NEURAL VARIATIONAL INFERENCE FOR TEXT PROCESSING

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ABSTRACT

Recent advances in neural variational inference have spawned a renaissance in deep latent variable models. In this paper we introduce a generic variational inference framework for generative and conditional models of text. While traditional variational methods derive an analytic approximation for the intractable distributions over latent variables, here we construct an inference network conditioned on the discrete text input to provide the variational distribution. We validate this framework on two very different text modelling applications, generative document modelling and supervised question answering. Our neural variational document model combines a continuous stochastic document representation with a bag-of-words generative model and achieves the lowest reported perplexities on two standard test corpora. The neural answer selection model employs a stochastic representation layer within an attention mechanism to extract the semantics between a question and answer pair. On two question answering benchmarks this model exceeds all previous published benchmarks.

1 INTRODUCTION

Probabilistic generative models underpin many successful applications within the field of natural language processing (NLP). Their popularity stems from their ability to use unlabelled data effectively, to incorporate abundant linguistic features, and to learn interpretable dependencies among data. However these successes are tempered by the fact that as the structure of such generative models becomes deeper and more complex, true Bayesian inference becomes intractable due to the high dimensional integrals required. Markov chain Monte Carlo (MCMC) (Neal, 1993; Andrieu et al., 2003) and variational inference (Jordan et al., 1999; Attias, 2000; Beal, 2003) are the standard approaches for approximating these integrals. However the computational cost of the former results in impractical training for the large and deep neural networks which are now fashionable, and the latter is conventionally confined due to the underestimation of posterior variance. The lack of effective and efficient inference methods hinders our ability to create highly expressive models of text, especially in the situation where the model is non-conjugate.

This paper introduces a neural variational framework for generative models of text, inspired by the variational auto-encoder (Rezende et al., 2014; Kingma & Welling, 2014). The principle idea is to build an inference network, implemented by a deep neural network conditioned on text, to approximate the intractable distributions over the latent variables. Instead of providing an analytic approximation, as in traditional variational Bayes, neural variational inference learns to model the posterior probability, thus endowing the model with strong generalisation abilities. Due to the flexibility of deep neural networks, the inference network is capable of learning complicated non-linear distributions and processing structured inputs such as word sequences. Inference networks can be designed as, but not restricted to, multilayer perceptrons (MLP), convolutional neural networks, and recurrent neural networks (RNN), approaches which are rarely used in conventional generative models. By using the reparameterisation method (Rezende et al., 2014; Kingma & Welling, 2014), the inference network is trained through back-propagating unbiased and low variance gradients w.r.t. the latent variables. Within this framework, we propose a Neural Variational Document Model (NVDM) for document modelling and a Neural Answer Selection Model (NASM) for question answering, a task that selects the sentences that correctly answer a factoid question from a set of candidate sentences.

The NVDM (Figure 1) is an unsupervised generative model of text which aims to extract a continuous semantic latent variable for each document. This model can be interpreted as a variational auto-encoder: an MLP encoder (inference network) compresses the bag-of-words document representation into a continuous latent distribution, and a softmax decoder (generative model) reconstructs the document by generating the words independently. A primary feature of NVDM is that each word is generated directly from a dense continuous document representation instead of the more common binary semantic vector (Hinton & Salakhutdinov, 2009; Larochelle & Lauly, 2012; Srivastava et al., 2013; Mnih & Gregor, 2014). Our experiments demonstrate that our neural document model achieves the state-of-the-art perplexities on the *20NewsGroups* and *RCV1-v2* datasets.

The NASM (Figure 2) is a supervised conditional model which imbues LSTMs (Hochreiter & Schmidhuber, 1997) with a latent stochastic attention mechanism to model the semantics of question-answer pairs and predict their relatedness. The attention model is designed to focus on the phrases of an answer that are strongly connected to the question semantics and is modelled by a latent distribution. This mechanism allows the model to deal with the ambiguity inherent in the task and learns pair-specific representations that are more effective at predicting answer matches, rather than independent embeddings of question and answer sentences. Bayesian inference provides a natural safeguard against overfitting, especially as the training sets available for this task are small. The experiments show that the LSTM with a latent stochastic attention mechanism learns an effective attention model and outperforms both previously published results, and our own strong non-stochastic attention baselines.

In summary, we demonstrate the effectiveness of neural variational inference for text processing on two diverse tasks. These models are simple, expressive and can be trained efficiently with the highly scalable stochastic gradient variational Bayes (SGVB) algorithm (Rezende et al., 2014; Kingma & Welling, 2014). Our neural variational framework is suitable for both unsupervised and supervised learning tasks, and can be generalised to incorporate any type of neural networks.

2 Neural Variational Inference Framework

Latent variable modelling is popular in many NLP problems, but it is non-trivial to carry out effective and efficient inference for models with complex and deep structure. In this section we introduce a generic neural variational inference framework that we apply to both the unsupervised NVDM and supervised NASM in the follow sections.

We define a generative model with a latent variable $\boldsymbol{h}$, which can be considered as the stochastic units in deep neural networks. We designate the observed parent and child nodes of $\boldsymbol{h}$ as $\boldsymbol{z}$ and $\boldsymbol{x}$ respectively. Hence, the joint distribution of the generative model is $p(\boldsymbol{z},\boldsymbol{x}) = \sum_{\boldsymbol{h}} p(\boldsymbol{x}|\boldsymbol{h})p(\boldsymbol{h}|\boldsymbol{z})p(\boldsymbol{z})$, and the variational lower bound $\mathcal{L}$ is derived as:

$$\begin{aligned} \log p_\theta(\boldsymbol{z},\boldsymbol{x}) &= \log \int \frac{q(\boldsymbol{h})}{q(\boldsymbol{h})} p_\theta(\boldsymbol{x}|\boldsymbol{h}) p_\theta(\boldsymbol{h}|\boldsymbol{z}) p(\boldsymbol{z}) d\boldsymbol{h} && (1) \\ &\geqslant \mathbb{E}_{q(\boldsymbol{h})}[\log p_\theta(\boldsymbol{x}|\boldsymbol{h}) p_\theta(\boldsymbol{h}|\boldsymbol{z}) p(\boldsymbol{z}) - \log q(\boldsymbol{h})] = \mathcal{L} && (2) \end{aligned}$$

where θ parameterises the generative distributions $p_\theta(\boldsymbol{x}|\boldsymbol{h})$ and $p_\theta(\boldsymbol{h}|\boldsymbol{z})$. In order to have a tight lower bound, the variational distribution $q(\boldsymbol{h})$ should approach the true posterior $p(\boldsymbol{h}|\boldsymbol{z},\boldsymbol{x})$. The three steps to construct the deep neural inference network $q_\phi(\boldsymbol{h}|\boldsymbol{z},\boldsymbol{x})$, which is a parameterised diagonal Gaussian distribution $\mathcal{N}(\boldsymbol{h}|\boldsymbol{\mu}(\boldsymbol{z},\boldsymbol{x}), \mathrm{diag}(\boldsymbol{\sigma}^2(\boldsymbol{z},\boldsymbol{x})))$, are:

1. Construct vector representations of the observed variables:
   $\boldsymbol{u} = f_z(\boldsymbol{z})$, $\boldsymbol{v} = f_x(\boldsymbol{x})$.
2. Assemble a joint representation:
   $\boldsymbol{\pi} = g(\boldsymbol{u},\boldsymbol{v})$.
3. Parameterise the variational distribution over the latent variable:
   $\boldsymbol{\mu} = l_1(\boldsymbol{\pi}), \log\boldsymbol{\sigma} = l_2(\boldsymbol{\pi})$.

