
Exploring Integrality Grip for Mixed-integer Programming by MCTS Planning

Anonymous Author(s)

Affiliation

Address

email

Abstract

1 In modern Mixed-integer Programming(MIP) solvers, the concept of heuristic is
2 well rooted as a principle underlying the search of high-quality solutions. In this
3 respect, Large Neighborhood Search (LNS) has been the first refinement heuristics
4 for improving existing solutions through a generic MIP solver used as a black box.
5 For a refinement heuristic, the quality of the search neighborhood is of critical
6 importance. However, existing methods have not fully investigated the strategy for
7 balancing exploration and exploitation of search spaces. In this work, we introduce
8 a novel refinement strategy for improving MIP solutions. The proposed framework
9 leverages the ideas of integrality grip to guide the neighborhood selection. More-
10 over, in order to achieve a good trade-off between exploration and exploitation
11 of the solution space, the LNS search is further improved by investigating the
12 convex relaxations of LNS sub-problems with Monte Carlo Tree Search (MCTS).
13 In particular, at each iteration of LNS, MCTS is firstly executed to evaluate the
14 integrality grip of the convex relaxations of next LNS sub-problems. Then the
15 expanded MCTS tree will select a promising solution neighborhood, which will be
16 solved to produce improving solutions. Our MCTS method reduces the challenging
17 LNS neighborhood selection problem to solving a series of LP relaxations. Those
18 LP problems are polynomial-time solvable, ensuring computational tractability.
19 We have conducted comprehensive computational experiments demonstrating sig-
20 nificant performance improvements of our proposed algorithms over existing LNS
21 methods, particularly in complex MIP scenarios.

22 1 Introduction

23 Combinatorial Optimization plays a crucial role in solving a wide range of complex decision-making
24 problems with discrete structures. At the core of modeling these problems lies Mixed-integer
25 Programming (MIP) [Jünger et al., 2009, Wolsey, 2020], a mathematical framework designed to
26 optimize specific objective functions subject to a set of constraints. Despite the broad applicability
27 of MIP for representing intricate scenarios, the inherent NP-hardness of many such problems poses
28 significant computational challenges, often requiring innovative algorithmic strategies to find feasible
29 solutions within reasonable time.

30 Recent advancements in solving MIPs involve integrating sophisticated heuristics [Blum and Roli,
31 2003, Berthold, 2006], Branch-and-Bound (B&B) [Land and Doig, 2010], Cutting Planes [Balas
32 et al., 1993, Marchand et al., 2002] and preprocessing [Achterberg et al., 2020] within modern MIP
33 solvers. These methods collectively aim to enhance the solver’s capability to manage large-scale
34 problems. Heuristic methods, in particular, are crucial for providing high-quality solutions rapidly,
35 complementing the slower, more resource-intensive exact methods such as B&B, which, although
36 powerful, may not be practical for very complex problems due to their computational demands.

37 Among heuristic approaches, Large Neighborhood Search (LNS) [Shaw, 1998] stands out for its
38 effectiveness in exploring extensive solution spaces. LNS iteratively modifies a subset of decision
39 variables within an existing solution to probe various neighborhood structures methodically. However,
40 traditional LNS implementations often struggle with efficiency and adaptability, as they typically
41 rely on manually crafted rules that may not fully leverage the problem’s structure and dynamic
42 information to select neighborhoods during the search process [Danna et al., 2005].

43 In response to these limitations, recent research has explored the integration of machine learning
44 techniques to inform the neighborhood selection process [Sonnerat et al., 2021, Wu et al., 2021a,b,
45 Liu et al., 2022, Huang et al., 2023]. Those approaches leverage available data and train deep learning
46 models to predict the most promising solution neighborhoods during the search. However, deep
47 learning based methods often face generalization issues and may not perform well on heterogeneous
48 datasets, such as those found in MIPLIB, due to the diversity of problem structures and instances.

49 To address these challenges, this paper introduces two LNS heuristics, designed to optimize dynamic
50 neighborhood selection and systematically explore the solution space for improving MIP solving. The
51 first LNS algorithm, called *Integrality Grip Induced LNS* (IG-LNS), utilizes the concept of integrality
52 grip—a metric that measures the closeness of a localized LP relaxation’s solution to integrality—to
53 dynamically select variables for refinement, focusing the search on regions likely to yield significant
54 improvements.

55 Further advancing this concept, the second algorithm, *MCTS Enhanced IG-LNS* (MIG-LNS), inte-
56 grates Monte Carlo Tree Search (MCTS) method into the IG-LNS framework. Specifically, at each
57 iteration of LNS, MCTS is firstly executed to assess the outcome of candidate solution neighborhoods
58 by solving the convex relaxations of expanded LNS sub-problems. Then the algorithm will select a
59 promising LNS neighborhood from the expanded MCTS tree, which will be solved by the off-the-shelf
60 MIP solver to produce improved solutions. By simulating various neighborhood configurations and
61 evaluating their potential outcomes through LP relaxations, the MIG-LNS algorithm optimizes the
62 neighborhood selection process to adapt to the evolving landscape of the solution space dynamically.

63 **Main Contributions:**

- 64 • We propose a new class of LNS heuristic for MIP that leverages the concept of integrality grip
65 to guide neighborhood selection, enhancing the traditional LNS approach.
- 66 • We design an efficient MCTS algorithm to improve IG-LNS heuristic. Our MCTS method is
67 more adaptable and efficient by reducing the challenging LNS neighborhood selection problem
68 to solving a series of LP relaxations. These LP problems are polynomial-time solvable, ensuring
69 computational tractability.
- 70 • Our methods do not require any machine learning pre-training, making the framework more
71 generalizable and adaptable to broader classes of MIP problems, particularly beneficial for new
72 problems with limited data availability.
- 73 • We conduct comprehensive computational experiments demonstrating significant performance
74 improvements of both proposed algorithms over existing LNS methods, particularly in complex
75 MIP scenarios.

76 The remainder of the paper is organized as follows: Section 2 reviews related works, Section 3
77 discusses preliminary concepts and foundational algorithms, Section 4 and 5 details the methodologies
78 of IG-LNS and MCTS Improved IG-LNS, including the integration of GNN-based techniques, Section
79 6 presents experimental results and discussions, and Section 7 concludes with final remarks and
80 future research directions.

81 **2 Related Work**

82 The progress in machine learning (ML) has stimulated increasing research interest in applying ML for
83 solving MIPs. These works can be broadly divided into two categories, *learning auxiliary strategies*
84 *within MILP solvers* and *learning heuristics*.

