Higher Degree Cubature Quadrature Kalman Filter for Randomly Delayed Measurements

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***Abstract*—In this work, it has been assumed that the state estimators are located remotely and measurements are received through a common unreliable network. In such scenario, due to limited communication capacity, measurements are generally delayed in a random manner. In this correspondence, the authors developed a higher degree cubature quadrature Kalman filter (HDCQKF) for a nonlinear system with arbitrary step randomly delayed measurements. With the help of two examples, it has been shown that the randomly delayed HDCQKF provides more accurate estimation compared with randomly delayed cubature Kalman filter (CKF).**

***Index Terms*—State estimation, randomly delayed measurements, nonlinear filter**

I. Introduction

Filtering is one of the most important tools in engineering for precise state estimation of a dynamic system. For a linear dynamic system with additive white Gaussian noise, optimal solution exists in the name of Kalman filter (KF) [1]. But in most of the real life state estimation problems, system dynamics and observation model are nonlinear in nature and no optimal solution is available. Initially, estimation of nonlinear system was done with the help of extended Kalman filter (EKF) [2], which had played a very significant role over the last 30-40 years. The EKF applies standard Kalman filtering algorithm over the linearized nonlinear system [3]. The EKF can easily lead to divergence for a highly nonlinear system. To avoid such limitations, several efficient filtering techniques are proposed in the literature. Among them, the unscented Kalman filter (UKF) [4], Gauss-Hermite filter (GHF) [5], particle filter (PF) [6], cubature quadrature Kalman filter [7] [8] [12], higher degree cubature quadrature Kalman filter [9]–[11] *etc.* are important.

As mentioned earlier, here we assume that estimators and controllers are remotely located and connected with the physical system through a common communication network, where delay can occur. Work on networked estimation has been started with the work of Ray *et al.* [13], where the authors have modified the standard Kalman filter for randomly delayed measurements. Later in subsequent publications, an optimal filter is proposed for randomly sampled and delayed measurement [14], delayed input [15], and with multiple packet dropouts [16]. For a nonlinear process and observation model, Hermoso-Carazo *et al.* proposed a nonlinear filtering algorithm for one step [17] and two-step [18] randomly delayed measurement. In [19], a nonlinear filtering algorithm is developed by using CKF for one step randomly delayed measurement. Very recently, Singh *et al.* [20] have introduced a sub optimal solution using CQKF for an arbitrary step randomly delayed measurements. In this paper, we have extended the work of [20], [21] by formulating the higher degree cubature quadrature Kalman filter (HDCQKF) for arbitrary step randomly delayed measurements. We call the proposed method as HDCQKF-RD and the method is applied to two nonlinear networked estimation problems. From the simulation results, it has been seen that HDCQKF-RD performs better than CKF-RD.

II. Bayesian Framework of Filtering for Randomly Delayed Measurements

Let us consider a dynamic system whose state equation is given by

$$x_k = \phi_{k-1}(x_{k-1}) + \eta_{k-1}, \tag{1}$$

and measurement equation is

$$z_k = \gamma_k(x_k) + v_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state of a dynamic system, $z_k \in \mathbb{R}^d$ is sensor output. $\phi_k(x_k)$ and $\gamma_k(x_k)$ are nonlinear function of x_k. The process noise, η_k and measurement noise, v_k are assumed to be uncorrelated, white, Gaussian with zero mean and covariance Q_k and R_k respectively.

In the Bayesian framework, state estimation consists of two steps: (i) prediction step (ii) update step. The prior pdf is obtained in prediction step while the posterior pdf is calculated in update step. For linear system, the prior and posterior pdfs are Gaussian in nature. For nonlinear system, the prior and posterior pdfs are not Gaussian and arbitrary in nature. But many times it is approximated as Gaussian and mean and covariance of prior and posterior pdfs are calculated.

