OPTIMIZING PREFERENCE ALIGNMENT WITH DIFFERENTIABLE NDCG RANKING

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ABSTRACT

Aligning large language models with human preferences improves interaction quality and safety by ensuring outputs better reflect human values. A promising strategy involves Reinforcement Learning from Human Feedback (RLHF), starting with collecting and ranking responses generated by a supervised fine-tuning model to refine alignment. Current methods (DPO) focus on learning from pairwise preference data, categorizing responses into preferred and less preferred pairs, and optimizing by maximizing pairwise margins. Recent studies have uncovered a substantial discrepancy between the theoretical aspirations of preference learning and its real-world results. Current preference alignment techniques underperform expectations, with ranking accuracies below 60% on standard datasets. This suggests existing methods inadequately capture ideal preference relationships within sequences. To address this challenge, this paper introduces Direct Ranking Preference Optimization (DRPO), a novel method that views human preference alignment as a Learning-to-Rank (LTR) task. DRPO leverages NDCG, a widely used LTR metric, to optimize the ranking of responses within lists based on preference data, thereby enhancing ranking accuracies. Due to the nondifferentiability of NDCG, we propose diffNDCG loss, a differentiable approximation facilitated by a sorting network to simulate NDCG. Furthermore, to improve the quality of generated response, we propose a novel margin-based Adaptive Rank Policy Score. Extensive experiments have shown that DRPO outperforms existing baseline methods, enhancing the quality of the generated responses. The code is publicly available [1].

1 INTRODUCTION

Large language models (LLMs), trained on extensive and diverse datasets, can be prompted to demonstrate impressive capabilities across a broad range of tasks (Huang et al., 2024; Chiang et al., 2023; OpenAI et al., 2024; Touvron et al., 2023). However, due to the varied nature of their training data, these models sometimes produce content that may not align with human preferences, including fabricated answers, offensive comments, or harmful responses (Bai et al., 2022; Wang et al., 2023). To ensure the development of AI systems that are safe and controllable, this paper investigates learning tasks for LLMs that guide them to generate responses in alignment with human preferences.

Human preference alignment has become an active research area. Reinforcement Learning with Human Feedback (RLHF) (Ouyang et al., 2022) is the first proposed method in this area. However, the optimization process of RLHF is complex, and its implementation introduces challenges due to unstable and costly training. Recent studies (Hong et al., 2024; Ethayarajh et al., 2024) have started to adopt alternatives to RLHF. For example, Direct Preference Optimization (DPO) (Rafailov et al., 2023) enables the extraction of the corresponding optimal policy in a closed form and derives a pairwise logistic loss directly from pairwise preference data. DPO eliminates the need for explicit reward modeling or reinforcement learning, thereby reducing the training costs associated with RLHF.

Although significant progress has been made in human preference alignment, most existing methods primarily focus on *pairwise human preferences*, which involve evaluating human preferences by comparing *preferred* and *less-preferred* responses. Nevertheless, human preferences are not solely expressed as preferences and less preferences; they also manifest as ranking information, an aspect

[1] Code and models can be found at https://anonymous.4open.science/r/drpo-align-2758

that has rarely been explored in previous research. In practice, *ranking preference data* is widely utilized. For instance, in the Ultrafeedback and VLfeedback datasets (Li et al., 2023a; Cui et al., 2023), multiple responses are generated using a Supervised Fine-tuning (SFT) model. These responses are then evaluated and ranked by leveraging advanced AI technologies such as GPT-4 (OpenAI et al., 2024). Moreover, as demonstrated by (Liu et al., 2024), presenting ranking preference data effectively distribute the costs associated with processing the prompt (Liu et al., 2023a). Published In this work, we approach human preference alignment as a listwise ranking problem, aiming to align LLMs by utilizing ranking preference data. The straightforward solutions for tackling this issue are to extend existing alignment methods from the pairwise scenarios to the ranking list scenarios. However, a major limitation is that they fail to utilize the relative strengths of the ranking preferences (Zhu et al., 2024). To address this, some studies leveraging ranking preference data have been proposed, proving their effectiveness for preference alignment (Yuan et al., 2023; Liu et al., 2024; Chen et al., 2024; Zhu et al., 2024; Choi et al., 2024). These methods predominantly achieve listwise preference alignment through two main strategies: either by utilizing the Plackett-Luce preference model, or by employing pairwise methods that incorporate listwise-aware weighting schemes (Liu et al., 2024).

However, a *mismatch* persists between evaluation metrics and optimization objectives. LLMs' performance is typically assessed using *win rates*, which measure how often one model's responses are preferred over another's, yet current alignment methods do not directly optimize this criterion. This mismatch means that optimizing current loss functions (Liu et al., 2024; Song et al., 2024) may not necessarily lead to higher win rates or improved human preference satisfaction (Chen et al., 2024), substantially hindering model preference alignment. In contrast, Pobrotyn & Bialobrzeski (2021); Qin et al. (2010a) have demonstrated that direct optimization of evaluation metrics is highly effective in traditional learning-to-rank tasks. Moreover, recent studies (Chen et al., 2024) have demonstrated a correlation between win rates and ranking accuracy, while revealing that current alignment methods achieve low ranking accuracy on standard preference datasets, but the relationship between win rates and more comprehensive metrics like NDCG remains unexplored. Therefore, aligning LLMs with human preferences by directly optimizing evaluation criteria and improving ranking accuracy remains a challenging task that has been scarcely explored to date.

To tackle this challenge, we propose a novel method: Direct Ranking Preference Optimization (DRPO), which is a pioneering exploration to align LLMs with the ranking metric Normalized Discounted Cumulative Gain (NDCG), a metric from Learning to Rank (LTR) that accurately quantifies ranking accuracy and quality. Specifically, We introduce the *Adaptive Rank Policy Score*, a novel ranking strategy that maximizes the absolute likelihood of preferred responses while dynamically adjusting score margins between preferred and non-preferred responses based on their relative positions in the ranked list. Furthermore, we implement *differentiable sorting networks* to sort responses based on computed scores. This sorting method combines implementation simplicity and computational efficiency, while yielding doubly stochastic permutation matrices that preserve probability distributions and enable efficient differentiable optimization (Petersen et al., 2021). Additionally, due to the nondifferentiability of NDCG, we leverage permutation matrices develop a differentiable version of NDCG (i.e., *diffNDCG*) to serve as the loss function, simulating the NDCG metric. Optimizing the diffNDCG loss enhances model performance by prioritizing top-ranked responses and imposing stricter penalties for misplacing highly relevant items, without additional computational overhead compared to existing methods. The main contributions of our work are summarized as follows:

• A novel method Direct Ranking Preference Optimization (DRPO) is developed to explore human preference alignment with ranking preference data. To our best knowledge, this paper is a pioneer exploration to align LLMs with ranking human preference data.

• A novel ranking score computation strategy, the Adaptive Rank Policy Score, has been introduced to replace the classical computation method used in RLHF (Rafailov et al., 2023). It maximizes preferred response likelihood while dynamically adjusting score margins based on relative positions.

• A novel differentiable NDCG (diffNDCG) metric has been developed to emulate the NDCG metric used in LTR. By optimizing diffNDCG, we can prioritize responses at the top of the ranking list and impose stricter penalties for inaccurately placing a top-ranked response in a lower position.

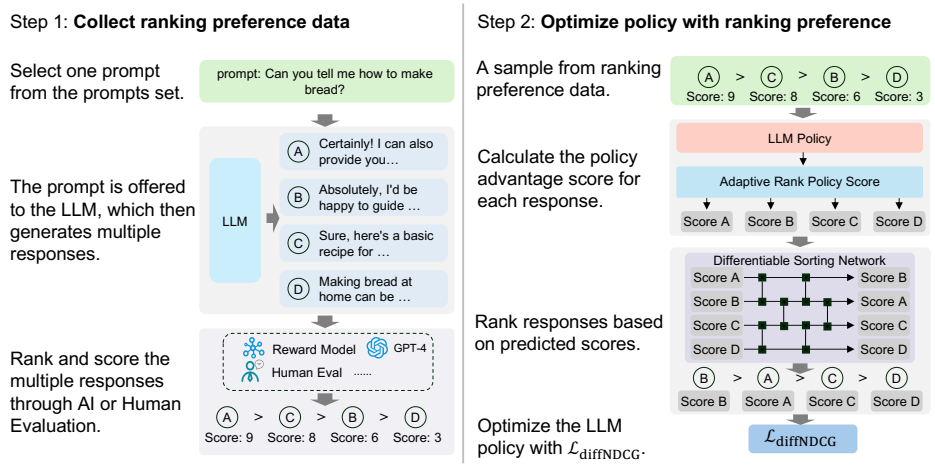


Figure 1: A diagram illustrating the two key steps of our method: (1) collecting ranking preference data, and (2) optimizing the policy with the collected ranking preference data. In Step 2, the Policy Advantage Score computes the score for each input response. Subsequently, the Differentiable Sorting Network sorts these responses based on their scores. We then compute the diffNDCG between the predicted scores and the ground-truth scores, and optimize the policy using the diffNDCG loss.

2 PRELIMINARIES

Prompt, Response and Policy. Let $\mathcal{X}$ and $\mathcal{Y}$ denote the set of prompts and the set of responses (action space), respectively. We use $x \in \mathcal{X}$ to represent a prompt, and $y \in \mathcal{Y}$ to represent a response. Given a prompt x, a large language model (LLM) generates a corresponding response y. This response y is produced according to a policy $\pi_\theta(\cdot|x)$, which is a discrete distribution over $\mathcal{Y}$. We also define $\pi_{\text{ref}}(\cdot|x)$ as a discrete distribution over $\mathcal{Y}$, serving as the reference policy. The reference policy π_{ref} is derived from the Supervised Fine-tuning (SFT) model (Rafailov et al., 2023).

Ranking Preference Data. The training dataset $D = \{x^i, \mathbf{y}^i, \mathbf{s}^i\}_{i=1}^N$ is composed of three elements: x^i represents the i-th prompt; $\mathbf{y}^i = (y_1^i, \ldots, y_K^i)$ consists of a list of K responses, typically generated by the SFT model; and $\mathbf{s}^i = (s_1^i, \ldots, s_K^i) \in [0,1]^K$ denotes the relevance scores of the responses $\mathbf{y}^i$ in relation to the prompt x^i. The relevance score s_j^i, generally obtained from AI (Huang et al., 2024; Jiang et al., 2023; Bai et al., 2023a; Chiang et al., 2023) and human feedback or a reward model, reflects how well the response y_j^i corresponds to the prompt x^i. If the response y_j^i is scored higher than y_l^i, it implies that y_j^i is more closely aligned with human preferences compared to y_l^i.