85 The first approach investigates the use of ML to learn to make algorithmic decisions within a MILP
86 solver, which is typically built upon a general B&B framework. The learned policies can be either
87 cheap approximations of existing expensive methods, or more sophisticated strategies that are new

88 to be discovered. Related works include: learning to select branching variables [Khalil et al., 2016,
 89 Balcan, 2018, Gasse et al., 2019], learning to select branching nodes [He et al., 2014], learning to
 90 select cutting planes [Tang et al., 2020], and learning to optimize the usage of primal heuristics
 91 [Khalil et al., 2017, Chmiela et al., 2021].

92 The *learning heuristics* approach is to learn algorithms to produce primal solutions for MIPs. There
 93 are a few works in this direction that typically use ML methods to develop LNS heuristics. Within an
 94 LNS scheme, ML models are trained to predict promising solution neighborhoods that are expected to
 95 contain improving solutions. Nair et al. [2020] used neural networks to predict partial solutions. The
 96 subproblems defined by fixing the predicted partial solutions are solved by a MIP solver. Song et al.
 97 [2020] proposed a decomposition-based LNS heuristic. They use imitation learning and reinforcement
 98 learning to decompose the set of integer variables into subsets of fixed size. Each subset defines a
 99 subproblem and the number of subsets is fixed as a hyperparameter. Sonnerat et al. [2021] proposed
 100 a LNS heuristic based on a “learn to destroy” strategy, which frees part of the current solution. The
 101 variables to be freed are selected by trained neural networks using imitation learning.

102 3 Preliminaries

103 3.1 Mixed-integer Programming

104 We consider a MIP problem of the form,

$$(P) \quad \min \quad \mathbf{c}^T \mathbf{x} \quad (1)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad (2)$$

$$x_j \in \{0, 1\}, \forall j \in \mathcal{B}, \quad (3)$$

$$x_j \in \mathbb{Z}^+, \forall j \in \mathcal{G}, \quad (4)$$

$$x_j \geq 0, \forall j \in \mathcal{C}, \quad (5)$$

105 where the index set of decision variables $\mathcal{N} := \{1, \dots, n\}$ is partitioned into $\mathcal{B}, \mathcal{G}, \mathcal{C}$, which are the
 106 index sets of binary, general integer and continuous variables, respectively.

107 3.2 Large Neighborhood Search

108 Large neighborhood search (LNS) is a refinement heuristic. In general, one iteration of LNS consists
 109 of 3 building blocks,

- 110 • **Destroy** function: destructs a part of the current solution \mathbf{x} by freeing a subset of variables and
 111 produces a solution neighborhood $N(\mathbf{x})$;
- 112 • **Repair** function: rebuilds the destroyed solution, typically by solving a sub-MIP defined by
 113 $N(\mathbf{x})$. Note: for some cases, the repaired solution can be worse than the destroyed solution;
- 114 • **Accept** function: decides whether the new solution should be accepted or rejected.

115 Given as an input a feasible solution $\bar{\mathbf{x}}$, it searches the best feasible solution in neighbourhood of $\bar{\mathbf{x}}$
 116 (the size of the neighborhood is a parameter). Once the best feasible solution $\tilde{\mathbf{x}}$ in the neighborhood
 117 is found, the procedure updates $\bar{\mathbf{x}} = \tilde{\mathbf{x}}$. The method keeps searching for the best feasible solution in
 118 the new neighborhood until the stopping criterion is reached.

119 3.3 Mont Carlo Tree Search

120 Monte Carlo Tree Search (MCTS) is an innovative search algorithm widely recognized for its effective-
 121 ness in handling complex decision-making processes, particularly in environments characterized
 122 by vast decision trees and stochastic outcomes. Initially popularized through its applications in board
 123 games like Go, MCTS has diversified its utility across a range of strategic and planning problems in
 124 artificial intelligence Browne et al. [2012].

125 Central to MCTS is its strategic use of random simulations to accumulate statistically meaningful
 126 data that informs robust decision-making. This algorithm diverges from conventional exhaustive
 127 search methods by preferentially expanding promising moves through an iterative process comprised
 128 of four key phases: *selection*, *expansion*, *simulation*, and *backpropagation*.

Algorithm 1 Basic LNS heuristic

Input: instance dataset $\mathcal{P} = \{p_j\}_{j=1}^M$
for instance $p_j \in \mathcal{P}$ **do**
 initialize the state s with an initial solution \bar{x} ;
 $x^* \leftarrow x$;
 repeat
 $N(x) \leftarrow \text{destroy}(x)$;
 $x' \leftarrow \text{repair}(N(x))$;
 if $\text{accept}(x', x)$ **then**
 $x \leftarrow x'$;
 end
 if $f(x) < f(x^*)$ **then**
 $x^* \leftarrow x$;
 end
 until *termination condition is reached*;
end
return x^*

- 129 • **Selection:** This phase involves traversing the tree from the root to a leaf node by selecting optimal
130 child nodes at each level. The selection strategy is governed by a balance between exploiting
131 nodes with high win ratios and exploring under-visited nodes to ensure a comprehensive search
132 distribution. This is typically guided by a policy like the Upper Confidence Bound (UCB)
133 applied to trees.
134 • **Expansion:** Upon reaching a leaf node that does not terminate the game, the tree is expanded by
135 adding one or more child nodes. This expansion is contingent upon the possible moves from the
136 current game state, thereby incrementally building the tree structure.
137 • **Simulation:** Also known as the playout or rollout phase, a simulation is conducted from the
138 newly expanded nodes using a default or random policy to play out the game until a terminal
139 state or predefined depth is reached. These simulations are crucial as they provide insights into
140 the potential outcomes of moves, which are otherwise not assessed through deep analytical
141 computations.
142 • **Backpropagation:** The outcomes of the simulations are propagated back through the tree, from
143 the leaf nodes up to the root. Each node visited during the simulation phase is updated to reflect
144 the new data, adjusting metrics such as average win rates and visit counts. This iterative updating
145 ensures that the tree gradually evolves to reflect more accurate assessments of potential moves.
146 The iterative four-step process continues until a termination condition is met, such as a time constraint.
147 Subsequently, the move associated with the highest-reward or most frequently visited child node of
148 the root is executed. The opponent then makes their move, and the cycle recommences with a fresh
149 search tree that reflects the current state of the game.

