Distributed Fixed-Time Control for Interconnected Systems

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***Abstract*—Large-scale interconnected systems have applications in various aspects of production and daily life, such as smart grids, industrial automation, transportation networks, and supply chain management. Therefore, this area has received significant attention in control research. Compared to other control systems, the structural coupling of interconnected systems increases the difficulty of research and can also affect control performance. Consequently, this paper designs a distributed dynamic state feedback fixed-time controller, ensuring that the state converges to zero within a fixed time (which is of great significance in practice, as it avoids the slow convergence associated with asymptotic stability). Additionally, we explore the impact of interconnectivity on convergence speed, aiming to provide a quantitative relationship between settling time and interconnectivity.**

We consider the following form of an interconnected system containing S subsystems indexed by $N = \{1, \ldots, S\}$:

$$\dot{x}_i(t) = A_{ii}x_i(t) + \sum_{j\in N, j\neq i} A_{ij}x_j(t) + u_i(t),$$

where x_i represents the node state and u_i is the control input. Considering that the communication topology is not equivalent to the physical topology, we design the following nonlinear dynamic state feedback control law:

$$\dot{x}_{ci}(t) = \dot{x}_{c1i}(t) + \dot{x}_{c2i}(t), \quad u_i(t) = x_{ci}(t).$$

In this equation, the first term on the right side of the first equation is a linear control term determined by LMI techniques, while the second term is a nonlinear acceleration term designed based on the Control Lyapunov Function (CLF) method, with the specific form:

$$\begin{aligned}\dot{x}_{c1i}(t) =& K_{1i}x_i(t) + K_{2i}x_{ci}(t), && \textbf{when } x_i \neq 0,\\ \dot{x}_{c2i}(t) =& (||z_i||^2)^{-1} z_i^T (F(V_i) + \sum_{j\in\tilde{N}_i} F(V_j) \\ &+ (||z_i||^4 + (F(V_i) + \sum_{j\in\tilde{N}_i} F(V_j))^2), && \textbf{when } z_i \neq 0.\end{aligned}$$

Here, $F(V_i)$ is the right side of the inequality for the fixed-time stability criterion based on the Lyapunov inequality, and z_i is the coefficient corresponding to the control rate u_i after differentiating the quadratic Lyapunov function $V_i(x_i, x_{ci})$. Under the influence of the controller, if $\tilde{N}_i = 0$ (indicating the distributed controller has degenerated into a decentralized one), then the overall Lyapunov function $V(t) = \sum_i V_i$ converges at time T_s, which can be determined by the fixed-time stability criterion. In other words, the state converges to zero for $t > T_s$.

Now, we focus on the convergence time under the influence of the distributed controller. Specifically, when $\tilde{N}_i \neq 0$, assuming that the minimum out-degree of all nodes in the network layer (or communication layer) of the aforementioned distributed controller is D_{min}, the convergence time of the entire system is $(1 + D_{min})^{-1}T_s$. When the communication graph of the system is fully connected, the convergence time can reach its minimum of $N^{-1}T_s$. Due to space limitations, the specific proof is omitted.

***Index Terms*—Fixed time control, interconnected system, dynamic nonlinear feedback.**

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