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# **CausalPlayground: Addressing Data-Generation Requirements in Cutting-Edge Causality Research**

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## Abstract

001 Research on causal effects often relies on synthetic data due 002 to the scarcity of real-world datasets with ground-truth effects. Since current data-generating tools do not always 003 004 meet all requirements for state-of-the-art research, ad-hoc methods are often employed. This leads to heterogeneity 005 006 among datasets and delays research progress. We address 007 the shortcomings of current data-generating libraries by in-800 troducing CausalPlayground, a Python library that provides a standardized platform for generating, sampling, and shar-009 010 ing structural causal models (SCMs). CausalPlayground offers fine-grained control over SCMs, interventions, and 011 the generation of datasets of SCMs for learning and quan-012 titative research. Furthermore, by integrating with Gym-013 nasium, the standard framework for reinforcement learning 014 015 (RL) environments, we enable online interaction with the 016 SCMs. Overall, by introducing CausalPlayground we aim to foster more efficient and comparable research in the field. 017 018 All code is available at https://anonymous.4open. science/r/CausalPlayground-D1B2/. The API 019 020 documentation will be released after de-anonymization.

#### 1. Introduction 021

022 Ever since the formalization of causality [18] the field has gained significant attention for improving inference from 023 024 data, scientific discovery, and others. Progress in causal-025 ity research relies heavily on data collection about the sce-026 nario that is being investigated. However, a major challenge is the lack of real-world data with known causal ground 027 truth [6]. As an example, even after decades of research, 028 only a few strong real-world benchmark datasets are avail-029 able for causal structure discovery [6, 32]. Consequently, 030 031 many state-of-the-art methods use synthetically generated 032 data, such as in causal representation learning [16], causal discovery [9], among others. 033

034 Naturally, each research comes with its specific require-035 ments for the data-generating process that is being investigated. In causality particularly though, some requirements can be observed that are common among many in-037 stances. We identify them as requirements for a causal data-generating library to be useful for a broad variety of 039 research questions as follows:

- R1 Interventional data generation: Intuitively, interventions are experiments in an environment that are crucial for identifying causal effects [4]. Therefore, a general causal data-generation framework must facilitate the sampling of interventional data.
- R2 Interaction with the causal model: It is becoming in-046 creasingly clear, that interacting with the causal model is 047 beneficial for many tasks. For example, intervening after 048 every sample can improve sample efficiency for causal 049 discovery methods [22, 23] and causal inference [29]. 050 This requirement is further highlighted by the popularity 051 of interactive frameworks like Causal World [1] and var-052 ious interactive causal models created ad-hoc for specific 053 research questions [15, 29, 35]. 054
- R3 Fine-grained control over the causal model: As causal inference and discovery often investigate settings with clearly defined assumptions on the data, like the popular linear functions with additive with non-Gaussian noise [19] assumption, a general data-generation library must provide detailed control over the functional relations of the causal model.
- R4 Causal model generation for quantitative results: Recent research in causal methods, such as [9, 17, 22, 23], emphasize the need for training and evaluation not merely on a single causal model, but rather on data-sets of models. This results in the requirement for a general causal data-generating library to be able to easily create many causal models.

As our comparison in Sec. 2 shows existing tools fall 069 short of addressing all current requirements within one 070 framework. This leads to many researchers having to rely 071 on ad-hoc data generation processes, introducing undesir-072 able heterogeneity of datasets amongst methods, more dif-073 ficult comparisons of results, and generally obstructing the 074 research progress of the field. 075 CVPR 2024 Submission #\*\*\*\*\*. CONFIDENTIAL REVIEW COPY. DO NOT DISTRIBUTE.

