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Entropy-based node centralities in networks: Targeted attacks on Watts–Strogatz and power grid systems

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One way to assess the importance of a node in a network is to observe what happens to the size of the network after a node has been removed. If we keep removing nodes we obtain a disconnected network [1]. Given a connected graph the size of Largest Connected Component (*LCC*) is the initial size N of the graph i.e. the total number of nodes. Normalized *LCC* that is 1 for the complete connected graph is used and it decreases as long as nodes keep being removed. It has been defined a centrality measure based on Shannon information theory: given a node i, how much information does it need in order to navigate the graph so it can reach any other node j ? Let's define some useful quantities: k_h is the degree of the node h, then it defines some derivative quantities: $K_{ij} = min(\prod_{h=i}^{j-1} k_h), \quad P_{ih} = \frac{1}{K_{ih}}, \quad Q_i = \sum_{h\neq i}^{N} P_{ih}$
With: $h \in \mathscr{P}(i,j)$ i.e. given a starting node i and a target node j, let's take the path that minimizes K_{ij} and among them then take the shortest path. Finally, it is possible to define the Generalized Shannon information:

$$I_i = -\sum_{h\neq i}^{N} \log_2 P_{ih} = \sum_{h\neq i}^{N} \log_2 K_{ih} \tag{1}$$

This quantity represents the amount of information in bits that is needed from the node i in order to be able to locate all the other nodes. Similar quantities have been studied in this paper: [2]. The Normalized version of I_i (*NGSI*) in which the probabilities are normalized can be defined. Another related quantity is the normalized generalized Shannon entropy S_i^N (*NGSE*).

$$I_i^N = -\sum_{h\neq i}^{N} \log_2 \frac{P_{ih}}{Q_i} = \sum_{h\neq i}^{N} \log_2 (Q_i K_{ih}) \tag{2}$$

$$S_i^N = -\sum_{h\neq i}^{N} \frac{P_{ih}}{Q_i} \log_2 \frac{P_{ih}}{Q_i} = \sum_{h\neq i}^{N} \frac{\log_2 (Q_i K_{ih})}{Q_i K_{ih}} \tag{3}$$

An important node from the point of view of *NGSI* is a node with the minimum value of *NGSI*, because it is the one that needs the least amount of bits to reach all the others. On the contrary, the most important node from the standpoint of entropy is the one with the largest value of *NGSE*. Those 2 quantities have been used to perform targeted attacks on Watts-Strogatz (*WS*) networks [1] along with classical network centralities such as Degree, Betweenness and Closeness. In addition, as a benchmark, it has been used random node removal. The more is important a category of nodes the less fraction of nodes (f_r) we have to remove to reach the 10% of the *LCC*. It has been generated 100 *WS* networks and the plot (1a)depict the average among the 100 sample. What is noticeable in the plot (a) and summarized in the table (1 a) is that S_i^N (S_{info} in the plots) seems to catch the most important nodes in the graphs. In fact, there is a significant drop in the *LCC* already after removing the 30% of the nodes and the target of 10% of *LCC* is reached after removing the 55.6% of the nodes. For a real world case scenario, it has been studied the power grid network of the Great Britain (*GBPN*) [3], the network is composed by the high voltage nodes and the connections among them are the links. It has been considered only the giant connected component of the network, ignoring isolated nodes . The

weight of nodes and links has been ignored and has been considered as a connected unweighted network. Since the network is uniquely defined, only the random strategy has been reiterated 100 times. The connected grid is composed by 494 nodes and 598 links. As shown in the plot (1b), Degree, Betweenness and *NGSE* are pretty good in disrupting the network, however *NGSE* won on the last mile as shown on the table (1b). It has been shown that *NGSE* could be used to assess the list of the most important nodes in *WS* networks and can be used to perform targeted attacks that outperform in the early phases the canonical network centralities. This attack strategy has been applied to a real world case of the Great Britain power grid, this study can be used to mitigate the risks protecting nodes that with other methods where not considered important.

References

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Table 1: **Fraction of nodes removed to reach the disruption of networks.** (a) Watts–Strogatz model, (b) Great Britain power grid.

(a) Watts–Strogatz model

Strategy	f_r to reach 10% *LCC*
Degree centrality	0.694
Betweenness centrality	0.720
Closeness centrality	0.736
NGSE centrality	0.556
NGSI centrality	0.750
Random removal	0.820

(b) Great Britain power grid

Strategy	f_r to reach 10% *LCC*
Degree centrality	0.221
Betweenness centrality	0.249
Closeness centrality	0.769
NGSE centrality	0.174
NGSI centrality	0.783
Random removal	0.328

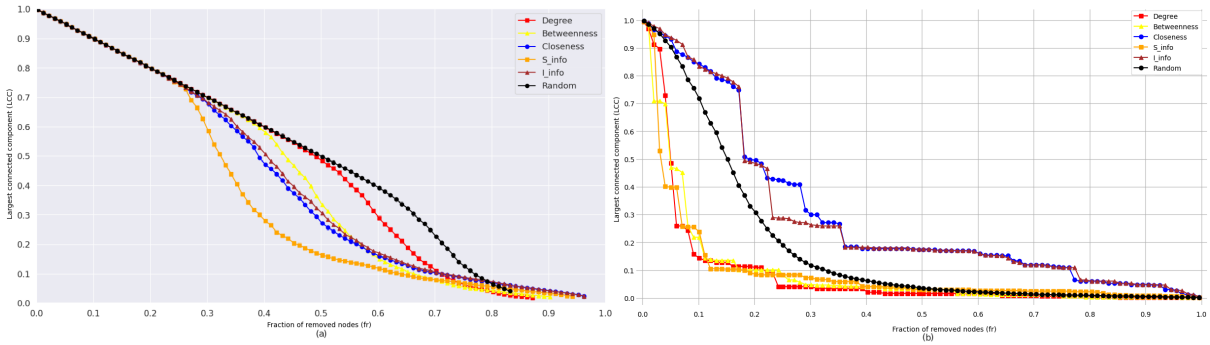

Figure 1: *LCC* in function of fr against different attack strategies. On the left: (a) *WS* networks with $N = 500$, $k = 16$, $p = 0.02$. On the right: (b) *GBPG* network