## Evaluating *n*-Gram Novelty of Language Models Using RUSTY-DAWG 🐼

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#### Abstract

How novel are texts generated by language models (LMs) relative to their training corpora? In this work, we investigate the extent to which modern LMs generate n-grams from their training data, evaluating both (i) the probability LMs assign to complete training *n*-grams and (ii) *n*-novelty, the proportion of *n*-grams generated by an LM that did not appear in the training data (for arbitrarily large n). To enable arbitrary-length *n*-gram search over a corpus in constant time, we develop RUSTY-DAWG, a novel search tool inspired by indexing of genomic data. We compare the novelty of LMgenerated text to human-written text and explore factors that affect generation novelty, focusing on the Pythia models. We find that, for n > 4, LM-generated text is *less novel* than human-written text, though it is more novel for smaller n. Larger LMs and more constrained decoding strategies both decrease novelty. Finally, we show that LMs complete *n*-grams with lower loss if they are less frequent in the training data. Overall, our results reveal factors influencing the novelty of LM-generated text, and we release RUSTY-DAWG to facilitate further pretraining data research.

#### 1 Introduction

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Despite an explosion of new applications of language models (LMs), a core question about LMs as text generators has not been fully answered: *how novel is the text they generate compared to their training corpus?* This question has both scientific value and practical relevance for LM deployment. From a scientific perspective, language understanding is often theorized as hinging on compositionality, meaning that an infinite range of meanings can be expressed by combining a small set of words or morphemes. If LMs were largely copying sentences or spans they had seen before, this would suggest they cannot compositionally generate new sentences like humans can. From a societal perspective, the novelty of LM-generated text may also be relevant to legal questions of whether copyrighted materials can be used in LM pretraining data. For instance, a lawsuit between the New York Times and OpenAI (ongoing at the time of writing) hinges on the legal ambiguity of whether including copyrighted material in training data is allowed under fair use (Klosek, 2024). Scientific evaluation of copying behavior in LMs may help guide the resolution of such questions. 043

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In past work, McCoy et al. (2021) evaluated the novelty of more typical text generated by sampling from small LMs, finding that small n-grams in LMgenerated text are less novel than in validation text, though larger n-grams are more novel. However, McCoy et al. (2021)'s LMs were trained on Web-Text (40 GB; Radford et al., 2019), which is 3% of the size of the Pile (1254 GB; Gao et al., 2020). Thus, it is unclear how their conclusions would transfer to larger-scale, modern LMs.

In this work, we evaluate the n-gram generation novelty of LMs of varying sizes trained on largescale web data. Specifically, we measure the proportion of generated *n*-grams that are novel against the training set across across many n, which we call *n*-novelty. Scaling the analysis of *n*-novelty to large corpora is challenging because measuring large-*n*-gram statistics over large corpora is infeasible when implemented naively. To solve this problem, we develop RUSTY-DAWG, a search tool that uses the Compacted Directed Acyclic Word Graph (CDAWG, Crochemore and Vérin, 1997; Inenaga et al., 2005) data structure for arbitrarylength *n*-gram matching over a corpus in *constant time* w.r.t. the corpus size and linear w.r.t the query size. While similar approaches were previously applied to genome data, we are the first, to the best of our knowledge, to scale them to LM pretraining data. We use RUSTY-DAWG to address the following research questions, focusing on the Pythia models (Biderman et al., 2023), trained on the Pile:

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zero (due to the way tokenizers work), and the curve will monotonically increase with n (since

RQ1. How novel is typical text generated by LMs

RQ2. How do model size, decoding strategies, and

RQ3. Across *n*-gram sizes, how does the occurrence

We make the following contributions and findings:

searches in massive pretraining datasets.

are more novel (RQ1, Section 5.1).

ing data also decreases novelty.

1. We find large *n*-grams (n > 4) are less novel

2. We show that **novelty decreases with larger** 

3. We show LMs complete frequent training

*n*-grams with lower loss (RQ3, Section 6).

**Operationalizing Novelty with** *n***-Grams** 

There are different ways to measure LM generation

novelty: one could assess the verbatim overlap

between the text and training data or attempt to

capture semantic and syntactic novelty. We target verbatim novelty via two *n*-gram-based metrics:

*n*-Novelty. Novelty can be evaluated at different

scales. For example, while all individual tokens in

a generated text will likely have occurred, it would

be notable if a 100-gram from the pretraining data

was generated verbatim. To capture novelty across

different *n*-gram lengths, we follow McCoy et al.