$f_z(\cdot)$ and $f_x(\cdot)$ can be any type of deep neural networks that are suitable for the observed data; $g(\cdot)$ is an MLP that concatenates the vector representations of the conditioning variables; $l(\cdot)$ is a linear transformation which outputs the parameters of the Gaussian distribution. By sampling from the

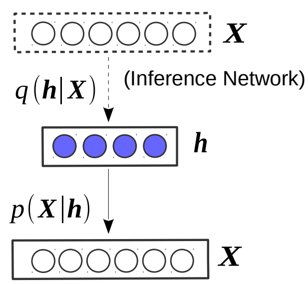

Figure 1: NVDM for document modelling.

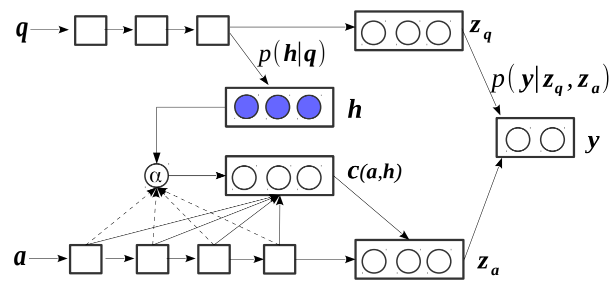

Figure 2: NASM for question answer selection.

variational distribution, $\boldsymbol{h} \sim q_\phi(\boldsymbol{h}|\boldsymbol{z}, \boldsymbol{x})$, we are able to carry out stochastic back-propagation to optimise the lower bound (Eq. 2).

During training, the model parameters θ together with the inference network parameters ϕ are updated by stochastic back-propagation based on the samples $\boldsymbol{h}$ drawn from $q_\phi(\boldsymbol{h}|\boldsymbol{z}, \boldsymbol{x})$. For the gradients w.r.t. θ, we have the form:

$$\nabla_\theta \mathcal{L} \simeq \frac{1}{L} \sum_{l=1}^{L} \nabla_\theta \log p_\theta(\boldsymbol{x}|\boldsymbol{h}^{(l)}) p_\theta(\boldsymbol{h}^{(l)}|\boldsymbol{z}) \tag{3}$$

For the gradients w.r.t. ϕ we reparameterise $\boldsymbol{h} = \boldsymbol{\mu} + \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}$ and sample $\boldsymbol{\epsilon}^{(l)} \sim \mathcal{N}(0, \boldsymbol{I})$ to reduce the variance in stochastic estimation (Rezende et al., 2014; Kingma & Welling, 2014). The update of ϕ can be carried out by back-propagating the gradients w.r.t. $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$:

$$\nabla_\mu \mathcal{L} \simeq \frac{1}{L} \sum_{l=1}^{L} \nabla_{\boldsymbol{h}} [\log p_\theta(\boldsymbol{x}|\boldsymbol{h}^{(l)}) p_\theta(\boldsymbol{h}^{(l)}|\boldsymbol{z}) - \log q_\phi(\boldsymbol{h}^{(l)}|\boldsymbol{z}, \boldsymbol{x})] \tag{4}$$

$$\nabla_\sigma \mathcal{L} \simeq \frac{1}{2L} \sum_{l=1}^{L} \boldsymbol{\epsilon}^{(l)} \nabla_{\boldsymbol{h}^{(l)}} [\log p_\theta(\boldsymbol{x}|\boldsymbol{h}^{(l)}) p_\theta(\boldsymbol{h}^{(l)}|\boldsymbol{z}) - \log q_\phi(\boldsymbol{h}^{(l)}|\boldsymbol{z}, \boldsymbol{x})] \tag{5}$$

It is worth mentioning that unsupervised learning is a special case of the variational autoencoder where $\boldsymbol{h}$ has no parent node $\boldsymbol{z}$. In that case $\boldsymbol{h}$ is directly drawn from the prior $p(\boldsymbol{h})$ instead of the conditional distribution $p_\theta(\boldsymbol{h}|\boldsymbol{z})$.

Here we only discuss the scenario where the latent variables are continuous and the parameterised diagonal Gaussian distribution is employed as the variational distribution. However the framework is also suitable for discrete units, and the only modification needed is to replace the Gaussian with a multinomial parameterised by the outputs of a softmax function. Though the reparameterisation trick for continuous variables is not applicable in this case, a policy gradient approach (Mnih & Gregor, 2014) can help to alleviate the problem of high variance during stochastic estimation.

3 Neural Variational Document Model

The Neural Variational Document Model (Figure 1) is a simple instance of unsupervised learning where a continuous hidden variable $\boldsymbol{h} \in \mathbb{R}^K$, which generates all the words in a document independently, is introduced to represent its semantic content. Let $\boldsymbol{X} \in \mathbb{R}^{|V|}$ be the bag-of-words representation of a document and $\boldsymbol{x}_i \in \mathbb{R}^{|V|}$ be the one-hot representation of the word at position i.

As an unsupervised generative model, we could interpret NVDM as a variational autoencoder: an MLP encoder $q(\boldsymbol{h}|\boldsymbol{X})$ compresses document representations into continuous hidden vectors ($\boldsymbol{X} \rightarrow \boldsymbol{h}$); a softmax decoder $p(\boldsymbol{X}|\boldsymbol{h}) = \prod_i p(\boldsymbol{x}_i|\boldsymbol{h})$ reconstructs the documents by independently generating the words ($\boldsymbol{h} \rightarrow \{\boldsymbol{x}_i\}$).

To maximise the document log-likelihood $\log \sum_{\boldsymbol{h}} p(\boldsymbol{X}|\boldsymbol{h}) p(\boldsymbol{h})$, we derive the lower bound:

$$\mathcal{L} = \mathbb{E}_{q_\phi(\boldsymbol{h}|\boldsymbol{X})} \left[\sum_{i=1}^{N} \log p_\theta(\boldsymbol{x}_i|\boldsymbol{h}) \right] - D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{X}) \| p(\boldsymbol{h})] \tag{6}$$

where N is the number of words in the document and $p(\boldsymbol{h})$ is a Gaussian prior for $\boldsymbol{h}$. The conditional probability over words $p_\theta(\boldsymbol{x}_i|\boldsymbol{h})$ (decoder) is modelled by multinomial logistic regression and shared across documents:

$$p_\theta(\boldsymbol{x}_i|\boldsymbol{h}) = \frac{\exp\{-E(\boldsymbol{x}_i; \boldsymbol{h}, \theta))\}}{\sum_{j=1}^{|V|} \exp\{-E(\boldsymbol{x}_j; \boldsymbol{h}, \theta)\}} \tag{7}$$

$$E(\boldsymbol{x}_i; \boldsymbol{h}, \theta) = -\boldsymbol{h}^T \boldsymbol{R}\boldsymbol{x}_i - \boldsymbol{b}_{x_i} \tag{8}$$

where $\boldsymbol{R} \in \mathbb{R}^{K\times|V|}$ learns the semantic word embeddings and $\boldsymbol{b}_{x_i}$ represents the bias term.

