# Local Fragments, Global Gains: Subgraph Counting using Graph Neural Networks

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## Abstract

Subgraph counting is a fundamental task for analyzing structural patterns in graph-structured data, particularly crucial for applications in computational biology and social network analysis, where identifying recurring motifs reveals functional properties and organizational structures. We propose a novel three-stage differentiable learning algorithm that computes the counts of various patterns by learning to combine the counts of its subpatterns. Our approach leverages localized versions of Weisfeiler-Leman (WL) algorithms and introduces a novel fragmentation technique that decomposes complex subgraphs into simpler patterns. This technique enables exact counting of all induced subgraphs of size at most 4 using just 1-WL. This method significantly improves upon existing Graph Neural Network(GNN) based approaches for subgraph counting, being computationally efficient, making it well-suited for learning combinatorial algorithms.

## 1 Introduction

Subgraph counting represents one of the most fundamental challenges when working with graph-structured datasets, with profound implications for understanding complex network structures across diverse domains. In computational biology, the ability to count specific molecular substructures such as benzene rings or protein binding motifs directly impacts drug discovery and molecular property prediction. Similarly, in social network analysis, counts of triangular patterns, stars, and other local structures provide insights into community formation, information propagation, and social capital dynamics.

The importance of accurate subgraph counting extends beyond mere pattern detection—it serves as a critical measure of a GNN's expressive power. Traditional message-passing neural networks (MPNNs) are fundamentally limited by their equivalence to the 1-dimensional Weisfeiler-Leman (1-WL) test, which prevents them from distinguishing between graphs that differ in their subgraph counts but are otherwise structurally similar. This limitation has significant practical consequences: standard GNNs cannot count triangles, cycles larger than 3 nodes, or other complex substructures that are essential for many real-world applications.

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Previous research [1, 2, 3, 4, 5] has primarily focused on two main paradigms to overcome these expressivity limitations. The first approach involves developing more powerful GNN architectures based on higher-order WL tests ($k$-WL), which can theoretically count larger substructures but suffer from prohibitive computational complexity scaling as $O(n^k)$, with $n$ being the number of nodes in the graph. The second paradigm employs nested GNN approaches, which extract local subgraphs around each node and apply base GNNs to these subgraphs. While nested GNNs can count certain substructures, they require expensive preprocessing steps and suffer from high time and memory costs when encoding large graphs. [6, 7] show that message-passing GNNs can count subgraphs by leveraging node individualization or learning sufficient graph statistics.

Subgraph GNNs represent another significant direction, where the input graph is partitioned into numerous subgraphs, and GNNs are applied to each subgraph to augment the overall graph representation. However, these methods require repetitive application of GNNs across all subgraphs, leading to substantial computational overhead that limits their practical applicability to large-scale problems.

Our proposed technique is motivated by the development of localized variants of the WL algorithms [8]. We use the term "local $k$-WL " to refer to such algorithms. These algorithms reduce the computational overhead of the standard $k$-WL approaches. The local $k$-WL algorithms operate on restricted neighbourhoods of a graph rather than the entire graph. Since it has been established that the expressiveness of standard GNNs is the same as 1-WL [9, 10], we can consider subgraph GNNs to have expressive power equivalent to local 1-WL.

Here, we introduce a novel **fragmentation** technique that represents a paradigm shift in how we approach complex subgraph counting. Instead of attempting to count complex subgraphs directly, our fragmentation method decomposes target subgraphs into simpler constituent patterns whose counts can be computed exactly using efficient algorithms.

The fragmentation approach is particularly well-suited for differentiable combinatorial algorithm learning frameworks because it naturally decomposes the complex counting problem into learnable components. Our three-stage learning algorithm first learns to identify the required subpatterns for any target subgraph, then accurately counts these subpatterns using our localized WL variants, and finally aggregates the subpattern counts into the final global count. This decomposition not only provides theoretical guarantees for correctness but also enables efficient gradient-based optimization of the entire counting pipeline.

Our method represents a significant advancement over existing approaches by combining the theoretical rigor of WL-based methods with the practical efficiency needed for real-world applications, while providing a natural fit for the emerging field of differentiable combinatorial algorithms.