(2021) in plotting the *n*-novelty curve, i.e., the nov-

elty of generated *n*-grams (where *n* varies) w.r.t.

some fixed corpus C. Formally, for any text query

Q (e.g., a model-generated document) we define

the *n*-novelty rate of Q as the proportion of *n*-

grams in Q that also occurred in C. We visualize

the n-novelty curve as a function of n as in Fig-

ure 1b. Intuitively, 1-novelty should be close to

substrings of a non-novel *n*-gram are non-novel).

in LM-generated text compared to human-

written text, though small *n*-grams ( $n \leq 4$ )

LMs and constrained decoding (RQ2, Sec-

tion 5.2). To an extent, prompting with train-

0. We introduce **RUSTY-DAWG**, an effi-

cient data structure based on CDAWG automata that enables unbounded-length n-gram

novelty of model-generated text?

impact their completion loss?

training distribution?

compared to new human-written text from the

prompting with training data influence the

and frequency of *n*-grams in the training set

**Non-Novel Suffix Length** (NNSL). We propose a new measure of aggregate novelty across different *n*. We define NNSL at token position i in Q as the length of the longest suffix of Q[: i] that appeared in C. We then aggregate by taking mean or max.

**Example.** Let C = hello\$world\$ be a character-tokenized corpus, where \$ is a document boundary. Query Q = 11 oyd has 1-gram novelty 1/5 (y is novel), 2-gram novelty 2/4 (oy and yd are novel), 3-gram novelty 2/3 (only 110 is non-novel), and 4-gram novelty 2/2. The NNSL at each position is  $\langle 1, 2, 3, 0, 1 \rangle$ , with mean 1.4 and max 3. We intuitively demonstrate this example in Figure 1b.

#### Measuring Novelty with CDAWGs 3

Naively computing our novelty metrics is prohibitively expensive over a large pretraining corpus like the Pile (334B tokens). To make the searches fast, we use a Compacted Directed Acyclic Word Graph (CDAWG; Crochemore and Vérin, 1997; Inenaga et al., 2005), a data structure which returns the NNSL at each position in Q against Cin constant time (w.r.t. the size of C), and linear time (w.r.t. the size of Q), from which nnovelty can be computed. We describe how to compute NSSL using CDAWG in Appendix A. This constant-time querying is crucial for our application of searching the Pile. We first discuss querying CDAWGs (Section 3.1), then their memory costs (Section 3.2), their construction (Section 3.3), and our open-source implementation (Section 3.4).

## 3.1 Querying CDAWGs

A CDAWG is a finite-state machine built for a corpus C that acts as a rich index for C (see Figure 1a). We use it to compute NNSL queries on C:

- INPUT: A string (e.g., Q = 11oyd).
- OUTPUT: NNSL at each position in Q (e.g.,  $L(Q) = \langle 1, 2, 3, 0, 1 \rangle$ ) as well as the training frequencies of the largest suffixes matched at each position (e.g., N(Q) = (3, 1, 1, 0, 1)).

An NNSL query is answered by passing Q as input through the CDAWG (shown in Figure 1a; see Appendix B.2 for details). Processing a single token takes constant time because it involves just following a single arc (potentially a failure arc; Allauzen, 2023). Thus, the query takes time O(|Q|) with no dependence on |C|. This makes CDAWGs useful for searching large corpora and faster than suffix arrays (Carlini et al., 2023; Liu et al., 2024).

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(a) CDAWG for C = hello\$world\$, where \$ is a document separator. Dashed arrows are failure arcs.



(b) Novelty curves computed from the CDAWG in Figure 1a, labeled by their corresponding queries.

Figure 1: Illustration of CDAWG and resulting novelty curves with character-level tokenization for simplicity.

For illustration (Figure 1), we use character-level tokenization, but this process can be applied with any tokenization. The *n*-novelty curve, as well as all other data presented in this paper, can be computed from non-novel suffix queries.

#### 3.2 Memory Overhead

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A practical concern for an indexing data structure is its memory overhead: how many bytes does it use on a corpus of size |C|? The CDAWG refines the earlier Directed Acyclic Word Graph (DAWG; Blumer et al., 1984) to reduce memory overhead. A DAWG contains at most 2|C| states and 3|C|arcs (Blumer et al., 1984), which, while linear, becomes impractical for large datasets. In contrast, a CDAWG achieves 0.18|C| states and 0.97|C|arcs on the Pile. As a result, we find the CDAWG takes ~50% as much memory to store as the vanilla DAWG in practice.<sup>1</sup> Still, the CDAWG takes 29|C|bytes vs. 7|C| for a suffix array, illustrating a time/space tradeoff between the two approaches.

Another factor that affects memory overhead is the choice of graph representation. We implemented the edge list for a node with an AVL tree to make transitions very fast, but at the cost of some memory overhead. Further details about the graph representation, memory overhead, and potential improvements can be found in Appendix B.3.