As there is no supervision information for the latent semantics, $\boldsymbol{h}$, the posterior approximation $q_\phi(\boldsymbol{h}|\boldsymbol{X})$ is only conditioned on the current document $\boldsymbol{X}$. The inference network $q_\phi(\boldsymbol{h}|\boldsymbol{X}) = \mathcal{N}(\boldsymbol{h}|\boldsymbol{\mu}(\boldsymbol{X}), diag(\boldsymbol{\sigma}^2(\boldsymbol{X})))$ is modelled as:

$$\boldsymbol{\pi} = g(f_X^{\text{MLP}}(\boldsymbol{X})) \tag{9}$$

$$\boldsymbol{\mu} = l_1(\boldsymbol{\pi}), \log \boldsymbol{\sigma} = l_2(\boldsymbol{\pi}) \tag{10}$$

For each document $\boldsymbol{X}$, the neural network generates its own parameters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ that parameterise the latent distribution over document semantics $\boldsymbol{h}$. Based on the samples $\boldsymbol{h} \sim q_\phi(\boldsymbol{h}|\boldsymbol{X})$, the lower bound (Eq. 6) can be optimised by stochastic back-propagation.

Since $p(\boldsymbol{h})$ is a standard Gaussian prior, $D_{\text{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{X})\|p(\boldsymbol{h})]$ is a Gaussian KL-Divergence which can be computed analytically to further lower the variance of the gradients. Following the neural variational framework, the parameters of inference network ϕ and the model parameters θ are updated by SGVB. Appendix B.1 provides the formal details of the neural network structure.

4 Neural Answer Selection Model

Answer sentence selection is a question answering paradigm where a model must identify the correct sentences answering a factual question from a set of candidate sentences. Assume a question $\boldsymbol{q}$ is associated with a set of answer sentences $\{\boldsymbol{a}_1, \boldsymbol{a}_2, ..., \boldsymbol{a}_n\}$, together with their judgements $\{\boldsymbol{y}_1, \boldsymbol{y}_2, ..., \boldsymbol{y}_n\}$, where $\boldsymbol{y}_m = 1$ if the answer $\boldsymbol{a}_m$ is correct and $\boldsymbol{y}_m = 0$ otherwise. This is a classification task where we treat each training data point as a triple $(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{y})$ while predicting $\boldsymbol{y}$ for the unlabelled question-answer pair $(\boldsymbol{q}, \boldsymbol{a})$.

The Neural Answer Selection Model (Figure 2) is a supervised model that learns the question and answer representations and predicts their relatedness. It employs two different LSTMs to embed raw question inputs $\boldsymbol{q}$ and answer inputs $\boldsymbol{a}$. We use $\boldsymbol{s}_q(j)$ and $\boldsymbol{s}_a(i)$ to represent the state outputs of the two LSTMs, and i, j denote the positions of the states. Conventionally, the last state outputs $\boldsymbol{s}_q(|\boldsymbol{q}|)$ and $\boldsymbol{s}_a(|\boldsymbol{a}|)$, as the independent question and answer representations, can be used for relatedness prediction. In NASM, however, we aim to learn pair-specific representations by a latent attention mechanism, which are more effective for the pair relatedness prediction.

The aim of the attention model is to focus on the words in the answer sentence that are prominent for predicting the answer match to the current question. Instead of using a deterministic question vector, such as $\boldsymbol{s}_q(|\boldsymbol{q}|)$, NASM employs a latent distribution $p_\theta(\boldsymbol{h}|\boldsymbol{q})$ to model the question semantics, which is a parameterised diagonal Gaussian $\mathcal{N}(\boldsymbol{h}|\boldsymbol{\mu}(\boldsymbol{q}), \text{diag}(\boldsymbol{\sigma}^2(\boldsymbol{q})))$. Therefore, the attention model extracts a context vector $\boldsymbol{c}(\boldsymbol{a}, \boldsymbol{h})$ by iteratively attending to the answer tokens based on the stochastic vector $\boldsymbol{h} \sim p_\theta(\boldsymbol{h}|\boldsymbol{q})$. In doing so the model is able to adapt to the ambiguity inherent in questions and obtain salient information through attention. Compared to its deterministic counterpart (applying deterministic question vector $\boldsymbol{s}_q(|\boldsymbol{q}|)$) , the stochastic units incorporated into NASM allow multi-modal attention distributions. Further, by marginalising over the latent variables, NASM is more robust against overfitting, which is important for small question answering training sets.

In this model, the conditional distribution $p_\theta(\boldsymbol{h}|\boldsymbol{q})$ is modelled as:

$$\boldsymbol{\pi}_\theta = g_\theta(f_q^{\text{LSTM}}(\boldsymbol{q})) = g_\theta(\boldsymbol{s}_q(|\boldsymbol{q}|)) \tag{11}$$

$$\boldsymbol{\mu}_\theta = l_1(\boldsymbol{\pi}_\theta), \log \boldsymbol{\sigma}_\theta = l_2(\boldsymbol{\pi}_\theta) \tag{12}$$

For each question $\boldsymbol{q}$, the neural network generates the corresponding parameters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ that parameterise the latent distribution over question semantics $\boldsymbol{h}$. The attention model is defined as:

$$\alpha(i) \propto \exp(\boldsymbol{W}_\alpha^T \tanh(\boldsymbol{W}_h \boldsymbol{h} + \boldsymbol{W}_s \boldsymbol{s}_a(i))) \tag{13}$$

$$\boldsymbol{c}(\boldsymbol{a}, \boldsymbol{h}) = \sum\nolimits_i \boldsymbol{s}_a(i)\alpha(i) \tag{14}$$

$$\boldsymbol{z}_a(\boldsymbol{a}, \boldsymbol{h}) = \tanh\left(\boldsymbol{W}_a \boldsymbol{c}(\boldsymbol{a}, \boldsymbol{h}) + \boldsymbol{W}_n \boldsymbol{s}_a(|\boldsymbol{a}|)\right) \tag{15}$$

where $\alpha(i)$ is the normalised attention score at answer token i, and the context vector $\boldsymbol{c}(\boldsymbol{a}, \boldsymbol{h})$ is the weighted sum of all the state outputs $\boldsymbol{s}_a(i)$. We adopt $\boldsymbol{z}_q(\boldsymbol{q}), \boldsymbol{z}_a(\boldsymbol{a}, \boldsymbol{h})$ as the question and answer representations for predicting their relatedness $\boldsymbol{y}$. $\boldsymbol{z}_q(\boldsymbol{q})$ is a deterministic vector that is equal to $\boldsymbol{s}_q(|\boldsymbol{q}|)$, while $\boldsymbol{z}_a(\boldsymbol{a},\boldsymbol{h})$ is a combination of the sequence output $\boldsymbol{s}_a(|\boldsymbol{a}|)$ and the context vector $\boldsymbol{c}(\boldsymbol{a},\boldsymbol{h})$ (Eq. 15). For the prediction of pair relatedness $\boldsymbol{y}$, we model the conditional probability distribution $p_\theta(\boldsymbol{y}|\boldsymbol{z}_q,\boldsymbol{z}_a)$ by sigmoid function:

$$p_\theta(\boldsymbol{y}=1|\boldsymbol{z}_q,\boldsymbol{z}_a)=\sigma\left(\boldsymbol{z}_q^T\mathbf{M}\boldsymbol{z}_a+b\right) \tag{16}$$

To maximise the log-likelihood $\log p(\boldsymbol{y}|\boldsymbol{q},\boldsymbol{a})$ we use the variational lower bound:

$$\begin{aligned}\log p(\boldsymbol{y}|\boldsymbol{q},\boldsymbol{a}) &= \log\sum\nolimits_{\boldsymbol{h}} p_\theta(\boldsymbol{y}|\boldsymbol{z}_q(\boldsymbol{q}),\boldsymbol{z}_a(\boldsymbol{a},\boldsymbol{h}))p_\theta(\boldsymbol{h}|\boldsymbol{q}) &(17)\\ &\geqslant \mathbb{E}_{q_\phi(\boldsymbol{h})}[\log p_\theta(\boldsymbol{y}|\boldsymbol{z}_q(\boldsymbol{q}),\boldsymbol{z}_a(\boldsymbol{a},\boldsymbol{h}))]-D_{\mathrm{KL}}(q_\phi(\boldsymbol{h})||p_\theta(\boldsymbol{h}|\boldsymbol{q}))=\mathcal{L} &(18)\end{aligned}$$

