Higher-order correlations reveal complex memory in temporal hypergraphs

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Extended Abstract

Temporal networks, where links connecting pairs of nodes are not continuously active, provide a framework to model how the interactions of a complex system evolve in time. They have revealed key in understanding how the time-varying interaction network of real-world social and biological systems affects the properties of dynamical processes, such as epidemic spreading, diffusion, synchronization, and others. Temporal network approaches, however, have a strong limitation. They are based on a graph description and, as such, they can only describe how dyadic interactions (i.e., links) vary in time, neglecting many-body interactions. Indeed, many real-world social, biological, neural or ecological systems also exhibit higher-order interactions, i.e., interactions involving groups of three or more units at the same time. Such many-body interactions are better modeled by higher-order networks, such as hypergraphs and simplicial complexes, where hyperedges and simplices encode interactions among an arbitrary number of units.

Some early works have already started to explore the temporal dimension of higher-order interactions[1]. Nevertheless, theoretical frameworks for modeling temporal group activation data have so far focused on uncorrelated time-varying hypergraphs[2]. To bridge this gap we introduce a general framework to study higher-order temporal dependencies in complex systems. First, we propose a simplified representation of a temporal hypergraph as a sequence of generalised adjacency matrices which encode the co-partipation of two nodes in interactions of a given order. This formalism allows to define a set of measures to extract higher-order temporal correlations, and characterize how the dynamics of hyperedges of different orders are correlated. By focusing on a variety of empirical social systems, our framework reveals the existence of long-range correlations at different group sizes, and their hierarchical organization. Furthermore, we uncover the presence of temporal correlations between groups of different sizes, i.e., between hyperedges of different orders, capturing emergent interactions between coherent mesoscopic structures. Finally, to gain intuition about the underlying microscopic mechanisms, we introduce novel models of temporal hypergraphs with higher-order complex memory (intra- and cross-order memory), able to explain the observed empirical patterns.

More details are provided in Figure 1. The work deals with anonymised data about human interactions, openly available and widely used by the community. The main contribution of the article is theoretical / mathematical. Taken together, the work does not raise ethical issues.

References

[1] G. Petri, A. Barrat, *Physical Review Letters* 121 (22), 228301 2018.
[2] L. Di Gaetano, F. Battiston, M. Starnini, *Physical Review Letters* 132 (3), 037401 2024.

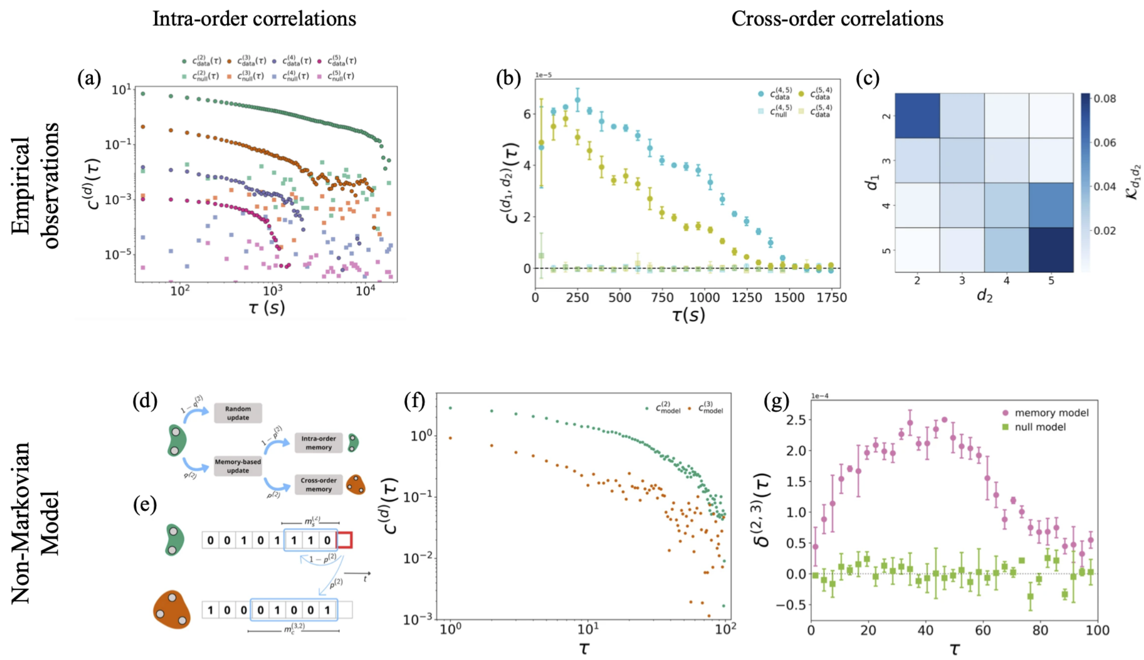

Figure 1: **Memory in temporal higher-order systems.** (a) Intra-order correlations in human face-to-face interactions (SocioPatterns data). Circles show the value of $c^{(d)}(\tau)$ for interactions in groups of size $d = 2, \ldots, 5$. Squares refer to a randomized null model where temporal correlations have been removed by reshuffling time steps. (b) Cross-order correlation functions in face-to-face interactions, $c^{(4,5)}(\tau)$ (cyan) and $c^{(5,4)}(\tau)$ (olive), describing the temporal dependencies between interactions of order four and interactions of order five. We compare the empirical system (circles) with a randomized null model with reshuffled time-steps (squares). (c) Normalized interaction matrix $\mathscr{K}_{d_1 d_2}(\tau)$, encoding the temporal dependencies between any pairs of order $d_1, d_2 \in \{2, \ldots, 4\}$ at time lag $\tau = 600s$. (d) An illustration of the proposed Non-Markovian higher-order model (cDARH model). At each time t, the state of a hyperedge of order d is updated either randomly, with a probability $1 - q^{(d)}$, or through a memory-based process, with a probability $q^{(d)}$ ($d = 2$ in the illustration). (e) In the latter case, the state is updated by copying either a previous state from the past of the hyperedge (intra-order memory) or a previous state of an overlapping hyperedge of a different order (cross-order memory), according to a probability $p^{(d)}$. (f) Intra-order correlations $c^{(d)}$, with $d \in \{2, 3\}$, for hyperedges of order two (green circles and squares) and three (orange circles and squares). The dashed vertical lines correspond to the value of the intra-order memory of hyperedges of order two (green) and three (orange), respectively. (g) Cross-order gap function $\delta^{(2,3)}$ between hyperedges of order two and three. We compare intra-order and cross-order correlations for the hypergraph generated using the cDARH model with a randomized null model.