# **Unified Interpretation of Smoothing Methods for Negative Sampling Loss** Functions in Knowledge Graph Embedding

**Anonymous ACL submission** 

### Abstract

Knowledge Graphs (KGs) are fundamental resources in knowledge-intensive tasks in NLP. Due to the limitation of manually creating KGs, KG Completion (KGC) has an important role in automatically completing KGs by scoring their links with KG Embedding (KGE). To handle many entities in training, KGE relies on Negative Sampling (NS) loss that can reduce the computational cost by sampling. Since the appearance frequencies for each link are at most one in KGs, sparsity is an essential and inevitable problem. The NS loss is no exception. As a solution, the NS loss in 013 KGE relies on smoothing methods like Self-Adversarial Negative Sampling (SANS) and subsampling. However, it is uncertain what kind of smoothing method is suitable for this 017 purpose due to the lack of theoretical understanding. This paper provides theoretical interpretations of the smoothing methods for the NS loss in KGE and induces a new NS loss, Triplet Adaptive Negative Sampling (TANS), that can cover the characteristics of the conventional smoothing methods. Experimental results of TransE, DistMult, ComplEx, RotatE, HAKE, and HousE on FB15k-237, WN18RR, and YAGO3-10 datasets and their sparser subsets show the soundness of our interpretation and performance improvement by our TANS.

#### 1 Introduction

027

041

Knowledge Graphs (KGs) represent human knowledge using various entities and their relationships as graph structures. KGs are fundamental resources for knowledge-intensive tasks like dialog (Moon et al., 2019), question answering (Reese et al., 2020), named entity recognition (Liu et al., 2019), open-domain questions (Hu et al., 2022), and recommendation systems (Gao et al., 2020), etc.

However, to create complete KGs, we need to consider a large number of entities and all their possible relationships. Taking into account the explosively large number of combinations between

entities, only relying on manual approaches is unrealistic to make complete KGs.

043

044

045

046

047

049

051

054

057

060

061

062

063

064

065

066

067

068

069

070

071

072

073

074

076

077

078

081

Knowledge Graph Completion (KGC) is a task to deal with this problem. KGC involves automatically completing missing links corresponding to relationships between entities in KGs. To complete the KGs, we need to score each link between entities. For this purpose, current KGC commonly relies on Knowledge Graph Embedding (KGE) (Bordes et al., 2011). KGE models predict the missing relations, named link prediction, by learning structural representations. In the current KGE, models need to complete a link (triplet)  $(e_i, r_k, e_i)$  of entities  $e_i$  and  $e_j$ , and their relationship  $r_k$  by answering  $e_i$  or  $e_j$  from a given query  $(?, r_k, e_j)$  or  $(e_i, r_k, ?)$ , respectively. Hence, KGE needs to handle a large number of entities and their relationships during its training.

To handle a large number of entities and relationships in KGs, Negative Sampling (NS) loss (Mikolov et al., 2013) is frequently used for training KGE models. The original NS loss is proposed to approximate softmax cross-entropy loss to reduce computational costs by sampling false labels from its noise distribution in training. Trouillon et al. (2016) import the NS loss from word embedding to KGE with utilizing uniform distribution as its noise distribution. Sun et al. (2019) extend the NS loss to Self-Adversarial Negative Sampling (SANS) loss for efficient training of KGE. Unlike the NS loss with uniform distribution, the SANS loss utilizes the training model's prediction as the noise distribution. Since the negative samples in the SANS loss become more difficult to discriminate for models in training, the SANS can extract models' potential compared with the NS loss with uniform distribution.

One of the problems left for KGE is the sparsity of KGs. Figure 1 shows the appearance frequency of queries and answers (entities) in the training data of FB15k-237, WN18RR and YAGO3-10 datasets.



Figure 1: Appearance frequencies of queries and answers in the training data of FB15k-237, WN18RR, and YAGO3-10. Note that the indices are sorted from high frequency to low.



Figure 2: Performances of KGE models HousE, HAKE, RotatE, ComplEx, DistMult, and TransE on datasets FB15k-237, WN18RR, and YAGO3-10 using NS, SANS, and subsampling methods (noted as *Base, Freq, Uniq*).

From the long-tail distribution of this figure, we can understand that both queries and answers necessary for training KGE models may suffer from the sparsity problem.

As a solution, several smoothing methods are used in KGE. Sun et al. (2019) import subsampling from word2vec (Mikolov et al., 2013) to KGE. Subsampling can smooth the appearance frequency of triplets and queries in KGs. Kamigaito and Hayashi (2022a) show a general formulation that covers the basic subsampling of Sun et al. (2019) (Base), their frequency-based subsampling (Freq) and uniquebased subsampling (Uniq) for KGE. Kamigaito and Hayashi (2021) indicate that SANS has a similar effect of using label-smoothing (Szegedy et al., 2016) and thus SANS can smooth the frequencies of answers in training. Figure 2 shows the effectiveness of SANS and subsampling in KGC performance. From the figure, since FB15k-237 is more sparse (imbalanced) than WN18RR and YAGO3-10 based on Figure 1, we can understand that strategy in choosing smoothing methods have more considerable influences than models when data is sparse.

094

100

102

104

105 106

107

108

109

While SANS and subsampling can improve model performance by smoothing the appearance frequencies of triplets, queries, and answers, their theoretical relationship is not clear, leaving their capabilities and deficiencies a question. For example, conventional works (Sun et al., 2019; Zhang et al., 2020b; Kamigaito and Hayashi, 2022a)<sup>1</sup> jointly use SANS and subsampling with no theoretical background. Thus, there is a call for further interpretability and performance improvement.

110

111

112

113

114

115

116

117

118

119

120

121

122

123

124

125

126

128

129

To solve the above problem, we theoretically and empirically study the differences of SANS and subsampling on three common datasets and their sparser subsets with six popular KGE models<sup>2</sup>. Our contributions are as follows:

- By focusing on the smoothing targets, we theoretically reveal the differences between SANS and subsampling and induce a new NS loss, Triplet Adaptive Negative Sampling (TANS), that can cover the smoothing target of both SANS and subsampling.
- We theoretically show that TANS with subsampling can potentially cover the conven-

<sup>&</sup>lt;sup>1</sup>Note that Sun et al. (2019); Zhang et al. (2020b) use subsampling in their released implementation without referring to it in their paper.

<sup>&</sup>lt;sup>2</sup>Our code and data are available at https://github.com/[innominated].

| • | We empirically verify that TANS improves  |
|---|---|
|   | KGC performance on sparse KGs in terms of |

tional usages of SANS and subsampling.

• We empirically verify that TANS with subsampling can cover the conventional usages of SANS and subsampling in terms of MRR.

# 2 Background

MRR.

130

131

132

133

134

135

136

137

138

139

140

141

142

143

144

145

146

147

148

149

150

152

153

154

156

157

158

160

163

164

165

166

168

1

In this section, we describe the problem formulation for solving KGC by KGE and explain the conventional NS loss functions in KGE.

# 2.1 Formulation of KGE

KGC is a research topic for automatically inferring new links in a KG that are likely but not yet known to be true. To infer the new links by KGE, we decompose KGs into a set of triplets (links). By using entities  $e_i$ ,  $e_j$  and their relation  $r_k$ , we represent the triplet as  $(e_i, r_k, e_j)$ . In a typical KGC task, a KGE model receives a query  $(e_i, r_k, ?)$  or  $(?, r_k, e_j)$  and predicts the entity corresponding to ? as an answer.

In KGE, a KGE model scores a triplet  $(e_i, r_k, e_j)$ by using a scoring function  $s_{\theta}(x, y)$ , where  $\theta$  denotes model parameters. Here, using a softmax function, we represent the existence probability  $p_{\theta}(y|x)$  for an answer y of the query x as follows:

$$p_{\theta}(y|x) = \frac{\exp(s_{\theta}(x,y))}{\sum_{y' \in Y} \exp(s_{\theta}(x,y'))}, \qquad (1)$$

where Y is a set of entities.

### 2.2 NS Loss in KGE

To train  $s_{\theta}(x, y)$ , we need to calculate losses for the observables  $D = \{(x_1, y_1), \dots, (x_n, y_n)\}$  that follow  $p_d(x, y)$ . Even if we can represent KGC by Eq. (1), it does not mean we can tractably perform KGC due to the large number of Y in KGs. For the reason of the computational cost, the NS loss (Mikolov et al., 2013) is used to approximate Eq. (1) by sampling false answers.

By modifying that of Mikolov et al. (2013), the following NS loss (Sun et al., 2019; Ahrabian et al., 2020) is commonly used in KGE:

$$\ell_{NS}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} \left[ \log(\sigma(s_{\theta}(x,y) + \tau)) + \frac{1}{\nu} \sum_{y_i \sim U}^{\nu} \log(\sigma(-s_{\theta}(x,y_i) - \tau)) \right], \quad (2)$$

where U is the noise distribution that follows uniform distribution,  $\sigma$  is the sigmoid function,  $\nu$  is the number of negative samples per positive sample (x, y), and  $\tau$  is a margin term to adjust the value range decided by  $s_{\theta}(x, y)$ . 172

173

174

175

177

178

179

180

181

182

183

184

185

186

187

188

189

190

191

192

193

194

195

198

199

200

201

202

203

204

205

206

207

208

209

211

212

# 2.3 Smoothing Methods for the NS Loss in KGE

As shown in Figure 1, KGC needs to deal with the sparsity problem caused by low frequent queries and answers in KGs. Imposing smoothing on the appearance frequencies of queries and answers can mitigate this problem. The following subsections introduce subsampling (Mikolov et al., 2013; Sun et al., 2019; Kamigaito and Hayashi, 2022a) and SANS (Sun et al., 2019), the conventional smoothing methods for the NS loss in KGE.

### 2.3.1 Subsampling

Subsampling (Mikolov et al., 2013) is a method to smooth the frequency of triplets or queries in the NS loss. Sun et al. (2019) import this approach from word embedding to KGE. Kamigaito and Hayashi (2022b,a) add some variants to subsampling for KGC and theoretically provide a unified expression of them as follows:

$$\ell_{ ext{SUB}}( heta)$$
 196

$$= -\frac{1}{|D|} \sum_{(x,y)\in D} \left[ A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y) + \tau)) \right]$$
197

$$+\frac{1}{\nu}\sum_{y_i\sim U}^{\nu}B(x,y;\alpha)\log(\sigma(-s_{\theta}(x,y_i)-\tau))], \quad (3)$$

where  $\alpha$  is a temperature term to adjust the frequecy of triplets and queries. Note that we incorporate  $\alpha$ into Eq. (3) to consider various loss functions even though Kamigaito and Hayashi (2022b,a) do not consider  $\alpha$ . In this formulation, we can consider several assumptions for deciding  $A(x, y; \alpha)$  and  $B(x, y; \alpha)$ . We introduce these assumptions in the following paragraphs:

**Base** As a basic subsampling approach, Sun et al. (2019) import the one originally used in word2vec (Mikolov et al., 2013) to KGE, defined as follows:

$$A(x, y; \alpha) = B(x, y; \alpha) = \frac{\#(x, y)^{-\alpha} |D|}{\sum_{(x'\!, y') \in D} \#(x', y')^{-\alpha}},$$
(4)
(4)
(4)

where # is the symbol for frequency and #(x, y)represents the frequency of (x, y). In word2vec,

291

292

251

252

253

subsampling randomly discards a word by a probability  $1 - \sqrt{t/f}$ , where t is a constant value and f is a frequency of a word. This is similar to randomly keeping a word with a probability  $\sqrt{t/f}$ . Thus, we can understand that Eq. (4) follows the original use in word2vec. Since the actual (x, y) occurs at most once in KGs, when  $(x, y) = (e_i, r_k, e_j)$ , they approximate the frequency of (x, y) as:

$$\#(x,y) \approx \#(e_i, r_k) + \#(r_k, e_j),$$
 (5)

based on the approximation of n-gram languagemodeling (Katz, 1987).

