

CoSineVerifier: Tool-Augmented Answer Verification for Computation-Oriented Scientific Questions

Anonymous ACL submission

Abstract

Answer verification methods are widely employed in language model training pipelines spanning data curation, evaluation, and reinforcement learning with verifiable rewards (RLVR). While prior work focuses on developing unified verifiers applicable across multiple reasoning scenarios, significant challenges remain in computation-oriented scientific domains, such as algebraic equivalence checking. In this paper, we introduce CoSineVerifier, a tool-augmented verifier that leverages external executors to perform precise computations and symbolic simplifications. CoSineVerifier enables robust verification that goes beyond simple semantic matching. To train this accurate tool-augmented verifier, we propose a novel verifier training data augmentation method and a two-stage training framework to increase the correctness of tool-invoked verifications on computation-heavy questions. Extensive experiments across STEM, QA, and long-form reasoning tasks demonstrate CoSineVerifier’s robust generalization, achieving state-of-the-art performance on VerifyBench-Hard and SCI-Bench. Furthermore, when employed as an RLVR reward model, CoSineVerifier consistently outperforms both rubric- and model-based verifiers on AIME’24, AIME’25 and GPQA-D, highlighting its potential to advance LLM reasoning. Our code and model are available in our anonymous [repository](#).

1 Introduction

The training methods for large language models (LLMs) have gradually shifted from using human-annotated data to model self-evolution approaches, which also imposes higher demands on how to evaluate the quality of generated content. In this context, answer verification, which compares a model output against a reference, has become a critical component throughout data curation, evaluation, and model training pipelines. In the data curation phase, answer verification methods are employed

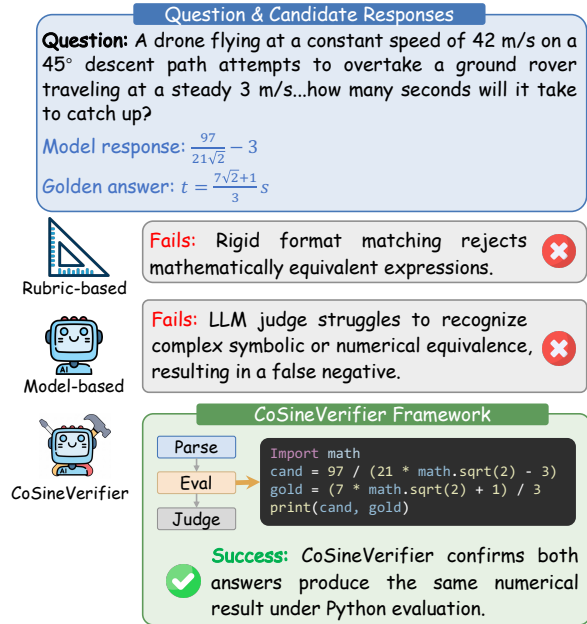


Figure 1: Comparison between CoSineVerifier and existing verifiers, where CoSineVerifier enables accurate judgment on calculation-intensive scenarios.

to guarantee the consistency, fidelity, and accuracy of synthetic corpora (He et al., 2025b; Yuan et al., 2025; Toshniwal et al., 2024; Albalak et al., 2025). At the evaluation stage, these methods provide a robust alternative to exact match (EM), enabling semantic-level comparison (Ma et al., 2025a; Li et al., 2024; Paech, 2023; Wu et al., 2025b). For model training, Reinforcement Learning with Verifiable Rewards (RLVR) relies on answer verifiers to produce reward signals (He et al., 2025a; Guha et al., 2025; Guo et al., 2025).

Rubric-based verifiers (Kydliček), which depend on structured outputs (e.g., Math-Verify evaluating equivalence in Markdown), usually fail even on simple cases such as comparing $2*2$ and 4 . Alternatively, many studies (Chen et al., 2025a; Liu et al., 2025a; Chen et al., 2025b) employ LLM-based verifiers that assess answers through rea-

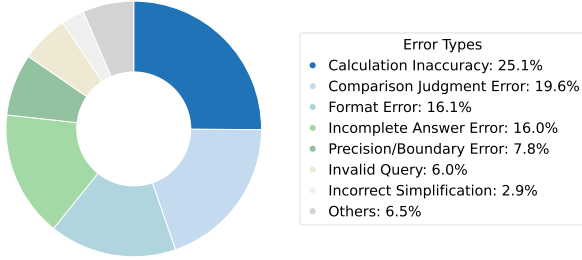


Figure 2: Error type distribution of verifiers.

soning. However, these verifiers are still prone to hallucination and inconsistencies in numerical or symbolic reasoning (e.g. comparing matrices with hundreds of entries). The lack of robust computation skills prevents verifiers from giving accurate rewards in RLVR, thereby limiting the performance when training LLMs.

As shown in Figure 1, to enable robust verification, a reliable verifier must be capable of performing arithmetic transformations before making a judgment. In this paper, we present CoSineVerifier, a tool-augmented LLM-based verifier designed for checking answers to computation-heavy questions. We equip the verifier with external tools, such as a Python interpreter, to perform precise computations on both formulas to derive aligned numerical results, thereby ensuring accurate verification. However, foundation models are not well-tuned on precise tool-augmented verification, which may issue inaccurate and redundant tool calls. To address this challenge, we propose a novel verifier training method. We construct a diverse set of questions spanning multiple domains, including mathematics, natural sciences, and logical reasoning.

Before constructing the dataset, we first conduct a comprehensive analysis of how common verification methods fail. As illustrated in Figure 2, we identify two major challenges: (i) Calculation inaccuracy represents the largest proportion of verification failures. (ii) Error types are diverse and follow a long-tail distribution, making it difficult for previous data augmentation approaches to achieve comprehensive coverage. Based on these insights, we design a long-tail and difficulty oriented data synthesis method to generate challenging computation-heavy instances. This approach increases both the density of queries prone to verification errors and the proportion of cases that require computational tool-augmented reasoning.

Finally, we employ a two-stage training strategy to equip the verifier with effective tool-use capabil-

ities in computation-heavy scenarios: (i) cold-start supervised fine-tuning, followed by (ii) reinforcement learning with tool-call encouragement. In the second stage, we propose an agentic reinforcement learning with rewards designed to explicitly encourage precise and effective tool usage. Extensive experiments conducted on two verifier benchmarks demonstrate that our proposed CoSineVerifier outperforms the state-of-the-art verifiers. To validate the effectiveness of CoSineVerifier when using it as a reward model in downstream tasks, we compare the performance of LLMs trained using CoSineVerifier and other verifiers on AIME24 and AIME25 benchmarks. And the results show that the LLM using CoSineVerifier as the reward model achieves the best performance compared to the current state-of-the-art verifier.

To summarize, our contributions are threefold:

- We propose **CoSineVerifier**, a tool-augmented answer verification model designed for providing precise training signals for computation-heavy scientific scenarios.
- We propose a novel verifier training data augmentation method and a two-stage training framework to increase the correctness of tool-invoked verifications on computation-heavy questions.
- Experimental results demonstrate that the CoSineVerifier achieves the state-of-the-art performance on both verification tasks and downstream RLVR applications.

2 Related Works

2.1 Answer Verification

Answer verification is the automated process of deciding whether a candidate response is factually correct to the provided reference answer. Significant progress has been made in the development of LLM-output verification (Liu et al., 2025b; Xia et al., 2025), which can broadly be divided into two main classes: rule-based verification and model-based verification. Early rule-based methods like Math-Verify (Kydliček) extract the final answer via heuristics and compare it against the ground truth using predefined rules. However, these approaches struggle with unstructured model outputs and tend to incur a high false-negative rate (Huang et al., 2025). To address these shortcomings, model-based verifiers exploit the flexibility of LLMs to interpret and evaluate unpredictable output. Xverify (Chen et al., 2025a) and CompasVerifier (Liu et al., 2025a) are designed

to provide efficient and robust answer verification by carefully constructing training examples. While effective, they are constrained by the verifier’s inherent reasoning and calculation competency—particularly in computation-intensive math and unit-sensitive STEM problems.

2.2 Tool Integrated Reasoning

A growing line of work investigates improving LLM reasoning by augmenting models with external tools, yielding consistent gains in accuracy, calibration, and sample efficiency across math, program, and multi-turn QA tasks (Mai et al., 2025; Chen et al., 2025c; Dong et al., 2025; Zhang et al., 2025; Ma et al., 2025b; Lin and Xu, 2025). ReTool (Feng et al., 2025), ToRL (Li et al., 2025), and SimpleTIR (Xue et al., 2025) train models to interleave real-time code execution with language reasoning and to learn when/how to call tools via reward signals, yielding strong gains on math-reasoning benchmarks. VTool-R1 (Wu et al., 2025a) integrates Python-based visual editing tools (box, mask, highlight) into the RFT loop so the model learns to interleave intermediate visual steps with textual chain-of-thoughts. Extending beyond code, Search-R1 (Jin et al., 2025) casts web search as an action space and trains query generation/revision via outcome rewards for multi-turn retrieval-augmented reasoning. Taken together, these approaches recast reasoning over a unified tool-action space, spanning code, vision, and search—where outcome-level rewards teach models when to plan, execute, and reflect, yielding stronger generalization and reliability than standard training beyond base models.