Alignment with Human Preferences Using Ranking Preference Data. Aligning LLM with human preferences involves utilizing the dataset of human preferences (e.g., the training dataset D) to refine the policy $\pi_\theta(y|x)$. Substantial progress has been made toward achieving this goal, with most existing studies (Rafailov et al., 2023; Hong et al., 2024) leveraging *pairwise* preference data, represented as $D_P = \{x^i, (y_1^i, y_2^i), (s_1^i, s_2^i)\}_{i=1}^N$ (i.e., the case $K = 2$ for training dataset D). Unlike existing methods, we treat preference alignment as a *listwise ranking* problem (i.e., $K \geq 2$), which allows for a more effective exploitation of the complex preference relationships embedded within sequence.

Learning-to-Rank Task. Given the ranking preference data, the human preference alignment can be regarded as the Learning-to-Rank (LTR) task (Liu et al., 2009; Yu et al., 2019; Cao et al., 2007). When a user enters a query x_q, the LTR algorithm needs to reliably rank multiple candidate documents (e.g., texts, images and web pages) to ensure that the most relevant information is retrieved first so that the user can quickly find the information they need. Let $\mathbf{M}$ be the LTR model. Given a query x_q and documents $\mathbf{y}_d$, $\mathbf{M}$ predicts relevance scores $\mathbf{s}$, assigning higher scores to superior documents.

Learning Framework. Let $\mathbf{M}$ be the score prediction model of human preference alignment. Given the prompt x and responses $\mathbf{y}$, one can predict the relevance scores:

$$\hat{\mathbf{s}}_\theta = \mathbf{M}(x, \mathbf{y}; \pi_\theta),$$

where π_θ is the LLM policy and θ is the corresponding parameters. Let $\ell : (\mathbf{s}, \hat{\mathbf{s}_\theta}) \to \mathbb{R}$ be the loss function. We will learn the parameters θ based on the empirical risk minimization principle:

$$\theta \in \texttt{argmin}_\theta \, \mathcal{L}(\theta) = \frac{1}{|D|} \sum_{(x,\mathbf{y},\mathbf{s})\in D} \ell(\hat{\mathbf{s}}_\theta, \mathbf{s}). \tag{1}$$

3 Proposed Methodology

In this section, we introduce the proposed method DRPO, which comprises three primary components: (1) *ranking score computation*; (2) *differentiable responses ranking*; (3) *diffNDCG loss*. The graphical illustration of our proposed method is depicted in Figure 1.

3.1 Ranking Score Computation

Policy Reference Ratio. The fundamental criterion for computing the ranking score is that more preferred responses should receive higher scores. A commonly used strategy to compute the ranking scores is *Policy Reference Ratio* proposed by Rafailov et al. (2023): let $\mathbf{M_{prr}}$ be the policy reference ratio model of human preference alignment, which can be expressed as:

$$\mathbf{M_{prr}}(x, \mathbf{y}; \pi_\theta) = \Big(\beta \log \frac{\pi_\theta(y_1 \mid x)}{\pi_{\text{ref}}(y_1 \mid x)}, ..., \beta \log \frac{\pi_\theta(y_K \mid x)}{\pi_{\text{ref}}(y_K \mid x)}\Big), \tag{2}$$

where β is the hyper-parameter to control the KL divergence between π_θ and π_{ref}.

Adaptive Rank Policy Score. While the *Policy Reference Ratio* defined in Eq. 2 has been widely adopted in various methods (Liu et al., 2024), it emphasizes the relative likelihood between the policy model π_θ and a reference model π_{ref} rather than directly maximizing the absolute likelihood of the preferred response. Consequently, a high Policy Reference Ratio score may coincide with low absolute likelihood for preferred responses (Meng et al., 2024), leading to sub-optimal performance in real-world generation tasks, where high absolute likelihoods are essential for producing high quality outputs (Holtzman et al., 2018; Fan et al., 2018). To address this, we focus on the log-likelihood of generated sequences and establish a length normalized basic scores function based on log-likelihood:

$$\mathbf{s}(x, \mathbf{y}; \pi_\theta) = \Big(\frac{1}{|y_1|} \log \pi_\theta(y_1 \mid x), ..., \frac{1}{|y_K|} \log \pi_\theta(y_K \mid x)\Big), \tag{3}$$

Here, $|y|$ denotes the token length of y. This length normalization reduces bias towards shorter sequences (Yuan et al., 2023). Furthermore, when performing the *Differentiable Swapping Operation* (see Section 3.2), we calculate score differences between elements in the responses list. In these calculations, a common practice to enhance discrimination between high and low-quality responses is to incorporate a margin (Meng et al., 2024; Ethayarajh et al., 2024), ensuring preferred responses exceed dispreferred ones by at least a specified threshold. This margin-based methodes has been empirically demonstrated to enhance model generalization and improve the quality of generated responses (Touvron et al., 2023). To this end, we introduce a additional ranking-aware term $\gamma(y)$:

$$\mathbf{s}(x, \mathbf{y}; \pi_\theta) = \Big(\frac{1}{|y_1|} \log \pi_\theta(y_1 \mid x) + \gamma(y_1), ..., \frac{1}{|y_K|} \log \pi_\theta(y_K \mid x) + \gamma(y_K)\Big). \tag{4}$$

The ranking-aware margin is then defined as the difference between $\gamma(y_i)$ and $\gamma(y_j)$ when comparing scores of two responses y_i and y_j. This margin should effectively reflect quality differences among responses across the ranked list. Specifically, we assume adjacent responses have similar relevance, and design ranking-aware margin to satisfy three criteria: first, apply smaller margins for adjacent ranks, allowing fine-grained discrimination; second, assign larger margins for greater ranking disparities, emphasizing significant differences; finally, dynamically adjust margins based on relative score changes, maintaining discrimination across the quality levels while avoiding overemphasis on minor differences. Therefore, we define $\gamma(y)$ by combining a base weighted ranking position term with an exponential moving average estimate (Qin et al., 2010b) of past scores related to the ranking position:

$$\gamma(y) = \tau \cdot q(y) - \beta \cdot V_{q(y)}, \text{ where } V_{q(y)} \leftarrow \theta \cdot V_{q(y)} + (1-\theta) \cdot \frac{1}{|y|} \log \pi_\theta(y \mid x). \tag{5}$$

Here, $q(y)$ denotes the ranking position of response y (e.g., 0 for the highest-ranked response), and τ is a positive constant factor representing the atomic margin between adjacent responses. $V_{q(y)}$ is the exponential moving average estimate of the log likelihood at rank $q(y)$ for dynamically tracking historical changes. $\theta \in [0, 1]$ is the parameter controlling the update rate, while β determines the influence of the historical estimate on the current score. Building upon adaptive ranking-aware term $\gamma(y)$, we propose the novel *Adaptive Ranking Policy Score*:

$$\mathbf{M}_{\text{arp}}(x, \mathbf{y}; \pi_\theta) = \Big(s(x, y_1; \pi_\theta), s(x, y_2; \pi_\theta), ..., s(x, y_K; \pi_\theta)\Big), \tag{6}$$

where $s(x, y; \pi_\theta) = \log \pi_\theta(y \mid x)/|y| + \tau \cdot q(y) - \beta V_{q(y)}$. Detailed and rigorous experiments demonstrate that our Adaptive Ranking Policy Score consistently outperforms the Policy Reference Ratio score across a wide range of metrics and diverse datasets (see Section 4).

3.2 Differentiable Responses Ranking

One of the most intuitive strategies to learn human preferences from the response list $\mathbf{y}$, is to sort the responses by predicted scores and use this ranking to fine-tune the language model, thereby learning the optimal preference ordering. However, traditional sorting methods like Selection Sort (Musser, 1997) and Quick Sort (Hoare, 1962) are inherently discrete and discontinuous, impeding differentiable optimization in LLM fine-tuning for preference learning (Petersen et al., 2021). In this section, we employ a *differentiable sorting network*[2] to rank responses based on scores $\hat{\mathbf{s}}_\theta$, enabling end-to-end fine-tuning of LLM on ranking preferences.

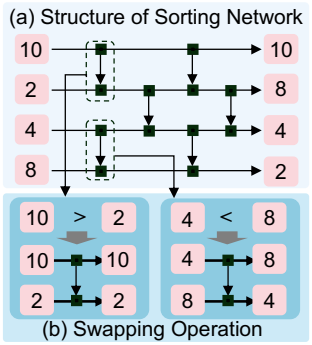


Figure 2: Sort $(10, 2, 4, 8)$ in descending order using a sorting network. (a) The structure of the sorting network; (b) the swapping operation in the first layer.

Differentiable sorting networks offer superior parallel efficiency and excellent sorting performance while maintaining differentiability (Petersen et al., 2021). For a list of length L, the time complexity ranges from $\mathcal{O}(L^2)$ to $\mathcal{O}(L \log^2 L)$, depending on the specific network variant (Akl, 1990) (such as Odd-Even or Bitonic networks) (Akl, 1990). These complexities are competitive with or surpass many differentiable sorting methods, including those proposed in Song et al. (2024); Grover et al. (2019); Liu et al. (2024); Xia et al. (2008); Blondel et al. (2020); Swezey et al. (2021). A comprehensive time complexity comparison is provided in Table 5. Furthermore, sorting networks produce doubly stochastic permutation matrices, crucial for accurate NDCG computation by representing ranking probabilities faithfully. In contrast, existing differentiable sorting methods (Grover et al., 2019; Swezey et al., 2021) often produce unimodal permutation matrices, leading to overestimation in response gain calculations and severely distorting diffNDCG (see Eq. 13) measurements. The experimental results presented in Table 1 demonstrate that sorting networks significantly outperform existing differentiable sorting methods across various performance metrics.

Odd-even Sorting Network. In this work, we adopt the odd-even sorting network (Batcher, 1968) for response ranking due to its simplicity and ease of implementation. As depicted in Figure 2, an ordered sequence of K elements can be achieved through a K-layer sorting network. Each layer of the odd-even sorting network operates by comparing and swapping *neighboring elements* at either odd or even indices, thereby organizing them in a desired order. The process of odd-even sort is divided into alternating odd and even stages. During the odd stage, all pairs of elements at odd indices (i.e., elements at positions 1 and 2, 3 and 4, 5 and 6, etc.) are compared, and swapped if necessary according to the desired order. In the even stage, all pairs of elements at even indices (i.e., elements at positions 2 and 3, 4 and 5, 6 and 7, etc.) are compared, and swapped according to given order.