## 2 Preliminaries

We consider a simple undirected graph $G(V, E)$. For basic definitions of graph theory, we refer the reader to [11]. The neighbourhood of a node $v \in V$ is denoted as $\mathcal{N}_G(v)$. The *closed* neighbourhood of $v$ is the set of all neighbours, including the node $v$ (denoted as $\mathcal{N}_G[v]$). We use the notation $d_v$ to refer to the degree of a node $v$. The radius of a graph is the minimum over all the nodes of a graph of the maximum distance from a node to any other node in the graph.

A graph $H$ is called a *subgraph* of $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. The subgraph induced on $S \subseteq V(G)$ is a graph whose node set $S$ contains all the edges in $G$ whose endpoints are in $S$ and is denoted by $G[S]$. The *induced subgraph* on an $r$-hop neighbourhood around node $v$ is denoted by $G_v^r$. The number of hops $r$ depends on the pattern of interest. For example, in order to count triangles, $r = 1$ is sufficient, whereas for counting $C_4$, we require $r = 2$. Attributed subgraphs are subgraphs where each node is marked with an attribute or a colour (also referred to as motifs). We note here that we count the number of patterns occurring as subgraphs in our work.

The Weisfeiler-Leman test [12] is a type of colour refinement algorithm used for testing the existence of an isomorphism between two graphs $G_1$ and $G_2$. Interested readers may refer to [13] for a detailed discussion. The local $k$-WL algorithm is applied to local subgraphs. Specifically, for each node $v \in V$, we extract $G_v^r$ and apply $k$-WL on it. While running $k$-WL on a graph with $n$ vertices require $O(n^{k+1} \log n)$ time, running local $k$-WL on any $G_v^r$ extracted from a graph of bounded degree $d$ requires times $O(n.d^{r(k+1)}. \log d)$.

Graph Neural Networks were first introduced by [14]. The model proposed a recursive use of two functions: MESSAGE and AGGREGATE to update the node embeddings $X^\ell$ (for the $\ell$th layer).

$$X_v^{(\ell)} = \texttt{UPDATE}^{(\ell)}\left(X_v^{(\ell-1)},\ \texttt{AGGREGATE}^{(\ell)}\left\{\texttt{MESSAGE}^{(\ell)}\left(X_v^{(\ell-1)},\ X_u^{(\ell-1)}\right) \mid u \in \mathcal{N}_G(v)\right\}\right) \quad (1)$$

## 3 Methodology

Before describing the fragmentation algorithm, we demonstrate the method using an example as shown in Figure 1. Suppose that the pattern to be counted is a tailed triangle. Rooted at a node $v$ (the red coloured node), we consider a 1-hop subgraph $G_v^1$. Now consider the subgraph $H_v = G_v^1 \setminus \{v\}$. If a tailed triangle is present, then $H_v$ must have an isolated node and an edge (Fragments 1 and 2 in Figure 1). Hence, the counts of such (isolated node, edge) pairs give a count of the number of tailed triangles.

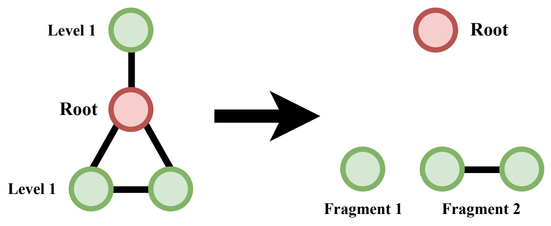


Figure 1: Fragmentation of a tailed triangle

Our proposed methodology comprises three components **(A) Pattern Learning**, **(B) Local Count Learning**, and **(C) Global Count Learning** as shown in Figure 2.

### 3.1 Pattern Learning

In the task of counting a particular pattern, such as a triangle or a 3-star, $G_v^1$ from each node $v$ is different. Counting triangles requires edges between the neighboring nodes of the root node only. Whereas, in the case of a 3-star, the subgraphs have edges between the root node and neighboring nodes. Therefore, we propose a learnable model that learns the required subgraph for a given pattern. For each node, we modify the $G_v^r$ by adding direction to the edges. We add a directed edge from the root node to other nodes. This modification is done to indicate which node is the root node in the subgraph. These modified subgraphs are then given as input to a GNN model, which updates the node embeddings. With the updated node embeddings, we construct the edge embeddings and classify the edges as 0 or 1, indicating which edges are required from $G_v^r$ as input for the next component.