150 4 Integrality Grip Enhanced LNS

151 In this section, we present a novel class of MIP LNS heuristic, *Integrality Grip Induced LNS* (IG-
152 LNS), through a combination of local constraints, LP relaxations and LNS strategies. By focusing on
153 the idea of integrality grip, which refers to the measure of how close the LP relaxation’s solution is
154 to being entirely integral, the algorithm effectively narrows the search space and improves solution
155 quality. The algorithm can utilize any local constraint to construct an LP relaxation around the current
156 incumbent solution and employs the fractionality of this solution to dynamically select variables for
157 constructing targeted LNS sub-problems.

158 4.1 Integrality Grip

159 The concept of “integrality grip” quantifies the closeness of a solution obtained from the LP relaxation
160 of a sub-MIP around an existing integer solution of the original MIP. The sub-MIP is typically defined
161 by some local constraints (e.g., local branching constraints [Fischetti and Lodi, 2003]) structured
162 from the current integer solution. This metric evaluates how closely the LP relaxation’s solution

163 approximates the current integral solution, emphasizing the influence of the integer solution in
 164 shaping the LP relaxation. We formally define the integrality grip as:

$$G(x', x^*) = 1 - \frac{1}{n} \sum_{i=1}^n |x'_i - x_i^*| \quad (6)$$

165 where $x' = (x'_1, x'_2, \dots, x'_N)$ is the solution vector from the LP relaxation of the sub-MIP, and
 166 $x^* = (x_1^*, x_2^*, \dots, x_N^*)$ is the initial integer solution. The term N denotes the number of variables.
 167 $|x'_i - x_i^*|$ calculates the deviation of each component x'_i from the corresponding integer component
 168 x_i^* , determining how far each component of the LP solution is from being integral. The integrality
 169 grip $G(x', x^*)$ varies from 0 to 1, where 0 indicates a solution with maximum fractionality and 1
 170 denotes a solution where all variables are integral.

171 **Example** Given an initial integer solution $x^* = (4, 2, 4, 3)$ of a simple MIP problem, and an
 172 LP relaxation of a sub-MIP formed with a local branching constraint around x^* , suppose this LP
 173 relaxation produces a solution vector $x' = (3.5, 1.8, 4.0, 2.9)$. Substituting these values into the
 174 integrality grip formula yields an integrality grip of 0.8, indicating that the solution x' is relatively
 175 close to the integral values specified by x^* .

176 Integrality grip is pivotal in determining the focus areas for the LNS, as it identifies where small,
 177 targeted modifications can potentially lead to substantial improvements in the overall solution quality.

178 4.2 The IG-LNS Heuristic

179 Now we present our IG-LNS heuristic, which consists of the following components.

180 • **Building LP Relaxation with Local Constraints** The IG-LNS algorithm starts by selecting
 181 a current integer solution, \bar{x} , around which the LP relaxation is constructed. This relaxation
 182 incorporates a local constraint that limits the search space to a neighborhood defined by a
 183 Hamming distance from \bar{x} . The constraint is formulated as:

$$\Delta(x', \bar{x}) = \sum_{i \in J} |x_i - \bar{x}_i| \leq k \quad (7)$$

184 where J is the set of indices of integer variables. Parameter k controls the neighborhood size.

185 • **The Upper Bound of Local Constraint** Let x' be the optimal LP solution of the original MIP
 186 model without any local constraint, and let k' be the value of the left-hand side of the local
 187 constraint evaluated using x' . Specifically, k' is computed by

$$k' = \Delta(x', \bar{x}). \quad (8)$$

188 If parameter k is greater than or equal to value k' , the LP solution is likely to remain unchanged
 189 after adding the local constraint. Consequently, k' serves as an upper bound for k . We can
 190 therefore parametrize k as

$$k = \phi k', \quad (9)$$

191 where $\phi \in [0, 1]$. Therefore, the value of ϕ should be determined when building a local LP
 192 relaxation constrained by (7).

193 • **Solving the LP Relaxation** The local LP relaxation is solved to produce an LP solution x' ,
 194 which is evaluated for its integrality grip. The fractional components of x' indicate the variables
 195 that are most challenging to integrate, providing a direct method to target the subsequent LNS
 196 steps.

197 • **Constructing and Solving the LNS Sub-MIP** Using the fractional variables identified from
 198 x' , a sub-MIP is constructed. This sub-MIP selectively “destroys” and “repairs” parts of x^*
 199 by allowing these variables to vary, while keeping others fixed. This targeted disruption aims
 200 to explore the solution space more deeply where the LP relaxation indicates potential for
 201 improvement.

202 • **Iterative Refinement** The process iterates with the newly found solutions from the LNS sub-
 203 MIPs being used to redefine the neighborhood in the LP relaxation, continually refining the
 204 approach towards an optimal solution. Each iteration recalibrates the parameter k based on the
 205 integrality grip and the outcomes of the LNS, adjusting the balance between exploration and
 206 exploitation.

207 **5 Exploring Integrality Grip by MCTS Planning**

208 To further enhance the IG-LNS algorithm, in this section, we study how to explore the integrality grip
 209 to improve the LNS search by MCTS planning. This approach is designed to dynamically explore and
 210 optimize the neighborhood through a guided search within the action space. We will first model the
 211 LNS into a Markov Decision Process (MDP) and then introduce how to efficiently integrate MCTS
 212 planning into the IG-LNS algorithm.

213 **5.1 Markov Decision Process Modeling of LNS**

214 Given an MIP instance with an initial feasible solution. The procedure can be formulated as a Markov
 215 Decision Process(MDP), wherein at each step, a LNS neighborhood is selected to build a sub-MIP
 216 and an off-the-shelf MIP solver is called to solve the LNS sub-problem.

- 217 - **State(\mathcal{S}):** The state consists of the solution status of sub-MIP and the encoded state of any MILP
 218 instance. After each LNS step, the resulting state can generally be classified into one of the four
 219 groups, depending on the status of the sub-MIP and the objective value, shown in Figure 1.
- 220 - **Action(a):** The set of possible actions consists of the portion d of variables to be destroyed from
 221 the current solution, where $d \in [0, 1]$. The number of destroyed variables n will be $n = d * N$,
 222 where N is the number of integer variables in the MIP model.
- 223 - **Policy(π):** The policy maps a state to an action in action space.
- 224 - **Reward(r):** By applying the updated k , the next iteration of LNS will be executed with time
 225 limit t_{limit} . Then the solution status of sub-MIP will be collected to create the next state. The
 226 reward will be formulated according to both the computing time and the quality of the new
 227 solution.