#### 3.3 Building CDAWGs

The naive way to build a CDAWG would involve enumerating all span in a corpus in quadratic time, which is infeasible for large corpora. Luckily, more refined algorithms for building DAWGs and CDAWGs have been developed that process each token in the corpus left to right, taking linear time overall (Blumer et al., 1984; Crochemore and Vérin, 1997; Inenaga et al., 2005). We implement Inenaga et al. (2005)'s linear-time algorithm. 213

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Once the CDAWG is built, we apply a postprocessing step to add frequency information to each node in the CDAWG via a depth-first traversal (Appendix B.1). Since edges dominate the memory overhead of the CDAWG, this only minimally increases the space overhead.

#### 3.4 RUSTY-DAWG Library

While there are some pre-existing open-source libraries for DAWGs,<sup>2</sup> we did not find a scalable open-source implementation of CDAWGs. To facilitate our research and other applications of CDAWGs to large text corpora, we implemented RUSTY-DAWG, a modern Rust library for building and using DAWGs and CDAWGs, which we will open-source. See Appendix B.4 for more details.

#### 4 Experimental Setup

#### 4.1 Building a CDAWG on the Pile

We focus our study on the copying behavior of the eight Pythia models (Biderman et al., 2023) trained on the Pile (Gao et al., 2020). The Pile contains many kinds of text, including web text, books, code, and email communication. We build our RUSTY-DAWG on the non-deduplicated version using the GPT-NeoX (Black et al., 2022) tokenization used by Pythia, under which it contains 334B tokens.

To parallelize building RUSTY-DAWG, we shard the Pile into 30 shards and build a CDAWG on each 11B-token shard on a different cloud machine. Each of the 30 created CDAWGs has 2B states and 11B arcs, taking 327 GB total memory. We store this in RAM during building. At inference time, we keep the CDAWG shards on

<sup>&</sup>lt;sup>1</sup>A CDAWG arc is larger than a DAWG arc. Hence, the CDAWG memory overhead is reduced 50% despite a larger reduction in the number of states and arcs.

<sup>&</sup>lt;sup>2</sup>https://github.com/elake/SuffixAutomaton



Figure 2: *n*-novelty curve for Pythia-12B with naive sampling. Compared to Dolma, LM-generated text is more novel for n > 4 and slightly less novel for  $n \le 4$ . The gap between the dark gray Dolma curve and the green Pythia-12B curve quantifies the novelty difference. LM-generated text is more novel than the Pile validation set across *n*-gram sizes due to contamination.

disk and execute NNSL queries on each of the 30 CDAWGs in parallel. We aggregate NNSL (by taking the max) and counts returned (by summing at maximum suffix lengths) to exactly simulate the output of a single CDAWG.

#### 4.2 Generating Text from LMs

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We evaluate the generation novelty of the Pythia models (Biderman et al., 2023), which were trained on the Pile (Gao et al., 2020) at different sizes up to 12B parameters. We consider two setups, (1) generating unmprompted texts, and (2) generating prompted texts, for which we sample 500 documents from the Pile validation set (trimmed to 1,000 tokens). In each setup we generate 500 documents of 1,000 tokens from each LM. We vary the model size (from 70M to 12B, 8 models in total) and decoding strategy, sweeping different parameters for top-p (Holtzman et al., 2020), top-k (Fan et al., 2018), temperature, and greedy beam search. Unless indicated otherwise, we use Pythia-12B and naive sampling with unconditioned prompt as defaults. We pass each generated text through the CDAWG to compute the NNSL at each position (cf. Section 3), from which the *n*-novelty curves as a function of *n*-gram size can be computed.

#### 4.3 Novelty Baselines

For small *n*, some *n*-grams will likely be repeated between a document and a large corpus by random chance. For large *n* this probability will decrease rapidly. Thus, to evaluate the novelty of LM generations, it is necessary to establish a baseline *n*-novelty curve. We consider two such baselines:



(a) n-novelty curves across model sizes.



(b) Mean NNSL across model sizes.

Figure 3: Both n-novelty and mean NNSL suggest larger LMs generate less novel text than smaller LMs.

Validation Text. Following McCoy et al. (2021), we use the novelty of text in the Pile's validation set as a baseline. If n-grams of a certain size are less novel in generated text compared to validation text, the LM is generating pretraining n-grams more commonly than expected for new documents from the pretraining distribution. This suggests the LM is copying from its pretraining corpus.

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**Text After Pile Date Cutoff.** The novelty of validation text may be artificially low if the training distribution contains duplicated documents (Lee et al., 2022a). To account for this, we filter text from Dolma (Soldaini et al., 2024) that was written after the Pile collection cutoff. Specifically, the two domains we use are Reddit and scientific texts (Pes2o; Soldaini and Lo, 2023), both of which are in-distribution for the Pile. Thus, we expect this baseline to represent natural overlap for human-written text without contamination. We report the *n*-novelty curve fit on both domains from Dolma together, though qualitatively we observe that the curve looks similar within each domain.

