
Sampling Language from Latent System 2 Reasoning

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Abstract

Modern language modeling datasets require models to handle system-2 compositional reasoning, fact recall, and task-specific constraints. While these tasks are expressed in natural language, they often imply an underlying symbolic representation. In this work, we consider methods for extracting a latent symbolic representation in an unsupervised manner. We introduce a latent variable modeling approach that models observed data as being generated by from a latent generative representation: an executable code program. Code as the latent symbolic representation offers two key advantages. First, code offers a structured space that can be explored via modular functions; second, code is interpretably executable using deterministic and neural interpreters, enabling compositional and programmatic decoding into text. By identifying and composing patterns in this latent space, we can sample programs that produce correct, diverse, and task-relevant text through program execution. We demonstrate how our method induces a latent space with modern LLMs, explore patterns discovered within it, and evaluate text data synthesized from our induced latent space.

1 Introduction

Many natural tasks expressed with language have an underlying symbolic process. These latent processes can be represented well by code. For example, consider the example in Figure 1. Correct interpretation and execution of the task requires one to identify processes useful for the task (parsing time strings, comparing timestamps), then compose them properly (compose identified relations). We can enable modeling of more complex tasks by pairing neural and symbolic processes: neural processes discover the underlying symbolic structure, and symbolic processes execute the identified representation.

Latent modeling is one way to uncover these symbolic representations. Mapping to and from a latent space for a given dataset exposes the underlying distribution, allowing us to manipulate latent processes to decode into domain-relevant text with desired attributes [3].

In this work, we propose to model observed text datasets as being generated from underlying code. Our method is non-parametric and leverages in-context learning (ICL) [4] with per-dataset demonstrations to model the encoder, decoder and prior of the latent codespace. Code as the latent representation allows us to use deterministic programmatic and neural interpreters [12] to map from latent space back into text. We sustain coverage over all task domains with code programs by interleaving compute operations, fact retrieval, and reasoning steps as necessary. Code programs are composed of modular function; we use these functions to conditionally sample from the latent space to sample and produce text according to target attributes.

Our method extracts the symbolic latent space underlying different datasets with minimal human effort by inducing an unsupervised non-parametric autoencoder from a set of given text examples. In this paper, we show that across a variety of datasets, our method effectively unveils underlying

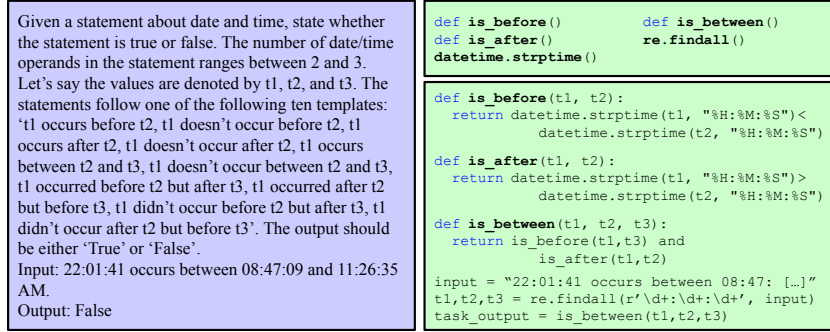


Figure 1: The text inquiry [22] (left) has an underlying symbolic representation represented by a task-level library of functions (top right) and an instance-level code program (right).

symbolic patterns in the datasets. Samples from the induced latent codespace produce more correct and domain-relevant text examples than do non-latent modeling approaches and latent modeling approaches without code as the latent representation. We find that leveraging an executable latent representation is particularly helpful in task settings that are compositional and algorithmic in nature. We also evaluate the effect of using our method to generate synthetic training data.

2 A Non-Parametric Latent Model for Datasets

Define a dataset as $x = \{x_i = (t_i, y_i)\}^N$, consisting of a sequence of text-output pairs. We are interested in defining a latent representation of this dataset, $z = \{z_\ell, z_1, \dots, z_N\}$ consisting of a global term z_ℓ and datapoint specific terms z_i . For each datapoint, we are interested in finding a latent symbolic representation $z_i \in \mathcal{Z}$. In addition, we posit that the dataset has a global shared library structure z_ℓ .

Our goal is to infer the latent representation z from the dataset. We write this as a posterior inference problem, $p(z|x)$. Inference of this distribution is intractable since it requires marginalizing over all representations. Instead, optimize a variational posterior q to approximate this objective:

$$\begin{aligned} \arg \min_q \text{KL}(q(z|x) || p(z|x)) &= \text{KL}(q(z_\ell|x) || p(z_\ell|x)) + \mathbb{E}_{z_\ell \sim q(z_\ell|x)} \sum_i \text{KL}(q(z_i | z_\ell, x_i) || p(z_i | z_\ell, x_i)) \\ &= \text{KL} + \mathbb{E}_{z_\ell, z_i \sim q} \log \frac{q(z_i | z_\ell, x_i)}{p(x_i | z_i, z_\ell)p(z_i | z_\ell)}. \end{aligned}$$

where the first step comes from KL identities and the assumption that each datapoint x_i is conditionally independent of z_j given the library z_ℓ , and the second step factors out each x_i .

To fully specify the variational objective, we need a variational family for q . In deep learning, it is common to parameterize q and use gradient descent to minimize this equation. Instead, we use a nonparametric approach to search over a subset of concrete exemplars \mathcal{D} , as example-latent representation pairs. Specifically \mathcal{D} consists of a base library and example-symbol pairs, i.e. $\mathcal{D} = \{\bar{z}_\ell, (\bar{x}_1, \bar{z}_1), \dots, (\bar{x}_M, \bar{z}_M)\}$. Given a specific set \mathcal{D} , the variational posterior can be defined through in-context learning (ICL) [4]. For each datapoint this is $q(z_i|x_i, z_\ell; \text{ICL}(\mathcal{D}))$, and the full objective is:

$$\arg \min_{\mathcal{D}} \text{KL}(q(z|x; \mathcal{D}) || p(z|x))$$

We need to solve this objective by finding the best subset \mathcal{D} for the dataset. This is a combinatorial optimization problem and is intractable to solve by search. We approximate this with rejection-sampling additions to the library and new \bar{z}_j examples. We apply an iterative optimization starting from \mathcal{D}^0 . For a given dataset example x_j , we sample \tilde{z}_ℓ and \tilde{z}_j from the variational posterior, as described above, and reject \tilde{z} that do not score well according to the log ratio. Library-representations pairs that score well are included in the next \mathcal{D}^{j+1} , roughly:

$$\mathcal{D}^{j+1} = \mathcal{D}^j \cup \{\tilde{z}_\ell, (x_j, \tilde{z}_j)\} \quad (1)$$

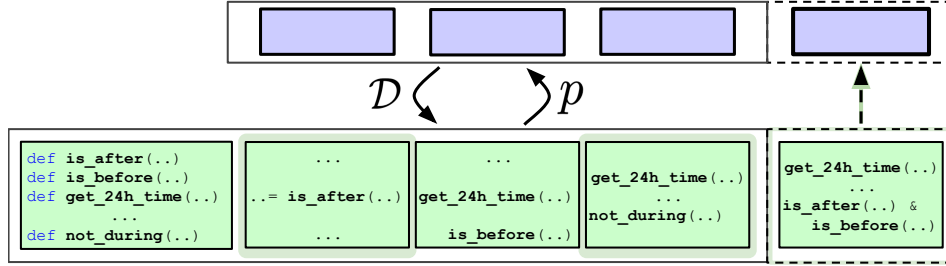


Figure 2: The underlying symbolic structure of observed text data (top) is represented by a shared function library and programs (bottom). We search for the set of demonstrations \mathcal{D} that optimizes the variational posterior. We sample from the induced posterior to synthesize new text (right).