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Do-Why [25]	$\checkmark$	Х	Х	Х	Х	$\checkmark$	Р
TETRAD [21, 24]	Х	$\checkmark$	Х	Х	Х	$\checkmark$	J/P/R
cause2e [8]	$\checkmark$	$\checkmark$	Х	Х	Х	$\checkmark$	Р
Lawrence et al. [14]	Х	$\checkmark$	Х	Х	$\sim$	Х	Р
MANM-CS [11]	$\checkmark$	$\checkmark$	Х	Х	$\sim$	Х	Р
BNlearn [27]	$\sim$	Х	Х	Х	Х	$\checkmark$	Р
causaldag [5]	$\sim$	$\checkmark$	Х	Х	Х	$\checkmark$	Р
gCastle [33]	Х	$\checkmark$	Х	Х	$\sim$	$\checkmark$	Р
CDT [12]	Х	$\checkmark$	Х	Х	$\sim$	$\checkmark$	Р
SCModels [2]	$\checkmark$	Х	Х	Х	$\checkmark$	Х	Р
R6causal [13]	$\checkmark$	Х	Х	Х	$\checkmark$	$\checkmark$	R
CausalWorld [1]	$\checkmark$	Х	Х	$\checkmark$	Х	$\checkmark$	Р
SynTReN [31]	Х	$\checkmark$	Х	Х	Х	Х	J
pcalg [3]	Х	$\checkmark$	Х	Х	Х	$\checkmark$	R
JustCause [10]	Х	Х	Х	Х	$\checkmark$	$\checkmark$	Р
CausalPlayground	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	Р

Table 1. Overview of software libraries that can be used for data-generation for causality research. Symbols  $\checkmark$ , X, and  $\sim$  indicate whether a requirement is met, not met, or partially met, respectively. J, P, R refer to Java, Python, and R, respectively. We evaluate the methods on the ability to sample interventional data, generate directed acyclic graphs (DAG), generate SCMs, interact with the SCM, the availability of a detailed API documentation, and the programming language that it is written in.

076 To offer a solution to this problem we created CausalPlayground, a Python library for generating syn-077 thetic data and providing generation processes for causal 078 079 models. Our library enables fine-grained control over the models, integrates processes within the popular RL frame-080 work Gymnasium [30] to accommodate the requirement for 081 082 interacting with the causal model, and implements the generation of many causal models at once to enable learn-083 ing and quantitative research. It thereby constitutes a 084 085 framework for causal data generation that allows for more standardized and easy-to-share causal data-generation pro-086 087 cesses.

In the remainder of the paper we will compare current
causal data-generating libraries in Sec. 2, introduce fundamental notions of causality more formally in Sec. 3, provide an overview of *CausalPlayground* in Sec. 4, outline
a simple use-case in Sec. 5, and give an outlook on future
versions of this library in Sec. 6.

# 094 **2. Related Work**

Based on the requirements we identified in Sec. 1, we investigate the current landscape of general purpose causal data-generation tools. We restrict our comparison to packages specifically designed for causality-related research, acknowledging that other general-purpose tools have been used, such as common RL environments like MuJoCo [28].

The comparison is summarized in Tab. 1.

Regarding interventional data-generation (R1), many available packages, including Do-Why [25], cause2e [8], MANM-CS [11], SCModels [2], R6causal [13], and CausalWorld [1], allow for sampling from interventional distributions for specifiable functional interventions. BNlearn [27] and causaldag [5] offer less versatile methods for intervening, and facilitating interventions by changing the node conditional distributions or the graphical structure, respectively.

Considering interaction with causal models (R2), only CausalWorld [1] provides out-of-the-box functionality for actively interacting with the model, highlighting an opportunity for useful new tools to meet this requirement.

With respect to generating causal models with specific assumptions on the causal functions (R3) SCModels [2], R6causal [13], and JustCause [10] give full control over functional relations and noise distributions, while Lawrence et al. [14], MANM-CS [11], gCastle [33], and CDT [12] provide coarse-grained control that simultaneously applies to all functions and distributions.

Lastly, to the best of our knowledge, no general-purpose framework for causal data-generation provides methods to generate sets of causal models with specific functional relations and noise distributions (R4).

Overall, this comparison shows a gap in libraries that simultaneously address some of the most important require-127

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ments for causal data-generation. This opens the opportunity to develop a tool that is relevant to a broader set of research questions.