### 3.2 Local Count Learning

From the previous section on pattern learning, we take the updated $G_v^r$ as input, where certain edges are removed based on the pattern that we want to take as input. For counting triangles, the local count (number of times a pattern appears in the local subgraph) is the number of edges in $G_v^r \setminus \{v\}$. For counting $k$-star graphs, the local count is the $\binom{d_v}{k}$, where $d_v$ is the degree of $v$. Likewise, we have different local counts as ground truth for different patterns. We input the updated $G_v^r$ to a GNN model and train the model based on the predictions of the local count.

### 3.3 Global Count Learning

While counting in different subgraphs, there is a potential for overcounting of the patterns. Therefore, we have a normalization model that takes the summation of all the local counts and learns the normalization factor required to counter the overcounted patterns.

### 3.4 Fragmentation

The fragmentation method, Algorithm 1, involves fragmenting the pattern $P$ into smaller subpatterns and counting these subpatterns to get the count of $P$ in the graph. The number of occurrences of

$P$ in $G_v^r$ can be computed by combining the counts of the simpler patterns (fragments). Instead of training a GNN for counting $P$, we can design GNNs for learning the easier tasks (i.e., for counting the fragments) and combine the outputs of those models. It should be noted that the fragmentation into smaller subgraphs depends on the structure of the pattern $P$.

Given a graph $G$ and a pattern $P$(whose number of occurrences is to be counted), we first fix a vertex $v \in V(G)$ as the key vertex. Now, assume that the radius of the pattern is $r$. Thus, for counting $P$ locally, it is sufficient to take $G_v^r$. Now, we fragment pattern $P$ into smaller subpatterns, say $\mathbb{P} = \{P_1, P_2, \dots P_l\}$. It should be noted that the decomposition of $P$ into $P_i$s is done in such a way that their counts can be calculated exactly (by a local 1-WL based trained GNN model). That is, we have learnt models $M_i^{\text{pattern}}$, corresponding to each $P_i$, which generates $\mathcal{P}_u$ for each node in $H_u = G_v^r \setminus \{v\}$. We also have learned models $M_i^{\text{count}}$, corresponding to each $P_i$, that count the number of subpatterns $P_i$ in $\mathcal{P}_u$. The array $c$ stores the count of $P_i$'s in each $H_u$. Now, for each subpattern $P_i$, we learn a function $\alpha$ as weights in a linear transformation to combine the counts in $c$ to get the count of $P_i$ in $H_u$. Then we learn the function $\beta$ to count $P$ for each root node. Finally, the function $\gamma$ finds the normalizing factor to get the actual count of the pattern in $G$.

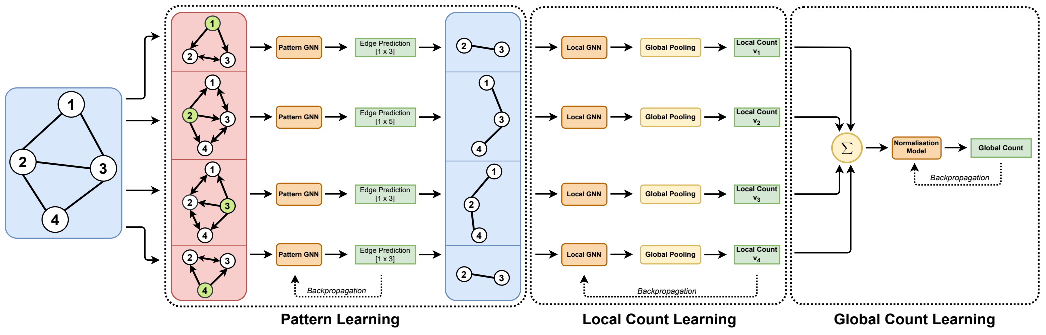


Figure 2: Overview of the framework. The pipeline is divided into three components – **(A) Pattern Learning:** $r$-hop neighborhood of each node is given as input with directed edges indicating the root node, where the model's task is to predict the required pattern corresponding to each node. **(B) Local Count Learning:** taking the predicted patterns as input from the **Pattern Learning** component, we predict the local count for each root node. **(C) Global Count Learning:** Upon addition of all local counts, we require to learn a normalization factor, which is learned in this section of the pipeline.

