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# Abstract

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Recommender systems relying on contextual multi-armed bandits continuously improve relevant item recommendations by taking into account the contextual information. The objective of bandit algorithms is to learn the best arm (e.g., best item to recommend) for each user and thus maximize the cumulative rewards from user engagement with the recommendations. The context that these algorithms typically consider are the user and item attributes. However, in the context of social networks where the action of one user can influence the actions and rewards of other users, neighbors' actions are also a very important context, as they can have not only predictive power but also can impact future rewards through spillover. Moreover, influence susceptibility can vary for different people based on their preferences and the closeness of ties to other users which leads to heterogeneity in the spillover effects. Here, we present a framework that allows contextual multi-armed bandits to account for such heterogeneous spillovers when choosing the best arm for each user. Our experiments on several semi-synthetic and real-world datasets show that our framework leads to significantly higher rewards than existing state-of-the-art solutions that ignore the network information and potential spillover.

#### Keywords

recommender systems, multi-armed bandits, information diffusion, social networks

### 1 Introduction

Contextual multi-armed bandit (CMAB) algorithms leverage user attributes and actions to optimize personalized recommendations over time and thus maximize rewards [1, 13, 22, 43]. Rewards can vary based on the application where the recommendations occur, including revenue from recommended products in e-commerce applications and clicks on recommended user-generated content on social media. When contextual bandit recommendations occur in social networks, they can spread from one user to another and overall rewards can be based on both direct recommendations and spillover. Network spillover refers to the phenomenon where the actions of one individual have an impact on the actions of others leading to the spread of information, ideas, attitudes, and behaviors. Understanding the effect of spillover is important in many fields, including psychology [9], marketing [12], public health [33], and economics [35].

To illustrate the motivation behind this paper, let's consider the following toy example: an advertisement company targets social network users with two types of ads, ads on politics and ads on sports. Alice is one of the targeted users and she regularly posts about and engages with fitness-related content. Therefore, when a sport-related ad about downloading a fitness app comes to Alice's news feed, Alice shares the ad link in her social media account. Due to sharing, the ad link appears in the news feed of two of her



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Figure 1: The workflow of the *NetCB* framework.

friends, namely Brian and Carla. Brian and Carla are not targeted users but Brian is also interested in fitness and downloads the app from the shared link while Carla is not interested in sports and does not download it. Rewards (i.e., downloads) occur both based on the direct recommendation (Alice's download) and based on spillover (Brian's download), therefore potential spillover should be taken into consideration when recommending items to Alice.

When different users respond differently to the sharing actions of their network contacts, this refers to spillover heterogeneity. For example, it was more likely for Alice to influence Brian to download the fitness app due to their shared interest than to influence Carla. Current research on contextual bandit recommendations [1, 13, 22, 43, 45] does not take into account spillover, much less heterogeneous spillover, to optimize rewards. Thus, in this paper we set out to examine whether heterogeneous spillover can be utilized to maximize the overall rewards during contextual bandit recommendation in social networks.

Present work. We develop a Network Contextual Bandit framework, NetCB, that leverages heterogeneous spillover and neighborhood knowledge for recommending items that maximize rewards in networks. As context for the bandit, we introduce a novel dynamic feature set for each user which captures the spillover dynamics over time and provides summary neighborhood statistics about the success of direct recommendations and spillovers. Another novel component of our framework is deciding when to recommend a different arm than the optimal arm predicted by the contextual multi-armed bandit if it leads to higher network rewards due to spillover. Figure 1 illustrates the workflow of the NetCB framework. NetCB recommends items in rounds where each round corresponds to a user arriving (e.g., opening a social network app). In the first step of each round, the CMAB uses the features to predict the best class to recommend for that user (e.g., politics, marked in blue). Next, NetCB checks whether the predicted class is optimal for the neighborhood by comparing the expected rewards due to spillover

for different preference classes. If the network rewards (i.e., direct recommendation and spillover rewards) are highest for a class dif-ferent from the one the CMAB suggests, then NetCB recommends the user that different class instead of the predicted one, going against the CMAB recommendation. In our example, estimated net-work rewards are higher for the alternate recommendation class, i.e., sports, shown in orange, and the user is recommended that class. In the final step of each round, neighbors' recommendations and spillover features are updated accordingly. NetCB can be seam-lessly integrated with any existing contextual multi-armed bandit algorithm. 

**Key idea and highlights.** To summarize, this paper makes the following contributions:

- We define a novel problem of maximizing *network rewards* for contextual bandit recommendation.
- We introduce a dynamic heterogeneous spillover model and develop a bandit framework which leverages spillover knowledge to increase long-term rewards.
- We perform a thorough evaluation of our framework on semi-synthetic and real-world datasets and compare it to state-of-the-art contextual bandit algorithms.

To the best of our knowledge, this is the first work that considers the impact of both recommendations and their heterogeneous spillover in networks when learning optimal recommendations and calculating bandit regrets. The only research that considers contextual bandits for networks is in the context of influence maximization [19, 36–38] where the goal is to find a set of influential users who will be treated with the goal of these users spreading information in the network. In contrast, our work assumes that anyone could influence others [4] and we focus on the choice of recommendations, not the choice of users. Moreover, unlike previous research on contextual bandit recommendations [10, 22, 26] which leverages only user and item characteristics, we leverage information about the user neighborhood.

#### 2 Related Work

Recommendations with contextual bandits. Contextual multi-armed bandit algorithms are widely used in recommender systems [10, 26]. LinUCB [22], Contextual Thompson Sampling (CTS) [1], and LinEI [34] algorithms assume a linear relationship between the expected reward of an action and its context. NeuralBandit1 [2], NeuralUCB [45], NeuralTS [43], NeuralEI [34], and EE-Net [7] use neural networks to remove the constraint of linearity. To lever-age the collaborative nature among users/items, different algorithms have been developed to model the dependency among item-s/users, e.g., GOB.Lin [11], CLUB [15], DYNUCB [27], COFIBA [24], CoLin [39], DCCB [20], CAB [14], SCLUB [23], GRC [40], ConUCB [44], DistCLUB [25], LOCB [5], HCB/pHCB [32], Meta-Ban [6], and GNB [28]. However, none of them consider the potential of spillover in maximizing rewards during recommendation. 

Contextual bandit for networks. The only research that con siders contextual bandits for networks is in the context of influence
 maximization [19, 36–38]. The influence maximization problem
 aims to maximize rewards in a social network by finding the most
 influential users that can maximize diffusion in the network. How ever, influence maximization differs from our work since we focus

on the choice of recommendations to users, not on the choice of users.

**Spillover.** Spillover is typically studied in the context of causal effect estimation in non-iid settings such as social networks and online marketplaces [16, 18, 21, 41]. Some example works in this space include characterization of the neighborhood information through exposure mappings [3] and machine learning [42], as well as using machine learning to estimate heterogeneous effects [8, 42]. All these works focus on spillover to avoid biased estimates of the treatment effect whereas our work focuses on leveraging the heterogeneity of network spillover to maximize rewards in recommendations.

## **Problem Description**

**Data model.** We define an attributed network graph G = (V, E), consisting of a set of *n* nodes  $V = \{v_1, v_2, \ldots, v_n\}$ , a set of edges  $E = \{e_{ij}, 1 \le i, j \le n\}$  where  $e_{ij}$  denotes the edge connecting node  $v_i \in V$  and node  $v_j \in V$ . The set of neighboring nodes of  $v_i$  is denoted with  $\mathcal{N}_i$  where  $\mathcal{N}_i = \{v_j : v_j \in V, e_{ij} \in E\}$ . We define  $\mathcal{N}_i$  as the neighborhood of node  $v_i$  and each node  $v_j \in \mathcal{N}_i$  is a neighbor of node  $v_i$ . Each node in the network has one latent preference from a set of *l* possible latent preferences (or classes),  $C = \{c_1, c_2, \ldots, c_l\}$ . In our toy example, the latent preferences are *sports* and *politics*.

We let  $\mathbf{X}_i$  denote the *d*-dimensional feature vector of node  $v_i$ and  $z_i \in C$  refers to its latent preference type. To capture neighborhood properties related to spillover, we introduce a 4*l*-dimensional dynamic neighborhood feature set for each node  $v_i$  of the network, denoted as  $\mathbf{X}_{N_i}$ . This is a novel component of our framework, described in detail in Section 4.1. Let  $y_i \in \{0, 1\}$  refer to the activation status of node  $v_i$  after a recommendation where  $y_i = 1$  means active node (e.g., downloading a fitness app) and  $y_i = 0$  means inactive.

The contextual bandit contains a set of arms  $\mathcal{A} = \{c_1, c_2, \ldots, c_l\}$  corresponding to user preferences. We denote the predicted by the contextual bandit preference of node  $v_i$  with  $arm_i \in \{\mathcal{A} \cup \{\emptyset\}\}$  and the recommended class of node  $v_i$  (which can sometimes be different from the predicted arm) with  $t_i \in \{\mathcal{A} \cup \{\emptyset\}\}$ .  $arm_i = \emptyset$  refers to no prediction made and  $t_i = \emptyset$  refers to no recommendation made to node  $v_i$ . We provide a detailed description of the bandit setup at the end of this section.