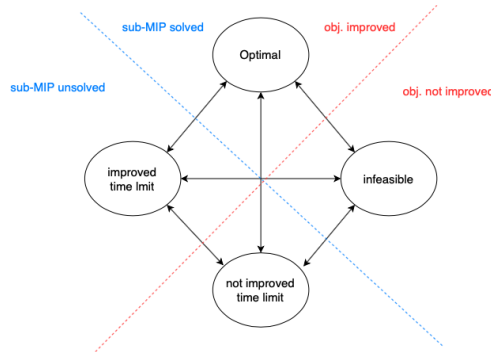


Figure 1: Solution states of sub-MIP in MDP of LNS

228 The definition above is just one possibility to build a MDP for this problem. In fact, how to define a
 229 compact MDP (e.g. state, reward) is crucial for constructing efficient LNS algorithms.

230 **5.2 MCTS Integration into IG-LNS**

231 MCTS is applied to the action space determined by the neighborhood size k , which is crucial for
 232 controlling the breadth of the search of LNS. In our method, we define the action a to determine the
 233 value of ϕ within the range $[0, 1]$. As introduced in (9), ϕ dictates the proportion of integer variables
 234 considered in each iteration, such that $k = \phi \cdot k'$, where k' represents the upper bound of parameter
 235 k computed by (8).

236 **MCTS Expansion Strategy** The tree in MCTS is expanded by selecting actions based on the
 237 proportion of variables to be included in k . At each node of the MCTS, an LNS sub-MIP is
 238 constructed and only the LP relaxation of this sub-MIP is solved. This strategy ensures that each
 239 MCTS iteration remains computationally efficient, as solving the LP relaxation is polynomially
 240 tractable, thereby allowing rapid generation of the tree.

241 **Reward Function in MCTS** The reward function in the MCTS framework is composed of three
 242 main components:

- 243 • **Objective Improvement:** This component measures the improvement in the objective value of
244 the LP relaxation of the LNS sub-MIP compared to the previous iterations.
- 245 • **Solving Time:** The computational time required to solve the LP relaxation of the LNS sub-MIP
246 is considered, emphasizing efficiency.
- 247 • **Integrality Grip:** Assesses how closely the LP solution approximates an integral solution,
248 providing insights into the quality of the LP solution in terms of feasibility.

249 These components collectively guide the search towards areas of the action space that potentially
250 yield significant improvements in solution quality and computational efficiency. The balance between
251 exploration and exploitation is managed through the selection and backpropagation steps of the
252 MCTS, ensuring that the algorithm progressively refines the neighborhood size K towards optimal
253 settings.

254 5.3 Integrality Grip Tailored MCTS

255 Our implementation of MCTS is customized to explore the action space of the neighborhood size k .

- 256 • **Selection Phase:** During the selection phase, the MCTS algorithm assesses each node by
257 exploring potential actions based on their historical success and exploratory value using the
258 Upper Confidence Bound (UCB) strategy. This phase is critical for navigating the expansive
259 solution space, aiming to incrementally approach the most promising areas that could yield
260 significant improvements in the heuristic’s performance.
- 261 • **Expansion Phase:** Upon selecting a node, the expansion phase involves introducing new child
262 nodes into the tree. Each node corresponds to a different action a , which variably adjusts the k
263 parameter of LNS neighborhood. This strategic expansion allows the heuristic to probe various
264 configurations of local constraints, enriching the diversity of solutions explored and identifying
265 potentially optimal neighborhood sizes.
- 266 • **Simulation Phase:** The simulation phase at each node involves solving only the LP relaxation
267 of the LNS sub-MIP associated with the current node configuration. This focused simulation is
268 key to maintaining the method’s efficiency, as solving the LP relaxation is polynomially solvable,
269 ensuring that the algorithm can rapidly evaluate a vast number of potential configurations without
270 excessive computational costs.
- 271 • **Backpropagation Phase:** Following the simulation, results are used to update the tree during
272 the backpropagation phase. This process adjusts the statistical values of the nodes, from the
273 expanded node back to the root, based on the outcome of the LP relaxation. These updates
274 refine the decision-making process, enhancing the algorithm’s capability to make more informed
275 selections in subsequent iterations.

276 By integrating MCTS into the IG-LNS heuristic, we obtain an extended LNS algorithm, namely
277 *MCTS-enhanced IG-LNS* (MIG-LNS). This extension provides a more adaptive and targeted approach
278 to managing the LNS neighborhood. This enhancement is expected to lead to faster convergence to
279 high-quality solutions, particularly in complex combinatorial optimization problems where traditional
280 LNS might stall or converge prematurely.

281 6 Experiments

282 In this section, we present our experimental results over three MIP benchmarks. We compare different
283 settings of our approach against the original LNS algorithm, using SCIP [Gerald et al., 2020] as the
284 underlying MILP solver.

285 6.1 Dataset

286 **MIP Benchmark** We first apply our framework to MIPLIB [Gleixner et al., 2021], the most well-
287 known state-of-the-art MIP benchmark. The MIPLIB instances are selected from the following
288 process: We want to collect a reasonable intermediate solution as the starting point for LNS search.
289 To get this, we run the SCIP 7.0.1 solver to solve the root node of each instance in MIPLIB2017 with
290 a time limit of 1 hour, and filter out all the instances that have reached to optimal on root node or still
291 can not find a solution after 1 hour. By this process, we filter out some instances that are too easy
292 or too difficult to find a starting solution for LNS search, which results in 126 instances. For each
293 instance, an initial feasible solution is required to start the LNS heuristic. We use an intermediate

Table 1: Statistics(mean, standard deviation (STD), maximum, minimum) on final primal gap and final primal integral over SC dataset.

Algorithm	Primal Gap			Primal Integral		
	Mean \pm STD	Max	Min	Mean \pm STD	Max	Min
Default SCIP	5.060 \pm 16.133	101.100	0.0	19.935 \pm 9.013	35.749	6.493
LB	18.705 \pm 23.765	58.966	0.0	19.901 \pm 11.168	36.780	4.045
Random-LNS	11.343 \pm 36.439	209.764	0.0	24.703 \pm 8.826	42.156	10.759
GNN-LNS	0.869 \pm 0.696	2.643	0.0	15.214 \pm 10.364	33.954	4.907
IG-LNS	0.241 \pm 0.497	2.217	0.0	13.071 \pm 10.331	28.613	3.945
MIG-INS	0.203 \pm 0.492	2.486	0.0	12.664 \pm 9.832	32.973	3.911

Table 2: Statistics(mean, standard deviation (STD), maximum, minimum) on final primal gap and final primal integral over GISP dataset.