3 Method: CoSineVerifier

In this section, as shown in Figure 3, we outline the workflow and training procedure of CoSineVerifier, which is endowed with agentic tool-use capabilities for accurate outcome verification.

3.1 Tool-augmented Verification

To facilitate the reliable outcome verification, we equip our verification model with external tools that can dynamically execute computations, enabling precise judgments on calculation-intensive and science tasks. Specifically, as shown in Figure 3 (a), when facing a verification problem, our CoSineVerifier first decides whether to use an external tool. If an external tool call is needed, CoSineVerifier first selects the tool and parses the tool arguments

(e.g., Python code), and then sends them to sandbox environments. After the sandbox environment completes the execution and returns the output, CoSineVerifier gives its final judgment based on the sandbox output.

3.1.1 Cold-start Training

To ensure verification accuracy and robustness, we equip CoSineVerifier with two executable tools: a Python interpreter and a unit-conversion utility. During the preliminary experiment of CoSineVerifier, we find that directly applying reinforcement learning to the base model yields performance degradation, due to the base model’s limited tool-calling capabilities. We therefore implement a cold-start stage to equip the model with foundational tool-use proficiency. Specifically, we first collect real-world tool-use traces from our training data and retain only those cases that (i) produce a correct final answer and (ii) contain explicit tool-call trajectories. To boost the efficiency of CoSineVerifier, we discard model internal `<think>...</think>` content, and further filter out responses longer than 200 tokens to keep the cold-start corpus compact and efficient. This procedure yields 29,339 tool-usage examples used to bootstrap training. During training CoSineVerifier, we mask the loss on tool-execution outputs (e.g., interpreter traces and unit-conversion results), which prevents the model from merely imitating tool dumps and improves optimization stability.

We also train a variant model named CoSineVerifier-Label, which is trained on the full 63,374 disagreement data together with 10k model-generated long-tail instances, and provides verdicts as binary classification with targets *Correct* and *Incorrect*. And in this variant model, we did not use tool calling in CoSineVerifier-Label.

3.2 Data Curation

The answer verification task presents a dilemma regarding model selection: large-scale LLMs (e.g., >100B parameters) are too inefficient for the pipeline, while smaller LLMs lack the proficiency for tasks involving complex computational tools. To address this, we need to construct a specialized dataset for fine-tuning models on answer verification.

Data Collection Answer verification in real-world applications is challenging for two reasons: (i) the diversity of questions encountered in prac-

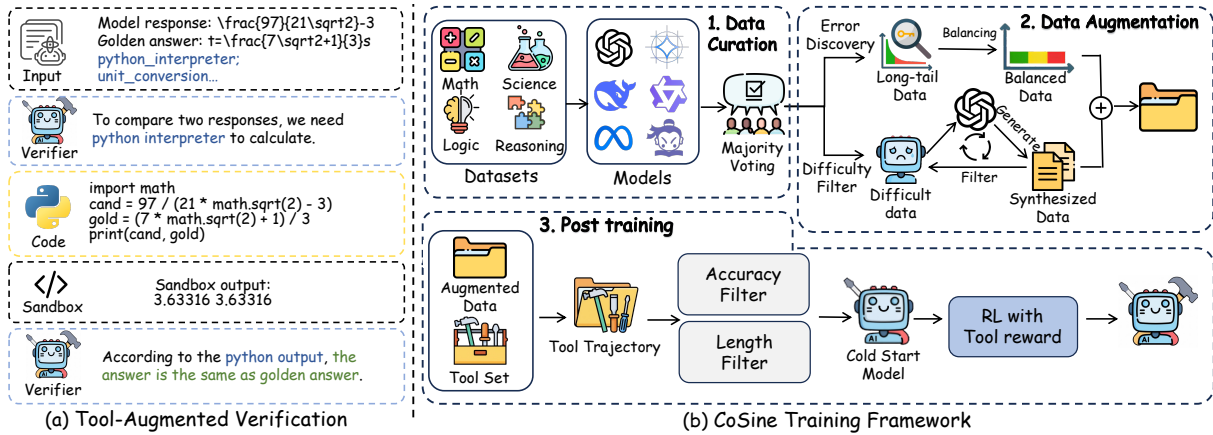


Figure 3: (a) Workflow of tool-augmented verification. (b) The overall training framework of CoSineVerifier.

250 tice, and (ii) the variability of model outputs (for- 286
 251 mat, length, and reasoning style). To address these 287
 252 challenges, we construct a large-scale candidate 288
 253 training dataset, comprising 1.14 million samples. 289
 254 These samples were collected from 15 datasets 290
 255 using 14 LLMs, spanning mathematics, science, 291
 256 logical reasoning, and commonsense knowledge 292
 257 domains. This provides broad coverage for train- 293
 258 ing robust verification models. In addition, to 294
 259 strengthen CosineVerifier on practical verifica- 295
 260 tion tasks, we deliberately go beyond static bench- 296
 261 marks (in contrast to prior collections such as Com- 297
 262 passVerifier (Liu et al., 2025a) and Xverify (Chen 298
 263 et al., 2025a)). A substantial portion of our data 299
 264 is sampled from competition-level math and science 300
 265 questions, where verification often fails in practice. 301
 266 During the construction of our data, we also en- 302
 267 sure cross-domain coverage, and to further increase 303
 268 robustness, we also prepend different prompt pre- 304
 269 fixes (e.g., Let’s think step by step) on a subset of 305
 270 training data, promoting generalization to diverse 306
 271 prompting styles. 307

272 **Data Annotation** After constructing a large and 308
 273 diverse training dataset, we introduce an itera- 309
 274 tive annotation pipeline that produces reliable ver- 310
 275 ification labels. We first employ a set of mod- 311
 276 els (Compass-Verifier-32B, qwen3-4b-2507, and 312
 277 Qwen3-32B), with each model providing three in- 313
 278 dependent annotations for a total of nine per sam- 314
 279 ple. We then retain 63,714 samples that show any 315
 280 disagreement among these nine annotations. For 316
 281 samples that showcase a high disagreement rate 317
 282 (defined as more than three out of nine annotations 318
 283 differing), we further send them to a more power- 319
 284 ful model (GPT-o3) with majority voting, ensuring 320
 285 high label fidelity on harder questions.

3.3 Data Augmentation

Long-tail data augmentation As shown in Fig- 287
 288 ure 2, all error types reveal a pronounced skew: 289
 290 the top five categories—dominated by Calculation 291
 292 Inaccuracy and Exact Match Failure, account for 292
 293 over 85% of errors, while the remaining ten cate- 294
 295 gories are sparsely represented. This distribution 295
 296 makes low-frequency errors harder to learn. To 296
 297 counter this imbalance, we use gpt-o3 to synthe- 297
 298 size targeted long-tail verification examples under 298
 299 constrained generation settings, yielding 10k long- 299
 300 tail synthesis instances. We further incorporate 300
 301 these samples into training for both the labeling 301
 302 verifier and the tool-augmented verifier, improving 302
 303 coverage of rare error modes. All training data 303
 304 statistics can be found in Appendix 7.4. 304

Difficulty-oriented data augmentation After 305
 306 cold-start training by using our constructed dataset, 305
 307 our model acquired foundational tool-use and ver- 306
 308 ification skills. To further amplify its perfor- 307
 309 mance, we focused on curating a high-difficulty 307
 310 post-training dataset using an iterative self-instruct 308
 311 framework. In each round, GPT-o3 synthesized 308
 312 new questions conditioned on the current hard 309
 313 cases. Our cold-start model then attempted these 309
 314 questions. We only retrained incorrect instances, 310
 315 and merged these instances with the reference set 311
 316 for the next iteration. After four such iterations, 312
 317 the procedure produced a reinforcement-learning 313
 318 corpus of 9,456 examples. A detailed algorithm is 314
 319 depicted in algorithm 1. 315

3.4 Reinforcement Learning

Overall framework For CoSineVerifier, we also 317
 318 undergone reinforcement learning to incentivize 318
 319 more reasoning ability. Specifically, we adopt the 319
 320

DAPO (Yu et al., 2025) algorithm, which is an improved variant of GRPO (Shao et al., 2024). Given a prompt-answer pair (q, a) , we draw G rollouts $\{s_i\}_{i=1}^G$ from the behavior policy $\pi_{\theta_{\text{old}}}$. With access to the grounded answer a , each rollout receives a scalar reward $R_i = R(s_i, a)$ and we then optimize the current policy π_{θ} using the clipped policy-gradient objective below:

$$\mathcal{J}_{\text{DAPO}}(\theta) = \mathbb{E}_{\substack{(q,a) \sim \mathcal{D}, \\ \{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}}} \left[\frac{1}{\sum |o_i|} \sum_{i=1}^G \sum_{t=1}^{|o_i|} \min \left(r_{i,t}(\theta) \hat{A}_{i,t}, \text{clip}(r_{i,t}(\theta), 1-\varepsilon_l, 1+\varepsilon_h) \hat{A}_{i,t} \right) \right]$$

$$\text{s.t. } 0 < \left| \{o_i \mid o_i \equiv a\} \right| < G, \quad (1)$$

where $o_i \equiv a$ denotes that the output o_i is equivalent to the ground truth a . The probability ratio and advantage function are defined as:

$$r_{i,t}(\theta) = \frac{\pi_{\theta}(o_{i,t} \mid q, o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t} \mid q, o_{i,<t})}, \quad (2)$$

$$\hat{A}_{i,t} = \frac{R_i - \text{mean}(\{R_i\}_{i=1}^G)}{\text{std}(\{R_i\}_{i=1}^G)}. \quad (3)$$

Reward design We decompose the CoSineVerifier’s outcome-based reward into two terms. The first is an answer-correctness component:

$$R_{\text{ans}} = \mathbb{I}\{\hat{y} = y\},$$

which assigns 1 when the model’s prediction \hat{y} matches the reference y , and 0 otherwise.