221

226

227

239

241

242

243

247

248

**Freq** Kamigaito and Hayashi (2022a) propose frequency-based subsamping (Freq) by assuming a case that (x, y) originally has a frequency, but the observed one in the KG is at most 1.

$$A(x, y; \alpha) = \frac{\#(x, y)^{-\alpha} |D|}{\sum_{(x', y') \in D} \#(x', y')^{-\alpha}},$$
  
$$B(x, y; \alpha) = \frac{\#x^{-\alpha} |D|}{\sum_{x' \in D} \#x'^{-\alpha}}.$$
 (6)

**Uniq** Kamigaito and Hayashi (2022a) also propose unique-based subsamping (Uniq) by assuming a case that the originally frequency and the observed one in the KG are both 1.

$$A(x, y; \alpha) = B(x, y; \alpha) = \frac{\#x^{-\alpha}|D|}{\sum_{x' \in D} \#x'^{-\alpha}}.$$
 (7)

### 2.3.2 SANS Loss

SANS is originally proposed as a kind of NS loss to train KGE models efficiently by considering negative samples close to their corresponding positive ones. Kamigaito and Hayashi (2021) show that using SANS is similar to imposing label-smoothing on Eq. (1). Thus, SANS is a method to smooth the frequency of answers in the NS loss. The SANS loss is represented as follows:

244 
$$\ell_{\text{SANS}}(\theta)$$
245 
$$= -\frac{1}{|D|} \sum_{(x,y)\in D} \left[ \log(\sigma(s_{\theta}(x,y) + \tau)) + \sum_{y_{i}\sim U}^{\nu} p_{\theta}(y_{i}|x;\beta) \log(\sigma(-s_{\theta}(x,y_{i}) - \tau)) \right], (8)$$

$$p_{\theta}(y_i|x;\beta) \approx \frac{\exp(\beta s_{\theta}(x,y_i))}{\sum_{j=1}^{\nu} \exp(\beta s_{\theta}(x,y_j))}, \qquad (9)$$

where  $\beta$  is a temperature to adjust the distribution of negative sampling. Different from subsampling, SANS uses  $p_{\theta}(y_i|x;\beta)$  that is predicted by a model  $\theta$  to adjust the frequency of the answer  $y_i$ . Since  $p_{\theta}(y_i|x;\beta)$  is essentially a noise distribution, it does not receive any gradient during training.

# **3** Triplet Adaptive Negative Sampling

In this section, we explain our proposed Triplet Adaptive Negative Sampling (TANS) in detail. We first show the overview of our TANS through the comparison with the conventional smoothing methods of the NS loss for KGE (See §2.3) in §3.1 and after that we explain the details of TANS through its mathematical formulations in §3.2 and §3.3.

### 3.1 Overview

TANS is fundamentally different from SANS, with SANS only taking into account the conditional probability of negative samples and TANS being a loss function that considers the joint probability of the pair of queries and their answers.

Table 1 shows the characteristics of TANS and the conventional smoothing methods of the NS loss for KGE introduced in §2.3. These characteristics are based on the decomposition of  $p_d(x, y)$ , the appearance probability for the triplet (x, y), into that of its answer  $p_d(y|x)$  and query p(x):

$$p_d(x,y) = p_d(y|x)p_d(x)$$
 (10)

In Eq. (10), smoothing both  $p_d(y|x)$  and  $p_d(x)$  is similar to smoothing  $p_d(x, y)$ . However, smoothing  $p_d(x, y)$  does not ensure smoothing both  $p_d(x)$ and  $p_d(y|x)$  considering the case of only one of them being smoothed, and the left one being still sparse. Similarly, smoothing only  $p_d(x)$  or  $p_d(y|x)$ does not ensure  $p_d(x, y)$  being smoothed due to the case where one of them is still sparse. In Table 1, we denote such a case where the method can influence the probability, but no guarantee of the probability be smoothed as  $\triangle$ .

In TANS, we aim to smooth  $p_d(x, y)$  by smoothing both  $p_d(y|x)$  and  $p_d(x)$  based on Eq. (10).

# 3.2 Formulation

Here, we induce TANS from SANS with targeting to smooth  $p_d(x, y)$  by smoothing both  $p_d(y|x)$  and  $p_d(x)$ . First, we assume a simple replacement from  $p_{\theta}(y|x)$  to  $p_{\theta}(x, y)$  in  $\ell_{\text{SANS}}(\theta)$  of Eq. (9):

$$-\frac{1}{|D|} \sum_{(x,y)\in D} \left[ \log(\sigma(s_{\theta}(x,y) + \tau)) \right]$$
293

$$+\sum_{y_i \sim U}^{} p_{\theta}(x, y_i) \log(\sigma(-s_{\theta}(x, y_i) - \tau)) \Big].$$
(11) 29

| Method                           | S | moothing   |                                     | Remarks                                   |   |  |  |  |
|----------------------------------|---|--|-------------------------------------|---|---|--|--|--|
|                                  |   | p(x,y)   | p(y x)                              | p(x)                                      |   |  |  |  |
| Base<br>Subsampling Uniq<br>Freq |   | $ \stackrel{\checkmark}{\bigtriangleup} \\ \stackrel{\checkmark}{\checkmark} $ | $\stackrel{\bigtriangleup}{\times}$ | $ \stackrel{\bigtriangleup}{\checkmark} $ | p(y x) and $p(x)$ are influenced by $p(x, y)$ .<br>p(x, y) is indirectly controlled by $p(x)$ .<br>p(y x) is indirectly controlled by $p(x, y)$ or $p(x)$ . |  |  |  |
| SANS                             |   | $\triangle$  | $\checkmark$                        | ×   | p(x, y) is indirectly controlled by $p(y x)$ .  |  |  |  |
| TANS                             |   | $\checkmark$   | $\checkmark$                        | $\checkmark$                              |   |  |  |  |

Table 1: The characteristics of each smoothing method for the NS loss in KGE (See §2.3 for the details.) and our proposed TANS.  $\checkmark$  and  $\bigtriangleup$  respectively denote the method smooths the probability directly and indirectly.  $\times$  denotes the method does not smooth the probability.

However, using Eq. (11) causes an imbalanced loss between the first and second terms since the sum of  $p_{\theta}(x, y_i)$  on all negative samples is not always 1. Thus, Eq. (11) is impractical as a loss function.

295

296

299

301

303

312

315

As a solution, we focus on the decomposition  $p_{\theta}(x,y) = p_{\theta}(y|x)p_{\theta}(x)$  and the fact that the sum of  $p_{\theta}(y|x)$  of all negative samples is always 1. By using  $p_{\theta}(x)$  to make a balance between the first and second loss term, we can modify Eq. (11) and induce our TANS as follows:

$$\ell_{\text{TANS}}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[ \log(\sigma(s_{\theta}(x,y) + \tau)) + \sum_{i=1}^{\nu} p_{\theta}(x;\gamma) \Big[ \log(\sigma(s_{\theta}(x,y) + \tau)) \Big]$$
(12)

$$+\sum_{y_i \sim U} p_{\theta}(y_i | x; \beta) \log(\sigma(-s_{\theta}(x, y_i) - \tau)) \Big],$$
(12)

308 
$$p_{\theta}(x;\gamma) = \sum_{y_i \in D} p_{\theta}(x,y_i;\gamma),$$
309 
$$p_{\theta}(x,y_i;\gamma) = \frac{\exp\left(\gamma s_{\theta}(x,y_i)\right)}{\exp\left(\gamma s_{\theta}(x,y_i)\right)}$$

$$p_{\theta}(x, y_i; \gamma) = \frac{\exp\left(\gamma s_{\theta}(x, y_i)\right)}{\sum_{(x', y') \in D} \exp\left(\gamma s_{\theta}(x', y')\right)}, \quad (13)$$

where  $\gamma$  is a temperature to smooth the frequency 310 of queries. Since TANS uses a noise distribution de-311 cided by  $p_{\theta}(x; \gamma)$  and  $p_{\theta}(y_i | x; \beta)$ , it does not propagate gradients through probabilities for negative 313 samples, and thus, memory usage is not increased. 314

#### 3.3 **Theoretical Interpretation**

In this subsection, we discuss the difference and similarities among TANS and other smoothing 317 methods for the NS loss in KGE. As shown in 318 Table 1, the subsampling methods, Base and Freq, 319 can smooth triplet frequencies similar to our TANS. To investigate TANS from the view point of subsampling, we reformulate Eq. (12) as follows:

$$\ell_{\text{TANS}}(\theta)$$
 323

322

327

328

331

332

333

334

335

336

337

338

339

340

341

342

343

344

345

347

349

350

351

352

$$+ \sum_{y_i \sim U}^{\nu} B(x, y; \beta, \gamma) \log(\sigma(-s_{\theta}(x, y_i) - \tau)) \Big], \tag{14}$$

$$A(x, y; \gamma) = p_{\theta}(x; \gamma),$$

$$B(x, y; \beta, \gamma) = p_{\theta}(y_i | x; \beta) p_{\theta}(x; \gamma).$$
(15)

Apart from the temperature terms,  $\alpha$ ,  $\beta$ , and  $\gamma$ , we can see that the general formulation of subsampling in Eq. (3) and the above Eq. (14) has the same formulation. Thus, TANS is not merely an extension of SANS but also a novel subsampling method.

Even though their similar characteristic, TANS and subsampling have an essential difference: TANS smooths the frequencies by model-predicted distributions as in Eq. (13), and the subsampling methods smooth them by counting appearance frequencies on the observed data as in Eq. (4), (5), (6), and (7). For instance, TANS can work even when the entity or relations included in the target triplet appear more than once, which is theoretically different from conventional approaches.

Since the superiority of using either model-based or count-based frequencies depends on the model and dataset, we empirically investigate this point through our experiments.