#### 5 Novelty of LM-Generated Text

#### 5.1 Novelty vs. Human-Written Text

To answer RQ1, we compare the novelty of LMgenerated text against novel human-written text. As such, we report novelty metrics for two humanwritten text baselines: validation text and Dolma documents written after the Pile cutoff.

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Setup	Param	Mean	Max
Dessline	Validation	29.94	1,000
Basenne	Reddit	4.74	66
	70M	4.18	187
	160M	4.07	207
	410M	4.61	191
Size	1 <b>B</b>	5.07	270
	1.4B	5.22	225
	2.8B	5.18	322
	6.9B	5.32	198
	12B	6.19	376
	1	5.83	624
Prompt	10	6.21	393
_	100	7.56	976

Table 1: NNSL results for human-written text baselines and different model sizes and prompt lengths.

**Validation Baseline.** Figure 2 shows that the validation *n*-novelty curve is very low across *n*. 2.4% of the 1,000-token validation documents are *exactly matched* somewhere in the Pile training set. 13.6% share a 100-gram with the training data, and 25.0% share a 50-gram. This suggests contamination, since we expect natural large-*n*-gram overlap should be vanishingly unlikely.

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To formally test this, we derive a lower bound on n-novelty assuming most next tokens are nondeterministic (Appendix C).<sup>3</sup> Under this assumption, the n-novelty curve for non-contaminated data should not enter the red region in Figure 2, i.e., almost all 23+-grams should be novel. The validation curve (but not Dolma) enters this region, suggesting many contaminated n-grams in the validation text. Thus, we turn to Dolma as a better representation of uncontaminated human-written text.

**Dolma Baseline.** Figure 2 shows that *n*-grams of size n > 4 are less novel in generated text compared to Dolma text, whereas *n*-grams of size  $n \le 4$  (median length) are slightly more novel. For instance, 8% of Pythia bigrams are novel (vs. 5% for Dolma), while 93% of Pythia 10-grams and 99% of 100-grams are novel (vs. 98% and 100%). This disagrees with McCoy et al. (2021)'s findings for small LMs trained on 40 GB of text, where LMs were more novel on small *n*-grams. One explanation for the difference may be the model and data scale, motivating us to more closely analyze the

Decoding	Param	Mean	Max
Baseline	Validation	29.94	1,000
	Reddit	4.74	66
Тор-р	0.85	15.02	992
	0.9	8.85	1000
	0.95	9.69	902
Top-k	20	11.34	507
	80	9.24	580
	160	8.17	386
Temperature	0.5	14.22	983
	0.85	10.18	969
	0.9	11.05	1,000
	0.95	6.55	418
	1.05	5.08	313
	1.1	4.34	375
Beam	8	192.03	408
	4	9.17	18
	1	8.40	19

Table 2: NNSL results for Pythia-12B with different decoding strategies. Across strategies, more constrained decoding leads to less novel text.

impact of model size on novelty in Section 5.2.

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**Examples of Copied** *n***-Grams.** We find that many non-novel *n*-grams generated by Pythia-12B are pieces of licenses and boilerplate code. For example, Pythia-12B generates a 64-gram with 45K occurrences in the Pile that starts:

//	34
// Licensed under the Apache License,	340
Version 2.0 (the "License");	34
// you may not use this file except in com-	34
pliance with the License	349

Another generated 64-gram (with 213K occurrences in the Pile) imports Linux libraries:

#include <sound core.h=""></sound>	352
<pre>#include <sound pcm.h=""></sound></pre>	353
<i>#include <sound soc.h=""></sound></i>	354

#### 5.2 Impact of Model Size and Decoding

Having explored the novelty of LM-generated text compared to human-written text, we assess the factors that influence the generation novelty of LMs (RQ2). We compare *n*-novelty curves varying model sizes, decoding strategies, and different amounts of training data used as a prompt.

**Larger LMs are Less Novel.** Figure 3a shows 362 that, across n, n-grams are less novel for larger 363

<sup>&</sup>lt;sup>3</sup>We assume 90% of tokens have entropy  $\ell \ge 1.8$  bits/token based on the best achieved Pile losses (Du et al., 2022).



Figure 4: Impact of decoding choices (top-k, top-p, temperature  $\tau$ , and beam size with  $\tau = 0$ ) on n-novelty. Less stochastic (darker) decoding choices *decrease novelty*; temperature and beam size have the strongest effect.

LMs than for smaller LMs. Similarly, Figure 3b shows that the mean NNSL increases linearly with log model size. Both metrics suggest that larger LMs are *less novel* than smaller LMs across all *n*gram sizes. This may indicate that larger LMs have more capacity to memorize *n*-grams from training.