**Node activation.** Nodes in the network can be activated in two ways, through direct recommendation by the system and through spillover. *Direct recommendation* refers to the system treating with a recommendation a particular node ( $t_i \neq \emptyset$ ) in the network. A spillover occurs when the activation of one node impacts the activation of another node. For example, when Alice is shown a fitness app ad that she downloads and shares with Brian, spillover occurs when Brian also downloads the app as a consequence of Alice's sharing.

**Recommendation types.** We define two types of direct recommendations based on the predicted preference class and the actual latent preference of the nodes. The recommendation to node  $v_i$  is *aligned* when its recommended class,  $t_i$  is the same as its latent preference,  $z_i$ . Similarly, the recommendation to node  $v_i$  is *misaligned* when its recommended class,  $t_i$  is different from its latent preference,  $z_i$ . We denote the probability of activating a particular node due to the aligned and misaligned recommendation as  $p_a$  and

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#### Figure 2: Recommendation-dependent heterogeneity in network spillover.

 $p_m$ , respectively:

$$p_a \leftarrow P(y_i = 1 | t_i = z_i); \qquad p_m \leftarrow P(y_i = 1 | t_i \neq z_i)$$

In our toy example, the activation probability of aligned recommendation would correspond to the probability of Alice downloading the fitness app when she was recommended the fitness app. Similarly, the activation probability of misaligned recommendation would correspond to the probability of Alice acting upon a politics ad when she was shown such an ad.

Spillover types. For ease of exposition, we model spillover with the widely used independent cascade model (ICM) [30], though our work can easily be adapted to incorporate other diffusion models. According to ICM, activated nodes have a probabilistic and independent chance of activating an inactive neighbor via spillover. This resembles contagious disease spread, where each social interaction may trigger an infection.

We define an activated node due to direct recommendation as a source node. Neighboring nodes that can get activated by the source node are considered recipient nodes. In our example, Alice is the source node and Brian is the recipient node. Just like direct recommendations, a spillover can be aligned or misaligned dependent on the latent preference type of nodes and the recommended class of the source node. A spillover is considered aligned when the recommended class of an activated source node is the same as that of the latent preference type of the recipient node. Similarly, a spillover is considered *misaligned* when the recommended class of a source node is different from the latent preference type of the recipient node.

We denote spillover probability with  $p_{sr}$  where  $s \in \{a, m\}$  refers to whether the recommendation of the source node is aligned or misaligned and  $r \in \{a, m\}$  refers to whether the spillover is aligned or misaligned. We formulate four types of heterogeneous spillover probabilities where  $v_i$  is the source node and  $v_j$  is the recipient node:

$$p_{aa} \leftarrow P(y_j = 1 | t_i = z_i, t_i = z_j)$$

$$p_{am} \leftarrow P(y_j = 1 | t_i = z_i, t_i \neq z_j)$$

$$p_{ma} \leftarrow P(y_j = 1 | t_i \neq z_i, t_i = z_j)$$

$$p_{mm} \leftarrow P(y_j = 1 | t_i \neq z_i, t_i \neq z_j)$$

An example of  $p_a$ ,  $p_m$ ,  $p_{aa}$ ,  $p_{am}$ ,  $p_{ma}$ , and  $p_{mm}$  is shown in Figure 2 where a toy network contains six nodes  $V \in \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and two possible preferences  $C \in \{P, S\}$  (e.g., *politics* and *sports*). Here,  $z_1 = P$ ,  $z_2 = S$ ,  $z_3 = P$ ,  $z_4 = S$ ,  $z_5 = P$ ,  $z_6 = P$ . In Figure 2(a), 290

 $t_1 = P$  and thus  $t_1 = z_1$ , therefore  $v_1$  gets aligned recommendation.  $v_3$  and  $v_6$  get the aligned spillover from the source node as  $t_1 = z_3$  and  $t_1 = z_6$ , respectively, from the source node  $v_1$  that is activated due to aligned recommendation.  $v_2$  and  $v_4$  get the misaligned spillover from the source node as  $t_1 \neq z_2$  and  $t_1 \neq z_4$ , respectively, from the source node v1 that is activated due to aligned recommendation. In Figure 2(b),  $t_1 = S$  and thus  $t_1 \neq z_1$ , therefore  $v_1$  gets misaligned recommendation.  $v_2$  and  $v_4$  get the aligned spillover from the source node as  $t_1 = z_2$  and  $t_1 = z_4$ , respectively, from the source node  $v_1$  that is activated due to misaligned recommendation.  $v_3$  and  $v_6$  get the misaligned spillover from the source node as  $t_1 \neq z_3$  and  $t_1 \neq z_6$ , respectively, from the source node  $v_1$  that is activated due to misaligned recommendation. In both networks, there is no spillover from node  $v_1$  to  $v_5$  since  $v_5$  is already activated and spillover can happen from the currently recommended and activated node to its inactive neighboring nodes.

We assume the heterogeneous spillover probabilities (i.e.,  $p_{sr}$ where  $s \in \{a, m\}$ ) are known in advance. An interesting follow-up work would be to learn them from data.

A contextual bandit with network rewards setup. We consider a stochastic *l*-armed contextual bandit setup with a total number of T rounds. In each round  $i \in \{1, 2, 3, ..., T\}$ , the learning agent receives an inactive node  $v_i \in V$  along with a context feature vector:  $\{\mathbf{X}_i, \mathbf{X}_{N_i}\}$ . The agent selects an action  $arm_i$  and receives network rewards  $R(v_i, arm_i)$ . The action of an arm is a direct recommendation of an item from one of the classes C. A reward refers to the activation of an inactive node due to direct recommendation or spillover. An example reward model is assigning a reward of 1 when a node is activated; otherwise, the reward is 0. The network rewards  $R(v_i, arm_i)$  is a non-negative integer which refers to the total number of newly activated nodes due to the selected arm's action, including the node's activation and potential spillover to its neighboring nodes. We assume that the network rewards are a function of the features that needs to be learned:  $R(v_i, arm_i) = F(\mathbf{X}_{i,arm_i}, \mathbf{X}_{\mathcal{N}_i,arm_i}) + \xi_i$ , where  $\xi_i$  is zeromean noise. The total T-round network rewards of the bandit are defined as  $R_{total} \leftarrow \sum_{i=1}^{T} R(v_i, arm_i)$ . Similarly, we define the optimal *T*-round network rewards as  $R_{total}^* \leftarrow \sum_{i=1}^{T} R(v_i, arm_i^*)$ , where  $arm_i^*$  is the arm with maximal expected network rewards in round i. The T-round cumulative regret of the bandit learning can be formulated as Regret  $\leftarrow \sum_{i=1}^{T} (R(v_i, arm_i^*) - R(v_i, arm_i))$ . We are now ready to formally define the problem:

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PROBLEM 1 (MAXIMIZING NETWORK REWARDS WITH CONTEXTUAL BANDITS). Given an attributed network graph G = (V, E), a set of attributes X associated with each node, and a set of arms  $\mathcal{A}$  associated with the user preferences, select a recommendation class for direct recommendation to each inactive node  $v_i$  at round  $i \in \{1, 2, 3, ..., T\}$ such that the network rewards  $R_{total}$  are maximized.

#### 4 Network Contextual Bandit framework

We design a contextual bandit framework that aims to select the optimal direct recommendation for each arriving node in a network by taking into account both the consequences of direct recommendation and indirect recommendations, i.e., spillover. Our **Net**work **Contextual Bandit** framework, *NetCB*, comprises two components, considered in each round. The first component (Section 4.1) integrates the use of dynamic neighborhood features in conjunction with the static features of the inactive node to make a prediction for the best direct recommendation. The second component (Section 4.2) leverages heterogeneous spillover and goes against the predicted direct recommendation if it achieves suboptimal expected network rewards. The two components of *NetCB* are described next and the pseudo-code is included in Algorithm 1.

#### 4.1 Dynamic neighborhood features per user

Bandit online learning is characterized by the increase in the ratio of successful recommendations to the total number of direct recommendations. To improve the learning, we introduce a set of novel dynamic features which captures the spillover dynamics over time and provides summary neighborhood statistics about the success of direct recommendations and spillovers. This set of features proves to be quite powerful, as we demonstrate in the Experiments section. Specifically, in addition to the static node features  $X_i \in \mathbb{R}^d$ , we consider dynamic neighborhood features  $X_{N_i} \in \mathbb{R}^{4l}$  of node  $v_i$ .