A. Delayed Measurement

In networked control systems, sensor output (z_k) reaches to remote estimator through a common, unreliable communication network. In this situation, not only measurement are, they can also be lost. The delay may be arbitrary step and random. The maximum number of delays considered by the

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algorithm will be fixed by the practitioner, and here we assume measurement can be delayed with maximum $(N-1)$ time step. At the time of fixing the largest number of delays, we need to consider two situations (i) if $N-1$ is very small, measurement can be lost (ii) if $N-1$ is very large, the complexity of the filtering algorithm will be increased. Delayed measurement (y_k) can be expressed as [20]

$$\begin{aligned} y_k &= (1-\beta_1)z_k + \beta_1(1-\beta_2)z_{k-1} + \beta_1\beta_2(1-\beta_3)z_{k-2} + \cdots \\ &+ (\prod_{j=1}^{N-1}\beta_j)(1-\beta_N)z_{k-N+1} + \Big[1-(1-\beta_1) - \beta_1(1-\beta_2) \\ &- \beta_1\beta_2(1-\beta_3) - \cdots - (\prod_{j=1}^{N-1}\beta_j)(1-\beta_N)\Big]y_{k-1} \\ &= \beta^0 z_k + \beta^1 z_{k-1} + \beta^2 z_{k-2} + \cdots + \beta^{N-1} z_{(k-N+1)} \\ &+ (1-\sum_{i=0}^{N-1}\beta^i)y_{k-1}, \end{aligned} \tag{3}$$

where $\beta^i = (\prod_{j=0}^{i}\beta_j)(1-\beta_{i+1})$ and $\beta_0 = 1$. β_j, $j \in \{1,2,3,\cdots,N\}$ are mutually independent Bernoulli random variables. As β_j are the Bernoulli random variables, the value of β_j will be either 0 or 1 and it satisfy the probability $P(\beta_j = 1) = \mathrm{p}$ and $P(\beta_j = 0) = 1-\mathrm{p}$. From now onwards, $\phi_{k-1}(x_{k-1})$ and $\gamma_k(x_k)$ will be replaced by $\phi_{k-1}(\cdot)$ and $\gamma_k(\cdot)$ respectively.

B. Time Update

In time update step, we evaluate prior mean $(\hat{x}_{k|k-1})$ and prior error covariance $(P_{k|k-1})$, which can be expressed as

$$\begin{aligned} \hat{x}_{k|k-1} &= E[\phi_{k-1}(\cdot) + \eta_{k-1}] \\ &= \int \phi_{k-1}(\cdot)p(x_{k-1}|y_{1:k-1})dx_{k-1} \\ &= \int \phi_{k-1}(\cdot)\aleph(x_{k-1};\hat{x}_{k-1|k-1},P_{k-1|k-1})dx_{k-1}, \end{aligned} \tag{4}$$

and

$$\begin{aligned} P_{k|k-1} &= E[(x_k-\hat{x}_{k|k-1})(x_k-\hat{x}_{k|k-1})^T] \\ &= \int \phi_{k-1}(\cdot)\phi_{k-1}^T(\cdot)\aleph(x_{k-1};\hat{x}_{k-1|k-1},P_{k-1|k-1})dx_{k-1} \\ &- \hat{x}_{k|k-1}\hat{x}_{k|k-1}^T + Q_{k-1}, \end{aligned} \tag{5}$$

where $p(\cdot)$ and $\aleph(\cdot)$ represent the probability density function and normal distribution respectively.

C. Measurement Update

To perform measurement update, we need to evaluate $\hat{z}_{k|k-1}$, $P_{k|k-1}^{zz}$ and $P_{k|k-1}^{xz}$. The expectation of the measurement can be expressed as

$$\begin{aligned} \hat{z}_{k|k-1} &= E[\gamma_k(\cdot) + v_k] \\ &= \int \gamma_k(\cdot)\aleph(x_k;\hat{x}_{k|k-1},P_{k|k-1})dx_k. \end{aligned} \tag{6}$$

The covariance of the measurement can be given as

$$\begin{aligned} P_{k|k-1}^{zz} &= E[(z_k-\hat{z}_{k|k-1})(z_k-\hat{z}_{k|k-1})^T] \\ &= \int \gamma_k(\cdot)\gamma_k^T(\cdot)\aleph(x_k;\hat{x}_{k|k-1},P_{k|k-1})dx_k \\ &- \hat{z}_{k|k-1}\hat{z}_{k|k-1}^T + R_k. \end{aligned} \tag{7}$$