3 A Codespace Autoencoder for Language

To apply our framework to extracting code representation, we describe how each term described in Equation 1 can be implemented with code LLMs and interpreters. First we establish the shared structure z_ℓ as the library of code functions used by latent code programs of the dataset. Each (x_i, z_i) instance of \mathcal{D} is a paired language example and corresponding code program.

The encoder is implemented as a LLM prompted by dataset-level demonstrations \mathcal{D} via ICL to sample code programs for a given language task instance. The $\text{ICL}(\mathcal{D})$ operation creates a prompt composed of a program induction instruction and example-program pairs of the demonstration set. We are limited by context window size, so we condition on a randomly selected subset rather than all exemplars in \mathcal{D} . We initialize program induction with a hand-written set of domain-general exemplars. These exemplars, prompt instructions, and templates are shared in the Appendix C.

The terms in the denominator are a prior $p(z_i | z_\ell)$ over code representations and a decoder $p(x_i | z_i, z_\ell)$ from latent representation back to text. For code, the prior is established by a compiler that rewards syntactically valid programs: $p(z_i | z_\ell) = \mathbb{1}(\text{compiles}(z))$.

The decoder term $p(x_i | z_i, z_\ell)$ measures how well the sampled code program reconstructs the observed data when executed with the given input. The term is decomposed into terms for the text input and output. The text input is given, so $p(t | z) = 1$.

$$p(x | z) = p(t | z) \times p(y | z, t) = p(y | z, t)$$

We approximate reconstruction accuracy $p(y | z, t)$ with text similarity metric threshold γ :

$$p(y | z, t) = \begin{cases} 1, & \text{textsimsim}(z(t), y) > \gamma \\ \epsilon & \text{otherwise} \end{cases}$$

For reconstruction $z(t)$, we use a combination of a real interpreter and LLM interpretation.

To cover the scope of NLP tasks that cannot easily be performed programmatically, we allow sampling of programs with interleaved real code lines and imagined functions. We follow the **Chain-of-Code** [12] method of using a Python interpreter as the real interpreter and a LLM as the imaginary interpreter. Specifically, for each line in a code program, we first attempt to execute it with a Python interpreter. If that fails, then we query a LLM to emulate the execution of that code line. The program state is then updated accordingly.

An overview of our method is sketched in Figure 2.

A Textspace Autoencoder for Language To isolate the benefits of code as the latent representation, we consider another representation modality for the latent space: text. Here, we describe the autoencoder model framework in terms of a text-based latent representation: chain-of-thought (CoT) [24]. Following the keyword prompting methodology of the TinyStories-Instruct [7] dataset, we describe the shared structure z_ℓ as a library of unique keywords used by latent CoT strings of the dataset. Each (x_i, z_i) instance of \mathcal{D} is a paired language example and the corresponding CoT string. The $\text{ICL}(\mathcal{D})$ operation creates a prompt in the same fashion, but using CoT-relevant domain-general

exemplars. These exemplars and prompts are shared in the Appendix C.2. The prior over CoT is established by a string checker that rewards strings with a certain CoT template:

$$p(z_i | z_\ell) = \mathbb{1}(z_i = \text{"* So the answer is *"})$$

Reconstruction accuracy is evaluated in the same way as for latent code representations, and the “execution” of the latent representation is string extraction.

4 Posterior Sampling

Once we find a \mathcal{D} that optimizes our variational objective, we use it to sample more synthetic examples. Sampling new text from a given dataset is factorized in our autoencoder as follows:

$$\begin{aligned} p(x_s | x_{1:N}) &= \sum_z p(x_s | z, x_{1:N}) p(z | x_{1:N}) \\ &= \mathbb{E}_{z \sim p(z | x_{1:N})} p(x_s | z, x_{1:N}) \\ &\approx \mathbb{E}_{z \sim q(z | x_{1:N}; \mathcal{D})} p(x_s | z, x_{1:N}) \\ &\approx \mathbb{E}_{z_\ell, z_s \sim q} p(x_s | z_s, z_\ell, x_{1:N}) \end{aligned}$$

The conditioning on $x_{1:N}$ gives us the \mathcal{D} terms. This is used to sample our library z_ℓ and to sample the program z_s . Since we do not have access to the observation x_s we sample the z_s using ICL that is unconditioned on the specific text datapoint.

Once sampled from the latent space, observed data x_s is decoded from z_s . A natural language input t_s is proposed for the sampled program, and the program z_s is executed to produce output y_s .

$$p(x_s | z_s, z_\ell, x_{1:N}) = p((t_s, y_s) | z_s, z_\ell, x_{1:N}) = p(t_s | z_s, z_\ell, \{t_i | (t_i, y_i) \in x_{1:N}\}) \times p(y_s | z_s, t_s)$$

5 Experimental Setup

Dataset: Super-NaturalInstructions (SNI) [22] is a dataset of expert-written instructions and over 5 million total data instances for 1616 diverse NLP tasks covering 76 task types, some of which can be solved deterministically by a simple computer program and some of which are natural language tasks. We study our method on this range of non- and algorithmic tasks.

We divide the SNI training tasks into sets of 100 in-domain (ID) and 100 out-of-domain (OOD) tasks. Both sets of tasks are composed of 33% tasks with a “synthetic” (algorithmic) source. A latent space is induced for each ID task. Evaluation on the held-out examples from ID tasks comprise ID evaluation, and evaluation with the OOD tasks comprise OOD evaluation.

Baselines: We compare against two baseline methods. To measure the value of latent modeling, we compare our method against a direct inference (no intermediate reasoning) interpolation baseline. Interpolation uses LLM prompting to generate new data points based on an ICL prompt of the dataset text instances, and does not use a latent space. To measure the value of using code programs as the latent representation, we compare our method against using chain-of-thought [24] as the latent representation.

Prompts: We prompt with instructions and 4 in-context exemplars, or up to the maximum context length. Specific prompts are shared in the Appendix C.

We use 4 hand-written examples of input, programs, and output from the original Chain of Code paper [12] as seed demonstrations to initialize the model fitting phase. Half of the examples are modified to use the internet search query `internet_lookup()` functionality that we add to LLM-emulation. These examples are shared in the Appendix C.1.

Autoencoding Parameters: We experiment with different demonstration set sizes $N \in \{12, 24\}$ and 2 LLMs of different size: Mistral AI’s Mistral 8x22b Instruction-tuned model [10] and Meta’s Llama3.1 8b Instruction-tuned model [6]. We use a temperature of 0.7 and top-p value of 0.9. For

Latent Rep.	Model	Recovery Rate (top-1 sampling)	
		Domain-general ICL	Induced \mathcal{D} ICL
CoT	L3.1 8B	67.25	86.33
Code	L3.1 8B	53.75	70.67

Table 1: Autoencoding recovery rate increases when conditioned on the induced demonstrations \mathcal{D} . Measured across 1,200 instances across 100 different tasks.

data generation, we sample $\{40, 240\}$ latent representations from the posterior and decode them into text examples.

Executed latent representations are validated using ROUGE-L [13] and BLEU [16] as the text similarity metrics, with corresponding thresholds $\gamma_R = 0.4$ and $\gamma_B = 0.3$. During posterior sampling, code execution is validated by ICL-prompting a verifier LLM with examples from the demonstration set. Specific details are shared in Appendix B.