Following the neural variational inference framework, we construct a deep neural network as the inference network $q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})=\mathcal{N}(\boldsymbol{h}|\boldsymbol{\mu}_\phi(\boldsymbol{q},\boldsymbol{a},\boldsymbol{y}),\mathrm{diag}(\boldsymbol{\sigma}^2_\phi(\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})))$:

$$\boldsymbol{\pi}_\phi=g_\phi(f_q^{\text{LSTM}}(\boldsymbol{q}),f_a^{\text{LSTM}}(\boldsymbol{a}),f_y(\boldsymbol{y}))=g_\phi(\boldsymbol{s}_q(|\boldsymbol{q}|),\boldsymbol{s}_a(|\boldsymbol{a}|),\boldsymbol{s}_y) \tag{19}$$

$$\boldsymbol{\mu}_\phi=l_3(\boldsymbol{\pi}_\phi),\log\boldsymbol{\sigma}_\phi=l_4(\boldsymbol{\pi}_\phi) \tag{20}$$

where $\boldsymbol{q}$ and $\boldsymbol{a}$ are modelled by LSTMs[1], and the relatedness label $\boldsymbol{y}$ is modelled by a simple linear transformation into the vector $\boldsymbol{s}_y$. According to the joint representation $\boldsymbol{\pi}_\phi$, we then generate the parameters $\boldsymbol{\mu}_\phi$ and $\boldsymbol{\sigma}_\phi$, which parameterise the latent distribution over the question semantics $\boldsymbol{h}$. To emphasise, though both $p_\theta(\boldsymbol{h}|\boldsymbol{q})$ and $q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})$ are modelled as parameterised Gaussian distributions, $q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})$ as an approximation only functions during inference by producing samples to compute the stochastic gradients, while $p_\theta(\boldsymbol{h}|\boldsymbol{q})$ is the generative distribution that generates the samples for predicting the question-answer relatedness $\boldsymbol{y}$.

Based on the samples $\boldsymbol{h}\sim q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})$, we use SGVB to optimise the lower bound (Eq.18). The model parameters θ and the inference network parameters ϕ are updated jointly using their stochastic gradients. In this case, similar to the NVDM, the Gaussian KL divergence $D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y}))\|p_\theta(\boldsymbol{h}|\boldsymbol{q})]$ can be analytically computed during training process. More details about the network structure and computational complexity can be found in Appendix B.2 and C.

5 Experiments

5.1 Dataset & Setup for Document Modelling

We experiment with NVDM on two standard news corpora: the *20NewsGroups*[2] and the Reuters *RCV1-v2*[3] datasets. The *20NewsGroups* dataset is a collection of newsgroup documents, consisting of 11,314 training and 7,531 test articles. The *RCV1-v2* dataset is a large collection from Reuters newswire stories with 794,414 training and 10,000 test cases. We apply the standard preprocessing procedure as Hinton & Salakhutdinov (2009); Mnih & Gregor (2014) and set the vocabulary size of the *20NewsGroups* and *RCV1-v2* datasets as 2,000 and 10,000 respectively.

To make a direct comparison with the prior work we follow the same setup as Hinton & Salakhutdinov (2009), Larochelle & Lauly (2012), Srivastava et al. (2013), and Mnih & Gregor (2014). We train NVDM models with 50 and 200 dimensional document representations respectively. For the construction of the inference network, we use an MLP (Eq. 9) with 2 layers and 500 dimension rectifier linear units, which converts document representations into embeddings. During training we carry out stochastic estimation by taking one sample for computing the expectation of the stochastic gradients, while in prediction we use 20 samples for predicting document perplexity. The model is trained by Adam (Kingma & Ba, 2015) and alternately optimise the generative model and the inference network by fixing the parameters of one while updating the parameters of the other.

5.2 Experimental Results on Document Modelling

Table 1a presents the results of document modelling on the test datasets. The first column lists the baseline models, and the second column shows the dimension of latent variables used in the

[1] In this case, the LSTMs for $\boldsymbol{q}$ and $\boldsymbol{a}$ are shared by the inference network and the generative model, but there is no restriction on using different LSTMs in the inference network.

[2] http://qwone.com/ jason/20Newsgroups

[3] http://trec.nist.gov/data/reuters/reuters.html

Model	WordDim	20News	RCV1
LDA	50	1091	1437
LDA	200	1058	1142
RSM	50	953	988
docNADE	50	896	742
SBN	50	909	784
fDARN	50	917	724
fDARN	200	—	598
NVDM	50	**836**	563
NVDM	200	852	**550**

(a) Perplexity on test dataset.

Space	***Religion***	***Encryption***	***Sport***	***Policy***
orbit	muslims	rsa	goals	bush
lunar	worship	cryptography	pts	resources
solar	belief	crypto	teams	charles
shuttle	genocide	keys	league	austin
moon	jews	pgp	team	bill
launch	islam	license	players	resolution
fuel	christianity	secure	nhl	mr
nasa	atheists	key	stats	misc
satellite	muslim	escrow	min	piece
japanese	religious	trust	buf	marc

(b) The topics learned by NVDM on 20News.

Table 1: (a) In the first group, LDA (Blei et al., 2003) is a traditional topic model that models documents by mixtures of topics, RSM (Hinton & Salakhutdinov, 2009) is an undirected topic model implemented by restricted Boltzmann machines, and docNADE (Larochelle & Lauly, 2012) is a neural topic model based on autoregressive assumption. In the second group, the models based on Sigmoid Belief Networks (SBN) and Deep AutoRegressive Neural Network (DARN) structures are implemented by Mnih & Gregor (2014), which employs an MLP to build a Monte Carlo control variate estimator for stochastic estimation. The third group presents the perplexity results of our NVDM. (b) The table lists the 5 topics and their 10 most indicative words on 20NewsGroups.

Word	**weapons**	**medical**	**companies**	**define**	**israel**	**book**	**windows**
NVDM	guns	medicine	expensive	defined	israeli	books	dos
	weapon	health	industry	definition	arab	reference	microsoft
	gun	treatment	company	printf	arabs	guide	ms
	militia	disease	market	int	lebanon	writing	file
	armed	patients	buy	sufficient	lebanese	pages	setup
NADE	weapon	treatment	demand	defined	israeli	reading	dos
	shooting	medecine	commercial	definition	israelis	read	microsoft
	firearms	patients	agency	refer	arab	books	version
	assault	process	company	make	palestinian	relevent	ms
	armed	studies	credit	examples	arabs	collection	pc

Table 2: The five nearest words based in the semantic space learned by NVDM and NADE.

experiments. The final two columns present the perplexity achieved by each topic model on the *20NewsGroups* and *RCV1-v2* datasets. In document modelling, perplexity is computed by $exp(-\frac{1}{D}\sum_n^{N_d}\frac{1}{N_d}\log p(\boldsymbol{X}_d))$, where D is the number of documents, N_d represents the length of the dth document and $\log p(\boldsymbol{X}) = \log \int p(\boldsymbol{X}|\boldsymbol{h})p(\boldsymbol{h})d\boldsymbol{h}$ is the log probability of the words in the document. Since $\log p(\boldsymbol{X})$ is intractable in the NVDM, we use the variational lower bound (which is an upper bound on perplexity) to compute the perplexity as in Mnih & Gregor (2014).

