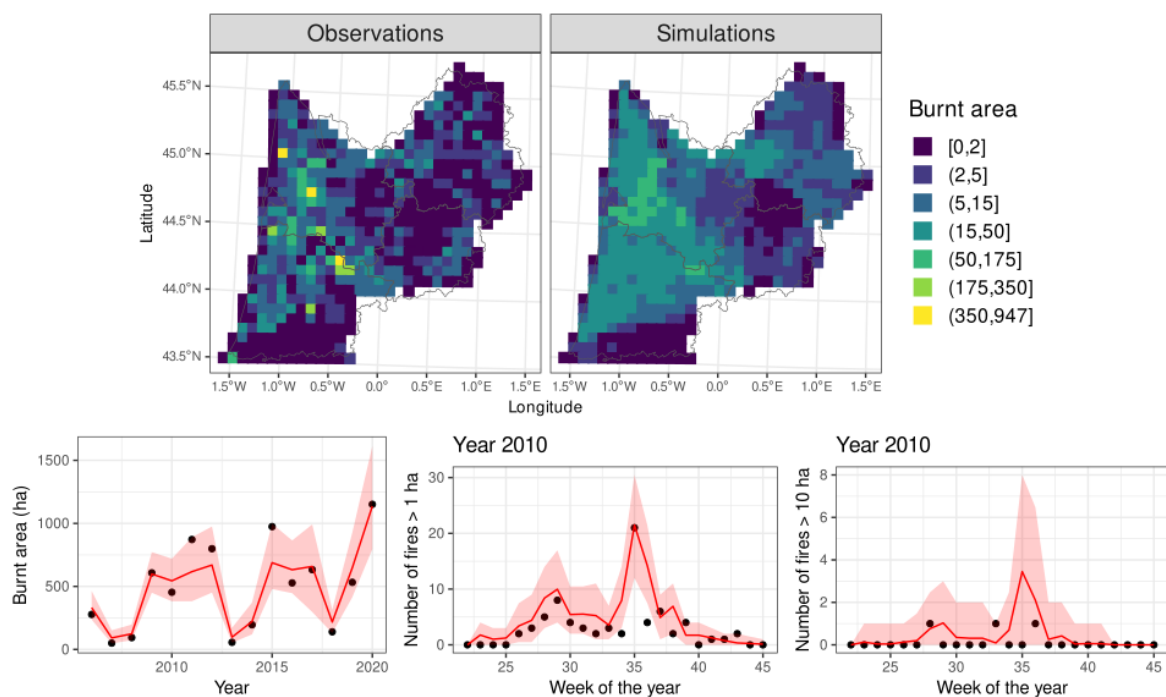


Figure 6: top: Spatial patterns of observed wildfire sizes (left) and simulated sizes (right). bottom: Simulated wildfire sizes with 95% pointwise confidence intervals (in red) and observations (black dots).

288 The size simulations depicted in Figure 6 show that the model successfully recovers the spatial
 289 pattern but misses some of the the most extreme values in terms of coverage of pointwise credible
 290 intervals. Unlike the wildfire occurrences, the distribution of burnt areas is heavy-tailed and more
 291 difficult to predict. Still, looking at the temporal aggregation the simulations provide satisfying
 292 results and almost all observations are within the uncertainty bands. We note that for the largest
 293 fires (i.e. greater than 10 ha), the sample size being much smaller, the uncertainties are larger.

5 Future wildfire simulations derived from climate model projections

We consider hereafter four different climate models under two climate scenarios RCP4.5 and RCP8.5 where the second one is more pessimistic in terms of expected global warming than the first one. The models, together with the institutes having developed them written in parentheses, are as follows: IPSL-CM5A (Institut Pierre-Simon Laplace, France), MPI-ESM (Max Planck Institut für Meteorologie, Germany), HadGEM2 (Met Office Hadley Center, UK) and CNRM (Météo-France, France).

For simulated FWI values in the climate change projections that fall outside the range of historical FWI values, we extrapolate the function f_{FWI} as a constant. For the forest surface FA in each pixel, we extrapolate the historical values in a constant manner, that is, we use the values available for 2018.