1) Neighborhood recommendations per class. The first l dimensions of  $X_{N_i}$  represent the ratios of direct recommendations to neighbors  $N_i$  for each of l arms  $c_k \in \mathcal{A}$  relative to the total direct recommendations to neighbors for all arms in  $N_i$ :

$$\mathbf{X}_{\mathcal{N}_{i}}[k] = \frac{\sum_{v_{j} \in \mathcal{N}_{i}} \mathbb{1}[t_{j} = c_{k}]}{\sum_{v_{j} \in \mathcal{N}_{i}} \mathbb{1}[t_{j} \neq 0]}$$

where  $k \in \{1, 2, ..., l\}$  and  $\mathbb{1}[.]$  is an indicator function.

2) Unsuccessful neighborhood recommendations per class. The second *l* dimensions represent the ratios of unsuccessful direct recommendations in  $N_i$ , i.e., direct recommendations that fail to activate a node, for each arm  $c_k \in \mathcal{A}$  relative to the total unsuccessful direct recommendations for all arms in  $N_i$ :

$$\mathbf{X}_{\mathcal{N}_{i}}[l+k] = \frac{\sum_{v_{j} \in \mathcal{N}_{i}} \mathbb{1}[t_{j} = c_{k} \land y_{j} = 0]}{\sum_{v_{i} \in \mathcal{N}_{i}} \mathbb{1}[t_{j} \neq \emptyset \land y_{j} = 0]}$$

Let's consider a case from Figure 2(a), where  $t_2 = t_5 = t_6 = P$ ,  $t_3 = \emptyset$ , and  $t_4 = S$ . Therefore,  $X_{N_1}[1] = \frac{3}{4}$  and  $X_{N_1}[2] = \frac{1}{4}$ . If the node  $v_5$  gets activated ( $y_5 = 1$ ) due to direct recommendation but all other neighboring nodes of node  $v_1$  remain inactive, then  $X_{N_1}[3] = \frac{2}{3}$  and  $X_{N_1}[4] = \frac{1}{3}$ .

A spillover is *successful* when a recipient node gets activated due to the activation of a source node. A spillover is considered *unsuccessful* when a source node fails to activate the recipient node. Each potential recipient node  $v_i \in V$  has two *l*-dimensional vectors, i.e.,  $S_i$  and  $\overline{S_i}$  to keep count of successful and unsuccessful spillovers per preference class, respectively.  $S_i = \vec{0}$  and  $\overline{S_i} = \vec{0}$  indicate that node  $v_i$  has received no successful or unsuccessful spillover, respectively.  $S_i[k] \in \{0, 1\}$  refers to whether  $v_i$  is activated with  $c_k$  through spillover from a neighboring source node.  $\overline{S_i}[k] \in \mathbb{Z}^+$  refers to the total unsuccessful spillover attempts to activate  $v_i$  with  $c_k$ .

3) Neighborhood spillovers per class. The third *l* dimensions of  $X_{N_i}$  represent the ratios of spillover attempts for each arm  $c_k \in \mathcal{A}$  relative to the total spillover attempts in  $N_i$  which can be written as follows for  $k \in \{1, 2, ..., l\}$ :

$$\mathbf{X}_{\mathcal{N}_{i}}[2l+k] = \frac{\sum_{v_{j} \in \mathcal{N}_{i}} (\mathcal{S}_{j}[k] + \mathcal{S}_{j}[k])}{\sum_{v_{j} \in \mathcal{N}_{i}} \sum_{r \in \{1,2,\dots,l\}} (\mathcal{S}_{j}[r] + \overline{\mathcal{S}}_{j}[r])}$$

4) Unsuccessful neighborhood spillovers per class. The fourth *l* dimensions represent the ratios of unsuccessful spillover attempts for each arm  $c_k \in \mathcal{A}$  relative to the total unsuccessful spillover attempts in  $N_i$  which can be written as follows:

$$\mathbf{X}_{\mathcal{N}_{i}}[3l+k] = \frac{\sum_{v_{j} \in \mathcal{N}_{i}} (\mathcal{S}_{j}[k])}{\sum_{v_{j} \in \mathcal{N}_{i}} \sum_{r \in \{1,2,\dots,l\}} (\overline{\mathcal{S}}_{j}[r])}$$

Let's consider another case from Figure 2(a), where the two neighboring nodes of node  $v_3$  get activated due to "P" direct recommendation and one other neighboring node gets activated due to "S" direct recommendation. Therefore, the node  $v_3$  receives spillover with "P" twice and "S" once. Similarly, the nodes  $v_4$  and  $v_5$  get spillover with "P" once. The nodes  $v_1$  and  $v_6$  do not receive any spillover. Therefore,  $X_{N_1}[5] = \frac{4}{5}$  and  $X_{N_1}[6] = \frac{1}{5}$ . If the node  $v_5$  gets activated  $(y_5 = 1)$  due to spillover but all other neighboring nodes of node  $v_1$  remain inactive, then  $X_{N_1}[7] = \frac{3}{4}$  and  $X_{N_1}[8] = \frac{1}{4}$ .

The neighborhood features are dynamic and change over time due to the arrival of new nodes and potential new activations in each round. The aggregated features  $X_i$  and  $X_{N_i}$  are passed to an off-the-shelf contextual multi-armed bandit (CMAB) algorithm, which predicts the optimal recommendation class  $(arm_i)$  for direct recommendation. The CMAB learns the parameters of the arms with the arrival of nodes and generalizes expected network rewards from direct recommendation and spillover.

#### 4.2 Spillover maximization

The second part of the algorithm considers the optimal recommendation arm, predicted by the off-the-shelf CMAB, and decides whether to follow the prediction and recommend that arm or go against the prediction and recommend a different arm to the node considered in that round. To do that, it estimates the potential rewards from spillover for all possible arms and picks the arm that gives the highest expected network reward. For example, NetCB may decide to show a politics ad to Alice instead of a sports one (even though she likes sports) if a lot of her inactive friends like politics and the expected network rewards are estimated to be higher. Since we cannot estimate the expected network rewards without knowing the true user preference, we first predict the preferred arm/class for all inactive nodes in the neighborhood and treat them as if they are the true arm (which is unknown). This allows us to

decide which heterogeneous spillover probability applies for each neighbor. The expected network rewards due to the direct recommendation of the predicted recommendation class  $arm_i$  for node  $v_i$  and spillover in its neighboring nodes is denoted with  $E[R_i | arm_i]$ , which is estimated as follows:

$$E[R_i \mid arm_i] = p_a + p_a * \sum_{v_j \in \mathcal{N}_i} (p_{aa} * \mathbb{1}[arm_i = arm_j \land y_j = 0] + p_{am} * \mathbb{1}[arm_i \neq arm_j \land y_j = 0]) \quad (1)$$

The expected network rewards due to the direct recommendation of each alternate arm,  $arm \in \mathcal{A}(arm \neq arm_i)$  for node  $v_i$ and spillover in its neighboring nodes is denoted with  $E[R_i | arm]$ , which is estimated as follows:

$$E[R_i \mid arm] = p_m + p_m * \sum_{v_j \in \mathcal{N}_i} (p_{ma} * \mathbb{1}[arm = arm_j \land y_j = 0] + p_{mm} * \mathbb{1}[arm \neq arm_j \land y_j = 0])$$
(2)

We denote the alternate arm with highest expected network reward for node  $v_i$  with  $\overline{arm_i}$ , i.e.,  $\overline{arm_i} = \arg \max E[R_i | arm]$ .

If the expected network rewards of the  $\overline{arm_i}$  is greater than that of the  $arm_i$ , the NetCB framework selects  $\overline{arm_i}$ ; otherwise, it selects  $arm_i$  for direct recommendation to node  $v_i$ . When the alternate arm  $\overline{arm_i}$  is selected, the arm parameters are not updated based on the recorded network rewards to avoid ambiguity in off-the-self CMAB bandit learning.

It is important to note that the success of this step depends on having good recommendation predictions for each node. To avoid considering poor predictions in the early learning stages of the CMAB, this step only applies after the direct activation rate (DAR) has stabilized. The *DAR* refers to the ratio of total activated nodes due to direct recommendations to the total direct recommendations made during contextual bandit learning.

### 4.3 Illustration of NetCB algorithm

Formally, the *NetCB* algorithm proceeds in discrete rounds i = 1, 2, 3, ..., T [Line: 9] following the initialization [Line: 3 – 8]. In each round *i*, it repeats the two steps described in Sections 4.1 and 4.2.