**Algorithm 1** Fragmentation Algorithm

```
Require: $G$; $\mathbb{P}$: List of patterns; $M_i^{\text{pattern}}$: learned model for generating pattern corresponding to
    $P_i \in \mathbb{P}$; $M_i^{\text{count}}$: learned model for counting $P_i \in \mathbb{P}$;
 1: $a \leftarrow []$
 2: for each node $v \in V(G)$ do
 3:     $H_v = G_v^r \setminus \{v\}$
 4:     $b \leftarrow []$
 5:     for each pattern $P_i \in \mathbb{P}$ do
 6:         $c \leftarrow []$
 7:         for each node $u \in H_v$ do
 8:             $\mathcal{P}_u = M_i^{\text{pattern}}(H_v, u)$
 9:             $c.append(M_i^{\text{count}}(\mathcal{P}_u))$
10:         end for
11:         $b.append(\alpha(c))$      #Learnable function
12:     end for
13:     $a.append(\beta(b))$      #Learnable function
14: end for
15: $Count = \gamma(a)$      #Learnable function
16: return $Count$
```

## 4 Experiments

### 4.1 Implementation Details

We refer to our proposed model as **InSig**. In the experiments, we predict the counts of the following substructures occurring as subgraphs: triangles, 3-Star, 2-Star, chordal $C_4$, $K_4$, $C_4$, and tailed triangles. When counting larger substructures like $K_4$, $C_4$, and Tailed Triangles, we use the

| **Task** | **Total pattern count** | **Zero Count graphs** | **Standard Deviation of Count** | **Average number of Nodes** | **Average number of Edges** | **Number of graphs** |
|---|---|---|---|---|---|---|
| *Triangle* | 25209 | 195 | 3.072 | | | |
| *2-Star* | 429463 | 0 | 18.015 | | | |
| *3-Star* | 309525 | 0 | 17.777 | | | |
| *Chordal* | 19088 | 1786 | 4.742 | 18.7976 | 62.678 | 5000 |
| *K4* | 643 | 4447 | 0.387 | | | |
| *C4* | 53002 | 16 | 6.938 | | | |
| *Tailed Triangle* | 177968 | 195 | 25.943 | | | |

Table 1: Dataset statistics. The total number of graphs in the dataset is 5000. We used 4000 graphs for training, 500 for validation, and 500 for testing.

**fragmentation** technique with the help of a model learned to predict triangles, 3-Star, 2-Star, and Chordal $C_4$. For counting $K_4$, the task of **Pattern Counting** Component is to learn which edges to prune. Later **Local Count Learning** component counts the number of substructures, which in the case of $K_4$, is triangles. Once the number of triangles is predicted using models learned to count triangles, the normalization factor is learned to output the global count. In other structures like tailed triangles, we have two patterns to learn: the nodes in the 1-hop neighborhood and the edges between them. During the inference phase, we use a rounding function, as the counts are integer numbers.

### 4.2 Hyperparameters

We use two GIN Convolutional layers for the Pattern Learning Local Count Learning component. We consider Linear transformations as readout layers in the previously mentioned components. Since we need to classify whether an edge should be present or not in a subgraph for counting a subpattern, we use Binary Cross-Entropy (BCE) loss for the pattern learning component. For the local counting and global counting components, we use Mean Absolute Error (MAE). For the models we have considered, as there can be paths of lengths more than 1 in the subgraph, 2 *GINConv* layers are sufficient to capture the information well.

We use a learning rate of $1e-4$ and a batch size of 1. We also experimented with different hidden dimensions for the node embeddings and obtained the best results when we used 4 as a hidden dimension size. The experiments were conducted using an NVIDIA A100 40GB GPU. The source code of the implementation is available at this Github Link.

