Autonomous Vehicles for Ride-Hailing

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Abstract

Problem definition: We consider a setting in which a ride-hailing platform operates a mixed fleet of conventional vehicles (CVs) and autonomous vehicles (AVs) over locations distributed spatially. The CVs are operated by human drivers who make independent decisions about whether to work for the platform and where to position themselves when they become idle. The AVs are under the control of the platform. The platform decides on the wage it pays the drivers, the size of the AV fleet and how AVs are positioned spatially when they are idle. The platform can also make decisions on how much demand to accept for each pair of origins and destinations and whether to prioritize AVs or CVs in assigning vehicles to customer requests. **Methodology:** We use a fluid model to characterize the optimal decisions of the platform in equilibrium and contrast those with the optimal decisions in the absence of AVs. We study outcomes in equilibrium in terms of platform profit, customer service level, driver welfare, and driver productivity. **Results:** Among our findings, we show that the platform, whenever possible, would deploy the AVs in such a way as to reduce repositioning by CVs from the low demand location to the high demand location (inducing such repositioning by drivers is costly for the platform). The presence of AVs can also eliminate driver incentives that would otherwise force the platform to reject demand from the low demand location to force drivers to relocate to the high demand location. Even though demand is no longer exclusively fulfilled by CVs, we show that the income of drivers may not necessarily be harmed, and that drivers may not necessarily experience more idleness or empty travel. Moreover, we find that drivers can be strictly better off if the platform prioritizes AVs in assigning customer requests to vehicles. **Managerial implications:** Our results uncover important ways the introduction of AVs affects the operation of a ride-hailing platform and highlight the nuanced impact of AVs on human drivers and customers. Our results are potentially useful to policy makers in deciding on regulatory interventions that can induce more socially desirable outcomes with the introduction of AVs.

Keywords: autonomous vehicles, ride-hailing, equilibrium fluid model, driver welfare, vehicle repositioning, admission control, assignment priorities

1 Introduction

Autonomous vehicle (AV) technology is an exciting new technology, though not fully mature yet, that some have argued will transform the transportation landscape. How this transformation will take place and how it would affect various stakeholders (riders, drivers, and service providers) continues to be a subject of vigorous debate (Iyer and Alton (2019) and Lalley (2017)). A potentially important application of AV technology is ride hailing (the provisioning of transportation services on-demand). The ride hailing industry, which currently relies mostly on independent drivers using conventional vehicles (CVs), has shown a particular interest in AV technology, with several of the leading platforms, such as Uber, Lyft, and Didi, making substantial investments in the research and development of AV technology (Uber (2019), Lyft (2021) and Didi (2021)). Under most scenarios, it is envisioned that the introduction of AVs will however be gradual and that ride hailing platforms are likely to operate initially with a mixed fleet of both AVs and CVs, with the latter owned and operated by human drivers (Iyer and Alton (2019)).

In this paper, we examine how the deployment of AVs, as part of a mixed fleet of AVs and CVs, impacts the operational decisions of a ride hailing platform, including how it manages the spatial mismatch between supply and demand through *vehicle repositioning*, demand curtailment through *admission control*, the matching of supply and demand through the *assignment* of vehicles to customers, and supply dimensioning through the *sizing* of the fleet of AVs and the level of wages paid to humans driving the CVs. We also examine how these decisions impact human drivers and riders.

We consider a setting where a ride-hailing platform operates a mixed fleet of AVs and CVs. The platform seeks to fulfill transportation requests from customers, who arrive continuously over time, so as to maximize profit. The platform operates over a network consisting of multiple locations. The rate at which customers arrive varies depending on the origin and destination of the requested trips. Customer requests that cannot be immediately matched with a vehicle are considered lost. The platform charges customers a price per unit of travel time. The CVs are driven by independent drivers, who are heterogeneous in their opportunity cost. The platform pays drivers a fixed wage per unit time of service (drivers are paid only when transporting a customer). Drivers decide whether or not to work for the platform depending on the expected earnings from working on the platform and their outside options. The platform incurs a fixed cost for purchasing AVs. AVs and CVs incur a variable cost per unit time of travel per vehicle. We assume that the platform pays for the AV travel cost and drivers pay for the CV travel cost.

Upon completing a trip transporting a customer, vehicles can either stay at the location where the trip terminated or reposition (drive empty) to another location. The repositioning of AVs is under the control of the platform while the repositioning of CVs is in the hands of the drivers who act strategically and reposition to the location that maximizes their expected earnings. The platform has other levers. It can curtail demand associated with a particular pair of origins and destinations (e.g., by rejecting a fraction of this demand). When a customer request arises, it can also decide on whether to prioritize AVs or CVs in assigning the request to a vehicle. Finally, the platform also decides on how many AVs to acquire and how much to pay the drivers.

From the perspective of the platform, autonomous vehicles and human drivers differ in two critical characteristics: cost structure and controllability. There is a fixed cost associated with AVs incurred regardless of how AVs are deployed during operation. There is only a variable cost associated with CVs in the form of payments to drivers when they have a customer on-board. AVs incur a variable cost when traveling independently of whether or not a customer is onboard. For a profit maximizing platform, we expect the variable cost associated with the AVs to be lower than that of CVs (otherwise, it would be more profitable for the platform to rely solely on CVs). AVs are under the control of the platform. Once acquired they are always available to the platform. The platform can dictate where the AVs position themselves once they complete a trip, giving the platform more latitude in its effort to most profitably match supply with demand.

To study such a setting, we adopt a fluid model based on the one recently introduced by Afèche et al. (2018). Afèche et al. (2018) consider a setting with two locations where the vehicles are all conventional and operated by human drivers. They primarily focus on admission control in situations where demand is unbalanced, with one high demand location and one low demand location, and where drivers act strategically in deciding where to reposition upon completing a trip. We extend the model in Afèche et al. (2018) in several important ways. We consider a setting with a mix of CVs and AVs with the repositioning of AVs under the control of the platform. We allow for the wage paid to drivers to be a decision made by the platform. We consider a broader range of decisions the platform can make, including how AVs should be repositioned, how to assign vehicles to incoming requests (vehicle assignment priorities) and how large the AV fleet size should be.

The following is a summary of our main findings (unless stated otherwise, the results hold under the platform's optimal policy).

- When the platform does not differentiate between AVs and CVs in assigning demand requests to vehicles, for a fixed supply of AVs and CVs, both AVs and CVs only reposition from the low demand location to the high demand location. The platform repositions AVs to minimize the repositioning of CVs. In particular, if the supply of AVs is sufficiently large, the platform repositions enough AVs from the low demand location to the high demand location to ensure that CVs have no incentive to reposition. Otherwise, the platform repositions AVs from the low demand location to the high demand location with probability 1 to minimize the CVs' likelihood of repositioning.

- When the supply of AVs and CVs are endogenously determined (the platform decides on the number of AVs and the wages paid to drivers), whether CVs reposition or not depends on the size of the driver pool and the purchase cost of AVs. If the driver pool size is sufficiently small (or equivalently the purchase cost of AVs is sufficiently low), CVs do not reposition. Otherwise, CVs reposition (along with the AVs, if any) with a positive probability.

- If the AV purchase cost is low, it can be beneficial for the platform to reject some of the demand for trips from the high demand location to the low demand location. The reverse is true when the AV purchase cost is high (i.e., it can be beneficial to reject demand for trips from the low demand location to the high demand location). In both cases, the platform seeks to balance the flows in the network, achievable either by curtailing the demand at the high demand location or through vehicle repositioning.

- It is optimal for the platform to prioritize AVs in assigning vehicles to customer requests. When the platform does, drivers are left fulfilling the demand that cannot be fulfilled by the AVs. From the perspective of the drivers this amounts to demand curtailment. The platform can use this form of demand curtailment to incentivize the drivers to reposition away from the low demand location to the high demand location. The fact that the platform prioritizes AVs does not necessarily harm drivers (relative to a policy where the platform does not prioritize AVs).

- If the platform does not differentiate between AVs and CVs when assigning vehicles to customers, the introduction of AVs does not necessarily harm drivers. This is because the introduction of AVs provides the platform with another means to serve otherwise unfulfilled demand due to demand imbalance. If the AV purchase cost is moderate, the platform uses AVs not to have them compete with CVs but as a complement. AVs can also be used strategically (e.g., by prioritizing AVs) to induce drivers to choose repositioning decisions that improve overall driver welfare. In both cases, the introduction of AVs may result in a reduction in idleness and empty travel for the CVs, making them more *productive*.

These findings suggest that there is nuance to how the introduction of AVs may impact the operation of a ride hailing service and how this will in turn impact the welfare of human drivers. Some of this nuance may carry over to other areas where automation is introduced. In particular, the results of this paper suggest that automation, when it is partial, can lead to improvements in the productivity or welfare of the humans involved.

The rest of the paper is organized as follows. In Section 2, we provide a review of related literature. In Section 3, we describe our model. In Section 4, we consider the case with a fixed supply of AVs and CVs. In Section 5, we treat the case where the supply of AVs and CVs is endogenized. In section 6, we study admission control. In Section 7, we study systems under different vehicle assignment priority policies. In Section 8, we offer concluding comments. Proofs for all the results, unless otherwise stated, are included in the Appendix.

2 Literature Review

This work contributes to the growing body of literature that studies spatial networks in which resources move from one location to another in the process of servicing demand that is also spatially distributed. Of particular relevance is literature that is motivated by applications in on-demand transportation services, including ride hailing and vehicle sharing (e.g., bike sharing); see Benjaafar and Hu (2020), Hu (2021), and Freund et al. (2019) for recent reviews. Our work is related to streams within this literature that focus on the operational control of these networks, where control levers include the assignment of resources with customers, the spatial repositioning of resources so as to better match supply with demand, and the shaping of demand, indirectly through pricing or directly through admission control. Some of this literature, particularly, as it relates to ride hailing accounts for the fact that the control of resources is distributed and in the hands of individuals who are strategic in their decision making.

Below we briefly review papers that are most salient to our work. We focus on papers that take, as we do, a queueing network (and its associated fluid model approximation) perspective. We refer the reader to Afèche et al. (2018) for a more comprehensive discussion of the literature.

Repositioning. Braverman et al. (2019) consider a vehicle sharing network where a platform controls the repositioning of all vehicles (this is akin to a system with only AVs in our setting). Using a fluid approximation, they show that the vehicle repositioning problem can be formulated as a linear program which can then be solved efficiently. Moreover, they prove that the optimal solution to this linear program specifies a repositioning strategy that is asymptotically optimal (namely, converges to the optimal solution for the non-fluid problem when the demand and number of vehicles are allowed to go to infinity). Afèche et al. (2018) consider a fluid model of a two-location, four-route network with strategic drivers. Strategic drivers are not controlled by the platform and make their own repositioning decisions to maximize their earnings (this corresponds to a system with only CVs in our setting). The platform maximizes profit by deciding on how much demand to accept from each location (i.e., the platform has control over admission). They characterize, in equilibrium, both the platform's optimal admission control and the drivers' optimal repositioning. In particular, they show that, under some conditions, it is optimal for the platform to reject demand in the low demand location in order to incentivize drivers to reposition to the high demand location. Hosseini et al. (2021) design a state-dependent vehicle repositioning policy based on structural properties of a fluid-based model. They provide numerical evidence showing that this state-dependent policy can outperform static policies. Benjaafar et al. (2021b) consider a repositioning problem where the objective is to minimize repositioning costs. They do so under a demand balance assumption and using an approximation for vehicle availability at each location.

There is extensive literature that deals with vehicle repositioning in non-queueing contexts, including problems with a single period or under multiple discrete periods and without strategic drivers; see for example, Benjaafar et al. (2021c), Akturk et al. (2021), He et al. (2020), and Zhao et al. (2020). A comprehensive review of this literature can be found in Benjaafar et al. (2021c).

Pricing. Waserhole and Jost (2016) and Banerjee et al. (2021) consider the optimal pricing problem in the context of a vehicle sharing system (modeled as a queueing network) with no repositioning. They study the asymptotic regime when the number of vehicles goes to infinity. They show that the resulting static pricing policy provides, under varying assumptions, guaranteed bounds for the finite system. Balseiro et al. (2021) consider a network with a hub-and-spoke structure. They develop a dynamic pricing policy and a performance bound based on a Lagrangian relaxation. Bimpikis et al. (2019) consider a ride-sharing network with strategic drivers. They characterize optimal prices and optimal wages. Courcoubetis and Dimakis (2019) show that a state-dependent policy that pays drivers in proportion to the number of passengers waiting to be picked up at a location maximizes throughput. Chen et al. (2020) study a vehicle sharing network using a discrete time framework. They propose several pricing heuristics which they show to be asymptotically optimal in the setting where the number of vehicles and customers are both large. Ma et al. (2020) consider a deterministic setting for a ride sharing network and propose a spatial-temporal mechanism for prices and wages with desirable properties; see also Besbes et al. (2021b). Besbes et al. (2021c) study a single location problem and show that a static pricing policy achieves nearly 80% of the performance (along several metrics) of an optimal policy.

Admission control and matching. Özkan and Ward (2020) study the problem of matching customer requests with nearby drivers in the context of a ride hailing network. They use a fluid model approximation and show that a static matching is asymptotically optimal under heavy traffic. Banerjee et al. (2020) consider a similar problem. They develop a family of state-dependent policies whose performance they show to improve exponentially as the number of vehicles scales to infinity. Kanoria and Qian (2020) study network control, using levers that include admission control, dispatching, and pricing. They develop a class of control policies that are nearly optimal under certain conditions for the discrete time version of the problem they consider.

Dimensioning. George and Xia (2011) develop exact and approximate solution algorithms to determine the optimal fleet size in a vehicle sharing network where the objective is to maximize system profit. George et al. (2012) derive the exact-order asymptotic growth rate of system throughput as the number of vehicles increases. Benjaafar et al. (2021c) develop an approximation for the number of vehicles in a vehicle sharing network needed to guarantee a specified service level (the fraction of demand fulfilled) at each location. They show that this approximation is optimal under various asymptotic regimes. Besbes et al. (2021a) study the problem of optimal service capacity for a ride-hailing system modeled as a single multi-server queue with a state-dependent service rate that account for pick up and travel times. For systems with strategic drivers where the vehicles are under the control of human drivers, service capacity is determined indirectly via the mechanism of the wage paid to drivers; see for example Taylor (2018), Benjaafar et al. (2021a) and Hu and Zhou (2020).

Finally, there is emerging literature that considers explicitly the role of AVs in ride sharing systems. Examples include Siddiq and Taylor (2021), Lian and Van Ryzin (2020), Baron et al. (2021) and Noh et al. (2021). This literature for the most part does not account for the spatial aspect of ride sharing. Our work is more broadly related to literature that examines the impact of automation. Much of that literature is focused on studying the extent to which automation will replace drivers; see for example Dixon et al. (2021), Acemoglu and Restrepo (2020), Graetz and Michaels (2018) and Frey and Osborne (2017).

3 The Basic Model

In this section, we provide a formal description of the model. Consider a platform that operates a mixed fleet of AVs and CVs and let M and N denote the amount of AVs and CVs respectively. The platform charges customers a price p per unit of travel time. That is, a customer pays amount pt_{ij} for a trip from location i to location j where t_{ij} is the duration of the trip from location i to location j. Customers generate demand for trips from location i to location j. If a customer arrives at a location and there are no empty vehicles available at that location, the customer leaves the system and the platform does not earn any revenue. The platform pays drivers a wage w per unit of time the driver spends transporting a customer. Therefore, a driver earns wt_{ij} from serving demand that originates at location i ends at location j.

We adopt the fluid model introduced by Afèche et al. (2018) of a stylized network consisting of two locations (indexed by $1, 2$) and four routes (two within-location routes and two cross-location routes). A within-location route represents trips within a location. A cross-location route represents a trip from one location to another. We allow $t_{12} \neq t_{21}$ to account, for example, for different road conditions of routes in opposite directions. Travel times are constant and exogenous. In particular, t_{ij} is independent of the decisions made by the platform. Though simple, this network captures many of the essential features of more complex settings, including imbalances in customer demand and vehicle supply across locations, strategic behavior on the part of the drivers, and multiple types of operational decisions on the part of the platform (see Afèche et al. (2018) for further discussion and motivation).

Upon completing service (a trip transporting a customer), vehicles can either stay at the location where the service terminated or drive empty to the other location. We denote by q_i^A and q_i^C the volume of AVs and CVs respectively queueing at location i and we let $q_i = q_i^A + q_i^C$ denote the sum of the two. When customers arrive, the platform selects vehicles among available AVs and CVs according to a specified priority policy to serve customers. Let W_i^A and W_i^C denote the steady-state delay experienced by AVs and CVs waiting to be matched with customers at location i. Let $\eta^C(\eta_1^C, \eta_2^C)$ denote the drivers' repositioning strategy, where η_i^C is the probability that a CV drives empty to location j after completing a service that ended at location i, where $i \neq j \in \{1, 2\}$.

We assume that drivers of CVs make their own decisions regarding repositioning in order to maximize their earnings. Following Afèche et al. (2018), we focus on the case where drivers adopt symmetric strategies. Therefore, we call $\eta^C = (\eta_1^C, \eta_2^C)$ a *CV equilibrium repositioning strategy* if it is the best response for every driver. We define the CV equilibrium repositioning strategy rigorously in subsequent sections under each of the vehicle assignment priority policies considered. For the AVs, we define $\eta^A(\eta_1^A, \eta_2^A)$ similarly. The platform owns the AVs and controls the AVs' repositioning strategy η^A. Let ν_i^A and ν_i^C denote the repositioning rates of AVs and CVs, respectively, from locations i to the other location.

AVs and CVs incur operational costs when they are traveling (for example, fuel costs and driving efforts). We use c_a and c_c to refer to the travel cost per unit time for AVs and CVs respectively. Because AVs are unmanned, we assume that $c_a < c_c$. This assumption is consistent with treatment elsewhere in the literature (see for example Baron et al. (2021)). We assume that drivers pay for the CV travel cost while the platform pays for the AV travel cost.

We consider a continuum of drivers with mass L, who are heterogeneous in their opportunity costs with a uniform distribution over $[0, \overline{w}]$, where $\overline{w}$ is the maximal opportunity cost for drivers. Note that because $w \leq p$, drivers with opportunity costs greater than $p - c_c$ never work for the platform. Therefore, we assume that $\overline{w} = p - c_c$. A driver works for the platform only if her expected earning in equilibrium exceeds her opportunity cost. The platform procures AVs at a fixed cost. Let I denote the AV purchase cost amortized over the AV's expected lifetime. We assume that $I + c_a \leq p$ (otherwise, the platform dose not procure any AVs).

The platform has several levers at its disposal. First, the platform determines the AV repositioning strategy, which has an effect on the CV's repositioining decisions. Second, the platform determines the amount of AVs to purchase and the wage it pays to drivers. Third, the platform employs admission control under which the platform can reject some demand. Lastly, the platform can decide on the priority policy in assigning customer requests to vehicles (e.g., prioritize AVs or CVs). We assume that drivers have (or can infer) full information, including the decisions made by the platform, when making their own decisions about whether to work for the platform and how to reposition.

Let $\overline{\Lambda}_{ij}$ denote the potential demand rate from location i to location j for $i, j \in \{1, 2\}$. Without loss of generality, we assume $\overline{\Lambda}_{12} < \overline{\Lambda}_{21}$ and thus we call location 1 the low demand location and location 2 the high demand location (note that when $\overline{\Lambda}_{12} = \overline{\Lambda}_{21}$, no repositioning is needed). To avoid trivial cases, we assume that $\overline{\Lambda}_{ij} > 0$. Let Λ_{ij} be the accepted demand arrival rate (not necessarily served) through admission control and let $S_{ij} = \Lambda_{ij} t_{ij}$. An accepted demand request is lost if there is no available vehicle at location i upon arrival. We use λ_{ij} to denote the effective demand rate from location i to location j (i.e., the rate of fulfilled demand that originates at location i and ends at location j). Let λ^A_{ij} and λ^C_{ij} denote the demand rate from location i to location j fulfilled by AVs and CVs respectively, and we have $\lambda_{ij} = \lambda^A_{ij} + \lambda^C_{ij}$. Let s^A_{ij} and s^C_{ij} denote, respectively the volume of AVs and CVs in service from location i to location j. By Little's law, $s^A_{ij} = \lambda^A_{ij} t_{ij}$ and $s^C_{ij} = \lambda^C_{ij} t_{ij}$. Let $s_{ij} = s^A_{ij} + s^C_{ij}$ refer to the total volume of vehicles in service from location i to location j. Denote by r^A_{ij} and r^C_{ij} the volume of AVs and CVs repositioning from location i to location j. By Little's law, $r^A_{ij} = \nu^A_{ij} t_{ij}$ and $r^C_{ij} = \nu^C_{ij} t_{ij}$. Let $r_{ij} = r^A_{ij} + r^C_{ij}$. Let s denote the pair

(s_{ij}^A, s_{ij}^C), r the pair (r_{ij}^A, r_{ij}^C) and q the pair (q_i^A, q_i^C). We refer to (s, r, q) as the capacity allocation of the system. In the rest of the paper, we use (s, r, q) to describe the system.

Let $a_i = \frac{\lambda_{ij}}{\Lambda_{ij}}$ for $j = 1, 2$ denote the fraction of accepted demand that is fulfilled. Let $F_i = \frac{s_{ij}^A}{s_{ij}}$ for $j = 1, 2$ denote the fraction of demand that is fulfilled by AVs at location i. In steady-state, η^A, η^C, and (s, r, q) must satisfy the following steady state fluid model equations.

$$\Lambda_{ij} a_i = \frac{s_{ij}}{t_{ij}}, \quad i, j = 1, 2, \tag{1}$$

$$\frac{r_{ij}^A}{t_{ij}} = \eta_i^A \sum_{k=1,2} \frac{s_{ki}}{t_{ki}} F_k, \tag{2}$$

$$\frac{r_{ij}^C}{t_{ij}} = \eta_i^C \sum_{k=1,2} \frac{s_{ki}}{t_{ki}} (1 - F_k), \quad i \neq j = 1, 2. \tag{3}$$

$$(\Lambda_{i1} + \Lambda_{i2}) a_i F_i = \frac{r_{ji}^A}{t_{ji}} + (1 - \eta_i^A) \sum_{k=1,2} \frac{s_{ki}}{t_{ki}} F_k, \tag{4}$$

$$(\Lambda_{i1} + \Lambda_{i2}) a_i (1 - F_i) = \frac{r_{ji}^C}{t_{ji}} + (1 - \eta_i^C) \sum_{k=1,2} \frac{s_{ki}}{t_{ki}} (1 - F_k), \quad i \neq j = 1, 2. \tag{5}$$

$$(1 - a_i) q_i = 0, \quad i = 1, 2. \tag{6}$$

$$\sum_{i,j=1,2} s_{ij} F_i + (r_{12}^A + r_{21}^A) + (q_1^A + q_2^A) = M, \quad \text{and} \tag{7}$$

$$\sum_{i,j=1,2} s_{ij} (1 - F_i) + (r_{12}^C + r_{21}^C) + (q_1^C + q_2^C) = N. \tag{8}$$

Equation (1) is a result of Little's law. Equation (2) and equation (3) are the repositioning flow balance equations for AVs and CVs respectively, i.e., $\nu_i^A = \eta_i^A (\lambda_{1i}^A + \lambda_{2i}^A)$ and $\nu_i^C = \eta_i^C (\lambda_{1i}^C + \lambda_{2i}^C)$. Equation (4) and equation (5) state that the rates of outflow and inflow at location i must be equal for both AVs and CVs. Equation (6) guarantees that the accepted demand originating at location i can only be lost if there are no vehicles queueing at location i. Equation (7) and equation (8) state that the amount of AVs and CVs in service, being repositioned, and queueing must be equal to the fleet size of AVs and CVs respectively. Additional equations will be discussed in subsequent sections accounting for the vehicle assignment priority policy that is in effect.

The goal of the platform is to maximize its profit, which is given by

$$\Pi = (p - w) \sum_{i,j=1,2} s_{ij}^C + (p - c_a) \sum_{i,j=1,2} s_{ij}^A - c_a(r_{12}^A + r_{21}^A) - IM.$$

4 Systems with Exogenous Supply of AVs and CVs

In this section, we consider the case where the supply of AVs and CVs are exogenous (we endogenize both in Section 5). Recall that M and N denote the amount of AVs and CVs respectively. In this section, we assume that $M > 0$ and $N > 0$. We limit our treatment to the case where demand is always fulfilled whenever there are available vehicles (i.e., the platform does not exercise admission control) and to where the platform does not differentiate between AVs and CVs in making vehicle assignments (i.e., the assignment priority policy is *random*). In Section 6, we study admission control and, in Section 7, we investigate assignment policies that prioritize AVs or CVs.

Given the above assumptions, by Little's law, the expected delay experienced by AVs and CVs queuing at location i is given by $W_i^A = W_i^C = \frac{q_i}{\Lambda_{i1}+\Lambda_{i2}}$. Additionally, in steady state, the amount of AVs and CVs in service from each location must be proportional to the amount of AVs and CVs queued at that location. That is, for $i, j \in \{1, 2\}$,

$$\begin{cases} s_{ij}^C = 0 & \text{if } q_i^C = 0 \\ s_{ij}^A = 0 & \text{if } q_i^A = 0 \\ \frac{s_{ij}^A}{s_{ij}^C} = \frac{q_i^A}{q_i^C}, & \text{otherwise.} \end{cases} \tag{9}$$

Because the platform dose not exercise admission control, we have

$$\Lambda_{ij} = \overline{\Lambda}_{ij} \quad \text{for} \quad i, j \in \{1, 2\}. \tag{10}$$

The platform decides on the AV repositioning strategy η^A to maximize its profit taking into account the repositioining decisions of the drivers. Note that, in this section, we assume that $t_{ij}c_c < t_{ji}(w - c_c)$, where $i \neq j \in \{1, 2\}$, to ensure that drivers can make a profit from repositioning.

The platform's problem can now be stated as follows:

$$\max_{\eta^A} \quad \Pi(\eta^A) = (p-w)\sum_{i,j=1,2} s_{ij}^C + (p-c_a)\sum_{i,j=1,2} s_{ij}^A - c_a(r_{12}^A + r_{21}^A), \tag{11}$$

subject to (1)–(10) and $\eta^C = (\eta_1^C, \eta_2^C)$ is a CV equilibrium repositioning strategy.

In the rest of this section, we first solve for the optimal AV repositioning strategy as well as the corresponding outcomes (Section 4.1). We then study how the supply of AVs and CVs affect the platform profit and the driver welfare (Section 4.2).

4.1 The Optimal AV Repositioning Strategy

We first show in Lemma 1 that for any AV repositioning strategy $\eta^A = (\eta_1^A, \eta_2^A)$, there exists a unique CV equilibrium repositioning strategy.

Lemma 1. *Given any AV repositioning strategy $\eta^A = (\eta_1^A, \eta_2^A)$, there exists a unique CV equilibrium repositioning strategy such that* (1)–(9) *hold.*

This result tells us that, given any AV repositioning strategy, the CV equilibrium repositioning strategy is well-defined (existence and uniqueness). Therefore, we let $\eta^C(\eta^A) = (\eta_1^C(\eta^A), \eta_2^C(\eta^A))$ denote the CV equilibrium strategy as a function of η^A. Theorem 1 describes the existence and uniqueness of the optimal AV repositioning strategy and its properties. Recall that we assume $\overline{\Lambda}_{12} < \overline{\Lambda}_{21}$.

Theorem 1. *Given (M, N, w), there exists a unique optimal AV repositioning strategy η^{A*} which satisfies the following properties:*

(i) $\eta_2^{A*} = 0$ and $\eta_2^C(\eta^{A*}) = 0$;

(ii) $\eta^{A*} \in \arg\min_{\eta^A}\{\eta_1^C(\eta^A)\}$.

Theorem 1 states that both AVs and CVs only reposition from location 1 to location 2. More importantly, Theorem 1 indicates that the platform repositions AVs to minimize the repositioning of CVs. In particular, if the supply of AVs is sufficiently large, the platform repositions enough AVs from location 1 to location 2 to ensure that drivers have no incentive to reposition. Otherwise, the platform repositions AVs from location 1 to location 2 with probability 1 to minimize the CVs' likelihood of repositioining.

The intuition behind Theorem 1 is as follows. Whenever CVs are repositioning and some AVs are queueing at location 1, the platform can reposition more AVs from location 1 to location 2 without changing the total demand fulfilled (CVs respond to the platform's change in strategy by reducing their repositioning). The platform increases in this way the amount of demand fulfilled by AVs. Because the platform makes a higher profit from demand fulfilled by AVs than that fulfilled by CVs, the platform benefits from increasing the repositioning of AVs until CVs do not reposition (the platform might further increase the repositioning of AVs depending on the parameters) or AVs reposition from location 1 to location 2 with probability 1.

The result in Theorem 1 describes a new mechanisms for the platform to increase its profit through the repositioning of AVs. A ride-hailing platform such as Uber and Lyft is typically concerned about the misaligned incentives with their drivers, who may make repositioning decisions that harm the platform profit. The introduction of AVs eliminates the platform's need to directly incentivize drivers to reposition.

4.2 Platform Profit and Driver Welfare

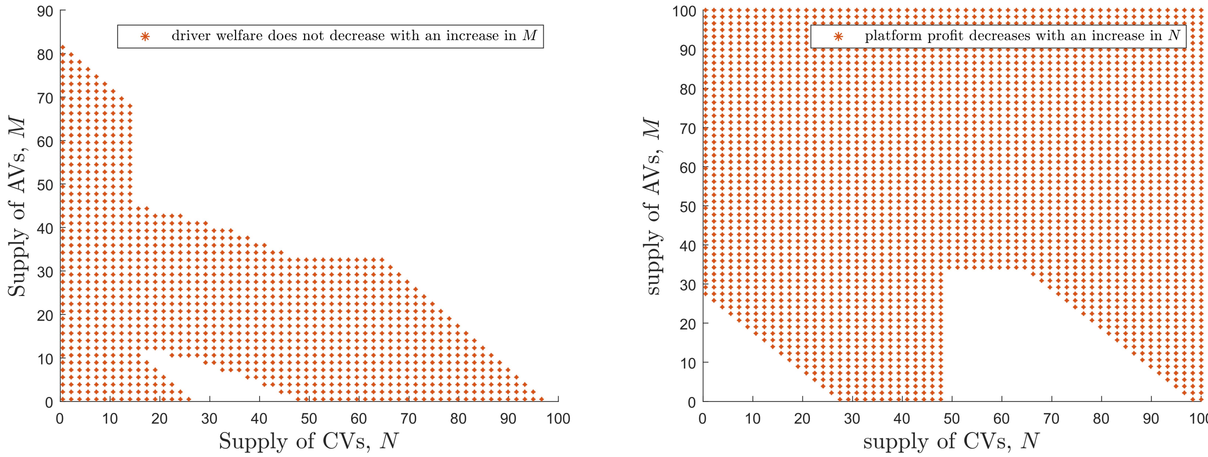
In this section, we examine how platform profit and driver welfare are affected by the supply of AVs and CVs. In particular, we address the question of whether drivers are helped or harmed by an increase in the supply of AVs and whether an increase in the supply of drivers is always beneficial to the platform. Let $\overline{\pi} = \frac{(w-c_c)\sum_{i,j=1,2} s_{ij}^C - c_c r_{12}^C}{N}$ denote average driver welfare.

Proposition 1. *Platform profit increases in M and is non-monotone in N. Average driver welfare (weakly) decreases in M and N.*

Proposition 1 indicates that the platform is always better off having more AVs but may be worse off having more drivers. Moreover, average driver welfare does not increase in the supply of either AVs or CVs but may not necessarily decrease. In the Appendix, we specify the ranges for parameter values under which the different outcomes described in Proposition 1 apply (see Remark 2). (A discussion of intuition behind these results, along with numerical illustration, is provided below.)

In Figure 1(a), we provide numerical results that illustrate how drivers are affected by the increase of AV supply. If (1) total service capacity (i.e., $M + N$) is sufficiently large such that the demand at both locations is fully fulfilled or (2) the service supply is moderate such that the demand at location 1 (the low demand location) is fully fulfilled but the platform has no incentive to reposition AVs, i.e., $\eta_1^A = 0$ (as shown in the white triangle shape region in Figure 1(a)), an increase in the supply of AVs intensifies the competition between AVs and CVs for demand at the low demand location and thus hurts drivers. Average driver welfare is not affected by the increase of AV supply if (1) the service supply is low such that the demand at neither location is fully fulfilled, or (2) the service supply is moderate such that the demand at location 1 is fully fulfilled and the platform has an incentive to reposition AVs, i.e., $\eta_1^A > 0$. In the later case, by virtue Theorem 1, when the supply of AVs increases, the platform adjusts the AV repositioning strategy so that more AVs are repositioning from location 1 to location 2 to serve demand at location 2. In these two cases, an increase in the supply of AVs does not intensify the competition between AVs and CVs. Instead, the platform uses the extra AVs to fulfill otherwise unfulfilled demand.

In Figure 1(b), the platform is always hurt by an increase in the supply of CVs if demand in both locations has already been fully served. In this case, an increase in the supply of CVs increases the fraction of demand served by CVs and thus lowers the platform profit. The platform may also be hurt when the supply of AVs and CVs are insufficient to fulfill all the demand. If the service supply is moderate such that the demand in location 2 is not fully fulfilled and the platform has an incentive to reposition AVs, an increase in driver supply potentially brings in two effects. On the one hand, the demand fulfilled by CVs increases and the total service level increases. On the other hand, by virtue of Theorem 1, in order to minimize CVs' repositioning, the platform repositions more AVs from location 1 to 2, which results in an increase in repositioning costs. Therefore, the platform serves more demand and generates more revenue, but pays more AV repositioning costs and more wages to drivers. When the AV repositioning cost and the wage paid to drivers is relatively high and the price is relatively low, the platform is hurt by an increase in the driver supply.



(a) Impact of increasing the supply of AVs on the average driver welfare

(b) Impact of increasing the supply of CVs on the platform profit

Figure 1: Impact of increasing the supply of AVs and CVs on average driver welfare and the platform profit. Model parameters: $\Lambda_{11} = 5$, $\Lambda_{12} = 10$, $\Lambda_{21} = 25$, $\Lambda_{22} = 5$, $t_{ij} = 1$ for $i, j \in \{1, 2\}$, $p = 1$, $w = 0.9$, $c_c = 0.2$ and $c_a = 0.1$.

5 Systems with Endogenous Supply of AVs and CVs

In this section, we endogenize the supply of AVs and CVs, allowing the platform to choose the amount of AVs to purchase and the wage it pays to drivers (in addition to the repositioning of AVs). The amount of supply the platform can induce is determined by the wage the platform pays. In particular, supply consists of drivers whose opportunity cost is less than their expected earnings net of costs incurred (i.e., average driver welfare $\overline{\pi}$). Because the fraction of drivers whose opportunity cost is smaller than average driver welfare is $\frac{\overline{\pi}}{\overline{w}}$, the amount of driver supply in equilibrium satisfies

$$N = \frac{\overline{\pi}}{\overline{w}} L. \tag{12}$$

In Lemma 2, we establish the existence and uniqueness of the driver supply given the supply of AVs M, the wage paid to drivers w, and the repositioning strategy for the AVs η^A.

Lemma 2. *Given (M, w, η^A), where $w \geq c_c$, there is a unique solution to* (12).

The platform's problem can then be stated as follows:

$$\max_{(M,w,\eta^A)} \Pi(M,w,\eta^A) = (p-w)\sum_{i,j=1,2} s_{ij}^C + (p-c_a)\sum_{i,j=1,2} s_{ij}^A - c_a(r_{12}^A + r_{21}^A) - I \times M, \tag{13}$$

subject to (1)–(10), (12) and $\eta^C = (\eta_1^C, \eta_2^C)$ is a CV equilibrium repositioning strategy. Theorem 2 establishes the existence of the optimal strategy of the platform.

Theorem 2. *There exists an optimal strategy* (M^*, w^*, η^{A*}) *for the platform.*[1] *Moreover, the corresponding capacity allocation* (s, r, q) *satisfies the following properties:*

(i) $q_2 = 0$*;*

(ii) either $q_1 = 0$ *and* $\eta_1^C = 0$ *or* $q_1 = q_1^*$, $\eta_1^C \in (0,1)$ *and* $\eta_1^A = 1$*, where* q_1^* *is specified in* (23).

Theorem 2 states that when the supply of AVs and CVs is endogenized, the equilibrium outcome satisfies two properties. First, no vehicles queue up at location 2. Second, the equilibrium outcome falls into one of the following two scenarios: either (i) no vehicles queue up at location 1 and CVs do not reposition or (2) CVs reposition from location 1 to location 2 with a positive probability and AVs (if any) reposition from location 1 to location 2 with probability 1. As we discuss in the Appendix, in which scenario the equilibrium falls depends on the AV purchase cost I and the driver pool size L. The second property is consistent with Theorem 1, and it implies that the platform has no incentive to let AVs compete with CVs for customers at the low demand location.