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**Decoding Constraints Decrease Novelty.** Prior work with small LMs and corpora suggests decoding choices could influence generation novelty (Mc-Coy et al., 2021). In particular, we expect more constrained decoding to decrease novelty (Liu et al., 2024). To evaluate this, we generate text with top-p, top-k, temperature (including greedy), and greedy beam search decoding setups, varying the parameter that constraints generation in each case. We hypothesize that the parameter choices that more constrained will result in lower generation novelty.

Indeed, Figure 4 shows that constrained decoding reduces *n*-novelty. The constrained decoding curves are consistently below the Dolma baseline, and, for small *n*, even below the validation baseline. The least *n*-novel approaches are low-temperature decoding and beam search. For 10-grams, temperature 0.5 reaches 71% novelty and temperature 0 reaches 69% novelty, while for 100-grams, temperature 0.5 reaches 98% and temperature 0 reaches 100%. Increasing beam size decreases novelty, with beam size 8 remaining near 0% novelty even up to 100-grams. With beam size 8, the LM deterministically generates a single document containing a 408-gram from training (see Appendix D for beam size results with nondeterministic conditioned generation). As we show in Section 5.1, temperature  $\leq 0.9$  and top- $p \leq 0.95$  dramatically increase both the mean NNSL and the max. Both novelty metrics suggest that constrained decoding reduces generation novelty.

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Long Training Prompts Slightly Decrease Novelty. To evaluate the impact of prompting with training data, we prompt the model with p tokens from the beginning of a training document before generating 1,000 additional tokens. We then evalute the n-novelty curve for these 1,000 tokens. Qualitative inspection reveal that the novelty curves look almost identical independent of the prompt length p. However, the NNSL statistics (Section 5.1) tell a more subtle story: the median NNSL remains unchanged, whereas the mean increases from 6.19 to 7.56 with 100 prompt tokens. This suggests that, while most n-grams do not become more novel when a prompt is given, the longest non-novel ngrams are longer when a longer prompt is given.

#### 6 Impact of *n*-Gram Training Frequency

Finally, regarding RQ3, we aim to test whether, at inference time, LMs assign higher probability



(a) Completion loss of Pythia-12B on n-grams in validation text based on whether the n-grams occurred in training. Across n-gram sizes, Pythia-12B assigns lower loss to n-grams seen during training.



(b) Completion loss as a function of n-gram training frequency for different n-gram sizes. Across n-gram sizes, more frequent n-grams have lower loss (with larger n being easier to predict).

Figure 5: *n*-gram completion loss based on presence in train and frequency.

419to *n*-grams from training, and how this interacts420with training frequency. We define the mean com-421**pletion loss** of  $x_1 \cdots x_n$  as the average probability422assigned to  $x_n$  when it occurs in validation text:

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$$\hat{\ell}(x) = \frac{1}{|V_x|} \sum_{i \in V_x} p_{\text{LM}}(v_i \mid v_{1:i-1}),$$

where  $V_x = \{i : v_{i+1-n:v_i} = x\}$ . This captures the LM's sensitivity to training *n*-grams in a way that is independent of the specific sampling choices made when decoding from the LM. It also captures use cases of LMs where the LM is used to assign probabilities to strings rather than as a text generator, such as in multiple-choice question answering like MMLU (Hendrycks et al., 2021) or evaluation of noun-verb agreement (Marvin and Linzen, 2018).

**Method.** We sample 5,000 documents of 1,000 tokens each, from the Pile validation set. We compute the per-token loss using Pythia-12B and use the CDAWG to find the non-novel suffixes at each position. For each n, we find tokens in the validation data that fall into two categories:

- In Train: The *n*-gram ending at the token occurred in the training data.
- Not in Train: The n-gram ending at the token did not occur in the training data, but the (n – 1)-gram ending at the previous token did.

We then compute the mean completion loss across all tokens in each condition with the same value of n, and plot this mean loss as a function of n. This quantity measures the surprisal of the LM when completing n-grams, with the two conditions differentiating whether the correct n-gram completion appeared in the training data. For the n-grams in the training data, we also investigate how their frequency affects completion loss.

a) $2.5 \times 10^7$	b) $2.3 \times 10^7$	c) $2.1 \times 10^7$	d) $1.1 \times 10^7$
	$\frac{\text{is}}{1.4\times 10^8}$	$\begin{array}{c} \mathrm{are} \\ 2.9 \times 10^7 \end{array}$	

Table 3: *n*-gram frequencies in the Pile computed by CDAWG. a) is more frequent than other options, and is is more frequent than are. Combined with Figure 5b, this suggests evaluations that use continuation probabilities may be susceptible to pretraining frequency effects.