The cross-covariance between the state and measurement can be expressed as

$$\begin{aligned} P_{k|k-1}^{xz} &= E[(x_k-\hat{x}_{k|k-1})(z_k-\hat{z}_{k|k-1})^T] \\ &= \int x_k\gamma_k^T(\cdot)\aleph(x_k;\hat{x}_{k|k-1},P_{k|k-1})dx_k - \hat{x}_{k|k-1}\hat{z}_{k|k-1}^T. \end{aligned} \tag{8}$$

Considering delay in measurement for a nonlinear system, Singh *et al.* [20] recently proposed a filtering algorithm, in which they considered $N-1$ as the largest number of delays and at k^{th} time step either any measurement z_{k-i} $(0 \le i \le N-1)$ arrives or measurement is lost. In case of measurement is lost, the estimator fetches the previous measurement from the buffer where the sensor output is stored, and it will be used for measurement update. The expectation of delayed measurement can be expressed as [20]

$$\hat{y}_{k|k-1} = (1-\mathrm{p})\sum_{i=0}^{N-1}\mathrm{p}^i\hat{z}_{k-i|k-i-1} + \mathrm{p}^N\hat{y}_{k-1|k-2}. \tag{9}$$

The covariance of the delayed measurement can be given as [20]

$$\begin{aligned} P_{k|k-1}^{yy} &= (1-\mathrm{p})\sum_{i=0}^{N-1}\mathrm{p}^i P_{k-i|k-i-1}^{zz} + (1-\mathrm{p})\sum_{i=0}^{N-1}\mathrm{p}^i(1-\mathrm{p}^i(1-\mathrm{p})) \\ &(\hat{z}_{k-i|k-i-1})(\hat{z}_{k-i|k-i-1})^T + \mathrm{p}^N P_{k-1|k-2}^{yy}. \end{aligned} \tag{10}$$

The cross-covariance between state and delayed measurement can be expressed as [20]

$$P_{k|k-1}^{xy} = (1-\mathrm{p})\sum_{i=0}^{N-1}\mathrm{p}^i P_{k-i|k-i-1}^{xz} + \mathrm{p}^N P_{k-1|k-2}^{xy}. \tag{11}$$

III. Higher Degree Cubature Quadrature Kalman Filter

To develop an algorithm of filtering under the Bayesian framework, we need to solve the intractable integrals, mentioned in Eqs. (4)-(8). These integrals cannot be solved analytically and generally be solved by using a numerical approximation method with the help of deterministic sample points and their corresponding weights. Here the integrals are solved with higher degree cubature quadrature (HDCQ) points [10]. In this method, these intractable integrals are decomposed into surface and line integral. Arbitrary odd degree spherical-radial cubature rule [9], [22] is used for calculating the surface integral over a unit hyper-sphere. The line integral is evaluated using the Gauss-Laguerre quadrature rule of integration [23]. With the help of the following theorems, HDCQ points and weights generation procedure could be understood.

Theorem 1 [8]: The integral, in Cartesian coordinate system

$$I(f) = \frac{1}{\sqrt{|\Sigma|(2\pi)^n}}\int_{\mathbb{R}^n} f(X)e^{-(1/2)(X-\mu)^T\Sigma^{-1}(X-\mu)}dX \tag{12}$$

can be written in spherical coordinate system as

$$I(f)=\frac{1}{\sqrt{(2\pi)^n}}\int_{r=0}^{\infty}\int_{U_n}[f(CrZ+\mu)d\sigma(Z)]\,r^{n-1}e^{-r^2/2}dr, \tag{13}$$

where $X\in\mathbb{R}^n$, $f(X)$ is an any arbitrary function, $X=CrZ+\mu$, C is the Cholesky decomposition of the covariance matrix Σ, $\parallel Z\parallel=1$, μ is the mean of the Gaussian distribution and U_n is the surface of a unit hyper-sphere.