Details about the characterization of the optimal strategy can be found in Appendix E.3. Here we summarize key aspects considered by the platform. First, note that the platform profit function can be decomposed into profit earned from AVs and profit earned from CVs. Second, observe that it is never optimal for the platform to procure excess supply of AVs and let them queue up. Hence, the platform only needs to consider two cases associated with the profit from AVs: (i) when AVs do not reposition and serve customers all the time and (ii) when AVs reposition from location 1 to location 2 with probability 1 and there is no queue at location 2.

The part of the profit function associated with CVs is more complicated because (i) CVs only reposition when $q_1 \geq q_1^*$, and (ii) q_1^* depends on the wage paid to drivers. Therefore, we need to

[1] In most cases, there exists a unique optimal strategy for the platform. In some boundary cases, the platform may have two different optimal strategies. See Appendix E.2 for more details.

analyze the profit function in two scenarios: (a) CVs do not reposition or queue up and (b) CVs reposition with a positive probability. Note that in order for drivers to reposition, there must be enough drivers queueing at location 1. Because the wage paid to drivers increases in the amount of drivers recruited, the platform only lets CVs reposition if the driver pool size is sufficiently large. We show in Appendix E.3 that the profit function in each scenario has some desirable properties (i.e., concavity or unimodality), allowing us to characterize the platform's optimal strategy.

Depending on the driver pool size and the AV purchase cost, different types of equilibria arise. We group these into the following types. Details about when these different types of equilibra arise can be found in the Appendix; see Remark 3.

- In type I and type II equilibra, the platform operates with only AVs. In type I equilibra, AVs serve all the demand at location 1 but do not reposition. In type II equilibra, AVs serve all the demand at location 1 and location 2 and reposition with a positive probability.
- In type III, type IV and type V equilibra, the platform operates with both AVs and CVs. In type III equilibra, AVs and CVs serve all the demand at location 1 but do not reposition. In type IV equilibra, AVs and CVs serve all the demand at location 1 and location 2. AVs reposition with a positive probability but CVs do not reposition. In type V equilibra, AVs and CVs serve all the demand at location 1 and location 2. CVs reposition with a positive probability and AVs reposition with probability 1.
- In type VI and type VII equilibra, the platform operates with only CVs. In type VI equilibra, CVs do not reposition and serve all the demand at location 1 while in type VII equilibra, CVs reposition with a positive probability.

In Proposition 2, we examine the impact of the AV purchase cost and the driver pool size on the optimal strategy of the platform.

Proposition 2. *The optimal amount of AVs M^* (weakly) decreases in I and L. The optimal wage w^* (weakly) increases in I and is non-monotone in L.*

As expected, the platform procures more AVs as the AV purchase cost or the driver pool size decreases. The optimal wage increases in the AV purchase cost because the platform is better off recruiting more drivers if the AV purchase cost increases. Lastly, the optimal wage is non-monotone in the driver pool size. This is because the average driver welfare depends on whether CVs are repositioning or not. When CVs are repositioning with a positive probability, drivers spend a fraction of their time queueing and repositioning. Therefore, the platform needs to pay a higher wage to compensate drivers for repositioning and for not being able to serve customers all the time. We can show that, given the AV purchase cost I, there exists a threshold (see Lemma E.8 in the Appendix) on the driver pool size such that when the driver pool size is smaller than the threshold, the platform does not let CVs reposition. Otherwise, the platform recruits enough drivers and lets them reposition which results in a jump in the optimal wage as a function of the driver pool size.

Comparing Systems with and without AVs. We conclude this section by comparing systems with and without AVs. In the system without AVs, the problem reduces to deciding on the optimal wage w^* paid to drivers so as to maximize the platform profit. The platform solves the following optimization problem:

$$\max_{w} \quad (p - w) \sum_{i,j=1,2} s_{ij}^C,$$

subject to (1), (3), (5), (6), (8), (10) and (12) with $M = 0$, $F_k = 0$ for $k = 1, 2$ and η^C is a CV equilibrium repositioning strategy.

A similar system with only CVs is considered by Afèche et al. (2018). We complement their work here by allowing the platform to decide on the wage paid to drivers. Theorem 3 establishes the existence of the platform's optimal strategy for the system without AVs.

Theorem 3. *In the system without AVs, there exists an optimal strategy w^* for the platform.*[2]

We define the *driver productivity* as the fraction of CVs in service (i.e., $\frac{\sum_{i,j=1,2} s_{ij}^C}{N}$). We compare the platform profit, service level, average driver welfare and the driver productivity between systems with and without AVs in following Propositions.

Proposition 3. *Compared to the system without AVs, the platform profit and service level (weakly) increase, the average driver welfare (weakly) decreases, and the driver productivity either increases or decreases. Moreover, it is possible for a system without AVs to be Pareto-improving.*

[2]Similar to the result in Theorem 2, there exists a unique optimal strategy for the platform in most cases. In some boundary cases, the platform may have two different optimal strategies.

Proposition 3 states that the introduction of AVs always (weakly) benefits the platform and customers because the platform can use the AVs to serve more demand. Moreover, the introduction of AVs may not necessarily harm drivers and a Pareto-improving outcome can be achieved. To achieve a Pareto-improving outcome, the driver pool size and the AV purchase cost need to be moderate (the parameter range is specified in (73) in Appendix E.5) because (i) if the driver pool size or the AV purchase cost is too small, the platform has an incentive to replace some CVs with AVs after the introduction of AVs; (ii) if the driver pool size or the AV purchase cost is too large, systems with and without AVs would result in the same outcome. To be more specific, when the driver pool size and the AV purchase cost are moderate (within the parameter range specified in (73)), the platform relies only on CVs to serve all the demand at location 1 (CVs do not queue up or reposition) in the system without AVs. In the system with AVs, because the AV purchase cost is moderate, the platform has no incentive to replace some CVs with AVs to serve customers at location 1. Instead, the platform repositions AVs with probability 1 to serve otherwise unfulfilled demand due to the imbalanced demand. In other words, when the driver pool size and the AV purchase cost are moderate, the platform uses AVs to complement CVs. Therefore, the introduction of AVs benefits the platform and the customer but does not harm drivers.

With the introduction of AVs, the platform replaces some CVs with AVs. Per the result in Theorem 1, the platform either lets CVs serve customers all the time, or lets AVs reposition with probability 1. In the first case, the introduction of AVs weakly increases driver productivity. In the second case, the introduction of AVs reduces the amount of demand served by CVs in the high demand location and thus decreases driver productivity. Because drivers experience idle time or empty travel while not serving consumers, Proposition 3 also implies that the introduction of AVs either increases or decreases the idleness and empty travel experienced by drivers.

6 Admission Control

In this section, we extend our analysis to the case where the platform is allowed to exercise admission control. Admission control enables the platform to strategically reduce some of the demand based on its origin and destination. That is, the platform decides on the demand rate Λ_{ij} originating at location i and ending at location j such that

$$\Lambda_{ij} \leq \bar{\Lambda}_{ij} \quad \text{for } i,j = \{1,2\}. \tag{14}$$

In settings where demand is sensitive to price, this can be viewed as the platform choosing prices that are origin- and destination-dependent.

The platform now decides on the supply of AVs, the wage paid to drivers, the AV repositioning strategy, and the demand rate associated with each origin and destination. Therefore, the platform's problem can be stated as follows:

$$\max_{(M,w,\eta^A,\Lambda_{ij})} \Pi(M,w,\eta^A,\Lambda_{ij}) = (p-w)\sum_{i,j=1,2} s_{ij}^C + (p-c_a)\sum_{i,j=1,2} s_{ij}^A - c_a(r_{12}^A + r_{21}^A) - I \cdot M, \tag{15}$$

subject to (1)–(9), (12), (14) and η^C is a CV equilibrium repositioning strategy.

Due to the complexity of the problem, we can only solve problem (15) when the AV purchase cost is below a well specified threshold, namely, $I < \frac{2C_2}{L}(p-c_c) + c_c - c_a$, where C_2 is defined in (59) in Appendix D. In this case, because the AV purchase cost is relatively low compared to the cost of CVs, the platform has no incentive to reposition CVs and relies on AVs (if any) to reposition. Nevertheless, this case is useful in illustrating how the presence of AVs modifies admission control decisions and in contrasting our results to those obtained by Afeche et al. (2018) for a system with only CVs.

Theorem 4. *If $I < \frac{2C_2}{L}(p-c_c) + c_c - c_a$, there exists a unique optimal strategy for the platform. Moreover, if $I \geq \frac{p}{1+\frac{t_{12}}{t_{21}}} - c_c$, it is optimal for the platform to reject demand that originates in location 2 and is destined to location 1 until $\Lambda_{21} = \Lambda_{12} = \overline{\Lambda}_{12}$; otherwise, it is optimal for the platform not to reject any type of demand.*

Afèche et al. (2018) study admission control for a similar system to the one we consider here but with only CVs and with wages being fixed. They show that, under some conditions, it is optimal for the platform to reject demand origination from the low demand location and destined to the high demand location. This is because by doing so the platform can incentivize drivers to reposition more and, as a consequence, more demand at the high demand location can be fulfilled. (Note that the strategy of rejecting demand originating at the low demand location and ending in the high demand location exacerbates the imbalance in the system. Nevertheless, it turns out that, within a certain parameter range, such imbalance is profitable.)

To what extent the result in Afèche et al. (2018) holds in our case depends on the AV purchase cost. From Theorem 4, when the AV purchase cost is low, i.e., $I < \frac{2C_2}{L}(p-c_c)+c_c-c_a$, the platform never benefits from rejecting demand at the low demand location. Moreover, when the AV purchase cost is not too low, i.e., $\frac{p}{1+\frac{t_{12}}{t_{21}}} - c_a \leq I < \frac{2C_2}{L}(p-c_c)+c_c-c_a$, it is optimal for the platform to reject demand for trips from the high demand location to the low demand location. The reason is as follows. Rejecting demand for trips from location 2 to location 1 increases the demand which can be fulfilled without repositioning. This is because by rejecting some demand for trips from location 2 to location 1, vehicles that just completed service at location 2 are more likely to serve within-location demand (for trips from location 2 to itself) than cross-location demand (for trips from location 2 to location 1). Therefore, more demand at location 2 can be served by rejecting some demand from location 2 destined to location 1.

In the Appendix, we show that, when the AV purchase cost is such that $ap - c_a < I < \frac{2C_2}{L}(p - c_c) + c_c - c_a$, where a is defined in (67) and $a > \frac{1}{1+\frac{t_{12}}{t_{21}}}$, the platform does not reposition AVs (neither do CVs, by virtue of Theorem 2)). In this case, by rejecting demand for trips from location 2 to location 1, the platform can procure more AVs to serve more demand without repositioning and realize more profit until the system is balanced, i.e., $\Lambda_{12} = \Lambda_{21}$. When the AV purchase cost is low such that $I \leq ap - c_a$, the platform is willing to reposition AVs to serve otherwise unfulfilled demand due to demand imbalance. In this case, the platform faces the trade-off between serving more demand and lowering the repositioning and purchasing cost. Therefore, if the AV purchase cost is even lower such that $I \leq \frac{p}{1+\frac{t_{12}}{t_{21}}} - c_a$, the platform has no incentive to reject any type of demand. Otherwise, it is optimal for the platform to reject demand for trips from location 2 to location 1 until the system is balanced.

When the AV purchase cost is high, i.e., $I \geq \frac{2C_2}{L}(p - c_c) + c_c - c_a$, We observe from numerical experiments that it is possible for the platform to benefit from rejecting demand at the low demand location (as shown in Figure 2, the platform profit can decrease in Λ_{12}), a result consistent with Afèche et al. (2018).

To summarize, when the AV purchase cost is low, the platform relies on AVs for serving cus-

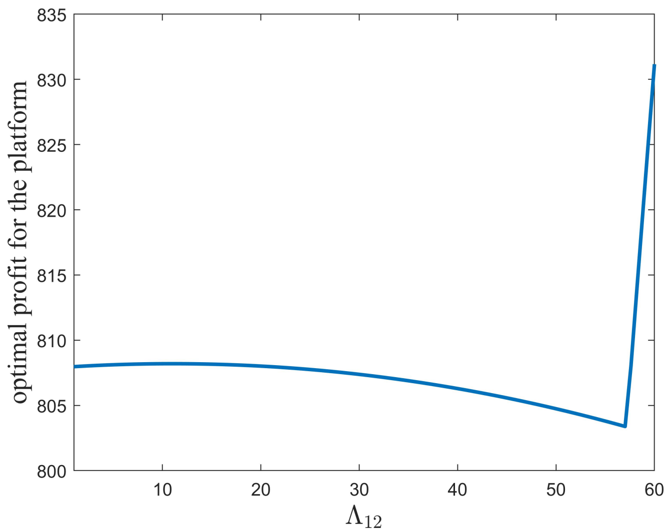


Figure 2: The optimal profit for the platform versus Λ_{12} when the system is CV-dominated. Model parameters: $\Lambda_{11} = 5$, $\Lambda_{21} = 100$, $\Lambda_{22} = 5$, $t_{11} = 1$, $t_{12} = 1$, $t_{21} = 5$, $t_{22} = 5$, $L = 1700$, $p = 3$, $I = 2.4$, $c_c = 0.2$, $c_a = 0.1$.

tomers. If the AV purchase cost is not excessively low, the platform has an incentive to achieve flow balancing through the curtailing of demand at the high demand location. When the AV purchase cost is high, the platform relies on CVs for serving customers. To incentivize the drivers to reposition away from the low demand location to the high demand location so as to serve more customers, it curtails the demand at the low demand location.

7 The Impact of Vehicle Assignment Priorities Policies

We have so far assumed that the platform does not differentiate between AVs and CVs when assigning vehicles to customer requests. In this section, we examine the impact of assignment policies that prioritize either AVs or CVs. One of our goals is to examine the extent to which such priority policies improve outcomes for the platform, the drivers, or both. For simplicity, we limit our discussion to the case where the platform does not exercise admission control.

7.1 The CV-Prioritized Policy

Under a CV-prioritized policy, CVs are given higher priority when assigning a vehicle to a customer. That is, if there are both AVs and CVs in the queue at a location, CVs are given priority in serving demand at that location. From the driver's perspective, the system under such a policy is identical to one without AVs. If, in steady state, $q_i^C > 0$, the expected delay experienced by AVs and CVs is given by $W_i^A = +\infty$ and $W_i^C = \frac{q_i^C}{\lambda_{i1}+\lambda_{i2}}$, respectively. Otherwise, $W_i^A = \frac{q_i^A}{(\Lambda_{i1}-\lambda_{i1}^C)+(\Lambda_{i2}-\lambda_{i2}^C)}$ and $W_i^C = 0$. Therefore, the demand at location i is assigned to AVs only if there are no CVs in the queue at location i. That is

$$(s_{i1}^A + s_{i2}^A)q_i^C = 0, \quad \text{for} \quad i \in \{1, 2\}. \tag{16}$$

The platform's problem can be stated as follows:

$$\max_{M,w,\eta^A} \quad \Pi(M, w, \eta^A)$$

$$\text{subject to} \quad (1)\text{–}(8),\ (10),\ (12),\ (16), \quad \eta^C \text{ is a CV equilibrium repositioning strategy, and}$$

$\Pi(M, w, \eta^A)$ is given in (13).

Theorem 5 characterizes the platform's optimal strategy.

Theorem 5. *Under the CV-prioritized policy, the optimal strategy for the platform and the corresponding outcomes are identical to those under the random priority policy.*

Theorem 5 indicates, perhaps surprisingly, that the optimal strategy and corresponding outcomes under the CV-prioritized policy are identical to those under the random priority policy obtained in Section 5. This is because, in accordance with Theorem 2, whenever CVs queue at location 1, the platform repositions AVs (if any) with probability 1 to meet otherwise unmet demand by CVs at location 2. In other words, under the random priority policy, the platform does not use AVs to compete with CVs for demand in the low-demand location, but rather repositions AVs to fulfill demand in the high-demand location, making the two policies equivalent in terms of outcomes.

7.2 The AV-Prioritized Policy

Under this policy, AVs are given higher priority when assigning a vehicle to a customer. That is, if there are both AVs and CVs in the queue at a location, AVs are given priority in serving demand at that location. From the driver's perspective, the system is equivalent to one in which the demand fulfilled by AVs is removed. If, in steady state, $q_i^A > 0$, the expected delay experienced by AVs and CVs is given by $W_i^A = \frac{q_i^A}{\Lambda_{i1}+\Lambda_{i2}}$ and $W_i^C = +\infty$ respectively. Otherwise, $W_i^A = 0$ and $W_i^C = \frac{q_i^C}{(\Lambda_{i1}-\lambda_{i1}^A)+(\Lambda_{i2}-\lambda_{i2}^A)}$. Therefore, the demand from location i is assigned to CVs only if no AVs are queued at location i. That is

$$(s_{i1}^C + s_{i2}^C)q_i^A = 0 \quad \text{for} \quad i \in \{1,2\}. \tag{17}$$

The platform's problem in this case can be stated as follows:

$$\max_{M,w,\eta^A} \quad \Pi(M,w,\eta^A)$$

$$\text{subject to} \quad (1)\text{–}(8),\ (10),\ (12), (17), \quad \eta^C \text{ is a CV equilibrium repositioning strategy, and}$$

$\Pi(M,w,\eta^A)$ is given in (13).

For simplicity, we consider in what follows the case where there is no travel cost for AVs and CVs (i.e. $c_c = c_a = 0$). [3] Note that the results we obtain (Theorem 6 and Proposition 4) still hold if we relaxed the assumption of no travel cost. However, due to the intractability of the profit function (e.g., lack of properties such as concavity or unimodality), we are not able to characterize the optimal strategy for the platform.

Theorem 6. *Under the AV-prioritized policy, there exists an optimal strategy for the platform.*[4] *Moreover, the corresponding capacity allocation (s, r, q) satisfies the following properties:*

(i) $q_2 = 0$*;*

(ii) either $\eta_1^C = 0$ *or* $\eta_1^C \vee a_2 = 1$*, where* a_2 *is the fraction of demand that is fulfilled at location* 2*.*

Theorem 6 shows that, under the AV-prioritized policy, vehicles do not queue up at the high demand location. Additionally, if CVs reposition, either they do so with probability 1 or the demand at location 2 is fully satisfied. This means that whenever the demand at location 2 (the high demand location) is not fully met and some CVs are repositioning, the platform has an incentive to encourage

[3]When $c_c = c_a = 0$, all of our results in the previous sections continue to hold without loss of optimality (i.e. there could exist other optimal strategies for the platform, but the platform profit, service level, and average driver welfare are the same).

[4]Within some parameter range, the platform has two different optimal strategies. Otherwise, the optimal strategy is unique. See Appendix G.2 for more details.

CVs to reposition more from the low demand location to the high demand location.

Details on the characterization of the optimal strategy can be found in Appendix G.2. The key takeaway is that by prioritizing AVs, the platform can indirectly reduce demand that can be fulfilled by CVs in the low demand location to incentivize drivers to reposition to the high demand location. The threshold for the queue length at location 1 above which CVs are willing to reposition (i.e., $\tilde{q}_1^*$) decreases as the demand fulfilled by AVs at location 1 increases. In the extreme case where all demand at location 1 is fulfilled by AVs, CVs reposition with probability 1 from location 1 to location 2.

As a result, in comparison to the equilibria under the CV-prioritized policy the AV-prioritized policy produces different equilibria types. To begin, recall from Section 5 that whenever CVs reposition, AVs (if any) reposition with probability 1. Under the AV-prioritized policy, we no longer have type V equilibria. Instead, we have two additional types of equilibria.

- Type VIII equilibria: the platform operates with both AVs and CVs, AVs reposition with a positive probability, and CVs reposition with probability 1.
- Type IX equilibria: the platform operates with both AVs and CVs, AVs do not reposition, and CVs reposition with a positive probability such that all the demand at location 1 and location 2 is fulfilled.

Furthermore, under the AV-prioritized policy, type VII equilibria encompasses only the case in which all demand at locations 1 and 2 is satisfied by CVs; otherwise, the platform can always increase its profit by purchasing additional AVs (which do not reposition) in order to force the CVs to reposition and fulfill more demand.

We conclude this section by examining the impact of introducing AVs on platform profit, customer service level, driver welfare, and driver productivity when the the vehicle assignment policy prioritizes AVs. The results are summarized in the following proposition.

Proposition 4. *Relative to a system without AVs, a system with AVs (weakly) increases platform profit and customer service level and either increases or decreases average driver welfare and driver productivity.*

Proposition 4 indicates that the introduction of AVs (when the vehicle assignment policy pri-

	Random priority policy & CV-prioritized policy	AV-prioritized policy
platform profit	(weakly) increases	(weakly) increases
Driver welfare	(weakly) decreases	either increases or decreases
Driver productivity	either increases or decreases	either increases or decreases
Service level	(weakly) increases	(weakly) increases

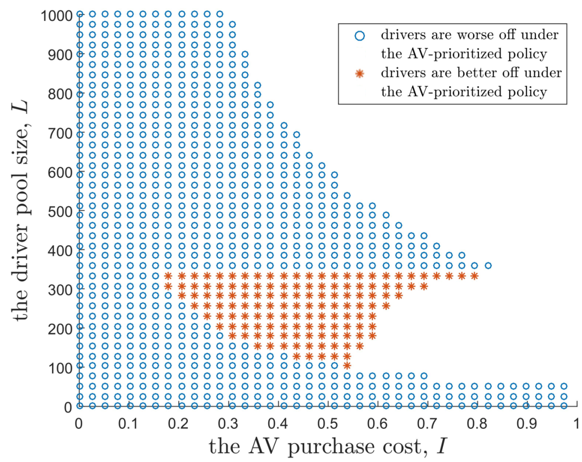
Table 1: Impact of introducing AVs on platform profit, driver welfare, driver's productivity and service level under various vehicle priority policy.

oritizes AVs) always benefits the platform and customers. Drivers can be more or less productive depending on the parameter values. Perhaps surprisingly, the result indicates that the AV-prioritized policy may not always be harmful to drivers and driver welfare may in fact improve as a result of the platform favoring AVs. Under the AV-prioritized policy, the platform can decide on the amount of demand that can be fulfilled by CVs at each location. Therefore, the platform has more control over the repositioning of CVs. Within a certain range of parameter values, the platform's interest (recruit more drivers and push them to reposition) is aligned with the interest of drivers as a whole (though not necessarily aligned with that of each individual driver). To be specific, drivers are better off when the driver pool size is moderate and the AV purchase cost is relatively high compared to the cost of CVs (as shown in Figure 3(a)). In this case, in the system without AVs, it is too expensive for the platform to incentivize CVs to reposition (because the driver pool size is moderate). After the introduction of AVs, the platform is able to push CVs to reposition with a much lower cost. Moreover, the platform is willing to rely more on CVs to serve customers because the AV purchase cost is relatively high.

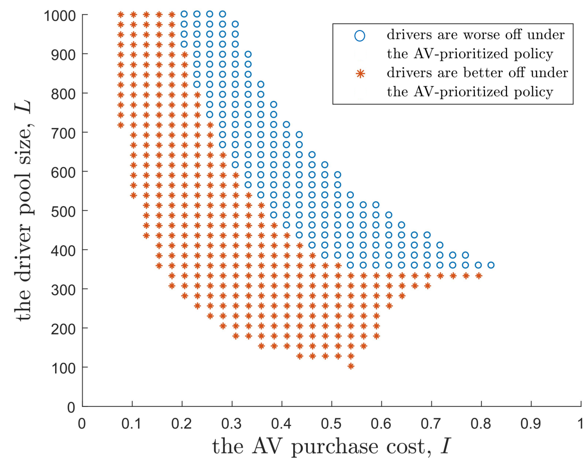
In table 1, we summarize the impact of the introduction of AVs on various outcomes (compared with a system without AVs) under the AV- and CV-prioritized policies.

7.3 AV-Prioritized Policy versus CV-Prioritized policy

In this section, we compare outcomes, with regard to platform profit, customer service level, driver welfare, and driver productivity under the AV- and CV-prioritized policies (assuming $c_c = c_a = 0$).



(a) Comparison result of driver welfare under the AV-prioritized policy and that in a system with only CVs.



(b) Comparison result of driver welfare under the AV-prioritized policy and the random priority policy.

Figure 3: Impact of vehicle priority policy on driver welfare. Model parameters: $\Lambda_{11} = 5$, $\Lambda_{12} = 10$, $\Lambda_{21} = 25$, $\Lambda_{22} = 5$, $t_{ij} = 1$ for $i, j \in \{1, 2\}$, $p = 1$, $c_c = 0$ and $c_a = 0$.

The results are summarized in the following proposition.

Proposition 5. *The AV-prioritized policy, relative to the CV-prioritized policy, (weakly) increases platform profit and customer service level, either increases or decreases average driver welfare, and (weakly) decreases driver productivity.*

Proposition 5 indicates that it is optimal for the platform to prioritize AVs when assigning vehicles to customers. Additionally, the AV-prioritized policy benefits customers with higher service levels. Similarly to Proposition 4, we also observe that the AV-prioritized policy may not always be harmful to drivers and driver welfare may in fact improve as a result of the platform's preference for AVs. This can happen when the labor pool size is moderate and AV purchase cost is relatively high (as shown in Figure 3(b)). The underlying explanation for these results are similarly to that of Proposition 4.

8 Concluding Comments

In this paper, we examined how the introduction of AVs may affect the decisions of a ride-hailing platform that chooses to operate with a mix of AVs and CVs, including decisions regarding capacity, spatial positioning of vehicles, admission control, and vehicle assignment policies. An important takeaway is that the introduction of AVs can significantly alter how the platform incentivizes human drivers and manages customer demand. In contrast to systems without AVs where the platform incentivizes drivers to reposition from the low demand location to the high demand location, we show that the platform may deploy AVs so as to reduce the need for the CVs to reposition. Consequently, the platform may no longer have an incentive to reject demand for trips from the low demand location to high demand location.

Another important takeaway is that the introduction of AVs may not necessarily be harmful to human drivers. A platform may deploy AVs to complement CVs (e.g., fulfill demand that may otherwise go unfulfilled or to avoid repositioning). Perhaps surprisingly, prioritizing AVs in fulfilling demand can be, under some conditions, beneficial to both the platform and the drivers (with more drivers deployed to fulfill more customers at the high demand location).

However, perhaps the most important takeaway is that the impact of AVs can be nuanced and crucially depends on both the fixed and variable costs of AVs and CVs. Even when the cost of AVs is relatively high (what is most likely to be the case in the short run), the deployment of AVs can be beneficial to both the platforms and the human drivers by creating operational efficiencies and expanding the demand that can be fulfilled. Moreover, the dependency of outcomes on the cost structures of AVs and CVs opens the door for possible regulatory interventions that can induce more socially desirable outcomes (e.g., a regulator may affect these costs via subsidies, taxes or the use of direct limits on the mix of AVs and CVs deployed). Finally, the results in this paper may also be useful in considering other applications where a mix of automation and humans are used.

This paper considers a rather stylized model of a spatial network, focusing on the dynamics of demand fulfillment and vehicle flows in a service region with pronounced differences between low demand and high demand zones. Although we expect the analysis to be considerably less tractable, a future research direction is to consider more general networks with multiple locations and varying distributions of demand across these locations (a hub and spoke network could be a first step toward to a more general treatment). It would also be useful to consider a mix of AVs and CVs in other contexts such as those of car sharing (services where customers drive themselves upon accessing a vehicle). In this case, AVs can be used to mitigate the need to manually reposition vehicles.

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Appendix

A Preamble

For ease of analysis, we reformulate the problem solved by the platform as a capacity allocation problem for each of the settings we considered. Following Afèche et al. (2018), we introduce the notion of *driver-incentive compatible capacity allocation*. When a capacity allocation (s, r, q) is driver-incentive compatible, no driver has an incentive to deviate from her strategy.

The overall Appendix is organized as follows. In Appendix B, we characterize the set of driver-incentive compatible capacity allocations. In Appendix C, we analyze the subgame and show that given any (M, N, η_A), there exists a unique driver-incentive compatible capacity allocation and the corresponding CV equilibrium repositioning strategy (Lemma 1). Appendix D provides proofs for systems with fixed supply of AVs and CVs (Theorem 1 and Proposition 1). Appendix E provides proofs for systems where the supply of AVs and CVs are endogenized (Lemma 2, Theorem 2, Theorem 3, Proposition 2 and Proposition 3). Appendix F provides proofs for systems with admission control (Theorem 4). Appendix G provides proofs for systems under the CV-prioritized policy and the AV-prioritized policy (Theorem 5, Theorem 6, Proposition 4 and Proposition 5).

We introduce additional notation that will be useful in various parts of the Appendix. Let $\Lambda_i = \Lambda_{i1} + \Lambda_{i2}$, $P_{ij} = \frac{\Lambda_{ij}}{\Lambda_i}$, and recall that $S_{ij} = \Lambda_{ij} t_{ij} = \Lambda_i P_{ij} t_{ij}$ for $i, j \in \{1, 2\}$. Let $q^A = q_1^A + q_2^A$ and $q^C = q_1^C + q_2^C$. For convenience, we use the shorthand DICCA to refer to the driver incentive compatible capacity allocation. For simplicity, under the setting where the platform does not exercise admission control (all the proofs except that for Theorem 4), we use Λ_{ij} to represent $\overline{\Lambda}_{ij}$.

For the convenience of the reader, we summarize important notation in Table 2.

Symbol	Meaning
M	Amount of AVs
N	Amount of CVs
t_{ij}	Travel time from location i to location j
$\overline{\Lambda}_{ij}$	Potential demand rate for trips from location i to location j
Λ_{ij}	Accepted demand rate for trips from location i to location j
Λ_i	Accepted demand rate at location i, i.e., $\Lambda_i = \Lambda_{i1} + \Lambda_{i2}$
λ_{ij}	Realized demand rate for trips from location i to location j
λ_{ij}^A	Rate of demand for trips from location i to location j served by AVs
λ_{ij}^C	Rate of demand for trips from location i to location j served by CVs
S_{ij}	Accepted amount of demand from location i to location j, i.e., $S_{ij} = \Lambda_{ij} t_{ij}$
s_{ij}	Realized amount of demand from location i to location j
s_{ij}^A	Amount of AVs in service from location i to location j
s_{ij}^C	Amount of CVs in service from location i to location j
ν_i^A	AV repositioning rate from location i
ν_i^C	CV repositioning rate from location i
r_{ij}	Amount of vehicles in repositioning from location i to location j
r_{ij}^A	Amount of AVs in repositioning from location i to location j
r_{ij}^C	Amount of CVs in repositioning from location i to location j
q_i	Amount of vehicles waiting at location i
q_i^A	Amount of AVs waiting at location i
q_i^C	Amount of CVs waiting at location i
W_i^A	Expected waiting time at location i for AVs
W_i^C	Expected waiting time at location i for CVs
W_i^C	Expected waiting time at location i for CVs
$\eta_i^A \in [0,1]$	AV repositioning probability from location i
$\eta_i^C \in [0,1]$	CV repositioning probability from location i
p	Price to charge customers per unit time of service
w	Wage to pay workers per unit time of service
c_a	Travel cost (per unit time) for AVs
c_c	Travel cost (per unit time) for CVs
I	AV purchase cost
L	Driver pool size
$\overline{w}$	The maximal driver opportunity cost ($\overline{w} = p - c_c$)
a_i	Service Level (fraction of accepted demand fulfilled) at location i, i.e., $a_i = \frac{\lambda_{ij}}{\Lambda_{ij}}$
F_i	Fraction of demand fulfulled by AVs at location i, i.e., $F_i = \frac{s_{ij}^A}{s_{ij}}$
P_{ij}	Fraction of demand generated at location i which is destined to location j, i.e., $P_{ij} = \frac{\Lambda_{ij}}{\Lambda_i}$
R_{imb}	Imbalanced amount of demand, i.e., $R_{imb} = \frac{t_{12}}{t_{21}} S_{21} - S_{12}$
C_1	Minimum amount of vehicles needed to cover all the repositioning of R_{imb}
C_2	Minimum amount of vehicles needed to serve all the demand at location 1
S	Total amount of accepted demand, i.e., $S = \sum_{i,j=1,2} S_{ij}$
a	Productivity for vehicles which reposition from location 1 to location 2 with probability 1

Table 2: Table of Important Notation.

B Characterization of DICCA

In this section, we characterize the set of DICCA. In Appendix B.1, we characterize the best response of a single driver given (s, r, q). We then obtain the set of DICCA in Appendix B.2

B.1 The Best Strategy of a Single Driver

Recall that drivers serving customers earn $w - c_c$ per unit time and incur a cost c_c per unit time when repositioning. Therefore, the earnings of a driver depend on the fraction of time she spends serving customers, repositioning, and queueing. For any driver, given her strategy $\eta = (\eta_1, \eta_2)$ and the system capacity allocation (s, r, q), her expected earning $\overline{\pi}(\eta, s, r, q)$ can be obtained using the Renewal Reward Theorem (Ross (1996)). Without loss of generality, we define the renewal cycle as the time between completing consecutive service at location 1. Then

$$\overline{\pi}(\eta, s, r, q) = \frac{(w - c_c)T^s(\eta, s, r, q) - c_c T^r(\eta, s, r, q)}{T(\eta, s, r, q)}, \tag{18}$$

where $T(\eta, s, r, q)$ is the expected time of a renewal cycle, $T^s(\eta, s, r, q)$ and $T^r(\eta, s, r, q)$ are the expected time the driver spends on serving customers and repositioning in each renewal cycle.

Let x_1 denote the expected time the driver experiences between starting repositioning from location 2 to location 1 and completing a service at location 1. Let x_2 denote the expected time the driver experiences between starting queueing at location 2 and completing a service at location 1. Recall that we use W_i^C to denote the steady-state delay experienced by a driver waiting to be matched with customers at location i. Then, $T(\eta, s, r, q)$, x_1 and x_2 satisfy

$$\begin{aligned}
&T(\eta, s, r, q) = (1 - \eta_1)\big[W_1^C + P_{11}t_{11} + P_{12}(t_{12} + \eta_2 x_1 + (1 - \eta_2)x_2)\big] + \eta_1\big[t_{12} + x_2\big],\\
&x_1 = t_{21} + W_1^C + P_{11}t_{11} + P_{12}[t_{12} + \eta_2 x_1 + (1 - \eta_2)x_2], \quad \text{and}\\
&x_2 = W_2^C + P_{21}t_{21} + P_{22}[t_{22} + \eta_2 x_1 + (1 - \eta_2)x_2].
\end{aligned}$$

Let x_1^s and x_1^r denote the expected time the driver spends on serving customers and repositioning between starting repositioning from location 2 to location 1 and completing a service at location 1 respectively. Let x_2^s and x_2^r denote the expected time the driver spends on serving customers and repositioning between starting queueing at location 2 and completing a service at location 1 respectively. Then, $T^s(\eta, s, r, q)$, x_1^s and x_2^s satisfy

$$
\begin{aligned}
T^s(\eta, s, r, q) &= (1-\eta_1)\big[P_{11}t_{11} + P_{12}(t_{12} + \eta_2 x_1^s + (1-\eta_2)x_2^s)\big] + \eta_1 x_2^s,\\
x_1^s &= P_{11}t_{11} + P_{12}\big[t_{12} + \eta_2 x_1^s + (1-\eta_2)x_2^s\big], \quad \text{and}\\
x_2^s &= P_{21}t_{21} + P_{22}\big[t_{22} + \eta_2 x_1^s + (1-\eta_2)x_2^s\big].
\end{aligned}
$$

Similarly, $T^r(\eta, s, r, q)$, x_1^r and x_2^r satisfy

$$
\begin{aligned}
T^r(\eta, s, r, q) &= (1-\eta_1)P_{12}(\eta_2 x_1^r + (1-\eta_2)x_2^r) + \eta_1\big[t_{12} + x_2^r\big],\\
x_1^r &= t_{21} + P_{12}\big[\eta_2 x_1^r + (1-\eta_2)x_2^r\big], \quad \text{and}\\
x_2^r &= P_{22}\big[\eta_2 x_1^r + (1-\eta_2)x_2^r\big].
\end{aligned}
$$

Because we assume that $\Lambda_{ij} > 0$ for $i, j \in \{1, 2\}$, the systems of equations characterized above admit unique solutions for $T(\eta, s, r, q)$, $T^s(\eta, s, r, q)$ and $T^r(\eta, s, r, q)$.