To promote tool use during training, we introduce a tool-use encouragement reward. Let u denote the number of tool invocations in the trajectory. We award a bonus when the model is correct and uses an external tool, and impose a penalty when the model is incorrect without relying on function calling:

$$R_t = \begin{cases} 0.5, & \text{if } \mathbb{I}\{\hat{y} = y\} = 1 \text{ and } u > 0, \\ -0.5, & \text{if } \mathbb{I}\{\hat{y} = y\} = 0 \text{ and } u = 0, \\ 0, & \text{otherwise.} \end{cases}$$

The final reward is the sum of the correctness and tool-encourage terms:

$$R = R_{\text{ans}} + R_t.$$

This shaping explicitly encourages effective tool use trajectories that leverage tools to reach correct answers and discourages failures that eschew tools.

4 Experimental Setup

Datasets and Baselines We evaluate the CoSineVerifier family on three benchmarks: VerifyBench, VerifyBench-Hard (Yan et al., 2025), and SciBench (Zheng et al., 2025), which are challenging verification benchmarks spanning math, science, commonsense knowledge, and general reasoning domains with 2000, 1000, and 2500 samples. Our baselines mainly include two kinds of models: labeling verifiers and COT verifiers, with a difference in whether they contain reasoning chains for verification, and we report accuracy as mean@3 and efficiency as average output tokens per verdict on these benchmarks.

Implementation details Our CoSineVerifier-4B-Tool, CoSineVerifier-4B-Label and CoSineVerifier-32B are trained from Qwen3-4B-Instruct-2507, Qwen3-4B, and Qwen3-32B (Yang et al., 2025) respectively. We use OpenRLHF (Hu et al., 2024) for supervised fine-tuning and verl (Sheng et al., 2025) framework for reinforcement learning. All models are trained for one epoch on 32xA800 80G GPUS during supervised finetuning with 1e-5 learning rate, and 60 steps for reinforcement learning with 1e-6 learning rate. All training details are listed in Appendix 7.4.

5 Experimental Results

5.1 Verification Performance

As shown in Table 1, our CoSineVerifier series outperforms all verifier baselines across three verification benchmarks. Specifically, for CoT Verifier, our CoSineVerifier-Tool-4B surpasses baselines by 0.5% - 14.7% with noticeably fewer tokens, especially on harder benchmarks like VerifyBench (hard) and Sci-Bench, where complex calculation and long-context string comparing are needed to verify the LLM-generated answer. In addition, for labeling verifiers, CoSineVerifier-4B-Label and CoSineVerifier-32B-Label also surpass other labeling verifiers by 0.7%-1.1%, demonstrating stronger verification skills under single-token scenarios.

Small models with external tools excel large models Figure 4(a) demonstrates the performance comparison of our CoSineVerifier against various model across different domains. Among three benchmarks, even closed-source SOTA models like GPT-o3 still struggle with reliable verification on compute-intensive and scientific reasoning tasks. In contrast, both CoSineVerifier-4B-Tool

Model	VerifyBench	VerifyBench-Hard	Sci-Bench	Avg. Tokens
CoT Verifier				
Closed-source Models				
o3(OpenAI, 2025)	<u>96.1</u>	<u>88.7</u>	<u>87.5</u>	206.7
GPT-4o(OpenAI, 2024)	96.0	84.6	86.0	192.4
Gemini2.5-Flash(Comanici et al., 2025)	96.0	86.0	85.9	193.0
Open-source Models				
GPT-oss-20B(Agarwal et al., 2025)	92.2	84.7	85.0	221.0
LLaMA3.3-70B-Instruct(Dubey et al., 2024)	94.8	77.2	84.8	347.3
Qwen3-4B(Yang et al., 2025)	92.6	80.3	82.0	1156.7
Qwen3-8B(Yang et al., 2025)	93.7	83.6	83.9	926.6
Qwen3-32B(Yang et al., 2025)	94.7	85.2	83.5	798.8
Qwen3-4B-Instruct-2507(Yang et al., 2025)	94.7	84.1	82.4	869.7
Qwen3-235B-A22B-2507(Yang et al., 2025)	94.4	87.7	82.6	1885.3
CompassVerifier-7B (CoT)(Liu et al., 2025a)	93.5	82.6	84.2	234.7
CompassVerifier-32B (CoT)(Liu et al., 2025a)	95.9	86.5	85.5	213.0
CoSineVerifier-4B-Tool	96.6	91.9	89.7	95.3
Labeling Verifier				
XVerify-8B-I(Chen et al., 2025a)	92.5	83.3	78.1	1.0
CompassVerifier-7B(Liu et al., 2025a)	93.5	85.2	85.7	1.0
CompassVerifier-32B(Liu et al., 2025a)	96.3	<u>88.9</u>	85.3	1.0
CoSineVerifier-4B-Label	<u>95.7</u>	85.4	<u>85.9</u>	1.0
CoSineVerifier-32B-Label	<u>95.7</u>	90.0	86.4	1.0

Table 1: Main results on three verify benchmarks. We separate CoT verifier and Labeling verifier for fair comparison, where CoT verifier outputs a reasoning chain before achieving the final answer while labeling verifier directly gives its verdict. **Bold** highlights the overall best accuracy, while underline highlights the second best accuracy. We report mean@3 accuracy and average token used per question.

and CoSineVerifier-32B-Label demonstrate exceptional proficiency in the Math and Science domains while achieving competitive or even superior performance on the remaining domains. Despite being much smaller in parameter size, the superior accuracy exhibited by CoSineVerifier-4B-Tool validates the efficacy of integrating external calculation tools to enhance the model’s verification capabilities. These results demonstrate the feasibility of tool-augmented, small-scale models to exceed the performance of large SOTA models in complex verification tasks.

5.2 Verification Efficiency Analysis

We further evaluate the efficiency of CoSineVerifier-4B-Tool against several baselines in Figure 5(b). To ensure a fair comparison, all models are tested on 4 NVIDIA A800 GPUs. We report the mean inference latency computed over 1000 randomly sampled prompts. While CoSineVerifier-4B-Tool achieves high accuracy, incorporating external tools does not incur signifi-

cant inference latency. We attribute this efficiency to two primary factors: (1) our efficiency-driven cold-start data construction methodology, which explicitly optimizes for short and precise tool-calling trajectories, resulting in 2.4x-19x times less token usage compared to other models. (2) the computational efficiency afforded by the model’s compact 4B parameter count, which minimizes the overhead of the backbone model during inference.

5.3 Ablation Study

Two-stage training is crucial for CoSineVerifier-4B-Tool As shown in Figure 6(a), we conduct ablation studies on each module’s contribution to our two-stage training framework. We first observe that after removing cold-start stage for CoSineVerifier-4B-Tool, the model’s performance drop sharply. We then discover this is because the model without cold-start fails to conduct accurate function calling and often becomes trapped in tool-use loops, harming performance while significantly increasing inference latency. In addition, model trained

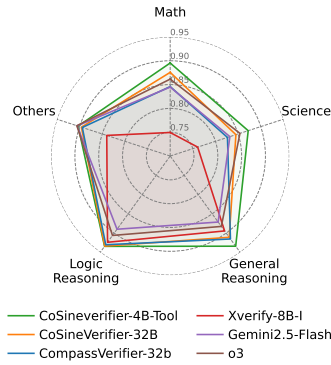


Figure 4: Analysis of CoSineVerifier-4B-Tool verification accuracy across different domains

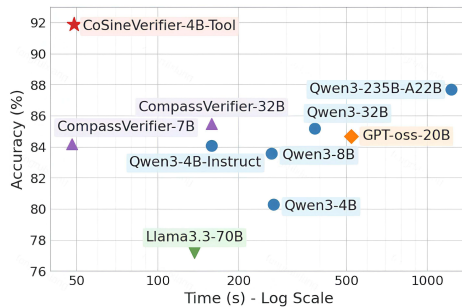


Figure 5: Analysis of CoSineVerifier-4B-Tool inference efficiency with other models.

without reinforcement learning (RL) also demonstrate weaker verification performance. To investigate this, we also analyze the accuracy on samples that have tool trajectories before and after RL stage. As shown in Figure 6(b), the accuracy on samples with tools improved markedly from 76.9% to 90.0%. This result indicates that RL teaches the model how to leverage tools for accurate verification, rather than merely to initiate a tool call. In conclusion, each stage of our two-stage framework is crucial for robust verifier training. Together, they equip the model to determine precisely when and how to use tool calls for verification.