LLM-Emulation Parameters: Programs are executed with interleaved execution by the Python interpreter, `internet_lookup()` with the duckduckgo search engine API [1], and LLM-emulation with the generating model, using code slightly modified from the Chain of Code [12] authors’.

Synthetic Data Generation Evaluation Training Parameters: Evaluation of synthetically-generated data for downstream model training is measured by fine-tuning a Pythia 1.4B model [2] on the synthesized text examples then evaluating the performance on the held-out test set. The model is trained with 12k steps, batch size 16, DeepSpeedFusedAdam optimizer, and learning rate of $1e-5$.

6 Results

In this section, we test the effectiveness of our method to identify the underlying symbolic space of a dataset. We measure this by evaluating first how well our symbolic representations are able to recover the original datasets. We then evaluate how well samples from the latent space cover the underlying task domain. Finally, we present results on using the latent space to generate synthetic training data.

6.1 Faithfulness of symbolic autoencoding

We evaluate the extent to which the induced exemplars \mathcal{D} optimizes the variational posterior. Using held-out datapoints, we compare the autoencoding recovery rate when conditioned on our induced \mathcal{D} versus conditioning on just domain-general demonstrations, i.e. the hand-written domain-generic seed ICL demonstrations. Table 1 shows that the variational method improves the autoencoding ability over the baseline method for code latent space. We also include evaluation using the CoT-based latent space, where inducing \mathcal{D} also leads to improved recovery.

6.2 Evaluating the identified latent space

Our goal is to find a latent space from which more programs z_s can be effectively sampled and executed to produce new dataset examples.

We aim for examples that are correct, in-domain, and diverse. We evaluate this by sampling points as described in Section 4 for different algorithmic and non-algorithmic tasks. For correctness and domain relevance, we use GPT-4o-mini evaluation¹ for 20 examples per method per task, for 6 total tasks: 3 algorithmic and 3 non-algorithmic. Human evaluation numbers are provided for the Llama3.1 8B generations as a control against the GPT-4o-mini judgements. For diversity, we measure average cosine similarity between the Sentence-BERT embedding [19] centroid for the synthesized dataset and each individual sentence embedding, a more tractable approximation for average pairwise cosine similarity used in prior works for diversity [21]. These results are summarized in Table 2.

Latent modeling methods produce more consistently domain-relevant and correct examples, though samples from the interpolation baseline become more domain-relevant with a larger model. Between

¹Prompt details in Appendix C.3

Task type	Method	Correctness %		Domain Relevance %		Avg. cossim (\downarrow)
		Human	GPT-4o	Human	GPT-4o	
Algo.	Gold	-	60	-	100	0.81
Non-algo.	Gold	-	61.7	-	90	0.53
Llama3.1 8B						
Algo.	Interpolation	61.7	18.3	65.0	48.3	0.59
	Latent CoT	50	20.0	93.3	90	0.89
	Latent Code	78.3	50	96.7	88.3	0.85
Non-algo.	Interpolation	65.0	38.3	43.3	56.7	0.52
	Latent CoT	60	43.3	96.7	85.0	0.78
	Latent Code	81.7	36.7	68.3	45.0	0.76
Mixtral 8x22b						
Algo.	Interpolation	-	20.0	-	100	0.94
	Latent CoT	-	38.3	-	98.3	0.94
	Latent Code	-	60.0	-	96.7	0.88
Non-algo.	Interpolation	-	60.0	-	100	0.86
	Latent CoT	-	63.3	-	86.7	0.89
	Latent Code	-	45.0	-	91.7	0.85

Table 2: Human, GPT-4o-mini, and diversity evaluation of algorithmic and non-algorithmic text samples from different methods. Each method synthesizes 240 examples. We examine 3 random algorithmic and 3 random non-algorithmic SNI tasks. Correctness and domain relevance is examined across 20 random samples per task. Diversity is measured with average embedding cosine similarity to centroid per task for all 240 synthesized instances.

using CoT and code as the latent representation, using code produces more correct data in algorithmic tasks and using CoT produces more correct data in non-algorithmic tasks. Diversity metrics, when observed with the domain relevance analysis, shows that the sampling baseline produces very diverse data to the point of being out-of-domain, and latent code produces slightly more diverse data than latent CoT.

6.3 Sampling for downstream tasks.

We compare the results of training a downstream model with data synthesized using our approach versus by baselines methods. For these experiments, we train the downstream model on instances synthesized for a group of tasks then evaluate the resulting model on both in-domain and out-of-domain tasks. Table 3 summarizes the resulting performance. In our experiments, data from the interpolation baseline generally outperforms data sampled from the latent modeling method. At times, data from the interpolation baseline even outperforms Gold data provided by the original dataset.

7 Qualitative Analysis

How often do we LLM-emulate? Table 4 shows that the vast majority of programs are fully deterministically executable, but non-algorithmic tasks tend to use more lines of code that must be LLM-emulated. Larger models perform LLM-emulation more accurately [12], and are more likely to propose lines that will be LLM-emulated.

An advantage of using code as the latent representation is the transparency of its execution. We can inspect the execution trace ² to gain insight into what makes the latent code programs correct or incorrect. Example code programs from the induced demonstration sets, along with paired snippets from their execution trace, are shared below for algorithmic, and non-algorithmic tasks:

²we follow the Chain-of-Code [12] template for LLM-emulated traces

Synthesis Parameters				OOD ROUGE		ID ROUGE		
Method	Model	#Seeds	#Synth	Algo.	Nonalgo.	Algo.	Nonalgo.	
Gold ; 40 examples				-	23.09	22.73	40.83	50.12
Interpolation	L3.1 8B	24	40	22.89	31.07	31.77	40.66	
	Mix 8x22b	24	40	23.89	31.24	35.97	44.39	
Latent CoT	Mix 8x22b	24	40	21.92	30.56	34.05	39.14	
Latent Code	L3.1 8B	24	40	25.64	28.33	33.22	38.62	
	Mix 8x22b	24	40	21.73	31.41	34.22	39.94	
Gold ; 240 examples				-	24.57	29.15	50.90	57.09
Interpolation	L3.1 8B	12	240	22.62	33.2	35.08	44.04	
	Mix 8x22b	12	240	20.93	30.43	43.13	48.67	
Latent CoT	Mix 8x22b	12	240	20.41	29.23	30.75	39.43	
Latent Code	L3.1 8B	12	240	21.72	30.33	32.65	38.22	
	Mix 8x22b	12	240	22.12	30.21	35.64	36.91	

Table 3: Downstream training results on SNI suggest that the interpolation baseline performs best, at times even yielding higher performance than using Gold data.

Model	Subset	% Programs Using LLM-emulation	Avg. % LLM-emulated lines
L3.1 8B	Algorithmic	0	0
	Non-algorithmic	1.8	0.2
M8x22B	Algorithmic	2.0	0.2
	Non-algorithmic	17.8	2.6

Table 4: Induced latent code programs tend to rarely require LLM-emulation. Non-algorithmic tasks and larger models are more likely to leverage LLM-emulation.