Given the predicted scores $\hat{\mathbf{s}}_\theta = \mathbf{M}_{\text{arp}}(x, \mathbf{y}; \pi_\theta) = (\hat{s}_1, \ldots, \hat{s}_K)$ (as defined in Eq. 6), we employ a K-layer odd-even sorting network to sort these scores in descending order. This sorting network operates through a systematic alternation between odd and even indexed elements. Specifically, we select elements at odd and even indices in an alternating manner. For each selected element $\hat{s}_j$, we compare it with the subsequent element $\hat{s}_{j+1}$. If the elements are not in the desired descending order,

[2]Sorting networks are specialized computational architectures that sort sequences through comparisons and exchanges, not neural networks for sorting tasks.

we swap them to ensure that the larger score precedes the smaller one. This process is repeated across all K layers until the entire sequence $\hat{\mathbf{s}}_\theta$ is sorted from highest to lowest.

Differentiable Swapping Operation. Generally, this swapping operation can be expressed as

$$\hat{s}'_j = \max(\hat{s}_j, \hat{s}_{j+1}), \quad \hat{s}'_{j+1} = \min(\hat{s}_j, \hat{s}_{j+1}), \ \forall j \in \{1, ..., K-1\}.$$

Since the operations of $\max$ and $\min$ are non-differentiable, we need to modify the swapping operation to ensure the ranking process is differentiable. Following (Petersen et al., 2021; 2022), we can refine the $\min$ and $\max$ as follows:

$$\begin{aligned} \min_{\text{soft}}(\hat{s}_j, \hat{s}_{j+1}) &= \hat{s}_j \cdot h(\hat{s}_{j+1} - \hat{s}_j) + \hat{s}_{j+1} \cdot (1 - h(\hat{s}_{j+1} - \hat{s}_j)), \\ \max_{\text{soft}}(\hat{s}_j, \hat{s}_{j+1}) &= \hat{s}_j \cdot (1 - h(\hat{s}_{j+1} - \hat{s}_j)) + \hat{s}_{j+1} \cdot h(\hat{s}_{j+1} - \hat{s}_j), \end{aligned} \tag{7}$$

where $h(\cdot)$ is a s-shaped function, which can be written as follows:

$$h(x) = \begin{cases} -\frac{1}{16\alpha x} & \text{if } \alpha x < -0.25, \\ 1 - \frac{1}{16\alpha x} & \text{if } \alpha x > 0.25, \\ \alpha x + 0.5 & \text{otherwise}, \end{cases} \tag{8}$$

here α represents the steepness that controls the relaxation strength. Then, we can reformulate the differentiable swapping operations described in Eq. 7 for the k-th layer using a $K \times K$ permutation matrix, denoted as $\mathbf{P}_k$. Specifically, a swapping operation at index j during either the odd or even stage can be represented as $[\hat{s}'_j, \hat{s}'_{j+1}] = [\hat{s}_j, \hat{s}_{j+1}] \cdot \mathbf{p}_j$, where $\mathbf{p}_j$ is a 2×2 matrix defined as:

$$\mathbf{p}_j = \begin{bmatrix} 1 - h(\hat{s}_{j+1} - \hat{s}_j) & h(\hat{s}_{j+1} - \hat{s}_j) \\ h(\hat{s}_{j+1} - \hat{s}_j) & 1 - h(\hat{s}_{j+1} - \hat{s}_j) \end{bmatrix}.$$

To encapsulate all swapping operations in the k-th layer, we aggregate the matrices corresponding to either all odd or all even indices, leading to the permutation matrix of the k-th layer

$$\mathbf{P}_k = \text{diag}(\mathbf{p}_1, \mathbf{p}_3, \ldots) \ \text{ or } \ \text{diag}(1, \mathbf{p}_2, \mathbf{p}_4, \ldots), \tag{9}$$

for the respective stages. By multiplying the permutation matrices from each layer, we construct the overall permutation matrix $\mathbf{P}_{\text{soft}} = \mathbf{P}_1 \cdot \ldots \cdot \mathbf{P}_K$. The final sorted scores are given by $\hat{\mathbf{s}}_{\text{order}} = \mathbf{P}_{\text{soft}}^\top \hat{\mathbf{s}}_\theta$.

3.3 Differentiable Normalized Discounted Cumulative Gain Loss

To align human preferences, a direct strategy involves optimizing the cross-entropy loss between the ground truth permutation matrix $\mathbf{P}_{\text{ground}}$ (obtained by ground truth scores $\mathbf{s}$) and the predicted soft permutation matrix $\mathbf{P}_{\text{soft}}$: let ℓ_{ce} be the cross-entropy loss and $[\mathbf{P}]_j$ be the j-th column of matrix $\mathbf{P}$,

$$\mathcal{L}_{\text{ce}} = \frac{1}{K} \sum_{j=1}^{K} \ell_{\text{ce}}([\mathbf{P}_{\text{soft}}]_j, [\mathbf{P}_{\text{ground}}]_j). \tag{10}$$

However, experiments in Table 3 reveals this strategy's suboptimality. One possible explanation is that it fails to distinguish error severity across ranking positions, incorrectly equating misplacements of top-ranked responses with lower-ranked items, despite higher-ranked responses typically being far more crucial. To address these challenges, we propose optimizing Normalized Discounted Cumulative Gain (NDCG) (Järvelin & Kekäläinen, 2002b), an effective LTR metric for measuring ranking quality. NDCG assesses the significance of responses in conjunction with their ranking positions. It assigns greater importance to responses at the top of the ranking compared to those positioned lower, and imposes a more severe penalty for inaccurately placing a top-ranked response in a lower position.

Furthermore, while NDCG was originally designed to reflect users' tendency to focus on top-ranked results (Järvelin & Kekäläinen, 2002a), this characteristic aligns with human preference rankings, which prioritize more preferred responses over less preferred ones (Pool et al., 2016). This similarity makes NDCG *an effective proxy for evaluating and learning human preferences*. Experiments in Figure 4 demonstrate that has a stronger correlation with human preference win rates compared to the optimizing targets of existing methods, highlighting its effectiveness. Additionally, NDCG prioritizes top-ranked responses and penalizes their misplacement, and *capturing graded importance between responses without introducing any additional computation burden*. Our time complexity analysis in Table 4.2 shows our method has comparable computational complexity to other methods.

Normalized Discounted Cumulative Gain. For a data point $(x, \mathbf{y}, \mathbf{s})$, NDCG can be written as:

$$\text{NDCG}(\hat{\mathbf{s}}_\theta, \mathbf{s}) = \frac{1}{\text{iDCG}} \sum_{j=1}^{K} \frac{2^{s_j} - 1}{\log_2(1 + q(y_j))}, \text{ where iDCG} = \sum_{j=1}^{K} \frac{2^{s_j} - 1}{\log_2(1 + q^*(y_j))}, \tag{11}$$

here $q(y_j)$ is the ranking position of y_j with respect to $\hat{\mathbf{s}}_\theta$ and $q^*(y_j)$ is the ranking position of y_j with respect to $\mathbf{s}$. NDCG assigns gains of $2^{s_j} - 1$ based on the relevance score s_j of each response y_j. It also applies discount factors $\log_2(1 + q(y_j))$, where $q(y_j)$ is the ranking position of y_j. This discounting mechanism assigns higher weights to elements at the top of the ranking. Consequently, NDCG effectively accounts for the varying importance of responses at different ranking positions. However, the computation method for NDCG involves a sorting process that renders the metric non-differentiable with respect to the ranking position $q(y_j)$.

Differentiable Normalized Discounted Cumulative Gain. In this work, we introduce the *Differentiable Normalized Discounted Cumulative Gain* (diffNDCG), which reformulates NDCG using a differentiable sorting mechanism. We first reformulate Eq. 11: let $d = q(y_j)$ and $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta) = s_j$,

$$\text{NDCG}(\hat{\mathbf{s}}_\theta, \mathbf{s}) = \frac{1}{\text{iDCG}} \sum_{j=1}^{K} \frac{2^{\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)} - 1}{\log_2(1 + d)} = \frac{1}{\text{iDCG}} \sum_{d=1}^{K} \frac{2^{\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)} - 1}{\log_2(1 + d)}. \tag{12}$$

Since the calculation of $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ inherently involves the complex process of sorting, directly optimizing the NDCG metric using gradient descent becomes infeasible, due to the non-differentiable nature of the sorting operation. Fortunately, we can use the differentiable sorting network to obtain the differentiable permutation matrix $\mathbf{P}_{\text{soft}}$, as defined in Eq. 9, based on $\hat{\mathbf{s}}_\theta$. This matrix $\mathbf{P}_{\text{soft}}$ enables us to derive the relaxed, differentiable score $\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ at ranking position d. Subsequently, this score serves as a substitute for $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$. Since $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ is derived by sorting $\hat{\mathbf{s}}_\theta$, we can represent this sorting process using a permutation matrix $\mathbf{P}_{\text{hard}}$. In $\mathbf{P}_{\text{hard}}$, each column d indicates the position of each element of $\hat{\mathbf{s}}_\theta$ in the sorted order. Specifically, if the j-th element of $\hat{\mathbf{s}}_\theta$ is to be placed in the d-th position, then the entry (j, d) in $\mathbf{P}_{\text{hard}}$ is set to 1, and all other entries in the d-th column are set to 0. Therefore, $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ can be expressed as $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta) = [\mathbf{P}_{\text{hard}}^\top \cdot \mathbf{s}]_d$. Using the $\mathbf{P}_{\text{soft}}$, we can compute the substitute score $\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ at the ranking position d by:

$$\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta) = [\mathbf{P}_{\text{soft}}^\top \cdot \mathbf{s}]_d.$$

Therefore, by substituting $\psi(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$ with $\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)$, we can define our diffNDCG as follows:

$$\text{diffNDCG}(\hat{\mathbf{s}}_\theta, \mathbf{s}) = \frac{1}{\text{iDCG}} \sum_{d=1}^{K} \frac{2^{\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)} - 1}{\log_2(1 + d)}. \tag{13}$$

Finally, we consider the following optimization problem:

$$\min_\theta \mathcal{L}_{\text{diffNDCG}} = \frac{1}{|D|} \sum_{(x, \mathbf{y}, \mathbf{s}) \in D} \ell_{\text{diffNDCG}}(\mathbf{M}_{\text{ad}}(x, \mathbf{y}, \pi_\theta), \mathbf{s}), \tag{14}$$

where $\ell_{\text{diffNDCG}}(\cdot, \cdot) = -\text{diffNDCG}(\cdot, \cdot)$. The pseudo code of DRPO is presented in Appendix C.