Taking partial derivatives with respect to $\overline{\pi}(\eta, s, r, q)$ (which is given in (18)), we can obtain that $\frac{\partial\overline{\pi}(\eta,s,r,q)}{\partial\eta_1} = [-1+\eta_2(P_{12}-P_{22})+P_{22}]\frac{A_{\eta_1}(\eta_1,\eta_2)}{(B_{\eta_1})^2}$ and $\frac{\partial\overline{\pi}(\eta,s,r,q)}{\partial\eta_2} = [-1+\eta_1(P_{21}-P_{11})+P_{11}]\frac{A_{\eta_2}(\eta_1,\eta_2)}{(B_{\eta_2})^2}$, where $-1+\eta_2(P_{12}-P_{22})+P_{22} < 0$, $-1+\eta_1(P_{21}-P_{11})+P_{11} < 0$,

$$
\begin{aligned}
&A_{\eta_1}(\eta_1, \eta_2)\\
&= c_c\big[\underbrace{-P_{11}t_{11}W_2^C + P_{22}t_{22}W_1^C}_{a_1} + P_{21}t_{21}W_1^C + P_{21}t_{12}W_1^C\big] + c_c\big[\eta_2(P_{22}t_{21}W_1^C + P_{22}t_{12}W_1^C - P_{12}t_{21}W_2^C - P_{12}t_{12}W_2^C)\big]\\
&+ w\big[P_{21}P_{12}t_{21}t_{12} + P_{12}P_{21}t_{12}^2 + P_{11}P_{21}t_{11}t_{12} + P_{22}P_{12}t_{22}t_{12}\big] + w\big[\underbrace{P_{12}t_{12}W_2^C + P_{11}t_{11}W_2^C - P_{21}t_{21}W_1^C - P_{22}t_{22}W_1^C}_{b_1}\big]\\
&+ w\big[\eta_2(-P_{22}P_{12}t_{22}t_{21} + P_{22}P_{12}t_{12}t_{21} + P_{22}P_{11}t_{11}t_{21} - P_{21}P_{12}t_{21}^2)\big]\\
&+ w\big[\eta_2(-P_{22}P_{12}t_{22}t_{12} + P_{22}P_{12}t_{12}^2 + P_{22}P_{11}t_{11}t_{12} - P_{21}P_{12}t_{21}t_{12})\big], \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&A_{\eta_2}(\eta_1,\eta_2)\\
&= c_c\big[\underbrace{-P_{22}t_{22}W_1^C + P_{11}t_{11}W_2^C}_{a_2} + P_{12}t_{12}W_2^C + P_{12}t_{21}W_2^C\big] + c\big[\eta_1(P_{11}t_{12}W_2^C + P_{11}t_{21}W_2^C - P_{21}t_{12}W_1^C - P_{21}t_{21}W_1^C)\big]\\
&+ w\big[P_{12}P_{21}t_{12}t_{21} + P_{12}P_{21}t_{21}^2 + P_{22}P_{12}t_{22}t_{21} + P_{11}P_{21}t_{11}t_{21}\big] + w\big[\underbrace{P_{21}t_{21}W_1^C + P_{22}t_{22}W_1^C - P_{12}t_{12}W_2^C - P_{11}t_{11}W_2^C}_{b_2}\big]\\
&+ w\big[\eta_1(-P_{11}P_{21}t_{11}t_{12} + P_{11}P_{21}t_{21}t_{12} + P_{11}P_{22}t_{22}t_{12} - P_{12}P_{21}t_{12}^2)\big]\\
&+ w\big[\eta_1(-P_{11}P_{21}t_{11}t_{21} + P_{11}P_{21}t_{21}^2 + P_{11}P_{22}t_{22}t_{21} - P_{12}P_{21}t_{12}t_{21})\big], \qquad (20)
\end{aligned}
$$

and B_{η_1}, B_{η_2} are some non-zero constants. Because η is defined on a compact set and $\overline{\pi}(\eta, s, r, q)$ is a continuous function in η, the maximum can be attained by the Extreme Value Theorem.

By some algebra, we can obtain that $A_{\eta_1}(0,0) + A_{\eta_2}(0,0) > 0$. Therefore, there exists $i \in \{1,2\}$ such that $A_{\eta_i}(0,0) > 0$. Without loss of generality, we assume $A_{\eta_2}(0,0) > 0$ and thus $\frac{\partial\pi}{\partial\eta_2}_+(0,0) < 0$, where we use a subscript + to denote the right-hand derivative because $(0,0)$ is a boundary point. For simplicity, we omit the subscript + in the rest of the proof. We then show that any optimal strategy $\eta^* = (\eta_1^*, \eta_2^*)$ for the driver must satisfy that $\eta_2^* = 0$. We first notice that $\frac{\partial\pi}{\partial\eta_2}(0,\eta_2) = \frac{\partial\pi}{\partial\eta_2}(0,0)$ as $\frac{\partial\pi}{\partial\eta_i}$ is independent of η_i. Therefore, $\pi(0,\eta_2) < \pi(0,0)$ for $\eta_2 \in (0,1]$. Suppose there exists $\tilde{\eta}_1 \in (0,1)$ and $\tilde{\eta}_2 \in (0,1)$ such that $\pi(\tilde{\eta}_1, \tilde{\eta}_2)$ achieves the maximum. Then we must have $\frac{\partial\pi}{\partial\eta_1}(\tilde{\eta}_1, \tilde{\eta}_2) = 0$. It follows that $\pi(0, \tilde{\eta}_2) = \pi(\tilde{\eta}_1, \tilde{\eta}_2)$, and thus $\pi(\tilde{\eta}_1, \tilde{\eta}_2)$ cannot be the maximum. We then notice that $A_{\eta_1}(1,1) + A_{\eta_2}(1,1) > 0$. Therefore, there exists $i \in \{1,2\}$ such that $\frac{\partial\pi}{\partial\eta_i}(1,1) < 0$, which implies that $\eta = (1,1)$ is dominated by either $(1,0)$ or $(0,1)$. Moreover, because neither $(0,1)$ nor $(1,1)$ is optimal, $(\eta_1, 1)$ cannot be optimal for any $\eta_1 \in (0,1)$ because $\pi(\eta_1, 1)$ is monotone in η_1 (as $\frac{\partial\pi}{\partial\eta_i}$ does not depend on η_i). It remains to show that $(1, \eta_2)$ cannot be an optimal point for any $\eta_2 \in (0,1)$. Similarly, because $\pi(1, \eta_1)$ is monotone in η_2, $(1, \eta_2)$ is weakly dominated by either $(1,0)$ or $(1,1)$, and thus it cannot be an optimal point. Therefore, we have shown that any optimal strategy η^* must satisfy that $\eta_2^* = 0$.

We define a_i and b_i for $i = 1,2$ as shown in (19)–(20). We can obtain that $A_{\eta_1}(0,0) = c_c a_1 + wb_1 + CT_1$ and $A_{\eta_2}(0,0) = c_c a_2 + wb_2 + CT_2$, where $CT_i > 0$ denotes the summation of all the terms in $A_{\eta_i}(0,0)$ other than $c_c a_i + w b_i$. Define the functions

$$g_1(s,r,q) = (w - c_c)(s_{22}q_1 - s_{11}q_2) + w(s_{21}q_1 - s_{12}q_2) \text{ and} \tag{21}$$

$$g_2(s,r,q) = (w - c_c)(s_{11}q_2 - s_{22}q_1) + w(s_{12}q_2 - s_{21}q_1). \tag{22}$$

Because $c_c(a_1 + a_2) + w(b_1 + b_2) = 0$, we have either $c_c a_1 + w b_1 \geq 0$, which is equivalent to $g_1(s,r,q) \leq 0$, or $c_c a_2 + w b_2 \geq 0$ which is equivalent to $g_2(s,r,q) \leq 0$. Without loss of generality, we assume that $c_c a_2 + w b_2 \geq 0$. Then we must have $A_{\eta_2}(0,0) > 0$, which implies that any optimal strategy for the driver must satisfy that $\eta_2^* = 0$. Moreover, because $A_{\eta_1}(0,0) \leq 0$ implies that $\frac{\partial \pi}{\partial \eta_i}(\eta_1, 0) \geq 0$, we have (i) $\eta_1^* = 0$ if $A_{\eta_1}(0,0) > 0$, (ii) $\eta_1^* \in [0,1]$ if $A_{\eta_1}(0,0) = 0$, and (iii) $\eta_1^* = 1$ if $A_{\eta_1}(0,0) < 0$, where

$$\begin{aligned} A_{\eta_1}(0,0) = w[P_{12}t_{12}(P_{21}t_{21} + P_{22}t_{22}) + P_{21}t_{12}(P_{12}t_{12} + P_{11}t_{11})] + \frac{w(P_{12}t_{12} + P_{11}t_{11}) - c_c P_{11}t_{11}}{\lambda_{21} + \lambda_{22}} q_2 \\ + \frac{c_c(P_{22}t_{22} + P_{21}t_{21} + P_{21}t_{12}) - w(P_{21}t_{21} + P_{22}t_{22})}{\lambda_{11} + \lambda_{12}} q_1. \end{aligned}$$

By simplifying the above expression of $A_{\eta_1}(0,0)$, we can obtain Lemma B.1 below, which characterizes the best response of a single driver given the capacity allocation of the whose system (s,r,q).

Lemma B.1. *Given any capacity allocation (s,r,q) of the network, if $g_i(s,r,q) \geq 0$, where $i \neq j \in \{1,2\}$, the optimal repositioning strategy $\eta^* = (\eta_1^*, \eta_2^*)$ for a driver satisfies*

- $\eta^* = (0,0)$ *if* $q_i < q_i^* + k_i^* q_j$;
- $\eta_j^* = 0$ *and* $\eta_i^* \in [0,1]$ *if* $q_i = q_i^* + k_i^* q_j$; *and*
- $\eta_j^* = 0$ *and* $\eta_i^* = 1$ *otherwise;*

where

$$q_i^* = \frac{s_{ij}(s_{ji} + s_{jj}) + s_{ji}\frac{t_{ij}}{t_{ji}}(s_{ii} + s_{ij})}{(s_{ji} + s_{jj}) - \frac{c_c}{w}(s_{jj} + s_{ji} + s_{ji}\frac{t_{ij}}{t_{ji}})}, \quad \text{and} \quad k_i^* = \frac{(s_{ij} + s_{ii}) - \frac{c_c}{w}s_{ii}}{(s_{ji} + s_{jj}) - \frac{c_c}{w}(s_{jj} + s_{ji} + s_{ji}\frac{t_{ij}}{t_{ji}})}. \tag{23}$$

Lemma B.1 is an analogue to Lemma 1 and Proposition 2 in Afèche et al. (2018). This result tells us that a driver's optimal repositioning strategy only depends on the capacity allocation of the system.

B.2 The Set of DICCA

Because we focus on the case where drivers adopt symmetric repositioning strategies, by Lemma B.1, we can obtain the set of DICCA per Proposition B.1 below.

Proposition B.1. *The capacity allocation (s, r, q) is driver-incentive compatible if*

$$(s, r, q) \in D = D_1 \cup D_2,$$

where

$$D_i := \left((s,r,q) \geq 0 : g_i(s,r,q) \geq 0, r_{ji}^C = 0, q_i \begin{cases} \leq q_i^* + k_i^* q_j & \text{if } r_{ij}^C = 0 \\ = q_i^* + k_i^* q_j & \text{if } \frac{r_{ij}^C}{t_{ij}} \in (0, \frac{s_{ii}^C}{t_{ii}} + \frac{s_{ji}^C}{t_{ji}}) \\ \geq q_i^* + k_i^* q_j & \text{if } s_{ii}^C = s_{ij}^C = 0 \text{ and } \frac{r_{ij}^C}{t_{ij}} = \frac{s_{ji}^C}{t_{ji}} \end{cases} \right),$$

$g_1(s, r, q)$ and $g_2(s, r, q)$ are given in (21)–(22)*, and q_i^*, k_i^* are given in* (23).

C Subgame Analysis

In this section, we conduct subgame analysis. In Appendix C.1, we investigate the capacity allocation of the system given η^A, η^C, $M > 0$ and $N > 0$ (the result is summarized in Lemma C.1 below). In Appendix C.2, we prove Lemma 1.

Lemma C.1. *Given η^A, η^C, $M > 0$, $N > 0$,* (1)–(9) *admit a unique solution for (s, r, q) if the parameters do not fall in a set E defined in Definition 1. Otherwise,* (1)–(9) *admit a unique solution for (s, r), while q_1 and q_2 can be any non-negative numbers such that $q_1 + q_2 = M + N - \sum_{i,j=1,2} s_{ij} - \sum_{i \neq j=1,2} r_{ij}$. Moreover, (s, r), q^A and q^C are continuous in η^A, η^C, M and N.*

C.1 Proof of Lemma C.1

We first introduce additional notation. Let

$$X = \begin{bmatrix} \Lambda_2 P_{21}(1-\eta_1^A) + \Lambda_2 P_{22}\eta_2^A \\ \Lambda_1 - \Lambda_1 P_{11}(1-\eta_1^A) - \Lambda_1 P_{12}\eta_2^A \end{bmatrix} \text{ and } Y = \begin{bmatrix} \Lambda_2 P_{21}(1-\eta_1^C) + \Lambda_2 P_{22}\eta_2^C \\ \Lambda_1 - \Lambda_1 P_{11}(1-\eta_1^C) - \Lambda_1 P_{12}\eta_2^C \end{bmatrix}. \tag{24}$$

Let X_i and Y_i denote the i-th element of X and Y respectively for $i \in \{1,2\}$.

Definition 1. *Define*

$$E = \{(\eta^A, \eta^C, \Lambda_{ij}, t_{ij}, M, N) \mid (X_1 = X_2, Y_1 = Y_2) \cap (\frac{M}{\sum_{k=1,2} A_k X_k} X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_1 > 1)\},$$

where A_k and B_k are defined in (30) *and* (31).

We first consider the case where (a) X_1, X_2, Y_1, Y_2 are all positive, and (b) $X_1 \neq X_2$ or $Y_1 \neq Y_2$. By (1)–(5), for $i \in \{1,2\}$ and $j \neq i$, we have

$$\Lambda_i a_i F_i = \eta_j^A \sum_{k=1,2} \Lambda_k P_{kj} a_k F_k + (1-\eta_i^A) \sum_{k=1,2} \Lambda_k P_{ki} a_k F_k \quad \text{and} \tag{25}$$

$$\Lambda_i a_i (1-F_i) = \eta_j^C \sum_{k=1,2} \Lambda_k P_{kj} a_k (1-F_k) + (1-\eta_i^C) \sum_{k=1,2} \Lambda_k P_{ki} a_k (1-F_k). \tag{26}$$

Let $D = diag(\Lambda_1, \Lambda_2)$, which is a diagonal matrix with Λ_1 and Λ_2 as its diagonal entries. Let (aF) be a 2-by-1 matrix with $(aF)_i = a_i F_i$. Let $(a(1-F))$ be a 2-by-1 matrix with $(a(1-F))_i = a_i(1-F_i)$. Let

$$B^A = \begin{bmatrix} \eta_2^A P_{12} + (1-\eta_1^A) P_{11}, & \eta_2^A P_{22} + (1-\eta_1^A) P_{21} \\ \eta_1^A P_{11} + (1-\eta_2^A) P_{12}, & \eta_1^A P_{21} + (1-\eta_2^A) P_{22} \end{bmatrix} \quad \text{and}$$

$$B^C = \begin{bmatrix} \eta_2^C P_{12} + (1-\eta_1^C) P_{11}, & \eta_2^C P_{22} + (1-\eta_1^C) P_{21} \\ \eta_1^C P_{11} + (1-\eta_2^C) P_{12}, & \eta_1^C P_{21} + (1-\eta_2^C) P_{22} \end{bmatrix}.$$

Then (25)–(26) can be rewritten in the following matrix form:

$$(I - B^A)D(aF) = 0 \quad \text{and} \tag{27}$$

$$(I - B^C)D(a(1-F)) = 0, \tag{28}$$

where I is a 2-by-2 identity matrix. By some algebra, we obtain that (27) admits a unique solution to (aF) up to some scale γ, and (28) admits a unique solution to $(a(1-F))$ up to some scale ζ. That is,

$$(aF) = \gamma X \quad \text{and} \quad (a(1-F)) = \zeta Y,$$

where X and Y are given by (24).

It remains to solve γ, ζ, q_i^A and q_i^C for $i \in \{1, 2\}$. By the definition of F_i and a_i (recall that a_i denotes the fraction of accepted demand fulfilled at location i), we require

$$\gamma \geq 0, \quad \zeta \geq 0 \quad \text{and} \quad \gamma X_i + \zeta Y_i = a_i \leq 1 \quad \text{for } i = 1, 2. \tag{29}$$

By (7)–(8), we have

$$\sum_{i,j=1,2} \Lambda_i P_{ij} t_{ij} a_i F_i + \eta_1^A t_{12} \sum_{i=1,2} \Lambda_i P_{i1} a_i F_i + \eta_2^A t_{21} \sum_{i=1,2} \Lambda_i P_{i2} a_i F_i + \sum_{i=1,2} q_i^A = M \quad \text{and}$$
$$\sum_{i,j=1,2} \Lambda_i P_{ij} t_{ij} a_i (1-F_i) + \eta_1^C t_{12} \sum_{i=1,2} \Lambda_i P_{i1} a_i (1-F_i) + \eta_2^C t_{21} \sum_{i=1,2} \Lambda_i P_{i2} a_2 (1-F_i) + \sum_{i=1,2} q_i^C = N.$$

Let

$$A = \begin{bmatrix} \Lambda_1 P_{11} t_{11} + \Lambda_1 P_{12} t_{12} + \eta_1^A \Lambda_1 P_{11} t_{12} + \eta_2^A \Lambda_1 P_{12} t_{21} \\ \Lambda_2 P_{21} t_{21} + \Lambda_2 P_{22} t_{22} + \eta_1^A \Lambda_2 P_{21} t_{12} + \eta_2^A \Lambda_2 P_{22} t_{21} \end{bmatrix} \quad \text{and} \tag{30}$$

$$B = \begin{bmatrix} \Lambda_1 P_{11} t_{11} + \Lambda_1 P_{12} t_{12} + \eta_1^C \Lambda_1 P_{11} t_{12} + \eta_2^C \Lambda_1 P_{12} t_{21} \\ \Lambda_2 P_{21} t_{21} + \Lambda_2 P_{22} t_{22} + \eta_1^C \Lambda_2 P_{21} t_{12} + \eta_2^C \Lambda_2 P_{22} t_{21} \end{bmatrix}. \tag{31}$$

Let A_i and B_i denote the i-th element of A and B respectively for $i \in \{1, 2\}$. Then (7)–(8) can be rewritten as:

$$\gamma \sum_{i=1,2} A_i X_i + \sum_{i=1,2} q_i^A = M, \quad \text{and} \tag{32}$$

$$\zeta \sum_{i=1,2} B_i Y_i + \sum_{i=1,2} q_i^C = N. \tag{33}$$

There are three types of capacity allocation depending on the queue length at each location. Type A: $q_1 = q_2 = 0$. Type B: $q_i > 0$ and $q_j = 0$ for $i \neq j$. Type C: $q_1 > 0$ and $q_2 > 0$. Therefore, we consider the following cases (parameter ranges) which lead to different types of capacity allocations.

Case (i): $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i \leq 1$, for $i = 1, 2$. By (32)–(33), we can see that $\frac{M}{\sum_{k=1,2} A_k X_k}$ and $\frac{N}{\sum_{k=1,2} B_k Y_k}$ are upper bounds for γ and ζ respectively. In this case, we can obtain that

$$\gamma = \frac{M}{\sum_{i=1,2} A_i X_i}, \quad \zeta = \frac{N}{\sum_{i=1,2} B_i Y_i}, \quad q_1 = 0, \quad \text{and} \quad q_2 = 0.$$

Otherwise, any solution to (32)–(33) must satisfy $q_1 > 0$ or $q_2 > 0$, which contradicts (6).

Case (ii): $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i > 1$ and $\frac{M}{\sum_{k=1,2} A_k X_k} X_j + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_j \leq 1$, where $i \neq j$. In this case, we shall show that the demand at location i is fully fulfilled (i.e., $a_i = 1$), while the demand at location j may not be fully served, i.e., $a_j \leq 1$. By (9) and (32)–(33), we have the following two sets of constraints (if $i = 1$, we consider (34)–(35) and we consider (36)–(37) otherwise):

$$\gamma X_1 + \zeta Y_1 = 1, \tag{34}$$

$$\frac{M - \gamma \sum_{k=1,2} A_k X_k}{\gamma X_1} = \frac{N - \zeta \sum_{i=1,2} B_k Y_k}{\zeta Y_1}; \tag{35}$$

$$\gamma X_2 + \zeta Y_2 = 1, \tag{36}$$

$$\frac{M - \gamma \sum_{k=1,2} A_k X_k}{\gamma X_2} = \frac{N - \zeta \sum_{i=1,2} B_k Y_k}{\zeta Y_2}. \tag{37}$$

In Lemma C.2, we show that (34)–(35) admit a unique solution of (γ, ζ) which satisfies (29). A similar result can be obtained for (36)–(37), which we omit here for simplicity.

Lemma C.2. *If X_1, X_2, Y_1, Y_2 are all positive and $\frac{M}{\sum_{k=1,2} A_k X_k} X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_1 > 1$, (34)–(35)*

admit a unique solution of (γ, ζ) *such that* $\gamma \leq \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta \leq \frac{N}{\sum_{i=1,2} B_i Y_i}$, *and* (29) *is satisfied. Moreover, the solution is continuous in* η^A, η^C, M *and* N.

Proof of Lemma C.2. Notice that $A \cdot X = \sum_{k=1,2} A_k X_k$ and $B \cdot Y = \sum_{k=1,2} B_k Y_k$, where " $\cdot$ " represents the dot product between vectors. If $\frac{Y_1^2}{X_1} A \cdot X \neq Y_1 B \cdot Y$, (34)–(35) admit two sets of solutions (ζ, γ) and (ζ', γ'), where

$$\zeta = \frac{-(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X) + \sqrt{(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X)^2 + 4N(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)}}{2(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)},$$

$$\gamma = \frac{1}{X_1} - \zeta \frac{Y_1}{X_1},$$

$$\zeta' = \frac{-(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X) - \sqrt{(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X)^2 + 4N(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)}}{2(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)},$$

$$\gamma' = \frac{1}{X_1} - \zeta' \frac{Y_1}{X_1}.$$

By some algebra, we can show that $\gamma \leq \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta \leq \frac{N}{\sum_{i=1,2} B_i Y_i}$ and (γ, ζ) satisfies (29). We then check (γ', ζ'). If $\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y > 0$, then $\zeta' < 0$, which contradicts to (29). Otherwise, we have

$$\begin{aligned}
&(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X)^2 + 4N(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y) \\
&\geq (MY_1 + NY_1)^2 + (B \cdot Y - \frac{Y_1}{X_1} A \cdot X)^2 + 2MY_1(B \cdot Y - \frac{Y_1}{X_1} A \cdot X) - 2(M+N)Y_1(B \cdot Y - \frac{Y_1}{X_1} A \cdot X) \\
&\geq (NY_1 - B \cdot Y + MY_1 + \frac{Y_1}{X_1} A \cdot X)^2.
\end{aligned}$$

It follows that $\zeta' > \frac{-(NY_1 + B \cdot Y + MY_1 - \frac{Y_1}{X_1} A \cdot X) - \sqrt{(NY_1 - B \cdot Y + MY_1 + \frac{Y_1}{X_1} A \cdot X)^2}}{2(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)}$. If $NY_1 + MY_1 \geq B \cdot Y - \frac{Y_1}{X_1} A \cdot X$, we have $\zeta' > \frac{-2(NY_1 + MY_1)}{2(\frac{Y_1^2}{X_1} A \cdot X - Y_1 B \cdot Y)}$ which contradicts (29). Otherwise, we can obtain that $\zeta' \geq \frac{1}{Y_1}$, which also contradicts (29). Therefore, (34)–(35) admit a unique solution which satisfies (29).

If $\frac{Y_1^2}{X_1} A \cdot X = Y_1 B \cdot Y$, (34)–(35) admit a unique solution such that

$$\zeta = \frac{N}{Y_1(M+N)} \quad \text{and} \quad \gamma = \frac{M}{X_1(M+N)},$$

which satisfies (29).

It remains to show that the solution is continuous in η^A, η^C, M and N. It suffices to check those points satisfying $\frac{Y_1^2}{X_1} A \cdot X = Y_1 B \cdot Y$, which are indeed continuous by checking the limits. $\square$

By (32)–(33) and Lemma C.2, we can obtain that if $\frac{M}{\sum_{k=1,2} A_k X_k} X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_1 > 1$, (γ, ζ) is the unique solution to (34)–(35) which satisfies (29), $q_1^A = M - \gamma \sum_{i=1,2} A_i X_i$, $q_2^A = 0$, $q_1^C = N - \zeta \sum_{i=1,2} B_i Y_i$ and $q_2^C = 0$. Otherwise, (γ, ζ) is the unique solution to (36)–(37) which satisfies (29), $q_1^A = 0$, $q_2^A = M - \gamma \sum_{i=1,2} A_i X_i$, $q_1^C = 0$ and $q_2^C = N - \zeta \sum_{i=1,2} B_i Y_i$.

Case (iii): $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i > 1$ for $i = 1, 2$. Let (γ_1, ζ_1) be the solution to (34)–(35) and (γ_2, ζ_2) the solution to (36)–(37). We first show that (γ_1, ζ_1) and (γ_2, ζ_2) do not satisfy the following constraints:

$$\gamma_1 X_1 + \zeta_1 Y_1 = 1, \tag{38}$$

$$\gamma_1 X_2 + \zeta_1 Y_2 \leq 1, \tag{39}$$

$$\gamma_2 X_2 + \zeta_2 Y_2 = 1, \tag{40}$$

$$\gamma_2 X_1 + \zeta_2 Y_1 \leq 1. \tag{41}$$

By doing so, we exclude the possibility that (1)–(9) adopt multiple solutions of (s, r, q). Recall that we assume $X_1 \neq X_2$ or $Y_1 \neq Y_2$. Without loss of generality, suppose $X_1 \neq X_2$. We first consider the case where $\frac{X_1}{Y_1} = \frac{X_2}{Y_2}$. Without loss of generality, suppose $X_1 > X_2$, which implies that $Y_1 > Y_2$. Then (40)–(41) lead to a contradiction. We then consider the case where $\frac{X_1}{Y_1} \neq \frac{X_2}{Y_2}$. Without loss of generality, we assume $\frac{X_1}{Y_1} > \frac{X_2}{Y_2}$. Let $\bar{\zeta}_1 = \frac{1 - \gamma_1 X_2}{Y_2}$. By (39), we have $\bar{\zeta}_1 \geq \zeta_1$. Therefore, by (38)–(41), the following constraints must be satisfied:

$$\gamma_1 X_2 + \bar{\zeta}_1 Y_2 = 1, \tag{42}$$

$$\gamma_2 X_2 + \zeta_2 Y_2 = 1, \tag{43}$$

$$\gamma_1 X_1 + \bar{\zeta}_1 Y_1 \geq 1, \tag{44}$$

$$\gamma_2 X_1 + \zeta_2 Y_1 \leq 1. \tag{45}$$

By (44)–(45), either $\gamma_1 \geq \gamma_2$ or $\bar{\zeta}_1 \geq \zeta_2$. By (42)–(43), we have $(\gamma_1 - \gamma_2) X_2 = (\zeta_2 - \bar{\zeta}_1) Y_2$. By (44)–(45), we have $(\gamma_1 - \gamma_2) X_1 \geq (\zeta_2 - \bar{\zeta}_1) Y_1$. It follows that $(\zeta_2 - \bar{\zeta}_1) \frac{Y_1}{X_1} \leq (\zeta_2 - \bar{\zeta}_1) \frac{Y_2}{X_2}$. If $\bar{\zeta}_1 > \zeta_2$, we have $\frac{X_1}{Y_1} \leq \frac{X_2}{Y_2}$, which contradicts to $\frac{X_1}{Y_1} > \frac{X_2}{Y_2}$. Therefore, we must have $\gamma_1 \geq \gamma_2$ and $\bar{\zeta}_1 \leq \zeta_2$, and thus $\zeta_2 \geq \zeta_1$. However, $\zeta_2 \geq \zeta_1$ and $\gamma_1 \geq \gamma_2$ contradict to (35) and (37). Because (38)–(41) cannot be satisfied simultaneously, we consider the following two sub-cases.

Case (iii.i): $\gamma_1 X_2 + \zeta_1 Y_2 \leq 1$ or $\gamma_2 X_1 + \zeta_2 Y_1 \leq 1$, but not both (this implies that (46)–(47) do not admit non-negative solutions). Without loss of generality, assume $\gamma_1 X_2 + \zeta_1 Y_2 \leq 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 > 1$. Then, (γ_1, ζ_1) is the desired solution as $\gamma_2 X_1 + \zeta_2 Y_1 > 1$ contradicts to (29). Moreover, $q_1^A = M - \gamma_1 \sum_{i=1,2} A_i X_i$, $q_2^A = 0$, $q_1^C = N - \zeta_1 \sum_{i=1,2} B_i Y_i$ and $q_2^C = 0$. A similar result can be obtained if $\gamma_2 X_1 + \zeta_2 Y_1 \leq 1$.

Case (iii.ii): $\gamma_1 X_2 + \zeta_1 Y_2 > 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 > 1$. In this case, neither (γ_1, ζ_1) nor (γ_2, ζ_2) satisfies (29). Moreover, $\gamma_1 X_2 + \zeta_1 Y_2 > 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 > 1$ imply that $\frac{X_1}{Y_1} \neq \frac{X_2}{Y_2}$. Therefore, (γ, ζ) must be the unique solution to

$$\gamma X_1 + \zeta Y_1 = 1 \quad \text{and} \tag{46}$$

$$\gamma X_2 + \zeta Y_2 = 1. \tag{47}$$

In this case, $q^A = q_1^A + q_2^A = M - \gamma \sum_{k=1,2} A_k X_k$ and $q^C = q_1^C + q_2^C = N - \zeta \sum_{k=1,2} B_k Y_k$. Then, it remains to show that (9) holds. That is, there exist $\alpha, \beta \in [0,1]$ such that

$$\frac{\alpha q^A}{\gamma X_1} = \frac{\beta q^C}{\zeta Y_1} \quad \text{and} \quad \frac{(1-\alpha) q^A}{\gamma X_2} = \frac{(1-\beta) q^C}{\zeta Y_2}. \tag{48}$$

Note that (48) implies that $\alpha = \frac{q^C/q^A - \zeta Y_2/\gamma X_2}{\zeta Y_1/\gamma X_1 - \zeta Y_2/\gamma X_2}$ and $\beta = \frac{q^A/q^C - \gamma X_2/\zeta Y_2}{\gamma X_1/\zeta Y_1 - \gamma X_2/\zeta Y_2}$. Therefore, it suffices to show that

$$\frac{q^A}{q^C} \in \big(\min\{\frac{\gamma X_1}{\zeta Y_1}, \frac{\gamma X_2}{\zeta Y_2}\}, \max\{\frac{\gamma X_1}{\zeta Y_1}, \frac{\gamma X_2}{\zeta Y_2}\}\big). \tag{49}$$

Without loss of generality, assume $\frac{X_1}{Y_1} > \frac{X_2}{Y_2}$. Let (γ_1, ζ_1) be the solution to (34)–(35). Then we have $\gamma_1 X_1 + \zeta_1 Y_1 = 1$, $\gamma X_1 + \zeta Y_1 = 1$, $\gamma_1 X_2 + \zeta_1 Y_2 > 1$ and $\gamma X_2 + \zeta Y_2 = 1$. It follows that

$$(\gamma_1 - \gamma) X_1 + (\zeta_1 - \zeta) Y_1 = 0 \quad \text{and} \quad (\gamma_1 - \gamma) X_2 + (\zeta_1 - \zeta) Y_2 > 0. \tag{50}$$

When $\frac{X_1}{Y_1} > \frac{X_2}{Y_2}$, for (50) to hold, we must have $\gamma_1 \leq \gamma$ and $\zeta_1 \geq \zeta$. Let (γ_2, ζ_2) be the solution to

(36)–(37). Similarly, we have $\gamma_2 \geq \gamma$ and $\zeta_2 \leq \zeta$. Combined with (35) and (37), we have

$$\frac{\gamma X_2}{\zeta Y_2} \leq \frac{M - \gamma \sum_{k=1,2} A_k X_k}{N - \zeta \sum_{k=1,2} B_k X_k} = \frac{q^A}{q^C} \leq \frac{\gamma X_1}{\zeta Y_1},$$

which implies (49).

We then consider the case where (a) X_1, X_2, Y_1, Y_2 are not all positive, and (b) $X_1 \neq X_2$ or $Y_1 \neq Y_2$. Without loss of generality, we investigate the following three cases.

Case (i): $X_1 = 0$ and $X_2, Y_1, Y_2 > 0$. We consider the following possibilities. Case (i.i) If $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i \leq 1$, for $i = 1, 2$, we can obtain that $\gamma = \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta = \frac{N}{\sum_{i=1,2} B_i Y_i}$, $q_1 = 0$ and $q_2 = 0$. Case (i.ii) If $\frac{M}{\sum_{k=1,2} A_k X_k} X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_1 > 1$ and $\frac{M}{\sum_{k=1,2} A_k X_k} X_2 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_2 \leq 1$, we can obtain that $\gamma = \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta = \frac{1}{Y_1}$, $q_1^A = q_2^A = q_2^C = 0$ and $q_1^C = N - \zeta \sum_{i=1,2} B_i Y_i$. Case (i.iii) If $\frac{M}{\sum_{k=1,2} A_k X_k} X_2 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_2 > 1$ and $\frac{M}{\sum_{k=1,2} A_k X_k} X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_1 \leq 1$, we can obtain that $\gamma = \gamma_2$, $\zeta = \zeta_2$, $q_1^A = q_1^C = 0$, $q_2^A = M - \gamma \sum_{i=1,2} A_i X_i$ and $q_2^C = N - \zeta \sum_{i=1,2} B_i Y_i$, where (γ_2, ζ_2) is the unique solution to (36)–(37). Case (i.iv) If $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i > 1$ for $i = 1, 2$, $\gamma_1 X_2 + \zeta_1 Y_2 \leq 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 > 1$, we can obtain that $\gamma = \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta = \frac{1}{Y_1}$, $q_1^A = q_2^A = q_2^C = 0$ and $q_1^C = N - \zeta \sum_{i=1,2} B_i Y_i$. Case (i.v) If $\frac{M}{\sum_{k=1,2} A_k X_k} X_i + \frac{N}{\sum_{k=1,2} B_k Y_k} Y_i > 1$ for $i = 1, 2$, $\gamma_1 X_2 + \zeta_1 Y_2 > 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 \leq 1$, we can obtain that $\gamma = \gamma_2$, $\zeta = \zeta_2$, $q_1^A = q_1^C = 0$, $q_2^A = M - \gamma \sum_{i=1,2} A_i X_i$ and $q_2^C = N - \zeta \sum_{i=1,2} B_i Y_i$, where (γ_2, ζ_2) is the unique solution to (36)–(37). Case (i.vi) If $\gamma_1 X_2 + \zeta_1 Y_2 > 1$ and $\gamma_2 X_1 + \zeta_2 Y_1 > 1$, we can obtain that (γ, ζ) is the unique solution to (46)–(47). Moreover, $q_1^A = 0$, $q_2^A = M - \gamma \sum_{i=1,2} A_i X_i$, $q_2^C = q_2^A \frac{\zeta Y_2}{\gamma X_2}$ and $q_1^C = N - \zeta \sum_{i=1,2} B_i Y_i - q_2^C$.

Case (ii): $X_1 = Y_2 = 0$ and $X_2, Y_1 > 0$. In this case, $\eta_1^A = 1$, $\eta_2^A = 0$, $\eta_1^C = 0$ and $\eta_2^C = 1$. We can obtain that $s_{21}^A = S_{21} \wedge \frac{S_{21} M}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{22}^A = S_{22} \wedge \frac{S_{22} M}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{21}^C = 0$, $s_{22}^C = 0$, $s_{12}^C = S_{12} \wedge \frac{S_{12} N}{S_{12} + S_{11} + S_{12} \frac{t_{21}}{t_{12}}}$, $s_{11}^C = S_{11} \wedge \frac{S_{11} N}{S_{12} + S_{11} + S_{12} \frac{t_{21}}{t_{12}}}$, $s_{12}^A = 0$, $s_{11}^A = 0$, $r_{12}^A = S_{21} \frac{t_{12}}{t_{21}} \wedge \frac{S_{21} \frac{t_{12}}{t_{21}} M}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $r_{12}^C = 0$, $r_{21}^C = S_{12} \frac{t_{21}}{t_{12}} \wedge \frac{S_{12} \frac{t_{21}}{t_{12}} N}{S_{11} + S_{12} + S_{12} \frac{t_{21}}{t_{12}}}$, $r_{21}^A = 0$, $q_1^A = 0$, $q_1^C = 0 \vee (N - S_{11} + S_{12} + S_{12} \frac{t_{21}}{t_{12}})$, $q_2^A = 0 \vee (M - S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}})$ and $q_2^C = 0$.