- An inactive node v<sub>i</sub> arrives to receive direct recommendation. The algorithm observes the context vector X<sub>i</sub> of the current node v<sub>i</sub> along with X<sub>Ni</sub> and bandit parameters for the set of arms, A [Line: 10].
- (2) An off-the-shelf CMAB algorithm predicts an arm,  $arm_i \in \mathcal{A}$  for direct recommendation to node  $v_i$  [Line: 11].
- (3) If *DAR* is stable, estimate expected network rewards for all arms [Line: 13-15] and find the maximum expected reward generating arm, *arm<sub>i</sub>* for node v<sub>i</sub> [Line: 16].
- (4) If *DAR* is not stable or the predicted arm *arm<sub>i</sub>* is the same as *ārm<sub>i</sub>*, recommend the predicted class, *t<sub>i</sub> = arm<sub>i</sub>* to node *v<sub>i</sub>*. Node *v<sub>i</sub>* then receives network rewards, *R(v<sub>i</sub>, arm<sub>i</sub>)*. The bandit parameters and neighborhood features, X<sub>N<sub>i</sub></sub> are updated. The algorithm then updates its arm-selection strategy with the observation, ({X<sub>i</sub>, X<sub>N<sub>i</sub></sub>}, arm<sub>i</sub>, *R(v<sub>i</sub>, arm<sub>i</sub>))* [Line: 19-20].

lg	<b>corithm 1</b> Maximizing network rewards with <i>NetCB</i>
1:	Input: Number of rounds <i>T</i> , off-the-shelf CMAB(e.g., LinUCB,
	NeuralTS etc.) hyperparameters
2:	<b>Output:</b> Direct recommendation $t_i$ for each inactive node $v_i$
	arrived in each round <i>i</i>
3:	for all $a \in \mathcal{A}$ do
4:	Initialize arm parameters
5:	end for
6:	for all $v_i \in V \underline{do}$
7:	$\mathbf{X}_{\mathcal{N}_i}, \boldsymbol{\mathcal{S}}_i, \boldsymbol{\mathcal{S}}_i \leftarrow 0$
8:	end for
9:	for $i \in \{1, 2, 3,, T\}$ do
10:	An inactive node $v_i$ arrives with a set of arm contexts
	$\{\mathbf{X}_{i,a}, \mathbf{X}_{\mathcal{N}_{i,a}}\}_{a \in \mathcal{A}}$
11:	Predict $arm_i$ for node $v_i$ using off-the-shelf CMAB
12:	if DAR is stable then
13:	for all $arm \in \mathcal{A}$ do
14:	Estimate expected network rewards of node $v_i$ for
	$arm, E[R_i \mid arm]$
15:	end for
16:	Find $arm_i = \arg\max_{arm} E[R_i \mid arm]$
17:	end if
18:	<b>if</b> <i>DAR</i> is not stable OR $arm_i = \overline{arm_i}$ <b>then</b>
19:	$t_i \leftarrow arm_i \ \#$ recommend CMAB's prediction
20:	Record $R(v_i, arm_i)$ ; update parameters for $arm_i$
21:	else
22:	$t_i \leftarrow \overline{arm_i} $ # go against CMAB's prediction
23:	Record $R(v_i, \overline{arm_i})$
24:	end if
25:	for $v_q \in \mathcal{N}_i$ do
26:	Update $X_{N_a}[k], X_{N_a}[l+k], S_a[k]$ , and $\overline{S_a}[k]$ where
	$k \in \{1, 2, \dots, l\}$
27:	for $v_r \in \mathcal{N}_a$ do
28:	Update $X_{N_{n}}[2l+k]$ and $X_{N_{n}}[3l+k]$ where $k \in$
	$\{1, 2, \dots, l\}$
29:	end for
30:	end for
31:	end for

(5) If the predicted arm *arm<sub>i</sub>* is different from *arm<sub>i</sub>*, recommend the direct recommendation class, *t<sub>i</sub>* = *arm<sub>i</sub>* to node *v<sub>i</sub>*. Node *v<sub>i</sub>* then receives network rewards, *R(v<sub>i</sub>, arm<sub>i</sub>)* [Line: 22-23].

(6) The neighborhood features are updated for each v<sub>q</sub> ∈ N<sub>i</sub> and v<sub>r</sub> ∈ N<sub>q</sub> [Line: 25-30].

The complexity of the algorithm depends on the complexity of the chosen off-the-shelf CMAB and the average degree of the network for updating the dynamic features in each round.

While the NetCB framework is designed to handle the arrival of one node at a time, as per the contextual multi-armed bandit literature, it may also be extended to accommodate batches of nodes arriving simultaneously. In this case, the spillover activation of a recipient node can depend on multiple source nodes trying to activate it.

#### 581 5 Experiments

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We evaluate the performance of *NetCB* on both real-world and semisynthetic network datasets using state-of-the-art CMAB methods showing whether *NetCB* improves these CMAB methods.

#### 5.1 Data representation

587 Real-world datasets. We consider four real-world attributed network datasets. BlogCatalog<sup>1</sup> is a network of social interactions 588 589 among bloggers on the BlogCatalog website. This dataset contains 590 5, 196 nodes, 343, 486 edges, 8, 189 attributes, and 6 labels. The labels represent topic categories inferred through the metadata of 591 the blogger interests. Flickr<sup>1</sup> is a benchmark social network dataset 592 which contains 7, 575 nodes, 479, 476 edges, 12, 047 attributes, and 9 593 labels. Each node in this network corresponds to a user, with each 594 edge representing a following relationship, and the labels indicating 595 596 the interests of groups of the users. The Hateful dataset is sampled from the Hateful Users on Twitter dataset [29] and the sample con-597 598 tains 3, 218 nodes, 9, 620 edges, 1, 036 attributes, and 2 labels. Each sample is classified as either "hateful" or "normal". Pubmed<sup>1</sup> is a 599 citation network where each node represents a scientific publica-600 tion related to diabetes and each directed edge represents a citation. 601 602 This dataset contains 19,717 nodes, 44,338 edges, 500 attributes, 603 and 3 labels. Each publication is classified into one of the 3 labels. The Shannon Equitability Index<sup>2</sup> values for the Blogcatalog, Flickr, 604 Hateful, and Pubmed datasets are 0.9992, 0.9996, 0.6314, and 0.9651, 605 respectively. As such, all datasets exhibit a high degree of balance, 606 except for the Hateful dataset. 607

Semi-synthetic datasets. We generate semi-synthetic dataset
 with different homophily for each real-world network dataset. Ho mophily is quantified by the proportion of edges connecting two
 nodes with the same label compared to the total number of edges
 in the network. The homophily scores in Blogcatalog, Flickr, Hate ful, and Pubmed network datasets are 0.40, 0.23, 0.73, and 0.80,
 respectively.

615 To increase homophily in a network dataset, we employ a random 616 edge removal of edges that connect two nodes with different labels, and where both nodes have a minimum degree of 2. By using this 617 method, we generate semi-synthetic Flickr and Blogcatalog datasets 618 with a homophily of 0.88. To decrease homophily in a network 619 dataset, we employ a random swapping of nodes that have different 620 labels within the network along with their associated attributes and 621 622 labels. We only consider pairs of nodes where swapping them leads to an increase in the number of edges connecting two nodes with 623 624 different labels. By using this method, we generate semi-synthetic 625 Pubmed and Hateful datasets with a homophilic score of 0.30 and 0.58, respectively. 626

Static features and latent preference of a node. The static 627 628 *d*-dimensions in feature vector  $X_i$  correspond to the attributes in 629 the datasets. To reduce computational complexity of the bandit algorithm, we reduce the dimension of  $X_i$  to 500 with truncated 630 631 SVD [17] for all datasets. The latent preference  $z_i$  of node  $v_i$  in 632 the network corresponds to its label in the dataset where l refers to the total number of labels. We aggregate *d*-dimensional static 633 node features with 4l dimensional dynamic neighborhood features 634

<sup>636</sup> <sup>2</sup>The Shannon Equitability Index [31] quantifies class imbalance in a dataset, with 0
 <sup>637</sup> being the most imbalanced and 1 being the most balanced.

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and consider it as a d + 4l dimensional feature vector for each node in the network. The context feature vector of each arm follow recommendation settings for classification datasets in previous works [7, 22, 28, 43, 45].

#### 5.2 Evaluation metrics

**Bandit accuracy.** The bandit accuracy,  $B_{acc}$  refers to the ratio of total aligned direct recommendation predictions to the total direct recommendation predictions made during contextual bandit learning, i.e.,  $B_{acc} = \frac{\sum_{o_i \in V} (\mathbbm{1}[arm_i = z_i])}{\sum_{v_i \in V} (\mathbbm{1}[arm_i \neq \emptyset])}$ Regrets. To evaluate regrets, we record network rewards for

**Regrets.** To evaluate regrets, we record network rewards for node  $v_i$  by counting newly activated nodes due to direct recommendation and spillover at round *i*. We compare them to the maximal network rewards for node  $v_i$  and report the T-round cumulative regrets defined in Section 3. The maximal network rewards is calculated through simulations.

#### 5.3 Main algorithms and baselines

**Baseline CMAB.** We consider several state-of-the-art CMAB algorithms, i.e., LinUCB [22], NeuralUCB [45], NeuralTS [43], and EE-Net [7] both as subroutines in our NetCB framework, and as baselines. We also include GNB [28] which has been shown to have better performance than CLUB [15], DYNUCB [27], COFIBA [24], SCLUB [23], and Meta-Ban [6]. The received reward is 1 if the learning agent predicts the aligned direct recommendation class; otherwise, the reward is 0. Following the literature of CMAB-based recommendation [7, 22, 28, 43, 45] system, we only consider CMAB-based recommendation systems.