Listing 1: To classify tweet toxicity, generated code imports and uses the NLTK sentiment analyzer.

```
import nltk
from nltk.sentiment import
    SentimentIntensityAnalyzer
nltk.download('vader_lexicon')
def solve_task(task_input):
    comment = extract_comment(task_input)
    sia = SentimentIntensityAnalyzer()
    scores = sia.polarity_scores(comment)
    if scores['compound'] < -0.5:
        answer = "Yes"
    else:
        answer = "No"
    return answer
```

```
[...]
line: comment = extract_comment(task_input)
explanation: Let's think step by step. The
    task_input variable has a full task description
    , then a specified input... So the answer is "I
    hate them"
delta state: {'comment': 'I hate them'}
line: sia = SentimentIntensityAnalyzer()
explanation: Python execution.
delta state: {'sia': nltk.sentiment.
    SentimentIntensityAnalyzer object}
line: scores = sia.polarity_scores(comment)
explanation: Python execution.
delta state: {'scores': {'neg': 0.787, 'neu': 0.213,
    'pos': 0.0, 'compound': -0.5719}}
[...]
final state: {'answer': 'Yes'}
```

Listing 2: To generate a question that every entity in an input list can be a valid answer to, generated code relies entirely on an imagined function: construct_question

```
def solve_task(task_input):
    answers = task_input.split(", ")
    question = construct_question(
        answers, ret_type="str")
    return question
```

```
delta_state: {'task_input': 'Construct a question
    that every answer in the list is correct and
    reasonable for it. input: airplanes, snowflakes
    , footballs, cards/valentines, paper dolls,
    crane/bird'}
[...]
line: question = construct_question(answers,
    ret_tye="str")
explanation: Let's think step-by-step. All of these
    [...] So the answer is name something that kids
    make out of paper
delta state: {'question': 'name something that kids
    make out of paper'}
```

Do functions used in the induced demonstration set reflect underlying patterns of different tasks? The latent code space of different tasks leverage different functions. Table 5 lists the most popular functions for some algorithmic and non-algorithmic SNI tasks. The most popular

functions correspond with reasoning steps relevant to the task. For example, evaluating arithmetic under swapped operator strings mandates heavy use of string substitution (`re.sub`) and expression evaluation (`eval`). For less algorithmic tasks such as abductive reasoning in generating text to support health facts, functions related to inferring conclusions from facts such as `complete_story` and `generate_sentence` are popular. Hybrid tasks such as typo identification use low-level operations such as `word_tokenize` and high-level functions such as `get_typo`.

Task	Most frequently used functions
Arithmetic under swapped operators	<code>eval, re.sub</code>
Identify typos	<code>word_tokenize, get_close_matches, get_typo</code>
Generate text to support health facts	<code>generate_claim, is_supported</code>
Write the middle sentence of a story	<code>complete_story, generate_sentence</code>

Table 5: Popular functions by task.

8 Related Works

Autoencoding Datasets Prior work has explored induction and analysis of latent variables for datasets. An RNN-based variational autoencoder has been used to expose a continuous posterior from which novel sentences are interpolated and sampled [3]. Our codespace autoencoder maintains a discrete latent space for direct inspection and execution. Grammars offer another form of structure—previous work diagnoses datasets by inducing underlying context-free grammars to identify “shortcuts” from which adversarial examples are constructed to challenge models trained on said datasets [9]. Other works [18, 14] use LLMs to propose and apply structured hypotheses over data. Though not explicitly framed as autoencoders, these methods, like ours, refine an “encoder” (hypothesis proposer) alongside a static “decoder” (hypothesis interpreter). But while their approaches are constrained by symbolic requirements for the hypotheses, we use deterministic and neural program execution, allowing us to model any text task.

Library Learning and Program Induction The advantages of breaking programming tasks into subprograms [25] and inducing a shared library of reusable subprograms have been demonstrated in prior research. For example, DreamCoder [8] is a wake-sleep algorithm for Bayesian program induction that cycles between searching for programs, building a library of common concepts in the domain, and training a neural search policy on recalled and sampled programs. Similarly, other work [23] develops a method to induce a library of verifiable and efficient subprograms during inference. In our work, we induce a development set that optimizes our autoencoding objectives, but unlike these prior works, our approach does not require code to be fully executable. We instead use imagined functions, as in the Chain-of-Code framework [12], which are emulated by a LLM.

Intermediate Reasoning with LLMs As LLMs become increasingly capable in language tasks, they have been used for more complex tasks requiring multi-step reasoning [27, 28]. Prior works show that prompting a model to “think aloud” during inference enhances their performance on tasks requiring reasoning [24, 15]. To address LLMs’ limitations in handling computational tasks or recalling updated facts, tools such as code execution, internet queries, and SMT solvers have been integrated into inference pipelines [12, 5, 20, 17, 11]. Reasoning has also been integrated back into the model parameters: the Self-Taught Reasoner [26] fine-tunes a model on its own produced and verified reasoning strings. While these works improve inference-time reasoning, they are constrained to the scope of the given data, and they do not attempt to infer structural properties of the dataset itself, which is a focus of our work.

9 Conclusion

In this work, we present and apply a framework for extracting a latent symbolic representation from any text dataset in an unsupervised manner and without updating any model parameters. We present results comparing a code-based to a text-based latent representation and to a non latent modeling approach. Our findings show that a codespace autoencoder for language tasks effectively extracts an underlying symbolic representation for language tasks. We show that code offers advantages in correctness, particularly for algorithmic tasks, and lends well to interpreting the induced latent space.

References

- [1] duckduckgo-search. <https://pypi.org/project/duckduckgo-search/>. Accessed: 2024-09-24.
- [2] Stella Biderman, Hailey Schoelkopf, Quentin Anthony, Herbie Bradley, Kyle O’Brien, Eric Hallahan, Mohammad Aflah Khan, Shivanshu Purohit, USVSN Sai Prashanth, Edward Raff, Aviya Skowron, Lintang Sutawika, and Oskar Van Der Wal. Pythia: a suite for analyzing large language models across training and scaling. In *Proceedings of the 40th International Conference on Machine Learning, ICML’23*. JMLR.org, 2023.
- [3] Samuel R Bowman, Luke Vilnis, Oriol Vinyals, Andrew M Dai, Rafal Jozefowicz, and Samy Bengio. Generating sentences from a continuous space. In *20th SIGNLL Conference on Computational Natural Language Learning, CoNLL 2016*, pages 10–21. Association for Computational Linguistics (ACL), 2016.
- [4] Tom B Brown. Language models are few-shot learners. *arXiv preprint arXiv:2005.14165*, 2020.
- [5] Wenhu Chen, Xueguang Ma, Xinyi Wang, and William W. Cohen. Program of thoughts prompting: Disentangling computation from reasoning for numerical reasoning tasks. *Transactions on Machine Learning Research*, 2023.
- [6] Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, Aurelien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Roziere, Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong, Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Allonsius, Daniel Song, Danielle Pintz, Danny Livshits, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino, Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael Smith, Filip Radenovic, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Anderson, Graeme Nail, Gregoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Korevaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel Kloumann, Ishan Misra, Ivan Evtimov, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason Park, Jay Mahadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee, Jeremy Fu, Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe Spisak, Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden Alwala, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, Khalid El-Arini, Krithika Iyer, Kshitiz Malik, Kuenley Chiu, Kunal Bhalla, Lauren Rantala-Yearry, Laurens van der Maaten, Lawrence Chen, Liang Tan, Liz Jenkins, Louis Martin, Lovish Madaan, Lubo Malo, Lukas Blecher, Lukas Landzaat, Luke de Oliveira, Madeline Muzzi, Mahesh Pasupuleti, Mannat Singh, Manohar Paluri, Marcin Kardas, Mathew Oldham, Mathieu Rita, Maya Pavlova, Melanie Kambadur, Mike Lewis, Min Si, Mitesh Kumar Singh, Mona Hassan, Naman Goyal, Narjes Torabi, Nikolay Bashlykov, Nikolay Bogoychev, Niladri Chatterji, Olivier Duchenne, Onur Çelebi, Patrick Alrassy, Pengchuan Zhang, Pengwei Li, Petar Vasic, Peter Weng, Prajjwal Bhargava, Pratik Dubal, Praveen Krishnan, Punit Singh Koura, Puxin Xu, Qing He, Qingxiao Dong, Ragavan Srinivasan, Raj Ganapathy, Ramon Calderer, Ricardo Silveira Cabral, Robert Stojnic, Roberta Raileanu, Rohit Girdhar, Rohit Patel, Romain Sauvestre, Ronnie Polidoro, Roshan Sumbaly, Ross Taylor, Ruan Silva, Rui Hou, Rui Wang, Saghar Hosseini, Sahana Chennabasappa, Sanjay Singh, Sean Bell, Seohyun Sonia Kim, Sergey Edunov, Shaoliang Nie, Sharan Narang, Sharath Rapparthi, Sheng Shen, Shengye Wan, Shruti Bhosale, Shun Zhang, Simon Vandenhende, Soumya Batra, Spencer Whitman, Sten Sootla, Stephane Collot, Suchin Gururangan, Sydney Borodinsky, Tamar Herman, Tara Fowler, Tarek Sheasha, Thomas Georgiou, Thomas Scialom, Tobias Speckbacher, Todor Mihaylov, Tong Xiao, Ujjwal Karn, Vedanuj Goswami, Vibhor Gupta, Vignesh Ramanathan, Viktor Kerkez, Vincent Gonguet, Virginie Do, Vish Vogeti, Vladan Petrovic, Weiwei Chu, Wenhan Xiong, Wenyan Fu, Whitney Meers, Xavier Martinet, Xiaodong Wang, Xiaoqing Ellen Tan, Xinfeng Xie, Xuchao Jia, Xuwei Wang, Yaelle Goldschlag, Yashesh Gaur, Yasmine Babaei, Yi Wen, Yiwen Song, Yuchen Zhang, Yue Li, Yuning Mao, Zacharie Delpierre Coudert, Zheng Yan, Zhengxing Chen, Zoe Papakipos, Aaditya Singh, Aaron Grattafiori, Abha Jain, Adam Kelsey,