## **3. Fundamental Concepts of Causality**

132 In this section, we provide an overview of the necessary for-133 mal concepts for our approach. Let an SCM M be a tuple  $M = (\mathcal{X}, \mathcal{U}, \mathcal{F}, \mathcal{P})$ , where  $\mathcal{X}$  and  $\mathcal{U}$  are sets of endoge-134 nous and exogenous random variables, respectively,  $\mathcal{F}$  is a 135 set of functions  $f_i$  that map the direct causes of  $X_i \in \mathcal{X}$ 136 to  $X_i$ , and  $\mathcal{P}$  is a set of pairwise independent probability 137 138 distributions  $P_i$  s.t.  $U_i$  follows distribution  $P_i$  with  $U_i \in \mathcal{U}$ . 139 Furthermore, we denote interventions as  $do(X_i = g(...))$ , where q is an arbitrary function that takes as input a sub-140 set of  $\mathcal{X} \cup \mathcal{U}$ . Such an intervention replaces function  $f_i$ 141 142 with function q. Applying a set of N arbitrary interventions  $a_t = \{do_0(\ldots), \ldots, do_N(\ldots)\}$  to an SCM, means replac-143 ing all corresponding functions simultaneously. Further-144 more, each SCM M induces a distribution  $P^M(\mathcal{X}, \mathcal{U} \mid a_t)$ 145 that we can sample via ancestral sampling. 146

# 147 4. CausalPlayground

In this section we provide an overview of the *CausalPlay- ground* library. By implementing the functionalities that address the requirements R1 - R4, we provide an off-the-shelf
causal data-generation platform that can be used for stateof-the-art causality research.

### **4.1. Fine-grained control over the SCM**

For each endogenous variable  $X_i$ , the structural equation  $f_i$ can be arbitrarily defined. Similarly, the distribution of the exogenous variables  $P_j$  can be arbitrarily defined. Notably, this entails the possibility of defining pixel-based environments. An example for creating the SCM with  $\mathcal{F} = \{A \leftarrow U + 5, Effect \leftarrow 2A\}$  and  $U \sim Uniform(3, 8)$  is provided below:

```
161
      scm = StructuralCausalModel()
      scm.add_endogenous_var('A',
162
           lambda noise: noise+5,
163
           { 'noise ': 'U' })
164
      scm. add_exogenous_var('U',
165
           random.randint,
166
167
           { 'a': 3, 'b': 8})
      scm.add_endogenous_var('Effect',
168
           lambda x: x*2,
169
           \{ x': A' \}
170
```

This level of control over the SCM is a significant improvement compared to other libraries, which often provide more coarse-grained control.

# 4.2. Arbitrary interventions

Interventions can be applied to existing SCMs using arbitrary functions g. The only restriction is that the new function does not induce cyclic causal relations. In the example code snippet:175177178

<pre>scm.do_interventions([("Effect",</pre>	179
( <b>lambda</b> a: a+1,	180
{ 'a ': 'A' } ))])	181

the intervention sets the value of the variable Effect to the value of A + 1, using a lambda function that takes a as an argument and a dictionary that maps a to the variable name A. This, effectively, applies do(Effect = A + 1) to the SCM.

#### **4.3. Interaction with the SCM**

Users can intervene actively in existing SCMs. At each step, 188 a set of interventions  $a_t$  can be applied, after which a sample 189 of the endogenous and exogenous variables is drawn from 190 the induced distribution. The interventions are undone be-191 fore the next step. The interaction is enabled by wrapping 192 the SCMs in the popular Gymnasium environment, facilitat-193 ing standardized integration with (deep) RL methods [20]. 194 In the example code snippet: 195

env = SCMEnvironment(scm,	196
possible_interventions=	197
$[("A", (lambda: 5, {})),$	198
("Effect", (lambda a: a+1,	199
{ 'a ': 'A' }))])	200

An SCMEnvironment object is created by passing the scm object and a list of two possible interventions with the same syntax as described before. The resulting env object can be used to interact with the SCM using the standard Gymnasium environment interface, allowing for interactive exploration and experimentation with the causal model.

More specifically, calling env.step(action) applies the interventions defined in the action via their index, samples the intervened SCM, determines the new observation, termination flag, truncated flag, and reward, and, finally undoes the interventions. Importantly, the precise observation, termination, truncated, and reward functions can be specified by the user.