A. High Degree Cubature Rule

An arbitrary but odd degree spherical cubature rule, which has been introduced by Genz [22], and later used by Jia *et al.* [9] is used to solve the surface integral.

Theorem 2 [9]: The spherical integral $I_{U_n}(f_{rZ})=\int_{U_n}f(rZ)d\sigma(Z)$ can be calculated for odd degree as:

$$I_{U_n,2m+1}(f_{rZ})=\sum_{|q|}w_q f\left\{ru_q\right\}. \tag{14}$$

Here $I_{U_n,2m+1}$, $(m\geq 1)$ represents the $(2m+1)^{th}$ degree spherical cubature rule used to imprecise the integral. Cubature points ru_q and their associated weights w_q are defined as

$$\left\{ru_q\right\}\triangleq\bigcup(\alpha_1 ru_{q_1},\alpha_2 ru_{q_2},\cdots,\alpha_n ru_{q_n}) \tag{15}$$

$$\text{and } w_q\triangleq 2^{-n(u_q)}\left(I_{U_n}\left(\prod_{i=1}^{n}\prod_{j=0}^{q_i-1}\frac{z_i^2-u_j^2}{u_{q_i}^2-u_j^2}\right)\right). \tag{16}$$

Here q is a set of positive integers, described as q=[q_1, q_2, $\cdots$, q_n] with $|q|=q_1+q_2+\cdots+q_n$. The superscript $n(u_q)$ is the number of non-zero element in set u_q (elements are non-negative), $\alpha=\pm 1$ and $u_{q_i}=\sqrt{q_i/m}$.

Theorem 3 [9]: The intermediate weight w_q can be computed with the help of the following formula.

$$\int_{U_n}z_1^{\theta_1}z_2^{\theta_2}\cdots z_n^{\theta_n}dZ=2\frac{\Gamma((\theta_1+1)/2)\cdots\Gamma((\theta_n+1)/2)}{\Gamma((\mid\theta\mid+n)/2)},$$

where $\Gamma(\cdot)$ represents the Gamma function and $\mid\theta\mid=\theta_1+\theta_2+\cdots+\theta_n$.

B. Gauss-Laguerre Quadrature Rule

The line integral can be approximately written as

$$\int_{\lambda=0}^{\infty}f(\lambda)\lambda^{\alpha}e^{-\lambda}d\lambda\approx\sum_{i'=1}^{n'}\omega_{i'}f(\lambda_{i'}), \tag{17}$$

where for an arbitrary n', quadrature points (λ_i') are the roots of the equation

$$L_{n'}^{\alpha}(\lambda_i')=(-1)^{n'}\lambda_{i'}^{-\alpha}\ e^{\lambda_i'}\frac{d^{n'}}{d\lambda_{i'}^{n'}}\lambda_{i'}^{\alpha+n'}e^{-\lambda_i'}=0, \tag{18}$$

and the corresponding weights are given by

$$\omega_{i'}=\frac{n'!\Gamma(\alpha+n'+1)}{\lambda_{i'}[\dot{L}_{n'}^{\alpha}(\lambda_{i'})]^2}. \tag{19}$$

C. Higher Degree Cubature Quadrature Rule

Theorem 4 [10]: With the assumption of zero mean and unity covariance, Eq. (13) can be approximately written as,

$$\begin{aligned}I(f)&=\frac{1}{\sqrt{(2\pi)^n}}\int_{r=0}^{\infty}\int_{U_n}[f(rZ)d\sigma(Z)]\,r^{n-1}e^{-r^2/2}dr\\&=\frac{1}{2\sqrt{\pi^n}}\sum_{i'=1}^{n'}\omega_{i'}\left[\sum_{|p|}w_q f\left\{\sqrt{2\lambda_{i'}}u_q\right\}\right].\end{aligned} \tag{20}$$

Note. The number of support points required for a n dimensional system, and arbitrary $(2m+1)$ degree cubature and n' order quadrature rule are provided in [10]. Unfortunately the table has some typographic error. Here we correct the expressions and presented in TABLE I.