**NetCB**<sub>CMAB</sub>. This method accounts only for the first component of our NetCB framework, utilizing dynamic neighborhood features, but not considering going against the bandit recommendation. The received network rewards is passed to the underlying CMAB subroutine of NetCB as implicit feedback which is a non-negative integer. When the underlying CMAB subroutine of NetCB framework is LinUCB, NeuralUCB, NeralTS, EE-Net, and GNB, we denote the methods with  $NetCB_{LinUCB}$ ,  $NetCB_{NeuralUCB}$ ,  $NetCB_{NeuralTS}$ ,  $NetCB_{EENet}$ ,  $NetCB_{GNB}$ , respectively.

**NetCB**<sub>*CMAB*</sub>. This method accounts for both components of our NetCB framework. The received network rewards is calculated similar to that of NetCB<sub>*CMAB*</sub>. This version of NetCB does not update parameters in the underlying CMAB subroutine for the received reward when the selected direct recommendation class contradicts the prediction from the first component. When the underlying off-the-shelf CMAB subroutine of NetCB framework is LinUCB, NeuralUCB, NeralTS, EE-Net, and GNB, we denote the methods with  $NetCB_{\overline{LinUCB}}$ ,  $NetCB_{\overline{NeuralUCB}}$ ,  $NetCB_{\overline{NeuralTS}}$ ,  $NetCB_{\overline{NeuralTS}}$ ,  $NetCB_{\overline{ReuralTS}}$ ,  $NetCB_{\overline{ReuralTS}$ 

## 5.4 Experimental setup

We consider single-hop spillover where only immediate neighbors can be activated. We consider a range of possible recommendation and spillover probabilities and show a representative set of results in our experiments using  $p_a = 0.7$ ,  $p_m = 0.5$ ,  $p_{aa} = 0.3$ ,  $p_{ma} = 0.3$ ,  $p_{am} = 0.0$ , and  $p_{mm} = 0.0$  for the first three experiments, and varying them for the others, as specified later. In all of our experiments, we set  $p_{mm} = 0$ ,  $p_{am} = 0$ . We do conduct a grid search for the exploration constant  $\alpha \in \{0.01, 0.1, 0.3, 0.5, 1, 2, 5\}$  of LinUCB. For

<sup>635 &</sup>lt;sup>1</sup>All datasets available at https://renchi.ac.cn/datasets/

Dataset	Flickr		Blogcatalog		Hateful		Pubmed	
Homophily	0.23	0.88	0.40	0.88	0.58	0.73	0.30	0.80
LinUCB [22]	$11097 \pm 686$	$11536 \pm 641$	$8421 \pm 375$	$9160 \pm 797$	$1574 \pm 89$	$1496 \pm 49$	$10862 \pm 104$	$10993 \pm 315$
NetCB <sub>LinUCB</sub>	<i>10012</i> ± 460	$8721 \pm 581$	<b>7040</b> ± 267	$6026\pm350$	<b>1520</b> ± 26	$1423 \pm 63$	<i>10817</i> ± 159	9186 ± 208
NetCB <sub>LinUCB</sub>	$10490 \pm 482$	<b>8506</b> ± 369	$7107 \pm 494$	<b>5856</b> ± 301	$1602 \pm 38$	<b>1409</b> ± 72	$10894 \pm 136$	<b>9163</b> ± 133
NeuralUCB [45]	$11033 \pm 1052$	$11002 \pm 841$	$8495 \pm 829$	$9116 \pm 436$	<b>1484</b> ± 78	<b>1538</b> ± 63	<i>11145</i> ± 97	$11263 \pm 386$
NetCB <sub>NeuralUCB</sub>	$12970 \pm 1169$	$9111 \pm 737$	<b>7003</b> ± 315	$5827 \pm 410$	$1510 \pm 20$	$1652 \pm 45$	$11477 \pm 298$	9586 ± 191
$NetCB_{\overline{NeuralUCB}}$	<b>10951</b> ± 806	<b>8578</b> ± 532	$7306 \pm 592$	$5318 \pm 488$	$1515 \pm 39$	$1691 \pm 66$	$11191 \pm 171$	<b>9371</b> ± 390
NeuralTS [43]	$9590 \pm 272$	$10301 \pm 461$	$8177 \pm 263$	$8604\pm753$	$1619 \pm 85$	$1647 \pm 43$	<b>11322</b> ± 161	11510 ± 198
NetCB <sub>NeuralTS</sub>	<b>9355</b> ± 307	$9326 \pm 222$	$7220 \pm 265$	$5974 \pm 325$	<b>1604</b> ± 73	$1482 \pm 53$	$11396 \pm 158$	$10704 \pm 383$
NetCB <sub>NeuralTS</sub>	$9425 \pm 342$	<b>8626</b> ± 544	<b>7189</b> ± 506	$5593 \pm 408$	$1637 \pm 48$	$1461 \pm 69$	$11789 \pm 415$	$10554 \pm 283$
EENet [7]	<b>9845</b> ± 414	$10206 \pm 477$	$8520 \pm 567$	$9049 \pm 557$	$1737 \pm 372$	$2033 \pm 98$	<b>10567</b> ± 183	$10375 \pm 315$
NetCB <sub>EENet</sub>	$12115 \pm 1931$	$8188 \pm 495$	$8356 \pm 183$	$6286 \pm 486$	<b>1509</b> ± 47	$1937 \pm 122$	$11026\pm200$	$9350 \pm 146$
NetCB <sub>EENet</sub>	$11523 \pm 707$	<b>8151</b> ± 684	<b>6953</b> ± 384	<b>6128</b> ± 293	$1532 \pm 37$	<b>1562</b> ± 75	$11152 \pm 209$	<b>9261</b> ± 227
GNB [28]	$9532 \pm 374$	$10686\pm503$	$8448 \pm 246$	$9993 \pm 501$	<b>1444</b> ± 65	$1478 \pm 86$	<b>11160</b> ± 142	$11217 \pm 298$
NetCB <sub>GNB</sub>	<b>9035</b> ± 377	$8292 \pm 500$	$6209 \pm 571$	$5136 \pm 282$	$1621 \pm 171$	$1435 \pm 87$	$11181 \pm 178$	$10045 \pm 430$
$NetCB_{\overline{GNB}}$	$9225 \pm 294$	<b>8113</b> ± 366	<b>5786</b> ± 234	$4854 \pm 120$	$1604 \pm 104$	<b>1416</b> ± 94	$11423 \pm 166$	<b>10009</b> ± 119

Table 1: Total regrets, at the last round in real-world (white) and semi-synthetic datasets (gray) with standard deviation.

NeuralUCB and NeuralTS, we use a grid search for the exploration parameter  $v \in \{0.001, 0.01, 0.1, 1\}$ , for the regularization parameter  $\lambda \in \{0.001, 0.01, 0.1, 1\}$  and for learning rate over  $\{0.001, 0.01, 0.1\}$ with a neural network width of 100. For EE-Net [7], we follow their default setup and do the grid search for learning rate over {0.0001, 0.001, 0.01, 0.1} for all neural networks. For GNB [28], we follow the default settings for classification dataset in their paper. We choose the best parameters from all grid-searched parameters for each dataset. To determine the stable point, we track the regression slope of direct activation rate (DAR) for the previous H rounds. In each round, the slope is calculated with the DARs of previous *G* rounds. If the slope remains within a threshold,  $|\theta|$ , during the previous H rounds, we say the bandit learning, as well as DAR, becomes stable, and consequently, we enable our strategy to go against the bandit. Before the start of a bandit experiment, we set  $arm_i = \emptyset, t_i = \emptyset, \mathbf{X}_{N_i} = \vec{0}, \mathbf{S}_i = \vec{0}, \overline{\mathbf{S}_i} = \vec{0}, \text{ and } y_i = 0 \text{ for all } \mathbf{S}_i = \vec{0}$  $v_i \in V$ . To determine the stable point for enabling our approach to go against the bandit prediction, we set G = 300, H = 300, and  $\theta$  = 0.00001. All experiments are repeated 10 times, and the average results for all methods are reported for comparison. We employ NVIDIA RTX A5000 GPU on Ubuntu 20.04 and Python 3.9.7 to run these experiments.

We run five different experiments. In the first experiment, we look at the effect of dynamic neighborhood knowledge on Regret by considering only the first step of NetCB. In the second experiment, we look at the effect of selecting direct recommendation against NetCB<sub>CMAB</sub> prediction on Regret by considering both steps of NetCB. To understand the effect of dynamic neighborhood knowledge on bandit accuracy,  $B_{acc}$  learning, we look at the bandit accuracy over time. In the fourth experiment, we look at the effect of activation probability due to direct recommendation on bandit accuracy, Bacc with NetCB<sub>NeuralTS</sub>. Finally, we look at the effect of dynamic neighborhood spillover knowledge on the bandit accuracy,  $B_{acc}$  in the fifth experiment.