Algorithm	Primal Gap			Primal Integral		
	Mean \pm STD	Max	Min	Mean \pm STD	Max	Min
Default SCIP	8.329 \pm 30.008	163.106	0.0	6.995 \pm 9.696	37.301	0.008
LB	12.809 \pm 21.585	88.829	0.0	8.901 \pm 10.761	40.905	0.011
Random-LNS	8.534 \pm 30.016	163.122	0.0	5.883 \pm 8.394	37.273	0.010
GNN-LNS	7.896 \pm 30.053	46.775	0.0	4.990 \pm 10.598	41.330	0.005
IG-LNS	7.728 \pm 34.153	87.857	0.0	4.813 \pm 8.925	40.055	0.006
MIG-LNS	5.949 \pm 26.410	45.140	0.0	4.385 \pm 8.581	29.705	0.006

294 solution found by SCIP, typically the best solution obtained by SCIP at the end of the root node
 295 computation, i.e., before branching.

296 **SC and GISP Benchmarks** In practice, there are many specific MIP applications where instances
 297 from the same class of problem are formulated and solved repeatedly. Therefore, in order to
 298 demonstrate how effective our approach is on those homogeneous problems, we conduct further
 299 computational experiments to two classes of MIP benchmarks: set covering (SC) [Balas and Ho,
 300 1980] and generalized independent set problem (GISP) [Hochbaum and Pathria, 1997, Colombi et al.,
 301 2017]. For SC benchmark, we generate 200 instances with 5000 rows and 2000 columns. For GISP,
 302 we use the public dataset from Chmiela et al. [2021].

303 6.2 Algorithmic Comparisons

304 We conduct experiments to compare the following algorithms:

- 305 • **SCIP**, the SCIP solver with default setting;
- 306 • **Random-LNS**, the LNS baseline algorithm;
- 307 • **LB**, the Local Branching heuristic [Fischetti and Lodi, 2003];
- 308 • **GNN-LNS**, the most commonly used state-of-the-art GNNs [Sonnerat et al., 2021] that have
 309 been trained for LNS neighborhood predictions;
- 310 • **IG-LNS**, basic version of our proposed Integrality Grip Enhanced LNS;
- 311 • **MIG-LNS**, our extended IG-LNS improved from exploring integrality grip by MCTS planning.

312 All the algorithms use SCIP as the underlying MIP solver.

313 We use the *primal integral* [Berthold, 2013] and standard *primal gap* to measure the performance of
 314 the compared MIP algorithms. Detailed information and formulations for computing the two metrics
 315 can be found in Appendix A.1.

316 6.3 Experimental Results

317 We evaluate the compared algorithms over the three benchmarks. The results of all the algorithms are
 318 shown in Table 1, 2, 3.

Table 3: Statistics(mean, standard deviation (STD), maximum, minimum) on final primal gap and final primal integral over MIPLIB dataset.

Algorithm	Primal Gap			Primal Integral		
	Mean \pm STD	Max	Min	Mean \pm STD	Max	Min
Default SCIP	8.257 \pm 29.894	163.024	0.0	6.912 \pm 9.654	37.222	0.001
LB	12.750 \pm 21.530	88.786	0.0	8.880 \pm 10.722	40.848	0.001
Random-LNS	8.485 \pm 29.959	163.0241	0.0	5.86516 \pm 8.367	37.245	0.001
GNN-LNS	7.865 \pm 30.005	46.729	0.0	4.969 \pm 10.574	41.305	0.001
IG-LNS	7.692 \pm 34.110	87.826	0.0	10.891 \pm 8.894	40.024	0.001
MIG-LNS	5.917 \pm 26.363	45.114	0.0	4.373 \pm 8.558	29.687	0.001

319 From the results, our IG-LNS and MIG-LNS algorithm presents the best heuristic behavior over all
320 the compared algorithms in terms of both primal integral and primal gap, showing that the proposed
321 local LP relaxation based LNS method is able to produce promising and robust LNS neighborhoods
322 by gripping the integrality of candidate solutions within the neighborhood. The results of MIG-LNS
323 also demonstrate that our MCTS algorithm achieves a better trade-off between exploitation and
324 exploration of the solution space during LNS search. The LNS behavior of our approach is robuster
325 than the compared baselines by improving both the feasibility and the objective of solutions within
326 the selected neighborhoods.

327 A significant advantage of our proposed methods is that they do not require any machine learning
328 pre-training. This feature enhances the generalizability and adaptability of our framework to a broader
329 range of MIP problems. It is particularly beneficial for new problems with limited data availability,
330 as our methods can be applied directly without the need for extensive training on large datasets.

331 7 Conclusion

332 In this work, we introduce the *Integrality Grip Induced Large Neighborhood Search* (IG-LNS)
333 algorithm, a novel class of LNS heuristic for Mixed-Integer Programming (MIP). Our approach
334 leverages the concept of integrality grip to dynamically guide neighborhood exploration, thereby
335 enhancing the effectiveness of the classic LNS method. The integrality grip measures how closely
336 an LP relaxation’s solution approximates integrality, enabling more targeted and efficient searches
337 within the solution space.

338 Building upon the IG-LNS framework, we integrate an efficient Monte Carlo Tree Search (MCTS)
339 algorithm to further refine and improve the heuristic. The MCTS method addresses the challenging
340 problem of LNS neighborhood selection by reducing it to solving a series of LP relaxations. These
341 LP problems are polynomial-time solvable, ensuring computational tractability and making the search
342 process more adaptable and efficient. We conduct comprehensive computational experiments to
343 validate our approaches, demonstrating significant performance improvements over existing LNS
344 methods.

345 While the IG-LNS algorithm with MCTS planning demonstrates significant improvements in solving
346 MIP problems, there are still limitations and open questions for future research. For example, although
347 the MCTS framework improves neighborhood selection efficiency, the computational overhead of
348 maintaining and updating the tree structure can be substantial for very large-scale problems. Future
349 research could focus on enhancing the scalability of the MCTS algorithm by exploring parallelization
350 techniques or hybrid approaches that combine MCTS with other metaheuristics. Another promising
351 direction is to investigate adaptive mechanisms that can dynamically adjust the parameters of the
352 integrality grip and MCTS based on problem characteristics. Finally, while our approach does not
353 require pre-training, exploring the integration of lightweight learning models to enhance decision-
354 making processes within the heuristic could offer additional performance gains without compromising
355 adaptability.