Case (iii): $X_1 = Y_1 = 0$ and $X_2, Y_2 > 0$. In this case, $\eta_1^A = 1$, $\eta_2^A = 0$, $\eta_1^C = 1$ and $\eta_2^C = 0$. We can obtain that $s_{21}^A = \frac{M}{M+N} S_{21} \wedge \frac{S_{21} M}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{22}^A = \frac{M}{M+N} S_{22} \wedge \frac{S_{22} M}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{21}^C = \frac{N}{M+N} S_{21} \wedge \frac{S_{21} N}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{22}^C = \frac{N}{M+N} S_{22} \wedge \frac{S_{22} N}{S_{21} + S_{22} + S_{21} \frac{t_{12}}{t_{21}}}$, $s_{12}^C = s_{11}^C = s_{12}^A = s_{11}^A = 0$, $r_{12}^A =$

$\frac{M}{M+N}S_{21}\frac{t_{12}}{t_{21}} \wedge \frac{S_{21}\frac{t_{12}}{t_{21}}M}{S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}}}$, $r^C_{12} = \frac{N}{M+N}S_{21}\frac{t_{12}}{t_{21}} \wedge \frac{S_{21}\frac{t_{12}}{t_{21}}N}{S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}}}$, $r^C_{21} = r^A_{21} = 0$, $q^A_1 = q^C_1 = 0$, $q^A_2 = 0 \vee \frac{M}{M+N}(M+N-S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}})$ and $q^C_2 = 0 \vee \frac{N}{M+N}(M+N-S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}})$.

Lastly, we consider the case where $\underline{X_1 = X_2 \text{ and } Y_1 = Y_2}$. Case (i): If $\frac{M}{\sum_{k=1,2} A_k X_k}X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k}Y_1 \leq 1$, we can obtain that $\gamma = \frac{M}{\sum_{i=1,2} A_i X_i}$, $\zeta = \frac{N}{\sum_{i=1,2} B_i Y_i}$, $q_1 = 0$ and $q_2 = 0$. Case (ii): If $\frac{M}{\sum_{k=1,2} A_k X_k}X_1 + \frac{N}{\sum_{k=1,2} B_k Y_k}Y_1 > 1$, (γ, ζ) is the unique solution to (34)–(35). Moreover, q_1 and q_2 can be any non-negative numbers such that $q_1 + q_2 = M + N - \sum_{i,j}(s^A_{ij} + s^C_{ij}) - (r^A_{12} + r^A_{21} + r^C_{12} + r^C_{21})$.

It remains to show that (s, r), q^A and q^C are continuous in η^A, η^C, M and N. Because $q^A = M - \sum_{i,j} s^A_{ij} - (r^A_{12} + r^A_{21})$, $q^C = N - \sum_{i,j} s^C_{ij} - (r^C_{12} + r^C_{21})$ and (s, r) are determined by γ and ζ, it suffices to check if γ and ζ are continuous in η^A, η^C, M and N. Through the derivation of γ and ζ, the continuity follows directly after carefully checking all the boundary cases (we omit the calculation here).

C.2 Proof of Lemma 1

Without loss of generality, we consider the case such that when $\eta^C_1 = \eta^C_2 = 0$, the corresponding q_1 and q_2 satisfy: $q_2 \leq q^*_2 + k^*_2 q_1$ (CVs do not have incentives to reposition from location 2 to location 1). We first consider the setting where $\underline{X_1 \neq X_2}$. In this case, by Lemma C.1, (s, r, q) are uniquely determined by η^A, η^C, M and N. Moreover, through the derivation of q^A_1, q^A_2, q^C_1 and q^C_2 in the proof of Lemma C.1, we can obtain that q^A_1, q^A_2, q^C_1 and q^C_2 are continuous in η^A, η^C, M and N. By Lemma B.1 and Proposition B.1, it suffices to show that q_1 (weakly) decreases in η^C_1 and q_2 (weakly) increases in η^C_1 given $\eta^C_2 = 0$. We consider three types of capacity allocation (type A: $q_1 = q_2 = 0$, the demand at location 1 and location 2 is not fully served; type B: $q_i > 0$, $q_j = 0$, the demand at location j is not fully served for $i \neq j$; type C: $q_1 > 0$ and $q_2 > 0$) and show that q_1 (weakly) decreases in η^C_1 and q_2 (weakly) increases in η^C_1 in each type. We omit the proofs for the boundary cases as they follow naturally.

In type A capacity allocation, $q_1 = q_2 = 0$ and thus the desired result follows immediately

In type B capacity allocation, without loss of generality, assume $(\gamma, \zeta) = (\gamma_1, \zeta_1)$, where (γ_1, ζ_1)

is the unique solution to (34)–(35). In this case, $q_1 > 0$ and $q_2 = 0$. By (32)–(33), we have

$$q_1 = q_1^C + q_1^A = M + N - \gamma A \cdot X - \zeta B \cdot Y = M + N - \frac{A \cdot X}{X_1} + \zeta(\frac{Y_1}{X_1} A \cdot X - B \cdot Y).$$

Because A and X are independent of η_1^C, it remains to show that

$$\zeta(\frac{Y_1}{X_1}A \cdot X - B \cdot Y) = -(N + \frac{1}{Y_1}B \cdot Y + M - \frac{1}{X_1}A \cdot X) + \sqrt{(N + \frac{1}{Y_1}B \cdot Y + M - \frac{1}{X_1}A \cdot X)^2 + 4N(\frac{1}{X_1}A \cdot X - \frac{1}{Y_1}B \cdot Y)}$$

decreases in η_1^C. Let $x = \frac{1}{Y_1}B \cdot Y - \frac{1}{X_1}A \cdot X$. We can obtain that $\frac{\partial B_1}{\partial \eta_1^C} = \Lambda_1 P_{11} t_{12} \geq 0$, $\frac{\partial B_2}{\partial \eta_1^C} = \Lambda_2 P_{21} t_{12} \geq 0$, $\frac{\partial Y_1}{\partial \eta_1^C} = -\Lambda_2 P_{21} \leq 0$ and $\frac{\partial Y_2}{\partial \eta_1^C} = \Lambda_1 P_{11} \geq 0$, which implies that x increases in η_1^C. Let $f(x) = -(M + N + x) + \sqrt{(M + N + x)^2 - 4Nx}$. It suffices to show that $f'(x) = -1 + \frac{(M+N+x)-2N}{\sqrt{(M+N+x)^2-4Nx}} < 0$, which is true as $\frac{(M+N+x)^2+4N^2-4N(M+N+x)}{(M+N+x)^2-4Nx} < 1$.

In type C capacity allocation, (γ, ζ) is the solution to (46)–(47). We have

$$\gamma = \frac{Y_2 - Y_1}{X_1 Y_2 - X_2 Y_1} \quad \text{and} \quad \zeta = \frac{X_1 - X_2}{X_1 Y_2 - X_2 Y_1}.$$

By (32)–(33) and (48), we have

$$F_1 = \frac{q^A \zeta Y_2 - q^C \gamma X_2}{q^A (Y_2 - \frac{X_2}{X_1} Y_1)\zeta} \quad \text{and} \quad F_2 = \frac{q^A \zeta Y_2 - q^C \gamma X_2}{q^C (\frac{Y_2}{Y_1} X_1 - X_2)\gamma}.$$

Therefore, we can obtain that

$$\begin{aligned} q_1 &= F_1 q^A + F_2 q^C = \frac{Y_2}{Y_2 - Y_1} q^A - \frac{X_2}{X_1 - X_2} q^C \quad \text{and} \\ q_2 &= (1 - F_1) q^A + (1 - F_2) q^C = \frac{X_1}{X_1 - X_2} q^C - \frac{Y_1}{Y_2 - Y_1} q^A. \end{aligned}$$

We then consider two cases. Case (i), $X_1 > X_2$. In this case, we must have $Y_2 > Y_1$. Because $\frac{Y_2}{Y_2 - Y_1}$ and $\frac{Y_1}{Y_2 - Y_1}$ decrease in η_1^C, it suffices to show that $q^A = M - \gamma A \cdot X$ decreases in η_1^C, and $q^C = N - \zeta B \cdot Y$ increases in η_1^C. Note that we have

$$\frac{\partial \gamma}{\partial \eta_1^C} = \frac{1}{(X_1 Y_2 - X_2 Y_1)^2} [\frac{\partial Y_2}{\partial \eta_1^C} Y_1 (X_1 - X_2) + \frac{\partial Y_1}{\partial \eta_1^C} Y_2 (X_2 - X_1)]. \tag{51}$$

When $X_1 \geq X_2$ and $Y_2 \geq Y_1$, we have $\frac{\partial \gamma}{\partial \eta_1^C} \geq 0$ and thus q^A decreases in η_1^C. Moreover, because

$$\frac{\partial \frac{B \cdot Y}{Y_2 - Y_1}}{\partial \eta_1^C} = -\frac{(\Lambda_2 P_{21} t_{21} + \Lambda_2 P_{22} t_{22})\Lambda_1 \Lambda_2 P_{21} + \Lambda_1^2 P_{11} t_{11} \Lambda_2 P_{21} + \Lambda_2^2 P_{21}^2 t_{12} \Lambda_1}{[\Lambda_1 - \Lambda_1 P_{11} - \Lambda_2 P_{21} + (\Lambda_1 P_{11} + \Lambda_2 P_{21})\eta_1^C]^2} \leq 0,$$

$$\frac{\partial \frac{B \cdot Y}{Y_2}}{\partial \eta_1^C} = -\frac{\Lambda_1^2 P_{11} t_{11} \Lambda_2 P_{21}}{(\Lambda_1 - \Lambda_1 P_{11} + \Lambda_1 P_{11} \eta_1^C)^2} \leq 0,$$

and $\zeta B \cdot Y = \frac{X_1 - X_2}{X_2(Y_2 - Y_1) + (X_1 - X_2)Y_2} B \cdot Y$, by Remark 1, we have $\zeta B \cdot Y$ decreases in η_1 and thus q^C increases in η_1^C.

Remark 1. *Let* $F(x) = \frac{f_1(x)}{f_2(x) + f_3(x)}$, $F_1(x) = \frac{f_1(x)}{f_2(x)}$, *and* $F_2(x) = \frac{f_1(x)}{f_3(x)}$.

- *If* $F_1'(x) \leq 0$ *and* $F_2'(x) \leq 0$, *then* $F'(x) \leq 0$.
- *If* $F_1'(x) \geq 0$ *and* $F_2'(x) \geq 0$, *then* $F'(x) \geq 0$.

Case (ii), $X_1 < X_2$, which implies $Y_2 < Y_1$. In this case, it suffices to show that q^A increases η_1^C and q^C decreases in η_1^C. By (51), $\frac{\partial \gamma}{\partial \eta_1^C} < 0$ and it follows that q^A increases in η_1^C. Because we have

$$\frac{\partial \frac{B \cdot Y}{Y_1 - Y_2}}{\partial \eta_1^C} = \frac{(\Lambda_2 P_{21} t_{21} + \Lambda_2 P_{22} t_{22})\Lambda_1 \Lambda_2 P_{21} + \Lambda_1^2 P_{11} t_{11} \Lambda_2 P_{21} + \Lambda_2^2 P_{21}^2 t_{12} \Lambda_1}{[\Lambda_1 - \Lambda_1 P_{11} - \Lambda_2 P_{21} + (\Lambda_1 P_{11} + \Lambda_2 P_{21})\eta_1^C]^2} \geq 0,$$

$$\frac{\partial \frac{B \cdot Y}{Y_1}}{\partial \eta_1^C} = \frac{\Lambda_2^2 P_{21}^2 t_{12} \Lambda_1 + (\Lambda_2 P_{21} t_{21} + \Lambda_2 P_{22} t_{22})\Lambda_1 \Lambda_2 P_{21}}{[\Lambda_2 P_{21}(1 - \eta_1^C)]^2} \geq 0,$$

and $\zeta B \cdot Y = \frac{X_2 - X_1}{X_1(Y_1 - Y_2) + (X_2 - X_1)Y_1}$, by Remark 1, $\zeta B \cdot Y$ increases in η_1^C and thus q^C decreases in η_1^C.

We then consider the setting where $\underline{X_1 = X_2}$. The proof is similar to that of the first setting, except the possibility that $(\eta^A, \eta^C, \Lambda_{ij}, t_{ij}, M, N) \in E$, where E is defined in Definition 1. If $(\eta^A, \eta^C, \Lambda_{ij}, t_{ij}, M, N) \in E$ for some $\eta_1^{C^*} \in [0, 1]$, we consider the following two cases. Case (i): $M + N - \sum_{i,j}(s_{ij}^A + s_{ij}^C) - (r_{12}^A + r_{21}^A + r_{12}^C + r_{21}^C) \geq q_1^*$ ((s, r) is obtained under $\eta_1^{C^*}$). In this case, because q_1 and q_2 can be any non-negative numbers splitting $M + N - \sum_{i,j=1,2}(s_{ij}^A + s_{ij}^C) - (r_{12}^A + r_{21}^A + r_{12}^C + r_{21}^C)$, we can choose q_1 and q_2 such that the corresponding capacity allocation is driver-incentive compatible and the choice is unique. Moreover, any other strategy $\eta_1^C \neq \eta_1^{C^*}$ cannot lead to a DICCA because of the following reasons. (1) If $\eta_1^C \neq \eta_1^{C^*}$, we must have $Y_1 \neq Y_2$. Therefore (46) and (47) do not have a solution which implies that $q_1 = 0$ or $q_2 = 0$. (2) If $\eta_1^C < \eta_1^{C^*}$, by the monotonicity of q_1 and q_2 in terms of η_1^C, we must have $q_1 > q_1^*$ which is not driver-incentive compatible. (3) If $\eta_1^C > \eta_1^{C^*}$, we have $Y_1 < Y_2$. Therefore, $\gamma X_1 + \zeta Y_1 = a_1 \leq a_2 = \gamma X_2 + \zeta Y_2$, which implies that $q_1 = 0$. Therefore the capacity allocation cannot be driver-incentive compatible. Case (ii): $M + N - \sum_{i,j=1,2}(s_{ij}^A + s_{ij}^C) - (r_{12}^A + r_{21}^A + r_{12}^C + r_{21}^C) < q_1^*$. In this case, if $\eta_1^C = 0$ leads to $q_1 \leq q_1^* + k_1^* q_2$, the corresponding capacity allocation is driver-incentive compatible. Otherwise, we can find a unique $\eta_1^C < \eta_1^{C^*}$ such that $q_1 = q_1^*$, which makes the corresponding capacity allocation driver-incentive compatible.

D Proofs for Systems with Fixed Supply of AVs and CVs

By proposition B.1, the optimization problem stated in (11) can be reformulated as the following capacity allocation problem:

$$\begin{aligned} &\max_{\eta^A} \;\; \Pi(\eta^A) \\ &\text{subject to} \;\; (1)\text{–}(10) \;\; \text{and} \;\; (s, r, q) \in D. \end{aligned} \tag{52}$$

D.1 Proof of Theorem 1

We prove Theorem 1 in three steps. In step (i), we show that any AV repositioning strategy η^A that makes AVs and CVs reposition in opposite directions is sub-optimal (Lemma D.1). Therefore, we can focus on the case where vehicles (both AVs and CVs) reposition in the same direction. In step (ii), we prove a monotonicity result for the platform profit with respect to η_1^A, which implies that the platform has an incentive to reposition AVs to minimize the repositioning of CVs. In step (iii), we obtain the optimal AV repositioning strategy and characterize the corresponding DICCA.

We first note that when (s, r, q) satisfies (1)–(9), q_1^* and k_1^* defined in (23) can be rewritten as

$$q_1^* = \frac{\Lambda_1 P_{12} t_{12}(P_{21} t_{21} + P_{22} t_{22}) + \Lambda_1 P_{21} t_{12}(P_{11} t_{11} + P_{12} t_{12})}{(P_{21} t_{21} + P_{22} t_{22}) - \frac{c_c}{w}(P_{22} t_{22} + P_{21} t_{21} + P_{21} t_{12})}, \quad \text{and} \tag{53}$$

$$k_1^* = \frac{\Lambda_{12} t_{12} + \Lambda_{11} t_{11} - \frac{c_c}{w}\Lambda_{11} t_{11}}{(\Lambda_{21} t_{21} + \Lambda_{22} t_{22}) - \frac{c_c}{w}(\Lambda_{22} t_{22} + \Lambda_{21} t_{21} + \Lambda_{21} t_{12})}. \tag{54}$$

Let $\Pi(s, r, q)$ be the platform profit as shown in (11) and $\pi(s, r, q) = (w - c_c)\sum_{i,j=1,2} s_{ij}^C - c_c(r_{12}^C + r_{21}^C)$

be the total driver welfare under (s, r, q).

D.1.1 Proof of Theorem 1 Step (i)

Lemma D.1. *η^A is sub-optimal if it results in a capacity allocation such that $r_{12} > 0$ and $r_{21} > 0$.*

Proof of Lemma D.1. By (2)–(5), we have

$$\frac{s_{12}^A}{t_{12}} - \frac{s_{21}^A}{t_{21}} = \frac{r_{21}^A}{t_{21}} - \frac{r_{12}^A}{t_{12}}, \quad \text{and} \tag{55}$$

$$\frac{s_{12}^C}{t_{12}} - \frac{s_{21}^C}{t_{21}} = \frac{r_{21}^C}{t_{21}} - \frac{r_{12}^C}{t_{12}}. \tag{56}$$

We first show that any η^A with $\eta_1^A > 0$ and $\eta_2^A > 0$ can not be optimal. Without loss of generality, assume $\frac{r_{12}^A}{t_{12}} > \frac{r_{21}^A}{t_{21}}$. By (55), we can adjust the the amount of AVs in repositioning (r_{12}^A, r_{21}^A) to $(\tilde{r}_{12}^A, \tilde{r}_{21}^A)$ such that $\tilde{r}_{21}^A = 0$ and $\tilde{r}_{12}^A = r_{12}^A - \frac{t_{12}}{t_{21}} r_{21}^A$. By doing so, we take out $r_{21}^A + \frac{t_{12}}{t_{21}} r_{21}^A$ amount of AVs from the system and constraints in (52) still hold. Moreover, the values of s_{ij}^A, s_{ij}^C, r_{ij}^C, q_i^A and q_i^C for $i, j \in \{1, 2\}$ remain the same, which implies that the platform profit remains the same. We can add $r_{21}^A + \frac{t_{12}}{t_{21}} r_{21}^A$ amount of AVs back to the system and optimize the platform profit over η_1^A given $\eta_2^A = 0$. By Proposition 1, our adjustment improves the platform profit and the desired result follows.

We then show that for any AV repositioning strategy η^A, if $\eta_i^A > 0$ and its corresponding CV equilibrium repositioning strategy $\eta_j^C > 0$ for $i \neq j \in \{1, 2\}$, η^A cannot be optimal. Similar to the proof of Lemma 1, we consider three types of capacity allocation (type A: $q_1 = q_2 = 0$, the demand at location 1 and location 2 is not fully served; type B: $q_i > 0$, $q_j = 0$, the demand at location j is not fully served for $i \neq j$; type C: $q_1 > 0$ and $q_2 > 0$). The boundary cases can be proved with similar arguments and thus we omit the details. By Proposition B.1, CVs only reposition under type B and type C capacity allocation.

Type B capacity allocation. Without loss of generality, we consider the case where $r_{21}^A > 0$ and $r_{12}^C > 0$. By Proposition B.1, we have $q_1 \geq q_1^*$ and $q_2 = 0$. Therefore, we consider two cases: (i) $q_1 > q_1^*$ and (ii) $q_1 = q_1^*$.

Case (i): $q_1 > q_1^*$. By Proposition B.1, $\eta_1^C = 1$, and thus $s_{11}^C = s_{12}^C = q_1^C = 0$. By (7)–(8) and

(55)–(56), we have

$$r_{12}^C = \frac{N}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{21}^C = \frac{\frac{t_{21}}{t_{12}}N}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{22}^C = \frac{\frac{P_{22}t_{22}}{P_{21}t_{12}}N}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{11}^A = S_{11}, \quad s_{12}^A = S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}}S_{12} - r_{21}^A \quad, s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}S_{12} - \frac{P_{22}t_{22}}{P_{21}t_{21}}r_{21}^A \text{ and } q_1^A = M - \sum_{i,j=1,2} s_{ij}^A - r_{21}^A.$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we decrease r_{21}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 > q_1^*$, and thus $\tilde{s}_{ij}^C = s_{ij}^C$ for $i \in \{1,2\}$, $\sum_{i,j=1,2}\tilde{s}_{ij}^A = \sum_{i,j=1,2}s_{ij}^A + (1+\frac{P_{22}t_{22}}{P_{21}t_{21}})\delta$. Therefore,

$$\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = [p + (p-c_a)\frac{P_{22}t_{22}}{P_{21}t_{21}}]\delta > 0,$$

which implies that η^A is not optimal.

Recall that we use F_1 to denote the fraction of demand served by AVs at location 1. By (7)–(8) and (55)–(56), we have

$$F_1 = \frac{M + \frac{P_{22}t_{22}}{P_{21}t_{21}}r_{21}^A}{S_{11} + S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1^*},$$
$$s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad q_1^A = F_1 q_1^*, \quad s_{21}^A = F_1\frac{t_{21}}{t_{12}}S_{12} - r_{21}^A, \quad s_{22}^A = F_1\frac{P_{22}t_{22}}{P_{21}t_{12}}S_{12} - \frac{P_{22}t_{22}}{P_{21}t_{21}}r_{21}^A,$$
$$r_{12}^C = \frac{N - (1-F_1)[S_{11} + S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1^*]}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12},$$
$$s_{21}^C = \frac{t_{21}}{t_{12}}[(1-F_1)S_{12} + r_{12}^C], \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}}[(1-F_1)S_{12} + r_{12}^C] \text{ and } q_1^C = (1-F_1)q_1^*.$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we decrease r_{21}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 = q_1^*$. By Lemma B.1, a driver is indifferent among all the repositioning strategy $\eta^* = (\eta_1^*, 0)$ with $\eta_1^* \in [0,1]$ under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$. Without loss of generality, we consider the case where $\eta^* = (0,0)$. Because $\tilde{q}_1 = q_1$ and $\tilde{q}_2 = q_2 = 0$, we have $\overline{\pi}(\eta^*, s, r, q) = \overline{\pi}(\eta^*, \tilde{s}, \tilde{r}, \tilde{q})$. It follows that $\pi(s, r, q) = \pi(\tilde{s}, \tilde{r}, \tilde{q})$, which implies that $\sum_{i,j=1,2}(\tilde{s}_{ij}^C - s_{ij}^C) = \frac{c_c}{w-c_c}(\tilde{r}_{12}^C - r_{12}^C)$. Therefore, we can obtain that

$$\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q)$$
$$= (p-c_a)\left(-\frac{\frac{P_{22}t_{22}}{P_{21}t_{21}}[S_{11} + S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})]}{S_{11} + S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1^*} + (1+\frac{P_{22}t_{22}}{P_{21}t_{21}})\right)\delta + c_a\delta - \frac{(p-w)c_c}{w-c_c}\frac{\frac{P_{22}t_{22}}{P_{21}t_{21}}}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}\delta > 0,$$

where the inequality is due to our assumption that $t_{ij}c_c < t_{ji}(w - c_c)$. Therefore, η^A is not optimal.

Type C capacity allocation. Without loss of generality, we consider the case where $r^A_{21} > 0$ and $r^C_{12} > 0$. By Proposition B.1, we consider two cases: (i) $q_1 > q^*_1 + k^*_1 q_2$ and (ii) $q_1 = q^*_1 + k^*_1 q_2$.

Case (i): $q_1 > q^*_1 + k^*_1 q_2$. By Proposition B.1, $\eta^C_1 = 1$, and thus $s^C_{11} = s^C_{12} = q^C_1 = 0$. Recall that we use F_2 to denote the fraction of demand served by AVs at location 2. By (7)–(9) and (55)–(56), we have

$$
\begin{aligned}
&F_2 = \frac{S_{12}t_{21}}{S_{21}t_{12}} - \frac{r^A_{21}}{S_{21}}, \quad s^A_{11} = S_{11}, \quad s^A_{12} = S_{12}, \quad s^A_{21} = F_2 S_{21}, \quad s^A_{22} = F_2 S_{22},\\
&q^A_2 = \frac{NF_2}{1-F_2} - F_2(1 + \frac{t_{12}}{t_{21}})S_{21} - F_2 S_{22}, \quad q^A_1 = M - \sum_{i,j=1,2} s^A_{ij} - r^A_{21} - q^A_2,\\
&s^C_{21} = (1-F_2)S_{21}, \quad s^C_{22} = (1-F_2)S_{22}, \quad r^C_{12} = \frac{t_{12}}{t_{21}} s^C_{21} \ \text{ and } \ q^C_2 = N - (1-F_2)(1+\frac{t_{12}}{t_{21}})S_{21} - (1-F_2)S_{22}.
\end{aligned}
$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we decrease r^A_{21} by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 > q^*_1 + k^*_1 \tilde{q}_2$. By Proposition B.1, under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$, CVs reposition from location 1 to location 2 with probability 1. It follows that $\tilde{s}^A_{11} = s^A_{11}$, $\tilde{s}^A_{12} = s^A_{12}$, $\tilde{s}^A_{21} = s^A_{21} + \delta$, $\tilde{s}^A_{22} = s^A_{22} + \frac{S_{22}}{S_{21}}\delta$, $\tilde{s}^C_{11} = s^C_{11}$, $\tilde{s}^C_{12} = s^C_{12}$, $\tilde{s}^C_{21} = s^C_{21} - \delta$ and $\tilde{s}^C_{22} = s^C_{22} - \frac{S_{22}}{S_{21}}\delta$. We can obtain that

$$
\Pi(\tilde{s}, \tilde{s}, \tilde{q}) - \Pi(s, r, q) = (p - c_a)(1 + \frac{S_{22}}{S_{21}})\delta - (p - w)(1 + \frac{S_{22}}{S_{21}})\delta + c_a\delta > 0.
$$

Therefore, η^A is not optimal.

Case (ii): $q_1 = q^*_1 + k^*_1 q_2$. By (7)–(9) and (55)–(56), we have

$$
\begin{aligned}
&r^C_{12} = \frac{t_{12}}{t_{21}}(S_{21} + r^A_{21}) - S_{12},\\
&q_1 = \frac{q^*_1}{1+k^*_1} + \frac{k^*_1(M + N - \sum_{i,j=1,2} S_{ij} - r^C_{12} - r^A_{21})}{1+k^*_1}, \quad q_2 = \frac{(M + N - \sum_{i,j=1,2} S_{ij} - r^C_{12} - r^A_{21} - q^*_1)}{1+k^*_1},\\
&F_1 = \frac{M - r^A_{21} + \frac{r^A_{21}}{S_{21}}(S_{21} + S_{22} + q_1)}{(S_{11} + S_{12} + q_1) + \frac{S_{12}t_{21}}{S_{21}t_{12}}(S_{21} + S_{22} + q_2)}, \quad F_2 = \frac{M - r^A_{21} - \frac{r^A_{21}t_{12}}{S_{12}t_{21}}(S_{11} + S_{12} + q_1)}{\frac{S_{21}t_{12}}{S_{12}t_{21}}(S_{11} + S_{12} + q_1) + (S_{21} + S_{22} + q_2)},\\
&s^C_{11} = (1-F_1)S_{11}, \quad s^C_{12} = (1-F_1)S_{12}, \quad s^C_{21} = (1-F_2)S_{21}, \quad s^C_{22} = (1-F_2)S_{22}, \quad q^C_1 = (1-F_1)q_1, \quad q^C_2 = (1-F_2)q_2,\\
&s^A_{11} = F_1 S_{11}, \quad s^A_{12} = F_1 S_{12}, \quad s^A_{21} = F_2 S_{21}, \quad s^A_{22} = F_2 S_{22}, \quad q^A_1 = F_1 q_1 \ \text{ and } \ q^A_2 = F_2 q_2.
\end{aligned}
$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we decrease r^A_{21} by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 > q_1$, $\tilde{q}_2 > q_2$ and $\tilde{q}_1 = q^*_1 + k^*_1 \tilde{q}_2$. By Lemma B.1, a driver is indifferent among all the reposition strategies $\eta^* = (\eta_1^*, 0)$ with $\eta_1^* \in [0,1]$ under both (s,r,q) and $(\tilde{s},\tilde{r},\tilde{q})$. Without loss of generality, we consider the case where $\eta^* = (0,0)$. Because $\tilde{q}_1 > q_1$ and $\tilde{q}_2 > q_2$, $\overline{\pi}(\eta^*, \tilde{s}, \tilde{r}, \tilde{q}) < \overline{\pi}(\eta^*, s, r, q)$. It follows that $\pi(\tilde{s},\tilde{r},\tilde{q}) = (w - c_c)\sum_{i,j=1,2} \tilde{s}_{ij}^C - c_c\tilde{r}_{12}^C < \pi(s,r,q) = (w - c_c)\sum_{i,j=1,2} s_{ij}^C - c_c r_{12}^C$. Because $\tilde{r}_{12}^C < r_{21}^C$, we must have $\Delta = \sum_{i,j=1,2}(\tilde{s}_{ij}^C - s_{ij}^C) < 0$. Because $\tilde{s}_{ij} = s_{ij}$, it follows that $\sum_{i,j=1,2}(\tilde{s}_{ij}^A - s_{ij}^A) = -\Delta > 0$ and thus

$$\Pi(\tilde{s},\tilde{r},\tilde{q}) - \Pi(s,r,q) = (p - c_a)\sum_{i,j=1,2}(\tilde{s}_{ij}^A - s_{ij}^A) - (p-w)\sum_{i,j=1,2}(s_{ij}^C - \tilde{s}_{ij}^C) + c_a\delta = -(p-c_a)\Delta + (p-w)\Delta + c_a\delta > 0.$$

Therefore, η^A is not optimal. □

D.1.2 Proof of Theorem 1 Step (ii)

By Lemma D.1, it suffices to focus on scenarios where AVs and CVs reposition (if at all) in the same direction. Because we assume that $\Lambda_{12} < \Lambda_{21}$, by Proposition B.1, AVs and CVs cannot reposition from location 2 to location 1 under any DICCA. Similar to the proof of Lemma 1, we consider three types of capacity allocation, namely type A: $q_1 = q_2 = 0$, the demand at location 1 and location 2 is not fully served; type B: $q_1 > 0$, $q_2 = 0$, the demand at location 2 is not fully served (we omit the case where $q_2 > 0$ and $q_1 = 0$ because this case cannot be optimal for the platform); type C: $q_1 > 0$ and $q_2 > 0$. We show that under type A capacity allocation, the platform profit decreases in η_1^A; under type B capacity allocation, the platform profit increases in η_1^A if $q_1 > \max(0, \min(\hat{q}_1, q_1^*))$ or $\eta_1^C > 0$, where $\hat{q}_1$ is given in (57), and decreases in η_1^A otherwise; under type C capacity allocation, the platform profit increases in η_1^A if $\eta_1^C > 0$, and decreases in η_1^A otherwise. Because the platform can be better off by increasing the repositioning of AVs if $\eta_1^C > 0$, the optimal strategy for the platform minimizes the repositioning of CVs.

Type A capacity allocation. In this case, we have $q_1 = q_2 = 0$. By Proposition B.1, $\eta_1^C = 0$.

By (7)–(9) and (55)–(56), we can obtain that

$$s_{11}^A = \frac{\frac{P_{11}t_{11}}{P_{12}t_{12}}[M - (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})r_{12}^A]}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{12}^A = \frac{M - (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})r_{12}^A}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{21}^A = \frac{\frac{t_{21}}{t_{12}}(M + \frac{P_{11}t_{11}}{P_{12}t_{12}}r_{12}^A)}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{22}^A = \frac{\frac{P_{22}t_{22}}{P_{21}t_{12}}(M + \frac{P_{11}t_{11}}{P_{12}t_{12}}r_{12}^A)}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{11}^C = \frac{\frac{P_{11}t_{11}}{P_{12}t_{12}}N}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{12}^C = \frac{N}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{21}^C = \frac{\frac{t_{21}}{t_{12}}N}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}} \quad \text{and} \quad s_{22}^C = \frac{\frac{P_{22}t_{22}}{P_{21}t_{12}}N}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}.$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we decrease r_{12}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 = 0$ and $\tilde{q}_2 = 0$. We have

$$\begin{aligned}\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) &= (p - c_a) \sum_{i,j=1,2} (\tilde{s}_{ij}^A - s_{ij}^A) - \sum_{i,j=1,2} (s_{ij}^C - \tilde{s}_{ij}^C) + c_a(\tilde{r}_{12}^A - r_{12}^A) \\ &= \frac{(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})(\frac{P_{11}t_{11}}{P_{12}t_{12}} + 1) - \frac{P_{11}t_{11}}{P_{12}t_{12}}(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}\delta + \delta > 0.\end{aligned}$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit decreases in η_1^A.

Type B capacity allocation. Assume $\eta_1^A \in (0, 1)$. We consider the following cases.

Case (i): $q_1 > q_1^*$. By Proposition B.1, we have $\eta_1^C = 1$, and thus $s_{11}^C = s_{12}^C = q_1^C = 0$. By (7)–(8) and (55)–(56), we can obtain that

$$r_{12}^C = \frac{N}{(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}, \quad s_{12}^C = \frac{\frac{t_{21}}{t_{12}}N}{(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}, \quad s_{22}^C = \frac{\frac{P_{22}t_{22}}{P_{21}t_{12}}N}{(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}$$
$$s_{11}^A = S_{11}, \quad s_{12}^A = S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}}(S_{12} + r_{12}^A), \quad S_{22} = \frac{P_{22}t_{22}}{P_{21}t_{12}}(S_{12} + r_{12}^A) \quad \text{and} \quad q_1^A = M - \sum_{i,j=1,2} s_{12}^A - r_{12}^A.$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we increases r_{12}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 > q_1^*$. We can obtain that $\tilde{s}_{ij}^C = s_{ij}$, $\tilde{r}_{12}^A = r_{12}^A + \delta$, $\tilde{s}_{11}^A = s_{11}^A$, $\tilde{s}_{12}^A = s_{12}^A$, $\tilde{s}_{21}^A = s_{21}^A + \frac{t_{21}}{t_{12}}\delta$, and $\tilde{s}_{22}^A = s_{22}^A + \frac{P_{22}t_{22}}{P_{21}t_{12}}\delta$. Because $p > w$, $t_{ij}c_c < t_{ji}(w - c_c)$ and $c_c \leq c_a$, we have

$$\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = p(\frac{t_{12}}{t_{21}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})\delta - c_a\delta > 0.$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit increases in η_1^A.

Case (ii): $q_1 = q_1^*$ and $r_{12}^C > 0$. By (7)–(8) and (55)–(56), we have

$$F_1 = \frac{M - (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})r_{12}^A}{S_{11} + S_{12}(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1^*},$$

$$s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, s_{21}^A = \frac{t_{21}}{t_{12}}(F_1 S_{12} + r_{12}^A), \quad s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}(F_1 S_{12} + r_{12}^A), \quad q_1^A = F_1 q_1^*,$$

$$r_{12}^C = \frac{N - (1 - F_1)[S_{11} + S_{12}(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1^*]}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad q_1^C = (1 - F_1)q_1^*,$$

$$s_{11}^C = (1 - F_1)S_{11}, \quad s_{12}^C = (1 - F_1)S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}}[(1 - F_1)S_{12} + r_{12}^C] \text{ and } s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}}[(1 - F_1)S_{12} + r_{12}^C].$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we increase r_{12}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 = q_1^*$, $\sum_{i,j=1,2} \tilde{s}_{ij} = \sum_{i,j=1,2} s_{ij}$, $\tilde{r}_{12}^A = r_{12}^A + \delta$ and $\tilde{r}_{12}^C = r_{12}^C - \delta$. By Lemma B.1, a driver is indifferent among all the repositioning strategies $\eta^* = (\eta_1^*, 0)$ with $\eta_1^* \in [0, 1]$ under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$. Without loss of generality, we consider the case where $\eta^* = (0, 0)$. Because $q_1 = \tilde{q}_1 = q_1^*$ and $q_2 = \tilde{q}_2 = 0$, the expected earnings for the driver remains the same, i.e., $\overline{\pi}(\eta^*, s, r, q) = \overline{\pi}(\eta^*, s, r, q)$. It follows that $\pi(s, r, q) = (w - c_c)\sum_{i,j-1,2} s_{ij}^C - c_c r_{12}^C = \pi(\tilde{s}, \tilde{r}, \tilde{q}) = (w - c_c)\sum_{i.j=1,2} \tilde{s}_{ij}^C - c_c \tilde{r}_{12}^C$. We can then obtain that $\sum_{i,j=1,2}(s_{ij}^C - \tilde{s}_{ij}^C) = \sum_{i,j=1,2}(\tilde{s}_{ij}^A - s_{ij}^A) = \frac{c_c}{w - c_c}\delta$. It follows that

$$\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = p \sum_{i,j=1,2} (\tilde{s}_{ij}^A - s_{ij}^A) - (p - w) \sum_{i,j=1,2} (s_{ij}^C - \tilde{s}_{ij}^C) - c_a \delta = (w \frac{c_c}{w - c_c} - c_a)\delta > 0.$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit increases in η_1^A.