We perform $n = 20$ realizations of pixel-day occurrences and generate for each occurrence its associated size. The occurrences and sizes can then be aggregated over various spatial and temporal scales to study the potential evolutions of future wildfire risk. In Figure 7, we depict the results at an annual scale. Interannual variability in simulated counts and sizes remains relatively high, even after averaging the twenty realizations over the whole study area. To smooth the projected curves and identify long-term trends in wildfire activity, we implemented a 1D-SPDE INLA model, given by Equation 5. The basis representation for the yearly effect is defined through a quadratic spline and we set PC priors for the prior function such that the probability to have a standard deviation larger than 100 is equal to 0.5 and we fix a range value of 30 years. This choice is motivated by the fact that a 30-year period is often used to calculate averages in climatological studies. Moreover, to assess if there is a general upward trend in the smoothed curve, we include the rescaled year as a linear covariate, i.e., as a fixed effect. The rescaling is constructed such as to interpret the coefficient β_1 as a decadal effect. Therefore, the role of the spline function f_{YEAR} is to model nonlinear deviations from the general linear trend, $\beta_0 + \beta_1 (\text{YEAR}_i - 2020) / 10$. For the sake of identifiability, we set Dirichlet boundary conditions for the spline function, such that it takes value zero at both boundaries.

$$\begin{aligned}
 Y_i \mid \mu_i, \boldsymbol{\theta}^{\text{YEAR}} &\sim \text{Lognormal}(\mu_i, \sigma) \\
 \mu_i &= \beta_0 + \beta_1 (\text{YEAR}_i - 2020) / 10 + f_{\text{YEAR}}(\text{YEAR}_i) \\
 \boldsymbol{\theta}^{\text{YEAR}} &\sim \text{Hyperprior}
 \end{aligned}
 \tag{5}$$

Since the posterior sampling from the fitted model takes a lot of memory to run, we have stored the results beforehand.

Looking at Figure 7, the four climate models provide substantially different results. The IPSL-CM5A and CNRM show no significant trend either for occurrences or for sizes under both scenarios. Among the two other models, MPI-ESM shows a clearer trend: by 2100, the number of wildfire occurrences can be expected to double on average under the most pessimistic emission scenario. The associated wildfire sizes are also increasing, going from 500 ha in 2020 to up to 1500 ha by the end of the century. This evolution can be better measured by looking at the posterior distribution of the decadal linear effect defined in Equation 5. Summary statistics for β_1 are reported in the Appendix A. Under the RCP8.5 scenario, the HadGEM-RCA4 and MPI-ESM models lead to significantly positive β_1 estimates since their 95% credible intervals do not contain zero.

To capture potential spatial variability of projected wildfire activities, we also considered the spatial aggregation of the simulated occurrences and sizes during the end of the projection period (2070–2100), results are depicted in Figure 8.

From Figure 8a, it can be seen that the spatial pattern of wildfire occurrences will remain essentially