While all the baseline models listed in Table 1a apply discrete latent variables, here NVDM employs a continuous stochastic document representation. The experiment results indicate that our NVDM achieves the best performance on both two datasets. For the experiments on *RCV1-v2* dataset, the NVDM with latent variable of 50 dimension performs even better than the fDARN with 200 dimension. It demonstrates that our document model with continuous latent variables has higher expressiveness and better generalisation ability.

In addition to the experiments on perplexity, we also qualitatively evaluate the semantic information learned by NVDM on the *20NewsGroups* dataset with latent variables of 50 dimension. We assume each dimension in the latent space represents a topic that corresponds to a specific semantic meaning. Table 1b presents 5 randomly selected topics with 10 words that have the strongest positive connection with the topic. Based on the words in each column, we can deduce their corresponding topics as: *Space, Religion, Encryption, Sport* and *Policy*. Table 2 compares the 5 nearest words selected according to the semantic vector learned from NVDM and docNADE. Although the model does not impose independent interpretability on the latent representation dimensions, we still see

Source	Set	Questions	QA Pairs	Judgement
QASent	Train	1,229	53,417	automatic
	Dev	82	1,148	manual
	Test	100	1,517	manual
WikiQA	Train	2,118	20,360	manual
	Dev	296	2,733	manual
	Test	633	6,165	manual

Table 3: Statistics of *QASent* and *WikiQA*. Judgement denotes whether correctness was determined automatically or by human annotators.

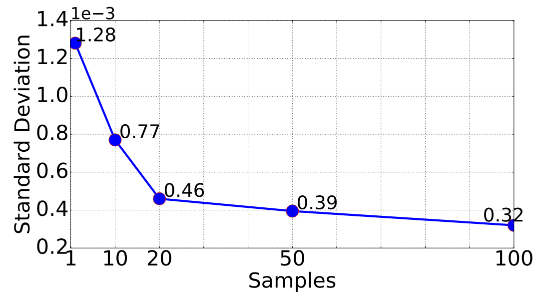

Figure 3: The standard deviations of MAP scores computed by running 10 NASM models on WikiQA with different numbers of samples.

that the NVDM learns locally interpretable structure. In addition to the listed results, Appendix A presents a t-SNE visualisation (Van der Maaten & Hinton, 2008) of the document representations.

5.3 Dataset & Setup for Answer Sentence Selection

We experiment on two answer selection datasets, the *QASent* and the *WikiQA* datasets. *QASent* (Wang et al., 2007) was created from the TREC QA track, and the *WikiQA* (Yang et al., 2015) is constructed from Wikipedia, which is less noisy and less biased towards lexical overlap[4]. Table 3 summarises the statistics of the two datasets.

In order to investigate the effectiveness of our NASM model we also implemented two strong baseline models — a vanilla LSTM model (LSTM) and an LSTM model with a deterministic attention mechanism (LSTM+Att). For the former, it directly applies the QA matching function (Eq. 16) on the last state outputs $\boldsymbol{s}_q(|\boldsymbol{q}|)$ and $\boldsymbol{s}_a(|\boldsymbol{a}|)$ from the question and answer LSTM models. For the latter, we add an attention model to learn pair-specific representation for prediction. Moreover, LSTM+Att is the deterministic counterpart of NASM, which has the same neural network architecture as NASM. The only difference is that it replaces the stochastic units $\boldsymbol{h}$ with deterministic ones, and no inference network is required to carry out stochastic estimation. Following previous work, for each of our models we also add a lexical overlap feature by combining a co-occurrence word count feature with the probability generated from the neural model. Besides, we adopt MAP and MRR as the evaluation metrics for this task.

To facilitate direct comparison with previous work we follow the same experimental setup as Yu et al. (2014) and Severyn (2015). The word embeddings ($K = 50$) are obtained by running the `word2vec` tool (Mikolov et al., 2013) on the English Wikipedia dump and the *AQUAINT*[5] corpus. We use LSTMs with 3 layers and 50 hidden units, and apply 40% dropout after the embedding layer. For the construction of the inference network, we use an MLP (Eq. 11) with 2 layers and tanh units of 50 dimension, and an MLP (Eq. 19) with 2 layers and tanh units of 150 dimension for the joint representation. In training we carry out stochastic estimation by taking one sample for computing the gradients, while in prediction, we use 20 samples to calculate the expectation of the lower bound. Figure 3 presents the standard deviation of NASM's MAP scores while using different numbers of samples. Considering the trade-off between computational cost and variance, we chose 20 samples for prediction in all the experiments. The models are trained using Adam (Kingma & Ba, 2015), with hyperparameters selected by optimising the MAP score on the development set.

5.4 Experimental Results on Answer Sentence Selection

Table 4 compares the results of our models with current state-of-the-art models on both answer selection datasets. As shown in Table 4a, on the *QASent* dataset, our vanilla LSTM model outperforms the deep CNN [6] model by approximately 7% on MAP and 6% on MRR. The LSTM+Att performs

[4] Yang et al. (2015) provide detailed explanation of the differences between the two datasets.

[5] `https://catalog.ldc.upenn.edu/LDC2002T31`

[6] As stated in (Yih et al., 2013) that the evaluation scripts used by previous work are noisy — 4 out of 72 questions in the test set are treated answered incorrectly. This makes the MAP and MRR scores $\sim 4\%$ lower

System	MAP	MRR
Published Models		
Bigram-CNN	0.5693	0.6613
Deep CNN	0.5719	0.6621
WA	0.7063	0.7740
LCLR	0.7092	0.7700
Bigram-CNN +Cnt	0.7113	0.7846
Deep CNN + Cnt	0.7186	0.7826
Our Models		
LSTM	0.6436	0.7235
LSTM + Att	0.6451	0.7316
NASM	**0.6501**	**0.7324**
LSTM + Cnt	0.7228	0.7986
LSTM + Att + Cnt	0.7289	0.8072
NASM + Cnt	**0.7339**	**0.8117**

(a) Experiment results on *QASent*

System	MAP	MRR
Published Models		
PV	0.5110	0.5160
Bigram-CNN	0.6190	0.6281
PV + Cnt	0.5976	0.6058
LCLR	0.5993	0.6068
Bigram-CNN + Cnt	0.6520	0.6652
Our Models		
LSTM	0.6552	0.6747
LSTM + Att	0.6639	0.6828
NASM	**0.6705**	**0.6914**
LSTM + Cnt	0.6820	0.6988
LSTM + Att + Cnt	0.6855	0.7041
NASM + Cnt	**0.6886**	**0.7069**

(b) Experiment results on *WikiQA*

Table 4: (a) Results of our models (LSTM, LSTM + Att, NASM) in comparison with other state of the art models on the *QASent* dataset. Bigram-CNN is the simple convolutional model reported in (Yu et al., 2014). Deep CNN is the deep convolutional model from (Severyn, 2015). WA is a model based on word alignment (Wang & Ittycheriah, 2015). LCLR is the SVM-based classifier trained using a set of features. Model + Cnt means that the result is obtained from a combination of a lexical overlap feature and the output from the distributional model. (b) Results of models on the *WikiQA* dataset. PV is the paragraph vector (Le & Mikolov, 2014).