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**Training** *n***-Grams are Easier to Complete.** Figure 5a shows that, across n-gram sizes, the completion loss for n-grams from the training set is smaller than for *n*-grams not in the training set (concretely, for n-grams above size 10, the completion loss is roughly 50% when the n-gram was in the training set vs. not). For n > 80, the loss curve for ngrams not in training becomes noisy, reflecting the rarity of such *n*-grams. These results suggest that Pythia-12B is upweighting tokens that complete ngrams from pretraining.<sup>4</sup> This finding potentially explains why more constrained decoding decreases novelty: while LMs assign probability to complete training n-grams, their next-token prediction with standard sampling also places a lot of probability mass on other tokens. Thus, training *n*-grams may not always get generated. However, the finding that training *n*-grams are upweighted in terms of probability suggests that pruning probability mass on other tokens (as approaches like top-p or top-kdo) would cause even more training n-grams to be generated, as found in Section 5.2.

<sup>&</sup>lt;sup>4</sup>While these results may be confounded (training *n*-grams may be easier to complete for other reasons besides their occurrence in the training set), we believe this is not a significant issue and leave the answer to this question for future work.

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**Frequent** *n*-**Grams are Easier to Complete.** Figure 5b shows that, across sizes, *n*-grams that are more frequent in the training data are easier for Pythia-12B to complete, implying LM predictions are sensitive to training data frequency effects. This is particularly relevant when specific token continuations are compared to assess multiple choice answers: e.g., a), b), c), and d) for MMLU evaluation (Hendrycks et al., 2021), or comparing i s/are to assess noun-verb agreement competence (Marvin and Linzen, 2018). Table 3 shows that the Pile frequency of these continuations are not uniform. Combined with Figure 5b, this suggests evaluating LMs by comparing these tokens may be susceptible to pretraining frequency effects.

#### 7 Related Work

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#### 7.1 Methods for Accessing Text Corpora

Data is becoming an important factor for understanding LM behavior (Elazar et al., 2024). As the scale of pretraining datasets continues to increase, naive search through these large datasets does not scale. As such, we need clever algorithms and data structures to interact with and study huge datasets.

McCoy et al. (2021), the first work to study the generation novelty of LMs, trained on Wikitext-103 (<1 GB) and WebText (40 GB). At this small data scale, they could run naive string matches over the data, a process that would not be feasible today with the Pile (1254 GB) or larger datasets. In contrast, Elazar et al. (2024) use an elastic search index based on an inverted index that allows a to search a corpus which depends on the number of documents in the corpus, making it much slower then our approach. Carlini et al. (2023); Liu et al. (2024) use a suffix array (Manber and Myers, 1990), allowing queries in logarithmic time w.r.t. corpus size. Another data structure previously used in the setting of text generation with retrieval is the FMindex (Ferragina and Manzini, 2000; Bevilacqua et al., 2022), a compressed suffix array.

In this work, we use a CDAWG (Crochemore and Vérin, 1997; Inenaga et al., 2005), which is a refinement of the earlier DAWG (Blumer et al., 1984), and part of a larger family of "\*DAWG" indices (Takagi et al., 2017; Inenaga, 2024). \*DAWGS use more memory than suffix automata but support faster membership and suffix overlap queries (cf. Section 3). \*DAWGs also support fast infinite *n*-gram queries (Liu et al., 2024), which could be useful for retrieval language modeling applications.

# 7.2 Memorization, Contamination, and Generalization

The increased use of LMs has raised concerns about memorization artifacts that might limit their generalization potential. For instance, Bender et al. (2021) draw a parallel of LMs to "stochastic parrots" that memorize and mimic their training data.

Memorization has been carefully studied and quantified (Zhang et al., 2021; Kandpal et al., 2022; Lee et al., 2022b; Magar and Schwartz, 2022; Carlini et al., 2023; Ippolito et al., 2023) and is often framed as a concerning property of model behavior. On the other hand, other works claim that memorization is integral for generalization (Feldman, 2020; Feldman and Zhang, 2020; Chatterjee, 2018). In this work, we do not take a stance on the importance or dangers of memorization, but rather quantify the novelty of LM-generated text vs. human text and investigate how different parameters affect novelty. In contrast to much previous work on memorization, we also focus on the novelty of typical text rather than text elicited in adversarial settings (Carlini et al., 2023; Ippolito et al., 2023).

Like McCoy et al. (2021), we focus on generation novelty rather than quality, and we are interested in the effect of different variables such as model size, and decoding strategies on the generation novelty. Due to the CDAWG, we are able to scale our analysis to larger datasets than McCoy et al. (2021). In addition to text diversity, Shaib et al. (2024) investigated the diversity of generated part-of-speech sequences rather than texts, as an abstract measurement over the raw texts.

#### 8 Conclusion

We introduce RUSTY-DAWG, an efficient index for finding *unbounded length n*-gram overlap against a pretraining corpus in *constant time*. Using RUSTY-DAWG, we show that, at large *n*, Pythia generates less novel *n*-grams than novel humanwritten text. We also find that increasing model size, constrained decoding (e.g., with temperature 0), or prompting with training data can decrease novelty. Finally, more frequent training *n*-grams are completed by LMs with *lower loss*. We hope RUSTY-DAWG enables further analysis of pretraining data as well as decontamination (Magnusson et al., 2023) and retrieval language modeling (Khandelwal et al., 2020; Liu et al., 2024) research.