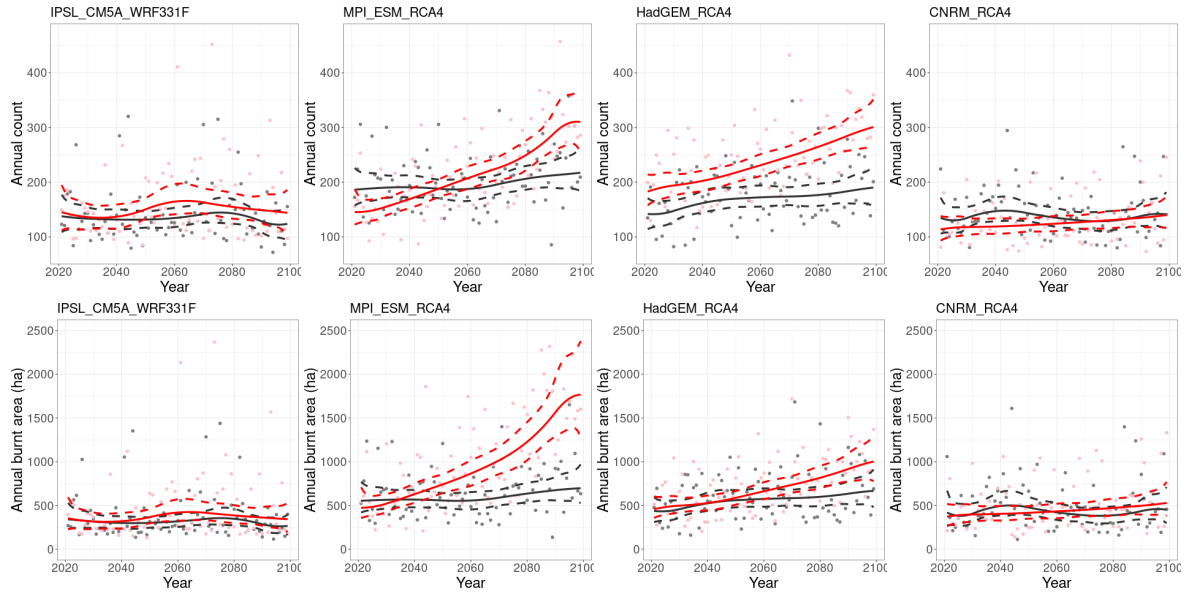


Figure 7: Annual means of wildfire occurrences (top-line) and sizes (bottom-line) for the four climate models and the two emission scenarios (RCP4.5 in black, RCP8.5 in red). Dots represent annual averages calculated over twenty samples from the full posterior model, continuous curves report the posterior mean fit of the INLA model used to smooth curves, and dashed lines indicate the corresponding 95% credible intervals.

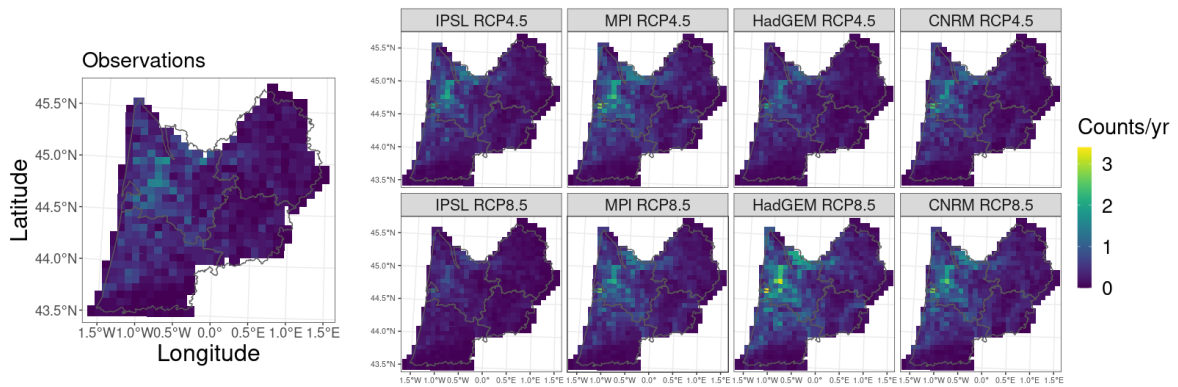
335 the same, but the intensities within each pixel may change. Under the scenario RCP 4.5 most models
 336 show no strong significant increasing trend. However, under the pessimistic scenario RCP 8.5,
 337 MPI-ESM model seems to predict a substantial increase in wildfire activities. To a lesser extent, a
 338 similar observation can be made for models CNRM and HadGEM2.

339 Looking at the projected burnt areas Figure 8b, similarly to the simulations performed over the
 340 observation period in Section 4.2, predictions are quite noisy which we attribute to the fact that burnt
 341 areas are relatively heavy-tailed, such that a relatively small number of values can have a relatively
 342 strong influence on the calculation of the mean. But again, under the scenario RCP 8.5, MPI-ESM
 343 model shows a significant increase in the average annual wildfire size compared to the observation
 344 period.