Q1	**how old was sue lyon when she made lolita**
A_{NASM}	**the actress who played lolita , sue lyon , was fourteen at the time of filming .**
A_{LSTM}	**the actress who played lolita , sue lyon , was fourteen at the time of filming .**
Q2	**how much is centavos in mexico**
A_{NASM}	**the peso is subdivided into 100 centavos , represented by " _UNK_ "**
A_{LSTM}	**the peso is subdivided into 100 centavos , represented by " _UNK_ "**
Q3	**how is jerky made**
A_{NASM}	**jerky is lean meat that has been trimmed of fat , cut into strips , and then dried to prevent spoilage .**
A_{LSTM}	**jerky is lean meat that has been trimmed of fat , cut into strips , and then dried to prevent spoilage .**

Figure 4: A visualisation of attention scores on answer sentences. A_{NASM} and A_{LSTM} visualise the attention scores by darkness of the colour, which are achieved from NASM and LSTM+Att. The questions and their corresponding correct answer sentences are selected from *WikiQA* test dataset.

slightly better than the vanilla LSTM model, and our NASM improves the results further. Since the *QASent* dataset is biased towards lexical overlapping features, after combining with a co-occurrence word count feature, our best model NASM outperforms all the previous models, including both neural network based models and classifiers with a set of hand-crafted features (e.g. LCLR). Similarly, on the *WikiQA* dataset, all of our models outperform the previous distributional models by a large margin. By including a word count feature, our models improve further and achieve the state-of-the-art. Notably, on both datasets, our two LSTM-based models have set strong baselines and NASM works even better, which demonstrates the effectiveness of introducing stochastic units to model question semantics in this answer sentence selection task. In addition, the Hinton diagrams in Appendix D reveals the interesting information captured by the latent distribution.

than the *true* scores. Severyn (2015) and Wang & Ittycheriah (2015), however, use a cleaned-up version of the evaluation scripts. In order to make our results directly comparable with previous work, we use the noisy evaluation scripts; and scale Severyn's and Wang's results by re-evaluating their outputs with the noisy scripts.

In Figure 4, we compare the effectiveness of the latent attention mechanism (NASM) and its deterministic counterpart (LSTM+Att) by visualising the attention scores on the answer sentences. For most of the sentences that are not answering the question, neither of the two attention models can attend to reasonable words that are beneficial for predicting relatedness. But for the correct answer sentences, such as the ones in Figure 4, both attention models are able to capture crucial information by attending to different parts of the sentence based on the question semantics. Interestingly, compared to the deterministic counterpart LSTM+Att, our NASM assigns higher attention scores on the prominent words that are relevant to the question, which forms a more peaked distribution and in turn helps the model achieve better performance.

6 Related Work

Training an inference network to approximate the variational distribution was first proposed in the context of Helmholtz machines (Hinton & Zemel, 1994; Hinton et al., 1995; Dayan & Hinton, 1996), but applications of these directed generative models come up against the problem of establishing low variance gradient estimators. Recent advances in neural variational inference mitigate this problem by reparameterising the continuous random variables via differentiable transformation (Rezende et al., 2014; Kingma & Welling, 2014), using control variates (Mnih & Gregor, 2014) or approximating the posterior with importance sampling (Bornschein & Bengio, 2015). The instantiations of these ideas (Gregor et al., 2015; Kingma et al., 2014; Ba et al., 2015) have demonstrated strong performance on the tasks of image generation, image classification and caption generation.

Another class of neural generative models make use of the autoregressive assumption to model high-dimensional input distributions (Larochelle & Murray, 2011; Uria et al., 2014; Germain et al., 2015). Deep AutoRegressive Networks (DARN) (Gregor et al., 2014) integrate this idea with variational inference. Applications of these models on document modelling achieve significant improvements on generating documents, compared to conventional probabilistic topic models (Hofmann, 1999; Blei et al., 2003) and also the RBMs (Hinton & Salakhutdinov, 2009; Srivastava et al., 2013). Different from these models that use binary semantic vectors, our NVDM employs dense continuous document representations which are both expressive and easy to train. The semantic word vector model (Maas et al., 2011) also employs a continuous semantic vector to generate words, but the model is trained by MAP inference which does not permit the calculation of the posterior distribution.

Prior work on question answering relies on classifiers with large numbers of hand-crafted syntactic and semantic features and various external resources. Only very recently have researchers started to apply deep learning to question answering. Relevant work includes mapping factoid questions with answer triples in the knowledge base by projecting them into a shared vector space using convolutional neural networks (Bordes et al., 2014a;b; Yih et al., 2014). Recently, the attention-based learning models (Bahdanau et al., 2015) are applied to QA, where long-term memories act as dynamic knowledge bases (Weston et al., 2015; Sukhbaatar et al., 2015; Kumar et al., 2015) or the attentive network helps read and comprehend (Hermann et al., 2015).

Our NASM effectively combines the strengths of LSTMs with attention mechanism and stochastic units, which equips our model with stronger capability to infer the correctness of an answer to a given question. Stochastic Feedforward Neural Networks (SFNN) (Tang & Salakhutdinov, 2013) applied similar idea of introducing stochastic units for expression classification. However, the inference is carried out by Monte Carlo EM algorithm with the reliance on importance sampling, which is less efficient and lack of scalability.

7 Conclusion

This paper introduces a deep neural variational inference framework for generative models of text. To demonstrate the effectiveness of this framework, we experimented on two diverse tasks, document modelling and question answer selection tasks, where in both cases our models achieve state of the art performance. Apart from the promising results, our model also has the advantages of (1) simple, expressive, and efficient when training with the SGVB algorithm; (2) suitable for both unsupervised and supervised learning tasks; (3) capable of generalising to incorporate any type of neural network.

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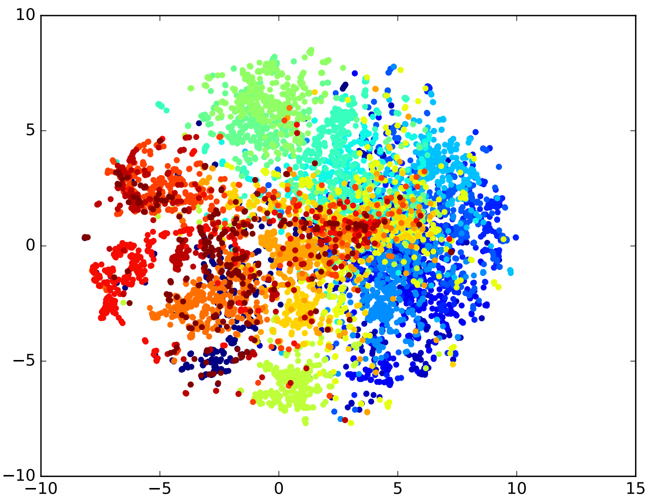
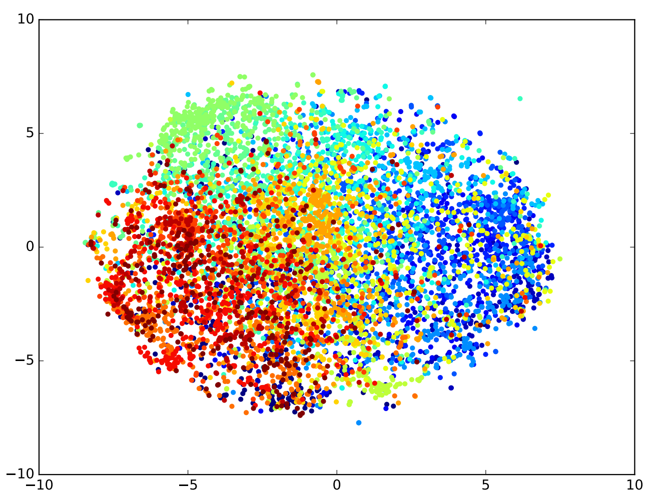
A t-SNE Visualisation of Document Representations



(a) Neural Variational Document Model



(b) Semantic Word Vector

Figure 5: t-SNE visualisation of the document representations achieved by (a) NVDM and (b) SWV (Maas et al., 2011) on the held-out test dataset of *20NewsGroups*. The documents are collected from 20 different news groups, which correspond to the points with different colour in the figure.