Data augmentation can further boost performance As shown in Figure 6(a), removing data augmentation also yields inferior performance. We attribute this to the fact that verification tasks often contain rare but difficult samples, such as truncated responses, multiple self-reflections, and meaningless repetitions. These samples are relatively more difficult to verify and are underrepresented in ablation training data, preventing the model from learning to handle them as effectively as common verification samples. Incorporating our data augmentation methods can expose the model to a more bal-

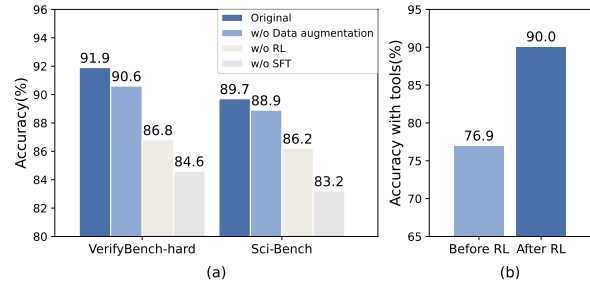


Figure 6: (a) Ablation study of CoSineVerifier-4B-Tool. (b) Average verification accuracy with tools before RL and after RL.

anced and challenging training distribution, which ensures our model has high verification accuracy in rare and difficult circumstances.

5.4 Evaluation on RLVR Task

One primary objective of our CoSineVerifier series is to establish a reliable and efficient outcome reward verifier tailored for real-world data processing and reinforcement learning. To avoid potential biases inherent in existing benchmarks and validate the efficacy of CoSineVerifier as a reward model with RL training, we further evaluate answer-verification methods in RLVR tasks. Specifically, we train Qwen3-4B-Instruct-2507 with an on-policy GRPO algorithm on math and science tasks in separate runs, each paired with different verifiers. For the math setting, we construct competition-math problems with 42K training data drawn from DAPO-Math-17k (Yu et al., 2025), OpenR1-Math-220k (Hugging Face, 2025), and DeepScaleR-Preview (Luo et al., 2025). And for the science setting, we directly leverage guru-RL(Cheng et al., 2025) as our training set. We compare our approach against various verifiers, including both rule-based and model-based methods, under identical training configurations. We report mean@32 accuracy on AIME24&25 for the math task and GPQA-D for the science task. Comprehensive training details are provided in Appendix 7.4.

CoSineVerifier enables robust reasoning improvements As illustrated in Figure 7, employing our CoSineVerifier as a reward model significantly enhances the base model’s reasoning capabilities. Specifically, in mathematical tasks, CoSineVerifier outperforms other verifiers by margins of 1.7%–6.6% on AIME 2024 and 1.8%–6.0% on AIME 2025. Similarly, in the science domain, our model demonstrates robust advantages, achieving improvements of 0.7%–2.9% over competing meth-

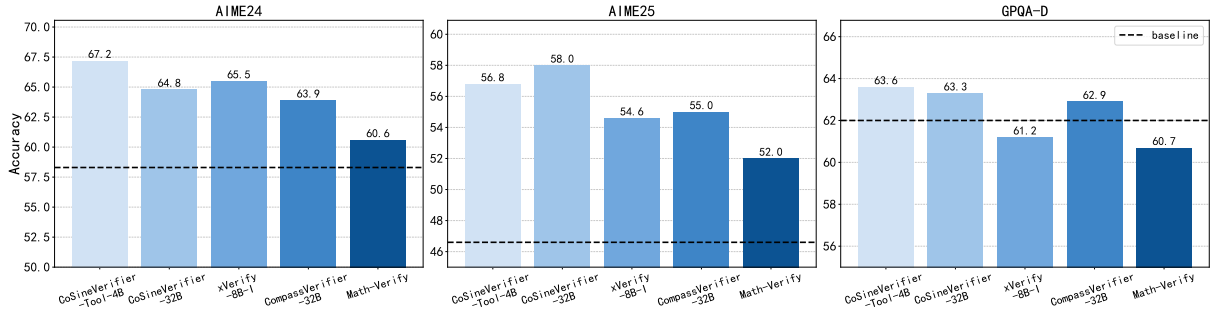


Figure 7: Results of using CoSineVerifier as a reward model on AIME24, AIME25 and GPQA-D.

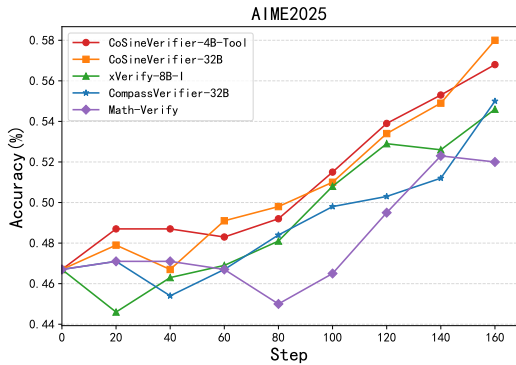


Figure 8: Accuracy of AIME 2025 during training.

ods. Notably, we observe a distinct performance divergence between model-based and rule-based verifiers. This gap is most pronounced in the challenging science benchmark (GPQA-D), where relying on the rule-based Math-Verify results in a 1.3% performance degradation compared to the baseline. This indicates that noisy or incorrect verification can misguide the policy, pushing it toward undesirable reasoning trajectories and ultimately degrading accuracy. These findings underscore the limitations of static verification methods in mapping diverse LLM outputs to reference answers. We believe precise and reliable reward signals are beneficial for RL training, especially when the model is already well-tuned and requires more challenging data to surpass its reasoning ceiling.

Verifier quality shapes performance and reasoning ceiling To investigate the impact of the verifier reward signal quality on the RL training dynamics, we analyze the model’s accuracy on AIME 2025 throughout the reinforcement learning process. Specifically, we aim to answer: *How does the choice of verifier influence the convergence and final performance of the base model under identical RL training conditions?* As shown in Figure 8, which plots the base model’s accuracy against train-

ing steps for different verifiers, our CoSineVerifier achieves a higher performance ceiling rather than uniformly faster convergence. It not only achieves higher final accuracy but also consistently outperforms all baselines at every stage of training. This clear lead indicates that a high-quality verifier provides more reliable reward signals, enabling more efficient and effective policy optimization. In contrast, weaker verifiers like Math-Verify result in slower improvement and lower performance ceiling, directly linking the verifier’s capability to the base model’s learning efficacy. As RL is increasingly applied to challenging datasets that elicit complex reasoning (Phan et al., 2025; Guan et al., 2025), robust verification mechanisms that ensure high-fidelity rewards become increasingly critical.

6 Conclusion

In this paper, we propose CoSineVerifier, a tool-augmented LLM-based verifier for computation-intensive scientific verification. To address the limitations of existing methods in precise verification, we equip the model with external tools, enabling accurate arithmetic transformation before judgment. We also introduce a novel data augmentation strategy and a two-stage training framework to teach the model when and how to invoke tools effectively. Experiments show that CoSineVerifier achieves state-of-the-art accuracy across multiple verification benchmarks while maintaining high efficiency. When used as a reward model in RL training, CoSineVerifier leads to higher final performance on challenging math and science benchmarks, outperforming both rule-based and existing model-based verifiers. Our work demonstrates that our lightweight, tool-integrated CoSineVerifier can help LLMs in tasks requiring rigorous computation, providing a reliable pathway toward more accurate and efficient evaluation systems for LLM training.

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Limitations

Despite strong empirical results, CoSineVerifier’s reliability is tightly coupled to its available tools and sandboxed execution environment, which may incur extra computational burden during RLVR training. Moreover, CoSineVerifier is fundamentally a reference-based verifier: it assesses correctness by comparing model outputs against a provided reference answer. As a result, it is less applicable to settings where ground-truth references are unavailable, ambiguous, or inherently subjective (e.g., open-ended generation, preference-based tasks, or partially specified problems), limiting its direct transfer to non-verifiable domains.

Ethical Considerations

CoSineVerifier is designed to improve the faithfulness of automated verification by grounding judgments in tool-executable computations, thereby making model training and evaluation more reliable. However, CoSineVerifier can still produce incorrect judgments during data curation or RLVR training; such errors may inject spurious reward signals and, in turn, steer optimization toward unintended behaviors.

References

Sandhini Agarwal, Lama Ahmad, Jason Ai, Sam Altman, Andy Applebaum, Edwin Arbus, Rahul K Arora, Yu Bai, Bowen Baker, Haiming Bao, et al. 2025. gpt-oss-120b & gpt-oss-20b model card. *arXiv preprint arXiv:2508.10925*.

Alon Albalak, Duy Phung, Nathan Lile, Rafael Rafailov, Kanishk Gandhi, Louis Castricato, Anikait Singh, Chase Blagden, Violet Xiang, Dakota Mahan, et al. 2025. Big-math: A large-scale, high-quality math dataset for reinforcement learning in language models. *arXiv preprint arXiv:2502.17387*.