#### Experimental results 5.5

Table 1 shows a summary of the regret comparison between each variant of  $NetCB_{CMAB}$  and  $NetCB_{\overline{CMAB}}$  with their corresponding CMAB. The best results are shown in bold; ones that are not statistically significantly better are also italicized. The table shows that in most cases (33/40), one of the NetCB variants performs better than its baseline CMAB. In 22 of the 40 cases, one of the NetCB variants has a lower regret that is also statistically significantly better than CMAB. In only 2 of the 40 cases, the baseline CMAB has a better performance that is statistically significant.

5.5.1 Effect of dynamic neighborhood knowledge on Regret. Net $CB_{CMAB}_{789}$ shows a noticeable decrease in Regret compared to its corresponding CMAB baseline, for almost all NetCB<sub>CMAB</sub> combinations in high-homophily datasets (19 out of 20) and for more than half of the combinations for low-homophily datasets (12 out of 20), as indicated in Table 1. On average NetCB<sub>CMAB</sub> decreases regret by 17.35% for high-homophily datasets and by 2.47% for low-homophily datasets.

5.5.2 Effect of selecting direct recommendation against NetCB<sub>CMAB</sub> prediction on Regret. NetCB $_{\overline{CMAB}}$  shows a decrease in Regret compared to its corresponding CMAB baseline, for 19 out of 20 NetCB combinations in high-homophily datasets and for 10 out of 20 combinations for low-homophily datasets, as indicated in Table 1. On average NetCB $_{\overline{CMAB}}$  decreases regret by 20.15% for high-homophily datasets and by 3.41% for low-homophily datasets. In comparison to NetCB<sub>CMAB</sub>, NetCB<sub>CMAB</sub> decreases regret by 3.52% for highhomophily datasets and by 0.92% for low-homophily datasets.

5.5.3 Effect of dynamic neighborhood knowledge on the bandit accuracy, Bacc. The incorporation of dynamic neighborhood knowledge helps the bandit learn faster in most cases, and thus increases the bandit accuracy, Bacc in NetCBCMABs compared to CMABs, particularly in the high homophilic datasets, as shown in Figure 3. NetCB<sub>CMAB</sub> shows an increase in bandit accuracy, Bacc for 19 out



Figure 3: Comparison of cumulative bandit accuracy, Bacc, in real-world and semi-synthetic (marked in blue) datasets.

of 20 NetCB<sub>CMAB</sub> combinations in high-homophily datasets and for 11 out of 20 combinations for low-homophily datasets. On average NetCB<sub>CMAB</sub> increases Bacc by 10.78% for high-homophily datasets and by 0.56% for low-homophily datasets.

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5.5.4 Effect of activation probability due to direct recommendation on bandit accuracy, Bacc. To understand how Bacc changes with the increase in difference between  $p_m$  and  $p_a$ , we run  $NetCB_{NeuralTS}$ with various combinations of  $p_m$  and  $p_a$ , i.e., (0.5, 0.7), (0.3, 0.7), (0.1, 0.7), in real-world datasets. We run these experiments with a spillover setting of  $p_{aa} = 0.3$ ,  $p_{ma} = 0.3$ ,  $p_{am} = 0.0$ ,  $p_{mm} = 0.0$  and observe the cumulative average of the bandit accuracy,  $B_{acc}$ .

The  $B_{acc}$  increases in all datasets as the difference between  $p_m$ and  $p_a$  increases. Therefore, the bandit can better differentiate among different types of nodes with higher difference between  $p_m$  and  $p_a$ . For example, when  $p_m = 0.1$  and  $p_a = 0.7$ , the bandit achieves around 20.19%, 43.40%, 77.42%, and 56.34% accuracy at the end of experiment in Flickr, Blogcatalog, Hateful, and Pubmed datasets, respectively. However, the bandit achieves 14.73%, 32.10%, 69.82%, and 47.06% accuracy in Flickr, Blogcatalog, Hateful, and Pubmed datasets, respectively, when  $p_m = 0.5$  and  $p_a = 0.7$ . The  $B_{acc}$  tends to decrease with the increase in the total number of labels, *l* of the datasets. For example, the Pubmed (l = 3) and Hateful (l = 2) datasets achieve higher accuracy than Blogcatalog (l = 6)and Flickr (l = 9) datasets at the end of the experiments. We show the details of these results in the Appendix.

5.5.5 Effect of dynamic neighborhood spillover knowledge on the 864 *bandit accuracy*,  $B_{acc}$ . We reduce the dimensions of  $\mathbf{X}_{N_i} \in \mathbb{R}^{4l}$  in 865 NetCB<sub>CMAB</sub> by removing the last 2l dimensions, which represent 866 past knowledge of spillovers in node  $v_i$ 's neighboring nodes. The 867 868 resulting 2l dimensional representation corresponds to past knowl-869 edge of direct recommendations in node  $v_i$ 's neighboring nodes. We

refer to this particular version of  $NetCB_{CMAB}$  as  $NetCB_{CMAB}^{direct}$ . To understand the specific impact of dynamic neighborhood spillover knowledge on the acceleration of bandit learning, we run each NetCB<sup>direct</sup> for real-world and semi-synthetic datasets and compare the results with their corresponding NetCB<sub>CMAB</sub>. We run all these experiments by setting  $p_a = 0.7$ ,  $p_m = 0.5$ ,  $p_{aa} = 0.3$ ,  $p_{ma} = 0.3$ ,  $p_{am} = 0.0$ , and  $p_{mm} = 0.0$ . In most experiments with high homophilic networks, dynamic neighborhood spillover information has shown a positive effect on raising the cumulative bandit accuracy,  $B_{acc}$ , e.g., 4.86% increase in  $NetCB_{GNB}$  compared to NetCBdirect for Pubmed (Homophily: 0.80) dataset. Nevertheless, the effect is diminished in the lower homophilic networks compared to higher homophilic networks, e.g., 0.12% increase in  $NetCB_{GNB}$  compared to  $NetCB_{GNB}^{direct}$  for semi-synthetic Pubmed (Homophily: 0.30) dataset.

#### 6 Conclusion

We presented NetCB which leverages dynamic neighborhood knowledge and the potential of heterogeneous spillover to maximize network rewards in bandit online learning. Our experiments on real-world and semi-synthetic datasets show a significant decrease in regret when considering neighborhood context in most cases and that it can be beneficial to make suboptimal direct recommendations, in order to maximize rewards from spillover. NetCB can be applied in practical recommendation applications in which the recommendation given to one individual can lead to network rewards when they share that recommendation with their social circles, e.g., video recommendations in social networks, targeted marketing in e-commerce, and healthcare interventions. Future work includes deriving regret bounds for NetCB and automatically learning the values of recommendation-dependent heterogeneous spillover probabilities.