Adam Shajnfeld, Adithya Gangidi, Adolfo Victoria, Ahuva Goldstand, Ajay Menon, Ajay Sharma, Alex Boesenberg, Alex Vaughan, Alexei Baeviski, Allie Feinstein, Amanda Kallet, Amit Sangani, Anam Yunus, Andrei Lupu, Andres Alvarado, Andrew Caples, Andrew Gu, Andrew Ho, Andrew Poulton, Andrew Ryan, Ankit Ramchandani, Annie Franco, Aparajita Saraf, Arkabandhu Chowdhury, Ashley Gabriel, Ashwin Bharambe, Assaf Eisenman, Azadeh Yazdan, Beau James, Ben Maurer, Benjamin Leonhardi, Bernie Huang, Beth Loyd, Beto De Paola, Bhargavi Paranjape, Bing Liu, Bo Wu, Boyu Ni, Braden Hancock, Bram Wasti, Brandon Spence, Brani Stojkovic, Brian Gamido, Britt Montalvo, Carl Parker, Carly Burton, Catalina Mejia, Changhan Wang, Changkyu Kim, Chao Zhou, Chester Hu, Ching-Hsiang Chu, Chris Cai, Chris Tindal, Christoph Feichtenhofer, Damon Civin, Dana Beaty, Daniel Kreymer, Daniel Li, Danny Wyatt, David Adkins, David Xu, Davide Testuggine, Delia David, Devi Parikh, Diana Liskovich, Didem Foss, Dingkan Wang, Duc Le, Dustin Holland, Edward Dowling, Eissa Jamil, Elaine Montgomery, Eleonora Presani, Emily Hahn, Emily Wood, Erik Brinkman, Esteban Arcaute, Evan Dunbar, Evan Smothers, Fei Sun, Felix Kreuk, Feng Tian, Firat Ozgenel, Francesco Caggioni, Francisco Guzmán, Frank Kanayet, Frank Seide, Gabriela Medina Florez, Gabriella Schwarz, Gada Badeer, Georgia Swee, Gil Halpern, Govind Thattai, Grant Herman, Grigory Sizov, Guangyi, Zhang, Guna Lakshminarayanan, Hamid Shojanazeri, Han Zou, Hannah Wang, Hanwen Zha, Haroun Habeeb, Harrison Rudolph, Helen Suk, Henry Aspegren, Hunter Goldman, Ibrahim Damlaj, Igor Molybog, Igor Tufanov, Irina-Elena Veliche, Itai Gat, Jake Weissman, James Geboski, James Kohli, Japhet Asher, Jean-Baptiste Gaya, Jeff Marcus, Jeff Tang, Jennifer Chan, Jenny Zhen, Jeremy Reizenstein, Jeremy Teboul, Jessica Zhong, Jian Jin, Jingyi Yang, Joe Cummings, Jon Carvill, Jon Shepard, Jonathan McPhie, Jonathan Torres, Josh Ginsburg, Junjie Wang, Kai Wu, Kam Hou U, Karan Saxena, Karthik Prasad, Kartikay Khandelwal, Katayoun Zand, Kathy Matosich, Kaushik Veeraraghavan, Kelly Michelena, Keqian Li, Kun Huang, Kunal Chawla, Kushal Lakhota, Kyle Huang, Lailin Chen, Lakshya Garg, Lavender A, Leandro Silva, Lee Bell, Lei Zhang, Liangpeng Guo, Licheng Yu, Liron Moshkovich, Luca Wehrstedt, Madian Khabsa, Manav Avalani, Manish Bhatt, Maria Tsimpoukelli, Martynas Mankus, Matan Hasson, Matthew Lennie, Matthias Reso, Maxim Groshev, Maxim Naumov, Maya Lathi, Meghan Keneally, Michael L. Seltzer, Michal Valko, Michelle Restrepo, Mihir Patel, Mik Vyatskov, Mikayel Samvelyan, Mike Clark, Mike Macey, Mike Wang, Miquel Jubert Hermoso, Mo Metanat, Mohammad Rastegari, Munish Bansal, Nandhini Santhanam, Natascha Parks, Natasha White, Navyata Bawa, Nayan Singhal, Nick Egebo, Nicolas Usunier, Nikolay Pavlovich Laptev, Ning Dong, Ning Zhang, Norman Cheng, Oleg Chernoguz, Olivia Hart, Omkar Salpekar, Ozlem Kalinli, Parkin Kent, Parth Parekh, Paul Saab, Pavan Balaji, Pedro Rittner, Philip Bontrager, Pierre Roux, Piotr Dollar, Polina Zvyagina, Prashant Ratanchandani, Pritish Yuvraj, Qian Liang, Rachad Alao, Rachel Rodriguez, Rafi Ayub, Raghotham Murthy, Raghu Nayani, Rahul Mitra, Raymond Li, Rebekkah Hogan, Robin Battey, Rocky Wang, Rohan Maheswari, Russ Howes, Ruty Rinott, Sai Jayesh Bondu, Samyak Datta, Sara Chugh, Sara Hunt, Sargun Dhillon, Sasha Sidorov, Satadru Pan, Saurabh Verma, Seiji Yamamoto, Sharadh Ramaswamy, Shaun Lindsay, Shaun Lindsay, Sheng Feng, Shenghao Lin, Shengxin Cindy Zha, Shiva Shankar, Shuqiang Zhang, Shuqiang Zhang, Sinong Wang, Sneha Agarwal, Soji Sajuyigbe, Soumith Chintala, Stephanie Max, Stephen Chen, Steve Kehoe, Steve Satterfield, Sudarshan Govindaprasad, Sumit Gupta, Sungmin Cho, Sunny Virk, Suraj Subramanian, Sy Choudhury, Sydney Goldman, Tal Remez, Tamar Glaser, Tamara Best, Thilo Kohler, Thomas Robinson, Tianhe Li, Tianjun Zhang, Tim Matthews, Timothy Chou, Tzook Shaked, Varun Vontimitta, Victoria Ajayi, Victoria Montanez, Vijai Mohan, Vinay Satish Kumar, Vishal Mangla, Vitor Albiero, Vlad Ionescu, Vlad Poenaru, Vlad Tiberiu Mihailescu, Vladimir Ivanov, Wei Li, Wenchen Wang, Wenwen Jiang, Wes Bouaziz, Will Constable, Xiao Cheng Tang, Xiaofang Wang, Xiaojuan Wu, Xiaolan Wang, Xide Xia, Xilun Wu, Xinbo Gao, Yanjun Chen, Ye Hu, Ye Jia, Ye Qi, Yenda Li, Yilin Zhang, Ying Zhang, Yossi Adi, Youngjin Nam, Yu, Wang, Yuchen Hao, Yundi Qian, Yuzi He, Zach Rait, Zachary DeVito, Zef Rosnbrick, Zhaoduo Wen, Zhenyu Yang, and Zhiwei Zhao. The llama 3 herd of models, 2024.