Ding Chen, Qingchen Yu, Pengyuan Wang, Wentao Zhang, Bo Tang, Feiyu Xiong, Xinchu Li, Minchuan Yang, and Zhiyu Li. 2025a. xverify: Efficient answer verifier for reasoning model evaluations. *arXiv preprint arXiv:2504.10481*.

Nuo Chen, Zhiyuan Hu, Qingyun Zou, Jiaying Wu, Qian Wang, Bryan Hooi, and Bingsheng He. 2025b. Judgelrm: Large reasoning models as a judge. *arXiv preprint arXiv:2504.00050*.

Yongchao Chen, Yueying Liu, Junwei Zhou, Yilun Hao, Jingquan Wang, Yang Zhang, Na Li, and Chuchu Fan. 2025c. R1-code-interpreter: LLMs reason with code via supervised and multi-stage reinforcement learning. *arXiv preprint arXiv:2505.21668*.

Zhoujun Cheng, Shibo Hao, Tianyang Liu, Fan Zhou, Yutao Xie, Feng Yao, Yuexin Bian, Yonghao Zhuang, Nilabjo Dey, Yuheng Zha, et al. 2025. Revisiting reinforcement learning for LLM reasoning from a cross-domain perspective. *arXiv preprint arXiv:2506.14965*.

Gheorghe Comanici, Eric Bieber, Mike Schaekermann, Ice Pasupat, Noveen Sachdeva, Inderjit Dhillon, Marcel Blistein, Ori Ram, Dan Zhang, Evan Rosen, et al. 2025. Gemini 2.5: Pushing the frontier with advanced reasoning, multimodality, long context, and next generation agentic capabilities. *arXiv preprint arXiv:2507.06261*.

Guanting Dong, Yifei Chen, Xiaoxi Li, Jiajie Jin, Hongjin Qian, Yutao Zhu, Hangyu Mao, Guorui Zhou, Zhicheng Dou, and Ji-Rong Wen. 2025. Tool-star: Empowering LLM-brained multi-tool reasoner via reinforcement learning. *arXiv preprint arXiv:2505.16410*.

Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, et al. 2024. The llama 3 herd of models. *arXiv e-prints*, pages arXiv–2407.

Jiazhan Feng, Shijue Huang, Xingwei Qu, Ge Zhang, Yujia Qin, Baoquan Zhong, Chengquan Jiang, Jinxin Chi, and Wanjun Zhong. 2025. Retool: Reinforcement learning for strategic tool use in LLMs. *arXiv preprint arXiv:2504.11536*.

Xinyu Guan, Li Lina Zhang, Yifei Liu, Ning Shang, Youran Sun, Yi Zhu, Fan Yang, and Mao Yang. 2025. rstar-math: Small LLMs can master math reasoning with self-evolved deep thinking. *arXiv preprint arXiv:2501.04519*.

Etash Guha, Ryan Marten, Sedrick Keh, Negin Raouf, Georgios Smyrnis, Hritik Bansal, Marianna Nezhurina, Jean Mercat, Trung Vu, Zayne Sprague, et al. 2025. Openthoughts: Data recipes for reasoning models. *arXiv preprint arXiv:2506.04178*.

Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, et al. 2025. Deepseek-r1: Incentivizing reasoning capability in LLMs via reinforcement learning. *arXiv preprint arXiv:2501.12948*.

Jujie He, Jiakai Liu, Chris Yuhao Liu, Rui Yan, Chaojie Wang, Peng Cheng, Xiaoyu Zhang, Fuxiang Zhang, Jiacheng Xu, Wei Shen, et al. 2025a. Skywork open reasoner 1 technical report. *arXiv preprint arXiv:2505.22312*.

Zhiwei He, Tian Liang, Jiahao Xu, Qiuzhi Liu, Xingyu Chen, Yue Wang, Linfeng Song, Dian Yu, Zhenwen Liang, Wenxuan Wang, et al. 2025b. Deepmath-103k: A large-scale, challenging, decontaminated, and verifiable mathematical dataset for advancing reasoning. *arXiv preprint arXiv:2504.11456*.

679	Jian Hu, Xibin Wu, Zilin Zhu, Weixun Wang, Dehao Zhang, Yu Cao, et al. 2024. Openrlhf: An easy-to-use, scalable and high-performance rlhf framework. <i>arXiv preprint arXiv:2405.11143</i> .	Xinji Mai, Haotian Xu, Zhong-Zhi Li, Xing W, Weinong Wang, Jian Hu, Yingying Zhang, and Wenqiang Zhang. 2025. Agent rl scaling law: Agent rl with spontaneous code execution for mathematical problem solving. <i>arXiv preprint arXiv:2505.07773</i> .	731
680			732
681			733
682			734
683	Yuzhen Huang, Weihao Zeng, Xingshan Zeng, Qi Zhu, and Junxian He. 2025. Pitfalls of rule-and model-based verifiers—a case study on mathematical reasoning. <i>arXiv preprint arXiv:2505.22203</i> .	OpenAI. 2024. GPT-4o system card . <i>arXiv preprint arXiv:2410.21276</i> .	736
684			737
685			
686			
687	Hugging Face. 2025. Open r1: A fully open reproduction of deepseek-r1 .	OpenAI. 2025. Openai o3 system card . Technical report, OpenAI.	738
688			739
689	Bowen Jin, Hansi Zeng, Zhenrui Yue, Jinsung Yoon, Sercan Arik, Dong Wang, Hamed Zamani, and Jiawei Han. 2025. Search-r1: Training llms to reason and leverage search engines with reinforcement learning. <i>arXiv preprint arXiv:2503.09516</i> .	Samuel J. Paech. 2023. Eq-bench: An emotional intelligence benchmark for large language models . <i>arXiv preprint arXiv:2312.06281</i> .	740
690			741
691			742
692			
693		Long Phan, Alice Gatti, Ziwen Han, Nathaniel Li, Josephina Hu, Hugh Zhang, Chen Bo Calvin Zhang, Mohamed Shaaban, John Ling, Sean Shi, et al. 2025. Humanity’s last exam. <i>arXiv preprint arXiv:2501.14249</i> .	743
694	Hynek Kydlíček. Math-Verify: Math Verification Library .		744
695			745
696			746
697	Tianle Li, Wei-Lin Chiang, Evan Frick, Lisa Dunlap, Tianhao Wu, Banghua Zhu, Joseph E. Gonzalez, and Ion Stoica. 2024. From crowdsourced data to high-quality benchmarks: Arena-hard and benchbuilder pipeline . <i>arXiv preprint arXiv:2406.11939</i> .	Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, YK Li, Yang Wu, et al. 2024. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. <i>arXiv preprint arXiv:2402.03300</i> .	748
698			749
699			750
700			751
701	Xuefeng Li, Haoyang Zou, and Pengfei Liu. 2025. Torl: Scaling tool-integrated rl. <i>arXiv preprint arXiv:2503.23383</i> .	Guangming Sheng, Chi Zhang, Zilingfeng Ye, Xibin Wu, Wang Zhang, Ru Zhang, Yanghua Peng, Haibin Lin, and Chuan Wu. 2025. Hybridflow: A flexible and efficient rlhf framework. In <i>Proceedings of the Twentieth European Conference on Computer Systems</i> , pages 1279–1297.	753
702			754
703			755
704	Heng Lin and Zhongwen Xu. 2025. Understanding tool-integrated reasoning. <i>arXiv preprint arXiv:2508.19201</i> .		756
705			757
706			758
707	Shudong Liu, Hongwei Liu, Junnan Liu, Linchen Xiao, Songyang Gao, Chengqi Lyu, Yuzhe Gu, Wenwei Zhang, Derek F Wong, Songyang Zhang, et al. 2025a. Compassverifier: A unified and robust verifier for llms evaluation and outcome reward. <i>arXiv preprint arXiv:2508.03686</i> .	Shubham Toshniwal, Wei Du, Ivan Moshkov, Branislav Kisanin, Alexan Ayrapetyan, and Igor Gitman. 2024. Openmathinstruct-2: Accelerating ai for math with massive open-source instruction data. <i>arXiv preprint arXiv:2410.01560</i> .	759
708			760
709			761
710			762
711		Mingyuan Wu, Jingcheng Yang, Jize Jiang, Meitang Li, Kaizhuo Yan, Hanchao Yu, Minjia Zhang, Chengxiang Zhai, and Klara Nahrstedt. 2025a. Vtool-r1: Vllms learn to think with images via reinforcement learning on multimodal tool use. <i>arXiv preprint arXiv:2505.19255</i> .	763
712			764
713	Zijun Liu, Peiyi Wang, Runxin Xu, Shirong Ma, Chong Ruan, Peng Li, Yang Liu, and Yu Wu. 2025b. Inference-time scaling for generalist reward modeling. <i>arXiv preprint arXiv:2504.02495</i> .		765
714			766
715			767
716			768
717	Michael Luo, Sijun Tan, Justin Wong, Xiaoxiang Shi, William Y. Tang, Manan Roongta, Colin Cai, Jeffrey Luo, Li Erran Li, Raluca Ada Popa, and Ion Stoica. 2025. Deepscaler: Surpassing o1-preview with a 1.5b model by scaling rl . Notion Blog.	Yuning Wu, Jiahao Mei, Ming Yan, Chenliang Li, Shaopeng Lai, Yuran Ren, Zijia Wang, Ji Zhang, Mengyue Wu, Qin Jin, et al. 2025b. Writingbench: A comprehensive benchmark for generative writing. <i>arXiv preprint arXiv:2503.05244</i> .	770
718			771
719			772
720			773
721			774
722	Ruotian Ma, Peisong Wang, Cheng Liu, Xingyan Liu, Jiaqi Chen, Bang Zhang, Xin Zhou, Nan Du, and Jia Li. 2025a. S²r: Teaching llms to self-verify and self-correct via reinforcement learning . <i>arXiv preprint arXiv:2502.12853</i> .	Yu Xia, Jingru Fan, Weize Chen, Siyu Yan, Xin Cong, Zhong Zhang, Yaxi Lu, Yankai Lin, Zhiyuan Liu, and Maosong Sun. 2025. Agentrm: Enhancing agent generalization with reward modeling. <i>arXiv preprint arXiv:2502.18407</i> .	775
723			776
724			777
725			778
726			779
727	Zexiong Ma, Chao Peng, Qunhong Zeng, Pengfei Gao, Yanzhen Zou, and Bing Xie. 2025b. Tool-integrated reinforcement learning for repo deep search. <i>arXiv preprint arXiv:2508.03012</i> .	Zhenghai Xue, Longtao Zheng, Qian Liu, Yingru Li, Xiaosen Zheng, Zejun Ma, and Bo An. 2025. Simpletir: End-to-end reinforcement learning for multi-turn tool-integrated reasoning. <i>arXiv preprint arXiv:2509.02479</i> .	780
728			781
729			782
730			783
			784