Case (iii): $q_1 \leq q_1^*$ and $r_{12}^C = 0$. By (56), we have $s_{11}^C = (1 - F_1)S_{11}$, $s_{12}^C = (1 - F_1)S_{12}$, $s_{21}^C = \frac{t_{21}}{t_{12}}(1 - F_1)S_{12}$, $s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}}(1 - F_1)S_{12}$ and $q_1^C = (1 - F_1)q_1$. By (8), we have

$$(1 - F_1)[S_{11} + S_{12}(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1] = N \quad \Rightarrow \quad F_1 = 1 - \frac{N}{S_{11} + S_{12}(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) + q_1}.$$

By (55)–(56), we have $s_{21} = \frac{t_{21}}{t_{12}}(S_{12} + r_{12}^A)$ and $s_{22} = \frac{P_{22}t_{22}}{P_{21}t_{12}}(S_{12} + r_{12}^A)$. Then by (7)–(8), we have

$$r_{12}^A = \frac{M + N - S_{11} - S_{12}(1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - q_1}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}.$$

We then formulate the platform profit as a function of q_1:

$$\begin{aligned}\Pi(q_1) &= (p-c_a)\sum_{i,j=1,2} s_{ij}^A + (p-w)\sum_{i,j=1,2} s_{ij}^C - c_a r_{12}^A \\ &= [(1-\frac{N}{S_{11}+S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})+q_1})(w-c_a)+(p-w)][S_{11}+S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})] \\ &\quad +[(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a]\frac{M+N-S_{11}-S_{12}(1+\frac{t_{12}}{t_{21}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-q_1}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}.\end{aligned}$$

We can obtain that $\Pi(q_1)$ is a concave function, and $\Pi'(\hat{q}_1)=0$, where

$$\hat{q}_1 = \sqrt{\frac{(w-c_a)N[S_{11}+S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})](1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})}{(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a}} - [S_{11}+S_{12}(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})]. \quad (57)$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit increases in η_1^A if $q_1 > \max(0, \min(\hat{q}_1, q_1^*))$, and decreases in η_1^A otherwise.

Type C capacity allocation. Assume $\eta_1^A \in (0,1)$. We consider the following cases.

Case (i): $q_1 > q_1^* + k_1^* q_2$. By Proposition B.1, $\eta_1^C = 1$, and thus $s_{11}^C = s_{12}^C = q_1^C = 0$. By (7)–(9) and (55)–(56), we have

$$\begin{aligned}&F_2 = \frac{t_{21}(S_{12}+r_{12}^A)}{t_{12}S_{21}}, \quad s_{11}^A = S_{11}, \quad s_{12}^A = S_{12}, \quad s_{21}^A = F_2 S_{21}, \quad s_{22}^A = F_2 S_{22}, \\ &q_2^A = \frac{F_2 N}{1-F_2} - F_2(1+\frac{t_{12}}{t_{21}})S_{21} - F_2 S_{22}, \quad q_1^A = M - \sum_{i,j=1,2} s_{ij}^A - r_{12}^A - q_2^A, \\ &s_{21}^C = (1-F_2)S_{21}, \quad s_{22}^C = (1-F_2)S_{22}, \quad r_{12}^C = \frac{t_{12}}{t_{21}} s_{21}^C \text{ and } q_2^C = N - (1-F_2)(1+\frac{t_{12}}{t_{21}})S_{21} - (1-F_2)S_{22}.\end{aligned}$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we increase r_{12}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{q}_1 > q_1^* + k_1^* \tilde{q}_2$. We can obtain that

$$\begin{aligned}\Pi(\tilde{s},\tilde{r},\tilde{q}) - \Pi(s,r,q) &= (p-c_a)\sum_{i,j=1,2}(\tilde{s}_{ij}^A - s_{ij}^A) - (p-w)\sum_{i,j=1,2}(s_{ij}^C - \tilde{s}_{ij}^C) - c_a(\tilde{r}_{12}^A - r_{12}^A) \\ &= (p-c_a)(\frac{t_{21}}{t_{12}}+\frac{t_{21}S_{22}}{t_{12}S_{21}})\delta - (p-w)(\frac{t_{21}}{t_{12}}+\frac{t_{21}S_{22}}{t_{12}S_{21}})\delta - c_a\delta > 0.\end{aligned}$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit increases in η_1^A.

Case (ii): $q_1 = q_1^* + k_1^* q_2$ and $\eta_1^C > 0$. By (7)–(9) and (55)–(56), we have

$$r_{12}^C = \frac{t_{12}}{t_{21}} S_{21} - S_{12} - r_{12}^A,$$
$$q_1 = \frac{q_1^*}{1+k_1^*} + \frac{k_1^*(M + N - \sum_{i,j=1,2} S_{ij} - r_{12}^A - r_{12}^C)}{1+k_1^*}, \quad q_2 = \frac{(M + N - \sum_{i,j=1,2} S_{ij} - r_{12}^A - r_{12}^C - q_1^*)}{1+k_1^*},$$
$$F_1 = \frac{M - r_{12}^A - \frac{t_{21} r_{12}^A}{t_{12} S_{21}}(S_{21} + S_{22} + q_2)}{(S_{11} + S_{12} + q_1) + \frac{t_{21} S_{12}}{t_{12} S_{21}}(S_{21} + S_{22} + q_2)}, \quad F_2 = \frac{M + \frac{r_{12}^A}{S_{12}}(S_{11} + q_1)}{\frac{t_{12} S_{21}}{t_{21} S_{12}}(S_{11} + S_{12} + q_1) + (S_{21} + S_{22} + q_2)},$$
$$s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12}, \quad s_{21}^C = (1-F_2)S_{21}, \quad s_{22}^C = (1-F_2)S_{22}, \quad q_1^C = (1-F_1)q_1, \quad q_2^C = (1-F_2)q_2,$$
$$s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad S_{21}^A = F_2 S_{21}, \quad s_{22}^A = F_2 S_{22}, \quad q_1^A = F_1 q_1 \text{ and } q_2^A = F_2 q_2.$$

Let $(\tilde{s}, \tilde{r}, \tilde{q})$ be the corresponding DICCA if we increase r_{12}^A by $\delta > 0$ where δ is sufficiently small such that $\tilde{r}_{12}^A = r_{12}^A + \delta$, $\tilde{r}_{12}^C = r_{12}^C - \delta$, $\tilde{s}_{ij} = s_{ij}$ and $\tilde{q}_i = q_i$ for $i, j = 1, 2$. By Lemma B.1, a driver is indifferent among all repositioning strategies $\eta^* = (\eta_1^*, 0)$ with $\eta^* \in [0, 1]$ under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$. Without loss of generality, we consider the case where the driver adopts $\eta^* = (0, 0)$ under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$. Because $\tilde{q}_i = q_i$ for $i = 1, 2$, the expected earnings for the driver are the same under (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$, i.e., $\overline{\pi}(\eta^*, s, r, q) = \overline{\pi}(\eta^*, \tilde{s}, \tilde{r}, \tilde{q})$. It follows that $\pi(s, r, q) = (w - c_c) \sum_{i,j=1,2} s_{ij}^C - c_c r_{12}^C = \pi(\tilde{s}, \tilde{r}, \tilde{q}) = (w - c_c) \sum_{i,j=1,2} \tilde{s}_{ij}^C - c_c \tilde{r}_{12}^C$, which implies that $\sum_{i,j=1,2} (\tilde{s}_{ij}^C - s_{ij}^C) = \frac{c_c \delta}{(w - c_c)}$. Then, we have

$$\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = (p - c_a) \sum_{i,j=1,2} (\tilde{s}_{ij}^A - s_{ij}^A) - (p - w) \sum_{i,j=1,2} (s_{ij}^C - \tilde{s}_{ij}^C) - c_a \delta = (w - c_a) \frac{c_c \delta}{(w - c_c)} - c_a \delta > 0.$$

By some algebra, we can show that η_1^A increases in r_{12}^A. Therefore, the platform profit increases in η_1^A.

Case (iii): $q_1 < q_1^* + k_1^* q_2$. By Proposition (B.1), we have $\eta_1^C = 0$. By (6), we have $s_{ij} = S_{ij}$ for $i = 1, 2$. By (55)–(56), we have $r_{12}^A = \frac{t_{12}}{t_{21}} S_{21} - S_{12}$ which is a constant. By (11), it suffices to consider $G(\eta^A) = (p - w) \sum_{i,j=1,2} s_{ij}^C + (p - c_a) \sum_{i,j=1,2} s_{ij}^A$, which can be rewritten as

$$G(\eta^A) = (p - c_a)[\Lambda_1(P_{11} t_{11} + P_{12} t_{12}) \gamma X_1 + \Lambda_2(P_{21} t_{21} + P_{22} t_{22}) \gamma X_2]$$
$$+ (p - w)[\Lambda_1(P_{11} t_{11} + P_{12} t_{12}) \zeta Y_1 + \Lambda_2(P_{21} t_{21} + P_{22} t_{22}) \zeta Y_2],$$

where (γ, ζ) is the solution to (46)–(47) We can obtain that $\gamma = \frac{Y_2 - Y_1}{X_1Y_2 - X_2Y_1}$, $\zeta = \frac{X_1 - X_2}{X_1Y_2 - X_2Y_1}$ and $\frac{\partial \zeta Y_i}{\partial \eta_1^A} = -\frac{\partial \gamma X_i}{\partial \eta_1^A}$ for $i = 1, 2$. Therefore, $\frac{\partial G(\eta^A)}{\partial \eta_1^A} = \Lambda_1(P_{11}t_{11} + P_{12}t_{12})(w - c_a)\frac{\partial c X_1}{\partial \eta_1^A} + \Lambda_2(P_{21}t_{21} + P_{22}t_{22})(w - c_a)\frac{\partial c X_2}{\partial \eta_1^A}$. Combined with the fact that $\eta^C = (0, 0)$, we can obtain that

$$\frac{\partial G(\eta^A)}{\partial \eta_1^A} = \frac{(w - c_a)\Lambda_1^2\Lambda_2^2}{(X_1Y_2 - X_2Y_1)^2}[P_{21}(P_{11}t_{11}+P_{12}t_{12})+P_{12}(P_{21}t_{21}+P_{12}t_{12})][P_{21}(1-\eta_2^A P_{12})+\eta_2^A P_{11}P_{22}](\Lambda_{12}-\Lambda_{21}) < 0.$$

Therefore, the platform profit decreases in η_1^A.

D.1.3 Proof of Theorem 1 Step (iii)

From the monotonicity result, we can characterize the optimal AV repositioning strategy. We first introduce additional notation. By (55)–(56), in order to fulfill all the demand at location 2, the amount of vehicles in repositioning R_{imb} satisfies $\frac{S_{12}}{t_{12}} + \frac{R_{imb}}{t_{12}} = \frac{S_{21}}{t_{21}}$, which implies that

$$R_{imb} = \frac{t_{12}}{t_{21}} S_{21} - S_{12}.$$

Let C_1 denote the minimum amount of vehicles (reposition with probability 1) needed to cover all the repositioning of R_{imb}. Then we have

$$C_1 = R_{imb} + \frac{t_{21}}{t_{12}} R_{imb} + \frac{P_{22}t_{22}}{P_{21}t_{12}} R_{imb} = (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})(\frac{t_{12}}{t_{21}} S_{21} - S_{12}). \tag{58}$$

Let C_2 denote the minimum amount of vehicles (do not reposition) needed to serve all the demand at location 1. Then we have

$$C_2 = S_{11} + (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) S_{12}. \tag{59}$$

Let $S = \sum_{i,j=1,2} S_{ij}$. Observe that $S + R_{imb} = C_1 + C_2$. From Lemma D.1 and the monotonicity result in Step (ii), we can characterize the optimal strategy with respect to the following thresholds.

(1) If $M + N < C_2$, the optimal AV repositioning strategy results in a type A capacity allocation, which is given by $r_{12}^A = r_{12}^C = q_1^A = q_1^C = q_2^A = q_2^C = 0$,

$$s_{12}^A = \frac{M}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{11}^A = \frac{P_{11}t_{11}}{P_{12}t_{12}} s_{12}^A, \quad s_{21}^A = \frac{t_{21}}{t_{12}} s_{21}^A, \quad s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^A, \tag{60}$$

$$s_{12}^C = \frac{N}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{11}^C = \frac{P_{11}t_{11}}{P_{12}t_{12}} s_{12}^A, \quad s_{21}^C = \frac{t_{21}}{t_{12}} s_{21}^C \text{ and } s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^C. \tag{61}$$

(2) If $C_2 \leq M + N < S + q_1^* + R_{imb}$. The optimal AV repositioning strategy results in a type B capacity allocation and vehicles only queue up at location 1 (if there is any). The optimal strategy then depends on the value of $\hat{q}_1$.

(2.1) $\hat{q}_1 < 0$. In this case, it is optimal for the platform to reposition AVs as much as possible until $q_1 = 0$ or the demand at location 2 is fully served. There are 5 possible outcomes.

(2.1.1) $M \leq C_1$ and $N \leq C_2$. In this case, $\eta_1^{A*} \in (0, 1)$. Moreover, $\eta_1^C = 0$, $q_1 = 0$, (s^C, r^C, q^C) is given in (61) and (s^A, r^A, q^A) is given by

$$r_{12}^A = \frac{M - (S_{11} - s_{11}^C) - (1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})(S_{12} - s_{12}^C)}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{11}^A = S_{11} - s_{11}^C, \quad s_{12}^A = S_{12} - s_{12}^C, \quad s_{21}^A = \frac{t_{21}}{t_{12}}(s_{12}^A + r_{12}^A) \text{ and } s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}(s_{12}^A + r_{12}^A).$$

(2.1.2) $M \leq C_1$ and $C_2 < N \leq C_2 + q_1^*$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C = 0$, $0 < q_1 < q_1^*$, and (s, r, q) is given by

$$s_{21}^A = \frac{M}{1 + \frac{t_{12}}{t_{21}} + \frac{P_{22}t_{22}}{P_{21}t_{21}}}, \quad r_{12}^A = \frac{t_{12}}{t_{21}} s_{21}^A, \quad s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{21}} s_{21}^A, \quad s_{11}^A = s_{12}^A = q_1^A = 0, \tag{62}$$
$$s_{11}^C = S_{11}, \quad s_{12}^C = S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}} S_{12}, \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} S_{12} \text{ and } q_1^C = N - \sum_{i,j=1,2} s_{ij}^C. \tag{63}$$

(2.1.3) $M \leq C_1$ and $N > C_2 + q_1^*$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C \in [0, 1)$, $q_1 = q_1^*$, (s^A, r^A, q^A) is given in (62) and (s^C, r^C, q^C) is given by

$$r_{12}^C = \frac{N - q_1^* - C_2}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}, \quad s_{11}^C = S_{11}, \quad s_{12}^C = S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}}(S_{12} + r_{12}^C), \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}}(S_{12} + r_{12}^C) \text{ and } q_1^C = q_1^*.$$

(2.1.4) $M > C_1$ and $M + N \leq S + R_{imb}$. In this case, $\eta_1^{A*} \in (0, 1)$. Moreover, $\eta_1^C = 0$ and (s, r, q) is the same as that in (2.1.1).

(2.1.5) $M > C_1$ and $M+N > S+R_{imb}$. In this case, $\eta_1^{A*} \in (0,1)$ and (s,r,q) is given by

$$F_1 = \frac{M-C_1}{M+N-C_1}, \quad q_1 = M+N-S-R_{imb},$$
$$r_{12}^A = R_{imb}, \quad s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}}(s_{12}^A + r_{12}^A), \quad s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}(s_{12}^A + r_{12}^A),$$
$$s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}} s_{12}^C, \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^C, \quad q_1^C = (1-F_1)q_1 \text{ and } r_{12}^C = 0.$$

(2.2) $0 \le \hat{q}_1 < q_1^*$. In this case, it is optimal for the platform to reposition AVs as much as possible until $q_1 = \hat{q}_1$ or the demand at location 2 is fully served. There are 6 possible outcomes.

(2.2.1) $M+N < C_2 + \hat{q}_1$. In this case, $\eta_1^{A*} = 0$ and (s,r,q) is given by

$$F_1 = \frac{M}{M+N}, \quad q_1 = M+N-C_2, \quad s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}} s_{12}^A, \quad s_{22}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^A, \quad q_1^A = F_1 q_1,$$
$$s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}} s_{12}^C, \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^C \text{ and } q_1^C = (1-F_1)q_1.$$

(2.2.2) $M+N \ge C_2 + \hat{q}_1$, $M \le C_1$ and $N \le C_2 + \hat{q}_1$. In this case, $\eta_1^{A*} \in (0,1)$. Moreover, $q_1 = \hat{q}_1$, $\eta_1^C = 0$, and (s,r,q) is given by

$$F_1 = 1 - \frac{N}{C_2 + \hat{q}_1}, \quad r_{12}^A = \frac{M+N-C_2-\hat{q}_1}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}}(s_{12}^A + r_{12}^A), \quad s_{21}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}(s_{12}^A + r_{12}^A), \quad q_1^A = F_1 \hat{q}_1,$$
$$s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}} s_{12}^C, \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^C, \quad q_1^C = (1-F_1)\hat{q}_1 \text{ and } r_{12}^C = 0.$$

(2.2.3) $M+N \ge C_2 + \hat{q}_1$, $M \le C_1$ and $C_2 + \hat{q}_1 < N < C_2 + q_1^*$. In this case, $\eta_1^{A*} = 1$, and (s,r,q) is the same as that in (2.1.2).

(2.2.4) $M+N \ge C_2+\hat{q}_1$, $M \le C_1$ and $N \ge C_2+q_1^*$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C \in (0,1)$, $q_1 = q_1^*$, and (s,r,q) is the same as that in (2.1.3).

(2.2.5) $M+N \ge C_2 + \hat{q}_1$, $M > C_1$ and $M+N \le S + R_{imb} + \hat{q}_1$. In this case, $\eta_1^{A*} \in (0,1)$. Moreover, $\eta_1^C = 0$, $q_1 = \hat{q}_1$, and (s,r,q) is the same as that in (2.2.2).

(2.2.6) $M > C_1$ and $M+N > S + R_{imb} + \hat{q}_1$. In this case, $\eta_1^{A*} \in (0,1)$ and (s,r,q) is the same as that in (2.1.5).

(2.3) $\hat{q}_1 \geq q_1^*$. In this case, it is optimal for the platform to reposition AVs as much as possible until $q_1 = q_1^*$ and $r_{12}^C = 0$. There are 4 possible outcomes.

(2.3.1) $M + N < C_2 + q_1^*$. In this case, $\eta_1^{A*} = 0$. Moreover, $\eta_1^C = 0$ and (s, r, q) is the same as that in (2.2.1).

(2.3.2) $M + N \geq C_2 + q_1^*$, $M \leq C_1$ and $N < C_2 + q_1^*$. In this case, $\eta_1^{A*} \in (0, 1)$. Moreover, $\eta_1^C = 0$, $q_1 = q_1^*$ and (s, r, q) is given by

$$F_1 = 1 - \frac{N}{C_2 + q_1^*}, \quad r_{12}^A = \frac{M + N - C_2 - q_1^*}{1 + \frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}},$$
$$s_{11}^A = F_1 S_{11}, \quad s_{12}^A = F_1 S_{12}, \quad s_{21}^A = \frac{t_{21}}{t_{12}}(s_{12}^A + r_{12}^A), \quad s_{21}^A = \frac{P_{22}t_{22}}{P_{21}t_{12}}(s_{12}^A + r_{12}^A), \quad q_1^A = F_1 q_1^*,$$
$$s_{11}^C = (1 - F_1)S_{11}, \quad s_{12}^C = (1 - F_1)S_{12}, \quad s_{21}^C = \frac{t_{21}}{t_{12}} s_{12}^C, \quad s_{22}^C = \frac{P_{22}t_{22}}{P_{21}t_{12}} s_{12}^C, \quad q_1^C = (1 - F_1)q_1^* \text{ and } r_{12}^C = 0.$$

(2.3.3) $M+N \geq C_2+q_1^*$, $M \leq C_1$ and $N \geq C_2+q_1^*$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C \in (0, 1)$, $q_1 = q_1^*$, and (s, r, q) is the same as that in (2.1.3).

(2.3.4) $M + N \geq C_2 + q_1^*$, $M > C_1$. In this case, $\eta_1^{A*} \in (0, 1)$. Moreover, $\eta_1^C = 0$, $q_1 = q_1^*$, and (s, r, q) is the same as that in (2.3.2).

(3) $M + N > S + q_1^* + R_{imb}$. In this case, it is optimal for the platform to reposition AVs such that $q_1 = q_1^* + k_1^* q_2$ which results in a type C capacity allocation. There are 3 possible outcomes.

(3.1) $M \leq C_1$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C \in (0, 1)$, $q_1 = q_1^* + k_1^* q_2$ and (s, r, q) is given by

$$q_1 = \frac{q_1^*}{1 + k_1^*} + \frac{k_1^*(M + N - S - R_{imb})}{1 + k_1^*}, \quad q_2 = \frac{M + N - S - R_{imb} - q_1^*}{1 + k_1^*}, \quad F_2 = \frac{M}{S_{21}(1 + \frac{t_{12}}{t_{21}}) + S_{22} + q_2}$$
$$s_{11}^C = S_{11}, \quad s_{12}^C = S_{12}, \quad s_{21}^C = (1 - F_2)S_{21}, \quad s_{22}^C = (1 - F_2)S_{22}, \quad r_{12}^C = \frac{t_{12}}{t_{21}} s_{21}^C - s_{12}^C, \quad q_1^C = q_1, \quad q_2^C = (1 - F_2)q_2,$$
$$s_{11}^A = s_{12}^A = 0, \quad s_{21}^A = F_2 S_{21}, \quad s_{22}^A = F_2 S_{22}, \quad r_{12}^A = \frac{t_{12}}{t_{21}} s_{21}^A, \quad q_1^A = 0 \text{ and } q_2^A = F_2 q_2.$$

(3.2) $M > C_1$ and $N - C_2 \leq q_1^* + \frac{k_1^* S_{21} + \frac{t_{21}}{t_{12}} S_{12}}{S_{21} - \frac{t_{21}}{t_{12}} S_{12}}(M - C_1)$. In this case, $\eta_1^{A*} \in (0, 1)$. Moreover, $q_1 = q_1^* + k_1^* q_2$, $\eta_1^C = 0$ and (s, r, q) is given by

$$q_1 = \frac{q_1^*}{1 + k_1^*} + \frac{k_1^*(M + N - S - R_{imb})}{1 + k_1^*}, \quad q_2 = \frac{M + N - S - R_{imb} - q_1^*}{1 + k_1^*},$$

$F_1 = 1 - \frac{N}{(S_{11}+S_{12}+q_1)+\frac{t_{21}S_{12}}{t_{12}S_{21}}(S_{21}+S_{22}+q_2)}, \quad F_2 = 1 - \frac{N}{\frac{t_{12}S_{21}}{t_{21}S_{12}}(S_{11}+S_{12}+q_1)+(S_{21}+S_{22}+q_2)},$

$s_{11}^C = (1-F_1)S_{11}, \quad s_{12}^C = (1-F_1)S_{12}, \quad s_{21}^C = (1-F_2)S_{21}, \quad s_{22}^C = (1-F_2)S_{22}, \quad q_1^C = (1-F_1)q_1, \quad q_2^C = (1-F_2)q_2,$

$r_{12}^C = 0, \quad s_{11}^A = F_1S_{11}, \quad s_{12}^A = F_1S_{12}, \quad s_{21}^A = F_2S_{21}, \quad s_{22}^A = F_2S_{22}, \quad q_1^A = F_1q_1, \quad q_2^A = F_2q_2$ and $r_{12}^A = R_{imb}$.

(3.3) $M > C_1$ and $N - C_2 > q_1^* + \frac{k_1^*S_{21}+\frac{t_{21}}{t_{12}}S_{12}}{S_{21}-\frac{t_{21}}{t_{12}}S_{12}}(M-C_1)$. In this case, $\eta_1^{A*} = 1$. Moreover, $\eta_1^C \in (0,1)$, $q_1 = q_1^* + k_1^*q_2$, and (s,r,q) is the same as that in (3.1).

D.2 Proof of Proposition 1

In the proof of Theorem 1, we characterize the close-form DICCA under the optimal AV repositioning strategy. From step (iii) in the proof of Theorem 1, we observe that the DICCA is continuous in M and N (all the expressions are continuous in M and N). Therefore, we study the effects of increasing M and N case by case (as defined in step (iii) in the proof of Theorem 1) and assume that the change in M or N is sufficiently small such that it results in the same case (the boundary cases can be proved with similar arguments and thus we omit the details). Let (s,r,q) denote the DICCA under the optimal AV repositioning strategy given (M,N). Let $(\tilde{s},\tilde{r},\tilde{q})$ denote the DICCA under the optimal AV repositioning strategy given $(M+\delta,N)$ or $(M,N+\delta)$, where δ is sufficiently small such that the parameters fall in the same case as discussed in Step (iii) in the proof of Theorem 1. We first study the effect of varing M and then the effect of N. We denote by $\overline{\pi}(s,r,q)$ the average driver welfare under (s,r,q).

D.2.1 The Effect of AV supply

In case (1), we have $\tilde{s}_{ij}^A > s_{ij}^A$ and $\tilde{s}_{ij}^C = s_{ij}^C$. Therefore, $\Pi(\tilde{s},\tilde{r},\tilde{q}) > \Pi(s,r,q)$ and $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) = \overline{\pi}(s,r,q)$.

In case (2.1.1) and (2.1.4), because CVs do not reposition and $\tilde{s}_{ij}^C = s_{ij}^C$ for $i,j=1,2$, we have $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) = \overline{\pi}(s,r,q)$. Moreover, we have $\Pi(\tilde{s},\tilde{r},\tilde{q}) - \Pi(s,r,q) = \frac{(p-c_a)(1+\frac{P_{22}t_{22}}{P_{21}t_{21}})-c_a\frac{t_{12}}{t_{21}}}{1+\frac{t_{12}}{t_{21}}+\frac{P_{22}t_{22}}{P_{21}t_{21}}}\delta > 0$.

In case (2.1.2) and (2.2.3), because $\tilde{s}_{ij}^C = s_{ij}^C$ for $i,j=1,2$, we have $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) = \overline{\pi}(s,r,q)$. Moreover, we have $\Pi(\tilde{s},\tilde{r},\tilde{q}) - \Pi(s,r,q) = \frac{(p-c_a)(1+\frac{P_{22}t_{22}}{P_{21}t_{21}})-c_a\frac{t_{12}}{t_{21}}}{1+\frac{t_{12}}{t_{21}}+\frac{P_{22}t_{22}}{P_{21}t_{21}}}\delta > 0$.

In case (2.1.3), (2.2.4) and (2.3.3), because $r^C_{12} = \tilde{r}^C_{12}$ and $\tilde{s}^C_{ij} = s^C_{ij}$ for $i, j = 1, 2$, we have $\overline{\pi}(s, r, q) = \overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q})$. Moreover, we have $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = \frac{(p-c_a)(1+\frac{P_{22}t_{22}}{P_{21}t_{21}})-c_a\frac{t_{12}}{t_{21}}}{1+\frac{t_{12}}{t_{21}}+\frac{P_{22}t_{22}}{P_{21}t_{21}}}\delta > 0$.

In case (2.1.5) and (2.2.6), because $\tilde{F}_1 > F_1$, where $\tilde{F}_1$ is the fraction of demand served by AVs at location 1 under $(\tilde{s}, \tilde{r}, \tilde{q})$, we have $\tilde{s}^A_{ij} > s^A_{ij}$ and $\tilde{s}^C_{ij} < s^C_{ij}$ for $i, j = 1, 2$. Moreover, Because $\tilde{s}_{ij} = s_{ij}$ for $i, j = 1, 2$ and $\tilde{r}^A_{12} = r^A_{12} = R_{imb}$, it follows that $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) > \Pi(s, r, q)$ and $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) < \overline{\pi}(s, r, q)$.

In case (2.2.1) and (2.3.1) because both AVs and CVs do not reposition and $\tilde{F}_1 > F_1$, we have $\tilde{s}^A_{ij} > \tilde{s}^A_{ij}$ and $\tilde{s}^C_{ij} < s^C_{ij}$ and $\tilde{s}_{ij} = s_{ij}$ for $i, j = 1, 2$. It follows that $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) > \Pi(s, r, q)$ and $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) < \overline{\pi}(s, r, q)$.

In case (2.2.2) and (2.2.5), we have $\tilde{F}_1 = F_1$, $\tilde{r}^A_{12} > r^A_{12}$ and $\tilde{r}^C_{12} = r^C_{12} = 0$. It follows that $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = (p - c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})(\tilde{r}^A_{12} - r^A_{12}) - c_a(\tilde{r}^A_{12} - r^A_{12}) > 0$ and $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) = \overline{\pi}(s, r, q)$.

In case (2.3.2) and (2.3.4), by the same argument as in case (2.2.2) and (2.2.5), we can obtain that $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) > \Pi(s, r, q)$ and $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) = \overline{\pi}(s, r, q)$.

In case (3.1) and (3.3), because $\tilde{F}_2 > F_2$, where $\tilde{F}_2$ is the fraction of demand served at location 2 by AVs under $(\tilde{s}, \tilde{r}, \tilde{q})$, we have $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = (\tilde{F}_2 - F_2)[(w - c_a)(S_{21} + S_{22}) - c_a\frac{t_{12}}{t_{21}}S_{21}] > 0$. Moreover, we have $\pi(\tilde{s}, \tilde{r}, \tilde{q}) - \pi(s, r, q) = [(w - c_c)(S_{21} + S_{22}) - c_c\frac{t_{12}}{t_{21}}S_{21}](F_2 - \tilde{F}_2) < 0$ which implies that $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) < \overline{\pi}(s, r, q)$.

In case (3.2), we have $\tilde{F}_1 > F_1$ and $\tilde{F}_2 > F_2$, and thus $\sum_{i,j=1,2} \tilde{s}^A_{ij} > \sum_{i,j=1,2} s^A_{ij}$ and $\sum_{i,j=1,2} \tilde{s}^C_{ij} < \sum_{i,j=1,2} s^C_{ij}$. Because $\tilde{r}^A_{12} = r^A_{12} = R_{imb}$, $\tilde{r}^C_{12} = r^C_{12} = 0$ and $\sum_{i,j=1,2} \tilde{s}_{ij} = \sum_{i,j=1,2} s_{ij}$, we have $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) > \Pi(s, r, q)$ and $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) < \overline{\pi}(s, r, q)$.

D.2.2 The Effect of CV Supply

We first notice that $\hat{q}_1$ increases in N.

In case (1), because $\tilde{s}^A_{ij} = s^A_{ij}$, $\tilde{s}^C_{ij} > s^C_{ij}$, $\tilde{r}^A_{12} = r^A_{12} = 0$ and $\tilde{r}^C_{12} = r^C_{12} = 0$, we have $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) > \Pi(s, r, q)$. Because CVs serve customers all the time under both (s, r, q) and $(\tilde{s}, \tilde{r}, \tilde{q})$, we have $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) = \overline{\pi}(s, r, q)$.

In case (2.1.1) and (2.1.4), because CVs serve customers all the time under both $(\tilde{s}, \tilde{r}, \tilde{q})$ and (s, r, q), we have $\overline{\pi}(\tilde{s}, \tilde{r}, \tilde{q}) = \overline{\pi}(s, r, q)$. Let $\Omega = (1 + \frac{t_{21}}{t_{12}} + \frac{P_{11}t_{11}}{P_{12}t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})$, we have $r^A_{12} = \frac{M-\Omega(S_{12}-s^C_{12})}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}$. It follows that $\Pi(\tilde{s}, \tilde{r}, \tilde{q}) - \Pi(s, r, q) = (p\frac{\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}} - w)\Omega(\tilde{s}^C_{12} - s^C_{12})$. Because $\tilde{s}^C_{12} > s^C_{12}$, we can obtain that $\Pi(\tilde{s},\tilde{r},\tilde{q}) > \Pi(s,r,q)$ if $p\frac{\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}} - w > 0$, and $\Pi(\tilde{s},\tilde{r},\tilde{q}) < \Pi(s,r,q)$ otherwise.

In case (2.1.2) and (2.2.3), we have $\tilde{s}^A_{ij} = s^A_{ij}$, $\tilde{s}^C_{ij} = s^C_{ij}$, $\tilde{r}^A_{12} = r^A_{12}$, $\tilde{r}^C_{12} = r^C_{12} = 0$, and $\tilde{q}_1 > q_1$, which implies that $\Pi(\tilde{s},\tilde{r},\tilde{q}) = \Pi(s,r,q)$ and $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) < \overline{\pi}(s,r,q)$.

In case (2.1.3), (2.2.4) and (2.3.3), because $\tilde{s}^A_{ij} = s^A_{ij}$ for $i,j = 1,2$, $\tilde{r}^A_{12} = r^A_{12}$ and $\sum_{i,j=1,2}\tilde{s}^C_{ij} > \sum_{i,j=1,2}s^C_{ij}$, we have $\Pi(\tilde{s},\tilde{r},\tilde{q}) > \Pi(s,r,q)$. By Lemma B.1, a driver is indifferent among all the repositioning strategy $\eta^* = (\eta^*_1, 0)$ with $\eta^{C*}_1 \in [0,1]$. Without loss of generality, we consider the case where $\eta^* = (0,0)$. Because $\tilde{q}_1 = q_1 = q^*_1$, the expected earning for the driver remains the same. It follows that $\overline{\pi}(s,r,q) = \overline{\pi}(\tilde{s},\tilde{r},\tilde{q})$.

In case (2.1.5) and (2.2.6), because $\tilde{F}_1 < F_1$, we have $\Pi(\tilde{s},\tilde{r},\tilde{q}) - \Pi(s,r,q) = (w-c_a)(\tilde{F}_1 - F_1)C_2 < 0$. Because $\overline{\pi}(s,r,q) = (w-c_c)\frac{C_2}{M+N-C_1}$ decreases in N, we conclude that $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) < \overline{\pi}(s,r,q)$.

In case (2.2.1) and (2.3.1), we have $\sum_{i,j=1,2}\tilde{s}_{ij} = \sum_{i,j=1,2}s_{ij} = C_2$, $\tilde{r}^A_{12} = r^A_{12} = 0$ and $\tilde{r}^C_{12} = \tilde{r}^C_{12} = 0$. Because $\tilde{s}^A_{ij} < s^A_{ij}$ and $s^C_{ij} > s^C_{ij}$, we have $\Pi(\tilde{s},\tilde{r},\tilde{q}) < \Pi(s,r,q)$. Because CVs do not reposition under both (s,r,q) and $(\tilde{s},\tilde{r},\tilde{q})$ and $\tilde{q}_1 > q_1$, we have $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) < \overline{\pi}(s,r,q)$.

In case (2.2.2) and (2.2.5), the platform profit can be written as

$$\begin{aligned}\Pi(s,r,q) &= (p-c_a)F_1C_2 + (p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})r^A_{12} + (p-w)(1-F_1)C_2 - c_a r^A_{12} \\ &= (p-c_a)C_2 + \frac{(M+N)[(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a]}{1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}} - 2\sqrt{\frac{(w-c_a)N[(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a]C_2}{1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}}}.\end{aligned} \tag{64}$$

In case (2.2.2), we have $N \leq C_2 + \hat{q}_1$, which implies that

$$N[(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a] < (w-c_a)C_2(1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}). \tag{65}$$

In case (2.2.5), Because $M > C_1$, it follows that $N \leq S + R_{imb} + \hat{q}_1 - M \leq S + R_{imb} + \hat{q}_1 - C_1 = C_2 + \hat{q}_1$, which also implies (65). Then, it follows that $\frac{\partial \Pi(s,r,q)}{\partial N} = \frac{(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a}{1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}} - \sqrt{\frac{(w-c_a)[(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a]C_2}{N(1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}} < 0$ by (65). Therefore, $\Pi(\tilde{s},\tilde{r},\tilde{q}) < \Pi(s,r,q)$. Because $\overline{\pi}(s,r,q) = \frac{(w-c_c)(1-F_1)C_2}{N} = (w-c_c)\sqrt{\frac{[(p-c_a)(\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}}) - c_a]C_2}{N(w-c_a)(1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{P_{21}t_{12}})}}$ decreases in N, we have $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q}) < \overline{\pi}(s,r,q)$.