356 References

- 357 Tobias Achterberg, Robert E Bixby, Zonghao Gu, Edward Rothberg, and Dieter Weninger. Presolve
358 reductions in mixed integer programming. *INFORMS Journal on Computing*, 32(2):473–506,
359 2020.
- 360 Egon Balas and Andrew Ho. Set covering algorithms using cutting planes, heuristics, and subgradient
361 optimization: a computational study. In *Combinatorial Optimization*, pages 37–60. Springer, 1980.
- 362 Egon Balas, Sebastián Ceria, and Gérard Cornuéjols. A lift-and-project cutting plane algorithm for
363 mixed 0–1 programs. *Mathematical programming*, 58(1):295–324, 1993.
- 364 Maria-Florina others Balcan. Learning to branch. In *International conference on machine learning*,
365 pages 344–353. PMLR, 2018.
- 366 Timo Berthold. *Primal heuristics for mixed integer programs*. PhD thesis, Zuse Institute Berlin
367 (ZIB), 2006.
- 368 Timo Berthold. Measuring the impact of primal heuristics. *Operations Research Letters*, 41(6):
369 611–614, 2013.
- 370 Christian Blum and Andrea Roli. Metaheuristics in combinatorial optimization: Overview and
371 conceptual comparison. *ACM computing surveys (CSUR)*, 35(3):268–308, 2003.
- 372 Cameron B Browne, Edward Powley, Daniel Whitehouse, Simon M Lucas, Peter I Cowling, Philipp
373 Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis, and Simon Colton. A survey
374 of monte carlo tree search methods. *IEEE Transactions on Computational Intelligence and AI in
375 games*, 4(1):1–43, 2012.
- 376 Antonia Chmiela et al. Learning to schedule heuristics in branch-and-bound. *arXiv preprint
377 arXiv:2103.10294*, 2021.
- 378 Marco Colombi et al. The generalized independent set problem: Polyhedral analysis and solution
379 approaches. *European Journal of Operational Research*, 260(1):41–55, 2017.
- 380 Emilie Danna, Edward Rothberg, and Claude Le Pape. Exploring relaxation induced neighborhoods
381 to improve mip solutions. *Mathematical Programming*, 102(1):71–90, 2005.
- 382 Matthias Fey and Jan Eric Lenssen. Fast graph representation learning with pytorch geometric. *arXiv
383 preprint arXiv:1903.02428*, 2019.
- 384 Matteo Fischetti and Andrea Lodi. Local branching. *Mathematical programming*, 98(1-3):23–47,
385 2003.
- 386 Gerald Gamrath et al. The scip optimization suite 7.0. 2020.
- 387 Maxime Gasse et al. Exact combinatorial optimization with graph convolutional neural networks. In
388 *Advances in Neural Information Processing Systems*, pages 15554–15566, 2019.
- 389 Gamrath Gerald et al. The SCIP Optimization Suite 7.0. ZIB-Report 20-10, Zuse Institute Berlin,
390 March 2020. URL <http://nbn-resolving.de/urn:nbn:de:0297-zib-78023>.
- 391 Ambros Gleixner, Gregor Hendel, Gerald Gamrath, Tobias Achterberg, Michael Bastubbe, Timo
392 Berthold, Philipp Christophel, Kati Jarck, Thorsten Koch, and Jeff Linderoth. Miplib 2017: data-
393 driven compilation of the 6th mixed-integer programming library. *Mathematical Programming
394 Computation*, pages 1–48, 2021.
- 395 He He et al. Learning to search in branch and bound algorithms. *Advances in neural information
396 processing systems*, 27:3293–3301, 2014.
- 397 Dorit S Hochbaum and Anu Pathria. Forest harvesting and minimum cuts: a new approach to handling
398 spatial constraints. *Forest Science*, 43(4):544–554, 1997.

- 399 Taoan Huang, Aaron M Ferber, Yuandong Tian, Bistra Dilkina, and Benoit Steiner. Searching
400 large neighborhoods for integer linear programs with contrastive learning. In Andreas Krause,
401 Emma Brunskill, Kyunghyun Cho, Barbara Engelhardt, Sivan Sabato, and Jonathan Scarlett,
402 editors, *Proceedings of the 40th International Conference on Machine Learning*, volume 202 of
403 *Proceedings of Machine Learning Research*, pages 13869–13890. PMLR, 23–29 Jul 2023. URL
404 <https://proceedings.mlr.press/v202/huang23g.html>.
- 405 Michael Jünger, Thomas M Liebling, Denis Naddef, George L Nemhauser, William R Pulleyblank,
406 Gerhard Reinelt, Giovanni Rinaldi, and Laurence A Wolsey. *50 Years of integer programming
407 1958-2008: From the early years to the state-of-the-art*. Springer Science & Business Media, 2009.
- 408 Elias Khalil et al. Learning to branch in mixed integer programming. *Proceedings of the AAAI
409 Conference on Artificial Intelligence*, 30(1), Feb. 2016. URL [https://ojs.aaai.org/index.
410 php/AAAI/article/view/10080](https://ojs.aaai.org/index.php/AAAI/article/view/10080).
- 411 Elias B Khalil et al. Learning to run heuristics in tree search. In *IJCAI*, pages 659–666, 2017.
- 412 Ailsa H Land and Alison G Doig. An automatic method for solving discrete programming problems.
413 In *50 Years of Integer Programming 1958-2008*, pages 105–132. Springer, 2010.
- 414 Defeng Liu et al. Learning to search in local branching. *Proceedings of the AAAI Conference
415 on Artificial Intelligence*, 36(4):3796–3803, Jun. 2022. doi: 10.1609/aaai.v36i4.20294. URL
416 <https://ojs.aaai.org/index.php/AAAI/article/view/20294>.
- 417 Stephen Maher et al. PySCIPopt: Mathematical programming in python with the SCIP optimization
418 suite. In *Mathematical Software – ICMS 2016*, pages 301–307. Springer International Publishing,
419 2016. doi: 10.1007/978-3-319-42432-3_37.
- 420 Hugues Marchand, Alexander Martin, Robert Weismantel, and Laurence Wolsey. Cutting planes in
421 integer and mixed integer programming. *Discrete Applied Mathematics*, 123(1-3):397–446, 2002.
- 422 Vinod Nair et al. Solving mixed integer programs using neural networks. *arXiv preprint
423 arXiv:2012.13349*, 2020.
- 424 Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor
425 Killeen, Zeming Lin, Natalia Gimelshein, and Luca Antiga. Pytorch: An imperative style, high-
426 performance deep learning library. *Advances in neural information processing systems*, 32:
427 8026–8037, 2019.
- 428 Paul Shaw. Using constraint programming and local search methods to solve vehicle routing problems.
429 In *International conference on principles and practice of constraint programming*, pages 417–431.
430 Springer, 1998.
- 431 Jialin Song et al. A general large neighborhood search framework for solving integer linear programs.
432 *arXiv preprint arXiv:2004.00422*, 2020.
- 433 Nicolas Sonnerat et al. Learning a large neighborhood search algorithm for mixed integer programs.
434 *arXiv preprint arXiv:2107.10201*, 2021.
- 435 Yunhao Tang et al. Reinforcement learning for integer programming: Learning to cut. In *International
436 Conference on Machine Learning*, pages 9367–9376. PMLR, 2020.
- 437 Laurence A Wolsey. *Integer programming*. John Wiley & Sons, 2020.
- 438 Yaoxin Wu, Wen Song, Zhiguang Cao, and Jie Zhang. Learning large neighborhood search policy
439 for integer programming. *Advances in Neural Information Processing Systems*, 34:30075–30087,
440 2021a.
- 441 Yaoxin Wu et al. Learning large neighborhood search policy for integer programming. In M. Ran-
442 zato, A. Beygelzimer, Y. Dauphin, P.S. Liang, and J. Wortman Vaughan, editors, *Advances in
443 Neural Information Processing Systems*, volume 34, pages 30075–30087. Curran Associates,
444 Inc., 2021b. URL [https://proceedings.neurips.cc/paper_files/paper/2021/file/
445 fc9e62695def29ccdb9eb3fed5b4c8c8-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2021/file/fc9e62695def29ccdb9eb3fed5b4c8c8-Paper.pdf).