B Details of the Deep Neural Network Structures

B.1 Neural Variational Document Model

(1) Inference Network $q_\phi(\boldsymbol{h}|\boldsymbol{X})$:

$$\boldsymbol{\lambda} = \mathrm{ReLU}(\boldsymbol{W}_1\boldsymbol{X} + \boldsymbol{b}_1) \tag{21}$$
$$\boldsymbol{\pi} = \mathrm{ReLU}(\boldsymbol{W}_2\boldsymbol{\lambda} + \boldsymbol{b}_2) \tag{22}$$
$$\boldsymbol{\mu} = \boldsymbol{W}_3\boldsymbol{\pi} + \boldsymbol{b}_3 \tag{23}$$
$$\log\boldsymbol{\sigma} = \boldsymbol{W}_4\boldsymbol{\pi} + \boldsymbol{b}_4 \tag{24}$$
$$\boldsymbol{h} \sim \mathcal{N}(\boldsymbol{\mu}(\boldsymbol{X}), \mathrm{diag}(\boldsymbol{\sigma}^2(\boldsymbol{X}))) \tag{25}$$

(2) Generative Model $p_\theta(\boldsymbol{X}|\boldsymbol{h})$:

$$\boldsymbol{e}_i = \exp(-\boldsymbol{h}^T\boldsymbol{R}\boldsymbol{x}_i + \boldsymbol{b}_{x_i}) \tag{26}$$
$$p_\theta(\boldsymbol{x}_i|\boldsymbol{h}) = \frac{\boldsymbol{e}_i}{\sum_j^{|V|}\boldsymbol{e}_j} \tag{27}$$
$$p_\theta(\boldsymbol{X}|\boldsymbol{h}) = \prod_i^N p_\theta(\boldsymbol{x}_i|\boldsymbol{h}) \tag{28}$$

(3) KL Divergence $D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{X})||p(\boldsymbol{h})]$:

$$D_{\mathrm{KL}} = \tfrac{1}{2}(K - \|\boldsymbol{\mu}\|^2 - \|\boldsymbol{\sigma}\|^2 + \log|\,\mathrm{diag}(\boldsymbol{\sigma}^2)|) \tag{29}$$

The variational lower bound to be optimised:

$$\mathcal{L} = \mathbb{E}_{q_\phi(\boldsymbol{h}|\boldsymbol{X})}\left[\sum_{i=1}^N \log p_\theta(\boldsymbol{x}_i|\boldsymbol{h})\right] - D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{X})||p(\boldsymbol{h})] \tag{30}$$
$$\approx \sum_{l=1}^L\sum_{i=1}^N \log p_\theta(\boldsymbol{x}_i|\boldsymbol{h}^{(l)}) - \frac{1}{2}(K - \|\boldsymbol{\mu}\|^2 - \|\boldsymbol{\sigma}\|^2 + \log|\,\mathrm{diag}(\boldsymbol{\sigma}^2)|) \tag{31}$$

B.2 Neural Answer Selection Model

(1) Inference Network $q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})$:

$$\boldsymbol{s}_q(|\boldsymbol{q}|) = f_q^{\mathrm{LSTM}}(\boldsymbol{q}) \tag{32}$$
$$\boldsymbol{s}_a(|\boldsymbol{a}|) = f_a^{\mathrm{LSTM}}(\boldsymbol{a}) \tag{33}$$
$$\boldsymbol{s}_y = \boldsymbol{W}_5\boldsymbol{y} + \boldsymbol{b}_5 \tag{34}$$
$$\boldsymbol{\gamma} = \boldsymbol{s}_q(|\boldsymbol{q}|)||\boldsymbol{s}_a(|\boldsymbol{a}|)||\boldsymbol{s}_y \tag{35}$$
$$\boldsymbol{\lambda}_\phi = \tanh(\boldsymbol{W}_6\boldsymbol{\gamma} + \boldsymbol{b}_6) \tag{36}$$
$$\boldsymbol{\pi}_\phi = \tanh(\boldsymbol{W}_7\boldsymbol{\lambda}_{\boldsymbol{\phi}} + \boldsymbol{b}_7) \tag{37}$$
$$\boldsymbol{\mu}_\phi = \boldsymbol{W}_8\boldsymbol{\pi}_{\boldsymbol{\phi}} + \boldsymbol{b}_8 \tag{38}$$
$$\log\boldsymbol{\sigma}_\phi = \boldsymbol{W}_9\boldsymbol{\pi}_{\boldsymbol{\phi}} + \boldsymbol{b}_9 \tag{39}$$
$$\boldsymbol{h} \sim \mathcal{N}(\boldsymbol{\mu}_\phi(\boldsymbol{q},\boldsymbol{a},\boldsymbol{y}), \mathrm{diag}(\boldsymbol{\sigma}_\phi^2(\boldsymbol{q},\boldsymbol{a},\boldsymbol{y}))) \tag{40}$$

(2) Generative Model

$p_\theta(\boldsymbol{h}|\boldsymbol{q})$:

$$\boldsymbol{\lambda}_\theta = \tanh(\boldsymbol{W}_1\boldsymbol{s}_q(|\boldsymbol{q}|) + \boldsymbol{b}_1) \tag{41}$$
$$\boldsymbol{\pi}_\theta = \tanh(\boldsymbol{W}_2\boldsymbol{\lambda}_\theta + \boldsymbol{b}_2) \tag{42}$$
$$\boldsymbol{\mu}_\theta = \boldsymbol{W}_3\boldsymbol{\pi}_\theta + \boldsymbol{b}_3 \tag{43}$$
$$\log\boldsymbol{\sigma}_\theta = \boldsymbol{W}_4\boldsymbol{\pi}_\theta + \boldsymbol{b}_4 \tag{44}$$

$p_\theta(\boldsymbol{y}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{h})$:

$$\boldsymbol{e}(i) = \boldsymbol{W}_\alpha^T \tanh(\boldsymbol{W}_h\boldsymbol{h} + \boldsymbol{W}_s\boldsymbol{s}_a(i)) \tag{45}$$

$$\alpha(i) = \frac{\boldsymbol{e}(i)}{\sum_j \boldsymbol{e}(j)} \tag{46}$$

$$\boldsymbol{c}(\boldsymbol{a},\boldsymbol{h}) = \textstyle\sum_i \boldsymbol{s}_a(i)\alpha(i) \tag{47}$$

$$\boldsymbol{z}_a(\boldsymbol{a},\boldsymbol{h}) = \tanh(\boldsymbol{W}_a\boldsymbol{c}(\boldsymbol{a},\boldsymbol{h}) + \boldsymbol{W}_n\boldsymbol{s}_a(|\boldsymbol{a}|)) \tag{48}$$

$$\boldsymbol{z}_q(\boldsymbol{q}) = \boldsymbol{s}_q(|\boldsymbol{q}|) \tag{49}$$

$$p_\theta(\boldsymbol{y}=1|\boldsymbol{q},\boldsymbol{a},\boldsymbol{h}) = \sigma(\boldsymbol{z}_q^T\boldsymbol{M}\boldsymbol{z}_a + b) \tag{50}$$