In case (2.3.2) and (2.3.4), r^A_{12} can be written as $r^A_{12} = \frac{M - F_1(C_2 + q^*_1)}{1+\frac{t_{21}}{t_{12}} + \frac{P_{22}t_{22}}{p_{21}t_{12}}}$. Therefore, we have

$\Pi(\tilde{s},\tilde{r},\tilde{q})-\Pi(s,r,q)=\{(w-c_a)C_2-[(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a]\frac{C_2+q_1^*}{1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}}}\}(\tilde{F}_1-F_1)$. Because $\hat{q}_1\geq q_1^*$ and $N<C_2+q_1^*$, we have

$$\sqrt{\frac{(w-c_a)NC_2(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})}{(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a}}-C_2>q_1^*,$$

which implies that

$$\sqrt{\frac{(w-c_a)C_2(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})}{(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a}(C_2+q_1^*)}>q_1^*+C_2.$$

Therefore, we have $\frac{(w-c_a)C_2(1+\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})}{[(p-c_a)(\frac{t_{21}}{t_{12}}+\frac{P_{22}t_{22}}{P_{21}t_{12}})-c_a](C_2+q_1^*)}>1$, which implies $\Pi(\tilde{s},\tilde{r},\tilde{q})<\Pi(s,r,q)$ as $\tilde{F}_1<F_1$. Because CVs do not reposition and $\tilde{q}_1=q_1=q_1^*$, the expected earning for a driver under both $(\tilde{s},\tilde{r},\tilde{q})$ and (s,r,q) are the same. It follows that $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q})=\overline{\pi}(s,r,q)$.

In case (3.1) and (3.3), we have $\tilde{F}_2<F_2$ and $\Pi(\tilde{s},\tilde{r},\tilde{q})-\Pi(s,r,q)=[(w-c_a)(S_{21}+S_{22})-c_a\frac{t_{12}}{t_{21}}S_{21}](\tilde{F}_2-F_2)<0$. By Lemma B.1, a driver is indifferent among all the repositioning strategy $\eta^*=(\eta_1^*,0)$ with $\eta^*\in[0,1]$ under both $(\tilde{s},\tilde{r},\tilde{q})$ and (s,r,q). Without loss of generality, we consider the case where $\eta^*=(0,0)$. Because $\tilde{q}_1>q_1$ and $\tilde{q}_2>q_2$, we have $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q})<\overline{\pi}(s,r,q)$.

In case (3.2), because $\tilde{r}_{12}^A=r_{12}^A=R_{imb}$, $\tilde{r}_{12}^C=r_{12}^C=0$, $\tilde{F}_1<F_1$ and $\tilde{F}_2<F_2$, we have $\Pi(\tilde{s},\tilde{r},\tilde{q})-\Pi(s,r,q)=(w-c_a)[(S_{11}+S_{12})(\tilde{F}_1-F_1)+(S_{21}+S_{22})(\tilde{F}_2-F_2)]<0$. Because CVs do not reposition under both (s,r,q) and $(\tilde{s},\tilde{r},\tilde{q})$, $\tilde{q}_1>q_1$ and $\tilde{q}_2>q_2$, it follows that $\overline{\pi}(\tilde{s},\tilde{r},\tilde{q})<\pi(s,r,q)$.

In Remark 2, We summarize conditions under which driver welfare does not decrease in M and the platform profit decreases in N.

Remark 2. *Driver welfare does not decrease in M in case (1), (2.1.1), (2.1.2), (2.1.3), (2.1.4), (2.2.2), (2.2.3), (2.2.4), (2.2.5),(2.3.2), (2.3.3) and (2.3.4) specified in Appendix D.1.3. The platform profit decreases in N in case (2.1.1) and (2.1.4) if $ap<w$ (a is defined in* (67)*), (2.1.5), (2.2.1), (2.2.2), (2.2.5), (2.2.6), (2.3.1), (2.3.2), (2.3.4), (3.1), (3.2) and (3.3) specified in Appendix D.1.3.*

E Proofs for Systems where the supply of AVs and CVs are Endogenized

In this section, we prove results for systems where the supply of AVs and CVs are endogenized. Similar to the proof of Theorem 1, by Proposition B.1, we can reformulate the problem faced by the platform as the following capacity allocation problem:

$$\max_{M,w,\eta^A} \quad \Pi(M, w, \eta^A)$$
$$\text{subject to} \quad (1)\text{–}(10),\ (12) \text{ and } \ (s, r, q) \in D.$$

We note that when w is endogenized, q_1^* depends on w. Throughout the remainder of the Appendix, we use $q_1^*(w)$ to indicate the dependence when needed, and use q_1^* when no confusion is caused.

We first demonstrate that any strategy resulting a capacity allocation which satisfies one of the properties listed in Lemma E.1 are dominated by other strategies (Lemma E.1 implies the properties of optimal strategies stated in Theorem 2). Based on Lemma E.1, we first characterize the optimal strategy in a system without AVs in Appendix E.2 and then in a system with AVs in Appendix E.3.

Lemma E.1. *When the supply of AVs and CVs are endogenized, any strategy of the platform resulting in the following capacity allocation cannot be optimal:*

(i) vehicles queue up at location 2 *(the high demand location);*

(ii) vehicles queue up at location 1 *and CVs do not reposition;*

(iii) vehicles queue up at location 1*, both AVs and CVs reposition with positive probabilities and AVs reposition with probability less than* 1.

Proof of Lemma E.1. In case (i), suppose there exists a DICCA (s, r, q) such that $q_2 > 0$. If CVs do not reposition, there exists a DICCA $(\tilde{s}, \tilde{r}, \tilde{q})$ such that $\tilde{s}_{ij}^A = s_{ij}^A$, $\tilde{s}_{ij}^C = s_{ij}^C$, $\tilde{r}_{ij}^A = r_{ij}^A$, $\tilde{r}_{ij}^C = r_{ij}^C = 0$ and $\tilde{q}_1 = \tilde{q}_2 = 0$. In this case, the demand served by AVs and CVs remains the same while the platform incurs a lower cost by recruiting less AVs and drivers. If CVs reposition, by Proposition B.1, we consider two scenarios. (a) $q_1 = q_1^* + k_1^* q_2$: then there exists a DICCA $(\tilde{s}, \tilde{r}, \tilde{q})$ such that $\tilde{s}_{ij}^A = s_{ij}^A$, $\tilde{s}_{ij}^C = s_{ij}^C$, $\tilde{r}_{ij}^A = r_{ij}^A$, $\tilde{r}_{ij}^C = r_{ij}^C$, $\tilde{q}_1 = q_1^*$ and $\tilde{q}_2 = 0$. In this case, the demand served by AVs and CVs are the same while the platform incurs a lower cost under $(\tilde{s}, \tilde{r}, \tilde{q})$. (b) $q_1^* > k_1^* + q_2$:

by Proposition B.1, CVs reposition with probability 1 and $q_1 = q_1^A$. In this case, the platform can improve its profit by purchasing less AVs to lower the queue length at location 1.

In case (ii), the platform can remove vehicles queueing at location 1 and the resulted capacity allocation is still driver-incentive compatible. By doing so, the demand fulfilled by AVs and CVs remains the same and the platform incurs a lower cost.

In case (iii), the platform can be better off from increasing the repositioning of AVs (see the proof of Theorem 1). □

E.1 Proof of Lemma 2

We denote by $\overline{\pi}(M, N, w, \eta^A)$ the average driver welfare $\overline{\pi}$ to indicate its dependence on (M, N, w, η^A). From the proof of Lemma C.1, Lemma 1 and Theorem 1, we observe that $\overline{\pi}(M, N, w, \eta^A)$ is continuous in N. Because $\lim_{N\to 0} \overline{\pi}(M, N, w, \eta^A) > 0$ and $\lim_{N\to\infty} \overline{\pi}(M, N, w, \eta^A) = 0$, it suffices to show that $\overline{\pi}(M, N, w, \eta^A)$ weakly decreases in N given any (M, w, η^A) such that $w \geq c_c$. Let (s, r, q) be the DICCA given (M, N, w, η^A). Let $\overline{\pi}'_+(M, N, w, \eta^A)$ be the right-hand derivative of $\overline{\pi}(M, N, w, \eta^A)$ with respect to N. We investigate all possible cases of the DICCA and show that $\overline{\pi}(M, N, w, \eta^A)$ weakly decreases in N in each case.

Case (i) $q_1 = q_2 = 0$ and the demand at location 1 is not fully served. Because the demand at location 1 is not fully served and CVs are serving customers all the time, $\overline{\pi}'_+(M, N, w, \eta^A) = 0$.

Case (ii.i) $q_1 \in [0, q_1^*)$, the demand at location 1 is fully served, and the demand at location 2 is not fully served. Because CVs have no incentive to reposition and q_1 increases in N (see case (ii) and case (iii.i) in the proof of Lemma C.1), $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (ii.ii) $q_2 \in [0, q_2^*)$, the demand at location 2 is fully served, and the demand at location 1 is not fully served. By a similar argument to that of Case (ii.i), we have $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (iii.i) $q_1 = q_1^*$ and the demand at location 2 is not fully served. Because the demand at location 2 is not fully served, the repositioning of CVs increases in N while q_1 does not change. Therefore, $\overline{\pi}'_+(M, N, w, \eta^A) = 0$.

Case (iii.ii) $q_2 = q_2^*$ and the demand at location 1 is not fully served. By a similar argument to that of Case (iii.i), we have $\overline{\pi}'_+(M, N, w, \eta^A) = 0$.

Case (iv.i) $q_1 > q_1^*$ and the demand at location 2 is not fully served. CVs reposition with probability 1 as N increases. Hence, $\overline{\pi}'_+(M, N, w, \eta^A) = 0$.

Case (iv.ii) $q_2 > q_2^*$ and the demand at location 1 is not fully served. By a similar argument to that of Case (iv.i), we have $\overline{\pi}'_+(M, N, w, \eta^A) = 0$.

Case (v) $q_1 < q_1^* + k_1^* q_2$, $q_2 < q_2^* + k_2^* q_1$, and the demand at both locations is fully served. CVs do not reposition the demand served by CVs remains the same (see case (iii.ii) in the proof of Lemma C.1). Therefore, $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (vi.i) $q_1 = q_1^* + k_1^* q_2$, and the demand at both locations is fully served. If CVs reposition, then both q_1 and q_2 increase in N, which implies that $\overline{\pi}'_+(M, N, w, \eta^A) < 0$. Otherwise, depending on the parameters, either both q_1 and q_2 increase in N, or q_1 decreases in N and q_2 increases in N and the demand served by CVs remains the same. In all scenarios, $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (vi.ii) $q_2 = q_2^* + k_2^* q_1$, and the demand at both locations is fully served. By a similar argument to that of Case (vi.i), we have $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (vii.i) $q_1 > q_1^* + k_1^* q_2$, and the demand at both locations is fully served. CVs reposition with probability 1, and q_2 increases in N. Therefore, $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Case (vii.ii) $q_2 > q_2^* + k_2^* q_1$, and the demand at both locations is fully served. By a similar argument to that of Case (vii.i), we have $\overline{\pi}'_+(M, N, w, \eta^A) < 0$.

Therefore, we conclude that $\overline{\pi}(M, N, w, \eta^A)$ weakly decreases in N and thus (12) has a unique solution.

E.2 Proof of Theorem 3

In a system without AVs, the problem faced by the platform can be reformulated as the following capacity allocation problem:

$$\max_{M,w,\eta^A} \quad \max_{w} \quad (p - w) \sum_{i,j=1,2} s_{ij}^C$$

subject to (1), (3), (5), (6), (8), (10) and (12) with $M = 0$, $F_k = 0$ for $k = 1, 2$ and $(s^C, r^C, q^C) \in D$.

We first characterize the profit function in terms of N in a system without AVs. By Lemma E.1, we consider the following two cases.

Case (1): CVs do not queue or reposition. In this case, drivers serve customers all the time. Given w, $\overline{\pi} = w - c_c$ and thus (12) can be simplified as $N = \frac{L(w-c_c)}{(p-c_c)}$. Therefore, w can be written as a function of N, which we denote by $w_1(N)$:

$$w_1(N) = \frac{N(p - c_c)}{L} + c_c. \tag{66}$$

We then use $\Pi_1(N)$ to denote the platform profit $N(p - w_1(N))$ as a function of N:

$$\Pi_1(N) = [(p - c_c) - \frac{N}{L}(p - c_c)]N.$$

Case (2): CVs reposition from location 1 to location 2 with a positive probability. In case (2), the demand at location 1 is fully fulfilled and $q_1 = q_1^*$ by Proposition B.1. Moreover, in steady state, an amount $C_2 + a(N - C_2 - q_1^*)$ of CVs is serving customers, an amount q_1^* of CVs is queueing at location 1, and an amount $(1 - a)(N - C_2 - q_1^*)$ of CVs is repositioning from location 1 to location 2, where C_2 and q_1^* are given in (59) and (23), and a is defined as

$$a = \frac{P_{21}t_{21} + P_{22}t_{22}}{P_{21}t_{21} + P_{22}t_{22} + P_{21}t_{12}}. \tag{67}$$

By Lemma B.1, when $q_1 = q_1^*$, a driver is indifferent among all the repositioning strategies $\eta^* = (\eta_1^*, 0)$ with $\eta_1^* \in [0, 1]$. Without loss of generality, we consider the case where $\eta^* = (1, 0)$ and the expected earning for the driver is

$$PF_1 = aw - c_c. \tag{68}$$

In this case, (12) can be simplified as $N = \frac{PF_1 L}{p-c_c}$. Therefore, w can be written as a function of N, which we denote by $w_2(N)$:

$$w_2(N) = \frac{1}{a}[\frac{N(p - c_c)}{L} + c_c]. \tag{69}$$

We then use $\Pi_2(N)$ to denote the platform profit as a function of N in case (2):

$$\Pi_2(N) = [C_2 + a(N - C_2 - q_1^*)](p - w_2(N)).$$

In Lemma E.2, we show that both $\Pi_1(N)$ and $\Pi_2(N)$ are concave.

Lemma E.2. $\Pi_1(N)$ *and* $\Pi_2(N)$ *are concave.*

Proof of Lemma E.2. Obviously, $\Pi_1(N)$ is concave. Taking the first order derivative for $\Pi_2(N)$ (for simplicity, we omit the subscript for w_2), we have

$$\Pi_2'(N) = a(1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})(p-w) - [C_2 + a(N - C_2 - q_1^*)]\frac{\partial w}{\partial N},$$

where $\frac{\partial w}{\partial N} = \frac{p-c_c}{aL} > 0$, q_1^* is given in (53) and

$$\frac{\partial q_1^*}{\partial w} = -\frac{\Lambda_1 P_{12}t_{12}(P_{21}t_{21} + P_{22}t_{22}) + \Lambda_1(P_{11}t_{11} + P_{12}t_{12})P_{21}t_{12}}{[(P_{21}t_{21} + P_{22}t_{22}) - \frac{c_c}{w}(P_{22}t_{22} + P_{21}t_{21} + P_{21}t_{12})]^2}\frac{c_c}{w^2}(P_{22}t_{22} + P_{21}t_{21} + P_{21}t_{12}). \quad (70)$$

Observe that $\frac{\partial q_1^*}{\partial w}$ increases in w, q_1^* decreases in w, w increases in N and $\frac{\partial w}{\partial N}$ is a constant. It follows that $\Pi_2'(N)$ decreases in N and thus $\Pi_2(N)$ is concave. □

In Lemma E.3, we show that if $\Pi_1'(C_2) \leq 0$, it is not optimal for the platform to let CVs reposition.

Lemma E.3. *If* $\Pi_1'(C_2) \leq 0$, *any strategy which results in a DICCA such that CVs reposition with a positive probability cannot be optimal.*

Proof of Lemma E.3. We first define a pseudo system under which CVs reposition to keep $q_1 = 0$ if the demand at location 1 is fully served. Under the pseudo system, if the demand at location 1 is fully served, $C_2 + a(N - C_2)$ amount of CVs are serving customers and $(1-a)(N - C_2)$ amount of CVs are repositioning. In this case, the average driver welfare is $\frac{C_2(w-c_c)+(N-C_2)PF_1}{N}$ and we can obtain that $N = \frac{L}{p-c_c}\frac{C_2(w-c_c)+(N-C_2)PF_1}{N}$, which implies that $w = \frac{\frac{N^2}{L}(p-c_c)+c_cN}{C_2+a(N-C_2)}$ and a is defined in (67). The platform profit can be written as $\hat{\Pi}(N) = [C_2 + a(N - C_2)](p - w)$ and its derivative is

$$\hat{\Pi}'(N) = a(p-w) - \frac{[2\frac{N}{L}(p-c_c) + c_c](C_2 - aC_2) + a\frac{N^2}{L}(p-c_c)}{C_2 + a(N - C_2)}. \quad (71)$$

When $N = C_2$, the corresponding wage $w = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$. It follows that $\Pi_1'(C_2) =$

$(p-c_c)(1-\frac{2C_2}{L})=(p-w)-\frac{p-c_c}{L}C_2$. We can obtain that

$$\begin{aligned}\hat{\Pi}'(C_2) &= a(p-w)-[2\frac{C_2}{L}(p-c_c)+c_c](1-a)+a\frac{C_2}{L}(p-c_c)\\ &= a(p-w)-c_c(1-a)-\frac{1}{L}(p-c_c)(C_2-aC_2)-\frac{p-c_c}{L}C_2\\ &\le (p-w)-\frac{p-c_c}{L}C_2=\Pi_1'(C_2).\end{aligned}$$

We then show that $\hat{\Pi}'(N)$ decreases in N. Taking the second order derivative, we can obtain

$$\begin{aligned}\hat{\Pi}''(N)=&-a\frac{[2\frac{N}{L}(p-c_c)+c_c](C_2-aC_2)+a\frac{N^2}{L}(p-c_c)}{[C_2+a(N-C_2)]^2}\\ &-\frac{[\frac{2}{L}(p-c_c)(C_2-aC_2)+\frac{2aN}{L}(p-c_c)][C_2+a(N-C_2)]-\left([2\frac{N}{L}(p-c_c)+c_c](C_2-aC_2)+a\frac{N^2}{L}(p-c_c)\right)a}{[C_2+a(N-C_2)]^2}\\ =&-\frac{\frac{2}{L}(p-c_c)(C_2-aC_2)+\frac{2aN}{L}(p-c_c)}{C_2+a(N-C_2)}<0.\end{aligned}$$

Therefore, in the pseudo system, if $\Pi_1'(C_2)\le 0$, the platform has no incentive to let CVs reposition. Because the platform makes a higher profit in the pseudo system than that in the original system under optimal strategies (recall that in the original system, CVs reposition only when $q_1\ge q_1^*$), the desired result follows. □

We then prove a useful inequality in Lemma E.4.

Lemma E.4. *Given any DICCA such that $\eta_1^C>0$, we have $\frac{C_2}{C_2+q_1^*}\le a$, where the equality holds if $c_c=0$.*

Proof of Lemma E.4. Under a DICCA such that $r_{12}^C>0$, a driver is indifferent between not repositioning and repositioning with probability 1. That is $\frac{C_2(w-c_c)}{C_2+q_1^*}=aw-c_c$, which implies the desired result. □

We define two parameters w_a and w_b. Recall that q_1^* depends on the wage paid to drivers w and we use $q_1^*(w)$ to indicate this dependence. If $L\ge\frac{p-c_c}{ap-c_c}(C_2+q_1^*(p))$, we let w_a denote the unique solution to $\frac{(aw-c_c)}{p-c_c}L=C_2+q_1^*(w)$. Otherwise, we let $w_a=p$. Similarly, if $L\ge\frac{p-c_c}{ap-c_c}(C_1+C_2+q_1^*(p))$, we let w_b denote the unique solution to $\frac{(aw-c_c)}{p-c_c}L=C_1+C_2+q_1^*(w)$. Otherwise, we let $w_b=p$.

Lemma E.5. *If $\Pi_1'(C_2) \geq 0$ and $\Pi_2'(C_2 + q_1^*(w_a)) \geq 0$, there exists a threshold L_1 such that $\Pi_1(C_2) \geq \Pi_2(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ if $L \leq L_1$ and $\Pi_1(C_2) < \Pi_2(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ otherwise, where $\tilde{N}$ is the unique solution to $\Pi_2'(N) = 0$.*

Proof of Lemma E.5. It suffices to show that if $\Pi_1(C_2) \leq \Pi_2(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$, $\frac{\partial \Pi_1(C_2)}{\partial L} \leq \frac{\partial \Pi_2(\tilde{N} \wedge (C_1+C_2+q_1^*(w_b)))}{\partial L}$. Recall the definitions of $w_1(N)$ and $w_2(N)$ in (66) and (69). We first consider the case where $\tilde{N} < (C_1 + C_2 + q_1^*(w_b))$. If $\Pi_1'(C_2) \geq 0$, which implies that $L \geq 2C_2$, we have $w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c \leq \frac{p-c_c}{2} + c_c$. Therefore, we can obtain that

$$\frac{\partial \Pi_1(C_2)}{\partial L} = \frac{C_2^2}{L^2}(p - c_c) = \frac{C_2}{L}(w_1(C_2) - c_c) \leq \frac{C_2}{L}(p - w_1(C_2)) = \frac{\Pi_1(C_2)}{L}.$$

Because $\tilde{N}$ is the unique solution to $\Pi_2'(N) = 0$, it follows that

$$\frac{\partial q_1^*}{\partial w} a(p - w_2(\tilde{N})) + [C_2 + (\tilde{N} - C_2 - q_1^*)a] = \frac{a(p - w_2(\tilde{N}))}{\frac{\partial w}{\partial N}} = a(p - w_2(\tilde{N}))\frac{La}{p - c_c}.$$

By the Envelope Theorem, we have

$$\frac{\partial \Pi_2(\tilde{N})}{\partial L} = -\frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial L} a(p - w_2(\tilde{N})) - [C_2 + (\tilde{N} - C_2 - q_1^*)a]\frac{\partial w}{\partial L} = \frac{\tilde{N}a}{L}(p - w_2(\tilde{N})) \geq \frac{\Pi_2(\tilde{N})}{L},$$

where the last inequality follows from Lemma E.4. Therefore, if $\Pi_2(\tilde{N}) \geq \Pi_1(C_2)$, we have $\frac{\partial \Pi_2(\tilde{N})}{\partial L} \geq \frac{\partial \Pi_1(C_2)}{\partial L}$, which leads to the desired result.

We then consider the case where $\tilde{N} \geq (C_1 + C_2 + q_1^*(w_b))$. The desired result follows as $\frac{\partial \Pi_2(C_2+C_1+q_1^*)}{\partial L} = \frac{(C_2+aC_1)(C_2+C_1+q_1^*)(p-c_c)}{aL^2} > \frac{\partial \Pi_1(C_2)}{\partial L} = \frac{C_2^2}{L^2}(p - c_c)$. □

By Lemma E.2, Lemma E.3, and Lemma E.5, we can characterize the optimal strategy for the platform in a system without AVs. From Lemma E.2, we know that the optimal strategy either leads to: (a) CVs do not reposition and the platform recruits $\min\{\tilde{N}, C_2\}$ amount of drivers where $\tilde{N}$ is the unique solution to $\Pi_1'(N) = 0$; or (b) CVs reposition with a positive probability and the platform recruits $\min\{\tilde{N}, C_1 + C_2 + q_1^*(w_b)\}$ amount of drivers where $\tilde{N}$ is the unique solution to $\Pi_2'(N) = 0$. If $\Pi_1'(C_2) \leq 0$, by Lemma E.3, the optimal strategy leads to scenario (a). Otherwise, by Lemma E.5, there exists a threshold L_1 on the driver pool size such that the optimal strategy leads to scenario (a) if $L \geq L_1$ and the optimal strategy leads to scenario (b) otherwise. In particular, we have the following possible outcomes.

Case (C.i): $\Pi_1'(C_2) < 0$. By Lemma E.3, the optimal strategy for the platform is to recruit N^* amount of drivers, where N^* is the unique solution to $\Pi_1'(N) = (p - c_c)(1 - \frac{2N}{L}) = 0$. It follows that $N^* = \frac{L}{2}$, $w^* = \frac{p+c_c}{2}$ and the corresponding profit is $\Pi^* = \Pi_1(N^*) = \frac{L(p-c_c)}{4}$.

Case (C.ii): $\Pi_1'(C_2) \geq 0$ and $\Pi_2'(C_2 + q_1^*(w_a)) < 0$. By Lemma E.3, the optimal strategy for the platform is to recruit C_2 amount of CVs. It follows that $N^* = C_2$, $w^* = \frac{C_2(p-c_c)}{L} + c_c$ and the corresponding profit is $\Pi^* = \Pi_1(C_2) = (p - c_c)(1 - \frac{C_2}{L})C_2$.

Case (C.iii): $\Pi_1'(C_2) \geq 0$, $\Pi_2'(C_2 + q_1^*(w_a)) \geq 0$ and $\Pi_2'(C_1 + C_2 + q_1^*(w_b)) < 0$. By Lemma E.5, there exists a threshold L_1 such that (a) if the driver pool size is small than L_1, $N^* = C_2$, $w^* = \frac{C_2(p-c_c)}{L} + c_c$ and $\Pi^* = \Pi_1(C_2) = (p - c_c)(1 - \frac{C_2}{L})C_2$; (b) otherwise, N^* is the unique solution to $\Pi_2'(N) = 0$, $w^* = w_2(N^*)$ and $\Pi^* = \Pi_2(N^*)$.

Case (C.iv): $\Pi_1'(C_2) \geq 0$ and $\Pi_2'(C_1 + C_2 + q_1^*(w_b)) \geq 0$. By Lemma E.5 There exists a threshold L_1 such that (a) if the driver pool size is smaller than L_1, $N^* = C_2$, $w^* = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$ and $\Pi^* = \Pi_1(C_2) = (p - c_c)(1 - \frac{C_2}{L})C_2$; (b) otherwise, $N^* = C_1 + C_2 + q_1^*(w_b)$, $w^* = w_2(N^*)$ and $\Pi = \Pi_2(C_1 + C_2 + q_1^*)$.

E.3 Proof of Theorem 2

We first introduce additional notation. The profit earned by the platform from one unit supply of AVs serving customers all the time is

$$MG_1^A = p - I - c_a.$$

The profit earned by the platform from one unit supply of AVs which reposition from location 1 to location 2 with probability 1 (assume the demand at location 2 is not fully served) is

$$MG_2^A = ap - c_a - I, \tag{72}$$

where a is defined in (67).

We then characterize the platform profit in terms of N (amount of CVs). By Lemma E.1, it suffices to consider the following cases.

Case (a): both AVs and CVs do not reposition and the demand at location 1 is fully served (because $MG_1^A > 0$ by assumption, the platform always fulfill the demand from location 1) and there is no queue at location 1. The profit earned by the platform from CVs and AVs is $\Pi_1(N)$ and $(C_2 - N) \cdot MG_1^A$ respectively, where C_2 is defined in (59). Therefore, the platform profit is

$$\Pi_3(N) = \Pi_1(N) + (C_2 - N) \cdot MG_1^A.$$

Case (b): CVs do not reposition and AVs reposition with a positive probability such that the demand at location 1 and location 2 are fully served (no queue at both locations). The profit earned by the platform from CVs and AVs is $\Pi_1(N)$ and $(C_2 - N)MG_1^A + C_1 \cdot MG_2^A$ respectively. Therefore, the platform profit is

$$\Pi_4(N) = \Pi_1(N) + (C_2 - N)MG_1^A + C_1 \cdot MG_2^A.$$

Case (c): CVs reposition with a positive probability and AVs reposition with probability 1 such that the demand at location 1 and location 2 are fully served. The profit earned by the platform from CVs and AVs is $\Pi_2(N)$ and $(C_1 + C_2 + q_1^* - N) \cdot MG_2^A$ respectively. Therefore, platform profit is

$$\Pi_5(N) = \Pi_2(N) + (C_1 + C_2 + q_1^* - N) \cdot MG_2^A.$$

In Lemma E.6, we show that if $\Pi_3'(C_2) = \Pi_4'(C_2) \leq 0$, it is not optimal for the platform to let CVs reposition.

Lemma E.6. *If $\Pi_3'(C_2) = \Pi_4'(C_2) \leq 0$, any strategy which results in a DICCA such that CVs reposition with a positive probability cannot be optimal.*

Proof of Lemma E.6. We consider a pseudo system which has been introduced in the proof of Lemma E.3. Under the pseudo system, CVs reposition when all the demand at location 1 is served to keep $q_1 = 0$. It suffices to show that $\Pi_3'(C_2) \leq 0$ implies $\hat{\Pi}'(C_2) \leq MG_2^A$ where $\hat{\Pi}'(N)$ is defined in (71). Because $\Pi_3'(C_2) \leq 0$ is equivalent to $\frac{p-c_c}{L}C_2 \geq I + c_a - w_1(C_2)$ and $\hat{\Pi}'(C_2) \leq MG_2^A$ is equivalent to $a(p - w_1(C_2)) - c_c(1-a) - \frac{1}{L}(p - c_c)(C_2 - aC_2) - \frac{p-c_c}{L}C_2 \leq ap - I - c_a$, it suffices to show that $I + c_a - w_1(C_2) \geq I + c_a - aw_1(C_2) - c_c(1-a) - \frac{1}{L}(p-c_c)(C_2 - aC_2)$, which is equivalent to $\frac{1}{L}(p-c_c)C_2 \geq w_1(C_2) - c_c$. This is true as $w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$. □

In Lemma E.7, we show that $\Pi_3(N)$, $\Pi_4(N)$ and $\Pi_5(N)$ have some good properties.

Lemma E.7. $\Pi_3(N)$ *and* $\Pi_4(N)$ *are concave. If* $\Pi_5'(N^*) > 0$*, then* $\Pi_5(N)$ *is unimodal over* $[N^*, \infty)$.

Proof of Lemma E.7. Obviously, $\Pi_3(N)$ and $\Pi_4(N)$ are concave. Taking derivative of $\Pi_5(N)$:

$$\Pi_5'(N) = (1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})(I + c_a - aw) - [C_2 + a(N - C_2 - q_1^*)]\frac{\partial w}{\partial N}.$$

Because $\frac{\partial q_1^*}{\partial w} < 0$, $\frac{\partial q_1^*}{\partial w}$ increases in w, and $\frac{\partial w}{\partial N}$ is a positive constant, if $\Pi_5'(N) \geq 0$, $\Pi_5''(N) < 0$, which implies the desired result. □

Lemma E.8. *Assume* $\Pi_4'(C_2) \geq 0$ *and* $\Pi_5'(C_2 + q_1^*(w_a)) \geq 0$. *When* I *is fixed, there exists a threshold* $L_2(I)$ *such that* $\Pi_4(C_2) \geq \Pi_5(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ *if* $L \leq L_2(I)$ *and* $\Pi_4(C_2) < \Pi_5(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ *otherwise, where* $\tilde{N}$ *is the unique solution to* $\Pi_5'(N) = 0$.

When L *is fixed, there exists a threshold* $I_2(L)$ *such that* $\Pi_4(C_2) \geq \Pi_5(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ *if* $I \leq I_2(L)$ *and* $\Pi_4(C_2) < \Pi_5(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))$ *otherwise.*

Proof of Lemma E.8. We notice that $\Pi_4(C_2) > \Pi_5(C_2 + q_1^*(w_a))$. Therefore, to obtain the first result, it suffices to show that when $\Pi_4(C_2) \leq \Pi_5\left(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b))\right)$, $\frac{\partial \Pi_5(\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b)))}{\partial L} \geq \frac{\partial \Pi_4(C_2)}{\partial L}$. When I is fixed, we first consider the case where $\tilde{N} < C_1 + C_2 + q_1^*(w_b))$. We have $\frac{\partial \Pi_4(C_2)}{\partial L} = \frac{C_2^2}{L^2}(p - c_c) = \frac{C_2}{L}(w_1(C_2) - c_c)$. Because $\tilde{N}$ is the unique solution to $\Pi_5'(N) = 0$, it follows that

$$\frac{\partial \Pi_5(\tilde{N})}{\partial N} = (1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})[a(p - w_2(\tilde{N})) - MG_2^A] - [C_2 + a(\tilde{N} - C_2 - q_1^*)]\frac{\partial w}{\partial N} = 0.$$

By the Envelope Theorem, we have

$$\begin{aligned}\frac{\partial \Pi_5(\tilde{N})}{\partial L} &= [MG_2^A - a(p - w_2(\tilde{N}))]\frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial L} - [C_2 + a(\tilde{N} - C_2 - q_1^*)]\frac{\partial w}{\partial L} \\ &= [a(p - w_2(\tilde{N})) - MG_2^A]\frac{\tilde{N}}{L}.\end{aligned}$$

Let Π_0 be the platform profit if all the demand at location 1 and location 2 is served by AVs. Then $\Pi_0 = C_2 \cdot MG_1^A + C_1 \cdot MG_2^A$.

Let $\Delta_1 = \Pi_4(C_2) - \Pi_0 = C_2(p - w_1(C_2) - MG_1^A) = C_2(I + c_a - w_1(C_2))$. If $\Pi_4'(C_2) \geq 0$, which is equivalent to $(p - c_c)(1 - \frac{2C_1}{L}) > p - I - c_a$, we have $I + c_a > c_c + 2(w_1(C_2) - c_c)$. It follows that $\Delta_1 > C_2(w_1(C_2) - c_c) \geq \frac{\partial \Pi_4(C_2)}{\partial L} L$.

Let $\Delta_2 = \Pi_5(\tilde{N}) - \Pi_0 = C_2(p - w_2(\tilde{N}) - MG_1^A) + (\tilde{N} - C_2 - q_1^*)[a(p - w_2(\tilde{N})) - MG_2^A]$. Because $(C_2 + q_1^*)[a(p - w_2(\tilde{N})) - MG_2^A] > (C_2 + q_1^*)a[(p - w_2(\tilde{N})) - MG_1^A] > C_2[(p - w_2(\tilde{N})) - MG_1^A]$, where the last inequality follows from Lemma E.4, it follows that $\Delta_2 \leq \tilde{N}[a(p - w_2(\tilde{N})) - MG_2^A] = \frac{\partial \Pi_5(\tilde{N}, L)}{\partial L} L$.

Therefore, if $\Pi_4(C_2) < \Pi_5(\tilde{N})$, we must have $\Delta_1 \leq \Delta_2$ which implies that $\frac{\partial \Pi_5(\tilde{N})}{\partial L} \geq \frac{\partial \Pi_4(C_2)}{\partial L}$. The desired result follows.

We then consider the case where $\tilde{N} \geq (C_1 + C_2 + q_1^*(w_b)))$. The desired result follows as $\frac{\partial \Pi_5(C_2 + C_1 + q_1^*)}{\partial L} \geq \frac{(C_2 + aC_1)(C_2 + C_1 + q_1^*)(p - c_c)}{aL^2} > \frac{\partial \Pi_4(C_2)}{\partial L} = \frac{C_2^2}{L^2}(p - c_c)$.

When L is fixed, we prove the desired result by contradiction. Let $\Pi_4(N, I)$ and $\Pi_5(N, I)$ denote the values of $\Pi_4(N)$ and $\Pi_5(N)$ to indicate their dependence on the AV purchase cost I. Let $\tilde{N}_I = \tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b))$, where $\tilde{N}$ is the unique solution to $\frac{\partial \Pi_5(N, I)}{\partial N} = 0$. Suppose there exist $I_1 < I_2$ such that $\Pi_4(C_2, I_2) > \Pi_5(\tilde{N}_{I_2}, I_2)$ and $\Pi_4(C_2, I_1) < \Pi_5(\tilde{N}_{I_1}, I_1)$. Because $\Pi_5(N, I)$ is unimodal in N, we must have $\Pi_5(\tilde{N}_{I_2}, I_2) \geq \Pi_5(\tilde{N}_{I_1}, I_2)$. Moreover, we have $\Pi_4(C_2, I_1) - \Pi_4(C_2, I_2) = C_1(I_2 - I_1)$ and $\Pi_5(\tilde{N}_{I_1}, I_1) - \Pi_5(\tilde{N}_{I_2}, I_2) < C_1(I_2 - I_1)$. It follows that $\Pi_4(C_2, I_1) = \Pi_4(C_2, I_2) + C_1(I_2 - I_1) > \Pi_5(\tilde{N}_{I_2}, I_2) + C_1(I_2 - I_1) > \Pi_5(\tilde{N}_{I_1}, I_1)$, which leads to a contradiction, and the desired result follows. □

By Lemma E.6, Lemma E.7 and Lemma E.8, we can characterize the optimal strategy for the platform in the system with AVs. First, we note that if $MG_2^A < 0$, the platform has no incentive to let AVs reposition. In this case, by Lemma E.6, if $\Pi_3'(C_2) \leq 0$, it suffices to investigate $\Pi_3(N)$. Otherwise, we also need to investigate $\Pi_2(N)$. Similarly, if $MG_2^A \geq 0$, the platform can make positive profit by repositioning AVs. In this case, by Lemma E.6, if $\Pi_4'(C_2) \leq 0$, it suffices to investigate $\Pi_4(N)$. Otherwise, we also need to investigate $\Pi_5(N)$. Therefore, we consider the following cases.