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#### 573 Limitations

When evaluating novelty, we focus on verbatim *n*-gram novelty rather than evaluating semantic 575 novelty, which would be harder to operationalize. Our analysis focuses on the non-deduplicated Pile, a primarily English dataset. There are many vari-579 ables about data curation or LM training that could affect generation novelty beyond the ones we have considered, which could be explored using similar methodology in future work. Finally, as discussed in Appendix B.3, one challenge with deploying 583 584 the CDAWG is the memory overhead, though we believe this can be optimized in future work.

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Output

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## **B** CDAWG Details

## **B.1** Populating Counts

documents before dividing.

Thus, the *n*-novelty is

We build the CDAWG according to Figure 17 of Inenaga et al. (2005). The final post-processing step we add is to populate the counts in the CDAWG via a depth-first traversal (cf. Algorithm 1). The idea is that the CDAWG represents the frequency of a string x in C by the number of paths from the node reached by x to a sink node. Further, the frequency of each node is the sum of the frequencies of its children. Thus, we can populate all the counts in the CDAWG via a depth-first traversal of its nodes, which takes time O(|C|).

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tion. Communications of the ACM, 64(3):107–115.

**Computing** *n***-Novelty** from NNSL

The direct output of the CDAWG is  $L_Q$ , the NNSL

vector across each position in Q. We now de-

scribe how to compute the n-novelty curve from

 $L_Q$ . First, we define c(n) as the the number of

 $c(n) = \sum_{\ell' \in L_Q[i]} \mathbb{1}[n = n'].$ 

 $\left(\sum_{n \in \mathcal{L}} c(n)\right) - (n-1).$ 

The total number of *n*-grams in Q is |Q| - (n-1).

 $n - \mathrm{novelty}(Q) = \frac{\left(\sum_{n' < n} c(n)\right) - (n-1)}{|Q| - (n-1)}.$ 

This can be extended to multiple documents by

summing the numerator and denominator across

Next, the number of novel n-grams in Q is

Retrieval, pages 304-316.

## **B.2** Querying the CDAWG

Algorithm 2 implements an NNSL query by greedily passing Q through the CDAWG one token at a time. We track the current state, any intermediate

### Algorithm 1: Add counts to CDAWG

Data: CDAWG G with source  $q_0$ create stack S;push  $\langle OPEN, q_0 \rangle$  onto S;while  $\langle o, q \rangle \leftarrow pop from S$  doif o = OPEN then| if count(q) > 0 then| continue;count(q)  $\leftarrow 1$ ;push  $\langle CLOSE, q \rangle$  onto S;for child q' of q do| push  $\langle OPEN, q' \rangle$  onto S;else| count(q)  $\leftarrow 0$ ;for child q' of q do| add count(q') to count(q);

progress along an arc represented by indices  $\langle \alpha, \gamma \rangle$ for a span in C, and the currently matched length. If no progress can be made along an arc by the next token, a failure arc (Allauzen, 2023) is followed to back off until a state with a defined transition is found (or to  $\emptyset$  if no such state exists). If some partial progress is matched along an arc, that progress must be matched at the arc out of the state backed off to as well. We refer to this as an *implicit failure transition*, denoted by  $\phi(q, \langle \alpha, \omega \rangle)$ .

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## **B.3** Graph Representation

An important detail for the memory usage of the CDAWG is it is represented as a graph. We represent the graph as a list of nodes and a list of edges. The edges at each node are represented by a binary AVL tree, which means the arc labelled by token  $\sigma \in \Sigma$  can be found in  $O(\log |\Sigma|)$  time. However, this representation means each edge takes 26 bytes (with 5 byte pointers), which leads to an overall size of 29|C| for the CDAWG. This is roughly  $4\times$  larger than the corresponding suffix array, meaning there is a time/space tradeoff between the two approaches. We believe the memory overhead factor of the CDAWG could be significantly optimized by refining this graph representation in future work.

The memory overhead of RUSTY-DAWG could be further reduced by implementing recent improvements of the CDAWG such as the linear-size CDAWG (LCDAWG; Takagi et al., 2017) and simplified LCDAWG (simLCDAWG; Inenaga, 2024).