### 4.3 Experimental Results

| **Models** | **Without Fragmentation** | | | | **Fragmentation** | | |
|---|---|---|---|---|---|---|---|
| | *Triangle* | *3-Star* | *2-Star* | *Chorcal C4* | $K_4$ | $C_4$ | *Tailed Triangle* |
| **ID-GNN** | 6.00E-04 | NA | NA | 4.52E-02 | 2.60E-03 | 2.20E-03 | 1.05E-01 |
| **NGNN** | 3.00E-04 | NA | NA | 3.92E-02 | 4.50E-03 | 1.30E-03 | 1.04E-01 |
| **GNNAK+** | 4.00E-04 | 1.50E-02 | NA | 1.12E-02 | 4.90E-03 | 4.10E-03 | 4.30E-03 |
| **PPGN** | 3.00E-04 | NA | NA | 1.50E-03 | 1.65E-01 | 9.00E-04 | 2.60E-03 |
| **I2-GNN** | 4.00E-04 | NA | NA | 1.00E-03 | 3.00E-04 | 1.60E-03 | 1.10E-03 |
| **InSig** | **0.00E-00** | **0.00E-00** | **0.00E-00** | **0.00E-00** | **0.00E-00** | **0.00E-00** | **0.00E-00** |

Table 2: MAE for the subgraph count of different patterns. Some results, such as 2-*star*, 3-*star*, are not conducted by the given baselines; therefore, it is mentioned NA.

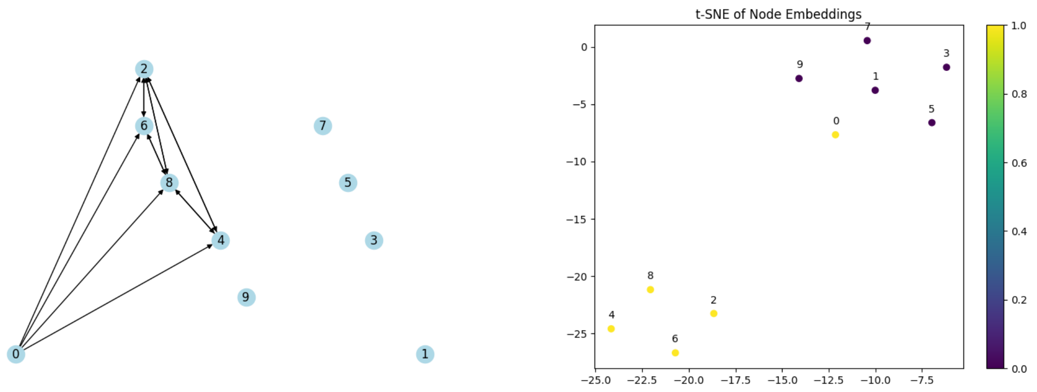
The dataset used for the experiments is a random graphs dataset prepared in [15]. In Table 1, we report the dataset statistics, specifically the counts of the various patterns used for our experiments, the number of graphs where these patterns do not appear, and so on. We report our experiments' Mean Absolute Error (MAE) in Table 2. We compare our results with those reported in *ID-GNN*

[2], *NGNN* [3], *GNNAK+* [4], *PPGN* [1] and *I2-GNN* [5]. We can observe that our approach of predicting substructures, the local counts, and then predicting the global counts, achieves zero error for all of the tests.

For all the patterns, we observed that the model gets to zero error after only 2 to 3 epochs. From Table 3, it can be observed that our approach requires a considerably smaller number of parameters and beats all the baselines. Our inference time comprises model inference as well as the graph preprocessing time, where it creates the set of subgraphs corresponding to each node in the graph.

| **Models** | **Number of Parameters** | **Inference Time (ms)** | **Memory Usage (GB)** |
|---|---:|---:|---:|
| **ID-GNN** | 102K | 5.73 | 2.35 |
| **NGNN** | 127K | 6.03 | 2.34 |
| **GNNAK+** | 251K | 16.07 | 2.35 |
| **PPGN** | 96K | 35.33 | 2.3 |
| **I2-GNN** | 143K | 20.62 | 3.59 |
| **InSig** | **268** | **20** | **1.2** |

Table 3: The table shows the comparison of the number of parameters required by the baselines and *InSig Model*. The values shown here correspond to the triangle counting task with the hidden dimension set as $4$.