### 4 **Unified Interpretation of SANS and** Subsampling

In the previous section, we understand that our TANS can smooth triplets, queries, and answers partially covered by SANS and subsampling methods. On the other hand, TANS only relies on modelpredicted frequencies to smooth the frequencies.

| Te       | mperati  | ıre      | Induced NS Loss  |  |  |  |  |  |  |  |
|----------|----------|----------|--|--|--|--|--|--|--|--|
| α        | $\beta$  | $\gamma$ |  |  |  |  |  |  |  |  |
| = 0      | = 0      | = 0      | Equivalent to $\ell_{NS}(\theta)$ , the basic NS loss in KGE (Eq. (2))   |  |  |  |  |  |  |  |
| = 0      | = 0      | $\neq 0$ | Currently does not exist   |  |  |  |  |  |  |  |
| = 0      | $\neq 0$ | = 0      | Proportional to $\ell_{\text{SANS}}(\theta)$ , the SANS loss (Eq. (9))   |  |  |  |  |  |  |  |
| = 0      | $\neq 0$ | $\neq 0$ | Equivalent to our $\ell_{\text{TANS}}(\theta)$ , the TANS loss (Eq. (12))  |  |  |  |  |  |  |  |
| $\neq 0$ | = 0      | = 0      | Proportional to $\ell_{NS}(\theta)$ , the basic NS loss in KGE (Eq. (2)) with subsampling in §2.3  |  |  |  |  |  |  |  |
| $\neq 0$ | = 0      | $\neq 0$ | Currently does not exist   |  |  |  |  |  |  |  |
| $\neq 0$ | $\neq 0$ | = 0      | Proportional to $\ell_{\text{SANS}}(\theta)$ , the SANS loss (Eq. (9)) with subsampling in §2.3  |  |  |  |  |  |  |  |
| $\neq 0$ | $\neq 0$ | $\neq 0$ | Equivalent to our $\ell_{\text{UNI}}(\hat{\theta})$ , the unified NS loss in KGE (Eq. (16))<br>and also equivalent to our $\ell_{\text{TANS}}(\hat{\theta})$ , the TANS loss (Eq. (12)) with subsampling in §2.3 |  |  |  |  |  |  |  |

Table 2: The relationship among the loss functions from the viewpoint of the unified NS loss,  $\ell_{\text{UNI}}(\theta)$  in Eq. (16).

Neubig and Dyer (2016) point out the benefits of
combining count-based and model-predicted frequencies in language modeling. This section integrates smoothing methods for the NS loss in KGE
from a unified interpretation.

# 4.1 Formulation

362

363

365

367

369

371

372

375

We formulate the unified loss function by introducing subsampling into our TANS as follows:

$$\ell_{\text{UNI}}(\theta) = -\frac{1}{|D|} \sum_{(x,y)\in D} p_{\theta}(x;\gamma) \Big[ A(x,y;\alpha) \log(\sigma(s_{\theta}(x,y)+\tau)) + \eta \sum_{y_{i}\sim U}^{\nu} B(x,y;\alpha) p_{\theta}(y_{i}|x;\beta) \log(\sigma(-s_{\theta}(x,y_{i})-\tau)) \Big],$$
(16)

where  $\eta$  is a hyperparamter that can be any value to absorb the difference among the three different subsampling methods, Base, Uniq, and Freq.

Here, we can induce the NS losses shown in our paper from Eq. (16) by changing the temperature parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ . Table 2 shows the induced losses from our  $\ell_{\text{UNI}}(\theta)$ . Note that since  $p_{\theta}(x;\gamma)$ only appears in our TANS, canceling  $p_{\theta}(x;\gamma)$  by  $\gamma = 0$  induces an inequivalent but a proportional relationship to the conventional NS loss.

### 4.2 Theoretical Interpretation

376As shown in Table 2, TANS w/ subsampling has<br/>characteristics of all smoothing methods for the NS<br/>loss in KGE introduced in this paper. Therefore,<br/>we can expect higher performance of TANS w/<br/>subsampling than the combination of conventional<br/>methods, the basic NS, SANS, and subsampling.<br/>However, because TANS w/ subsampling uses sub-<br/>sampling in §2.3, we need to choose the one from<br/>Base, Uniq, and Freq for TANS w/ subsampling.<br/>Since this part is out of the scope of our theoret-

ical interpretation, we investigate this part in the experiments.

386

387

388

390

391

392

393

394

395

396

397

398

400

401

402

403

404

405

406

407

408

409

410

411

412

413

414

### **5** Experiments

In this section, we investigate our theoretical interpretation in §3.3 and §4.2 through experiments.

### 5.1 Experimental Settings

**Datasets** We used three common datasets, FB15k-237 (Toutanova and Chen, 2015), WN18RR, and YAGO3-10 (Dettmers et al., 2018) <sup>3</sup>.

Comparison Methods As comparison methods, we used TransE (Bordes et al., 2013), Dist-Mult (Yang et al., 2015), ComplEx (Trouillon et al., 2016), RotatE (Sun et al., 2019), HAKE (Zhang et al., 2020a), and HousE (Li et al., 2022). We followed the original settings of Sun et al. (2019) for TransE, DistMult, ComplEx, and RotatE with their implementation<sup>4</sup>, the original settings of Zhang et al. (2020a) for HAKE with their implementation<sup>5</sup>, and the original settings of Li et al. (2022) for HousE with their implementation<sup>6</sup>. We tuned temperature  $\gamma$  on the validation split for each dataset. Metrics We employed conventional metrics in KGC, i.e., MRR, Hits@1 (H@1), Hits@3 (H@3), and Hits@10 (H@10) and reported the average scores and their standard deviations by three different runs with fixed random seeds.

### 5.2 Results

Since the result tables are large<sup>7</sup>, we discuss them individually, focusing on important information in

<sup>3</sup>Table 3 in Appendix A shows the dataset statistics.

<sup>4</sup>https://github.com/DeepGraphLearning/ KnowledgeGraphEmbedding <sup>5</sup>https://github.com/MIRALab-USTC/ KGE-HAKE

<sup>&</sup>lt;sup>6</sup>https://github.com/rui9812/HousE

<sup>&</sup>lt;sup>7</sup>The full experimental results are listed in Appendix B. The scores are included in Table 5, 6, and 7 of Appendix B.1. The training loss curves and validation MRR curves for each smoothing method are in Figure 6, 7, and 8 of Appendix B.2.



(a) Results on datasets FB15k-237, WN18RR, YAGO3-10 using NS, SANS, TANS, and NS with subsampling.



(b) Results on datasets FB15k-237, WN18RR, YAGO3-10 using SANS, TANS, and those with subsampling.

Figure 3: KGC performance on common KGs (Notations are the same as in Figure 2).

the following subsections. 415

416

417

418

419

420

421

422

423

494

425

426

427

428

429

430

#### 5.2.1 Effectiveness of TANS

Figure 3a shows the MRR scores of each method. From the result, we can understand the effectiveness of considering triplet information in SANS as conducted in TANS. Thus, the result is along with our expectation in §3.3 that TANS can cover the role of subsampling methods. However, as the result of HAKE on WN18RR shows, there is a case that subsampling methods outperform TANS. As discussed in §3.3, using only TANS does not cover all combinations of NS loss and subsampling. Considering this theoretical fact, we further compare TANS with subsampling and the NS loss with subsampling.

5.2.2 Validity of the Unified Interpretation

Figure 3b shows the result for each configuration. 431 We can see performance improvements by using 432 subsampling in both SANS and TANS. Further-433 more, in almost all cases, TANS with subsampling 434 achieve the highest MRR. This observation is along 435 with the theoretical conclusion in §3.3 that TANS 436

with subsampling can cover the characteristic of other NS loss in terms of smoothing. On the other 438 hand, the results of HAKE on YAGO3-10 show the 439 different tendency that SANS with subsampling 440 achieves the best MRR instead of TANS. Because 441 the model prediction estimates the triplet frequencies, TANS is influenced by the selected model. Therefore, carefully choosing the combination of a loss function and model is still effective in im-445 proving KGC performance on the NS loss with 446 subsampling.

437

442

443

444

447

448

### Analysis

We analyze how TANS mitigates the sparsity prob-449 lem in imbalanced KGs commonly caused by low 450 frequent triplets in KGC. By considering that all 451 triplets in KGs appear at most once, we focus on 452 queries. We extracted 0.5% triplets with the highest 453 or lowest frequent queries in training, validation, 454 and test splits as the sparser subsets FB15k-237-455 HL, WN18RR-HL, and YAGO3-10-HL, respec-456

6



Figure 4: KGC performance on filtered sparser KGs, i.e., FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL (Notations are the same as in Figure 2).

tively <sup>8</sup> from original data, for the investigation.

Figure 4 shows MRRs for each model on each sparser dataset. From the result, we can understand that TANS can perform even much better in KGC when KGs get more imbalanced. You can see further detailed results in Table 8, 9, and 10 of Appendix C.3.

# 7 Related Work

Mikolov et al. (2013) initially propose the NS loss of the frequent words to train their word embedding model, word2vec. Trouillon et al. (2016) introduce the NS loss to KGE to speed up training. Melamud et al. (2017) use the NS loss to train the language model. In contextualized pre-trained embeddings, Clark et al. (2020a) indicate that a BERT(Devlin et al., 2019)-like model ELECTRA (Clark et al., 2020b) uses the NS loss to perform better and faster than language models.

Sun et al. (2019) extend the NS loss to SANS loss for KGE and proposed their noise distribution, which is subsampled by a uniformed probability  $p_{\theta}(y_i|x)$ . Kamigaito and Hayashi (2021) point out the sparseness problem of KGs through their theoretical analysis of the NS loss in KGE. Furthermore, Kamigaito and Hayashi (2022a) reveal that subsampling (Mikolov et al., 2013) can alleviate the sparseness problem in the NS for KGE and conclude three assumptions for subsampling, i.e., Base, Freq, and Uniq.

Through our work, we theoretically clarify the position of the previous works on SANS loss and subsampling from the viewpoint of smoothing methods for the NS loss in KGE. Since our work unitedly interprets SANS loss and subsampling, our proposed TANS inherits the advantages of conventional works and can deal with the sparsity problem in the NS loss for KGE.

489

490

491

492

493

494

495

496

497

498

499

500

501

502

503

504

505

506

507

508

509

510

511

512

513

514

515

516

517

518

519

520

521

522

523

### 8 Conclusion

We reveal the relationships between SANS loss and subsampling for the KG completion task through theoretical analysis. We explain that SANS loss and subsampling under three assumptions, Base, Freq, and Uniq have similar roles to mitigate the sparseness problem of queries and answers of KGs by smoothing the frequencies of queries and answers. Furthermore, based on our interpretation, we induce a new loss function, Triplet Adaptive Negative Sampling (TANS), by integrating SANS loss and subsampling. We also introduce a theoretical interpretation that TANS with subsampling can cover all conventional combinations of SANS loss and subsampling.

We verified our interpretation by empirical experiments in three common datasets, FB15k-237, WN18RR, and YAGO3-10, and six popular KGE models, TransE, DistMult, ComplEx, RotatE, HAKE, and HousE. The experimental results show that our TANS loss can outperform subsampling and SANS loss with many models in terms of MRR as expected by our theoretical interpretation. Furthermore, the combinatorial use of TANS and subsampling achieved comparable or better performance than other combinations and showed the validity of our theoretical interpretation that TANS with subsampling can cover all conventional combinations of SANS loss and subsampling in KGE.