TABLE I: Number of support points required for odd degree cubature and n' order quadrature rule

Degree of cubature $(2m+1)$ rule	No. of support points (n_s)
3	$2n\times n'$
5	$2n^2\times n'$
7	$\frac{2n(1+2n^2)\times n'}{3}$
9	$\frac{2n^2(2+n^2)\times n'}{3}$
11	$\frac{2n(2n^4+10n^2+3)\times n'}{15}$

D. Calculation of HDCQ Points and their Corresponding Weights

The HDCQ points and their related weights can be calculated as follows:

- Find all the feasible sets of q=[q_1, q_2, $\cdots$, q_n] with $|q|=q_1+q_2+\cdots+q_n$, where q_i is positive integer and n is the dimension of the system.
- Set $u_q=[u_{q_1},u_{q_2},\cdots,u_{q_n}]$ where $u_{q_i}=\sqrt{q_i/m}$ for each sets of q.
- Find the cubature points for $(2m+1)$ degree of cubature rule, $\xi=[\alpha_1 u_{q_1},\alpha_2 u_{q_2},\cdots,\alpha_n u_{q_n}]$, where $\alpha_i=\pm 1$.
- Determine the intermediate weights, w_q, for their associated cubature points with the help of Eq. (16).
- Calculate the quadrature points (λ_i') and their associated weight by using Eqs. (18)-(19) respectively.
- Determine the cubature quadrature points $\xi_j=\sqrt{2\lambda_{i'}}\xi_i$ and their associated weights $W_j=\dfrac{\omega_{i'}w_q}{2\sqrt{\pi^n}}$, where $i=1,2,\cdots,n_k$, $i'=1,2,\cdots,n'$, $j=1,2,\cdots,n_s$, $n_k=n_s/n'$ and n_s is the number of support points.

Once the HDCQ points and weights are generated, the working equations for randomly delayed measurements (summarized in section II) could be realized. The algorithm of HDCQKF-RD is summarized in Algorithm 1.

Algorithm 1

- `Initialize with` $\hat{x}_{0|0}$, $P_{0|0}$ `and` $x_{0|0}$.
- `Generate` ξ_j `and` W_j $\quad j=1\cdots n_s$.

Prediction step

- This step is same as ordinary HDCQKF [10].

Measurement update step

- Cholesky decomposition of prior error covariance, $P_{k|k-1} = S_{k|k-1}S_{k|k-1}^T$.
- HDCQ points, $\chi_{j,k|k-1}^* = S_{k|k-1}\xi_j + \hat{x}_{k|k-1}$.
- Propagated HDCQ points, $z_{j,k|k-1} = \gamma(\chi_{j,k|k-1}^*)$.
- Estimate the predicted measurement at current time-step, $\hat{z}_{k|k-1} = \sum_{j=1}^{n_s} W_j z_{j,k|k-1}$.
- Estimate the actual delayed measurement, $\hat{y}_{k|k-1}$ by using Eq. (9).
- Calculate the covariance of the measurement, $P_{k|k-1}^{zz} = \sum_{j=1}^{n_s} W_j(z_{j,k|k-1} - \hat{z}_{k|k-1})(z_{j,k|k-1} - \hat{z}_{k|k-1})^T + R_k$.
- Calculate the covariance of the delayed measurement, $P_{k|k-1}^{yy}$ with the help of Eq. (10).
- Calculate the cross-covariance of state and measurement, $P_{k|k-1}^{xz} = \sum_{j=1}^{n_s} W_j(\chi_{j,k|k-1}^* - \hat{z}_{k|k-1})(\chi_{j,k|k-1}^* - \hat{z}_{k|k-1})^T$.
- Calculate the cross-covariance of state and delayed measurement, $P_{k|k-1}^{xy}$ by using Eq. (11).
- Estimate the Kalman gain, $K = P_{k|k-1}^{xy}(P_{k|k-1}^{yy})^{-1}$.
- Estimate the posterior state, $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K(y_k - \hat{y}_{k|k-1})$.
- Posterior error covariance, $P_{k|k} = P_{k|k-1} - KP_{k|k-1}^{yy}K^T$.