Case (AC.i): $MG_2^A \leq 0$ and $\Pi_2'(C_2 + q_1^*(w_a)) \leq 0$. In this case, the platform has no incentive to let CVs or AVs reposition. It suffices to consider the following subcases.

Case (AC.i.i): $\Pi_3(0)' \leq 0$. The platform operates with only AVs. The demand at location 1 is fully served while AVs do not queue or reposition. We have $M^* = C_2$, $w^* = 0$, $N^* = 0$ and the corresponding profit is $\Pi^* = \Pi_3(0)$.

Case (AC.i.ii): $\Pi_3'(C_2) < 0 < \Pi_3'(0)$. It is optimal for the platform to recruit N^* amount of drivers and purchase $C_2 - N^*$ amount of AVs where N^* is the unique solution to $\Pi_3'(N) = 0$. It follows that $N^* = \frac{L(I+c_a-c_c)}{2(p-c_c)}$, $w^* = \frac{I+c_a+c_c}{2}$, $M^* = C_2 - N^*$ and the corresponding profit is $\Pi^* = \Pi_3(N^*)$.

Case (AC.i.iii): $\Pi_3'(C_2) \geq 0$. It is optimal for the platform to operate with only CVs such that the demand at location 1 is fully served and CVs do not queue or reposition. Therefore, $N^* = C_2$, $w^* = \frac{C_2(p-c_c)}{L} + c_c$, $M^* = 0$ and the corresponding profit is $\Pi^* = \Pi_3(C_2)$.

Case (AC.ii): $MG_2^A > 0$ and $\Pi_5'(C_2 + q_1^*(w_a)) \leq 0$. In this case, the platform has an incentive to let AVs reposition but not CVs. It suffices to consider the following subcases.

Case (AC.ii.i): $\Pi_4'(0) \leq 0$. It is optimal for the platform to operate with only AVs such that all the demand at location 1 and location 2 is fully served. It follows that $w^* = 0$, $N^* = 0$, $M^* = C_1 + C_2$ and the corresponding profit is $\Pi^* = \Pi_4(0)$.

Case (AC.ii.ii): $\Pi_4(C_2) < 0 < \Pi_4(0)$. It is optimal for the platform to recruit N^* amount of drivers and purchase $C_1 + C_2 - N^*$ amount of AVs such that all the demand at location 1 and location 2 is fully served while CVs do not reposition, where N^* is the unique solution to $\Pi_4'(N) = 0$. It follows that $N^* = \frac{L(I+c_a-c_c)}{2(p-c_c)}$, $w^* = \frac{I+c_a+c_c}{2}$, $M^* = C_1 + C_2 - N^*$ and the corresponding profit is $\Pi^* = \Pi_4(N^*)$.

Case (AC.ii.iii): $\Pi_4'(C_2) \geq 0$. It is optimal for the platform to recruit C_2 amount of drivers and purchase C_1 amount of AVs such that all the demand at location 1 is served by CVs and AVs reposition with probability 1. Therefore, $N^* = C_2$, $w^* = \frac{C_2(p-c_c)}{L} + c_c$, $M^* = C_1$ and the corresponding profit is $\Pi^* = \Pi_4(C_2)$.

Case (AC.iii): $MG_2^A \leq 0$ and $\Pi_2'(C_2 + q_1^*(w_a)) > 0$. In this case, the platform has no incentive to let AVs reposition, but might let CVs reposition depending on the parameters.

Case (AC.iii.i): $\Pi_3'(0) \leq 0$. By Lemma E.6, the optimal strategy and outcomes are exactly the same as that in case (AC.i.i).

Case (AC.iii.ii): $\Pi_3'(C_2) \leq 0 < \Pi_3'(0)$. By Lemma E.6, the optimal outcomes is exactly the same as that in case (AC.i.ii).

Case (AC.iii.iii): $\Pi_3'(C_2) > 0$ and $\Pi_2'(C_1+C_2+q_1^*(w_b)) < 0$. The optimal strategy and outcomes are the same as that in case (C.iii) (in the proof of Theorem 3).

Case (AC.iii.iv): $\Pi_3'(C_2) > 0$ and $\Pi_2'(C_1+C_2+q_1^*(w_b)) \geq 0$. The optimal strategy and outcomes are the same as that in case (C.iv) (in the proof of Theorem 3).

Case (AC.iv): $MG_2^A > 0$ and $\Pi_5'(C_2 + q_1^*(w_a)) \geq 0$. In this case, the platform might let AVs and CVs reposition depending on the parameters. It suffices to consider the following subcases.

Case (AC.iv.i): $\Pi_4'(0) \leq 0$. By Lemma E.6, the optimal strategy and outcomes are exactly the same as that in case (AC.ii.i).

Case (AC.iv.ii): $\Pi_4'(C_2) \leq 0 < \Pi_4'(0)$. By Lemma E.6, the optimal strategy and outcomes are exactly the same as that in case (AC.ii.ii).

Case (AC.iv.iii): $\Pi_4'(C_2) > 0$ and $\Pi_5'(C_1 + C_2 + q_1^*(w_b)) \leq 0$. In this case, the optimal strategy of the platform falls into one of the following scenarios whichever gives a higher profit.

(a) All the demand at location 1 is served by CVs, CVs do not queue or reposition and AVs reposition with probability 1. That is: $N^* = C_2$, $w^* = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$, $M^* = C_1$ and the corresponding profit is $\Pi^* = \Pi_4(N^*)$.

(b) All the demand at locating 1 is served by CVs, CVs reposition with a positive probability and AVs reposition with probability 1. That is: N^* is the unique solution to $\Pi_5'(N) = 0$, $w^* = w_2(N_b)$, $M^* = C_1 + C_2 + q_1^* - N^*$ and the corresponding profit is $\Pi^* = \Pi_5(N^*)$, where $w_2(N)$ is given in (69).

By Lemma E.8, given any I, there exists a threshold $L_2(L)$ such that if $L \leq L_2(I)$, the optimal strategy falls into scenario (a) and it falls into scenario (b) otherwise.

Case (AC.iv.iv): $\Pi_4'(C_2) > 0$ and $\Pi_5'(C_1 + C_2 + q_1^*(w_b)) > 0$. In this case, the optimal strategy of the platform falls into one of the following scenarios whichever gives a higher profit:

(a) All the demand at locating 1 is served by CVs, CVs do not queue or reposition and AVs reposition with probability 1. That is: $N^* = C_2$, $w^* = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$, $M^* = C_1$ and the corresponding profit $\Pi^* = \Pi_4(N^*)$.

(b) The platform operates with only CVs to serve all the demand at both locations. That is: $N^* = C_1 + C_2 + q_1^*(w_b)$, $w^* = w_b$, $M^* = 0$, and the corresponding profit is $\Pi^* = \Pi_5(N^*)$.

By Lemma E.8, given any I, there exists a threshold $L_2(I)$ such that if $L \leq L_2(I)$, the optimal strategy falls into scenario (a) and it falls into scenario (b) otherwise. □

In Remark 3, we group possible cases of optimal outcomes into 7 types of equilibria introduced in Section 5

Remark 3. *Case (AC.i.i) and (AC.iii.i) belong to type I equilibra. Case (AC.ii.i) and (AC.vi.i) belong to type II equilibra. Case (AC.i.ii) and (AC.iii.ii) belong to type III equilibra. Case (AC.ii.ii), (AC.ii.iii), (AC.iv.ii), (AC.iv.iii) when $L \leq L_2(I)$ and (AC.iv.iv) when $L \leq L_2(I)$ belong to type IV equilibra. Case (AC.iv.iii) when $L \geq L_2(I)$ belongs to Type V equilibra. Case (AC.i.iii), (AC.iii.iii) when $L \leq L_1$ and (AC.iii.iv) when $L \leq L_1$ belong to type VI equilibra. Case (AC.iii.iii) when $L \geq L_1$, (AC.iii.iv) when $L \geq L_1$ and (AC.iv.iv) when $L \geq L_2(I)$ belong to type VII equilibra.*

E.4 Proof of Proposition 2

It follows directly from the mathematical expression of the optimal strategy in the proof of Theorem 2 that w^* (weakly) increases in I and (weakly) decreases in L; M^* (weakly) decreases in I and L in case (AC.i.i), (AC.i.ii), (AC.i.iii), (AC.ii.i), (AC.ii.ii), (AC.ii.iii), (AC.iii.i), (AC.iii.ii), (AC.iii.iv) (AC.iv.i), (AC.iv.ii), and (AC.iv.iv).

In case (AC.iii.iii), the platform operates with only CVs. Therefore, $M^* = 0$ is constant. By Lemma E.5, there exists a threshold L_1 such that if $L \leq L_1$, the optimal strategy is given by $N^* = C_2$, $w^* = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$. Otherwise, N^* is the unique solution to $\Pi_2'(N) = 0$ and $w^* = w_2(N^*)$. When $L \leq L_1$, w^* decreases in L. When $L > L_1$, recall that $N = \frac{L(aw-c_c)}{p-c_c}$. Let

$$F(w, L) = \Pi_2'(N) = a(1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})(p - w) - [C_2 + a(\frac{L(aw - c_c)}{p - c_c} - C_2 - q_1^*)]\frac{\partial w}{\partial N}.$$

We can obtain $\frac{\partial w^*}{\partial L}$ through Implicit function theorem given $F(w, L) = 0$. Because $\frac{\partial q_1^*}{\partial w} \leq 0$ and

$\frac{\partial^2 q_1^*}{\partial w^2} > 0$, we have

$$\frac{\partial F(w,L)}{\partial w} = -a\frac{\partial^2 q_1^*}{\partial w^2}\frac{\partial w}{\partial N}(p-w) - a(1-\frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N}) - a(\frac{aL}{p-c_c} - \frac{\partial q_1^*}{\partial w})\frac{\partial w}{\partial N} < 0 \text{ and}$$
$$\frac{\partial F(w,L)}{\partial L} = -a\frac{\partial q_1^*}{\partial w}\frac{\partial w^*}{\partial N \partial L}(p-w) - a\frac{aw-c_c}{p-c_c}\frac{\partial w}{\partial N} - [C_2 + a(\frac{L(aw-c_c)}{p-c_c} - C_2 - q_1^*)]\frac{\partial^2 w}{\partial N \partial L}$$
$$= \frac{a}{L}(p + \frac{c_c}{a} - 2w),$$

where the last equality is implied by $F(w,L) = 0$. Moreover, because $F(w,L) = 0$ and

$$q_1^* > \frac{S_{12}(P_{21}t_{21} + P_{22}t_{22}) + P_{21}t_{12}(S_{11} + S_{12})}{P_{21}t_{21} + P_{22}t_{22}} > \frac{P_{21}t_{12}}{P_{21}t_{21} + P_{22}t_{22}}C_2,$$

we have

$$a(p + \frac{c_c}{a} - 2w) \leq a(1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})(p-w) - (aw - c_c) = [(1-a)C_2 - aq_1^*]\frac{\partial w}{\partial N} < 0.$$

Therefore, by Implicit function theorem, $\frac{\partial w}{\partial L} = -\frac{\partial F}{\partial L}/\frac{\partial F}{\partial w} < 0$. It follows that w^* decreases in L. Moreover, because $\tilde{N} > C_2$ where $\tilde{N}$ is the unique solution to $\Pi_2'(N) = 0$, we have $\lim_{L \uparrow L_1} w^* = \lim_{L \uparrow L_1} w_1(C_2) < \lim_{L \downarrow L_1} w_2(\tilde{N}) = \lim_{L \downarrow L_1} w^*$. Therefore, there is a jump at L_1 and thus w^* is non-monotone in L.

In case (AC.iv.iii), by Lemma E.8, there exists a threshold $L_2(I)$ such that if $L \leq L_2(I)$, the optimal strategy is given by $M^* = C_1$, $w^* = w_1(C_2) = \frac{C_2(p-c_c)}{L} + c_c$. Otherwise, N^* is the unique solution to $\Pi_5'(N) = 0$, $w^* = w_2(N^*)$ and $M^* = C_1 + C_2 - N^*$. When $L \leq L_2(I)$, M^* is a constant, w^* decreases in L and does not depend on I. When $L > L_2(I)$, we first show that w^* decreases in L. Let

$$F(w,L) = \Pi_5'(N) = (1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})(I + c_a - aw) - [C_2 + (\frac{L(aw-c_c)}{p-c_c} - C_2 - q_1^*)a]\frac{\partial w}{\partial N}.$$

By a similar analysis to that in case (AC.iii.i), we have

$$\frac{\partial F(w,L)}{\partial w} = -\frac{\partial^2 q_1^*}{\partial w^2}\frac{\partial w}{\partial N}(I + c_a - aw) - a(1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N}) - a(\frac{aL}{p-c_c} - \frac{\partial q_1^*}{\partial w})\frac{\partial w}{\partial N} < 0, \text{ and}$$
$$\frac{\partial F(w,L)}{\partial L} = -\frac{\partial q_1^*}{\partial w}\frac{\partial^2 w}{\partial N \partial L}(I + c_a - aw) - a\frac{aw-c_c}{p-c_c}\frac{\partial w}{\partial N} - [C_2 + (\frac{L(aw-c_c)}{p-c_c} - C_2 - q_1^*)a]\frac{\partial^2 w}{\partial w \partial L}$$

$$= \frac{1}{L}(I + c_a + c_c - 2aw) < 0.$$

By implicit function theorem, we have $\frac{\partial w}{\partial L} = \frac{\partial F}{\partial L} / \frac{\partial F}{\partial w} \leq 0$. Therefore, w^* decreases in L. Moreover, by the same argument as in case (AC.iii.i), $\lim_{L \uparrow L_2(I)} w^* = \lim_{L \uparrow L_2(I)} w_1(C_2) < \lim_{L \downarrow L_2(I)} w_2(\tilde{N}) = \lim_{L \downarrow L_2(I)} w^*$, where $\tilde{N}$ is the unique solution to $\Pi_5'(N) = 0$. Therefore, there is a jump at $L_2(I)$ and thus w^* is non-monotone in L.

We then show that M^* decreases in L. Let

$$F(N, L) = \Pi_5'(N) = (1 - \frac{\partial q_1^*}{\partial w} \frac{\partial w}{\partial N})(I + c_a - \frac{N(p - c_c)}{L} - c_c) - [C_2 + (N - C_2 - q_1^*)a] \frac{\partial w}{\partial N}.$$

Then we have

$$\begin{aligned}
\frac{\partial F(N, L)}{\partial N} &= -\frac{\partial^2 q_1^*}{\partial w^2} (\frac{\partial w}{\partial N})^2 (I + c_a - \frac{N(p - c_c)}{L} - c_c) - (1 - \frac{\partial q_1^*}{\partial w} \frac{\partial w}{\partial N}) \frac{(p - c_c)}{L} - a(1 - \frac{\partial q_1^*}{\partial w} \frac{\partial w}{\partial N}) \frac{\partial w}{\partial N} < 0, \quad \text{and} \\
\frac{\partial F(N, L)}{\partial L} &= -\frac{\partial q_1^*}{\partial w} \frac{\partial^2 w}{\partial N \partial L} (I + c_a - \frac{N(p - c_c)}{L} - c_c) + (1 - \frac{\partial q_1^*}{\partial w} \frac{\partial w}{\partial N}) \frac{N(p - c_c)}{L^2} - [C_2 + (N - C_2 - q_1^*)] \frac{\partial^2 w}{\partial N \partial L} \\
&= \frac{1}{L}(I + c_a - \frac{N(p - c_c)}{L} - c_c) + (1 - \frac{\partial q_1^*}{\partial w} \frac{\partial w}{\partial N}) \frac{N(p - c_c)}{L^2} > 0
\end{aligned}$$

By the Implicit function theorem, $\frac{\partial N}{\partial L} = -\frac{\partial F}{\partial L} / \frac{\partial F}{\partial N} > 0$. It follows that M^* decreases in L.

Lastly, because $\Pi_5'(N)$ increases in I, we have w^* and N^* increases in I and thus M^* decreases in I.

E.5 Proof of Proposition 3

Because the platform always has the option of using only CVs (i.e., does not purchase any AVs), the platform profit in the system with AVs is (weakly) higher than that in the system without AVs.

We then show that the average driver welfare is (weakly) lower in the system with AVs. By (12), to compare the average driver welfare, it suffices to compare the amount of drivers recruited by the platform. The desired result follows immediately by observing that $\Pi_1'(N) \geq \Pi_3'(N)$ if $MG_2^A < 0$, $\Pi_1'(N) \geq \Pi_4'(N)$ and $\Pi_2'(N) \geq \Pi_5'(N)$ if $MG_2^A \geq 0$.

Next, we show that the service level is (weakly) higher in the system with AVs. The desired result follows directly if $MG_2^A \geq 0$ as the service level is 1 in the system with AVs. If $MG_2^A < 0$, we consider two cases: (a) CVs do not reposition in the system without AVs. In this case, the demand fulfilled is less than (or equal to) C_2. In the system with AVs, the demand fulfilled is C_2. It follows that the service level weakly increases in the system with AVs. (b) CVs reposition in the system without AVs, which implies that $\Pi_1'(C_2) \geq 0$ and $\Pi_2'(C_1 + q_1^*(w_a)) \geq 0$ and $\Pi_1(C_2) \leq \Pi_2(\tilde{N})$, where $\tilde{N}$ is the unique solution to $\Pi_2(\tilde{N}) = 0$. Because $\Pi_3'(C_2) \leq \Pi_1'(C_2)$, we consider two subcases. (b.1) $\Pi_3'(C_2) \geq 0$, then it is optimal for the platform to only use CVs which results in the same service level. (b.2) $\Pi_3'(C_2) < 0$. By the proof of Lemma E.6, we have $\hat{\Pi}'(C_2) \leq MG_2^A$, where $\hat{\Pi}(N)$ is the profit function for the pseudo system with only CVs and CVs reposition to keep $q_1 = 0$ if the demand at location 1 is fully fulfilled. Therefore, CVs do not reposition in the pseudo system, which implies that CVs do not reposition in the system without AVs, and thus this case is not possible.

We then investigate the driver productivity. By the analysis in Appendix E.2, in a system without AVs, we consider two cases. Case (i): CVs neither queue nor reposition and Case (2): CVs reposition with a positive probability and there are q_1^* amount of AVs queue at location 1. In case (i), because CVs serve customers all the time, the driver productivity is 1. Therefore, the introduction of AVs (weakly) decreases driver productivity. In case (ii), the amount of drivers recruited by the platform $N^C = \tilde{N}^C \wedge (C_1 + C_2 + q_1^*(w_b))$ where $\tilde{N}^C$ is the unique solution to $\Pi_2'(N) = 0$ and $N^C > C_2 + q_1^*$. According to the analysis in Appendix E.3, we consider three possibilities after the introduction of AVs. Case (ii.1): CVs neither queue nor reposition. In this case, the productivity is 1 and thus the introduction of AVs increases driver productivity. Case (ii.2): CVs reposition with a positive probability and $MG_2^A < 0$. In this case, the platform operates with only CVs. Therefore, the outcomes in systems with and without AVs are the same and thus the driver productivity remains the same. Case (ii.3): CVs reposition with a positive probability and $MG_2^A \geq 0$. In this case, the amount of drivers recruited by the platform $N^{AC} = \tilde{N}^{AC} \wedge (C_1 + C_2 + q_1^*(w_b))$ where $\tilde{N}^{AC}$ is the unique solution to $\Pi_5'(N) = 0$. When CVs reposition with a positive probability, we can define driver productivity $\rho(N) = \frac{C_2 + a(N - C_2 - q_1^*(\tilde{w}))}{N}$ as as a function of N. Taking the first order derivative, we have $\rho'(N) = \frac{1}{N^2}[a(1 - \frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N})N - C_2 - a(N - C_2 - q_1^*)] = \frac{1}{N^2}[-a\frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N}N - C_2 + a(C_2 + q_1^*)] \geq \frac{1}{N^2}[-a\frac{\partial q_1^*}{\partial w}\frac{\partial w}{\partial N}] \geq 0$, where the first inequality is due to Lemma E.4. Because $N^C \geq N^{AC}$, we have $\rho(N^C) \geq \rho(N^{AC})$.

Lastly, we specify conditions under which a Pareto-improving outcome can be achieved. In case (C.ii) and (C.iii) scenario (a) (in the proof Theorem 3 for the system without AVs), all the demand at location 1 is served and CVs do not reposition. In case (AC.ii.iii), (AC.iv.iii) scenario (a) (in the proof of Theorem 2 for the system with AVs), all the demand at location 1 is served by CVs and CVs do not reposition, AVs reposition with probability 1 to fulfill otherwise unfulfilled demand due to demand imbalance. In these cases, the amount of drivers recruited is C_2. In the system with AVs, the platform purchase C_1 amount of AVs such that all the demand at location 1 and location 2 is served. It follows that the service level increases, the platform makes $C_1 \cdot MG_2^A$ more profit in the system with AVs and the average driver welfare remains the same. Therefore, the parameter range such that a Pareto-improving outcome can be achieved is given by

$$(E_1 \cup E_2) \cap (E_3 \cup E_4), \tag{73}$$

where

$$E_1 = \{\Lambda_{ij}, t_{ij}, I, L, p, c_c, c_a \mid \Pi_1'(C_2) \geq 0, \Pi_2'(C_2 + q_1^*(w_a)) \leq 0\},$$
$$E_2 = \{\Lambda_{ij}, t_{ij}, I, L, p, c_c, c_a \mid \Pi_1'(C_2) \geq 0, \Pi_2'(C_2 + q_1^*(w_a)) \geq 0, L \leq L_1\},$$
$$E_3 = \{\Lambda_{ij}, t_{ij}, I, L, p, c_c, c_a \mid MG_2^A \geq 0, \Pi_3'(C_2) \geq 0, \Pi_5'(C_2 + q_1^*(w_a)) \leq 0\} \text{ and}$$
$$E_4 = \{\Lambda_{ij}, t_{ij}, I, L, p, c_c, c_a \mid MG_2^A \geq 0, \Pi_3'(C_2) \geq 0, \Pi_5'(C_2 + q_1^*(w_a)) \geq 0, L \leq L_2(I)\}.$$

F Proof of Theorem 4

By Theorem 2, we can obtain the optimal strategy and the corresponding optimal profit Π^* for the platform given Λ_{ij} for $i, j \in \{1, 2\}$. From the proof of Theorem 2, we can obtain that if $I < \frac{2C_2}{L}(p - c_c) + c_c - c_a$, the platform's optimal strategy could lead to four possible types of equilibra (Type I through Type IV). In order to obtain the optimal admission control strategy, we analyze Π^* with respect to S_{ij} for $i, j \in \{1, 2\}$, where $S_{ij} = \Lambda_{ij} t_{ij}$. We conduct the analysis for each type of equilibra separately, and consider the case that an infinitesimal change in S_{ij} does not change the resulted equilibra type (the boundary cases can be analyzed similarly).

First, we recall that $C_1 = (1 + \frac{t_{21}}{t_{12}} + \frac{S_{22}}{S_{21}\frac{t_{12}}{t_{21}}})(\frac{t_{12}}{t_{21}} S_{21} - S_{12})$, $C_2 = S_{11} + (1 + \frac{t_{21}}{t_{12}} + \frac{S_{22}}{S_{21}\frac{t_{12}}{t_{21}}})S_{12}$ and

$a = \frac{S_{21}+S_{22}}{S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}}}$. We can obtain that

$$\begin{aligned}
&\frac{\partial C_1}{\partial S_{11}} = 0, \quad \frac{\partial C_2}{\partial S_{11}} = 1, \quad \frac{\partial a}{\partial S_{11}} = 0, \\
&\frac{\partial C_1}{\partial S_{12}} = -(1 + \frac{t_{21}}{t_{12}} + \frac{S_{22}}{S_{21}\frac{t_{12}}{t_{21}}}), \quad \frac{\partial C_2}{\partial S_{12}} = 1 + \frac{t_{21}}{t_{12}} + \frac{S_{22}}{S_{21}\frac{t_{12}}{t_{21}}}, \quad \frac{\partial a}{\partial S_{12}} = 0, \\
&\frac{\partial C_1}{\partial S_{21}} = \frac{S_{22}S_{12}}{S_{21}^2\frac{t_{12}}{t_{21}}} + \frac{t_{12}}{t_{21}} + 1, \quad \frac{\partial C_2}{\partial S_{21}} = -\frac{S_{22}S_{12}}{S_{21}^2\frac{t_{12}}{t_{21}}}, \quad \frac{\partial a}{\partial S_{21}} = -\frac{\frac{t_{12}}{t_{21}}S_{22}}{(S_{21}+S_{22}+S_{21}\frac{t_{12}}{t_{21}})^2}, \\
&\frac{\partial C_1}{\partial S_{22}} = 1 - \frac{S_{12}}{S_{21}\frac{t_{12}}{t_{21}}}, \quad \frac{\partial C_2}{\partial S_{22}} = \frac{S_{12}}{S_{21}\frac{t_{12}}{t_{21}}} \quad \text{and} \quad \frac{\partial a}{\partial S_{22}} = \frac{\frac{t_{12}}{t_{21}}S_{21}}{(S_{21}+S_{22}+\frac{t_{12}}{t_{21}}S_{21})^2}.
\end{aligned}$$

Type I equilibra. In this case, the platform profit is $\Pi^* = \Pi_3(0) = C_2 \cdot MG_1^A$. It follows that

$$\frac{\partial \Pi^*}{\partial S_{ij}} = MG_1^A \cdot \frac{\partial C_2}{\partial S_{ij}}.$$

Therefore, in Type I equilibra, the platform can be better off from rejecting demand from location 2 to location 1 and has no incentive to reject other types of demand.

Type II equilibra. In this case, the platform profit is $\Pi^* = \Pi_4(0) = C_2 \cdot MG_1^A + C_1 \cdot MG_2^A$. It follows that

$$\begin{aligned}
&\frac{\partial \Pi^*}{\partial S_{11}} = MG_1^A > 0, \\
&\frac{\partial \Pi^*}{\partial S_{12}} = p(1-a)(1 + \frac{t_{21}}{t_{12}} + \frac{S_{22}}{\frac{t_{12}}{t_{21}}S_{21}}) > 0 \\
&\frac{\partial \Pi^*}{\partial S_{21}} = p - (1 + \frac{t_{12}}{t_{21}})(I + c_a), \\
&\frac{\partial \Pi^*}{\partial S_{22}} = \frac{S_{12}}{S_{21}\frac{t_{12}}{t_{21}}}MG_1^A + (1 - \frac{S_{12}}{S_{21}\frac{t_{12}}{t_{21}}})MG_2^A + pC_1\frac{\partial a}{\partial S_{22}} > 0.
\end{aligned}$$

Therefore, in Type II equilibra, if $p - (1 + \frac{t_{12}}{t_{21}})(I + c_a) < 0$ the platform can be better off from rejecting demand from location 2 to location 1. Otherwise, the platform has no incentive to reject any type of demand.

Type III equilibra. In this case, the platform profit is $\Pi^* = \Pi_3(N^*) = N^*(p - w_1(N^*)) + (C_2 - N^*)MG_1^A$ where $N^* = \frac{L(I+c_a-c_c)}{2(p-c_c)}$ is the unique solution to $\Pi_3'(N) = 0$ and $w_1(N^*) = \frac{I+c_a+c_c}{2}$. We can obtain that $\frac{\partial \Pi^*}{\partial S_{ij}}$ for $i, j = 1, 2$ are the same as that in Type I equilibra. Therefore, in Type

III equilibra, the platform can be better off from rejecting demand from location 2 to location 1 and has no incentive to reject other types of demand.

Type IV equilibra ($\Pi_4'(C_2) < 0$). The platform profit is $\Pi^* = \Pi_4(N^*) = N^*(p - w_1(N^*)) + (C_2 - N^*)MG_1^A + C_1 \cdot MG_2^A$, where $N^* = \frac{L(I+c_a-c_c)}{2(p-c_c)}$ is the unique solution to $\Pi_4'(N) = 0$ and $w_1(N^*) = \frac{I+c_a+c_c}{2}$. We can obtain that $\frac{\partial \Pi^*}{\partial S_{ij}}$ for $i,j = 1,2$ are the same as that in Type II equilibra. Therefore, if $p - (1 + \frac{t_{12}}{t_{21}})(c_a + I) < 0$, the platform can be better off from rejecting demand from location 2 to location 1. Otherwise, the platform has no incentive to reject any type of demand.

Because $\frac{1}{a} < 1 + \frac{t_{12}}{t_{21}}$, we conclude that if $p-(1+\frac{t_{12}}{t_{21}})(c_a+I) < 0$, the platform can be better off from rejecting demand from location 2 to location 1. Moreover, we observe that rejecting demand from location 2 to location 1 increases C_2, which implies that the condition $I < \frac{2C_2}{L}(p-c_c)+c_c-c_a$ continues to hold. Therefore, it remains to show that any admission control strategy which results in other types of equilibria cannot be optimal. For simplicity, we refer to Type I, II, III, IV ($\Pi_4'(C_2) < 0$) as Type A equilibra, and refer to Type IV ($\Pi_4'(C_2) \geq 0$), V, VI, VII equilibria as Type B equilibria. We first consider the case where $MG_2^A \geq 0$. Without admission control, the equilibra can be either Type II or Type IV. The first case is trivial because the platform has no incentive to recruit CVs under any admission control strategy. For the second case, we assume that the platform rejects some demand which leads to a Type B equilibria. The resulted equilibra can be either Type IV (with $\Pi_4'(C_2) \geq 0$), V, VI, or VII equilibra. We notice that the platform is never better off from rejecting demand from location 2 to itself. Because rejecting demand from location 2 to location 1 solely still leads to a Type A equilibria, it suffices to show that when $I < \frac{2C_2}{L}(p-c_c)+c_c-c_a$, the platform cannot be better off from rejecting demand at location 1. We consider a pseudo system which has been introduced in the proof of Lemma E.3. It is easy to see when $I < \frac{2C_2}{L}(p-c_c)+c_c-c_a$, the optimal strategy for the platform without admission control is the same as that under the pseudo system. We then notice that rejecting demand from location 1 never benefits the platform in the pseudo system, which leads to the desired result.

We then consider the case where $MG_2^A < 0$. Without admission control, the optimal equilibra can be either Type I or Type III. The first case is trivial because the platform has no incentive to recruit CVs under any admission control strategy. For the second case, we can apply a similar argument to the pseudo system which concludes the proof.

G Proofs for Systems under Different Vehicle Priority Policies

G.1 Proof of Theorem 5

Under the CV-prioritized policy, from the driver's perspective, the system is identical to that without AVs. We can reformulate the problem faced by the platform as the following capacity allocation problem:

$$\begin{aligned} \max_{M,w,\eta^A} \quad & \Pi(M, w, \eta^A) \\ \text{subject to} \quad & (1)\text{–}(8),\ (10),\ (12),\ (16) \ \text{ and } \ (s^C, r^C, q^C) \in D. \end{aligned}$$

In this case, the platform has no reason to allow AVs to queue up anywhere (otherwise, the platform can be better off by removing the the amount of AVs queued in both locations). Additionally, pursuant to Proposition B.1, drivers reposition only when $q_1^C \geq q_1^*$. The optimal outcome must fall in one of the following scenarios:

(a) CVs do not reposition and $q_1^C = 0$; or

(b) CVs reposition with a positive probability, $q_1^C = q_1^*$ and AVs (if any) reposition with probability 1.

As a result, we can use a similar argument to that used in the proof of Theorem 2 to obtain the same optimal strategy for the platform.

G.2 Proof of Theorem 6

Under the AV prioritized policy, from the driver's perspective, the system is identical to one in which the demand fulfilled by AVs is removed. Therefore, by Proposition B.1, the capacity allocation (s, r, q) is driver-incentive compatible under the AV-prioritized policy if

$$(s^C, r^C, q^C) \in \tilde{D} = \tilde{D}_1 \cup \tilde{D}_2,$$

where

$$\tilde{D}_i := \left((s,r,q) \geq 0 : g_i(s^C, r^C, q^C) \geq 0, r^C_{ji} = 0, q_i \begin{cases} \leq \tilde{q}^*_i + \tilde{k}^*_i q_j & \text{if } r^C_{ij} = 0 \\ = \tilde{q}^*_i + \tilde{k}^*_i q_j & \text{if } \frac{r^C_{ij}}{t_{ij}} \in (0, \frac{s^C_{ii}}{t_{ii}} + \frac{s^C_{ji}}{t_{ji}}) \\ \geq \tilde{q}^*_i + \tilde{k}^*_i q_j & \text{if } s^C_{ii} = s^C_{ij} = 0 \text{ and } \frac{r^C_{ij}}{t_{ij}} = \frac{s^C_{ji}}{t_{ji}} \end{cases} \right),$$

$$\tilde{q}^*_i = \frac{(S_{ij} - s^A_{ij})[(S_{ji} - s^A_{ji}) + (S_{jj} - s^A_{jj})] + (S_{ji} - s^A_{ji})\frac{t_{ij}}{t_{ji}}[(S_{ii} - s^A_{ii}) + (S_{ij} - s^A_{ij})]}{[(S_{ji} - s^A_{ji}) + (S_{jj} - s^A_{jj})] - \frac{c_c}{w}[(S_{jj} - s^A_{jj}) + (S_{ji} - s^A_{ji}) + (S_{ji} - s^A_{ji})\frac{t_{ij}}{t_{ji}}]}, \quad \text{and}$$

$$\tilde{k}^*_i = \frac{[(S_{ij} - s^A_{ij}) + (S_{ii} - s^A_{ii})] - \frac{c_c}{w}(S_{ii} - s^A_{ii})}{[(S_{ji} - s^A_{ji}) + (S_{jj} - s^A_{jj})] - \frac{c_c}{w}[(S_{jj} - s^A_{jj}) + (S_{ji} - s^A_{ji}) + (S_{ji} - s^A_{ji})\frac{t_{ij}}{t_{ji}}]}.$$

We can then reformulate the problem faced by the platform as the following capacity allocation problem:

$$\max_{M,w,\eta^A} \quad \Pi(M, w, \eta^A)$$
$$\text{subject to } \; (1)\text{–}(8),\, (10),\, (12), (17) \;\text{ and }\; (s^C, r^C, q^C) \in \tilde{D}.$$

Let $\tilde{S}_{ij}$ for $i \in \{1,2\}$ be the corresponding amount of demand after the demand served by AVs is removed from the system. We define $\tilde{q}^*_1$ and $\tilde{k}^*_1$ with respect to $\tilde{S}_{ij}$ as counterparts to q^*_1 and k^*_1 defined in (53)–(54). For example, when a fraction $1-\theta$ of demand at location 1 is served by AVs and AVs do not reposition, we have $\tilde{S}_{11} = \theta S_{11}$, $\tilde{S}_{12} = \theta S_{12}$, $\tilde{S}_{21} = S_{21} - (1-\theta)\frac{t_{21}}{t_{12}}S_{12}$, $\tilde{S}_{22} = S_{22} - (1-\theta)\frac{P_{22}t_{22}}{P_{21}t_{12}}S_{12}$, and $\tilde{q}^*_1 = \theta q^*_1$. When all the demand at location 1 and a fraction $1-\theta$ of the demand at location 2 is served by AVs, we have $\tilde{S}_{11} = \tilde{S}_{12} = 0$, $S_{21} = \theta S_{21}$, $\tilde{S}_{22} = \theta S_{22}$ and $\tilde{q}^*_1 = 0$.