457

458

459

<sup>&</sup>lt;sup>8</sup>Note that we show their appearance frequencies of queries and answers in the training data in Figure 5 and detailed statistics in Table 4 of Appendix C.1 and C.2, respectively.

# 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622

623

624

625

626

627

628

629

573

### 524 Limitations

530

531

532

533

538

539

540

541

542

543

545

547

548

551

554

555 556

557

558

559

560

561

562

564

565

566

567

568

570

525 Our experiments are conducted exclusively on pub526 lic datasets, which are relatively well-balanced.
527 Consequently, we anticipate that our TANS will
528 perform better on real-world KGs.

# **Ethics Statement**

We used the publicly available datasets, FB15k-237, WN18RR, and YAGO3-10, to train and evaluate KGE models, and there is no ethical consideration.

# 534 Reproducibility Statement

We used the publicly available code to implement KGE models, TransE, DistMult, ComplEx, RotatE, HAKE, and HousE with the author-provided hyperparameters as described in §5.1. Regarding the temperature parameter  $\gamma$ , we tuned it on the validation split for each dataset and reported the values in Table 5, 6, and 7 of Appendix B. Our code and data are available at https://github.com/[innominated].

### References

- Kian Ahrabian, Aarash Feizi, Yasmin Salehi, William L. Hamilton, and Avishek Joey Bose. 2020. Structure aware negative sampling in knowledge graphs. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 6093–6101, Online. Association for Computational Linguistics.
- Antoine Bordes, Nicolas Usunier, Alberto García-Durán, Jason Weston, and Oksana Yakhnenko.
   2013. Translating embeddings for modeling multirelational data. In Advances in Neural Information Processing Systems 26: 27th Annual Conference on Neural Information Processing Systems 2013, pages 2787–2795.
- Antoine Bordes, Jason Weston, Ronan Collobert, and Yoshua Bengio. 2011. Learning structured embeddings of knowledge bases. In *Proceedings of the AAAI conference on artificial intelligence*, volume 25, pages 301–306.
- Kevin Clark, Minh-Thang Luong, Quoc Le, and Christopher D. Manning. 2020a. Pre-training transformers as energy-based cloze models. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 285–294, Online. Association for Computational Linguistics.
- Kevin Clark, Minh-Thang Luong, Quoc V. Le, and Christopher D. Manning. 2020b. Electra: Pretraining text encoders as discriminators rather than

generators. In International Conference on Learning Representations.

- Tim Dettmers, Pasquale Minervini, Pontus Stenetorp, and Sebastian Riedel. 2018. Convolutional 2d knowledge graph embeddings. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence,* (AAAI-18), pages 1811–1818.
- Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. BERT: Pre-training of deep bidirectional transformers for language understanding. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4171–4186, Minneapolis, Minnesota. Association for Computational Linguistics.
- Yang Gao, Yi-Fan Li, Yu Lin, Hang Gao, and Latifur Khan. 2020. Deep learning on knowledge graph for recommender system: A survey.
- Ziniu Hu, Yichong Xu, Wenhao Yu, Shuohang Wang, Ziyi Yang, Chenguang Zhu, Kai-Wei Chang, and Yizhou Sun. 2022. Empowering language models with knowledge graph reasoning for question answering.
- Hidetaka Kamigaito and Katsuhiko Hayashi. 2021. Unified interpretation of softmax cross-entropy and negative sampling: With case study for knowledge graph embedding. In Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing (Volume 1: Long Papers), pages 5517–5531, Online. Association for Computational Linguistics.
- Hidetaka Kamigaito and Katsuhiko Hayashi. 2022a. Comprehensive analysis of negative sampling in knowledge graph representation learning. In Proceedings of the 39th International Conference on Machine Learning, volume 162 of Proceedings of Machine Learning Research, pages 10661–10675. PMLR.
- Hidetaka Kamigaito and Katsuhiko Hayashi. 2022b. Erratum to: Comprehensive analysis of negative sampling in knowledge graph representation learning. *ResearchGate*.
- Slava Katz. 1987. Estimation of probabilities from sparse data for the language model component of a speech recognizer. *IEEE transactions on acoustics, speech, and signal processing*, 35(3):400–401.
- Rui Li, Jianan Zhao, Chaozhuo Li, Di He, Yiqi Wang, Yuming Liu, Hao Sun, Senzhang Wang, Weiwei Deng, Yanming Shen, Xing Xie, and Qi Zhang. 2022. House: Knowledge graph embedding with householder parameterization.
- Weijie Liu, Peng Zhou, Zhe Zhao, Zhiruo Wang, Qi Ju, Haotang Deng, and Ping Wang. 2019. K-bert: Enabling language representation with knowledge graph.

Oren Melamud, Ido Dagan, and Jacob Goldberger. 2017. A simple language model based on PMI matrix approximations. In *Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing*, pages 1860–1865, Copenhagen, Denmark. Association for Computational Linguistics.

631

640

641

642

646

647

651

653

654

657

666

669

670

671

672

673

674

675

679 680

683

- Tomás Mikolov, Ilya Sutskever, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. Distributed representations of words and phrases and their compositionality. *CoRR*, abs/1310.4546.
- Seungwhan Moon, Pararth Shah, Anuj Kumar, and Rajen Subba. 2019. OpenDialKG: Explainable conversational reasoning with attention-based walks over knowledge graphs. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, pages 845–854, Florence, Italy. Association for Computational Linguistics.
  - Graham Neubig and Chris Dyer. 2016. Generalizing and hybridizing count-based and neural language models. In *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pages 1163–1172, Austin, Texas. Association for Computational Linguistics.
- Justin Reese, Deepak Unni, Tiffany Callahan, Luca Cappelletti, Vida Ravanmehr, Seth Carbon, Kent Shefchek, Benjamin Good, James Balhoff, Tommaso Fontana, Hannah Blau, Nicolas Matentzoglu, Nomi Harris, Monica Munoz-Torres, Melissa Haendel, Peter Robinson, Marcin Joachimiak, and Christopher Mungall. 2020. Kg-covid-19: a framework to produce customized knowledge graphs for covid-19 response. *Patterns*, 2:100155.
- Zhiqing Sun, Zhi-Hong Deng, Jian-Yun Nie, and Jian Tang. 2019. Rotate: Knowledge graph embedding by relational rotation in complex space. In *Proceedings of the 7th International Conference on Learning Representations, ICLR 2019.*
- Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jon Shlens, and Zbigniew Wojna. 2016. Rethinking the inception architecture for computer vision. In *Proceedings of the IEEE conference on computer* vision and pattern recognition, pages 2818–2826.
- Kristina Toutanova and Danqi Chen. 2015. Observed versus latent features for knowledge base and text inference. In *Proceedings of the 3rd Workshop on Continuous Vector Space Models and their Compositionality*, pages 57–66, Beijing, China. Association for Computational Linguistics.
- Théo Trouillon, Johannes Welbl, Sebastian Riedel, Éric Gaussier, and Guillaume Bouchard. 2016. Complex embeddings for simple link prediction. In Proceedings of the 33nd International Conference on Machine Learning, ICML 2016, volume 48 of JMLR Workshop and Conference Proceedings, pages 2071– 2080. JMLR.org.

Bishan Yang, Wen-tau Yih, Xiaodong He, Jianfeng Gao, and Li Deng. 2015. Embedding entities and relations for learning and inference in knowledge bases. In *Proceddings of the 3rd International Conference on Learning Representations, ICLR 2015.* 

685

686

687

688

689

690

691

692

693

694

695

696

697

698

699

700

701

702

704

705

706

707

708

710

711

712

713

714

715

716

717

718

720

721

722

723

724

725

727

728

729

730

- Zhanqiu Zhang, Jianyu Cai, Yongdong Zhang, and Jie Wang. 2020a. Learning hierarchy-aware knowledge graph embeddings for link prediction. In *Proceedings of the Thirty-Fourth AAAI Conference on Artificial Intelligence, (AAAI20)*, pages 3065–3072.
- Zhiyuan Zhang, Xiaoqian Liu, Yi Zhang, Qi Su, Xu Sun, and Bin He. 2020b. Pretrain-KGE: Learning knowledge representation from pretrained language models. In *Findings of the Association for Computational Linguistics: EMNLP 2020*, pages 259–266, Online. Association for Computational Linguistics.

# **A** Dataset Statistics

Table 3 shows the dataset statistics for dataset FB15k-237, WN18RR, and YAGO3-10, introduced in §5.1.

### **B** Full Experimental Results

### **B.1** Results Tables

Table 5, 6, and 7 list all results on FB15k-237, WN18RR, and YAGO3-10, explained in §5.2. In these tables, the bold scores are the best results for each subsampling type (e.g. *None*, *Base*, *Freq*, and *Uniq*.),  $\dagger$  indicates the best scores for each model, *SD* denotes the standard deviation of the three trials, and  $\gamma$  denotes the temperature chosen by development data.

### **B.2** Training Loss and Validation MRR Curve

Figure 6, 7, and 8 show the training loss curves and validation MRR curves for each smoothing method. From these figures, we can understand that the convergence of TANS loss is as well as SANS and NS loss on datasets FB15k-237, WN18RR, and YAGO3-10 for each KGE model. Meanwhile, the time complexity of TANS is the same with SANS and NS loss too.

# **C** Sparse Queries

# C.1 Appearance Frequencies of Queries and Answers

Figure 5 shows the appearance frequencies of queries and answers in the training set of our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6.



Figure 5: Appearance frequencies of queries and answers in the training data of the sparser subsets FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL. Note that the indices are sorted from high frequency to low.

| Dataset    | Split  | Tuple     | Query   | Entity  | Relation |             | Dataset      | Split  | Tuple   | Query   | Entity |
|------------|--------|-----------|---------|---------|----------|-------------|--------------|--------|---------|---------|--------|
|            | Total  | 310,116   | 150,508 | 14,541  | 237      |             |              | Total  | 111,631 | 63,330  | 11,828 |
| ED 151 007 | #Train | 272,115   | 138,694 | 14,505  | 237      |             |              | #Train | 95,244  | 55,923  | 11,600 |
| FB15K-237  | #Valid | 17,535    | 19,750  | 9,809   | 223      |             | FB15K-237-HL | #Valid | 7,571   | 6,918   | 4,933  |
|            | #Test  | 20,466    | 22,379  | 10,348  | 224      |             |              | #Test  | 8,816   | 7,830   | 5,406  |
|            | Total  | 93,003    | 77,479  | 40,943  | 11       |             | WN18RR-HL    | Total  | 14,697  | 14,675  | 12,973 |
| WALLORD    | #Train | 86,835    | 74,587  | 40,559  | 11       |             |              | #Train | 13,758  | 13,785  | 12,275 |
| WN18KK     | #Valid | 3,034     | 5,431   | 5,173   | 11       |             |              | #Valid | 465     | 619     | 613    |
|            | #Test  | 3,134     | 5,565   | 5,323   | 11       |             |              | #Test  | 474     | 623     | 619    |
|            | Total  | 1,089,040 | 372,775 | 123,182 | 37       |             |              | Total  | 366,079 | 182,274 | 95,788 |
| NA 602 10  | #Train | 1,079,040 | 371,077 | 123,143 | 37       |             | VACO2 10 HI  | #Train | 362,728 | 181,196 | 95,432 |
| YAGO3-10   | #Valid | 5,000     | 8,534   | 7,948   | 33       | IAGO5-10-HL | #Valid       | 1,662  | 2,316   | 2,113   |        |
|            | #Test  | 5,000     | 8,531   | 7,937   | 34       |             |              | #Test  | 1,689   | 2,359   | 2,135  |

Table 3: Statistics for each public dataset.