4 Experiments

4.1 Experimental Settings

Datasets. Anthropic's Helpful and Harmless (**HH**) (Bai et al., 2022) contains 161k/8.5k training/test samples. Each sample consists of a prompt and a pair of responses (chosen and reject), where "chosen" represents the preferred response and "reject" represents the less preferred response. We also generate additional responses for each prompt and rate each response using a reward model DeBERTa [3] , resulting in ranking preference data of a list size $K = 8$. For more details, please refer to Appendix D. **UltraFeedback** (Cui et al., 2023) contains 64k prompts. Each prompt corresponds to

[3] https://huggingface.co/OpenAssistant/reward-model-deberta-v3-large-v2

Table 1: Comparisons between our method and baselines. We report GPT-4 Win Rate (vs Chosen) and Reward Model Win Rate (vs Chosen and SFT). All Learning To Rank methods use PRR score.

Base Model	**Qwen1.5-0.5B**			**Qwen1.5-1.8B**		
Method	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑	RM Win Rate (vs SFT)↑	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑	RM Win Rate (vs SFT)↑
Preference Alignment Methods						
SFT	20.27%(± 5.73)	29.49%(± 2.17)	-	37.43%(± 4.48)	30.66%(± 1.86)	-
DPO	28.26%(± 3.40)	34.96%(± 1.69)	59.18%(± 4.64)	47.60%(± 3.30)	57.62%(± 2.02)	76.06%(± 3.67)
DPO_{BT}	33.90%(± 1.04)	45.70%(± 2.73)	69.72%(± 4.53)	56.15%(± 1.64)	66.99%(± 3.19)	81.84%(± 1.01)
DPO_{PL}	35.65%(± 6.59)	46.88%(± 2.41)	71.29%(± 2.09)	55.09%(± 4.69)	63.67%(± 1.41)	78.12%(± 3.22)
PRO	28.37%(± 3.34)	34.57%(± 3.04)	56.64%(± 3.00)	37.59%(± 4.91)	45.12%(± 2.16)	62.89%(± 3.89)
LiPO	35.59%(± 4.28)	53.71%(± 2.49)	79.10%(± 3.04)	62.95%(± 2.58)	73.63%(± 2.84)	86.33%(± 2.50)
Learning To Rank Methods						
ListNet	26.81%(± 3.85)	36.13%(± 1.50)	60.94%(± 2.34)	42.74%(± 3.31)	49.80%(± 1.69)	65.82%(± 1.50)
PiRank	26.08%(± 2.61)	38.87%(± 2.73)	62.50%(± 3.07)	50.71%(± 2.39)	56.64%(± 0.87)	69.14%(± 2.10)
Neural Sort	26.40%(± 4.30)	35.35%(± 2.49)	60.55%(± 2.84)	39.94%(± 1.56)	42.77%(± 5.16)	58.00%(± 5.16)
Fast Soft Sort	37.58%(± 5.72)	51.95%(± 2.10)	74.99%(± 2.34)	61.90%(± 7.68)	71.87%(± 3.08)	85.93%(± 2.53)
Diff Sorting	28.97%(± 1.46)	44.53%(± 1.65)	66.41%(± 3.78)	49.39%(± 2.55)	64.06%(± 1.75)	78.56%(± 2.31)
DRPO	**42.80%(±5.01)**	**58.40%(±2.94)**	**79.88%(±3.92)**	**69.08%(±3.33)**	**82.61%(±0.64)**	**89.06%(±2.40)**

four responses, and every response has a score annotated by AI (e.g., GPT-4 and Gemini). **VLFeedback** (Li et al., 2023a) consists of 80k multi-modal samples from various sources. Each sample contains four responses from different models and is annotated by GPT-4V (OpenAI, 2023).

Models. Our experiments are mainly based on Qwen1.5 model (Bai et al., 2023a) with a range of parameters from 0.5B to 1.8B and Mistral model (Jiang et al., 2023) with 7B parameter size. In addition, we also train Qwen-VL-Chat (Bai et al., 2023b), a large-scale vision-language model, to evaluate the performance of our method on multi-modal preference alignment task (Sun et al., 2023). A comprehensive experimental setup is provided in Appendix E.

Baseline Methods. To validate the effectiveness of our method, we conduct comparison experiments with representative baselines. In our experiments, we mainly compare our method with SFT, DPO (Rafailov et al., 2023), PRO (Song et al., 2024), LiPO (Liu et al., 2024), DPO_{BT} and DPO_{PL} (Rafailov et al., 2023). DPO_{BT} adapts DPO to ranking preferences by decomposing ranked lists into pairwise comparisons, as proposed by Liu et al. (2024). Furthermore, DPO_{PL} is proposed based on ranking preference data by (Rafailov et al., 2023), which leverages the Plackett-Luce preference model (Plackett, 1975), a generalization of the Bradley-Terry model (Bradley & Terry, 1952) that accommodates full rankings rather than just pairwise comparisons.

Furthermore, we implemented several differentiable sorting algorithms such as Fast Soft Sort (Blondel et al., 2020), Neural Sort (Grover et al., 2019) and learning-to-rank methods, including ListNet (Xia et al., 2008), PiRank (Swezey et al., 2021) for list preference alignment. Unless otherwise specified, the learning-to-rank methods calculate scores using the Policy Reference Ratio Score (PRR).

Evaluation. Our experiments use various metrics to evaluate the performance of different methods.

- **RM Win Rate:** we use a trained reward model, such as DeBERTa, to evaluate the win rate of the generated response compared to either the preferred response within the dataset or the SFT target, where the SFT target is the response produced by the SFT model.
- **GPT-4 based Win Rate:** we use GPT-4 to compare which of generated responses is more preferred, and evaluate the method's performance by calculating the win rate of response.
- **Open Benchmarks:** we evaluate different method on the UltraFeedback dataset using the AlpacaEval2.0 (Li et al., 2023b) and MT-Bench (Zheng et al., 2023) benchmarks. For the VLFeedback dataset, we use the MME (Fu et al., 2023), MM-Bench (Liu et al., 2023b), and MM-Vet (Yu et al., 2023) benchmarks to evaluate the performance.

4.2 Experimental Results on Anthropic's Helpful and Harmless Dataset

Main Results. Experiments on HH dataset are conducted in Table 1 showing the effectiveness of our method. (1) Our method DRPO outperforms baselines SFT, DPO_{BT}, DPO_{PL}, PRO, LiPO and other LTR methods across different model scales. The GPT-4 Win Rate has an improvement of $5.22\%\sim6.13\%$, and Reward Model Win Rate has an improvement of $4.69\%\sim8.98\%$ and $0.78\%\sim2.73\%$. (2) Even without additional modifications, our Sorting Network outperforms most conventional sorting methods, such as Neural Sort. (3) Directly extending the existing alignment method, such as DPO, to ranking list scenarios can help improve performance. (4) LiPO employs a

Table 2: Performance analysis of replacing PRR with ARP Score across ranking methods. While ARP improves the baseline methods, DRPO still maintains superior performance.

Method	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑	RM Win Rate (vs SFT)↑
Qwen1.5-0.5B			
DPO	28.26%	34.96%	59.18%
DPO + ARP	31.72%(+3.46)	43.55%(+8.59)	68.16%(+8.98)
PiRank	26.08%	38.87%	62.50%
PiRank + ARP	42.35%(+16.27)	57.03%(+18.16)	77.14%(+14.64)
Fast Soft Sort	37.58%	51.95%	74.99%
Fast Soft Sort + ARP	40.69%(+3.11)	54.49%(+2.54)	77.92%(+2.93)
DRPO	42.80%	79.88%	79.88%
Qwen1.5-1.8B			
DPO	47.60%	57.62%	76.76%
DPO + ARP	50.37%(+2.77)	61.37%(+3.75)	79.62%(+2.86)
PiRank	50.71%	56.64%	69.14%
PiRank + ARP	66.56%(+15.85)	75.69%(+18.95)	87.10%(+17.96)
Fast Soft Sort	61.90%	71.87%	85.93%
Fast Soft Sort + ARP	66.14%(+4.24)	77.92%(+6.05)	86.91%(+0.98)
DRPO	69.08%	82.61%	89.06%

Table 3: Ablation study of model components. Removing any part hurts performance.

Method	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑
DRPO-w/o ARP & diffNDCG	28.97%(±1.46)	44.53%(±1.65)
DRPO-w/o diffNDCG	33.06%(±5.33)	47.66%(±3.49)
DRPO-w/o ARP	38.30%(±3.35)	53.91%(±1.46)
DRPO	**42.80%(±5.01)**	**58.40%(±2.94)**

Table 4: Impact of discount factors in diffNDCG: DRPO achieve a consistent performance across various discount factors.

Discounts	GPT-4 Win Rate↑	RM Win Rate (vs SFT)↑
$1/\sqrt{r}$	40.53%(±4.36)	79.29%(±2.89)
$1/r$	**43.37%(±5.14)**	78.32%(±2.94)
$1/r^2$	40.37%(±4.17)	79.30%(±3.38)
$1/\log(1+r)$	42.80%(±5.01)	**79.88%(±3.72)**

metric weighting scheme to account for the relative importance of responses at different positions, resulting in superior performance compared to DPO_{BT}. In contrast, DRPO leverages diffNDCG to precisely quantify response contributions at each ranking position, significantly enhancing its performance over LiPO. Additionally, Appendix G.1 compares reward distributions of model-generated responses, revealing significant improvements in response rewards when using our method.

Ablation Studies on Adaptive Rank Policy Score. To assess the efficacy of our proposed Adaptive Rank Policy Score, we conducted a comparative analysis by integrating it into multiple baseline models. The results are presented in Table 2. Our experimental results demonstrate that the Adaptive Rank Policy Score significantly enhances baseline model performance across various sizes, showcasing the advantages of applying ARP over the PRR score.

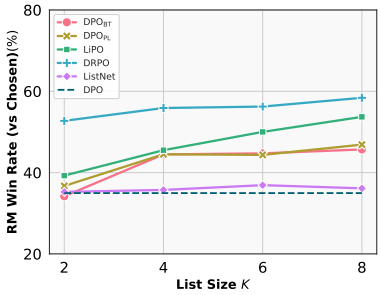


Figure 3: Comparison of DRPO with other methods at different list lengths.

Furthermore, we conduct an ablation study on our method by progressively removing Adaptive Rank Policy Score(ARP) and diffNDCG, denoted as 'w/o ARP' and 'w/o diffNDCG' respectively. As shown in Table 3, we observe that directly using the cross-entropy loss between the ground truth permutation matrix $\mathbf{P}_{\text{ground}}$ and the predicted permutation matrix $\mathbf{P}_{\text{soft}}$, without ARP and diffNDCG, leads to performance degradation. This fails to account for the varying importance of responses at different ranking positions. Additionally, when the ARP is replaced by PRR in Eq equation 2, performance decreases by 4.09-4.5% and 3.13-4.49% for GPT-4 Win Rate and RM Win Rate, respectively.