IV. Simulation Results

In this section, the superiority of HDCQKF-RD is shown with the help of two examples.

Problem 1: Here we consider a discrete time system with process model

$$x_{k+1} = \phi_k x_k + \eta_k,$$

and measurement model

$$y_k = \sqrt{1 + x_k^T x_k} + v_k,$$

where $x_k \in \mathbb{R}^2$, $y_k \in \mathbb{R}$ is the system output, ϕ_k is time varying function given by

$$\phi_k = \begin{bmatrix} 0.8 & 0 \\ 1 + \sin(2\pi k/N_1) & 0.6 \end{bmatrix},$$

where $N_1 = 100$. Process noise η_k is Gaussian with mean zero and covariance $Q_k = I_2$ and measurement noise v_k is also Gaussian with mean zero and covariance $R_k = 1.25$. Initial truth value of the state is $x_0 = [0\ 0]^T$. The filter is initialized with an initial estimate of the state $\hat{x}_0 = [0\ 0]^T$ and initial error covariance $P_{0|0} = 0.1I_2$. The simulation is carried out for 100 time-step, and 100 Monte Carlo runs are performed for unbiased comparison. In this problem, measurement is assumed as one step delayed $(N-1=1)$.

For this two-dimensional problem, points and their corresponding weights used by respective filters are as shown in Fig. 1. It is observed that, during simulation, sometimes algorithm stops due to negative-definite error covariance matrix. This problem is common and to avoid it, square root version of CKF-RD (SRCKF-RD) [7], [24] is used. Here fifth degree $(m=2)$ and seventh degree $(m=3)$ of cubature rule are used for implementation of SRHDCQKF-RD. The quadrature rule is considered as 2 *i.e.* $n'=2$. We abbreviated the filter as SRHDCQKF-RD(d), where d is the degree of cubature rule.

For comparison, averaged RMSE is calculated for each state. Averaged RMSE is the mean value of RMSE over time. For each state, the averaged RMSE against the probability ($\mathrm{p} = P(\beta_j = 1)$) are plotted in Fig 2. From Eq. (3), it has been seen that if $\mathrm{p}=0$ no delay in measurement occurs and if $\mathrm{p}=1$ no measurement is received (*i.e.* packet drop condition). The value of probability (p) is varied from 0.1 to 0.9. From the figure, it can be seen that SRHDCQKF-RD(5,7) performs better than SRCKF-RD.

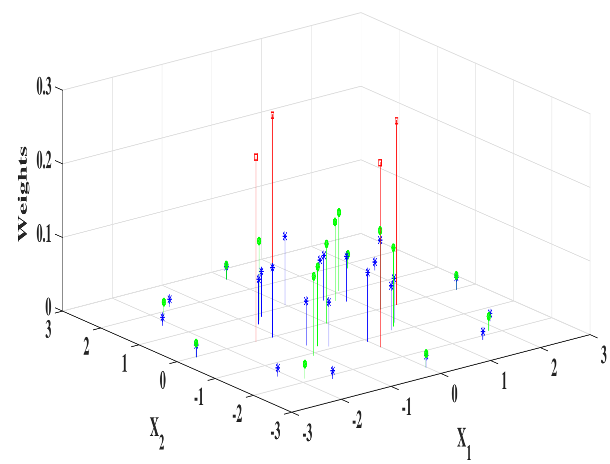

Fig. 1: Plot of weights against points (a) SRCKF-RD (red) (b) SRHDCQKF-RD(5) (green) (c) SRHDCQKF-RD(7) (blue)