Table 4: Statistics of the filtered sparser datasets.

Relation

155

155

90

89 10

10

9 8

29

29

13

14

### C.2 Data Statistics

Table 4 shows detailed statistics of our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6.

### C.3 Detailed Results

Table 8, 9, and 10 shows the detailed results on our filtered sparser data FB15k-237-HL, WN18RR-HL, and YAGO3-10-HL, expained in §6. Notations are as those described in §B.1.

733 734

731

737

738

739

| FB15k-237   |                              |             |                   |     |                   |     |                   |     |                   |     |        |
|-------------|------------------------------|-------------|-------------------|-----|-------------------|-----|-------------------|-----|-------------------|-----|--------|
| N 11        | Subsampling MRR H@1 H@3 H@10 |             |                   |     |                   |     |                   |     |                   |     |        |
| wodel       | Assumption                   | Loss        | Mean              | SD  | Mean              | SD  | Mean              | SD  | Mean              | SD  | ſγ     |
|             |                              | NS          | 23.9              | 0.2 | 15.8              | 0.1 | 26.1              | 0.3 | 40.0              | 0.2 | -      |
|             | None                         | SANS        | 22.3              | 0.1 | 13.8              | 0.1 | 24.2              | 0.0 | 39.5              | 0.2 | -      |
|             | -                            | TANS        | 32.8              | 0.2 | 23.2              | 0.1 | 36.2              | 0.2 | 52.2              | 0.1 | -2     |
|             |                              | NS          | 27.2              | 0.1 | 19.1              | 0.1 | 29.5              | 0.1 | 43.0              | 0.2 | -      |
|             | Base                         | SANS        | 32.3              | 0.0 | 23.0              | 0.1 | 35.4              | 0.1 | \$1.2             | 0.1 | -      |
| ComplEx     |                              | TANS        | 133.3             | 0.0 | 123.8             | 0.1 | 136.9             | 0.1 | 152.7             | 0.0 | -1     |
|             | _                            | NS<br>CANC  | 25.1              | 0.2 | 17.1              | 0.3 | 27.4              | 0.2 | 41.0              | 0.2 | -      |
|             | Freq                         | SANS        | 32.7              | 0.1 | 23.6              | 0.1 | 36.0              | 0.1 | 51.2              | 0.1 | -      |
|             |                              | TANS        | 33.3              | 0.0 | 23.8              | 0.0 | 36.8              | 0.1 | 52.1              | 0.2 | -0.5   |
|             | •• •                         | NS          | 22.8              | 0.4 | 14.7              | 0.5 | 24.7              | 0.4 | 39.0              | 0.1 | -      |
|             | Uniq                         | SANS        | 32.6              | 0.0 | 23.5              | 0.1 | 35.8              | 0.1 | 51.2              | 0.1 | -      |
|             |                              | IANS        | 22.2              | 0.1 | 15.6              | 0.1 | 25.7              | 0.1 | 28.1              | 0.1 | -0.5   |
|             | None                         | SANS        | 23.3              | 0.1 | 14.0              | 0.1 | 23.7              | 0.1 | 30.4              | 0.1 | -      |
|             | None                         | TANS        | 31.0              | 0.1 | 21.7              | 0.2 | 34.0              | 0.1 | 49.6              | 0.0 | -1     |
|             |                              | NS          | 25.4              | 0.1 | 17.9              | 0.1 | 27.6              | 0.1 | 40.4              | 0.1 | -      |
|             | Base                         | SANS        | 30.8              | 0.1 | 21.9              | 0.1 | 33.6              | 0.1 | 48.4              | 0.1 | -      |
| DistMult    | Duse                         | TANS        | <sup>†</sup> 31.5 | 0.1 | <sup>†</sup> 22.4 | 0.1 | <sup>†</sup> 34.6 | 0.1 | † <b>49</b> 7     | 0.0 | -0.5   |
| Distivituit |                              | NS          | 24.0              | 0.1 | 16.7              | 0.2 | 25.9              | 0.1 | 38.4              | 0.1 | -      |
|             | Freq                         | SANS        | 29.9              | 0.0 | 21.2              | 0.1 | 32.8              | 0.0 | 47.5              | 0.1 | -      |
|             | Ticq                         | TANS        | 30.7              | 0.0 | 21.6              | 0.0 | 34.0              | 0.0 | 49.0              | 0.0 | -1     |
|             | -                            | NS          | 21.0              | 0.1 | 13.5              | 0.2 | 22.8              | 0.2 | 36.3              | 0.2 | -      |
|             | Uniq                         | SANS        | 29.2              | 0.0 | 20.5              | 0.1 | 31.9              | 0.0 | 46.7              | 0.0 | -      |
|             | 1                            | TANS        | 30.7              | 0.1 | 21.5              | 0.1 | 33.8              | 0.1 | 49.3              | 0.1 | -2     |
|             |                              | NS          | 30.4              | 0.0 | 21.3              | 0.1 | 33.4              | 0.1 | 48.5              | 0.0 | -      |
|             | None                         | SANS        | 33.0              | 0.1 | 22.9              | 0.1 | 37.2              | 0.1 | <sup>†</sup> 53.0 | 0.1 | -      |
|             |                              | TANS        | 33.6              | 0.0 | 23.9              | 0.0 | 37.3              | 0.0 | <sup>†</sup> 53.0 | 0.1 | -0.5   |
|             |                              | NS          | 29.4              | 0.1 | 20.0              | 0.1 | 32.8              | 0.0 | 48.1              | 0.0 | -      |
|             | Base                         | SANS        | 33.0              | 0.1 | 23.1              | 0.1 | 36.8              | 0.1 | 52.7              | 0.1 | -      |
| TransE      |                              | TANS        | 33.0              | 0.0 | 23.1              | 0.0 | 36.8              | 0.1 | 52.7              | 0.1 | -0.1   |
|             |                              | NS          | 29.3              | 0.1 | 20.0              | 0.1 | 32.8              | 0.1 | 47.8              | 0.1 | -      |
|             | Freq                         | SANS        | 33.5              | 0.0 | 23.9              | 0.1 | 37.2              | 0.1 | 52.8              | 0.1 | -      |
|             |                              | TANS        | 33.5              | 0.1 | 23.9              | 0.1 | 37.2              | 0.0 | 52.8              | 0.1 | -0.1   |
|             |                              | NS          | 30.1              | 0.1 | 21.0              | 0.1 | 33.6              | 0.0 | 48.0              | 0.0 | -      |
|             | Uniq                         | SANS        | 33.5              | 0.0 | 23.9              | 0.0 | 37.3              | 0.2 | 52.7              | 0.1 | -      |
|             |                              | TANS        | <sup>†</sup> 34.0 | 0.1 | <sup>†</sup> 24.5 | 0.1 | <sup>†</sup> 37.7 | 0.1 | <sup>†</sup> 53.0 | 0.1 | 0.5    |
|             |                              | NS          | 30.3              | 0.0 | 21.4              | 0.1 | 33.2              | 0.1 | 48.4              | 0.1 | -      |
|             | None                         | SANS        | 32.9              | 0.1 | 22.8              | 0.1 | 36.8              | 0.0 | 53.1              | 0.2 | -      |
|             |                              | TANS        | 34.1              | 0.1 | 24.6              | 0.1 | 37.7              | 0.1 | ⁺53.3             | 0.1 | -0.5   |
|             |                              | NS          | 29.5              | 0.0 | 20.3              | 0.0 | 32.7              | 0.1 | 47.9              | 0.0 | -      |
| D ( )F      | Base                         | SANS        | 33.6              | 0.1 | 23.9              | 0.1 | 37.3              | 0.1 | 53.1              | 0.0 | -      |
| RotatE      |                              | TANS        | 33.8              | 0.0 | 24.2              | 0.0 | 37.4              | 0.0 | 53.0              | 0.1 | -0.5   |
|             | г                            | INS<br>CANC | 29.4              | 0.1 | 20.2              | 0.1 | 32.6              | 0.1 | 47.6              | 0.1 | -      |
|             | Freq                         | TANS        | 34.0              | 0.1 | 24.0              | 0.0 | 37.7              | 0.0 | 53.0              | 0.0 | - 0.01 |
|             |                              | NS          | 30.1              | 0.0 | 24.0              | 0.0 | 33.3              | 0.0 | 47.7              | 0.1 | -0.01  |
|             | Unia                         | SANS        | 33.9              | 0.0 | 24.4              | 0.1 | 37.6              | 0.1 | 52.9              | 0.1 |        |
|             | Uniq                         | TANS        | 134.2             | 0.0 | 124.7             | 0.1 | 137.8             | 0.0 | 53.1              | 0.1 | 0.5    |
|             |                              | NS          | 30.8              | 0.0 | 21.8              | 0.1 | 33.8              | 0.0 | 48.6              | 0.1 | -      |
|             | None                         | SANS        | 32.8              | 0.2 | 22.7              | 0.3 | 36.9              | 0.1 | 52.8              | 0.1 | -      |
|             | None                         | TANS        | 34.4              | 0.1 | 24.9              | 0.1 | 37.9              | 0.2 | 53.6              | 0.0 | -0.5   |
|             |                              | NS          | 30.4              | 0.1 | 21.6              | 0.1 | 33.3              | 0.1 | 48.2              | 0.0 | -      |
|             | Base                         | SANS        | 34.1              | 0.1 | 24.4              | 0.1 | 37.9              | 0.1 | 53.6              | 0.2 | -      |
| HAKE        |                              | TANS        | 34.1              | 0.0 | 24.4              | 0.0 | 37.9              | 0.0 | 53.7              | 0.0 | -0.05  |
|             |                              | NS          | 30.2              | 0.1 | 21.5              | 0.0 | 33.1              | 0.0 | 47.7              | 0.1 | -      |
|             | Freq                         | SANS        | 34.7              | 0.0 | 25.2              | 0.1 | 38.2              | 0.0 | 53.8              | 0.1 | -      |
|             |                              | TANS        | 34.6              | 0.0 | 25.0              | 0.1 | 38.2              | 0.2 | 53.7              | 0.1 | 0.05   |
|             |                              | NS          | 30.7              | 0.1 | 22.2              | 0.1 | 33.5              | 0.1 | 48.0              | 0.1 | -      |
|             | Uniq                         | SANS        | 34.7              | 0.1 | 25.1              | 0.1 | 38.3              | 0.1 | 53.9              | 0.1 | -      |
|             |                              | TANS        | <sup>†</sup> 34.9 | 0.0 | <sup>†</sup> 25.4 | 0.0 | <sup>†</sup> 38.6 | 0.1 | <sup>†</sup> 54.0 | 0.1 | 0.5    |
|             |                              | NS          | 29.1              | 0.1 | 20.6              | 0.1 | 31.6              | 0.1 | 46.3              | 0.1 | -      |
|             | None                         | SANS        | 34.7              | 0.2 | 24.8              | 0.2 | 38.5              | 0.3 | 54.4              | 0.2 | -      |
|             |                              | TANS        | 35.6              | 0.1 | 26.1              | 0.1 | 39.4              | 0.1 | 54.5              | 0.1 | -1     |
|             |                              | NS          | 28.1              | 0.1 | 19.6              | 0.1 | 30.9              | 0.2 | 45.1              | 0.2 | -      |
|             | Base                         | SANS        | 35.2              | 0.2 | 25.6              | 0.2 | 39.0              | 0.2 | 54.4              | 0.3 | -      |
| HousE       |                              | TANS        | 35.6              | 0.1 | 26.1              | 0.1 | 39.4              | 0.2 | 54.5              | 0.1 | -0.5   |
|             |                              | NS          | 27.9              | 0.1 | 19.2              | 0.1 | 30.7              | 0.2 | 45.2              | 0.1 | -      |
|             | Freq                         | SANS        | 35.9              | 0.2 | 26.4              | 0.2 | 39.5              | 0.2 | 54.7              | 0.1 | -      |
|             |                              | IANS        | 35.8              | 0.2 | 20.4              | 0.2 | 39.0              | 0.2 | 54.7              | 0.1 | -0.01  |
|             | TT?                          | 0.1270      | 26.8              | 0.1 | 20.2              | 0.2 | 20.0              | 0.1 | 43./              | 0.0 | -      |
|             | Uniq                         | SANS        | 36.1              | 0.1 | 26.7              | 0.2 | 39.8              | 0.1 | * 54.8            | 0.2 | -      |
|             |                              | TANS        | 36.2              | 0.1 | 26.7              | 0.2 | 39.9              | 0.1 | 54.8              | 0.1 | 0.1    |