Lemma G.1. *Under the AV-prioritized policy, any platform strategy which results in the following capacity allocation cannot be optimal.*

(i) Both AVs and CVs reposition from location 1 *to location* 2*, and CVs reposition with a probability less than* 1 *(i.e.,* $r^A_{12} > 0$*,* $r^C_{12} > 0$ *and* $\eta^C_1 \neq 1$*).*

(ii) CVs reposition from location 1 *to location* 2 *with a positive probability less than* 1 *and the demand at location* 2 *is not fully served (i.e.,* $\eta^C \in (0,1)$ *and* $a_2 < 1$*).*

Proof of Lemma G.1. In case (i), it suffices to consider the case where all the demand at location 1 and location 2 is served. Otherwise, the platform can be better off by purchasing more AVs to serve unfulfilled demand at location 2. Suppose a fraction $1-\theta$ of demand at location 1 is served by AVs, then we must have $M+N=S+\theta q_1^*(w_1)$ where $w_1=\frac{N(p-c_c)}{aL}+\frac{c_c}{a}$. The platform can increase its profit by (1) fixing M, (2) decreasing θ to $\theta-\Delta$ where Δ is a sufficiently small positive number, and (3) recruiting $\theta q_1^*(w_1)-(\theta-\Delta)q_1^*(w_2)$ less drivers where w_2 is the unique solution to $N-\theta q_1^*(w_1)+(\theta-\Delta)q_1^*(w_2)=\frac{aw_2-c_c}{p-c_c}L$. By doing so, the platform can use AVs to serve more customers and pay less repositioning cost and also pay a lower wage to drivers which improves its profit.

In case (ii), it suffices to consider the case where $MG_2^A<0$, which implies that AVs do not reposition. (Otherwise, the platform can be better off by purchasing more AVs to serve unfulfilled demand at location 2 and such setting has been considered in case (i).) Let $1-\theta$ be the fraction of demand fulfilled by AVs at location 1, we have $N=\sum_{i,j=1,2}s_{ij}^C+\theta q_1^*(w_1)$ where $w_1=\frac{N(p-c_c)}{aL}+\frac{c_c}{a}$. The platform can increase its profit by (1) decreasing θ to $\theta-\Delta$ where Δ is a sufficiently small positive number, (2) increasing M to $M+\Delta C_2$, and (3) fixing the wage w_1 and N. By doing so, the platform can procure more AVs and use AVs to serve more demand (without repositioning) and also use CVs to serve more demand (it can be shown by noticing that CVs reposition more and the total driver welfare remains the same) with the same wage w_1 which improves the platform profit. □

By Lemma G.1, under the AV-prioritized policy, it suffices to investigate the following cases: (1) CVs do not reposition; (2) AVs do not reposition, CVs reposition with a positive probability and all the demand at location 2 is served; and (3) CVs reposition with probability 1. If CVs do not reposition, the profit function for the platform is given by $\Pi_3(N)$ if $MG_2^A<0$ and $\Pi_4(N)$ otherwise. If CVs reposition with a positive probability, we characterize the profit function for the platform in the following scenarios.

Scenario (a): $MG_2^A<0$, all the demand at location 1 is served by AVs and CVs reposition with probability 1. Given N, the profit function for the platform is

$$\Pi_6(N)=C_2\cdot MG_1^A+aN(p-w),$$

where $w = w_2(N)$ and $w_2(N)$ is given in (69).

Scenario (b): $MG_2^A \geq 0$, all the demand at location 1 is served by AVs, AVs reposition with a positive probability and CVs reposition with probability 1. Given N, the profit function for the platform is

$$\Pi_7(N) = C_2 \cdot MG_1^A + (C_1 - N)MG_2^A + aN(p - w),$$

where $w = w_2(N)$ and $w_2(N)$ is given in (69).

Scenario (c): AVs do not reposition and CVs reposition with a positive probability such that all the demand at location 1 and location 2 is served. In this case, the average driver welfare is given by PF_1 (defined in (68)). Let θ be the fraction of demand served by CVs at location 1, then we have

$$N = \theta(q_1^* + C_2) + C_1 = \frac{PF_1 L}{p - c_c}. \tag{74}$$

The profit for the platform is

$$\Pi_8(N) = (aC_1 + \theta C_2)(p - w) + (1 - \theta)C_2 \cdot MG_1^A,$$

where $w = w_2(N)$ and $w_2(N)$ is given in (69).

Lemma G.2. *If* $MG_2^A < 0$, $\Pi_6'(C_1) < 0$ *implies that* $\Pi_8'(C_1) < 0$*; otherwise,* $\Pi_7'(C_1) < 0$ *implies that* $\Pi_8'(C_1) < 0$.

Proof of Lemma G.2. $\Pi_6'(C_1) < 0$ is equivalent to

$$ap < c_c + \frac{2C_1(p - c_c)}{L}. \tag{75}$$

$\Pi_7'(C_1) < 0$ implies that

$$I + c_a < c_c + \frac{2C_1(p - c_c)}{L} < \frac{c_c}{a} + \frac{1 + a}{a}\frac{C_1(p - c_c)}{L}. \tag{76}$$

$\Pi_8'(C_1) < 0$ is equivalent to

$$\frac{C_2}{q_1^* + C_2}[(p - w) - MG_1^A] - \frac{C_1(p - c_c)}{L} < 0$$

$$\Leftrightarrow I + c_a < w + \frac{q_1^* + C_2}{C_2}\frac{C_1(p-c_c)}{L} = \frac{c_c}{a} + \frac{C_1(p-c_c)}{aL} + \frac{q_1^* + C_2}{C_2}\frac{C_1(p-c_c)}{L}. \tag{77}$$

Therefore, if $MG_2^A < 0$, (75) implies (77) because $p > I + c_a$ and $a = \frac{C_2}{q_1^*+C_2}$. If $MG_2^A \geq 0$, (76) also implies (77). □

Lemma G.3. *If $c_c = 0$, $\Pi_6(N)$, $\Pi_7(N)$ and $\Pi_8(N)$ are concave.*

Proof of Lemma G.3 . Obviously, $\Pi_6(N)$ and $\Pi_7(N)$ are concave. If $c_c = 0$, we can obtain from (74) that $\theta = \frac{N-C_1}{q_1^*+C_2}$, where q_1^* is a constant. Therefore, we have

$$\Pi_8'(N) = \frac{C_2}{q_1^* + C_2}(p - w - MG_1^A) - (aC_1 + \theta C_2)\frac{p-c_c}{aL}, \quad \text{and}$$
$$\Pi_8''(N) = -2\frac{C_2}{q_1^* + C_2}\frac{p-c_c}{aL} < 0,$$

which implies that $\Pi_8(N)$ is concave. □

By Lemma G.1, Lemma G.2 and Lemma G.3, we can characterize the optimal strategy for the platform under the AV-prioritized policy. First, we note the following. When $MG_2^A < 0$, the platform has no incentive to let CVs reposition if $\Pi_6'(0) < 0$. Moreover, if $\Pi_6'(C_1) < 0$, by Lemma G.2, either CVs do not reposition or CVs reposition with probability 1 under the optimal strategy. Similarly, when $MG_2^A \geq 0$, the platform has no incentive to let CVs reposition if $\Pi_7'(0) < 0$. Moreover, if $\Pi_7'(C_1) < 0$, either CVs do not reposition or CVs reposition with probability 1 under the optimal strategy.

Therefore, given $MG_2^A < 0$, if $\Pi_6'(0) < 0$, it suffices to investigate $\Pi_3(N)$. If $\Pi_6'(C_1) < 0 \leq \Pi_6'(0)$, we need to compare the profit function $\Pi_6(N)$ when CVs reposition with probability 1 with the profit function $\Pi_3(N)$ when CVs do not reposition. If $\Pi_6'(C_1) \geq 0$, we need to compare the profit function $\Pi_8(N)$ when AVs do not reposition and CVs reposition such that the all demand at location 1 and location 2 is served with the profit function $\Pi_3(N)$ when CVs do not reposition.

Similarly, given $MG_2^A \geq 0$, if $\Pi_7'(0) < 0$, it suffices to investigate $\Pi_4(N)$. If $\Pi_7'(C_1) < 0 \leq \Pi_7(0)$, we need to compare the profit function $\Pi_7(N)$ when CVs reposition with probability 1 with the profit function $\Pi_4(N)$ when CVs do not reposition. If $\Pi_7'(C_1) \geq 0$, we need to compare the profit function $\Pi_8(N)$ when AVs do not reposition and CVs reposition such that all the demand at location 1 and location 2 is served with the profit $\Pi_4(N)$ when AVs reposition while CVs do not.

We then introduce Lemma G.4, which provides some comparison results on profit functions.

Lemma G.4. *(i) When* $MG_2^A < 0$ *and* $\Pi_6'(C_1) < 0 \le \Pi_6'(0)$*, we have* $\Pi_6(\frac{aL}{2}) < \Pi_3(\frac{IL}{2p})$*, where* $\frac{aL}{2}$ *is the unique solution to* $\Pi_6'(N) = 0$ *and* $\frac{IL}{2p}$ *is the unique solution to* $\Pi_3'(N) = 0$*.*

(ii) When $MG_2^A < 0$ *and* $\Pi_8'(C_1) \ge 0$*, we have* $\Pi_8(\frac{aIL}{2p}) < \Pi_3(\frac{IL}{2p})$*, where* $\frac{aIL}{2p}$ *is the unique solution to* $\Pi_8'(N) = 0$ *and* $\frac{IL}{2p}$ *is the unique solution to* $\Pi_3'(N) = 0$*.*

(iii) When $MG_2^A \ge 0$ *and* $\Pi_7'(C_1) < 0 \le \Pi_7'(0)$*, we have* $\Pi_7(\frac{IL}{2p}) = \Pi_4(\frac{IL}{2p})$*, where* $\frac{IL}{2p}$ *is the unique solution to* $\Pi_7'(N) = 0$ *and* $\Pi_4'(N) = 0$*.*

(iv) When $MG_2^A \ge 0$ *and* $\Pi_8'(C_1) \ge 0$*, we have* $\Pi_8(\frac{aIL}{2p}) < \Pi_4(\frac{IL}{2p})$ *where* $\frac{aIL}{2p}$ *is the unique solution to* $\Pi_8'(N) = 0$ *and* $\frac{IL}{2p}$ *is the unique solution to* $\Pi_4'(N) = 0$*.*

Proof of Lemma G.4. In case (i), we can obtain that $\Pi_6(\frac{aL}{2}) - \Pi_3(\frac{IL}{2p}) = \frac{a^2Lp}{4} - \frac{I^2L}{4p} < 0$ as $MG_2^A < 0$.

In case (ii), because $\Pi_8'(C_1) \ge 0$, we have $C_1 \le \frac{aIL}{2p}$. It follows that

$$\begin{aligned}\Pi_8(\frac{aIL}{2p}) - \Pi_3(\frac{IL}{2p}) &= aC_1 \cdot MG_1^A + \frac{a^2I^2L}{4p} - \frac{I^2L}{4p} \\ &\le \frac{a^2IL}{2} - \frac{a^2I^2L}{4p} - \frac{I^2L}{4p} \\ &\le \frac{IL}{4p}(2a^2p - 2\sqrt{a^2I^2}) < 0,\end{aligned}$$

where the last inequality is due to the Cauchy–Schwarz inequality and $MG_2^A < 0$.

In case (iii), the desired result follows directly from the definitions of Π_7 and Π_4.

In case (iv), because $\Pi_8'(C_1) \ge 0$, we have $C_1 \le \frac{aIL}{2p}$. It follows that

$$\Pi_8(\frac{aIL}{2p}) - \Pi_4(\frac{IL}{2p}) = -aIC_1 + \frac{a^2I^2L}{4p} - \frac{I^2L}{4p} + IC_1 = (1-a)I\left[C_1 - \frac{IL}{4p}(1+a)\right] < 0.$$

□

Through out the remainder of the proof, we shall frequently refer to cases (labeled by "AC") discussed in the proof of Theorem 2. Because $c_c = c_a = 0$, we have $\Pi_3'(0) = \Pi_4'(0) = I > 0$. Now we can characterize the optimal strategy and its corresponding outcome under the AV-prioritized policy.

Case (AP.i): $MG_2^A < 0$ and $\Pi_6'(C_1) < 0$. In this case, the platform has no incentive to let AVs reposition. We consider the following subcases.

Case (AP.i.i): $\Pi_6'(0) < 0$. It suffices to investigate $\Pi_3(N)$. If $\Pi_3'(C_2) > 0$, the optimal strategy and corresponding outcomes are identical to those in case (AC.i.ii). Otherwise, they are identical to those in case (AC.i.iii).

Case (AP.i.ii): $\Pi_6'(C_1) < 0 \leq \Pi_6'(0)$. By Lemma G.2, CVs either do not reposition or reposition with probability 1. The optimal strategy falls into one of the following scenarios whichever gives a higher profit.

(a) The platform recruits $\frac{aL}{2}$ amount of drivers and drivers reposition with probability 1, where $\frac{aL}{2}$ is the unique solution to $\Pi_6'(N) = 0$. The platform purchases C_2 amount of AVs and AVs do not queue or reposition. It follows that $M = C_2$, $w = w_2(\frac{aL}{2})$ and the corresponding profit is $\Pi_6(\frac{aL}{2})$.

(b) CVs do not reposition, and we consider the following possibilities.

(b.1) If $\Pi_3'(C_2) < 0$, the strategy and is identical to that in case (AC.i.ii). That is, $N = \frac{IL}{2p}$, $w = w_1(\frac{IL}{2p})$, $M = C_2 - \frac{IL}{2p}$ and the corresponding profit is $\Pi_3(\frac{IL}{2p})$. By Lemma G.4, because $\Pi_3(\frac{IL}{2p}) > \Pi_6(\frac{aL}{2})$, the strategy in scenario (b.1) dominates that in scenario (a).

(b.2) If $\Pi_3'(C_2) \geq 0$, the strategy is identical to that in case (AC.i.iii). That is $N = C_2$, $M = 0$, $w = w_1(C_1)$ and the corresponding profit is $\Pi_3(C_2)$.

Case (AP.ii): $MG_2^A \geq 0$ and $\Pi_7'(C_1) < 0$. In this case, the platform can make positive profit by repositioning AVs. We consider the following subcases.

Case (AP.ii.i): $\Pi_7'(0) < 0$. It suffices to investigate $\Pi_4(N)$. If $\Pi_4'(C_2) < 0$, the optimal strategy and outcomes are identical to those in case (AC.ii.ii). Otherwise, they are identical to those in case (AC.ii.iii).

Case (AP.ii.ii): $\Pi_7'(C_1) < 0 \leq \Pi_7'(0)$. By Lemma G.2, CVs either do not reposition or reposition with probability 1. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) The platform recruits $\frac{IL}{2p}$ amount of drivers and drivers reposition with probability 1 where

$\frac{IL}{2p}$ is the unique solution to $\Pi_7'(N) = 0$. The platform purchases $C_1 + C_2 - \frac{IL}{2p}$ amount AVs and AVs reposition with a positive probability. It follows that $M = C_1 + C_2 - \frac{IL}{2p}$, $w = w_2(\frac{IL}{2p})$ and the profit is $\Pi_7(\frac{IL}{2p})$.

(b) CVs do not reposition, and we consider the following possibilities.

(b.1) If $\Pi_4'(C_2) < 0$, the strategy is identical to that in case (AC.ii.ii). That is, $N = \frac{IL}{2p}$, $M = C_1 + C_2 - \frac{IL}{2p}$, $w = w_1(\frac{IL}{2p})$ and the profit is $\Pi_4(\frac{IL}{2p})$. By Lemma G.4, $\Pi_4(\frac{IL}{2p}) = \Pi_7(\frac{IL}{2p})$. Therefore, strategies in scenario (a) and scenario (b.1) are both optimal.

(b.2) If $\Pi_4'(C_2) \geq 0$, the strategy is identical to that in case (AC.ii.iii). That is $N = C_2$, $M = C_1$, $w = w_1(C_2)$ and the profit is $\Pi_4(C_2)$. By Lemma G.4, $\Pi_4(C_2) < \Pi_4(\frac{IL}{2p}) = \Pi_7(\frac{IL}{2p})$, where $\frac{IL}{2p}$ is the solution to $\Pi_4'(N) = 0$, the strategy in scenario (a) dominates that in scenario (b.2).

Case (AP.iii): $MG_2^A < 0$ and $\Pi_6'(C_1) \geq 0$. We consider the following subcases.

Case (AP.iii.i): $\Pi_8'(C_1 + C_2 + q_1^*) \geq 0$. By Lemma E.4, we can obtain that $\frac{(C_1+C_2+q_1^*)(p-c_c)}{L} - c_c + a(I + c_a) - \frac{(aC_1+C_2)(p-c_c)}{aL} > 0$, which implies that $\Pi_3'(C_2) = -\frac{2C_2(p-c_c)}{L} - c_c + I + c_a \geq 0$. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) All the demand at location 1 and location 2 is served by CVs. That is: $N = C_1 + C_2 + q_1^*$, $M = 0$, $w = w_2(C_1 + C_2 + q_1^*)$ and the profit is $\Pi_8(C_1 + C_2 + q_1^*)$.

(b) The strategy and outcomes are identical to that in case (AC.i.iii).

Case (AP.iii.ii): $\Pi_8'(C_1 + C_2 + q_1^*) < 0 \leq \Pi_8'(C_1)$. By Lemma G.2, CVs either reposition with a positive probability such that all the demand at location 2 is fulfilled or do not reposition. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) The platform recruits $\frac{aIL}{2p}$ amount drivers and drivers reposition with a positive probability, where $\frac{aIL}{2p}$ is the unique solution to $\Pi_8'(N) = 0$. The platform purchases $(1-\theta)C_2$ amount of AVs and Avs do not reposition, where $\theta = \frac{\frac{aIL}{2p} - C_1}{q_1^* + C_2}$. It follows that $M = (1-\theta)C_2$, the wage paid to drivers is $w_2(\frac{aIL}{2p})$ and the profit is $\Pi_8(\frac{aIL}{2p})$.

(b) CVs do not reposition, and we consider the following possibilities.

(b.1) If $\Pi_3(C_2) < 0$, the strategy is identical to that in case (AC.i.ii). That is $N = \frac{IL}{2p}$, $w = w_1(\frac{IL}{2p})$, $M = C_2 - \frac{IL}{2p}$ and the profit is $\Pi_3(\frac{IL}{2p})$. By Lemma G.4, because $\Pi_3(\frac{IL}{2p}) > \Pi_8(\frac{aIL}{2p})$, the optimal strategy in scenario (b.1) dominates that in scenario (a).

(b.2) If $\Pi_3(C_2) \geq 0$, the strategy is identical to that in case (AC.i.iii). That is $N = C_2$, $M = 0$, $w = w_1(C_1)$ and the profit is $\Pi_3(C_2)$.

Case (AP.iii.iii): $\Pi_8'(C_1) < 0$. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) The platform purchases C_2 amount of AVs and AVs do not queue or reposition. The platform recruits C_1 amount of drivers and drivers reposition with probability 1. It follows that $M = C_2$, $N = C_1$, $w = w_2(C_1)$ and the profit is $\Pi_6(C_1)$.

(b) If $\Pi_3'(C_2) < 0$, the strategy and outcomes are identical to those in case (AC.i.ii). Otherwise, they are identical to those in case (AC.i.iii).

Case (AP.iv): $MG_2^A \geq 0$ and $\Pi_7'(C_1) \geq 0$. We consider the following subcases.

Case (AP.iv.i): $\Pi_8'(C_1 + C_2 + q_1^*) \geq 0$. In this case, we have $\Pi_4'(C_2) > 0$. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) The strategy and outcome are identical to those in case (AP.iii.i) scenario (a).

(b) The strategy and outcome are identical to those in case (AC.ii.iii).

Case (AP.iv.ii): $\Pi_8'(C_1 + C_2 + q_1^*) < 0 \leq \Pi_8'(C_1)$. In this case, we have $\Pi_4'(0) > 0$. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) CVs reposition with a positive probability. The strategy and outcomes are identical to those in case (AP.iii.ii) scenario (a). That is, $N = \frac{aIL}{ap}$, $\theta = \frac{\frac{aIL}{ap} - C_1}{q_1^* + C_2}$, $M = \theta C_2$, $w = w_2(\frac{aIL}{ap})$ and the profit is $\Pi_8(\frac{aIL}{2p})$.

(b) CVs do not reposition, and we consider the following possibilities.

(b.1) If $\Pi_4'(C_2) < 0$, the strategy is identical to that in case (AC.ii.ii). That is, $N = \frac{IL}{2p}$, $M = C_1 + C_2 - \frac{IL}{2p}$, $w = w_1(\frac{IL}{2p})$ and the profit is $\Pi_4(\frac{IL}{2p})$. By Lemma G.4, because $\Pi_4(\frac{IL}{2p}) > \Pi_8(\frac{aIL}{2p})$, the optimal strategy in scenario (b.1) dominates that in scenario (a).

(b.2) If $\Pi_4'(C_2) \geq 0$, the strategy is identical to that in Case (AC.ii.iii). That is, $N = C_2$, $M = C_1$, $w = w_1(C_2)$ and the profit is $\Pi_4(C_2)$.

Case (AP.iv.iii): $\Pi_8'(C_1) \leq 0$. The optimal strategy is determined by which of the following scenarios yields a higher profit.

(a) The strategy and outcomes are identical to those in case (AP.iii.iii) scenario (a).

(b) If $\Pi_4'(C_2) < 0$, the strategy and outcomes are identical to ~~that~~ those in case (AC.ii.ii). Otherwise, they are identical to those in case (AC.ii.iii).

G.3 Proof of Proposition 5

According to the proof of Theorem 2, allowing AVs to queue at any location is not optimal for the platform under the random priority policy. Therefore, under the AV-prioritized policy, the platform can still use the optimal strategy under the random priority policy, indicating that the platform profit is (weakly) higher under the AV-prioritized policy than under the random priority policy.

We then show that the service level is (weakly) higher under the AV-prioritized policy. The desired result follows directly if $MG_2^A \geq 0$ as the service level is 1 under both priority policies. If $MG_2^A < 0$, we consider two cases: (a) CVs do not reposition under the random priority policy. In this case, the desired result also follows naturally. (b) CVs reposition under the random priority policy, which implies that the platform recruits $\tilde{N} \wedge (C_1 + C_2 + q_1^*(w_b))$ amount of drivers, where $\tilde{N}$ is the unique solution to $\Pi_2'(N) = 0$. Because $\frac{\partial \Pi_2(N+C_2+q_1^*)}{\partial N} < \Pi_6'(N)$, the desired result follows.

Next, we investigate the average driver welfare. By (12), to compare the average driver welfare, it suffices to compare the amount of drivers recruited by the platform. Let N_1 be the amount of CVs under the random priority policy, and N_2 be that under the AV-prioritized policy. If $MG_2^A < 0$, the optimal outcome under the random priority policy could fall into the following cases.

Case (i): the platform operates with both AVs and CVs, while AVs and CVs do not reposition. In this case, we have $N_1 = \frac{LI}{2p}$. According to the Proof of Theorem 6, under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (i.1): AVs and CVs do not reposition. We have $N_2 = N_1$. Case (i.2): AVs serve C_2, CVs reposition with probability 1, and $\Pi_6'(C_1) > 0$. Because $N_2 = C_1 < \frac{aL}{2} < N_1$ as $MG_2^A < 0$, where $\frac{aL}{2}$ is the solution to $\Pi_6'(N) = 0$, we can obtain that $N_2 < N_1$.

Case (ii): the platform only operates with CVs and CVs do not reposition. In this case, we have $N_1 = C_2$. Under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (ii.1): The platform only operates with CVs and CVs do not reposition. We have $N_2 = N_1$. Case (ii.2): AVs serve C_2, CVs reposition with probability 1, and $\Pi_6'(C_1) \leq 0$. We can obtain that $N_2 = \frac{aL}{2}$. Moreover, because $\Pi_6(N_2) \geq \Pi_3(N_1)$, we have $C_2 < \frac{aL}{2}$ and thus $N_1 < N_2$.

Case (ii.3): AVs serve C_2, CVs reposition with probability 1, and $\Pi_6'(C_1) > 0$. Because $N_1 = C_2$, we must have $\Pi_3'(C_2) > 0$, which implies that $I > \frac{2pC_2}{L}$. Moreover, because $\Pi_6(C_1) > \Pi_3(N_1)$, we have $-C_2 I + aC_1 p > \frac{p}{L}(C_1^2 - C_2^2)$, which implies that $I(C_1 - C_2) > \frac{p}{L}(C_1 - C_2)(C_1 + C_2)$. Suppose $C_2 \geq C_1$, then we must have $I \leq \frac{p}{L}(C_1 + C_2) \leq \frac{2p}{L}C_2$, which leads to a contradiction. Therefore, we must have $C_1 > C_2$ and thus $N_2 > N_1$. Case (ii.4): CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We have $N_2 = \frac{aIL}{2p}$. Therefore, if $\frac{aIL}{2p} > C_2$, we have $N_2 > N_1$, and $N_2 \leq N_1$ otherwise.

Case (iii): the platform only operates with CVs and CVs reposition with a positive probability. In this case, we have $N_1 = \frac{aL}{2} \wedge (C_1 + C_2 + q_1^*(w_b))$, where $\frac{aL}{2}$ is the unique solution to $\Pi_2(N) = 0$ (because $\frac{C_2}{C_2 + q_1^*} = a$ by Lemma E.4 when $c_c = 0$). Under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (iii.1): AVs serve C_2, CVs reposition with probability 1, and $\Pi_6'(C_1) \leq 0$. We can obtain that $N_2 = \frac{aL}{2} = N_1$. Case (iii.2): AVs serve C_2, CVs reposition with probability 1, and $\Pi_6'(C_1) > 0$. Because $C_1 \leq \frac{aL}{2}$ by Case (iii.1) we can obtain that $N_2 \leq N_1$. Case (iii.3): CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. Because $\frac{aIL}{2p} < \frac{aL}{2}$ where $\frac{aIL}{2p}$ is the solution to $\Pi_8'(N) = 0$, we have $N_2 \leq N_1$.

If $MG_2^A \geq 0$, the optimal outcome under the random priority policy could fall into the following cases.

Case (iv): the platform operates with both CVs and AVs, CVs do not reposition and AVs reposition with a positive probability. In this case, we have $N_1 = \frac{LI}{2p}$. According to the proof of Theorem 2, under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (iv.1): CVs do not reposition and AVs reposition with a positive probability. We have $N_2 = N_1$. Case (iv.2): CVs reposition with probability 1, and $\Pi_7'(C_1) \leq 0$. We can obtain that $N_2 = \frac{LI}{2p} = N_1$, where $\frac{LI}{2p}$ is the solution to $\Pi_7'(N) = 0$. Case (iv.3): AVs serve C_2, CVs reposition with probability 1, and $\Pi_7'(C_1) > 0$. Because $C_1 \leq \frac{LI}{2p}$, by Case (iv.2), we can obtain that $N_2 < N_1$.

Case (v): the platform operates with both AVs and CVs, CVs do not reposition, AVs reposition with probability 1. In this case, we have $N_1 = C_2$. Under the AV-prioritized policy, the optimal outcomes could fall into the following subcases. Case (v.1): CVs do not reposition, AVs reposition with probability 1. We have $N_2 = N_1$. Case (v.2): AVs reposition with a positive probability, CVs reposition with probability 1. We can obtain that $N_2 = \frac{LI}{2p}$. Because $\Pi_4'(C_2) > 0$, $C_2 < \frac{LI}{2p}$ where $\frac{LI}{2p}$ is the solution to $\Pi_4'(N) = 0$, we have $N_1 < N_2$. Case (v.3) AVs do not reposition, CVs reposition with probability 1. We have $N_2 = C_1$. Because $\Pi_7(C_1) > \Pi_4(C_2)$, we have $I(C_1 - C_2) > \frac{p}{L}(C_1 + C_2)(C_1 - C_2)$. Then, by the same analysis as in case (ii.3), we can obtain that $N_2 > N_1$. Case (v.4) CVs reposition with a positive probability less than 1 and AVs do not reposition. We have $N_2 = \frac{aIL}{2p}$. Therefore, if $\frac{aIL}{2p} > C_2$, we have $N_2 > N_1$. Otherwise, we have $N_2 \leq N_1$.

Case (vi): the platform operates with both CVs and AVs, CVs reposition with a positive probability and AVs reposition with probability 1. In this case, we have $N_1 = \frac{LI}{2p}$. Under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (vi.1): CVs reposition with a positive probability and AVs reposition with probability 1. We have $N_2 = N_1$. Case (vi.2): CVs reposition with probability 1, and $\Pi_7'(C_1) \leq 0$. We can obtain that $N_2 = \frac{LI}{2p} = N_1$, where $\frac{LI}{2p}$ is the solution to $\Pi_7'(N) = 0$. Case (vi.3): AVs serve C_2, CVs reposition with probability 1, and $\Pi_7'(C_1) > 0$. Because $C_1 < \frac{LI}{2p}$, by Case (vi.2), we can obtain that $N_2 < N_1$. Case (vi.4): CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. Because $\frac{aIL}{2p} < \frac{aL}{2p}$, where $\frac{aIL}{2p}$ is the solution to $\Pi_8'(N) = 0$, we have $N_2 < N_1$.

Lastly, we consider the driver productivity. The optimal outcome under the random priority policy could fall into the following cases.

Case (i) CVs do not reposition. In this case, the driver productivity is 1 and the AV-prioritized policy must weakly decrease driver productivity.

Case (ii) CVs reposition with a positive probability. In this case, the driver productivity is a (this is because $\frac{C_2}{C_2+q_1^*} = a$ by Lemma E.4). By Theorem 6, after the priority policy is switched to the AV-prioritized policy, it is optimal for the platform to let CVs reposition (if the platform does not reposition CVs, this strategy must be dominated by the optimal strategy under the random priority policy). Because $\frac{C_2}{C_2+q_1^*} = a$ by Lemma E.4 when $c_c = c_a = 0$, we conclude that the driver productivity is still a under the AV-prioritized policy.

G.4 Proof of Proposition 4

The desired results regarding the platform profit and service level can be obtained by the same argument in the proof of Proposition 5.

Next, we investigate the average driver welfare. By (12), to compare the average driver welfare, it suffices to compare the amount of drivers recruited by the platform. Let N_1 be the amount of CVs in the system without AVs, and N_2 be that in the system with AVs under the AV-prioritized policy. The optimal outcome in the system without AVs could fall into the following cases.

Case (i) CVs do not reposition and $\Pi_1'(C_2) < 0$. In this case, we have $N_1 = \frac{L}{2} < C_2$. According to the Proof of Theorem 6, under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (i.1): $MG_2^A < 0$, AVs and CVs do not reposition. We have $N_2 < N_1$. Case (i.2): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi_6'(C_1) \leq 0$. We can obtain that $N_2 = \frac{aL}{2} < N_1$, where $\frac{aL}{2}$ is the solution to $\Pi_6'(N) = 0$. Case (i.3): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi_6'(C_1) > 0$. We can obtain that $N_2 = C_1 < \frac{aL}{2} < N_1$. Case (i.4): $MG_2^A < 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 = \frac{aLI}{2p} < N_1$, where $\frac{aLI}{2p}$ is the solution to $\Pi_8'(N) = 0$. Case (i.5): $MG_2^A \geq 0$, CVs do not reposition and AVs reposition with a positive probability. We have $N_2 < N_1$. Case (i.6): $MG_2^A \geq 0$, CVs reposition with probability 1, and $\Pi_7'(C_1) \leq 0$. We can obtain that $N_2 = \frac{LI}{2p} < N_1$, where $\frac{LI}{2p}$ is the solution to $\Pi_7'(N) = 0$. Case (i.7): $MG_2^A \geq 0$, AVs serve C_2, CVs reposition with probability 1, and $\Pi_7'(C_1) > 0$. We can obtain that $N_2 = C_1 \leq \frac{LI}{2p} < N_1$. Case (i.8): $MG_2^A \geq 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 \leq \frac{aLI}{2p} < N_1$.

Case (ii) CVs do not reposition and $\Pi_1'(C_2) \geq 0$. In this case, we have $N_1 = C_2$. According to the Proof of Theorem 6, under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (ii.1): $MG_2^A < 0$, CVs do not reposition. We have $N_2 \leq N_1$. Case (ii.2): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi_6'(C_1) \leq 0$. We can obtain that $N_2 = \frac{aL}{2}$. Moreover, because $\Pi_6(N_2) \geq \Pi_3(N_1)$, we have $C_2 < \frac{aL}{2}$ and thus $N_1 < N_2$. Case (ii.3): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi_6'(C_1) > 0$. We can obtain that $N_2 = C_1$. If $C_1 \geq C_2$, $N_2 \geq N_1$. Otherwise, $N_2 < N_1$. Case (ii.4): $MG_2^A < 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 = \frac{aLI}{2p}$, where $\frac{aLI}{2p}$ is the solution to $\Pi_8'(N) = 0$. Therefore, if $\frac{aIL}{2p} \geq C_2$, we have $N_2 \geq N_1$, and $N_2 < N_1$ otherwise. Case (ii.5): $MG_2^A \geq 0$, CVs do not reposition and AVs reposition with a positive probability. We have $N_2 \leq N_1$. Case (ii.6): $MG_2^A \geq 0$, CVs reposition with probability 1, and $\Pi_7'(C_1) \leq 0$. We can obtain that $N_2 = \frac{LI}{2p}$, where $\frac{LI}{2p}$ is the solution to

$\Pi'_7(N) = 0$. Therefore, if $\frac{IL}{2p} > C_2$, we have $N_2 > N_1$, and $N_2 \leq N_1$ otherwise. Case (ii.7): $MG_2^A \geq 0$, AVs serve C_2, CVs reposition with probability 1, and $\Pi'_7(C_1) > 0$. We can obtain that $N_2 = C_1$. Therefore, if $C_1 > C_2$, we have $N_2 > N_1$, and $N_2 \leq N_1$ otherwise. Case (i.8): $MG_2^A \geq 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 = \frac{aLI}{2p}$. Therefore, if $\frac{aLI}{2p} > C_2$, we have $N_2 > N_1$, and $N_2 \leq N_1$ otherwise.

Case (iii) CVs reposition with a positive probability. In this case, we have $N_1 = \frac{aL}{2} \wedge (C_1 + C_2 + q_1^*(w_b))$, where $\frac{aL}{2}$ is the unique solution to $\Pi'_2(N) = 0$ (because $\frac{C_2}{C_2+q_1^*} = a$ by Lemma E.4 when $c_c = 0$). According to the Proof of Theorem 6, under the AV-prioritized policy, the optimal outcome could fall into the following subcases. Case (iii.1): $MG_2^A < 0$, CVs do not reposition. We have $N_2 < N_1$. Case (iii.2): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi'_6(C_1) \leq 0$. We can obtain that $N_2 = \frac{aL}{2} = N_1$. Case (iii.3): $MG_2^A < 0$, AVs serve C_2, CVs reposition with probability 1 and $\Pi'_6(C_1) > 0$. We can obtain that $N_2 = C_1 < \frac{aL}{2}$. Therefore, $N_2 < N_1$. Case (iii.4): $MG_2^A < 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 = \frac{aLI}{2p} < \frac{aL}{2}$, where $\frac{aLI}{2p}$ is the solution to $\Pi'_8(N) = 0$. Therefore, $N_2 \leq N_1$. Case (iii.5): $MG_2^A \geq 0$, CVs do not reposition and AVs reposition with a positive probability. We have $N_2 < N_1$. Case (iii.6): $MG_2^A \geq 0$, CVs reposition with probability 1, and $\Pi'_7(C_1) \leq 0$. We can obtain that $N_2 = \frac{LI}{2p} \leq \frac{aL}{2}$, where $\frac{LI}{2p}$ is the solution to $\Pi'_7(N) = 0$. Therefore $N_2 \leq N_1$. Case (iii.7): $MG_2^A \geq 0$, AVs serve C_2, CVs reposition with probability 1, and $\Pi'_7(C_1) > 0$. Because $C_1 < \frac{LI}{2p}$, we conclude that $N_2 < N_1$. Case (iii.8): $MG_2^A \geq 0$, CVs reposition with a positive probability less than 1 and all the demand at location 1 and location 2 is served. We can obtain that $N_2 = \frac{aLI}{2p} < \frac{aL}{2}$. Therefore, we have $N_2 \leq N_1$.

Lastly, we consider the driver productivity. The optimal outcome in the system without AVs could fall into the following cases.

Case (i) CVs do not reposition. In this case, the driver productivity is 1 and the introduction of AVs must weakly decrease driver productivity.

Case (ii) CVs reposition with a positive probability. In this case, the driver productivity is a (this is because $\frac{C_2}{C_2+q_1^*} = a$ by Lemma E.4). The introduction of AVs increases driver productivity if CVs do not reposition in the optimal outcome under the AV-prioritized policy. Otherwise, the introduction of AVs does not change driver productivity (this is because $\frac{C_2}{C_2+q_1^*} = a$ by Lemma E.4).