- [7] Ronen Eldan and Yuanzhi Li. Tinstories: How small can language models be and still speak coherent english?, 2023.
- [8] Kevin Ellis, Catherine Wong, Maxwell Nye, Mathias Sablé-Meyer, Lucas Morales, Luke Hewitt, Luc Cary, Armando Solar-Lezama, and Joshua B. Tenenbaum. Dreamcoder: bootstrapping inductive program synthesis with wake-sleep library learning. In *Proceedings of the 42nd ACM*

- SIGPLAN International Conference on Programming Language Design and Implementation*, PLDI 2021, page 835–850, New York, NY, USA, 2021. Association for Computing Machinery.
- [9] Dan Friedman, Alexander Wettig, and Danqi Chen. Finding dataset shortcuts with grammar induction. 2022.
 - [10] Albert Q. Jiang, Alexandre Sablayrolles, Antoine Roux, Arthur Mensch, Blanche Savary, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Emma Bou Hanna, Florian Bressand, Gianna Lengyel, Guillaume Bour, Guillaume Lample, L lio Renard Lavaud, Lucile Saulnier, Marie-Anne Lachaux, Pierre Stock, Sandeep Subramanian, Sophia Yang, Szymon Antoniak, Teven Le Scao, Th ophile Gervet, Thibaut Lavril, Thomas Wang, Timoth e Lacroix, and William El Sayed. Mixtral of experts, 2024.
 - [11] Celine Lee, Abdulrahman Mahmoud, Michal Kurek, Simone Campanoni, David Brooks, Stephen Chong, Gu-Yeon Wei, and Alexander M Rush. Guess & sketch: Language model guided transpilation. In *The Twelfth International Conference on Learning Representations*, 2024.
 - [12] Chengshu Li, Jacky Liang, Andy Zeng, Xinyun Chen, Karol Hausman, Dorsa Sadigh, Sergey Levine, Li Fei-Fei, Fei Xia, and Brian Ichter. Chain of code: Reasoning with a language model-augmented code emulator. In Ruslan Salakhutdinov, Zico Kolter, Katherine Heller, Adrian Weller, Nuria Oliver, Jonathan Scarlett, and Felix Berkenkamp, editors, *Proceedings of the 41st International Conference on Machine Learning*, volume 235 of *Proceedings of Machine Learning Research*, pages 28259–28277. PMLR, 21–27 Jul 2024.
 - [13] Chin-Yew Lin. ROUGE: A package for automatic evaluation of summaries. In *Text Summarization Branches Out*, pages 74–81, Barcelona, Spain, July 2004. Association for Computational Linguistics.
 - [14] Aman Madaan, Shuyan Zhou, Uri Alon, Yiming Yang, and Graham Neubig. Language models of code are few-shot commonsense learners. In Yoav Goldberg, Zornitsa Kozareva, and Yue Zhang, editors, *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing*, pages 1384–1403, Abu Dhabi, United Arab Emirates, December 2022. Association for Computational Linguistics.
 - [15] Maxwell Nye, Anders Johan Andreassen, Guy Gur-Ari, Henryk Michalewski, Jacob Austin, David Bieber, David Dohan, Aitor Lewkowycz, Maarten Bosma, David Luan, Charles Sutton, and Augustus Odena. Show your work: Scratchpads for intermediate computation with language models, 2021.
 - [16] Kishore Papineni, Salim Roukos, Todd Ward, and Wei-Jing Zhu. Bleu: a method for automatic evaluation of machine translation. In *Proceedings of the 40th Annual Meeting on Association for Computational Linguistics*, ACL ’02, page 311–318, USA, 2002. Association for Computational Linguistics.
 - [17] Ofir Press, Muru Zhang, Sewon Min, Ludwig Schmidt, Noah Smith, and Mike Lewis. Measuring and narrowing the compositionality gap in language models. In Houda Bouamor, Juan Pino, and Kalika Bali, editors, *Findings of the Association for Computational Linguistics: EMNLP 2023*, pages 5687–5711, Singapore, December 2023. Association for Computational Linguistics.
 - [18] Linlu Qiu, Liwei Jiang, Ximing Lu, Melanie Sclar, Valentina Pyatkin, Chandra Bhagavatula, Bailin Wang, Yoon Kim, Yejin Choi, Nouha Dziri, and Xiang Ren. Phenomenal yet puzzling: Testing inductive reasoning capabilities of language models with hypothesis refinement. In *The Twelfth International Conference on Learning Representations*, 2024.
 - [19] Nils Reimers and Iryna Gurevych. Sentence-bert: Sentence embeddings using siamese bert-networks. In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing*. Association for Computational Linguistics, 11 2019.
 - [20] Timo Schick, Jane Dwivedi-Yu, Roberto Dessi, Roberta Raileanu, Maria Lomeli, Eric Hambro, Luke Zettlemoyer, Nicola Cancedda, and Thomas Scialom. Toolformer: Language models can teach themselves to use tools. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023.