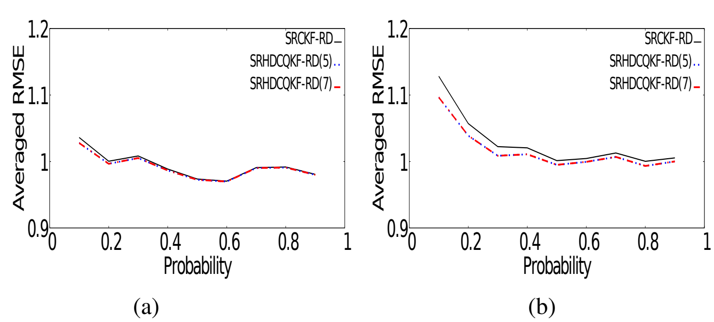

Fig. 2: Averaged RMSE against probability plot, for (a) state-1 (b) state-2

Problem 2: The process model is given by [20]

$$x_k = 2\cos(x_{k-1}) + \eta_{k-1},$$

and measurement model is given by

$$y_k = \sqrt{1 + x_k^T x_k} + v_k.$$

The system dimension is considered as 6. The process noise, η_{k-1} and the measurement noise, v_k are assumed to be white, uncorrelated and normally distributed with covariance $5I_6$ and 5 respectively. The initial truth value of the filter is $x_0 = 0.1_{6\times1}$. The initial estimate of the state and error covariance are $\hat{x}_{0|0} = 15_{6\times1}$ and $P_{0|0} = 5I_6$ respectively.

Here states are estimated by using CKF-RD and HDCQKF-RD. The maximum number of delay in measurement is considered as 1. The simulation is done for 200 time-steps. For a fair comparison, 200 independent Monte Carlo runs are performed. The filtering performance has been compared using averaged RMSE. From the Fig 3, it can be seen that HDCQKF-RD(7) has lowest averaged RMSE while CKF-RD has most. So for a higher degree of cubature rule, the accuracy of the filter is more.

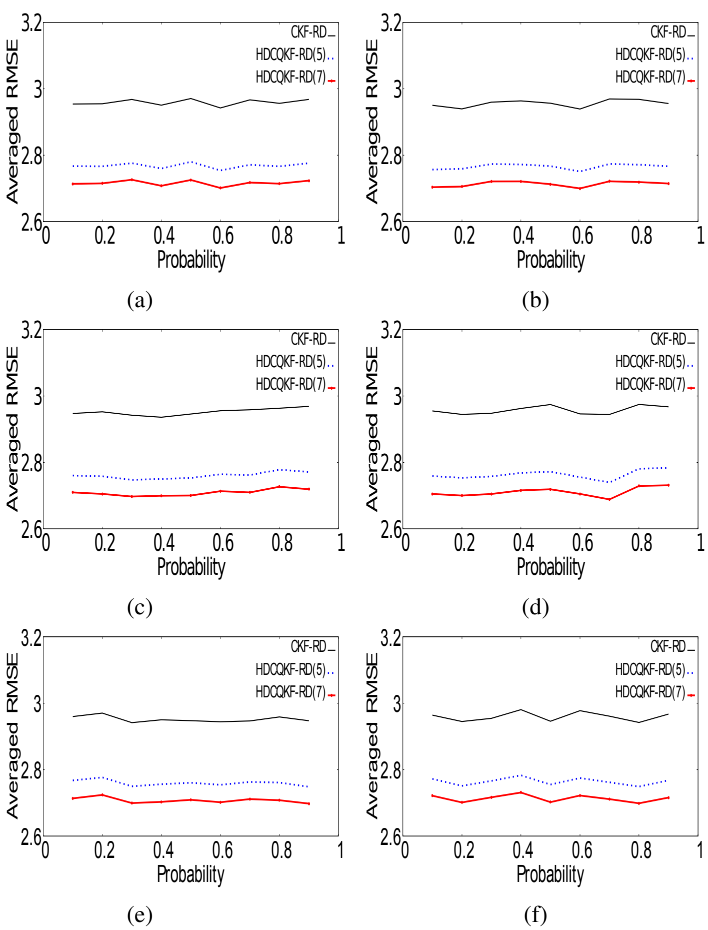

Fig. 3: Averaged RMSE against probability plot, for (a) state-1 (b) state-2 (c) state-3 (d) state-4 (e) state-5 (f) state-6

V. Discussions and Conclusions

In networked control system where the estimators are remotely located, delay in measurement is very common. In this paper, a higher degree CQKF is formulated for arbitrary delayed measurement systems. The superiority of HDCQKF-RD in comparison with CKF-RD has been demonstrated with the help of two nonlinear estimation problems.

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