(a) Figure shows an example of $G_0^1$ for a random graph with 10 nodes

(b) t-SNE plot of the updated node embeddings from the Pattern learning component

Figure 3: Figure showing the input and updated node embedding from the pattern learning component

In Figure 3, we show an example of a graph sent as input to the pattern learning component and the updated node embeddings output from the model. In Figure 3a, we see that the root node is node $0$ and there are directed edges from the root node to its neighboring nodes. Figure 3b shows the updated node embeddings of each node in the graph. We can observe that $8, 2, 4, 6$ nodes can be separated using some separator from the rest of the nodes. This indicates that the model is able to learn the updated node embedding such that we can distinguish nodes which has edges between them.

## 5 Conclusion

Subgraph counting is a fundamental combinatorial problem that arises in the study of graphs and graph-structured problems. Exactly counting the number of subgraphs in a graph is also a computationally hard problem. In this paper, we present a learnable algorithm that is able to compute exact counts of a number of commonly occurring patterns. The proposed fragmentation method has proven to be beneficial for the task of counting subgraphs. Additionally, since fragmentation results in smaller subgraphs, the parameter requirements of the proposed GNN-based architecture are orders of magnitude less than those of previous methods. As future work, we plan to analyse the fragmentation algorithm from a theoretical perspective. We also plan to investigate the effectiveness of this technique for increasing the expressiveness of GNNs for downstream tasks. Along with that, the current approach is limited to patterns that can be computed using 1-hop or 2-hop subgraphs; hence, we can study our algorithm for bigger patterns.

## References

[1] Haggai Maron, Heli Ben-Hamu, Hadar Serviansky, and Yaron Lipman. Provably powerful graph networks, 2019.

[2] Jiaxuan You, Jonathan Gomes-Selman, Rex Ying, and Jure Leskovec. Identity-aware graph neural networks, 2021.

[3] Muhan Zhang and Pan Li. Nested graph neural networks. *Advances in Neural Information Processing Systems*, 34:15734–15747, 2021.

[4] Lingxiao Zhao, Wei Jin, Leman Akoglu, and Neil Shah. From stars to subgraphs: Uplifting any GNN with local structure awareness. In *International Conference on Learning Representations*, 2022.

[5] Yinan Huang, Xingang Peng, Jianzhu Ma, and Muhan Zhang. Boosting the cycle counting power of graph neural networks with $i^2$-gnns, 2023.

[6] Paolo Pellizzoni, Till Hendrik Schulz, Dexiong Chen, and Karsten Borgwardt. On the expressivity and sample complexity of node-individualized graph neural networks. In *Proceedings of the 38th International Conference on Neural Information Processing Systems*, NIPS '24, Red Hook, NY, USA, 2024. Curran Associates Inc.

[7] Paolo Pellizzoni, Till Hendrik Schulz, and Karsten Borgwardt. Graph neural networks can (often) count substructures. In *The Thirteenth International Conference on Learning Representations*, 2025.

[8] Christopher Morris, Kristian Kersting, and Petra Mutzel. Glocalized weisfeiler-lehman graph kernels: Global-local feature maps of graphs. In *2017 IEEE International Conference on Data Mining (ICDM)*, pages 327–336, 2017.

[9] Christopher Morris, Martin Ritzert, Matthias Fey, William L. Hamilton, Jan Eric Lenssen, Gaurav Rattan, and Martin Grohe. Weisfeiler and leman go neural: Higher-order graph neural networks, 2021.

[10] Keyulu Xu, Weihua Hu, Jure Leskovec, and Stefanie Jegelka. How powerful are graph neural networks?, 2019.

[11] Douglas Brent West. *Introduction to graph theory*. Prentice Hall, Upper Saddle River, N.J., 2nd ed edition, 2001.

[12] Boris Weisfeiler and A. A. Lehman. A Reduction of a Graph to a Canonical Form and an Algebra Arising During This Reduction. *Nauchno-Technicheskaya Informatsia*, Ser. 2(N9):12–16, 1968.

[13] Ningyuan Teresa Huang and Soledad Villar. A short tutorial on the weisfeiler-lehman test and its variants. In *ICASSP 2021 - 2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, page 8533–8537. IEEE, June 2021.

[14] Thomas N. Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. *CoRR*, abs/1609.02907, 2016.

[15] Zhengdao Chen, Lei Chen, Soledad Villar, and Joan Bruna. Can graph neural networks count substructures? *Advances in neural information processing systems*, 33:10383–10395, 2020.

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