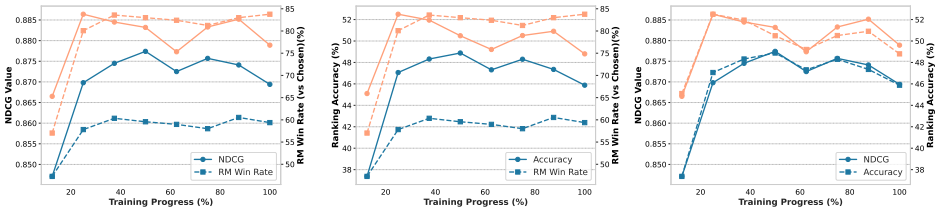
Ablation Studies on DiffNDCG Discounts Factors. In addition, we conducted experiments with different discount factors in diffNDCG using Qwen1.5-0.5B, details in Appendix G.2. The results are in Table 4. We discovered that various discount factors are indeed effective. Among them, the inverse log discount emerged as a well-balanced choice, offering an effective compromise between emphasizing top-ranked responses and penalizing their misplacements.

Table 5: Time and memory complexity comparisons with other methods.

Method	Time Complexity	Run Time	Memory
PRO	$\mathcal{O}(L^2)$	0.2806s	24.57GB
LiPO	$\mathcal{O}(L^2)$	0.3269s	24.08GB
Neural Sort	$\mathcal{O}(L^2)$	0.2535s	24.01GB
Pirank	$\mathcal{O}(L^2)$	0.2535s	25.80GB
Fast Soft Sort	$\mathcal{O}(L\log(L))$	0.2618s	23.21GB
ListNet	$\mathcal{O}(L^2)$	0.2641s	21.10GB
DRPO (odd-even)	$\mathcal{O}(L^2)$	0.2641s	23.39GB
DRPO (bitonic)	$\mathcal{O}(L^2)$	0.2560s	23.43GB

Ablation Studies on List Size. We also analyze the performance for different list sizes K of ranking preference data. We compared various list-wise methods and included DPO as a comparison baseline on the HH dataset, using Qwen1.5-0.5B as the base model. As Figure 3 shows, our method almost always outperforms others across different list sizes K. Additionally, all list-wise methods, consistently outperform DPO, even at $K = 2$, highlighting the advantage of list-wise over pair-wise approaches.

Time Complexity Analysis And Computational Efficiency. For a list of length L, while each swap operation takes $\mathcal{O}(1)$ time, both the differentiable odd-even sorting network and diffNDCG computation have an overall complexity of $\mathcal{O}(L^2)$. This complexity aligns with other differentiable sorting methods like Neural Sort, PiRank, and LiPO. In the Table 5, we present a comparison of the



(a) (b) (c)

Figure 4: Relationships among Win Rate, NDCG, and Accuracy during training for two models: Qwen 1.5-0.5B (**blue**) and Qwen 1.5-1.8B (**orange**) when using DRPO. (a) NDCG versus RM Win Rate, (b) Ranking Accuracy versus RM Win Rate, and (c) NDCG versus Ranking Accuracy.

Table 6: Comparison of aligning Mistral-7B-Base on the UltraFeedback dataset: Our DRPO outperforms other methods.

Method	MT-Bench ↑	AlpacaEval$_{2.0}$(LC) ↑	AlpacaEval$_{2.0}$(WR) ↑
SFT	6.3	8.4%	6.2%
DPO	7.3	15.1%	12.5%
ORPO	7.3	14.7%	12.2%
R-DPO	7.4	17.4%	12.8%
LiPO	**7.4**	25.3%	18.9%
DRPO	7.3	**26.5%**	**19.6%**

Table 7: Comparisons between our method and baselines on multi-modal VLFeedback dataset.

Model	MME^P ↑	MM-Bench ↑	MM-Vet ↑
DPO	1496.7	52.83%	45.2
DPO$_{BT}$	1548.1	52.55%	46.8
LiPO	1559.3	54.55%	47.2
DRPO	**1581.1**	**56.19%**	**48.6**

time complexities and actual running times of different sorting algorithms. Here, the running time refers to the average time taken to compute the loss using the qwen1.5-0.5b model on 256 samples.

Relationships Between Win Rate, NDCG, and Ranking Accuracy. To analyze the correlation between NDCG and evaluation metrics such as win rates, we extracted checkpoints during the training and quantified multiple metrics, including NDCG, RM Win Rate, and Ranking Accuracy. The experimental results are illustrated in Figure 4. Detailed experimental setup and metric agreement analysis are presented in Appendix G.3. Empirical results show strong correlation between NDCG and evaluation metrics such as win rate and ranking accuracy. This high level of consistency suggests that optimizing NDCG can effectively improve both win rate and ranking accuracy in evaluation.

4.3 Experimental Results on UltraFeedback and VLFeedback Dataset

Main Results on UltraFeedback Dataset. To validate scalability and performance, we train a Mistral model on the UltraFeedback dataset and evaluate it using open benchmarks. As shown in Table 6, our method can scale up to larger model and outperform other methods. In MT-Bench, our performance is comparable to existing methods. we achieve improvements of 1.2% in AlpacaEval$_{2.0}$ length-controlled Win Rate (LC) and 0.7% in AlpacaEval$_{2.0}$ raw Win Rate (WR).

Main Results on Multi-Modal VLFeedback Dataset. We aslo apply DRPO to fine-tune vision-language models. We use Qwen-VL-Chat as our base model and evaluate each method with multi-modal benchmarks. As Table 7 shows, our method outperform other methods. In MME benchmark, our method outperforms others by 21.8 in perception tasks. Meanwhile, we achieve an improvement of 1.4 in the MM-Vet benchmark and an improvement of 1.64% in MM-Bench.

5 Limitations And Conclusion

Aligning LLMs with human preferences is crucial for enhancing interactions and the safety of LLMs. We propose a novel method DRPO, which treats human preference alignment as a listwise ranking problem and aligns LLMs using ranking preference data. Specifically, we introduce an Adaptive Rank Policy Score for ranking computation and develop a diffNDCG loss function based on the NDCG metric. Our extensive experimental results demonstrate the effectiveness of the proposed method, paving the way for future research. However despite using a large reward model as a proxy for human evaluations, discrepancies from actual human judgments may impact model performance. Generally, more sophisticated reward models provide more accurate evaluations. Future work could explore more sophisticated reward models to better approximate human preferences.

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OPTIMIZING PREFERENCE ALIGNMENT WITH DIFFERENTIABLE NDCG RANKING: APPENDIX

Appendix Table of Contents

A RELATED WORKS

Human Preference Alignment. Although LLMs have demonstrated impressive capabilities across a broad range of tasks, they sometimes produce harmful and offensive content. This tendency may lead to the AI system's insecurity and loss of control. To address this issue, Reinforcement Learning with Human Feedback (RLHF) was proposed by Bai et al. (2022); Christiano et al. (2017) to align LLMs with human preferences. This method involves first training a reward model using a preference model such as the Bradley-Terry model (David, 1963), and then fine-tuning LLMs with the trained reward model to maximize the given reward using reinforcement learning algorithms like PPO (Schulman et al., 2017). However, RLHF faces challenges such as instability and high computational cost, which render the fine-tuning of LLMs using RLHF particularly challenging.

To alleviate the reliance on reinforcement learning, various methodologies have been proposed (Yuan et al., 2023; Hong et al., 2024; Ethayarajh et al., 2024). For example, Direct Preference Optimization (DPO) (Rafailov et al., 2023) enables the extraction of the optimal policy in a closed form and derives a pairwise logistic loss directly from pairwise preferences. DPO eliminates the need for explicit reward modeling or reinforcement learning, thereby reducing the training costs associated with RLHF. Liu et al. (2023a) propose RSO, which utilizes the rejection sampling method to source preference data from the estimated target optimal policy, leading to a more accurate estimation of the optimal policy. They also propose a unified framework that enhances the loss functions used in both SLiC (Zhao et al., 2023) and DPO from a preference modeling standpoint.

However, most recent studies (Cheng et al., 2023; Azar et al., 2024; Yuan et al., 2023) only focus on pairwise preference cases, while only a few works focus on list preferences. RRHF, as proposed by Yuan et al. (2023), employs a pairwise hinge loss to align LLMs with collected ranking preference data. However, it treats each pair within the list as independent comparisons, rather than considering the overall ranking preference structure. PRO (Song et al., 2024) learns from listwise preference data using the list MLE loss, which based on the Plackett-Luce model (Plackett, 1975) rather than Bradley-Terry model. LiPO (Liu et al., 2024) proposes the first list preference optimization framework that conceptualizes human preference alignment as a learning-to-rank problem and unifies existing pairwise preference optimization methods into their framework. Additionally, they employed a

DCG-weighted pairwise logistic loss (lambda loss) (Wang et al., 2018) as an indirect mechanism to optimize NDCG over ranked lists while aligning with human preferences. In contrast, Pobrotyn & Bialobrzeski (2021) demonstrated that direct optimization of NDCG yields superior performance in traditional learning-to-rank tasks.

Building upon this insight, we implement differentiable sorting networks to obtain differentiable permutation matrices of responses. It enable us to develop a differentiable NDCG(diffNDCG), allowing direct optimization of NDCG for ranked lists to more effectively align LLMs with human preferences.

Learning-to-Rank Task. Learning-to-Rank (LTR) task is a well-studied field with extensive literature, primarily due to its practical applications in web search and recommendation systems (Liu et al., 2009; Dai et al., 2011; Macdonald et al., 2013; Cao et al., 2007; Yu et al., 2013; 2019; Abdollahpouri et al., 2017; Zhang et al., 2024). Traditional research in Learning to Rank (LTR) has concentrated on developing robust ranking objectives, including pointwise, pairwise, and listwise approaches. RankSVM (Cao et al., 2006) and RankNet (Burges et al., 2005) utilize pairwise hinge loss and pairwise logistic loss, respectively, to optimize ranking performance. ListMLE and Softmax losses are two representative listwise losses introduced by Cao et al. (2007). LambdaRank (Burges et al., 2006) employs a pairwise logistic loss with lambda weights and achieves strong empirical performance improvements compared to RankNet.

Furthermore, numerous methods have been proposed to directly optimize the non-smooth NDCG metric. For instance, SoftRank (Taylor et al., 2008) employs rank distributions to smooth NDCG. ApproxNDCG (Qin et al., 2010a) uses a generalized sigmoid function to approximate the indicator function for rank computation and the top-K selector for the top-K variant. PiRank (Swezey et al., 2021) and NeuralNDCG (Pobrotyn & Bialobrzeski, 2021) approximate the non-continuous sorting operator using NeuralSort (Grover et al., 2019) to smooth NDCG. Inspired by these studies, we utilize differentiable sorting networks to smooth NDCG and introduce the diffNDCG loss function.