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#### References

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- Shipra Agrawal and Navin Goyal. 2013. Thompson sampling for contextual bandits with linear payoffs. In *International conference on machine learning*. PMLR, 127–135.
- [2] Robin Allesiardo, Raphaël Féraud, and Djallel Bouneffouf. 2014. A neural networks committee for the contextual bandit problem. In *International Conference* on Neural Information Processing. Springer, 374–381.
- [3] Peter M Aronow and Cyrus Samii. 2017. Estimating average causal effects under general interference with application to a social network experiment. *The Annals* of Applied Statistics 11, 4 (2017), 1912–1947.
- [4] Eytan Bakshy, Jake M Hofman, Winter A Mason, and Duncan J Watts. 2011. Everyone's an influencer: quantifying influence on twitter. In Proceedings of the fourth ACM international conference on Web search and data mining. 65–74.
- [5] Yikun Ban and Jingrui He. 2021. Local clustering in contextual multi-armed bandits. In Proceedings of the Web Conference 2021. 2335–2346.
- [6] Yikun Ban, Yunzhe Qi, Tianxin Wei, and Jingrui He. 2022. Neural collaborative filtering bandits via meta learning. arXiv preprint arXiv:2201.13395 (2022).
- [7] Yikun Ban, Yuchen Yan, Arindam Banerjee, and Jingrui He. 2022. EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits. In International Conference on Learning Representations.
- [8] Falco J Bargagli-Stoffi, Costanza Tortu, and Laura Forastiere. 2020. Heterogeneous treatment and spillover effects under clustered network interference. arXiv preprint arXiv:2008.00707 (2020).
- [9] Rosalind C Barnett and Nancy L Marshall. 1992. Worker and mother roles, spillover effects, and psychological distress. Women & Health 18, 2 (1992), 9–40.
- [10] Djallel Bouneffouf and Irina Rish. 2019. A survey on practical applications of multi-armed and contextual bandits. arXiv preprint arXiv:1904.10040 (2019).
- [11] Nicolo Cesa-Bianchi, Claudio Gentile, and Giovanni Zappella. 2013. A gang of bandits. Advances in neural information processing systems 26 (2013).
- [12] Inyoung Chae, Andrew T Stephen, Yakov Bart, and Dai Yao. 2017. Spillover effects in seeded word-of-mouth marketing campaigns. *Marketing Science* 36, 1 (2017), 89–104.
- [13] Wei Chu, Lihong Li, Lev Reyzin, and Robert Schapire. 2011. Contextual bandits with linear payoff functions. In Proceedings of the Fourteenth International Conference on Artificial Intelligence and Statistics. JMLR Workshop and Conference Proceedings, 208–214.
- [14] Claudio Gentile, Shuai Li, Purushottam Kar, Alexandros Karatzoglou, Giovanni Zappella, and Evans Etrue. 2017. On context-dependent clustering of bandits. In International Conference on machine learning. PMLR, 1253–1262.
- [15] Claudio Gentile, Shuai Li, and Giovanni Zappella. 2014. Online clustering of bandits. In International conference on machine learning. PMLR, 757–765.
- [16] Andrei Hagiu and Julian Wright. 2015. Marketplace or reseller? Management Science 61, 1 (2015), 184–203.
- [17] Nathan Halko, Per-Gunnar Martinsson, and Joel A Tropp. 2009. Finding structure with randomness: Stochastic algorithms for constructing approximate matrix decompositions. (2009).
- [18] David Holtz, Ruben Lobel, Inessa Liskovich, and Sinan Aral. 2020. Reducing interference bias in online marketplace pricing experiments. arXiv preprint arXiv:2004.12489 (2020).
- [19] Alexandra Iacob, Bogdan Cautis, and Silviu Maniu. 2022. Contextual Bandits for Advertising Campaigns: A Diffusion-Model Independent Approach. In Proceedings of the 2022 SIAM International Conference on Data Mining (SDM). SIAM, 513–521.
- [20] Nathan Korda, Balazs Szorenyi, and Shuai Li. 2016. Distributed clustering of linear bandits in peer to peer networks. In *International conference on machine learning*. PMLR, 1301–1309.
- [21] Hannah Li, Geng Zhao, Ramesh Johari, and Gabriel Y Weintraub. 2022. Interference, bias, and variance in two-sided marketplace experimentation: Guidance for platforms. In *Proceedings of the ACM Web Conference 2022*. 182–192.
- [22] Lihong Li, Wei Chu, John Langford, and Robert E Schapire. 2010. A contextualbandit approach to personalized news article recommendation. In Proceedings of the 19th international conference on World wide web. 661–670.
- [23] Shuai Li, Wei Chen, Shuai Li, and Kwong-Sak Leung. 2019. Improved Algorithm on Online Clustering of Bandits. In Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19. International Joint Conferences on Artificial Intelligence Organization, 2923–2929.
- [24] Shuai Li, Alexandros Karatzoglou, and Claudio Gentile. 2016. Collaborative filtering bandits. In Proceedings of the 39th International ACM SIGIR conference on Research and Development in Information Retrieval. 539–548.
- [25] Kanak Mahadik, Qingyun Wu, Shuai Li, and Amit Sabne. 2020. Fast distributed bandits for online recommendation systems. In Proceedings of the 34th ACM international conference on supercomputing. 1–13.
- [26] Jérémie Mary, Romaric Gaudel, and Philippe Preux. 2015. Bandits and recommender systems. In Machine Learning, Optimization, and Big Data: First International Workshop, MOD 2015, Taormina, Sicily, Italy, July 21-23, 2015, Revised Selected Papers 1. Springer, 325–336.

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- [27] Trong T Nguyen and Hady W Lauw. 2014. Dynamic clustering of contextual multi-armed bandits. In Proceedings of the 23rd ACM international conference on conference on information and knowledge management. 1959–1962.
- [28] Yunzhe Qi, Yikun Ban, and Jingrui He. 2023. Graph neural bandits. In Proceedings of the 29th ACM SIGKDD Conference on Knowledge Discovery and Data Mining. 1920–1931.
- [29] Manoel Horta Ribeiro, Pedro H Calais, Yuri A Santos, Virgílio AF Almeida, and Wagner Meira Jr. 2018. Characterizing and detecting hateful users on twitter. In *Twelfth international AAAI conference on web and social media.*
- [30] Paulo Shakarian, Abhinav Bhatnagar, Ashkan Aleali, Elham Shaabani, and Ruocheng Guo. 2015. The independent cascade and linear threshold models. In Diffusion in Social Networks. Springer, 35–48.
- [31] Claude Elwood Shannon. 1948. A mathematical theory of communication. The Bell system technical journal 27, 3 (1948), 379–423.
- [32] Yu Song, Shuai Sun, Jianxun Lian, Hong Huang, Yu Li, Hai Jin, and Xing Xie. 2022. Show me the whole world: Towards entire item space exploration for interactive personalized recommendations. In *Proceedings of the Fifteenth ACM International Conference on Web Search and Data Mining*. 947–956.
- [33] McKenzie N Sparrer, Natasha F Hodges, Tyler Sherman, Susan VandeWoude, Angela M Bosco-Lauth, and Christie E Mayo. 2023. Role of spillover and spillback in SARS-CoV-2 transmission and the importance of one health in understanding the dynamics of the COVID-19 pandemic. *Journal of Clinical Microbiology* 61, 7 (2023), e01610–22.
- [34] Hung Tran-The, Sunil Gupta, Santu Rana, Tuan Truong, Long Tran-Thanh, and Svetha Venkatesh. 2022. Expected Improvement for Contextual Bandits. In Advances in Neural Information Processing Systems, Vol. 35. Curran Associates, Inc., 22725–22738.
- [35] Nguyen Ba Trung. 2019. The spillover effects of US economic policy uncertainty on the global economy: A global VAR approach. *The North American Journal of Economics and Finance* 48 (2019), 90–110.
- [36] Sharan Vaswani, Branislav Kveton, Zheng Wen, Mohammad Ghavamzadeh, Laks VS Lakshmanan, and Mark Schmidt. 2017. Model-independent online learning for influence maximization. In *International Conference on Machine Learning*. PMLR, 3530–3539.
- [37] Zheng Wen, Branislav Kveton, Michal Valko, and Sharan Vaswani. 2017. Online influence maximization under independent cascade model with semi-bandit feedback. Advances in neural information processing systems 30 (2017).
- [38] Bryan Wilder, Nicole Immorlica, Eric Rice, and Milind Tambe. 2018. Maximizing influence in an unknown social network. In Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 32.
- [39] Qingyun Wu, Huazheng Wang, Quanquan Gu, and Hongning Wang. 2016. Contextual bandits in a collaborative environment. In Proceedings of the 39th International ACM SIGIR conference on Research and Development in Information Retrieval. 529–538.
- [40] Xian Wu, Suleyman Cetintas, Deguang Kong, Miao Lu, Jian Yang, and Nitesh Chawla. 2020. Learning from cross-modal behavior dynamics with graphregularized neural contextual bandit. In *Proceedings of The Web Conference 2020*. 995–1005.
- [41] Yingchen Yan, Ruiqing Zhao, and Zhibing Liu. 2018. Strategic introduction of the marketplace channel under spillovers from online to offline sales. *European Journal of Operational Research* 267, 1 (2018), 65–77.
- [42] Yuan Yuan and Kristen M Altenburger. 2023. A Two-Part Machine Learning Approach to Characterizing Network Interference in A/B Testing. arXiv preprint arXiv:2308.09790 (2023).
- [43] Weitong Zhang, Dongruo Zhou, Lihong Li, and Quanquan Gu. 2021. Neural thompson sampling. In International Conference on Learning Representations.
- [44] Xiaoying Zhang, Hong Xie, Hang Li, and John CS Lui. 2020. Conversational contextual bandit: Algorithm and application. In *Proceedings of the web conference* 2020. 662–672.
- [45] Dongruo Zhou, Lihong Li, and Quanquan Gu. 2020. Neural contextual bandits with ucb-based exploration. In *International Conference on Machine Learning*. PMLR, 11492–11502.

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#### 1045 A Appendix

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The Appendix contains additional details about the results presented in Section 5.5.

# A.1 Effect of dynamic neighborhood knowledge on Regret

1052 *Regret* in *NetCB<sub>LinUCB</sub>*, is reduced by 24.40%, 34.21%, 4.87%, and 1053 16.43% when compared to LinUCB in the semi-synthetic Flickr (Ho-1054 mophily: 0.88), semi-synthetic Blogcatalog (Homophily: 0.88), Hate-1055 ful (Homophily: 0.73), and Pubmed (Homophily: 0.80) datasets, re-1056 spectively. The datasets show respective decreases of 9.47%, 30.57%, 1057 10.01%, and 7.00% in NetCB<sub>NeuralTS</sub> compared to NeuralTS. In comparison to EENet, the datasets show decreases in NetCBEENet 1058 1059 of 19.77%, 30.54%, 4.72%, and 9.88%, respectively. The datasets also 1060 exhibit respective decreases of 22.41%, 48.60%, 2.9%, and 10.45% 1061 in  $NetCB_{GNB}$  compared to GNB. In these datasets, a reduction in NetCB<sub>NeuralUCB</sub> is observed in Flickr, Blogcatalog, and Pubmed, 1062 1063 with decreases of 17.19%, 36.08%, and 14.89%, respectively, in com-1064 parison to NeuralUCB.