446 **A Appendix**

447 **A.1 Metrics for Measuring the Performance of MIP Algorithms**

448 The *primal integral* was originally proposed to measure the performance of primal heuristics for
449 solving mixed-integer programs. The metric takes into account both the quality of solutions and the
450 computing time spent to find those solutions during the solving process. To define the primal integral,
451 we first consider a *scaled primal gap function* $p(t)$ as a function of time, defined as

$$p(t) = \begin{cases} 1, & \text{if no incumbent until time } t, \\ \bar{\gamma}(\tilde{\mathbf{x}}(t)), & \text{otherwise,} \end{cases}$$

452 where $\tilde{\mathbf{x}}(t)$ is the incumbent solution at time t , and $\bar{\gamma}(\cdot) \in [0, 1]$ is the *scaled primal gap*

$$\bar{\gamma}(\tilde{\mathbf{x}}) = \frac{|f(\tilde{\mathbf{x}}_{\text{opt}}) - f(\tilde{\mathbf{x}})|}{|f(\tilde{\mathbf{x}}_{\text{opt}}) - f(\tilde{\mathbf{x}}_{\text{init}})|},$$

453 where $f(\tilde{\mathbf{x}})$ denotes the objective value given solution $\tilde{\mathbf{x}}$, $\tilde{\mathbf{x}}_{\text{opt}}$ is either the optimal solution or the
454 best one known for the instance and $\tilde{\mathbf{x}}_{\text{init}}$ is the initial solution.

455 The standard *primal gap* without scaling is defined as

$$\gamma(\tilde{\mathbf{x}}) = \frac{|f(\tilde{\mathbf{x}}_{\text{opt}}) - f(\tilde{\mathbf{x}})|}{|f(\tilde{\mathbf{x}}_{\text{opt}})|}.$$

456 Let $t_{\text{max}} > 0$ be the time limit for executing the heuristic. The primal integral measure is then defined
457 as

$$P(t_{\text{max}}) = \int_0^{t_{\text{max}}} p(t) dt.$$

458 **A.2 Experimental Settings and Hyperparameters**

459 For training GNN models, we used the focal loss as the loss function. For tuning the learning rate,
460 we have experimented different learning rates from 10^{-5} to 10^{-1} and have chosen a learning rate of
461 10^{-4} . We trained the model with a limit of 500 epochs.

462 For the LNS hyperparameters, we set a time limit of 3 seconds for each LNS iteration for all the
463 compared algorithms. The global time limit for all algorithms is set to 3600 seconds.

464 For the baselines, we compare the performance of our extended GNNs against state-of-the-art
465 message-passing based GNNs used in other works and also against classic LNS algorithm and default
466 SCIP solver baseline. We are aware of the fact that there are more learning-based LNS baselines in
467 the literature which could be potentially added to the list for a fair comparison. However, some of
468 existing works have not revealed their code to public and it is challenging to fairly implement their
469 models.

470 Our code is written in Python 3.8 and we use Pytorch 1.7.1 Paszke et al. [2019], Pytorch Geometric
471 2.0.2 Fey and Lenssen [2019], PySCIPOpt 3.1.1 Maher et al. [2016], SCIP 7.01 Gamrath et al. [2020]
472 for developing our models and solving MIPs. Our experiments were conducted on 2.70 GHz Intel
473 Xeon Gold 6258R machines with 8 cores.

474 **NeurIPS Paper Checklist**

475 **1. Claims**

476 Question: Do the main claims made in the abstract and introduction accurately reflect the
477 paper's contributions and scope?

478 Answer: [\[Yes\]](#)

479 Justification: We have carefully write our main claims in the abstract and introduction to
480 reflect the paper's contributions and scope.

481 Guidelines:

- 482 • The answer NA means that the abstract and introduction do not include the claims
483 made in the paper.
- 484 • The abstract and/or introduction should clearly state the claims made, including the
485 contributions made in the paper and important assumptions and limitations. A No or
486 NA answer to this question will not be perceived well by the reviewers.
- 487 • The claims made should match theoretical and experimental results, and reflect how
488 much the results can be expected to generalize to other settings.
- 489 • It is fine to include aspirational goals as motivation as long as it is clear that these goals
490 are not attained by the paper.