785 Yuchen Yan, Jin Jiang, Zhenbang Ren, Yijun Li,
786 Xudong Cai, Yang Liu, Xin Xu, Mengdi Zhang,
787 Jian Shao, Yongliang Shen, et al. 2025. Verify-
788 bench: Benchmarking reference-based reward sys-
789 tems for large language models. *arXiv preprint*
790 *arXiv:2505.15801*.

791 An Yang, Anfeng Li, Baosong Yang, Beichen Zhang,
792 Binyuan Hui, Bo Zheng, Bowen Yu, Chang Gao,
793 Chengen Huang, Chenxu Lv, et al. 2025. Qwen3
794 technical report. *arXiv preprint arXiv:2505.09388*.

795 Qiyong Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan,
796 Xiaochen Zuo, Yu Yue, Weinan Dai, Tiantian Fan,
797 Gaohong Liu, Lingjun Liu, et al. 2025. Dapo: An
798 open-source llm reinforcement learning system at
799 scale. *arXiv preprint arXiv:2503.14476*.

800 Weizhe Yuan, Jane Yu, Song Jiang, Karthik Padthe,
801 Yang Li, Ilya Kulikov, Kyunghyun Cho, Dong Wang,
802 Yuandong Tian, Jason E Weston, et al. 2025. Natu-
803 ralreasoning: Reasoning in the wild with 2.8 m chal-
804 lenging questions. *arXiv preprint arXiv:2502.13124*.

805 Shaokun Zhang, Yi Dong, Jieyu Zhang, Jan Kautz,
806 Bryan Catanzaro, Andrew Tao, Qingyun Wu, Zhiding
807 Yu, and Guilin Liu. 2025. Nemotron-research-tool-
808 n1: Exploring tool-using language models with rein-
809 forced reasoning. *arXiv preprint arXiv:2505.00024*.

810 Shenghe Zheng, Chenyu Huang, Fangchen Yu, Junchi
811 Yao, Jingqi Ye, Tao Chen, Yun Luo, Ning Ding,
812 Lei Bai, Ganqu Cui, et al. 2025. Sci-verifier:
813 Scientific verifier with thinking. *arXiv preprint*
814 *arXiv:2509.24285*.

7 Appendix

7.1 Error Analysis

Table 2 demonstrates the detailed types and distribution of our summarized errors. We run 3 times for each verification method and leverage Qwen3-32b to identify their errors. We then send these errors to GPT-o3 to summarize these errors into 15 error types. After obtaining these error types, we use Qwen3-32b to annotate errors for each verification method and derive the statistics.

7.2 Difficulty-oriented Data Augmentation

We provide the detailed algorithm for difficulty-oriented data augmentation below:

Input: Initial hard set \mathcal{D}_0 ; questioner Q ; solver S ; rounds T .

Output: Reinforcement-learning corpus \mathcal{R} .

```

 $\mathcal{R} \leftarrow \{\}$ 
 $\mathcal{D} \leftarrow \mathcal{D}_0$ 
for  $t \leftarrow 1$  to  $T$  do
     $\mathcal{Q}_t \leftarrow Q(\text{conditioned on } \mathcal{D})$ 
    // Conditioned generation by Questioner
     $\mathcal{F}_t \leftarrow \{x \in \mathcal{Q}_t \mid S(x) \text{ is incorrect}\}$ 
    // Accumulate failures as next-round context
     $\mathcal{R} \leftarrow \mathcal{R} \cup \mathcal{F}_t$ 
     $\mathcal{D} \leftarrow \mathcal{D} \cup \mathcal{F}_t$ 
end
return  $\mathcal{R}$ 

```

Algorithm 1: Iterative Difficult Data Synthesis

7.3 Training Data Statistics

In this section, we provide the relevant statistics of our CoSineVerifier training data.

Source	Count	Rate
Science	452661	39.70%
Math	271415	23.80%
DROP	103070	9.04%
SuperGPQA	60356	5.29%
Ceval	59501	5.21%
Cmmlu	52222	4.58%
MMLU_pro	50301	4.41%
ARC	33148	2.90%
SimpleQA	18103	1.58%
ChineseSimpleQA	12720	1.11%
BBH	11794	1.03%
Korbench	9561	0.83%
GAOKAOBench	3281	0.28%
GPQA	1913	0.16%
Total	1,140,046	100.00%

Table 3: Candidate training data distribution

Source	Count	Rate
Python interpreter data	2500	29.33%
Unit conversion data	1500	17.60%
No Tool use data	3000	35.20%
Long-tail augment data	1523	17.87%
Total	8,523	100.00%

Table 4: Cold start data distribution

Source	Count	Rate
Science	20044	31.63%
Math	8789	13.87%
SuperGPQA	8409	13.27%
MMLU_pro	6740	10.64%
Korbench	5877	9.27%
BBH	5745	9.07%
DROP	2974	4.69%
SimpleQA	1757	2.77%
ChineseSimpleQA	1215	1.92%
GAOKAOBench	849	1.34%
Ceval	484	0.76%
Cmmlu	308	0.49%
ARC	145	0.23%
GPQA	38	0.06%
Total	63,374	100.00%

Table 5: Disagreement data distribution

Error Name	Description	Percentage
Calculation Inaccuracy	The final numerical value is mathematically incorrect.	25.089%
Comparison Judgment Error	The response (e.g., a sequence) does not perfectly match the reference.	19.618%
Format Error	The answer’s structure violates the required output format.	16.085%
Incomplete Answer Error	Fails to provide all required answers, e.g., missing candidates or sub-parts.	15.957%
Precision/Boundary Error	Incorrect rounding or definitions of interval endpoints, e.g., open/closed.	7.843%
Invalid Query	The error originates from a defective question or reference answer.	6.007%
Incorrect Simplification	Fails to simplify to minimal form or simplifies erroneously.	2.938%
Constraint Violation	The answer ignores a specific rule, e.g., "use each number once" .	2.166%
Missing Final Result	Provides only reasoning or code without the conclusive answer.	1.836%
Refusal or Inconclusive Response	Fails to provide a clear answer or states it is unanswerable.	1.355%
Extraneous Content Error	The response includes correct data mixed with unrelated, incorrect information.	0.677%
Self-Correction Failure	The model initially answers correctly but changes to an incorrect answer.	0.285%
Truncated Response	The response is clearly cut off and incomplete.	0.085%
Garbled or Corrupted Output	The response contains unreadable characters or significant noise.	0.046%
Meaningless Repetition Error	The response repeats text extensively, hiding any valid answer.	0.013%

Table 2: Error Types and Distributions

Model	Count	Rate
NBG-Family	13308	21.00%
DeekSeek-Qwen-7B	6337	10.00%
GPT-OSS-20B	5300	8.36%
DeepSeek-Qwen-1.5B	4874	7.69%
Qwen3-30B-A3B	4488	7.08%
Llama3.1-8B	4056	6.40%
MiMo-7b	3581	5.65%
Qwen3-4B	3405	5.37%
Qwen2.5-7B	2931	4.62%
Qwen3-32B	2917	4.60%
Qwen3-1.7B	2763	4.36%
Gemma2-9B	2474	3.90%
Gemma2-2B	2415	3.81%
InternLM-7B	2277	3.59%
Qwen3-8B	2248	3.55%
Total	63,374	100.00%

7.4 Training Details

7.4.1 Training CoSineVerifier-4B-Label and CoSineVerifier-32B-Label

For labeling verifier CoSineVerifier-4B and CoSineVerifier-32B, we conducted supervised fine-tuning on 73,714 samples, which contains 63,714 disagreement data, 2000 long-tail augmentation data and 8000 difficult-oriented augmentation data. These two verifiers are designed to output binary verification (*Correct* and *Incorrect*) without reasoning, maintaining superior efficiency while having performance comparable to CoSineVerifier-4B-Tool. Detailed training parameters are listed in Table 7:

Table 6: Model responses count on disagreement data distribution (NBG-Family aggregated)

Parameter	Value
BF16	True
Learning Rate	1×10^{-5}
LR Scheduler Type	cosine_with_min_lr
Maximum Sequence Length	16384
Training Epochs	1
Use Liger Kernel	True
Warmup Ratio	0.01

Table 7: Cold start training configurations of CoSineVerifier-4B-Tool and supervised fine tuning configurations of CoSineVerifier-4B-Label and CoSineVerifier-32B-Label

7.4.2 Ablation Study of Labeling Verifier

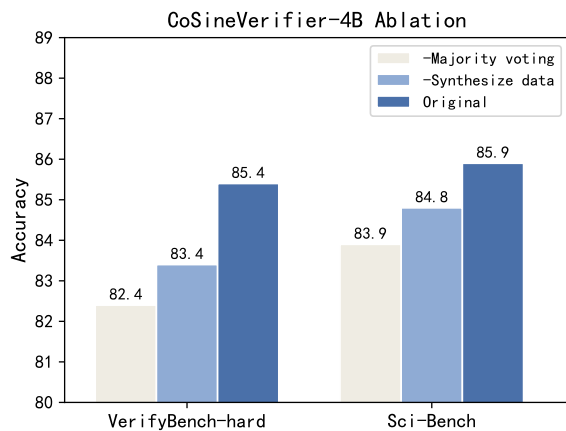


Figure 9: Ablation study of CoSineVerifier-4B.

For CoSineVerifier-4B, we investigate the impact of removing: (1) **multi-turn majority voting** and (2) **long-tail data synthesis**. As illustrated in Figure 9, removing any model from CoSineVerifier consistently leads to performance degradation, demonstrating the necessity of our holistic framework design. Specifically, excluding multi-turn majority data results in the most significant performance deterioration across two major benchmarks. This finding strongly underscores the imperative of collecting reliable “golden” answers from diverse, real-world domains. In addition, removing long-tail synthesis data also causes a marked decline, showing the effectiveness of long-tail alignment for rare but high-value cases. In conclusion, only combining two components together yields the best performance

7.4.3 Training Details of CoSineVerifier-4B-Tool

We detail the training configurations of CoSineVerifier-4B-Tool. For cold-start process, we use 8,523 samples for cold-start with

long-tail augmentation data, and we then use 9,456 difficulty-oriented augmentation data for reinforcement learning. All training configurations are listed below.

Parameters	Value
BF16	True
Train batch size	128
Train epochs	10
Learning rate	1e-6
Max prompt length	16384
Max response length	2048
n	8
ϵ_{\min}	0.2
ϵ_{\max}	0.28
Temperature	1.0

Table 8: RL training configurations of CoSineVerifier-4B-Tool.

Parameters	Value
BF16	True
Train batch size	128
Train epochs	3
Learning rate	1e-6
Max prompt length	4096
Max response length	16384
n	8
ϵ_{\min}	0.2
ϵ_{\max}	0.28
Temperature	1.0

Table 9: Training configurations of RLVR application.

7.5 Case Study

CosineVerifier-4B-Tool for calculation

Question: A block of mass $m = 100$ g and of negligible size slides down a wedge of mass $M = 10$ kg, with angle above the horizontal $\theta = 30^\circ$, and maximum height $h = 2$ m. The block is free to move as well, and there is a coefficient of friction $\mu = \frac{1}{2}$ between the wedge and block. Initially, the block is at rest with respect to the wedge and sits at the top of the wedge. The wedge is moving at $v_0 = 1$ m/s with the point of the wedge moving “forward.”

For this problem, use $g = 10 \text{ m/s}^2$. Find the exact amount of time it takes for the block to slide all the way to the bottom of the wedge. Remember to put your final answer within boxed

model_answer:

"model_ans": "To solve this problem, we need to analyze the motion of the block and the wedge, taking into account the forces acting on them and the constraints of the system.

Step 1: Analyze the forces and motion

1. **Forces on the block:** - Gravity: mg acting downward. - Normal force: N perpendicular to the wedge's surface. - Friction force: $f = \mu N$ acting up the wedge.
2. **Forces on the wedge:** - Gravity: Mg acting downward. - Normal force: N from the block. - Friction force: $f = \mu N$ acting down the wedge. - Initial velocity: $v_0 = 1 \text{ m/s}$.

Step 2: Set up the equations of motion

1. **For the block:** - Along the wedge: $ma_{\text{block}} = mg \sin \theta - f$ - Perpendicular to the wedge: $N = mg \cos \theta$

2. **For the wedge:** - Horizontal direction: $Ma_{\text{wedge}} = f \cos \theta - N \sin \theta$

Step 3: Substitute the known values

1. **Normal force:**

$$N = mg \cos \theta = 0.1 \times 10 \times \cos 30^\circ = 0.1 \times 10 \times \frac{\sqrt{3}}{2} = 0.5\sqrt{3} \text{ N}$$

2. **Friction force:**

$$a_{\text{block}} = g \sin \theta - \frac{f}{m} = 10 \times \frac{1}{2} - \frac{0.25\sqrt{3}}{0.1} = 5 - 2.5\sqrt{3} \text{ m/s}^2$$

Step 4: Calculate the time to slide down

The block slides down a distance $s = \frac{h}{\sin \theta} = \frac{2}{\sin 30^\circ} = 4 \text{ m}$ along the wedge.

Using the kinematic equation:

$$s = \frac{1}{2} a_{\text{block}} t^2$$

$$4 = \frac{1}{2} (5 - 2.5\sqrt{3}) t^2$$

$$t^2 = \frac{8}{5 - 2.5\sqrt{3}}$$

$$t = \sqrt{\frac{8}{5 - 2.5\sqrt{3}}}$$

Step 5: Simplify the expression To simplify, we can rationalize the denominator:

$$t = \frac{8\sqrt{5 + 2.5\sqrt{3}}}{5}$$

Final Answer:

$$\boxed{\frac{8\sqrt{5 + 2.5\sqrt{3}}}{5}}$$

"

reference answer:

$$t = \sqrt{((1604(2 + \sqrt{3}))/505)} \text{ s}$$

CosineVerifier-4B-Tool

verification:

```
<tool_call>
{"name": "python_interpreter",
"arguments": {"code": "def
main():\n import math\n
cand = 8*math.sqrt(5 +
2.5*math.sqrt(3))/5\n gold =
math.sqrt(1604*(2+math.sqrt(3))/505)
\n print(cand, gold)\n\nif __name__
== \"__main__\":\n main()"}}
</tool_call>
<tool_response>
{"compile_result": null,
"execution_info": {"code_length":
183, "execution_phases":
["execution_start",
"execution_end"],
"execution_start_time":
1760942845.014331, "language":
```

```

"python",          "stdin_length":
4,                "stdin_provided":
true,             "temp_file":
"/tmp/tmposhv6sxt.py",
"total_execution_time":
0.028516054153442383,
"warnings":      [], "run_result":
{"command":      ["python3",
"/tmp/tmposhv6sxt.py"],
"execution_time":
0.028277158737182617,
"exit_success":  true,
"return_code":  0,  "status":
"Finished",     "stderr":      "",
"stdin_used":   "None",
"stdout":       "4.887241058965765
3.4429464400122507\n",
"working_directory": "/tmp"},
"status": "Success"}
</tool_response>

```

assistant

The candidate's final expression $t = 8\sqrt{5} + 2.5\sqrt{3}/5$ is numerically ≈ 4.89 s.

The reference expression $t = \sqrt{(1604(2 + \sqrt{3})/505)}$ is numerically ≈ 3.44 s.

Since $4.89 \neq 3.44$, the candidate answer does not match the gold answer.