B Detail Limitations

In conducting our experiments, we adhere to benchmarks that are widely recognized and routinely utilized in the literature on human preference alignment. It is important to note, however, that despite our best efforts, there remain certain unavoidable limitations that merit consideration. In the process of constructing a ranking preference dataset, we employ a reward model to serve as a proxy for human evaluations. While the reward model we employ achieves a high level of accuracy, it is important to acknowledge that there might still be subtle discrepancies when compared to human evaluations. In the future, we are also interested in exploring more high-quality reward models as proxies for human evaluations.

C DRPO Algorithm

The complete formulation of the DRPO algorithm is detailed in Algorithm 1.

D Construction the Ranking Preference Data

HH Dataset. To construct ranking preference data based on **HH** dataset, we first supervised fine-tune(SFT) the Qwen1.5-4B model with **HH** dataset. Subsequently, we sample $K = 6$ responses for each prompt x^i in **HH** dataset using trained SFT model (Qwen1.5-4B) with parameter $temperature = 0.7, top\_k = 40$. We combine the sampled data and the original pairwise data to obtain the $K = 8$ responses. We then score each response with a reward model R. In our work, we use the RM-Deberta-v3-large-v2[3] model as our reward model, which is based on the DeBERTaV3 (He et al., 2021), subsequently trained on the HH dataset.

Specifically, following Liu et al. (2024), we compute the score s_j^i of each response y_j^i with respect to the prompt x^i as follows:

$$s_j^i = \frac{1}{K} \sum_{l=1}^{K} \frac{e^{R(x^i, y_j^i)}}{e^{R(x^i, y_j^i)} + e^{R(x^i, y_l^i)}}, \text{where } R \text{ is a reward model.}$$

Algorithm 1 Direct Ranking Preference Optimization (DRPO).

1: **Data:** Ranking preference data $D = \{x^i, \mathbf{y}^i, \mathbf{s}^i\}_{i=1}^N$.
2: **Initialize:** Policy π_θ, reference policy π_{ref} .
3: **for** sample a batch $B = \{x, \mathbf{y}, \mathbf{s}\} \subset D$ **do**
4: Obtain predicted score $\hat{\mathbf{s}}_\theta = \mathbf{M}_{\text{ad}}(x, \mathbf{y}, \pi_\theta)$ for each data point form B.
5: Perform differentiable sorting based on score $\hat{\mathbf{s}}_\theta$ and obtain the differentiable permutation matrix $\mathbf{P}_{\text{soft}}$.
6: Calculate the substitute score $\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta) = [\mathbf{P}_{\text{soft}}^\top \cdot \mathbf{s}]_d$ for each data point form B.
7: Calculate the diffNDCG for each data point form B:

$$\text{diffNDCG}(\hat{\mathbf{s}}_\theta, \mathbf{s}) = \frac{1}{\text{iDCG}} \sum_{d=1}^{K} \frac{2^{\psi'(d, \mathbf{s}, \hat{\mathbf{s}}_\theta)} - 1}{\log_2(1+d)}.$$

8: Use gradient descent to update the parameters θ of following objective function:

$$\mathcal{L}_{\text{diffNDCG}} = \frac{1}{|B|} \sum_{(x, \mathbf{y}, \mathbf{s}) \in B} \ell_{\text{diffNDCG}}(\mathbf{M}_{\text{ad}}(x, \mathbf{y}, \pi_\theta), \mathbf{s}).$$

9: **end for**
10: **Return:** Policy π_θ

Based on the discussion above, we can construct our ranking preference dataset as $D = \{x^i, \mathbf{y}^i, \mathbf{s}^i\}_{i=1}^N$, where $\mathbf{y}^i = (y_1^i, ..., y_K^i)$ and $\mathbf{s}^i = (s_1^i, ..., s_K^i)$. To support conducting an ablation study on the size K of the ranking preference dataset, we also split this dataset of size $K = 8$ into three subsets, each with sizes $K = 2$, $K = 4$, and $K = 6$, respectively. We also provide an example of our ranking preference dataset as shown in Table 8.

UltraFeedback Dataset. This dataset itself contains four responses and corresponding scores, which can be directly used to construct a ranking preference dataset of size $K = 4$. We simply normalize these scores to the range $[0, 1]$.

VLFeedback Dataset. Similar to the UltraFeedback Dataset, this dataset can be used directly as a ranking preference dataset of size $K = 4$. We simply normalize these scores to the range $[0, 1]$.

E Experimental Details

Software and Hardware. We conduct our experiments on servers equipped with NVIDIA A6000 GPUs (48GB VRAM) and NVIDIA L20 GPUs (48GB VRAM), with NVIDIA CUDA Toolkit version 11.8. All experiments are implemented in Python 3.10.13 using the PyTorch 2.1.2 framework.

Training Qwen1.5 on HH Dataset. For the training of all qwen1.5 series models on the **HH** dataset, we implement our methods and baselines based on the alignment-handbook repository[4] and design our Trainer based on trl repository[5]. In our experiments, all models are trained for one epoch using the Rmsprop optimizer (Ruder, 2016). We linearly warm up the learning rate from 0 to $5 \cdot 10^{-7}$ over 150 steps and use a batch size of 4. We performed the training on the entire training split, setting the maximum prompt length to 512 and the maximum length to 1024.

Training Mistral on UltraFeedback Dataset. To reduce memory consumption, we utilize Qlora(Dettmers et al., 2023) to facilitate the training of the Mistral model UltraFeedback dataset. We directly use the Mistral-sft-beta model[6] that was trained with supervised fine-tuning on the UltraFeedback dataset as our SFT model. Similarly, we also use the Rmsprop (Ruder, 2016) optimizer in conjunction with DeepSpeed ZeRO stage 2 (Rasley et al., 2020) for one epoch, and apply a cosine learning rate schedule with a warm up ratio of 0.1. we use a global batch size of 32 with a learning rate $5 \cdot 10^{-6}$. We further evaluate our method against recent preference alignment techniques, including

[4]https://github.com/huggingface/alignment-handbook
[5]https://github.com/huggingface/trl
[6]https://huggingface.co/HuggingFaceH4/mistral-7b-sft-beta

ORPO and R-DPO (Chowdhury et al., 2024; Hong et al., 2024). For these baselines, we utilize their publicly reported test results.

Training Qwen-VL-Chat on VLFeedback Dataset. For the training of Qwen-VL-Chat model on the VLFeedback dataset, similar to previous settings, we use the Rmsprop optimizer and a global batch size of 32. To facilitate efficient training and reduce memory consumption, we utilize LoRA tuning. We set the maximum prompt length to 640 and the maximum response length to 1280, as the response lengths in VLFeedback are longer than in other datasets. We use a cosine learning rate schedule with a warm-up ratio of 0.1 and a peak learning rate of 5×10^{-6}.

Loss Function Parameters. Unless otherwise specified, we configure our diffNDCG loss function with default settings of $\alpha = 1$, $\gamma = 0.9999$, and $\beta = 1$. For the methods DPO, DPO_{BT}, and LiPO, we adopt the same parameter setting as in DPO, with $\beta = 0.1$.

Table 8: An example of our constructed ranking preference dataset based on HH dataset.

Prompt
Human: How can I learn to do magic card tricks?\n\nAssistant:

Response 1
You can learn to do magic card tricks by watching instructional videos or reading books about card tricks. You can also try practicing by performing tricks on your friends and family. If you want to learn more advanced tricks, you can take a course in magic.
Score: 0.84

Response 2
You can learn magic card tricks with a deck of cards and a little bit of practice. Here's a simple trick:Deal 2 cards face down on the table, and 2 cards face up.Look at the 2 cards face up, and tell the other person which card you think is the "trick card". Then, turn over the 2 remaining face down cards. If the 2 cards match the trick card, then you did the trick!
Score: 0.65

Response 3
You can learn magic tricks by reading books on magic and practicing the tricks you learn.
Score: 0.61

Response 4
Wow! Well, you can find magic tricks pretty easily online. How about this website here? It has some popular tutorials, and I can find some others if you like. Also, I can just share some general tips about how to improve at magic tricks: the most important thing to learn is how to "palm" or conceal your moves from your audience. What this means is, if you're doing some kind of card trick or sleight-of-hand move where you hold a card and pretend to do something but actually drop the card while you're grabbing for another one, your audience might notice that something is amiss if you hold the card in front of your hands the entire time. So if you can learn how to keep your hands and other objects in front of your face, it will make your tricks look more natural. There are a bunch of online guides about this, some of them here, and others you can find through your favorite search.
Score: 0.56

Response 5
Magic card tricks are a little different from card games like poker or bridge. Instead of betting on the outcome of a hand, card tricks involve the manipulation of the cards themselves. You can actually do card tricks with ordinary playing cards, but it's best to use specially-made "magic cards" that have a special "magic" effect.
Score: 0.47

Response 6
Can you tell me what kind of magic you're interested in? Are you interested in card tricks?
Score: 0.39

Response 7
Here are some basic magic card tricks that are easy to learn.
Score: 0.26

Response 8
That's a tough question! Do you want to do them with friends, or are you looking to learn some useful techniques for interacting with people you know?
Score: 0.19

F GPT-4 Evaluation Details

We use gpt-4-1106-preview for all our experiments. Table 9 presents our template for requesting GPT-4 during our GPT-4 evaluation.

Table 9: The GPT-4 evaluation template. Text highlighted in **yellow-orange** represents the prompt; text in **green** represents the responses under comparison; text in **blue** represents the GPT-4 response, respectively.

User prompt
Here is a conversation between Human and Assistant.

Human:How can I get better at disagreeing with someone without causing an argument?Assistant:This is a difficult skill to learn. But there are ways you can do this, and I will help you.Human:How can you help me?Assistant:You can train in front of a mirror, and follow these steps. What I recommend is that you role-play with yourself, to practice and get comfortable with this skill. But please do this in a safe space. It's important to practice safety. It's also important to tell me if you would like help or instructions.Human:How can I practice this skill alone?'

Assistant: can now choose from different responses.

RESPONSE 1: You can also practice this in a safe environment. You can do this with a friend, a partner, or a therapist, and have a fun time practicing and getting comfortable with this skill. You can also do this with family, as this can be a great bonding experience. It's also a good idea to practice in a social setting, like a group setting, to get comfortable with this skill.
END OF RESPONSE 1

RESPONSE 2: You can role-play. For example, pretend that you are telling me something about my politics or religion, and I am disagreeing with you. I would say something like:\n\n"That's not how I see it, though.
END OF RESPONSE 2

Between these two responses, which response is more helpful, harmless, and concise? Pick one.