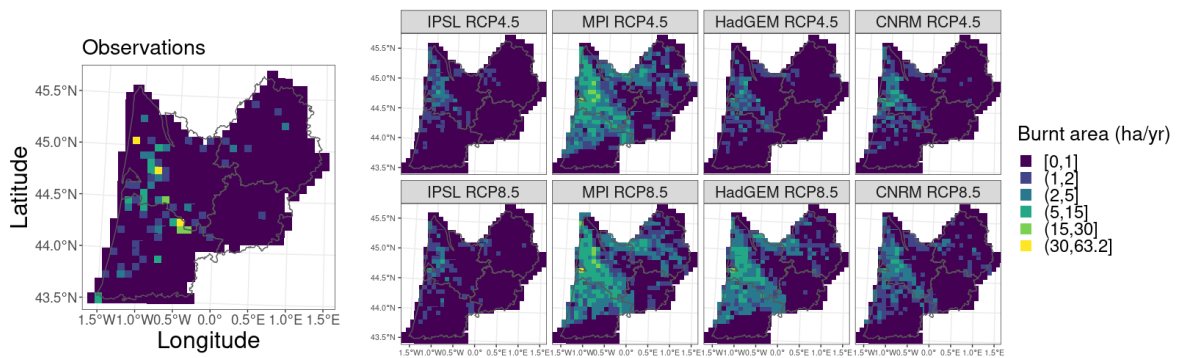
345 These experiments seem to indicate that for the Aquitaine region the climate-related vulnerability of
 346 forests to wildfires could increase to a lesser extent in the future than in the historical core wildfire
 347 area in the Southeastern France.

348 6 Discussion

349 A first remark should be made concerning the most recent wildfire events: as the 2022 weather data
 350 are not yet available, the 2022 summer fire season was not considered in this study. Therefore, the
 351 projections obtained above do not take into account the relatively extreme wildfire activities with
 352 several very large fires that have been observed in the Western part of France during this period, and
 353 the resulting projections could potentially have been more pessimistic in terms of the future increase
 354 of wildfire risk in the study region. The results we obtain point towards an increase in future wildfire
 355 risk. However, the uncertainty about the future climate remains large and propagates through to
 356 projected wildfire risk. Indeed, the weather simulations of the four climate models considered here
 357 lead to clearly significant increases only in some cases for rather pessimistic scenarios of greenhouse



(a) Projected wildfire occurrences



(b) Projected burnt area

Figure 8: Spatial patterns of wildfire activity during the end of the projection period (2070–2100) for each climate models and associated climate scenarios. Values correspond to the mean values over the period and are compared to the mean values during the observation period (2006-2020) displayed on the left-hand side.

Table 2: Estimated slopes of the linear predictor defined in Eq. (5) for each climate model

	Occurrence			Size		
	mean	0.025quant	0.975quant	mean	0.025quant	0.975quant
IPSL_CM5A_WRF331F 4.5	-0.013	-0.073	0.039	-0.029	-0.132	0.061
IPSL_CM5A_WRF331F 8.5	-0.003	-0.077	0.055	-0.007	-0.137	0.090
MPI_ESM_RCA4 4.5	0.020	-0.020	0.061	0.029	-0.037	0.096
MPI_ESM_RCA4 8.5	0.094	0.033	0.133	0.164	0.069	0.228
HadGEM_RCA4 4.5	0.037	-0.008	0.082	0.052	-0.021	0.124
HadGEM_RCA4 8.5	0.064	0.030	0.098	0.101	0.042	0.157
CNRM_RCA4 4.5	0.003	-0.053	0.058	0.003	-0.105	0.100
CNRM_RCA4 8.5	0.027	-0.014	0.077	0.046	-0.025	0.131

358 gas emissions.

359 This work presented a step-by-step methodology for the modelling of spatiotemporal marked point
360 processes, that has been applied to the modelling of wildfire activities in the Southwest region
361 of France. Due to the high stochasticity involved in wildfire activity but also in climate-change
362 projections, and due to the complex processes and data that have to be modeled, Bayesian hierarchical
363 modeling provides an appropriate framework for including various observed predictors and random
364 effects into a model that allows for accurate predictions with precise uncertainty assessment. Our
365 model includes additive random effects for various components of the linear predictors, such as
366 nonlinear effects of continuous covariates, spatial random effects and temporal random effects. The
367 SPDE approach provides flexible Gaussian prior distributions for such effects with two hyperparam-
368 eters for the variance and the correlation range, and the INLA method allows for fast and reliable
369 Bayesian inference even with complex and high-dimensional structures of the latent linear predictor
370 and the likelihood model of the data.

371 In addition, we also presented how INLA can be used to smooth relatively noisy simulations of
372 projected time series of risk occurrences, here based on combining posterior simulations of model
373 parameters with new weather-related covariates obtained from climate model output. Our smoothing
374 approach based on a Bayesian hierarchical model is an attractive statistical alternative to the classical
375 filtering approaches from signal processing, since it can lead to more interpretable results while at
376 the same time providing uncertainty envelopes.

