1)

(1) $g(0) = \dim(\mathrm{Null}(A - 0I)) = \dim(\mathrm{Null}(A))$

$$= \dim\left(\mathrm{span}\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}\right) = 1$$

(2) $\because \mathrm{Null}(A^3) \neq \mathrm{Null}(A^2) \wedge \mathrm{Null}(A^4) = \mathrm{Null}(A^3)$

$\therefore n_1(0) = 3$

(3) $A^2 t_3 = k\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow t_3 = k\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \quad k \in \mathbb{R}$

(4) $t_2 = At_3 = k\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad k \in \mathbb{R}$

(5) $t_1 = At_2 = A^2 t_3 = k\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad k \in \mathbb{R}$

(6) $g(1) = \dim(\mathrm{Null}(A - I)) = \dim\left(\mathrm{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}\right)$

$$= 2$$

(7) $\because \mathrm{Null}((A-I)^2) \neq \mathrm{Null}(A-I) \wedge \mathrm{Null}((A-I)^3)$

$= \mathrm{Null}((A-I)^2)$

$\therefore n_1(1) = 2$

8. similar to 3, $V_3 = k\begin{bmatrix}0\\0\\0\\-1\\0\\0\end{bmatrix}$, $k \in \mathbb{R}$

9. $V_2 = k\begin{bmatrix}0\\1\\-1\\0\\0\\0\end{bmatrix}$, $k \in \mathbb{R}$

10. $V_1 = k\begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}$, $k \in \mathbb{R}$

11. $J_1 = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$, $J_2 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$

12.
$$e^{J_1 t} = I + J_1 t + \frac{J_1^2 t^2}{2} \quad (\lambda_1 = 0)$$
$$e^{J_2 t} = e^t\left(I + J_2' t + \frac{J_2'^2 t^2}{2}\right) \quad (\lambda_2 = 1)$$
$$\Rightarrow e^{Jt} = \begin{bmatrix} I + J_1 t + \frac{J_1^2 t^2}{2} & 0 \\ 0 & e^t\left(I + J_2' t + \frac{J_2'^2 t^2}{2}\right) \end{bmatrix}$$

$$\Rightarrow e^{At} = T e^{Jt} T^{-1}$$

2)

(a)

$$zX(z) - zX(0) = AX(z) + BU(z)$$

$$\Rightarrow X(z) = z(zI-A)^{-1}X(0) + (zI-A)^{-1}BU(z)$$

$$z(zI-A)^{-1} = z\begin{bmatrix} z-0.6 & -0.4 \\ -0.1 & z-0.6 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \left(\frac{1}{1-0.4z^{-1}} + \frac{1}{1-0.8z^{-1}}\right)/2 & \frac{-1}{1-0.4z^{-1}} + \frac{1}{1-0.8z^{-1}} \\ \left(\frac{-1}{1-0.4z^{-1}} + \frac{1}{1-0.8z^{-1}}\right)/4 & \left(\frac{1}{1-0.4z^{-1}} + \frac{1}{1-0.8z^{-1}}\right)/2 \end{bmatrix}$$

$$\Rightarrow A^k = \mathcal{Z}^{-1}\left[z(zI-A)^{-1}\right] = \begin{bmatrix} \frac{1}{2}(0.4^k + 0.8^k) & 0.8^k - 0.4^k \\ \frac{1}{4}(0.8^k - 0.4^k) & \frac{1}{2}(0.4^k + 0.8^k) \end{bmatrix}$$

(b) $$\lim_{k\to\infty} x(k) = \lim_{k\to\infty} \sum_{l=1}^{k} A^{l-1}BU(l) = \sum_{l=0}^{\infty} A^l \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot 1$$

$$= \sum_{l=0}^{\infty} \begin{bmatrix} 0.8^l - 0.4^l \\ \frac{1}{2}(0.8^l + 0.4^l) \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ \frac{25}{6} \end{bmatrix}$$

3、

$$G(z) = (1-z^{-1}) Z\left[\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]\right]$$

$$= (1-z^{-1}) Z\left[\mathcal{L}^{-1}\left\{\frac{w^2}{s^2+w^2}\right\}\right]$$

$$= (1-z^{-1}) Z\left[1-\cos wt\right]$$

$$= (1-z^{-1}) \cdot \frac{1-\cos(wt)\, z^{-1}}{1-2\cos(wt) z^{-1} + z^{-2}}$$

(a) $wT = \pi$, $G(z) = (1-z^{-1}) \cdot \dfrac{1+z^{-1}}{1+2z^{-1}+z^{-2}} = \dfrac{1-z^{-1}}{1+z^{-1}}$

$$= \frac{z-1}{z+1}$$

(b) $wT = 2\pi/3$, $G(z) = \dfrac{(z-1)(2z+1)}{2(z^2+z+1)}$

(c) $wT = 4\pi/3$, $G(z) = \dfrac{(z-1)(2z+1)}{2(z^2+z+1)}$

4.