1065 While the Regret is decreased on average by 17.35% in higher ho-1066 mophilic networks, the impact of incorporating dynamic neighbor-1067 hood knowledge in reducing Regret diminishes in lower homophilic 1068 network datasets as shown in Table 1. For instance, all NetCB<sub>CMAB</sub>s 1069 generate higher *Regret* compared to *CMABs* in the semi-synthetic 1070 Pubmed (Homophily: 0.30) dataset except in NetCB<sub>LinUCB</sub>, which 1071 shows 0.41% decrease in Regret compared to LinUCB. However, the *Regret* decreases by 9.78%, 2.45%, 5.21% in *NetCB<sub>LinUCB</sub>*, 1072

1073 NetCB<sub>NeuralTS</sub>, and NetCB<sub>GNB</sub>, respectively, when compared 1074 to their respective CMAB baselines for the Flickr (Homophily: 1075 0.23) dataset. These decreases in the Blogcatalog (Homophily: 0.40) 1076 dataset are 16.4%, 11.71%, and 26.50%, respectively. The dataset also 1077 shows a decrease of 17.56% and 1.9% in Regret for NetCB<sub>NeuralUCB</sub> 1078 and NetCB<sub>EENet</sub>, respectively, in comparison to their correspond-1079 ing CMAB baselines. The semi-synthetic Hateful (Homophily: 0.58) 1080 dataset indicates a decrease of 3.44%, 0.94%, 13.11% in Regret for 1081  $NetCB_{LinUCB}, NetCB_{NeuralTS}, and NetCB_{EENet}, respectively, com-$ 1082 pared to their respective CMAB baselines. 1083

# A.2 Effect of selecting direct recommendation against NetCB<sub>CMAB</sub> prediction on Regret

Our strategy to leverage spillover in the second component helps 1087 to decrease the *Regrets* for most NetCB $_{\overline{CMAB}}$ s in the datasets with 1088 high homophily compared to their respective NetCB<sub>CMAB</sub>s as 1089 shown in Table 1. For example, *Regret* in  $NetCB_{LinUCB}$ , is reduced 1090 by 2.47%, 2.82%, 0.98%, and 0.25% when compared to NetCBLinUCB 1091 in the semi-synthetic Flickr (Homophily: 0.88), semi-synthetic Blog-1092 catalog (Homophily: 0.88), Hateful (Homophily: 0.73), and Pubmed 1093 (Homophily: 0.80) datasets, respectively. The datasets show respec-1094 tive decreases of 7.51%, 6.38%, 1.42%, and 1.40% in  $NetCB_{\overline{NeuralTS}}$ 1095 compared to NetCB<sub>NeuralTS</sub>. In comparison to NetCB<sub>EENet</sub>, the 1096 datasets show decreases in  $NetCB_{\overline{EENet}}$  of 0.45%, 2.51%, 19.36%, 1097 and 0.95%, respectively. The datasets also exhibit respective de-1098 creases of 2.16%, 5.49%, 1.32%, and 0.36% in  $NetCB_{\overline{GNB}}$  compared 1099 to  $NetCB_{GNB}$ . In these datasets, a reduction in  $NetCB_{\overline{NeuralUCB}}$ 1100 is observed in Flickr, Blogcatalog, and Pubmed, with decreases of 1101 1102

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# 5.85%, 8.74%, and 2.24%, respectively, in comparison to $NetCB_{NeuralUCB}$ .

The strategy to leverage spillover also helps to decrease the *Regret* in some lower homophilic networks, as shown in Table 1. For example, Flickr (Homophily: 0.23) dataset shows a decrease of 15.57% and 4.89% in *Regret* for *NetCB*<sub>*NeuralUCB*</sub> and *NetCB*<sub>*EENet*</sub>, respectively, in comparison to their corresponding NetCB<sub>*CMAB*</sub>s. The *Regret* decreases by 0.43%, 16.79%, 6.81% in *NetCB*<sub>*CMAB*</sub>s. The *Regret* decreases by 0.43%, 16.79%, 6.81% in *NetCB*<sub>*CMAB*</sub>s. The *Regret* decreases by 0.43%, 16.79%, 6.81% in *NetCB*<sub>*CMAB*</sub>s. NetCB<sub>*EENet*</sub>, and *NetCB*<sub>*GNB*</sub>, respectively, when compared to their respective NetCB<sub>*CMAB*</sub>s generate higher *Regret* compared to NetCB<sub>*CMAB*</sub>s in the semi-synthetic Pubmed (Homophily: 0.30) dataset except in *NetCB*<sub>*NeuralUCB*</sub>. The same goes for the semi-synthetic Hateful (Homophily: 0.58) dataset except in *NetCB*<sub>*GNB*</sub>, which shows a 1.05% decrease in *Regret* compared to *NetCB*<sub>*GNB*</sub>.

# A.3 Effect of dynamic neighborhood knowledge on the bandit accuracy, *B<sub>acc</sub>*

The bandit accuracy, *Baccs* in *NetCBLinUCB*, increases by 11.48%, 16.31%, 6.45%, and 7.14% when compared to LinUCB in the semisynthetic Flickr (Homophily: 0.88), semi-synthetic Blogcatalog (Homophily: 0.88), Hateful (Homophily: 0.73), and Pubmed (Homophily: 0.80) datasets, respectively. The Baccs of NetCB<sub>NeuralUCB</sub> increase by 19.70%, 21.15%, 5.84%, and 12.10%, respectively, compared to NeuralUCB in these datasets. The datasets show respective increases of 5.42%, 17.55%, 4.29%, and 4.02% in  $NetCB_{NeuralTS}$  compared to NeuralTS. In comparison to GNB, the datasets show increases in NetCBGNB of 11.88%, 27.87%, 3.49%, and 6.77%, respectively. In case of *NetCB<sub>EENet</sub>*, the *B<sub>acc</sub>* in the Hateful (Homophily: 0.73) dataset decreases by 0.52% compared to EENet. However, Baccs increase by 12.93%, 16.84%, and 4.93% in the semi-synthetic Flickr (Homophily: 0.88), semi-synthetic Blogcatalog (Homophily: 0.88), and Pubmed (Homophily: 0.80) datasets, respectively, compared to EENet.

The impact of dynamic neighborhood knowledge is diminished in lower homophilic datasets compared to higher homophilic datasets as shown in Figure 3. For example, the bandit accuracy,  $B_{acc}$  increases by 1.65%, 3.84%, 0.67%, 6.32% in NetCB<sub>LinUCB</sub>, NetCB<sub>NeuralUCB</sub>, NetCB<sub>NeuralTS</sub>, and NetCB<sub>GNB</sub>, respectively, when compared to LinUCB, NeuralUCB, NeuralTS, and GNB, respectively, in the Flickr (Homophily: 0.23) dataset. These increases in the Blogcatalog (Homophily: 0.40) dataset are 9.01%, 12.79%, 7.90%, and 18.74%, respectively. All NetCB<sub>CMAB</sub>s yield lower B<sub>acc</sub>s compared to their respective CMABs in the semi-synthetic Hateful (Homophily: 0.58) dataset, except for NetCB<sub>NeuralUCB</sub> and NetCB<sub>EENet</sub>, which show 1.72% and 13.54% rise relative to NeuralUCB and EENet, respectively. The NetCB<sub>EENet</sub> also shows 4.88% rise relative to EENet in the Blogcatalog (Homophily: 0.40) dataset. All NetCB<sub>CMAB</sub>s generate lower B<sub>acc</sub>s compared to their respective CMABs in the semi-synthetic Pubmed (Homophily: 0.30) datasets.



Figure 4: Comparison of cumulative bandit accuracy, Bacc of NetCBNeuralTS by varying activation probabilities due to direct recommendations.

#### Effect of activation probability due to direct A.4 recommendation on bandit accuracy, Bacc

The difference in the activation probabilities for aligned and misaligned direct recommendation plays a role in how well the bandit can learn to distinguish between different types of nodes. The bandit learning becomes harder with smaller difference between the activation probabilities as shown in Figure 4. However, neighborhood information helps the bandit learn to distinguish among them,

regardless of the difference in the activation probabilities. In realworld scenarios,  $p_m$  and  $p_a$  are unknown and therefore it is very important to incorporate neighborhood information.

#### A.5 Runtime

The runtime of these experiments depends on the choice of CMAB as well as the density and size of network datasets. For example, NetCB<sub>LinUCB</sub> requires around 6, 2.5, 4, and 1 hrs to complete a bandit experiment on Flickr, Blogcatalog, Hateful, and Pubmed datasets, respectively. However, the datasets require around four times more hours for *NetCB<sub>NeuralUCB</sub>*.