377 We want to emphasize that our modeling approach for spatiotemporal marked point processes can
378 also be used in other contexts. In ecology, for example, researchers are interested in modelling the dis-
379 tribution of species in space and time over a given study area: the occurrences of the spatio-temporal
380 process could be the observation locations, and the marks could refer to certain characteristics (traits)
381 of the observed individuals. In particular, we plan to construct INLA-SPDE models similar to the one
382 presented here to project how species distributions evolve in response to present and future climatic
383 change (see e.g. Guillot et al. 2022; Laxton et al. 2023).

384 Appendix A

385 Estimates of β_1 as defined in Equation 5 are depicted in the following table, highlighting for which
386 climate model there is a positive trend in wildfire activities (in red).

387 Appendix B

388 The results depicted in Figures Figure 5 and Figure 6 are obtained with the original wildfire data. Since
389 these data are confidential, the code presented in this paper should be applied to a simulated data set,
390 obtained with the model developed above using the original data (see Supplementary materials).

391 In the following, we show the figures that should be obtained using the proposed simulated data set
392 and the code developed in the previous sections.

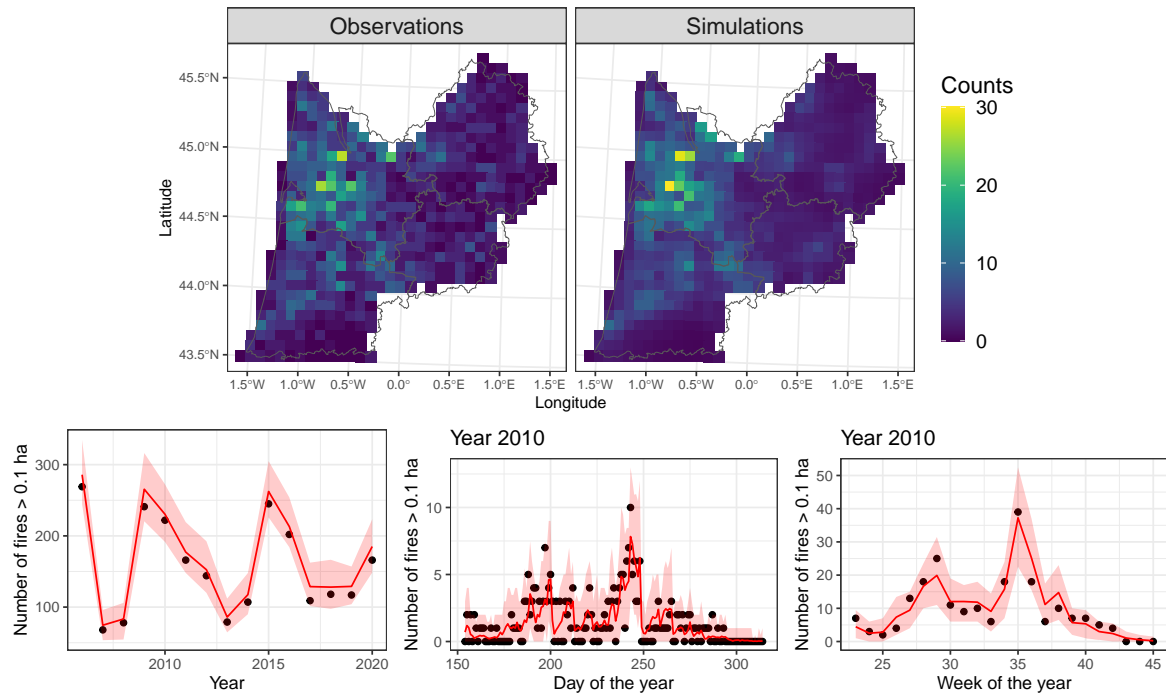


Figure 9: top: Spatial patterns of observed wildfire occurrences (left) and simulated occurrences (right). bottom: Simulated wildfire occurrences with 95% pointwise confidence intervals (in red) and observations (black dots).

393 Acknowledgments

394 The authors are grateful to Météo-France for making the SAFRAN data available.

395 Supplementary materials

396 All data used in this study are available at the following link: <https://doi.org/10.5281/zenodo.7870592>.
397 Note that for confidentiality reasons, the wildfire data provided correspond to simulated data from
398 the model developed in this work.