(3) KL Divergence $D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})||p_\theta(\boldsymbol{h}|\boldsymbol{q})]$:

$$\begin{aligned} D_{\mathrm{KL}} \;=\;& \frac{1}{2}(K + \log|\operatorname{diag}(\boldsymbol{\sigma}_\phi^2)| - \log|\operatorname{diag}(\boldsymbol{\sigma}_\theta^2)| - \operatorname{Tr}(\operatorname{diag}(\boldsymbol{\sigma}_\phi^2)\operatorname{diag}^{-1}(\boldsymbol{\sigma}_\theta^2)) \\ & -(\boldsymbol{\mu}_\phi - \boldsymbol{\mu}_\theta)^T\operatorname{diag}^{-1}(\boldsymbol{\sigma}_\theta^2)(\boldsymbol{\mu}_\phi - \boldsymbol{\mu}_\theta)) \end{aligned} \tag{51}$$

The variational lower bound to be optimised:

$$\mathcal{L} \;=\; \mathbb{E}_{q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})}[\log p_\theta(\boldsymbol{y}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{h})] - D_{\mathrm{KL}}[q_\phi(\boldsymbol{h}|\boldsymbol{q},\boldsymbol{a},\boldsymbol{y})||p_\theta(\boldsymbol{h}|\boldsymbol{q})] \tag{52}$$

$$\begin{aligned} \approx\;& \sum_{l=1}^{L} \boldsymbol{y}\log\sigma(\boldsymbol{z}_q^T\boldsymbol{M}\boldsymbol{z}_a^{(l)} + b) + (1-\boldsymbol{y})\log(1-\sigma(\boldsymbol{z}_q^T\boldsymbol{M}\boldsymbol{z}_a^{(l)} + b)) \\ & -\frac{1}{2}(K + \log|\operatorname{diag}(\boldsymbol{\sigma}_\phi^2)| - \log|\operatorname{diag}(\boldsymbol{\sigma}_\theta^2)| - \operatorname{Tr}(\operatorname{diag}(\boldsymbol{\sigma}_\phi^2)\operatorname{diag}^{-1}(\boldsymbol{\sigma}_\theta^2)) \\ & -(\boldsymbol{\mu}_\phi - \boldsymbol{\mu}_\theta)^T\operatorname{diag}^{-1}(\boldsymbol{\sigma}_\theta^2)(\boldsymbol{\mu}_\phi - \boldsymbol{\mu}_\theta)) \end{aligned} \tag{53}$$

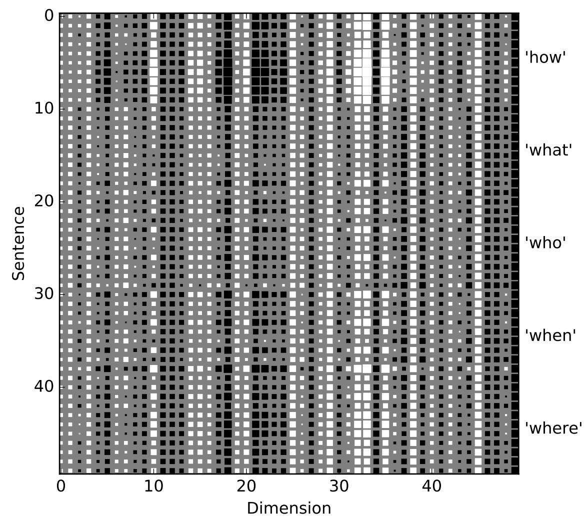
C Computational Complexity

The computational complexity of NVDM for a training document is $C_\phi + C_\theta = O(LK^2 + KSV)$. Here, $C_\phi = O(LK^2)$ represents the cost for the inference network to generate a sample, where L is the number of the layers in the inference network and K is the average dimension of these layers. Besides, $C_\theta = O(KSV)$ is the cost of reconstructing the document from a sample, where S is the average length of the documents and V represents the volume of words applied in this document model, which is conventionally much lager than K.

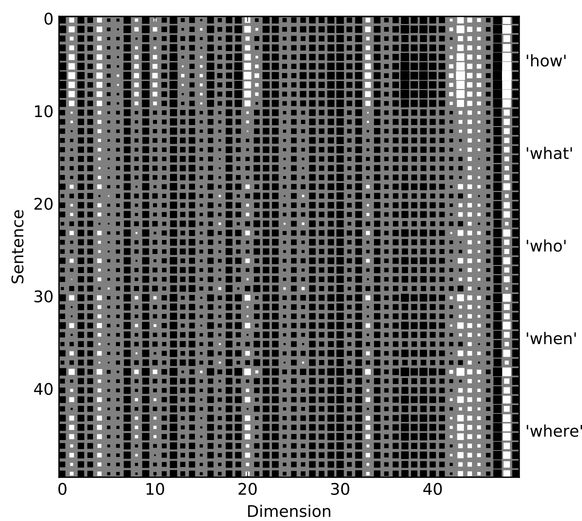
The computational complexity of NASM for a training question-answer pair is $C_\phi + C_\theta = O((L+S)K^2 + SW)$. The inference network needs $C_\phi = 2SW + 2K + LK^2 = O(LK^2 + SW)$. It takes $2SW + 2K$ to produce the joint representation for a question-answer pair and its label, where W is the total number of parameters of an LSTM and S is the average length of the sentences. Based on the joint representation, an MLP spends LK^2 to generate a sample, where L is the number of layers and K represents the average dimension. The generative model requires $C_\theta = 2SW + LK^2 + SK^2 + 5K^2 + 2K^2 = O((L+S)K^2 + SW)$. Similarly, it costs $2SW + LK^2$ to construct the generative latent distribution , where $2SW$ can be saved if the LSTMs are shared by the inference network and the generative model. Besides, the attention model takes $SK^2 + 5K^2$ and the relatedness prediction takes the last $2K^2$.

Since the computations of NVDM and NASM can be parallelised in GPU and only one sample is required during training process, it is very efficient to carry out the neural variational inference. As NVDM is an instantiation of variational auto-encoder, its computational complexity is the same as the deterministic auto-encoder. In addition, the computational complexity of LSTM+Att, the deterministic counterpart of NASM, is also $O((L+S)K^2 + SW)$. There is only $O(LK^2)$ time increase by introducing an inference network for NASM when compared to LSTM+Att.

D HINTON DIAGRAMS OF THE GAUSSIAN PARAMETERS IN NASM



(a) Mean



(b) Log standard deviation

Figure 6: Hinton diagrams of the means and log standard deviations, which parameterise the latent distribution over question semantics. In a Hinton diagram, the size of a square is proportional to a value's magnitude, and the colour (black/white) indicates its sign (positive/negative). In this case, we visualise 50 instances that are conditional distributions $p(\boldsymbol{h}|\boldsymbol{q})$ given the questions from 5 different groups, which start with 'how', 'what', 'who', 'when' and 'where'. The means are randomly initialised while the initial log standard deviations are set as zero. According to (b), we can see that the questions starting with 'how' have more white areas, which indicates higher variances or more uncertainties are in these dimensions. By contrast, the questions starting with 'what' have black squares in almost every dimension. Intuitively, it is more difficult to understand and answer the questions starting with 'how' than the others, while the 'what' questions commonly have explicit words indicating the possible answers. Interestingly, the questions with 'when', 'who' and 'where' have similar distributions on both mean and variance.