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Algorithm 2: NNSL query with CDAWG

**Data:** CDAWG G with source  $q_0$ **Input:** query Q **Output:** NNSL vector  $L_Q$  and counts  $N_Q$ , emitted pairwise  $s.q \leftarrow q_0;$  $s.\langle \alpha, \omega \rangle \leftarrow \langle 0, 0 \rangle;$  $s.\ell \leftarrow 0;$ for token  $\sigma$  of Q do  $s \leftarrow \operatorname{trans}(s, \sigma);$ emit  $\langle s.\ell, \mathsf{count}(s.q) \rangle$ ; **fn** trans( $s, \sigma$ ): if  $s.q = \emptyset$  then  $s.q \leftarrow q_0;$  $s.\langle \alpha, \omega \rangle \leftarrow \langle 0, 0 \rangle;$  $s.\ell \leftarrow 0;$ else if  $\alpha = \omega$  then  $q' \leftarrow$  target of completed arc; if  $e \leftarrow \sigma$ -edge out of q' then  $s.q \leftarrow q';$  $s.\langle \alpha, \omega \rangle \leftarrow$  weight of e;  $s.\ell \leftarrow s.\ell + 1;$ else  $s.q \leftarrow \phi(q', \langle \alpha, \omega \rangle);$  $s \leftarrow \mathsf{trans}(s, \sigma);$ else  $\sigma' \leftarrow \text{token } s.\alpha \text{ of } C;$ if  $\sigma = \sigma'$  then  $s.\alpha \leftarrow s.\alpha + 1;$  $s.\ell \leftarrow s.\ell + 1;$ else  $s.q \leftarrow \phi(q, \langle \alpha, \omega \rangle, \ell);$  $s \leftarrow \mathsf{trans}(s, \sigma);$ return s;

#### B.4 RUSTY-DAWG Library

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DAWGs and CDAWGs can be stored in either RAM or disk to accomodate different resource constraints (building and inference are faster in RAM, but for very large datasets, using disk may 865 be preferable due to resource constraints, especially for inference). Rust was chosen as a language so runtime and memory overhead could be optimized, though we also created Python bindings for easy integration with machine learning workflows. All 870 experiments in the paper were carried out using the 871 Python bindings to access a CDAWG built with the **RUSTY-DAWG** library. 873

#### C Lower Bound on Novelty Without Duplication

Our theoretical lower bound baseline is based on the idea that the next token is fundamentally nondeterministic, and, therefore, long n-gram spans should be unlikely.

#### C.1 Warmup: Always Nondeterministic Case

Say that we sample a corpus C of strings from some distribution p and then denote by  $\mathcal{D}_n$  the set of all n-grams in C. We then let X be a random string of length n sampled from p. We say that X is n-novel if  $X \notin \mathcal{D}_n$  and we are interested in analyzing this probability. The probability of this event is:

$$p(X \ n-\text{novel}) = 1 - p\left(\bigvee_{d \in \mathcal{D}_n} \bigwedge_{i=1}^n \delta[X_i = d_i]\right).$$

By the union bound,

$$p(X \ n-\text{novel}) \ge 1 - \sum_{d \in \mathcal{D}_n} p\left(\bigwedge_{i=1}^n \delta[X_i = d_i]\right)$$
<sup>89</sup>

$$= 1 - \sum_{d \in \mathcal{D}_n} \prod_{i=1}^n p(X_i = d_i \mid X_{< i}).$$
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Assume p is always nondeterministic at every position, so there is some q < 1 such that, for all i,

$$p(X_i = d_i \mid X_{\le i}) \le q.$$

Then it follows that:

$$p(X \ n-\text{novel}) \ge 1 - |\mathcal{D}_n| \cdot q^n$$

$$\ge 1 - |C| \cdot q^n$$
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$$= 1 - |C| \cdot \exp(-n\ell), \tag{898}$$

where  $\ell$  is the inherent entropy of any token (defined implicitly by q). A first observation here is that  $p(X \ n-\text{novel})$  should exponentially decay 1 quickly with n.

#### C.2 Probably Nondeterministic Case

Say more generally that with probability p (over tokens in C), we have entropy  $\ell$  about the next token. Then the above becomes:

$$p(X \ n-\text{novel}) \ge 1 - |C| \cdot p^n \cdot a^n$$

$$= 1 - |C| \cdot \exp(n(\log p - \ell)).$$
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This is the form of the lower bound invoked in the main plots (cf. Section 4.3).



Figure 6: Beam decoding results with different amounts of training tokens used as a prompt: 1 token (left), 10 tokens (center), and 100 tokens (right).

#### D Beam Search Results Elaboration

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The beam search decoding used in Figure 4 is de-912 terministic because the temperature is 0 and the 913 prompt is null. To complement these results, we 914 also include additional results in Figure 6 where a 915 prompt of length p (taken from the training data) 916 is used. In this regime, we find that, similar to 917 the promptless results, beam search decreases n-918 novelty. However, the novelty curve is not so ex-919 treme for beam size 8. This indicates that, with 920 beam size 8, the LM does not always copy very 921 large chunks of training documents like in Figure 4. 922