- [21] Guy Tevet and Jonathan Berant. Evaluating the evaluation of diversity in natural language generation. In Paola Merlo, Jorg Tiedemann, and Reut Tsarfaty, editors, *Proceedings of the 16th Conference of the European Chapter of the Association for Computational Linguistics: Main Volume*, pages 326–346, Online, April 2021. Association for Computational Linguistics.
- [22] Yizhong Wang, Swaroop Mishra, Pegah Alipoormolabashi, Yeganeh Kordi, Amirreza Mirzaei, Atharva Naik, Arjun Ashok, Arut Selvan Dhanasekaran, Anjana Arunkumar, David Stap, Eshaan Pathak, Giannis Karamanolakis, Haizhi Lai, Ishan Purohit, Ishani Mondal, Jacob Anderson, Kirby Kuznia, Krma Doshi, Kuntal Kumar Pal, Maitreya Patel, Mehrad Moradshahi, Mihir Parmar, Mirali Purohit, Neeraj Varshney, Phani Rohitha Kaza, Pulkit Verma, Ravsehaj Singh Puri, Rushang Karia, Savan Doshi, Shailaja Keyur Sampat, Siddhartha Mishra, Sujan Reddy A, Sumanta Patro, Tanay Dixit, and Xudong Shen. Super-NaturalInstructions: Generalization via declarative instructions on 1600+ NLP tasks. In Yoav Goldberg, Zornitsa Kozareva, and Yue Zhang, editors, *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing*, pages 5085–5109, Abu Dhabi, United Arab Emirates, December 2022. Association for Computational Linguistics.
- [23] Zhiruo Wang, Daniel Fried, and Graham Neubig. Trove: Inducing verifiable and efficient toolboxes for solving programmatic tasks, 2024.
- [24] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, brian ichter, Fei Xia, Ed Chi, Quoc V Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models. In S. Koyejo, S. Mohamed, A. Agarwal, D. Belgrave, K. Cho, and A. Oh, editors, *Advances in Neural Information Processing Systems*, volume 35, pages 24824–24837. Curran Associates, Inc., 2022.
- [25] Eric Zelikman, Qian Huang, Gabriel Poesia, Noah D Goodman, and Nick Haber. Parsel : Algorithmic reasoning with language models by composing decompositions, 2022.
- [26] Eric Zelikman, Yuhuai Wu, Jesse Mu, and Noah Goodman. STar: Bootstrapping reasoning with reasoning. In Alice H. Oh, Alekh Agarwal, Danielle Belgrave, and Kyunghyun Cho, editors, *Advances in Neural Information Processing Systems*, 2022.
- [27] Wenting Zhao, Justin Chiu, Claire Cardie, and Alexander Rush. Abductive commonsense reasoning exploiting mutually exclusive explanations. In Anna Rogers, Jordan Boyd-Graber, and Naoaki Okazaki, editors, *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 14883–14896, Toronto, Canada, July 2023. Association for Computational Linguistics.
- [28] Wenting Zhao, Justin Chiu, Claire Cardie, and Alexander Rush. Hop, union, generate: Explainable multi-hop reasoning without rationale supervision. In Houda Bouamor, Juan Pino, and Kalika Bali, editors, *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pages 16119–16130, Singapore, December 2023. Association for Computational Linguistics.

A Regularization in our ELBO

We do not follow the classic KL divergence framework of traditional VAE ELBOs. Since we operate in discrete space, our prior over latent code representations z cannot be Gaussian. In our space, a similar idea of “smoothness” is that the latent code should not overfit to a single input-output example—it should generalize well to other examples in the dataset. Instead of minimizing KL divergence between the variational posterior and the prior, this regularization is represented:

$$\operatorname{argmax}_D \left[\mathbb{E}_{z \sim p(z|D)} \sum_i^N \{\log p(x^i, y^i | z)\} \right]$$

In experiments for this paper, this regularization is implemented by rejecting z s that do not pass some generalization threshold $R \in \mathbb{Z}$:

$$\underbrace{\left[\sum_n^N \mathbb{E}_{z \sim p(z|x^n, y^n, \mathcal{D})} \{\log p(x^n, y^n | z)\} \right]}_{\text{auto-encoding objective}} + \underbrace{\left[\mathbb{E}_{z \sim p(z|\mathcal{D})} \sum_i^N \{\log p(x^i, y^i | z)\} \right]}_{\text{regularization}}$$

$$\approx \sum_n^N \mathbb{E}_{z \sim p(z|x^n, y^n, \mathcal{D})} \{\log p(x^n, y^n | z) \times \mathbb{1} [|\{x^i, y^i : p(y^i | x^i, z)\}| \geq R]\}$$

B Verifying Synthesized Examples

A challenge of synthesizing correct examples is the lack of a ground truth output against which to verify the execution of the sampled latent code program. For this, we parameterize a verifier LLM with the learned demonstration set \mathcal{D} by collecting seed demonstrations (z, x, y) according to their reconstruction success $p(x, y | z)$. Successful reconstructions are saved as positive examples $v^+ = \{(z, x, y : z(x) \equiv y)\}$ and failed reconstructions as negative $v^\times = \{(x, y_\times, z_\times : z_\times(x) \not\equiv y_\times)\}$. The verifier LLM is parameterized using ICL with these positive and hard negative examples to predict the correctness of an input demonstration:

$$p(x_s, y_s, z_s | \mathcal{D}) := p(x_s, z_s(x_s), z_s | v^+, v^\times)$$

C Prompts

Listing 3: Program induction instruction

Write the Python code to perform the given task. The 'task_output' variable at the end should contain the response to the instruction.

Listing 4: Input generation instruction

Write the natural language instruction that describes what is done by the given Python code. Provide the instruction and all relevant inputs.

Listing 5: Instruction for posterior sampling of latent code programs

Write a snippet of Python code to complete some task. The code should use the following functions: {keywords}.

C.1 Hand-written exemplar demonstrations

Listing 6: Demo examples for latent code variational inference.

Which sentence has the correct adjective order:
Options:
(A) rubber terrible ship
(B) terrible rubber ship
--> (B)
CODE START
import numpy as np
def solve_task(task_input):
 options = extract_options(task_input)
 priority = {"opinion": 1, "size": 2, "age": 3, "shape": 4, "
 color": 5, "origin": 6, "material": 7, "purpose": 8}
 valid_types = list(priority.keys())

```

scores = []
for option, sentence in options.items():
    adjectives = sentence.split(" ")[:-1]
    order = [priority[get_adjective_type(adjective, valid_types,
        ret_type=str)] for adjective in adjectives]
    scores.append([order[i+1] > order[i] for i in range(len(order)
        - 1)].count(True))
answer = list(options.keys())[np.argmax(scores)]
return answer
# CODE END

How many countries have I been to? I've been to Bilbao, Death
Valley, Paris, Honolulu, Skye.
--> 4
# CODE START
def solve_task(task_input):
    places = extract_places(task_input)
    countries = set()
    for place in places:
        search_result = lookup_on_internet(f"what country is {place}
            in?")
        country = extract_country(search_result)
        countries.add(country)
    answer = len(countries)
    return answer
# CODE END

Today is Christmas Eve of 1937. What is the date 10 days ago in MM
/DD/YYYY?
--> 12/14/1937
# CODE START
import datetime
def solve_task(task_input):
    parsed_task_input = extract_arguments(task_input)
    today = datetime.date(year=parsed_task_input['year'], month=
        parsed_task_input['month'], day=parsed_task_input['day'])
    date = today - datetime.timedelta(days=parsed_task_input['
        days_ago'])
    answer = date.strftime("%m/%d/%Y")
    return answer
# CODE END

Recommend a movie similar to Star Wars Episode IV - A New Hope,
Indiana Jones and the Last Crusade, Star Wars Episode V - The
Empire Strikes Back, The Big Lebowski:
--> Blade Runner (1982)
# CODE START
def solve_task(task_input):
    ref_movies = extract_reference_movies(task_input)
    ref_movie_infos = []
    for movie in ref_movies:
        search_result = lookup_on_internet(f"{movie} release year and
            genre")
        release_year = extract_year(search_result)
        genre = extract_genre(search_result)
        ref_movie_infos.append((genre, release_year))
    answer = get_most_similar_movie(ref_movies, ref_movie_infos,
        ret_type="str")
    return answer

```

```
# CODE END
```

C.2 Chain-of-thought Prompts

Listing 7: CoT induction instruction

```
Write the thought process to perform the given task.
```

Listing 8: Input generation instruction for latent CoT

```
Write the natural language instruction that describes what is shown by the given thought process. Provide the instruction and all relevant inputs.
```

Listing 9: Instruction for posterior sampling of CoT

```
Write out the thought process that completes some task. The thought process should include the following terms: {keywords }.
```

