# Machine Learning-Based Coarse-Scale Identification and Bifurcation Analysis for Modelling Tipping Points: a Financial Market Agent-Based Illustration

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**Abstract**

We present a machine learning framework that targets the systematic identification and bifurcation analysis of effective coarse-grained dynamical laws describing the emergent abrupt changing behavior of agent-based models near tipping points. In particular, based on high-fidelity spatio-temporal data, obtained in correspondence of different initial condition and different value of a tracked parameter, we solve the inverse problem to learn reduced effective low-dimensional macroscopic laws in the form of (a) Integro-Partial differential Equation for coarse-scale space-dependent fields and/or (b) Stochastic differential equation (SDE) for global variables. We illustrate and compare the two approaches through an event-driven agent-based financial model describing the mimetic behavior of many interacting investors. Specifically, employing Diffusion Maps and Gaussian Processes for discovering low-dimensional manifold embeddings and Deep feedforward neural networks (DFNN) and single-layer Random Projection Neural Networks(RPNNs) for learning effective coarse-scale differential operators. Based on the learned black-box Integro or StochasticDifferential model, we construct the corresponding bifurcation diagram, thus we investigate the stability of its multiple solution branches exploiting the numerical bifurcation analysis tools. Finally, close to the tipping point we compute the escaping time exploiting Monte-Carlo simulation and a quadrature approach.