399 The R codes are available on GitHub: https://github.com/jlegrand35/wildfire_activities

400 References

401 Session information

402 R version 4.3.1 (2023-06-16)

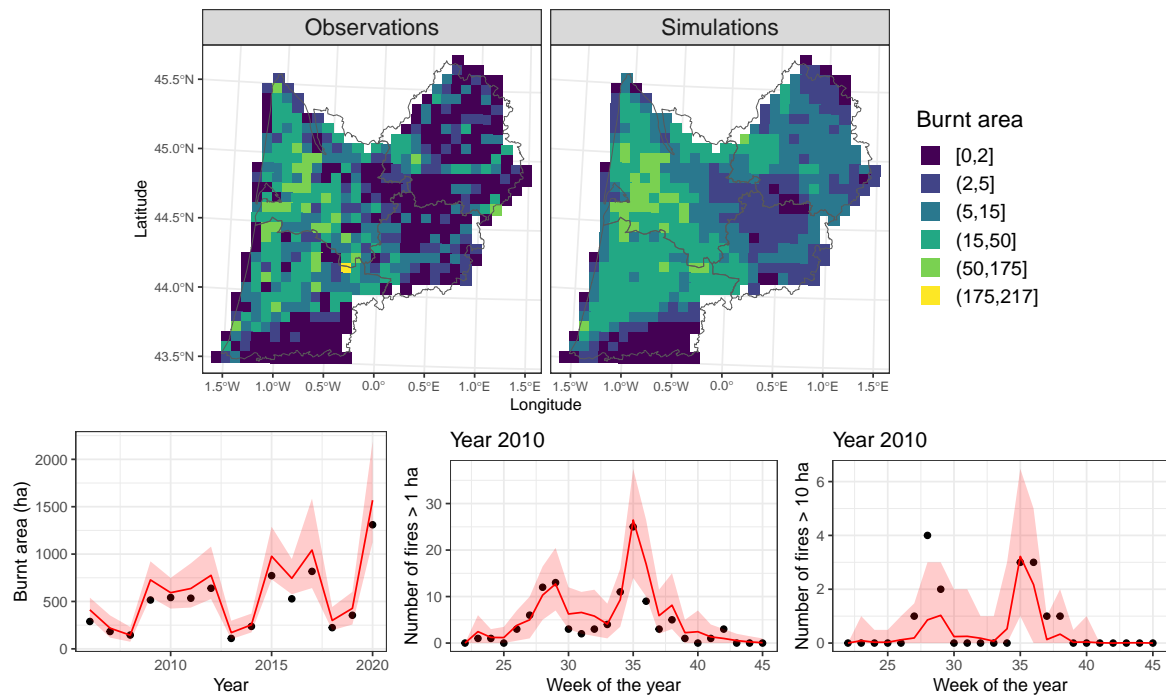


Figure 10: top: Spatial patterns of observed wildfire sizes (left) and simulated sizes (right). bottom: Simulated wildfire sizes with 95% pointwise confidence intervals (in red) and observations (black dots).

```

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404 Running under: Ubuntu 22.04.3 LTS
405
406 Matrix products: default
407 BLAS: /usr/lib/x86_64-linux-gnu/openblas-pthread/libblas.so.3
408 LAPACK: /usr/lib/x86_64-linux-gnu/openblas-pthread/libopenblaspr0.3.20.so; LAPACK version 3.10.0
409
410 locale:
411 [1] LC_CTYPE=C.UTF-8 LC_NUMERIC=C LC_TIME=C.UTF-8
412 [4] LC_COLLATE=C.UTF-8 LC_MONETARY=C.UTF-8 LC_MESSAGES=C.UTF-8
413 [7] LC_PAPER=C.UTF-8 LC_NAME=C LC_ADDRESS=C
414 [10] LC_TELEPHONE=C LC_MEASUREMENT=C.UTF-8 LC_IDENTIFICATION=C
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416 time zone: UTC
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421 [8] base
422
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424 [1] kableExtra_1.3.4 knitr_1.43 INLA_23.10.28 Matrix_1.6-1
425 [5] viridis_0.6.4 spdep_1.2-8 sf_1.0-14 spData_2.3.0
426 [9] dplyr_1.1.3 gridExtra_2.3 ggplot2_3.4.3 stringr_1.5.0
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