[Incorrect]

CosineVerifier-4B-Tool for string match

Question: "Please generate the corresponding RNA sequence directly based on the dot-bracket notation of the secondary structure:((((.....))).((((.....(((.....(((.....(((.....)))))))).)))).)).....((((((((((.....)))))))))).....))))))"

model_answer:

```

"AUUGCUCGAAUUUAUAGGACUUUUUCUAUAAA
GAAUAGUUUGGACUUGAAAUG
UAUUUAAAAACAAGAGGUUGGUAGAUUAUCAGCC
UCUUUCUUGUCGUUGAAAAAG",

```

reference_answer:

```

ACGGGUUCCCGGGAAACCCCAAAAUGGGGCCCC
UGUCGGGUUUUAUCCCUUGUCGUCGCCCUUUUUG
GGA

```

CosineVerifier-4B-Tool

verification:

```

<tool_call>
{"name": "python_interpreter",
"arguments": {"code":
"def main():\n    correct =
\"AUUGCUCGAAUUUAUAGGACUUUUUCUAUAAA
GAAUAGUUUGG
ACUUGAAAUGUAUUUAAAAACAAGAGGUUGGUAGAU
UAUCAGCCCUUUUCUUGUCGUUGAAAAAG\"
\n
candidate =
\"ACGGGUUCCCGGGAAACCCCAAAAUGGGGCCCC
UGUCGGGUUUUAUCCCUUG
UCGUCGCCCUUUUUGGGA\"
\nprint('match'
if correct == candidate
else 'no')\n\nif __name__ ==
\"__main__\":\n    main()"}
</tool_call>

```

```

<tool_response>
{"compile_result": null,
"execution_info": {"code_length":
411, "execution_phases":
["execution_start",
"execution_end"],
"execution_start_time":
1761468426.650931, "language":
"python", "stdin_length":
4, "stdin_provided":
true, "temp_file":
"/tmp/tmp4b51vbkx.py",
"total_execution_time":
0.028637170791625977,
"warnings": [], "run_result":
{"command": ["python3",
"/tmp/tmp4b51vbkx.py"],
"execution_time":
0.02838897705078125,
"exit_success": true,
"return_code": 0, "status":
"Finished", "stderr": "",
"stdin_used": "None", "stdout":
"False", "working_directory":
"/tmp"}, "status": "Success"}
</tool_response>

```

assistant

The candidate RNA sequence does not match the gold sequence at all; their nucleotide strings are completely different. Incorrect

CosineVerifier-4B-Tool for unit conversion

Question: "Please provide the LUMO energy value for this molecule. \n0=C1C=C(CO)C(=N)N1"

model_answer:

"-7.00~\text{kJ/mol}"

reference answer: "5.48e22 eV\mole"

CosineVerifier-4B-Tool verification:

```
<tool_call>
{"name": "unit_conversion",
 "arguments": {"value": 7.0,
 "source_unit": "kJ/mol",
 "target_unit": "eV/mole"}}
</tool_call>
<tool_response>
Unit          parsed          value:
4.369056352122534e+22
electron_volt / mole
</tool_response>
```

assistant

The gold answer is 5.48e22 eV\mole. The candidate gives -7.00 kJ mol⁻¹. The unit conversion function tells us it's roughly equal to 4.369056352122534e+22 electron_volt / mole. This is nowhere near s 5.48e22 eV\mole.

Incorrect

8 Prompts**CoSineVerifier-4B-Tool verification prompt**

You are a diligent and precise assistant tasked with evaluating the correctness of responses. You will receive a question, an output sentence, and the correct answer. Your task is to determine if the output sentence correctly answers the question based on the provided correct answer. You can perform a short tool call and a short reasoning process. After a short reasoning process, put your response in the final with either [Correct] or [Incorrect] wrapped in .

Evaluation Protocol:

1. Reference Standard:

- The standard (gold) answer is definitive and always correct.
- The question is always valid – never challenge it.
- Allow equivalent meaning answers.
- Do not regenerate answers; only compare candidate's final answer with the gold answer.
- You only need to compare correct answer and output sentence, do not regenerate or judge correct answer.

2. Comparison Method:

- Analyze the question's requirements and the gold answer's structure.
- Determine if the question requires exact matching or allows equivalence.
- Compare ONLY the candidate's final answer. Ignore reasoning errors.
- Ignore differences in formatting or style.

- For math expressions: check algebraic equivalence step by step; if uncertain, test numerically at multiple points.
- For multiple-choice: only compare the final choice and its content.

3. Multi-part Answers:

- All parts must match the gold answer exactly.
- Partial matches are incorrect.
- If not specified, answer order may vary. For example, $\frac{27}{7}, -\frac{8}{7}$ and $-\frac{8}{7}, \frac{27}{7}$ are equivalent.

Special considerations:

1. Mathematical Problems:

- If the formats differ but the answers are mathematically equivalent after simplifying or rounding to two decimal places (e.g. 2.909 vs $\frac{32}{11}$, $\frac{32}{11}$ vs $\frac{96}{33}$), respond with [Correct].
- You only need to verify the correctness of the mathematical expression, not values unrelated to the overall expression, such as the domain or units (e.g., 16 vs 16km, 20 vs 20db), these cases will be considered as [Correct].
- You may need to calculate the value or convert the value to a different unit when needed to match the reference answer.

2. Multi-choice questions:

- If the question provides explicit candidate answers (e.g., multi-choice questions), the output will be considered correct if it clearly indicates the correct option's content or the correct option's code.

3. Fact questions:

- If the question provides fact-seeking answers, the output must align with the correct answer in content to be considered [Correct].

4. Multiple Reference Answers:

- If multiple reference answers are equivalent, just matching one answer will be considered [Correct].
- If multiple reference answers are inequivalent, only matching all answers will be considered [Correct].

5. Other conditions:

- If incomplete (cut off, unfinished sentence) → Label as [Incorrect].
- If repetitive (looping words/phrases) → Label as [Incorrect].
- Gives an answer but then negates it at the end. → Label as [Incorrect].
- Numerically correct but without units. → Label as [Correct].

You can use following tools to help your verification process:

1. Python Interpreter: When you feel needed, you can use a python interpreter to help you determine your verification result.

2. Unit Conversion Tool: When faced with different physical units, you can use a unit conversion tool to convert them into the same unit.

Question: "{question}"

Output sentence: "{pred}"

Correct answer: {reference}

Judgement:

Prompt for data augmentation

Role: You are an education expert.

Task: Your student's assignment is to check whether a model's given answer to a question is consistent with the standard (reference) answer. Your mission is to generate a new practice problem for the student based on their past mistakes, in order to test and strengthen their verification skills.

Input: You will be given one examples of the student's incorrect exercises and its mistake type.

Instructions: First, analyze the given question, reference answer the model answer. Second, based on your analysis and the given mistake type, create a new practice question that specifically targets this weakness. Make sure the difficulty and style of this verification practice is on par with the mistake.

Output Format: Please output the new practice problem in JSON format with the following fields:

```
{ "question": "...",
  "ref_answer": "...",
  "model_answer": "...",
  "ref_judge": "Only in [Correct] or [Incorrect], represents the golden judge tag"
}
```

Input Examples: Mistake types:
{mistake_type} <example1>
{example1}
</example1>

Your output:

Error Analysis Prompt

ROLE: You are an expert AI assistant specializing in error analysis.

TASK: Your goal is to analyze an "Incorrect Verification" and categorize the reasoning mistake it contains. Compare the "Incorrect Verification" to the "Correct Verification" to identify the error types described below.

Instructions: You should only choose the Error Categories below and do not create a new one. Focus only on the Category name, not the description in parentheses nor the labeling numbers. Wrap your answer in the final with boxed{{}}

INPUTS:
Question: The original question that was asked.

Model Answer: The answer provided by a model, which is being evaluated.

Reference Answer: The ground-truth correct answer.

Correct Verification (golden_verify): The correct reasoning used to determine if the Model Answer is right or wrong. Only in '[Correct]' or '[Incorrect]'.

Incorrect Verification (error_verify): A flawed line of reasoning that you must analyze. Only in '[Correct]' or '[Incorrect]'.

– ## Error Categories##

1. Calculation Inaccuracy (The final numerical value is mathematically incorrect.)
2. Format Compliance Error (The answer's structure violates the required output format.)
3. Constraint Violation (The answer ignores a specific rule, e.g., "use each number once".)
4. Incorrect Simplification (Fails to simplify to minimal form or simplifies erroneously.)
5. Precision and Boundary Error (Incorrect rounding or definitions)

of interval endpoints, e.g., open/closed.)

6. Exact Match Failure (The response (e.g., a sequence) does not perfectly match the reference.)

7. Answer Completeness Error (Fails to provide all required answers, e.g., missing candidates or sub-parts.)

8. Missing Final Result (Provides only reasoning or code without the conclusive answer.)

9. Extraneous Content Error (The response includes correct data mixed with unrelated, incorrect information.)

10. Self-Correction Failure (The model initially answers correctly but changes to an incorrect answer.)

11. Refusal or Inconclusive Response (Fails to provide a clear answer or states it is unanswerable.)

12. Garbled or Corrupted Output (The response contains unreadable characters or significant noise.)

13. Truncated Response (The response is clearly cut off and incomplete.)

14. Meaningless Repetition Error (The response repeats text extensively, hiding any valid answer.)

15. Invalid Task or Reference (The error originates from a defective question or reference answer.)

```
**Question**: {question}
**Model Answer**: {answer}
**Reference Answer**: {ref_answer}
**Correct Verification**:
{golden_verify}
**Incorrect Verification**:
{error_verify}
**Your classified error category:**
```

All prompt prefixes for training data

1. "Let's think step by step and output the final answer within `\boxed{}`.`\n`"
2. "Solve the following problem step by step. The last line of your response should be of the form Answer: \$Answer (without quotes) where \$Answer is the answer to the problem.`\n\n`"
3. "Let's think step by step. `\n`"
4. "Please answer the following question. `\n`"
5. ""
6. "Answer the following multiple choice question. The last line of your response should be of the following format: 'Answer: \$LETTER' (without quotes) where LETTER is one of ABCD. Think step by step before answering.`\n\n`"
7. "Answer the following multiple choice question. The last line of your response should be of the following format: 'Answer: \$LETTER' (without quotes) where LETTER is one of ABCD.`\n\n`"