### 4.4. SCM generation

CausalPlayground's SCM generation procedure provides a<br/>flexible approach to creating datasets of SCMs. Users can<br/>specify the number of endogenous variables and exogenous<br/>variables, the presence of confounders, and define the class<br/>of possible causal relations. This is exemplified by the fol-<br/>lowing code:215<br/>216217<br/>218<br/>219218<br/>219

gen = SCMGenerator(all\_functions = 221

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222	{ 'linear ': f_linear })
223	scm_unconfounded = gen.create_random(
224	<pre>possible_functions =["linear"],</pre>
225	$n_endo=5$ , $n_exo=4$ ,
226	exo_distribution=random.gauss,
227	exo_distribution_kwargs =
228	$\{mu=0, sigma=1\},\$
229	$allow_exo_confounders =$
230	False)[0]

that creates a random SCM with the causal relations follow-231 232 ing a predefined f\_linear function, 5 endogenous vari-233 ables, 4 exogenous variables, a Gaussian exogenous distri-234 bution with  $\mu = 0, \sigma = 1$  for each exogenous variable, and 235 no confounders. Similarly, an SCM can also be generated 236 from a given causal structure.

This comprehensive control over the SCM generation 237 process facilitates the systematic evaluation and compari-238 son of causal models. 239

#### 4.5. Further Implementation Details 240

*CausalPlayground* is implemented in Python with 241 five main classes. The StructuralCausalModel 242 class represents an SCM and provides methods for 243 manipulation and sampling. The SCMGenerator 244 creates random SCMs based on specified param-245 while the CausalGraphGenerator 246 eters, and 247 CausalGraphSetGenerator generate random causal 248 graphs and sets of unique causal graphs, respectively. The 249 SCMEnvironment class, allows interaction with an SCM. Furthermore, CausalPlayground relies on external classes 250 such as networkx.DiGraph, pandas.DataFrame, 251 252 Gymnasium.Box, and Gymnasium.Sequence for All code is published under various functionalities. 253 254 the open-source MIT license and we highly welcome 255 contributions.

#### 5. Use-Case: Comparison of Causal Discovery 256 Algorithms 257

258 As one of many use-cases, we outline how to use the CausalPlayground library to compare different causal dis-259 covery algorithms. We generate synthetic data using the 260 261 library's functions and evaluate the performance of the algorithms on both confounded and unconfounded, automat-262 263 ically generated SCMs. We use the causal discovery algorithms provided by the gCastle [33] library. The detailed 264 265 implementation can be found in the repository examples.

First, we generate distinct confounded 266 and unconfounded graphs using the 267 causal CausalGraphSetGenerator class from CausalPlay-268 ground. Each of the sets has 4 endogenous variables and 269 270 4 exogenous variables. Based on these graphs we generate 271 the SCMs for evaluation using the SCMGenerator class.



Figure 1. Experimental results of the causal discovery algorithm comparison use-case.

The functional relations are determined by specifiable 272 functional relations. For our experiment we use linear 273 additive functions and functions that add all causes and 274 multiply the value of two randomly selected causes of a 275 variable. More specifically, for every endogenous variable, 276 the function is drawn from either of the two classes. 277

We then iterate over the 30 generated SCMs and draw 278 100 samples per SCM. We evaluate the performance of each 279 causal discovery algorithm on both confounded and uncon-280 founded datasets. More specifically, we compare PC [26], 281 GES [7], and NOTEARS [34] and measure F1 score and the 282 true-positive rate with gCastle [33]. The results can be seen 283 in Fig. 1. 284

This use-case provides a glimpse into the usefulness of CausalPlayground for causality research, not the least because the data-generating processes could now be shared among experiments and researchers.

# 6. Conclusion and Outlook

In this work, we identified the core requirements for data-290 generation in causal research and found that they are unsat-291 isfactorily met by the current landscape of data-generation 292 tools. To offer a solution, we introduced CausalPlayground, 293 a Python library that allows for sampling of, generation of, 294 and online interaction with SCMs. By providing functional-295 ity to address some of the main requirements of causality re-296 search, this library can enable a broad research community 297 to easily share their SCMs and more rigorously compare its 298 methods. 299

For future versions of this library, we are anticipating the 300 integration of more diverse causal models such as Bayesian 301 networks, and optimizations regarding parallelization for 302 fast deployment on GPUs. Ultimately, we envision that our 303 proposed library contributes to faster scientific progress in 304 fields that rely on causality. 305

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