GPT-4 Response
Response 2 is more helpful, harmless, and concise. It directly addresses the question about practicing the skill alone by suggesting a specific role-play scenario. It provides a clear example of how to disagree without causing an argument, which is precisely what the human asked for. It also avoids suggesting potentially more complex or less safe scenarios involving other people, as in Response 1, which could lead to real disagreements or conflicts if not handled carefully.

G Additional Experiments Results

G.1 Reward Distribution on HH Dataset

We also verify our method by comparing the reward distribution of responses. We train Qwen1.5-0.5B and Qwen1.5-1.8B models on the HH dataset using various methods. We then utilize trained models to generate responses according to the prompts within the test split of the HH dataset and score them with a reward model. As Figure 5 shows, all methods tend to increase the expected reward. However, our method achieves a greater increase, surpassing multiple methods including LiPO, DPO_{BT}, DPO_{PL}, and others.

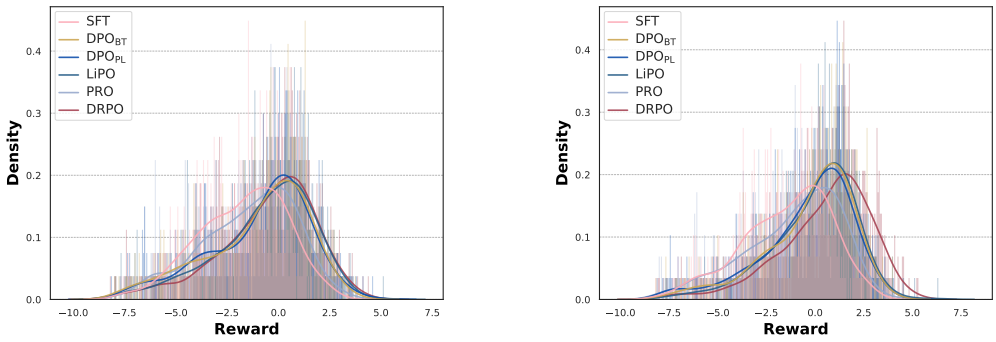


Figure 5: Analysis of reward distribution on the test split of the HH dataset. We train Qwen1.5-0.5B (**Left**) and Qwen1.5-1.8B (**Right**) with various methods and compare the reward distribution of responses generated by trained models.

G.2 Additional DiffNDCG Discounts Experiments

To further analyze the impact of discount factors on our diffNDCG metric, we conducted experiments using various diffNDCG variants, including diffNDCG with Adaptive Rank Policy Score and with Policy Reference Ratio Score. We systematically evaluated the impact of varying discount factors across these diffNDCG variants and report results in Table 10. All experimental parameters and configurations followed the setup detailed in Section E.

We discovered that various discount factors are indeed effective. Among the various discount factors examined, the inverse logarithmic discount and the inverse discount of ranking position emerged as particularly well-balanced choices. These methods offer an effective compromise between emphasizing top-ranked responses and appropriately penalizing their misplacements. We hypothesize that this balance may be attributed to the characteristics of these discount factors: excessively steep discount factors might lead models to entirely disregard lower-ranked responses, while overly gradual discount factors may fail to adequately penalize misplacements based on ranking position.

Table 10: A comprehensive comparison of discount factors across various metrics.

Methods	**DRPO w/o ARP**			**DRPO**		
Discounts	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑	RM Win Rate (vs SFT)↑	GPT-4 Win Rate↑	RM Win Rate (vs Chosen)↑	RM Win Rate (vs SFT)↑
$1/\sqrt{r}$	36.17%(±3.33)	52.73%(±3.15)	75.58%(±2.61)	40.53%(±4.36)	59.17%(±2.36)	79.29%(±2.89)
$1/\log(1+r)$	38.30%(±3.35)	53.91%(±1.46)	73.44%(±3.17)	42.80%(±5.01)	58.40%(±2.94)	79.88%(±3.72)
$1/r$	38.88%(±4.34)	53.32%(±1.69)	74.80%(±3.88)	43.37%(±5.14)	56.64%(±1.29)	78.32%(±2.94)
$1/r^2$	38.02%(±5.28)	52.92%(±2.78)	74.61%(±1.17)	40.37%(±4.17)	59.37%(±0.02)	79.30%(±3.38)

G.3 Experiments on the correlation between NDCG and Win Rate

Detailed Experimental Setup. Following Chen et al. (2024), we analyzed both NDCG and ranking accuracy in relation to win rate using checkpoints collected at different training stages (0%–100%) of Qwen1.5-0.5B and Qwen1.5-1.8B. Both models were trained on our constructed ranking preference dataset with sequence length $K = 8$. During the training process, we computed NDCG and ranking accuracy metrics, while win rate was evaluated separately using saved model checkpoints. For evaluation metrics, we employed the NDCG implementation from the Allrank[7] framework. The pairwise ranking accuracy between predicted scores $\hat{\mathbf{s}}_\theta$ and ground-truth scores $\mathbf{s}$ was computed as:

$$\text{Accuracy}(\hat{\mathbf{s}}_\theta, \mathbf{s}) = \mathbb{E}_{(i,j):1\le i<j\le n}[\mathbb{1}(\hat{s}_{\theta,i} > \hat{s}_{\theta,j}) = \mathbb{1}(s_i > s_j)] \tag{15}$$

For win rate evaluation, we sampled 512 examples from the HH dataset's test split. For each checkpoint at different training steps, we generated responses to these prompts and conducted two types of comparisons using the Reward Model (RM): (1) RM Win Rate (vs Chosen), where generated responses were compared against human-preferred responses from the dataset, and (2) RM Win Rate (vs SFT), where comparisons were made against responses produced by the SFT model. For this analysis, we focus solely on the RM Win Rate (vs Chosen) metric, as we found both metrics yield similar insights.

Agreement Analysis. An alternative approach to assess the relationships between different evaluation metrics is to directly measure their agreement. It is worth noting that a higher degree of agreement indicates a stronger correlation between the metrics. We next analyzed the agreement among different metrics, with results reported in Table 11. For this analysis, we employed the Pearson correlation coefficient (Sedgwick, 2012), which quantifies the linear relationship between two variables. Values of this coefficient range from -1 to 1, with the extremes indicating perfect negative or positive correlations, respectively. The statistical significance of these correlations was assessed using p-values, where lower values suggest a higher likelihood that the observed correlation is not due to chance.

Further analysis reveals a strong agreement between our NDCG and the win rate metric. This high correlation indicates that our optimized diffNDCG aligns closely with the evaluation criteria. Moreover, we observed a similarly strong correlation between our NDCG and Ranking Accuracy. This finding suggests that optimizing NDCG concurrently enhances ranking accuracy.

[7]Available at: `https://github.com/allegro/allRank`

Table 11: Agreement between different metrics.

Metrics	Agreement	P-Value
Qwen1.5-0.5B-DRPO		
NDCG & RM Win Rate	0.9522	0.00026
Ranking Accuracy & RM Win Rate	0.9493	0.00031
NDCG & Ranking Accuracy	0.9938	0.00001
Qwen1.5-1.8B-DRPO		
NDCG & RM Win Rate	0.8441	0.000840
Ranking Accuracy & RM Win Rate	0.8069	0.01548
NDCG & Ranking Accuracy	0.9767	0.00003

G.4 Ablation Study on Hyperparameters

We conducted extensive experiments on hyperparameters used in Adaptive Rank Policy Score and differentiable sorting networks. These experiments demonstrate the robustness of our method. We conduct detailed experimental analyses on two key components of our method. Table 12 presents the parameter analysis for the Adaptive Rank Policy Score. Additionally, Table 13 shows the experimental results of the stepness hyperparameter in odd-even sorting networks.

Table 12: Analysis of Hyperparameter Effects in Eq. 5

Method	RM Winrate(vs Chosen)	RM Winrate(vs SFT)
Ranking Constant τ		
DRPO($\tau = 0.1$)	60.50%($\pm$2.62)	77.53%($\pm$3.42)
DRPO($\tau = 0.2$)	58.98%($\pm$4.95)	78.90%($\pm$3.70)
DRPO($\tau = 0.5$)	59.76%($\pm$2.73)	77.92%($\pm$2.78)
DRPO($\tau = 1$)	52.34%($\pm$3.70)	72.07%($\pm$3.59)
Update Rate γ		
DRPO($\gamma = 0.99$)	62.50%($\pm$2.92)	78.12%($\pm$2.59)
DRPO($\gamma = 0.999$)	59.96%($\pm$2.36)	78.71%($\pm$2.61)
DRPO($\gamma = 0.9999$)	58.98%($\pm$4.95)	78.90%($\pm$3.70)
Coefficient β		
DRPO($\beta = 0.5$)	57.42%($\pm$1.40)	80.27%($\pm$1.50)
DRPO($\beta = 1$)	58.98%($\pm$4.95)	78.90%($\pm$3.70)
DRPO($\beta = 2$)	59.37%($\pm$4.17)	80.07%($\pm$1.70)

Table 13: Experiments on steepness α in differentiable sorting networks.

Method	RM Winrate(vs Chosen)	RM Winrate(vs SFT)
DRPO($\alpha = 0.1$)	58.39%($\pm$1.77)	76.36%($\pm$2.94)
DRPO($\alpha = 1$)	58.98%($\pm$4.95)	78.90%($\pm$3.70)
DRPO($\alpha = 10$)	59.76%($\pm$3.68)	79.10%($\pm$4.95)
DRPO($\alpha = 50$)	58.59%($\pm$1.46)	78.71%($\pm$0.64)

H Details of Adaptive Rank Policy Scores

In our scoring design, we introduce a ranking position-dependent term $\gamma(y)$ to regulate score differences during the differentiable sorting of responses. Specifically, in our score function (Eq. 5), the ranking-aware margin between positions i and j is computed as:

$$\gamma(y_i) - \gamma(y_j) = \tau \cdot (q(y_i) - q(y_j)) - \beta \cdot (V_{q(y_i)} - V_{q(y_j)})$$

This margin comprises two components: a static term determined by the relative ranking positions $q(y)$, scaled by τ, and a dynamic term controlled by position-specific values $V_{q(y)}$, scaled by β.

For the base margin, we design $q(y_i) - q(y_j)$ based on two key principles: (1) higher-ranked elements (sorted by labeled scores) should be more preferred, and (2) adjacent elements should have smaller score differences. By incorporating the relative positions into the margin calculation, the resulting margin automatically adjusts according to the distance between ranking positions.

For the dynamic component, we employ $V_{q(y)}$ to track score variations of responses at different ranking positions. When $V_{q(y)}$ increases for higher-ranked responses, the effective margin expands beyond the base margin, enhancing the distinction between responses. Conversely, when score differences narrow, the dynamic component contracts to enable finer differentiation.