Table 5: Results on FB15k-237.

| WN18RR   |            |              |                   |     |                   |     |                   |     |                   |     |       |
|----------|------------|--------------|-------------------|-----|-------------------|-----|-------------------|-----|-------------------|-----|-------|
| Madal    | Subsam     | npling       | MR                | R   | H@                | 21  | H@                | 3   | H@                | 10  |       |
| Model    | Assumption | Loss         | Mean              | SD  | Mean              | SD  | Mean              | SD  | Mean              | SD  | .1    |
|          |            | NS           | 44.5              | 0.1 | 38.1              | 0.2 | 48.3              | 0.2 | 55.5              | 0.1 | -     |
|          | None       | SANS         | 45.0              | 0.1 | 41.0              | 0.1 | 46.5              | 0.3 | 53.3              | 0.3 | -     |
|          |            | NS           | 47.5              | 0.0 | 38.0              | 0.0 | 49.1              | 0.1 | 55.7              | 0.1 | -2    |
|          | Pasa       | SANS         | 46.9              | 0.1 | 42.7              | 0.1 | 48.0              | 0.2 | 55.5              | 0.1 |       |
| ComplEx  | Base       | TANS         | 47.7              | 0.2 | 43.6              | 0.1 | 49.3              | 0.2 | 55.9              | 0.3 | -2    |
| Compilia |            | NS           | 45.1              | 0.1 | 38.9              | 0.1 | 48.8              | 0.2 | 56.0              | 0.2 | -     |
|          | Freq       | SANS         | 47.4              | 0.1 | 43.2              | 0.1 | 49.2              | 0.2 | 56.0              | 0.2 | -     |
|          | 1          | TANS         | 48.0              | 0.1 | 43.9              | 0.1 | <sup>†</sup> 49.7 | 0.1 | 56.1              | 0.1 | -2    |
|          |            | NS           | 45.0              | 0.1 | 38.7              | 0.1 | 48.8              | 0.1 | 56.0              | 0.3 | -     |
|          | Uniq       | SANS         | 47.5              | 0.1 | 43.3              | 0.1 | 49.1              | 0.2 | 56.2              | 0.2 | -     |
|          |            | TANS         | † <b>48.3</b>     | 0.1 | <sup>†</sup> 44.4 | 0.2 | 49.6              | 0.1 | <sup>†</sup> 56.3 | 0.2 | -1    |
|          |            | NS           | 38.5              | 0.2 | 30.6              | 0.3 | 42.9              | 0.2 | 52.5              | 0.1 | -     |
|          | None       | SANS         | 42.4              | 0.0 | 38.2              | 0.1 | 43.7              | 0.0 | 51.0              | 0.2 | -     |
|          |            | TANS         | 44.2              | 0.1 | 40.1              | 0.1 | 45.3              | 0.1 | 53.2              | 0.2 | -2    |
|          | _          | NS           | 39.3              | 0.2 | 31.9              | 0.2 | 43.3              | 0.1 | 53.0              | 0.2 | -     |
| S        | Base       | TANE         | 43.9              | 0.1 | 39.4              | 0.1 | 45.2              | 0.1 | 53.5              | 0.2 | -     |
| DistMult |            | NS           | 39.0              | 0.0 | 31.2              | 0.2 | 43.7              | 0.1 | 52.9              | 0.1 | -2    |
|          | Freq       | SANS         | 44.5              | 0.1 | 40.0              | 0.1 | 46.0              | 0.1 | 54.2              | 0.2 | -     |
|          | Treq       | TANS         | 44.7              | 0.1 | 40.5              | 0.2 | 45.8              | 0.0 | 54.0              | 0.2 | -2    |
|          |            | NS           | 38.8              | 0.2 | 30.8              | 0.2 | 43.1              | 0.1 | 53.0              | 0.2 | -     |
|          | Uniq       | SANS         | 44.7              | 0.1 | 40.1              | 0.1 | <sup>†</sup> 46.2 | 0.3 | 54.3              | 0.0 | -     |
|          | -          | TANS         | <sup>†</sup> 45.0 | 0.1 | <sup>†</sup> 40.7 | 0.1 | 46.1              | 0.2 | <sup>†</sup> 54.5 | 0.2 | -0.5  |
|          |            | NS           | 21.1              | 0.0 | 2.1               | 0.1 | 36.5              | 0.2 | 50.4              | 0.2 | -     |
|          | None       | SANS         | 22.5              | 0.1 | 1.7               | 0.1 | 40.2              | 0.1 | 52.5              | 0.2 | -     |
|          |            | TANS         | 22.7              | 0.0 | 2.5               | 0.0 | 39.5              | 0.2 | 53.4              | 0.1 | 0.5   |
|          |            | NS           | 20.3              | 0.1 | 1.6               | 0.1 | 35.1              | 0.2 | 49.9              | 0.2 | -     |
|          | Base       | SANS         | 22.3              | 0.0 | 1.3               | 0.1 | 40.2              | 0.1 | 52.9              | 0.1 | -     |
| TransE   |            | TANS         | 22.4              | 0.1 | 1.4               | 0.1 | 40.1              | 0.1 | 53.0              | 0.1 | 0.1   |
|          | _          | NS<br>CANC   | 21.0              | 0.1 | 1.8               | 0.1 | 36.4              | 0.2 | 51.0              | 0.2 | -     |
|          | Freq       | SANS         | 23.0              | 0.0 | 1.9               | 0.1 | 40.9              | 0.2 | 53.0              | 0.0 | -     |
|          |            | IANS         | 23.1              | 0.0 | 2.1               | 0.0 | 41.0              | 0.1 | 53.8              | 0.0 | 0.1   |
|          | TTala      | EANS<br>CANS | 21.5              | 0.1 | 2.2               | 0.0 | 37.2              | 0.1 | 52.6              | 0.2 | -     |
|          | Uniq       | TANS         | 123.2             | 0.0 | 13.0              | 0.1 | 40.2              | 0.2 | †54.4             | 0.1 | 0.5   |
|          |            | NS           | 47.0              | 0.1 | 42.5              | 0.0 | 48.6              | 0.2 | 55.8              | 0.1 | -     |
|          | None       | SANS         | 47.2              | 0.1 | 42.6              | 0.1 | 49.1              | 0.1 | 56.7              | 0.0 | -     |
|          | rtone      | TANS         | 47.3              | 0.1 | 42.6              | 0.1 | 49.1              | 0.1 | 56.7              | 0.1 | -0.01 |
|          |            | NS           | 47.0              | 0.0 | 42.2              | 0.1 | 48.7              | 0.1 | 56.3              | 0.1 | -     |
|          | Base       | SANS         | 47.5              | 0.1 | 42.7              | 0.2 | 49.3              | 0.1 | 57.2              | 0.1 | -     |
| RotatE   |            | TANS         | 47.5              | 0.1 | 42.7              | 0.2 | 49.3              | 0.1 | 57.1              | 0.1 | 0.01  |
|          |            | NS           | 47.1              | 0.1 | 42.3              | 0.1 | 48.7              | 0.1 | 56.4              | 0.1 | -     |
|          | Freq       | SANS         | 47.7              | 0.1 | <sup>†</sup> 42.9 | 0.2 | 49.6              | 0.0 | 57.4              | 0.1 | -     |
|          |            | TANS         | 47.7              | 0.1 | 42.8              | 0.2 | 49.7              | 0.1 | 57.4              | 0.1 | 0.1   |
|          |            | NS           | 47.2              | 0.2 | 42.7              | 0.2 | 48.7              | 0.1 | 56.3              | 0.1 | -     |
|          | Uniq       | SANS         | 47.7              | 0.1 | 42.9              | 0.1 | 49.6              | 0.1 | 57.2              | 0.1 | -     |
|          |            | TANS         | <sup>†</sup> 47.8 | 0.2 | 42.8              | 0.3 | <sup>†</sup> 49.8 | 0.1 | *57.6             | 0.1 | 0.5   |
|          |            | NS           | 48.8              | 0.1 | 44.5              | 0.1 | 50.5              | 0.2 | 57.3              | 0.1 | -     |
|          | None       | SANS         | 48.9              | 0.0 | 44.5              | 0.2 | 50.6              | 0.3 | 57.7              | 0.1 | -     |
|          |            | IANS         | 40.9              | 0.0 | 44.4              | 0.1 | 51.1              | 0.5 | 57.0              | 0.1 | 0.01  |
|          | Bace       | SANS         | 49.5              | 0.0 | 45.0              | 0.1 | 51.2              | 0.2 | 58.2              | 0.2 |       |
| HAKE     | Dase       | TANS         | 49.5              | 0.1 | 45.0              | 0.2 | 51.2              | 0.3 | 58.4              | 0.2 | 0.1   |
| mate     |            | NS           | 49.3              | 0.1 | 44.8              | 0.1 | 51.3              | 0.2 | 58.0              | 0.2 | -     |
|          | Freq       | SANS         | 49.7              | 0.1 | 45.2              | 0.2 | 51.5              | 0.1 | 58.4              | 0.2 | -     |
|          |            | TANS         | 49.7              | 0.0 | 45.2              | 0.2 | 51.6              | 0.3 | 58.4              | 0.2 | -0.01 |
|          |            | NS           | 49.4              | 0.2 | 44.9              | 0.2 | 51.3              | 0.2 | 57.8              | 0.2 | -     |
|          | Uniq       | SANS         | † <b>49.9</b>     | 0.0 | 45.3              | 0.1 | <sup>†</sup> 51.8 | 0.2 | <sup>†</sup> 58.6 | 0.2 | -     |
|          |            | TANS         | † <b>49.9</b>     | 0.1 | <sup>†</sup> 45.4 | 0.1 | <sup>†</sup> 51.8 | 0.2 | 58.5              | 0.2 | 0.05  |
|          |            | NS           | 47.4              | 0.1 | 41.7              | 0.1 | 50.2              | 0.1 | 57.3              | 0.1 | -     |
|          | None       | SANS         | 49.7              | 0.1 | 44.8              | 0.2 | 51.5              | 0.1 | 59.5              | 0.1 | -     |
|          |            | TANS         | 50.2              | 0.1 | 45.3              | 0.1 | 52.0              | 0.1 | 60.0              | 0.1 | -0.5  |
|          | _          | NS           | 48.1              | 0.1 | 42.4              | 0.1 | 50.9              | 0.1 | 58.5              | 0.2 | -     |
|          | Base       | SANS         | 51.2              | 0.1 | 40.7              | 0.1 | 53.0              | 0.2 | 60.3              | 0.1 | -     |
| HousE    |            | NS           | 48 1              | 0.1 | 40.7              | 0.2 | 50.0              | 0.0 | 58.5              | 0.1 | 0.05  |
|          | Freq       | SANG         | †51 A             | 0.1 | 146.8             | 0.1 | 152.2             | 0.2 | †60 5             | 0.1 |       |
|          | inq        | TANC         | 51.9              | 0.1 | 40.0<br>A6 7      | 0.1 | 52.1              | 0.5 | †60.5             | 0.1 | 0.05  |
|          |            | NS           |                   | 0.2 | 40.7              | 0.2 | 50.8              | 0.5 | 58.1              | 0.1 | 0.05  |
|          | Unio       | SANG         | 51 2              | 0.1 | †46 S             | 0.1 | 52.7              | 0.1 | 60.1              | 0.1 |       |
|          | omq        | TANS         | 51.1              | 0.3 | 46.7              | 0.5 | 52.7              | 0.1 | 60.0              | 0.1 | -0.1  |
|          |            |              |                   |     |                   |     |                   |     |                   |     |       |