Method 1 — State space model

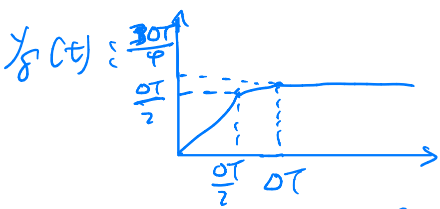
(a) $a=0$, $b=1$, $c=1$

(b)
$$x((k+1)\Delta T)=e^{0}x(k\Delta T)+\int_{k\Delta T}^{(k+1)\Delta T} e^{0(t-\tau)}\, b\, u(\tau)\, d\tau$$
$$=x(k\Delta T)+\int_{k\Delta T}^{(k+\frac{1}{2})\Delta T} u(k)\, d\tau+\int_{(k+\frac{1}{2})\Delta T}^{(k+1)\Delta T} u(k)\, d\tau$$
$$=x(k\Delta T)+\frac{3}{4}u(k)\Delta T$$
$$\Rightarrow a_d=1,\quad b_d=\frac{3}{4}\Delta T$$

(c) $$G(z)=\frac{\frac{3}{4}\Delta T}{z-1}$$

Method 2 — Direct Computation of $G(z)$

(a) $$u(t)=1(t)-\frac{1}{2}1(t-\frac{1}{2}\Delta T)-\frac{1}{2}1(t-\Delta T)$$



(b) $$y_\delta(k)=y_\delta(\Delta t\, k)=\begin{cases}0, & k=0\\ \frac{3}{4}\Delta T, & k\geq 1\end{cases}$$

(c) $$G(z)=\mathcal{Z}\{y_\delta(k)\}=\frac{\frac{3}{4}\Delta T}{z-1}$$

5、 $a_4 = 1$, $a_3 = 2.9$, $a_2 = 2.7$, $a_1 = 0.7$,

$a_0 = K - 0.1$

$$\begin{array}{c|cc} S_4 & 1 & 2.7 \\ S_3 & 2.9 & 0.7 \end{array}$$

$$b_1 = \frac{2.9 \times 2.7 - 1 \times 0.7}{2.9} = \frac{7.13}{2.9} \approx 2.459 > 0 \quad ①$$

$$b_2 = \frac{2.9(K-0.1) - 1 \times 0}{2.9} = K - 0.1 > 0 \quad ②$$

$$\begin{array}{c|cc} S_2 & 2.459 & K-0.1 \end{array}$$

$$c_1 = \frac{2.459 \times 0.7 - 2.9 \times (K-0.1)}{2.459} > 0 \quad ③$$

Combine ①、②、③, we have

$0.1 < K < 0.693$

6.

$$A(z) = z^4 + 0.8z^3 + 0.79z^2 - 0.008z - 0.008$$

Let $z = \frac{1+s}{1-s}$, we have

$$\frac{q}{p} = \frac{0.99s^4 + 2.416s^3 + 4.372s^2 + 5.648s + 2.574}{s^4 - 4s^3 + 6s^2 - 4s + 1}$$

consider q part of $\frac{q}{p}$,

$$\begin{array}{c|ccc} s^4 & 0.99 & 4.372 & 2.574 \\ s^3 & 2.416 & 5.648 & 0 \\ s^2 & -2.058 & 0 & 0 \end{array}$$

$\because$ s^2 has a negative value

$\therefore$ unstable

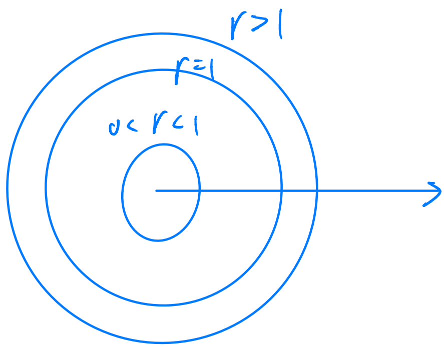
7.

Let $s = jw$

$$\Rightarrow z = \frac{r(1+jw)}{1-jw} = \frac{r(1+jw)^2}{1+w^2}$$

$$= \frac{r(1-w^2+2jw)}{1+w^2}$$

$$= \frac{r(1-w^2)}{1+w^2} + j\,\frac{2rw}{1+w^2}$$



8.

$$Y(z) = G(z)\,C(z)\,(1 - Y(z))$$

$$\Rightarrow \quad Y(z) = \frac{G(z)\,C(z)}{1 + G(z)\,C(z)} = \frac{k}{z(z-0.8)+k}$$

$$\Rightarrow \quad A(z) = z(z - 0.8) + k$$

$$\Rightarrow \quad A^*(z) = A\left(\frac{r(1+s)}{1-s}\right)(1-s)^n$$

$$= (k + 0.65)\,s^2 + (0.5 - 2k)\,s + (k - 0.15)$$

s_2	$k + 0.65$	$k - 0.15$
s_1	$0.5 - 2k$	0
s_0	$k - 0.15$	

$$\Rightarrow \quad 0.15 < K < 0.25$$