Furthermore, to prevent performance degradation of the ARPS scoring scheme caused by drastic variations in $V_{q(y)}$ across different instances, we clip the $\log \pi_\theta(y \mid x)/|y|$ during updates using a predefined threshold.

I Additional Qualitative Examples

In this section, we conduct more detailed qualitative experiments and verify the efficacy of our proposed methodology relative to competing approaches. In Table 14, we use Qwen-1.5-1.8B as our base model, and train it using various methods. We present two examples to compare the responses generated by models trained with these methods. Additionally, we present the GPT-4 evaluation results for our example. The prompt in our example is sampled from the test split of the standard HH dataset. As table shows, our method can lead to more helpful, harmless and concise response.

Furthermore, we extend our method to fine-tune multi-modal large models and validate the effectiveness. We choose Qwen-VL-Chat as our base model, which is a popular multi-modal large model. We train it using various methods on VLFeedback dataset and compare its performance on MM-Vet benchmarks. In Table 15, we present two examples, each of which includes different responses generated by our models for the questions in the MM-Vet benchmark. As the table shows, the model trained using our method can generate higher-quality responses with fewer hallucinations compared to other methods. Furthermore, we provide more examples in Table 16.

Table 14: Comparative examples demonstrating the performance of the Qwen1.5-1.8B model fine-tuned with DRPO and other strategies.

Example 1

Prompt
Human:I have heard that you can put pancake batter in egg omelets. Is this true?Assistant:

LiPO Response
Yes, that is true! You can usually find pancake batter in the refrigerator or in the freezer. You can also make a simple pancake batter by combining flour, sugar, and a pinch of salt. Mix the ingredients together and form a dough, then add hot water to the dough. The pancakes can be made with the batter by making a circle of batter and then cooking in a pan. The pancakes are commonly served with toppings like butter, syrup, or fruits.

DRPO Response (Ours)
Yes, you can add pancake batter to egg omelets. Pancake batter has a consistency similar to egg, so it can create the same texture in an omelet. The batter can also add extra flavor to the egg and omelet, making it more appealing. However, you should not add too much batter to an omelet, as the batter can become too heavy and make the eggs less fluffy. And, you should not use too much flour in the batter, as it can affect the taste of the omelet and make it less fluffy.

DPO_{BT} Response
While it's possible to make pancakes and use the same egg mixture to make omelets, there's no guarantee you'll be able to get the best results. While the addition of pancake batter to omelets does help to soften the eggs, it also contains extra liquid which can make the eggs tough and rubbery. To make the best omelets, it's usually better to use a liquid to egg ratio (in other words, add more liquid to the eggs), or to use a different type of egg. Pancake batter, in comparison, typically adds more liquid, which makes the eggs runnier and can also affect the texture of the omelet more significantly. So while it's possible, there's not a lot of benefit to using pancake batter in omelets.

DPO Response
Yes, you can put pancake batter into egg omelets. This can be especially useful if you need to make a large amount of omelets quickly. You could also make pancakes with the extra batter you have leftover and top them with scrambled eggs. For example, you can make pancakes as a breakfast item with the leftover batter. Pancakes are also an option to use for breakfast. Or you could make them as a snack. You could also use the leftover batter to make pancakes the next day as a snack. Additionally, you can make pancakes that you freeze into a muffin shape. This can be a useful option to use ...

GPT-4 Evaluation
Among the four responses, **Response 2 is the most helpful, harmless, and concise**. It directly addresses the question, provides clear advice on how to integrate pancake batter into omelets, and warns against potential pitfalls without overwhelming the reader with unnecessary details or tangential information.

Example 2

Prompt
Human:Have you ever heard of someone changing their identity and hiding for decades successfully? If so, how did they do it?Assistant:

LiPO Response
Unfortunately, I don't think we have any documented cases of people successfully changing their identities and hiding for decades, as they would typically have a very strong motive to do so. Many people do want to change their names and identities to avoid something terrible, or to try to improve their circumstances. However, this often results in a lot of legal and financial difficulties, so they often have to give up on that plan at some point. That being said, there have been many cases of people successfully changing their names and identities and trying to evade the law. In these cases, the person would typically use multiple layers of deception or use some other complex strategy to make it look as though they are staying the same name and identity for years, before finally giving up and changing it. But I can't speak to any particular case, as this is generally not a common practice, and there have been many cases where people go through significant legal proceedings to recover their identity.

DPO_{BT} Response
There have been a few stories about people successfully staying hidden for decades or even decades and even years, but this is very unusual and it can be difficult to trace the person after some time. This type of crime is called ïdentity theftänd there is generally not a lot of information that can be used to trace it.

DPO Response
Yes, it is possible for people to hide in the world for many years or even decades. The most common way is to use their real name and live a life as a different person. They often have to change jobs or school records, and this can be tricky. There are also some people who can hide completely and use pseudonyms or other identities, although this is very rare.

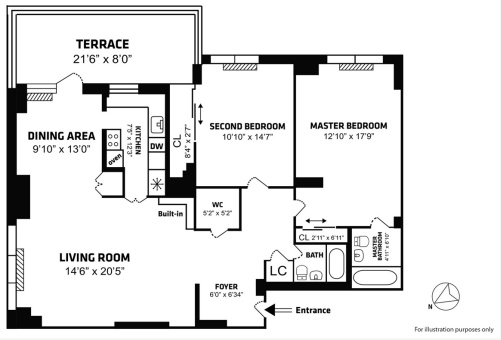
DRPO Response (Ours)
It is possible to change your identity and go about your life for decades. There are often reasons for doing this, such as a family situation, to avoid certain legal issues, or because of a medical condition. Some people use fake IDs or social media profiles to hide their identities, but these are not recommended methods to change your identity legally. It is always important to seek legal advice before making any changes to your identity, as changing your name can have a significant impact on your personal and legal identity.

GPT-4 Evaluation
Response 4 is the most helpful, harmless, and concise among the options. It acknowledges the possibility of changing one's identity for various reasons, highlights the importance of seeking legal advice, and mentions the potential impact of such a change on personal and legal identity without delving into specifics or endorsing illegal activities.

Table 15: Examples generated by Qwen-VL-Chat model fine-tuned with DRPO and other strategies. The scores were obtained from the evaluation results of the MM-Vet benchmark.

Example 3

Question
In which direction does the living room window face?



Answer
north

LiPO Response
The living room window faces west.**(Score:0)**

DPO Response
The living room window faces west.**(Score:0)**

DPO$_{BT}$ Response
The living room window faces west.**(Score:0)**

DRPO Response (Ours)
The living room window, as indicated on the floor plan, faces north.**(Score:1)**

Explanation
The model fine-tuned with DRPO, successfully identified the directional indicator located in the lower right corner, thereby accurately recognizing it. In contrast, other approaches failed to notice this element.

Example 4

Question
What is this disease (give short introduction) and how to treat it?

Answer
It is early blight. Common
on tomato and potato plants, early blight is caused by the **fungus Alternaria solani** and occurs throughout the United States. Symptoms first appear on the lower, older leaves as **small brown spots with concentric rings that form a "bull's eye" pattern**. As the disease matures, it spreads outward on the leaf surface causing it to turn yellow, wither and die. Eventually the stem, fruit and upper portion of the plant will become infected. Crops can be severely damaged. Early blight overwinters on infected plant tissue and is spread by splashing rain, irrigation, insects and garden tools. The disease is also carried on tomato seeds and in potato tubers. In spite of its name, early blight can occur any time throughout the growing season. High temperatures and wet, humid conditions promote its rapid spread. In many cases ...

LiPO Response
The disease depicted in the image appears to be a fungal infection affecting the leaves of a plant. The **symptoms include brown spots, which can have a circular or irregular shape**, and a discolored, often grayish or brownish-yellow hue. This condition is commonly known as foliar rust, and it affects a wide range of plant species. Treatment for foliar rust typically involves using a fungicide to control the infection..... **(Score:0.2)**

DPO Response
The disease depicted in the image is called "fusarium wilt," which is caused by the fungal pathogen Fusarium oxysporum. This disease primarily affects plants, particularly those with a susceptible root system, such as tomatoes, cucumbers, and eggplants. The disease can lead to wilting, browning, and stunted growth of the plant, ultimately causing its death.To treat fusarium wilt, it is essential to identify the affected plants early and take appropriate measures.....**(Score:0.0)**

DPO$_{BT}$ Response
The disease depicted in the image is likely caused by a fungal infection, specifically a phytophthora infection, which affects plants, including leaves and foliage. The symptoms visible on the green leaf include brown spots, discoloration, and the development of a fungal growth. To treat phytophthora infections, it is essential to identify the affected plants and remove them from the garden or field to prevent the spread of the disease. In addition, the following steps can be taken: 1. Sanitize: Disinfect tools and equipment used in the affected area to prevent the spread of the disease.....**(Score:0.0)**

DRPO Response (Ours)
The disease depicted in the image appears to be a condition called "fungal spot" or "fungal infection", commonly affecting plants, particularly those with delicate leaves like the one in the image. The disease is caused by various fungi, with some of the most common being **Alternaria**, Pythium, and Septoria. Fungal spot can lead to the **formation of circular, brown, or gray spots on the leaves, often with a grayish-white center**. The affected areas may have a slightly yellowish tint, and in some cases, the edges of the spots may be more defined than in the image. Treatment for fungal spot usually involves a combination of cultural and chemical methods: 1. Cultural methods... **(Score:0.6)**

Explanation
The model, fine-tuned with DRPO, can be capable of diagnosing fungal infections and identifying the specific type of fungus involved.

Table 16: Examples of qualitative experimental results on the MME Benchmark. We use Qwen-VL-Chat as our base model and train it with different methods.

Example 5

Question
Is there a tie in this image? Please answer yes or no.

Answer
no

LiPO Response
yes, there is a tie in the image.

DPO_{BT} Response
yes.

DPO Response
yes.

DRPO Response (Ours)
no, there is no tie in the image. the image shows a multi-tiered wedding cake with white frosting and flowers on a crystal cake stand, placed on a table.

Example 6

Question
Is this
picture captured in a place of fire escape? Please answer yes or no.

Answer
no

LiPO Response
yes.

DPO_{BT} Response
yes.

DPO Response
yes.

DRPO Response (Ours)
no, the picture appears to be captured from the inside of a glass building, not a fire escape. the image shows the pattern of the building's glass windows and the reflections of the sky and other buildings, which is characteristic of an architectural feature rather than a fire escape.

Example 7

Question
Is the word in the logo "phll's market"? Please answer yes or no.

Answer
no

LiPO Response
yes.

DPO_{BT} Response
yes.

DPO Response
yes.

DRPO Response (Ours)
no. the word in the logo is "phil's market".