Table 6: Results on WN18RR.

|        |            |      |                   | YA  | GO3-10            |     |                   |     |                   |     |          |
|--------|------------|------|-------------------|-----|-------------------|-----|-------------------|-----|-------------------|-----|----------|
|        | Subsamp    | ling | MR                | R   | H@                | 1   | H@                | 3   | H@                | 10  |          |
| Model  | Assumption | Loss | Mean              | SD  | Mean              | SD  | Mean              | SD  | Mean              | SD  | $\gamma$ |
|        |            | NS   | 43.5              | 0.1 | 32.8              | 0.2 | 49.1              | 0.2 | 63.7              | 0.3 | -        |
|        | None       | SANS | 49.6              | 0.2 | 39.9              | 0.1 | 55.3              | 0.3 | 67.3              | 0.2 | -        |
|        |            | TANS | 49.6              | 0.2 | 40.0              | 0.2 | 55.4              | 0.5 | 67.2              | 0.3 | -0.05    |
|        |            | NS   | 44.8              | 0.1 | 34.5              | 0.3 | 50.0              | 0.2 | 64.7              | 0.2 | -        |
|        | Base       | SANS | 49.6              | 0.3 | 40.1              | 0.3 | 55.2              | 0.4 | 67.4              | 0.3 | -        |
| RotatE |            | TANS | 49.5              | 0.3 | 40.1              | 0.3 | 55.0              | 0.5 | 67.3              | 0.3 | -0.05    |
|        |            | NS   | 44.8              | 0.2 | 34.5              | 0.3 | 50.0              | 0.1 | 64.7              | 0.2 | -        |
|        | Freq       | SANS | 49.9              | 0.2 | 40.5              | 0.3 | 55.5              | 0.5 | 67.4              | 0.3 | -        |
|        |            | TANS | 49.9              | 0.2 | 40.5              | 0.3 | 55.5              | 0.5 | 67.4              | 0.2 | 0.01     |
|        |            | NS   | 44.4              | 0.2 | 34.0              | 0.3 | 49.8              | 0.2 | 64.3              | 0.2 | -        |
|        | Uniq       | SANS | 50.0              | 0.3 | 40.6              | 0.2 | 55.6              | 0.3 | 67.5              | 0.2 | -        |
|        | _          | TANS | <sup>†</sup> 50.1 | 0.2 | † <b>40.7</b>     | 0.1 | <sup>†</sup> 55.7 | 0.3 | <sup>†</sup> 67.6 | 0.3 | 0.05     |
|        |            | NS   | 47.4              | 0.3 | 36.6              | 0.5 | 53.9              | 0.1 | 67.0              | 0.1 | -        |
|        | None       | SANS | 53.5              | 0.2 | 44.6              | 0.3 | 59.1              | 0.4 | 69.0              | 0.2 | -        |
|        |            | TANS | 53.7              | 0.1 | 45.3              | 0.3 | 59.0              | 0.1 | 68.8              | 0.1 | 0.05     |
|        |            | NS   | 48.8              | 0.3 | 38.4              | 0.4 | 55.0              | 0.2 | 68.1              | 0.3 | -        |
|        | Base       | SANS | 54.6              | 0.2 | 46.2              | 0.3 | 59.9              | 0.2 | 69.6              | 0.2 | -        |
| HAKE   |            | TANS | 54.5              | 0.2 | 45.9              | 0.3 | 59.9              | 0.2 | 69.9              | 0.1 | -0.1     |
|        |            | NS   | 49.3              | 0.2 | 39.1              | 0.3 | 55.4              | 0.1 | 68.1              | 0.2 | -        |
|        | Freq       | SANS | 54.6              | 0.4 | 46.0              | 0.7 | 60.2              | 0.1 | 69.6              | 0.3 | -        |
|        |            | TANS | 54.8              | 0.2 | 46.4              | 0.3 | 60.1              | 0.1 | 69.6              | 0.3 | 0.05     |
|        |            | NS   | 45.2              | 0.1 | 34.3              | 0.1 | 51.1              | 0.1 | 65.8              | 0.3 | -        |
|        | Uniq       | SANS | <sup>†</sup> 55.2 | 0.3 | <sup>†</sup> 46.8 | 0.5 | <sup>†</sup> 60.5 | 0.2 | † <b>70.0</b>     | 0.3 | -        |
|        |            | TANS | 55.1              | 0.2 | † <b>46.8</b>     | 0.3 | 60.3              | 0.1 | 69.9              | 0.2 | -0.1     |
|        |            | NS   | 29.2              | 0.0 | 18.3              | 0.1 | 33.6              | 0.2 | 50.1              | 0.2 | -        |
|        | None       | SANS | 54.8              | 1.3 | 46.8              | 1.3 | 59.7              | 1.2 | 68.9              | 1.2 | -        |
|        |            | TANS | 54.8              | 1.2 | 46.9              | 1.2 | 59.6              | 1.2 | 68.8              | 1.1 | 0.01     |
|        |            | NS   | 29.6              | 0.1 | 19.8              | 0.1 | 33.6              | 0.2 | 48.9              | 0.1 | -        |
|        | Base       | SANS | 56.7              | 0.1 | 48.6              | 0.2 | 61.7              | 0.2 | 71.3              | 0.1 | -        |
| HousE  |            | TANS | 57.0              | 0.2 | 49.0              | 0.4 | 61.9              | 0.3 | <sup>†</sup> 71.5 | 0.2 | -0.1     |
|        |            | NS   | 27.3              | 0.8 | 17.5              | 0.9 | 31.0              | 0.8 | 46.6              | 0.8 | -        |
|        | Freq       | SANS | 57.0              | 0.1 | 49.0              | 0.2 | 62.0              | 0.1 | 71.4              | 0.1 | -        |
|        | inq        | TANS | 57.2              | 0.1 | 49.3              | 0.1 | <sup>†</sup> 62.3 | 0.1 | 71.4              | 0.1 | -0.1     |
|        |            | NS   | 28.1              | 0.2 | 18.2              | 0.4 | 31.8              | 0.1 | 47.6              | 0.0 | -        |
|        | Unia       | SANS | 57.2              | 0.1 | 49.3              | 0.2 | 62.0              | 0.0 | 71.4              | 0.2 | _        |
|        | Uniq       | TANS | <sup>†</sup> 57.3 | 0.2 | <sup>†</sup> 49.5 | 0.3 | 62.2              | 0.1 | <sup>†</sup> 71.5 | 0.1 | -0.05    |

Table 7: Results on YAGO3-10.