Listing 10: Demo examples for variational inference of latent CoT.

```
Which sentence has the correct adjective order:
Options:
(A) rubber terrible ship
(B) terrible rubber ship
--> (B)
Let's think step by step.
The priority of adjective categories is, in order: opinion, size,
age, shape, color, origin, material, purpose.
rubber: material
terrible: opinion
The opinion adjective should come before the material adjective. '
terrible rubber ship' follows the correct adjective order. So
the answer is (B).

How many countries have I been to? I've been to Bilbao, Death
Valley, Paris, Honolulu, Skye.
--> 4
Let's think step by step.
We'll group by countries and count:
1. Spain: Bilbao
2. USA: Death Valley, Honolulu
3. France: Paris
4. UK: Skye
There are 4 countries in total. So the answer is 4.

Today is Christmas Eve of 1937. What is the date 10 days ago in MM
/DD/YYYY?
--> 12/14/1937
Let's think step by step.
Christmas Eve of 1937 is 12/24/1937.
To get the date 10 days ago, subtract 10 from the date section. So
the answer is 12/14/1937.

Recommend a movie similar to Star Wars Episode IV - A New Hope,
Indiana Jones and the Last Crusade, Star Wars Episode V - The
Empire Strikes Back, The Big Lebowski:
--> Blade Runner (1982)
```

```

Let's think step by step.
The genre and release year of each of the reference movies are:
Star Wars Episode IV - A New Hope: Sci-fi/Action (1977)
Indiana Jones and the Last Crusade: Adventure/Action (1989)
Star Wars Episode V - The Empire Strikes Back: Family/Sci-fi
(1980)
The Big Lebowski: Comedy/Crime (1998)
A similar movie should have a release date in 1970-2000 and be Sci
-fi/Action/Crime. So the answer is Blade Runner (1982).

```

C.3 Measuring Data Quality with GPT-4o-mini

Listing 11: Instruction prompt to verify data quality

```

You are tasked with evaluating the following instances for correctness
and domain relevance.

Please:
1. In [CORRECT] and [/CORRECT] tags, state if the output seems correct
based on the input and domain task description. Write Yes or No.
2. In [RELEVANT] and [/RELEVANT] tags, assess if the input and output are
relevant to the specified domain (e.g., the context of the
experiment). Write Yes or No.
Provide your reasoning for each answer, if necessary.

Domain task description: Provide a movie recommendation.
Input: Recommend a movie similar to Star Wars Episode IV - A New Hope,
Indiana Jones and the Last Crusade, Star Wars Episode V - The Empire
Strikes Back, The Big Lebowski:
Output: Blade Runner (1982)
1. All of these movies are action movies released around 1990 to 2010. So
Blade Runner (1982) is an appropriate recommendation. [CORRECT]Yes[/
CORRECT]
2. The task is to provide a movie recommendat. This is domain-relevant. [
RELEVANT]Yes[/RELEVANT]

Domain task description: Identify whether the given comment is severely
toxic.
Input: Sammy wanted to go to where the people were. Where might he go?
Options: (a) race track (b) populated areas (c) desert (d) apartment
(e) roadblock
Output: (b)
1. People are at populous places, so if Sammy wants to go where the
people are, he should go there. Option (b) is correct. [CORRECT]Yes[/
CORRECT]
2. The task is to identify whether a given comment is severely toxic.
Determining where Sammy wants to go has nothing to do with
identifying comment toxicity. [RELEVANT]No[/RELEVANT]

Domain task description: {task_description}
Input: {instance_input}
Output: {instance_output}

```

D Synthesized Examples

Examples generated by the sampling baseline tend to be easier, out-of-domain, or incorrect. This is likely because sampling simply relies on surface-level generation and prefers generating strings with low perplexity, i.e. it is comfortable with.

Listing 12: Mixtral 8x22b Sampling (incorrect)


```
In this task you will be given a list of integers. You should only return
an integer if the first digit is the same as the last digit in the
number. If an integer has a single digit, then it should be returned.
If there are no integers that start and end with the same digit then
an empty list ([]) should be returned.
input: [989, 597, 930, 637, 648, 647, -972, 943, 269, -486, 530, 974,
993, -631, 431, 979, -701, 560, -928, 935]
output: [989, 993]
```

Listing 13: Llama3 8.1b Sampling (very easy, OOD)

```
In this task you will be given a list of integers. You should only return
an integer if the first digit is the same as the last digit in the
number. If an integer has a single digit, then it should be returned.
If there are no integers that start and end with the same digit then
an empty list ([]) should be returned.
input: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
output: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

Examples generated by the latent CoT baseline are good at generating in-domain data, but are often wrong in algorithmic settings.

Listing 14: Llama3 8.1b Latent CoT Baseline (non-algorithmic, correct)

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In this task, you are given a description of a person (PersonX) and a
description of an event. You have to determine whether the
description of the person necessarily implies the description of the
event, or whether the description of the person may imply the
description of the event, or whether the description of the person
does not necessarily imply the description of the event. Classify
your answers into "Yes", "No", and "Maybe". The description of
the person and the event may also contain "____", a placeholder that
can be an object, a person, and/or an action.
input: PersonX: PersonX bakes a cake<sep>Event: PersonX accidentally
drops the cake"
COT: Let's think step by step.
1. The description of PersonX is that they bake a cake. The description
of the event is that they accidentally drop the cake. The two
concepts do not match because the description of the event does not
imply that PersonX has a certain quality, which is baking a cake.
3. The description of PersonX is that they bake a cake. The description
of the event is that they accidentally drop the cake. The two
concepts do not match because the description of the event does not
imply that PersonX has a certain quality, which is baking a cake.
Therefore, the description of PersonX does not necessarily imply the
description of the event.
However, the description of PersonX may imply the description of the
event, because the description of the event is a direct result of the
description of PersonX. The description of PersonX is that they bake
a cake. The description of the event is that they accidentally drop
the cake. The two concepts may match because the description of the
event is a direct result of the description of PersonX. Therefore,
the description of PersonX may imply the description of the event.
However, the description of PersonX does not necessarily imply the
description of the event. The description of PersonX is that they
bake a cake. The description of the event is that they accidentally
drop the cake. The two concepts do not match because the description
of the event does not imply that PersonX has a certain quality, which
is baking a cake.
Therefore, the description of PersonX does not necessarily imply the
description of the event.
However, the description of PersonX may imply the description of the
event, because the description of the event is a direct result of the
description of PersonX. The description of PersonX is that they bake
a cake. The description of the event is that they accidentally drop
```

the cake. The two concepts may match because the description of the event is a direct result of the description of PersonX. Therefore, the description of PersonX may imply the description of the event. However, the description of PersonX does not necessarily imply the description of the event. So the answer is Maybe.

Listing 15: Llama3 8.1b Latent CoT Baseline (algorithmic, wrong)

In this task, you need to remove all words of a given list in the sentence. The list of words to be removed is given, for example, the list ['bear', 'scissors', 'beach']. Ignore the words that are not present in the sentence.

input: Sentence: 'a group of people are playing with food on the beach near a dark place hanging by a seat on the ocean with sand near a computer'. Remove all words in the given list: ['bear', 'scissors', 'beach'].

input: a group of people are playing with food near a dark place hanging by a seat on the ocean with sand near a computer

output: Let's think step by step.

The 3 words to remove are 'bear', 'scissors', 'beach'. However, 'beach' is mentioned several times in the sentence, so we have to remove the first 'beach'.

The sentence is: 'a group of people are playing with food on the beach near a dark place hanging by a seat on the ocean with sand near a computer'.

So the result is 'a group of people are playing with food near a dark place hanging by a seat on the ocean with sand near a computer'.

So the answer is: a group of people are playing with food near a dark place hanging by a seat on the ocean with sand near a computer.