

# Representing Complex Shapes with Conceptual Spaces

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## 1 Motivation

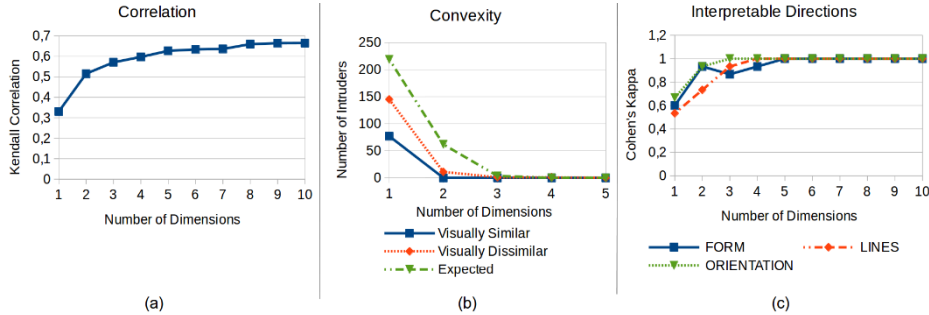
Shape representations play a central role in our interaction with the world [1]. Nevertheless, we have only a vague understanding of how shape knowledge is represented. In this paper, we use the framework of conceptual spaces [2] as a modeling tool for representing complex shapes. In this framework, each object is represented as a point in the conceptual space whose coordinates represent the object specific values of the employed quality dimensions. The geometric distance between two points represents the similarity between two objects (the smaller the distance, the more similar the objects). Higher-order concepts can be identified as convex, non-overlapping regions.

Our study aims at discovering structural characteristics of a shape space that can explain human perception and categorizations of complex shapes. This can contribute to new insights and a deeper understanding of perceptual processes but also allows for constructive uses, e.g. in the context of cognitive AI.

Based on human similarity ratings for pictures of common objects, shape spaces of varying dimensionality were constructed and validated by considering how well the similarities are reflected in the distances. Moreover, the conceptual regions for example categories were analyzed. In a second analysis step, we tested whether primitive shape features are detectable as quality dimensions in the shape spaces. The analysis scripts used in our study are available at <https://github.com/lbechberger/LearningPsychologicalSpaces>.

## 2 Conceptual Spaces for Shapes

Our set of test shapes included 60 standardized black-and-white line drawings of common objects (6 visually similar and 6 visually dissimilar categories with 5 objects each). A web-based survey (n subj = 62) was conducted to obtain 15 shape similarity ratings for all pairwise combinations of the images. Image pairs were presented in isolation on the screen (in random order). The shape similarity had to be judged on a Likert scale ranging from 1 (totally dissimilar) to 5 (very similar). Results of a control study verified that the shape similarity ratings differed significantly from semantic similarity ratings (Spearman correlation of  $r_s = 0.44$ ). Moreover, the mean category internal shape similarity was significantly higher for visually similar categories ( $M = 4.18$ ) than visually dissimilar categories ( $M = 2.56$ ), while the mean internal semantic similarity did not differ significantly. For the data analysis, the shape similarity ratings were



**Fig. 1.** Results of our analyses of the shape spaces.

aggregated into a global matrix of dissimilarities by taking the mean over the individual responses and by inverting the scale.

We used the SMACOF algorithm [3] for performing MDS on the dissimilarity matrix. The SMACOF algorithm uses an iterative process of matrix multiplications to minimize the differences between pairwise distances in the space and the pairwise dissimilarities from the ratings. Figure 1a shows the Kendall correlation of distances and dissimilarities as a function of the number of dimensions. Conceptual spaces with 2 to 5 dimensions seem to be good candidates for representing complex shapes. A one-dimensional space is insufficient, and from 5 dimensions onwards the correlation's improvement stagnated. In a control analysis, we used the distances between the raw pixels of differently downscaled versions of the images. The pixel-based distances reached only a Kendall correlation of 0.35 to the dissimilarities. This indicates that raw pixel information is not sufficient for representing complex shapes.

Visually similar categories like birds are expected to be represented as convex, non-overlapping regions in the shape space because all category members have a common global shape structure which is category distinctive. For visually dissimilar categories, this does not hold. We constructed a convex hull for each of the categories and counted the number of intruder objects from other categories. Figure 1b shows the results as a function of the number of dimensions, including the expected number of intruders for randomly chosen points as a comparison. Similar categories seem to be represented as convex regions in all spaces considered as potential candidates (2- to 5-dimensional spaces), i.e. as higher-order shape concepts.

### 3 Quality Dimensions of Shape Spaces

The framework of conceptual spaces assumes that dimensions represent meaningful qualities in which two objects can be judged to be similar or different. When modeling cognitive structures, these quality dimensions should correspond to phenomenal (psychological) dimensions. Inspired by findings from perceptual psychology (e.g., [4]), we tested whether primitive shape features which are considered to be relevant in early visual processing qualify for dimensions of shape spaces. We considered three shape primitives, namely line shape (LINES), global shape structure (FORM), and orientation (ORIENTATION).

A web-based survey ( $n_{\text{subj}} = 27$ ) was conducted to obtain 9 ratings for each primitive feature for each shape. Feature judgments had to be given on a continuous scale with labeled endpoints (LINES: absolutely straight to strongly curved, FORM: elongated to blob-like, ORIENTATION: horizontal via diagonal to vertical). Images were presented in groups of four on the screen (in random order). Subjects had to arrange the images on the scale such that the final positions reflect their values. Feature values were aggregated per object by using the median. The 15 objects with the highest and the lowest values were used as positive and negative examples for the respective feature.

The solutions provided by MDS are invariant under rotation. Thus, the axes of the coordinate system of MDS-generated spaces do not necessarily coincide with interpretable features. We therefore trained a linear support vector machine to separate positive from negative examples for each feature. The normal vector of the separating hyperplane can be interpreted as a direction representing the respective feature [5]. Figure 1c shows the quality of the separations (measured with Cohen's kappa) as a function of the number of dimensions. The results suggest that the tested primitive shape features are good candidates for quality dimensions of shape spaces, especially the features FORM and ORIENTATION which were found quite early. From 3-dimensional spaces onwards, high-quality directions were found for all three primitive shape features.

## 4 Conclusion

The conceptual spaces approach seems to be promising for modeling human representation of complex shapes. Conceptual spaces with 2 to 5 dimensions were identified as adequate for representing shape perceptions. Moreover, convexity as criterion for concept formation was observed for example categories. The primitive features FORM, ORIENTATION and LINES were identified as good candidates for quality dimensions.

In order to understand the structure of shape spaces even better, additional primitive features should be investigated. Moreover, the explanatory use of the spaces we obtained through MDS is limited to our fixed set of test shapes. In future work, we aim to train an artificial neural network on mapping novel images to points in the shape space (cf. [6]) to expand the use and allow for predictions about generalizations.

## References

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