|        |            | FB15 | WN18RR-HL         |     |                   |     |          |        |            |      |                   |     |                   |     |          |
|--------|------------|------|-------------------|-----|-------------------|-----|----------|--------|------------|------|-------------------|-----|-------------------|-----|----------|
|        | Subsamp    | ling | MR                | R   | H@                | 1   |          |        | Subsamp    | ling | MR                | R   | H@                | 1   |          |
| Model  | Assumption | Loss | Mean              | SD  | Mean              | SD  | $\gamma$ | Model  | Assumption | Loss | Mean              | SD  | Mean              | SD  | $\gamma$ |
|        |            | NS   | 38.1              | 0.3 | 28.4              | 0.5 | -        |        |            | NS   | 10.8              | 0.1 | 8.7               | 0.2 | -        |
|        | None       | SANS | 35.2              | 0.2 | 24.5              | 0.3 | -        |        | None       | SANS | 10.3              | 0.1 | 7.8               | 0.1 | -        |
|        |            | TANS | 41.1              | 0.1 | 33.0              | 0.1 | -1       |        |            | TANS | 13.9              | 0.2 | <sup>†</sup> 12.1 | 0.2 | -2       |
|        |            | NS   | 40.5              | 0.1 | 31.8              | 0.2 | -        |        |            | NS   | 12.1              | 0.2 | 9.5               | 0.3 | -        |
|        | Base       | SANS | 38.4              | 0.2 | 28.9              | 0.2 | -        |        | Base       | SANS | 11.1              | 0.1 | 9.1               | 0.1 | -        |
|        |            | TANS | 41.8              | 0.1 | 33.6              | 0.2 | -1       |        |            | TANS | 13.7              | 0.1 | 11.7              | 0.3 | -2       |
| HAKE   |            | NS   | 41.1              | 0.1 | 32.8              | 0.1 | -        | HAKE   |            | NS   | 12.4              | 0.1 | 10.4              | 0.1 | -        |
|        | Freq       | SANS | 40.2              | 0.0 | 31.5              | 0.1 | -        |        | Freq       | SANS | 11.9              | 0.2 | 9.5               | 0.2 | -        |
|        | 1          | TANS | <sup>†</sup> 42.0 | 0.1 | <sup>†</sup> 33.7 | 0.1 | -1       |        | . 1        | TANS | <sup>†</sup> 14.2 | 0.5 | 11.9              | 0.4 | -2       |
|        |            | NS   | 41.5              | 0.1 | 33.2              | 0.1 | -        |        |            | NS   | 13.3              | 0.3 | 11.3              | 0.3 | -        |
|        | Uniq       | SANS | 41.1              | 0.0 | 32.8              | 0.0 | -        |        | Uniq       | SANS | 11.9              | 0.2 | 9.7               | 0.2 | -        |
|        |            | TANS | 41.9              | 0.2 | 33.5              | 0.2 | -0.1     |        |            | TANS | 14.1              | 0.2 | 11.7              | 0.2 | -2       |
|        |            | NS   | 40.0              | 0.1 | 30.8              | 0.1 | -        |        |            | NS   | 14.2              | 0.2 | 11.8              | 0.3 | -        |
|        | None       | SANS | 36.3              | 0.1 | 25.3              | 0.2 | -        |        | None       | SANS | 13.9              | 0.3 | 11.7              | 0.3 | -        |
|        |            | TANS | 41.5              | 0.0 | 33.1              | 0.1 | -1       |        |            | TANS | 14.4              | 0.1 | 11.8              | 0.2 | -2       |
|        |            | NS   | 41.8              | 0.1 | 33.6              | 0.1 | -        | RotatE |            | NS   | 13.9              | 0.2 | 11.5              | 0.2 | -        |
|        | Base       | SANS | 40.7              | 0.1 | 31.7              | 0.2 | -        |        | Base       | SANS | 14.1              | 0.3 | 11.7              | 0.3 | -        |
|        |            | TANS | 42.0              | 0.1 | 33.8              | 0.1 | -0.5     |        |            | TANS | 14.5              | 0.1 | 11.7              | 0.1 | -2       |
| RotatE | Freq       | NS   | 41.3              | 0.1 | 33.2              | 0.1 | -        |        |            | NS   | 14.4              | 0.1 | 12.0              | 0.1 | -        |
|        |            | SANS | 42.0              | 0.2 | 33.6              | 0.3 | -        |        | Freq       | SANS | 14.3              | 0.4 | 12.0              | 0.3 | -        |
|        | -          | TANS | † <b>42.3</b>     | 0.0 | <sup>†</sup> 34.1 | 0.1 | -0.5     |        |            | TANS | <sup>†</sup> 15.1 | 0.1 | 12.2              | 0.1 | -2       |
|        |            | NS   | 41.7              | 0.1 | 33.7              | 0.2 | -        |        |            | NS   | 14.4              | 0.2 | 12.2              | 0.1 | -        |
|        | Uniq       | SANS | 42.2              | 0.1 | 33.8              | 0.2 | -        |        | Uniq       | SANS | 14.2              | 0.2 | 11.9              | 0.2 | -        |
|        |            | TANS | 42.1              | 0.1 | 33.8              | 0.2 | -0.05    |        |            | TANS | <sup>†</sup> 15.1 | 0.2 | <sup>†</sup> 12.3 | 0.3 | -2       |
|        |            | NS   | 39.1              | 0.2 | 29.8              | 0.2 | -        |        |            | NS   | 10.7              | 1.8 | 8.4               | 1.4 | -        |
|        | None       | SANS | 37.0              | 0.2 | 26.2              | 0.4 | -        |        | None       | SANS | 11.7              | 1.1 | 9.5               | 0.9 | -        |
|        |            | TANS | 42.3              | 0.1 | 34.1              | 0.2 | -2       |        |            | TANS | 13.4              | 0.4 | 11.0              | 0.4 | -2       |
|        |            | NS   | 40.3              | 0.1 | 31.3              | 0.2 | -        |        |            | NS   | 9.9               | 0.4 | 8.4               | 0.4 | -        |
|        | Base       | SANS | 40.5              | 0.4 | 31.3              | 0.4 | -        |        | Base       | SANS | 11.5              | 0.2 | 9.5               | 0.2 | -        |
|        |            | TANS | 42.4              | 0.2 | 34.2              | 0.3 | -2       |        |            | TANS | 13.4              | 0.2 | 11.3              | 0.3 | -2       |
| HousE  |            | NS   | 39.8              | 0.3 | 31.0              | 0.3 | -        | HousE  |            | NS   | †1 <b>3.</b> 9    | 0.1 | 11.8              | 0.2 | -        |
|        | Freq       | SANS | 42.1              | 0.2 | 33.8              | 0.2 | -        |        | Freq       | SANS | 13.8              | 0.2 | 11.9              | 0.3 | -        |
|        |            | TANS | † <b>42.8</b>     | 0.3 | † <b>34.8</b>     | 0.4 | -1       |        |            | TANS | †1 <b>3.</b> 9    | 0.3 | <sup>†</sup> 12.0 | 0.2 | 0.1      |
|        |            | NS   | 40.5              | 0.2 | 31.9              | 0.2 | -        |        |            | NS   | 13.7              | 0.1 | 11.6              | 0.1 | -        |
|        | Uniq _     | SANS | 42.4              | 0.2 | 34.4              | 0.2 | -        |        | Uniq       | SANS | 13.8              | 0.2 | 11.6              | 0.2 | -        |
|        |            | TANS | 42.5              | 0.1 | 34.5              | 0.0 | -1       |        |            | TANS | 13.8              | 0.2 | 11.7              | 0.3 | -0.05    |

Table 8: Results on FB15k-237-HL.

Table 9: Results on WN18RR-HL.

| YAGO3-10-HL |            |      |                   |     |                   |     |          |  |  |  |  |  |
|-------------|------------|------|-------------------|-----|-------------------|-----|----------|--|--|--|--|--|
|             | Subsamp    | ling | MR                | R   | H@                | 1   |          |  |  |  |  |  |
| Model       | Assumption | Loss | Mean              | SD  | Mean              | SD  | $\gamma$ |  |  |  |  |  |
|             |            | NS   | 45.9              | 0.0 | 36.9              | 0.1 | -        |  |  |  |  |  |
|             | None       | SANS | 47.8              | 0.4 | 40.0              | 0.6 | -        |  |  |  |  |  |
|             |            | TANS | 49.2              | 0.4 | 39.8              | 0.7 | -0.5     |  |  |  |  |  |
|             |            | NS   | 50.2              | 0.3 | 43.0              | 0.3 | -        |  |  |  |  |  |
|             | Base       | SANS | 47.7              | 0.4 | 40.5              | 0.7 | -        |  |  |  |  |  |
|             |            | TANS | 50.1              | 0.3 | 41.4              | 0.3 | -0.5     |  |  |  |  |  |
| HAKE        |            | NS   | <sup>†</sup> 50.8 | 0.3 | † <b>43.3</b>     | 0.2 | -        |  |  |  |  |  |
|             | Freq       | SANS | 48.8              | 0.1 | 41.3              | 0.2 | -        |  |  |  |  |  |
|             | 1          | TANS | 49.7              | 0.3 | 41.0              | 0.2 | -0.5     |  |  |  |  |  |
|             |            | NS   | 49.4              | 0.2 | 40.8              | 0.2 | -        |  |  |  |  |  |
|             | Uniq       | SANS | 46.9              | 0.4 | 39.8              | 0.5 | -        |  |  |  |  |  |
|             |            | TANS | 49.4              | 0.6 | 40.6              | 0.8 | -0.5     |  |  |  |  |  |
|             |            | NS   | 38.0              | 0.1 | 28.7              | 0.3 | -        |  |  |  |  |  |
|             | None       | SANS | 41.3              | 0.1 | 32.3              | 0.2 | -        |  |  |  |  |  |
|             |            | TANS | 43.5              | 0.1 | 34.8              | 0.2 | -0.5     |  |  |  |  |  |
|             |            | NS   | 40.6              | 0.2 | 31.8              | 0.5 | -        |  |  |  |  |  |
|             | Base       | SANS | 43.8              | 0.2 | 35.1              | 0.1 | -        |  |  |  |  |  |
|             |            | TANS | 43.8              | 0.2 | 35.2              | 0.1 | -0.05    |  |  |  |  |  |
| RotatE      | Freq       | NS   | 40.3              | 0.2 | 31.4              | 0.4 | -        |  |  |  |  |  |
|             |            | SANS | 43.5              | 0.2 | 34.6              | 0.1 | -        |  |  |  |  |  |
|             |            | TANS | 43.7              | 0.0 | 35.1              | 0.1 | -0.1     |  |  |  |  |  |
|             |            | NS   | 40.2              | 0.0 | 31.3              | 0.2 | -        |  |  |  |  |  |
|             | Uniq       | SANS | 43.9              | 0.1 | 35.1              | 0.2 | -        |  |  |  |  |  |
|             | - 1        | TANS | † <b>44.1</b>     | 0.1 | <sup>†</sup> 35.4 | 0.3 | -0.1     |  |  |  |  |  |
|             |            | NS   | 37.8              | 0.3 | 26.9              | 0.4 | -        |  |  |  |  |  |
|             | None       | SANS | 50.3              | 0.1 | 40.7              | 0.3 | -        |  |  |  |  |  |
|             |            | TANS | <sup>†</sup> 52.5 | 0.5 | <sup>†</sup> 45.4 | 0.3 | -0.5     |  |  |  |  |  |
|             |            | NS   | 42.8              | 1.2 | 34.3              | 1.9 | -        |  |  |  |  |  |
|             | Base       | SANS | 51.9              | 0.3 | 44.4              | 0.2 | -        |  |  |  |  |  |
|             |            | TANS | 51.9              | 0.6 | 44.3              | 0.8 | 0.05     |  |  |  |  |  |
| HousE       |            | NS   | 39.7              | 0.8 | 29.9              | 1.5 | -        |  |  |  |  |  |
|             | Frea       | SANS | 48.6              | 1.7 | 40.0              | 1.4 | -        |  |  |  |  |  |
|             | - 1        | TANS | 52.0              | 0.1 | 44.5              | 0.3 | -1       |  |  |  |  |  |
|             |            | NS   | 41.0              | 0.1 | 31.6              | 0.1 | -        |  |  |  |  |  |
|             | Uniq       | SANS | 49.4              | 0.3 | 41.1              | 1.1 | -        |  |  |  |  |  |
|             | Uniq       | TANS | 52.2              | 0.1 | 44.7              | 0.1 | -0.05    |  |  |  |  |  |

Table 10: Results on YAGO3-10-HL.



Figure 6: Training loss and validation MRR Curve on FB15k-237.



Figure 7: Training loss and validation MRR Curve on WN18RR.



Figure 8: Training loss and validation MRR Curve on YAGO3-10.