491 **2. Limitations**

492 Question: Does the paper discuss the limitations of the work performed by the authors?

493 Answer: [\[Yes\]](#)

494 Justification: We have discussed the limitations of the work and also provide perspectives
495 for future research.

496 Guidelines:

- 497 • The answer NA means that the paper has no limitation while the answer No means that
498 the paper has limitations, but those are not discussed in the paper.
- 499 • The authors are encouraged to create a separate "Limitations" section in their paper.
- 500 • The paper should point out any strong assumptions and how robust the results are to
501 violations of these assumptions (e.g., independence assumptions, noiseless settings,
502 model well-specification, asymptotic approximations only holding locally). The authors
503 should reflect on how these assumptions might be violated in practice and what the
504 implications would be.
- 505 • The authors should reflect on the scope of the claims made, e.g., if the approach was
506 only tested on a few datasets or with a few runs. In general, empirical results often
507 depend on implicit assumptions, which should be articulated.
- 508 • The authors should reflect on the factors that influence the performance of the approach.
509 For example, a facial recognition algorithm may perform poorly when image resolution
510 is low or images are taken in low lighting. Or a speech-to-text system might not be
511 used reliably to provide closed captions for online lectures because it fails to handle
512 technical jargon.
- 513 • The authors should discuss the computational efficiency of the proposed algorithms
514 and how they scale with dataset size.
- 515 • If applicable, the authors should discuss possible limitations of their approach to
516 address problems of privacy and fairness.
- 517 • While the authors might fear that complete honesty about limitations might be used by
518 reviewers as grounds for rejection, a worse outcome might be that reviewers discover
519 limitations that aren't acknowledged in the paper. The authors should use their best
520 judgment and recognize that individual actions in favor of transparency play an impor-
521 tant role in developing norms that preserve the integrity of the community. Reviewers
522 will be specifically instructed to not penalize honesty concerning limitations.

523 **3. Theory Assumptions and Proofs**

524 Question: For each theoretical result, does the paper provide the full set of assumptions and
525 a complete (and correct) proof?

526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579

Answer: [NA]

Justification: This paper does not provide theoretical results.

Guidelines:

- The answer NA means that the paper does not include theoretical results.
- All the theorems, formulas, and proofs in the paper should be numbered and cross-referenced.
- All assumptions should be clearly stated or referenced in the statement of any theorems.
- The proofs can either appear in the main paper or the supplemental material, but if they appear in the supplemental material, the authors are encouraged to provide a short proof sketch to provide intuition.
- Inversely, any informal proof provided in the core of the paper should be complemented by formal proofs provided in appendix or supplemental material.
- Theorems and Lemmas that the proof relies upon should be properly referenced.

4. Experimental Result Reproducibility

Question: Does the paper fully disclose all the information needed to reproduce the main experimental results of the paper to the extent that it affects the main claims and/or conclusions of the paper (regardless of whether the code and data are provided or not)?

Answer: [Yes]

Justification: We have provided all the information needed to reproduce the main experimental results of the paper. Code is provided and detailed hyperparameter settings can be found in Appendix.

Guidelines:

- The answer NA means that the paper does not include experiments.
- If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
- If the contribution is a dataset and/or model, the authors should describe the steps taken to make their results reproducible or verifiable.
- Depending on the contribution, reproducibility can be accomplished in various ways. For example, if the contribution is a novel architecture, describing the architecture fully might suffice, or if the contribution is a specific model and empirical evaluation, it may be necessary to either make it possible for others to replicate the model with the same dataset, or provide access to the model. In general, releasing code and data is often one good way to accomplish this, but reproducibility can also be provided via detailed instructions for how to replicate the results, access to a hosted model (e.g., in the case of a large language model), releasing of a model checkpoint, or other means that are appropriate to the research performed.
- While NeurIPS does not require releasing code, the conference does require all submissions to provide some reasonable avenue for reproducibility, which may depend on the nature of the contribution. For example
 - (a) If the contribution is primarily a new algorithm, the paper should make it clear how to reproduce that algorithm.
 - (b) If the contribution is primarily a new model architecture, the paper should describe the architecture clearly and fully.
 - (c) If the contribution is a new model (e.g., a large language model), then there should either be a way to access this model for reproducing the results or a way to reproduce the model (e.g., with an open-source dataset or instructions for how to construct the dataset).
 - (d) We recognize that reproducibility may be tricky in some cases, in which case authors are welcome to describe the particular way they provide for reproducibility. In the case of closed-source models, it may be that access to the model is limited in some way (e.g., to registered users), but it should be possible for other researchers to have some path to reproducing or verifying the results.

5. Open access to data and code

580 Question: Does the paper provide open access to the data and code, with sufficient instruc-
581 tions to faithfully reproduce the main experimental results, as described in supplemental
582 material?

583 Answer: [Yes]

584 Justification: We have provided code and data.

585 Guidelines:

- 586 • The answer NA means that paper does not include experiments requiring code.
- 587 • Please see the NeurIPS code and data submission guidelines ([https://nips.cc/
588 public/guides/CodeSubmissionPolicy](https://nips.cc/public/guides/CodeSubmissionPolicy)) for more details.
- 589 • While we encourage the release of code and data, we understand that this might not be
590 possible, so “No” is an acceptable answer. Papers cannot be rejected simply for not
591 including code, unless this is central to the contribution (e.g., for a new open-source
592 benchmark).
- 593 • The instructions should contain the exact command and environment needed to run to
594 reproduce the results. See the NeurIPS code and data submission guidelines ([https:
595 //nips.cc/public/guides/CodeSubmissionPolicy](https://nips.cc/public/guides/CodeSubmissionPolicy)) for more details.
- 596 • The authors should provide instructions on data access and preparation, including how
597 to access the raw data, preprocessed data, intermediate data, and generated data, etc.
- 598 • The authors should provide scripts to reproduce all experimental results for the new
599 proposed method and baselines. If only a subset of experiments are reproducible, they
600 should state which ones are omitted from the script and why.
- 601 • At submission time, to preserve anonymity, the authors should release anonymized
602 versions (if applicable).
- 603 • Providing as much information as possible in supplemental material (appended to the
604 paper) is recommended, but including URLs to data and code is permitted.

605 6. Experimental Setting/Details

606 Question: Does the paper specify all the training and test details (e.g., data splits, hyper-
607 parameters, how they were chosen, type of optimizer, etc.) necessary to understand the
608 results?

609 Answer: [Yes]

610 Justification: We have provided those experimental details in the paper and code package.

611 Guidelines:

- 612 • The answer NA means that the paper does not include experiments.
- 613 • The experimental setting should be presented in the core of the paper to a level of detail
614 that is necessary to appreciate the results and make sense of them.
- 615 • The full details can be provided either with the code, in appendix, or as supplemental
616 material.

617 7. Experiment Statistical Significance

618 Question: Does the paper report error bars suitably and correctly defined or other appropriate
619 information about the statistical significance of the experiments?

620 Answer: [Yes]

621 Justification: We have reported statistical details in the report of the experimental results.

622 Guidelines:

- 623 • The answer NA means that the paper does not include experiments.
- 624 • The authors should answer "Yes" if the results are accompanied by error bars, confi-
625 dence intervals, or statistical significance tests, at least for the experiments that support
626 the main claims of the paper.
- 627 • The factors of variability that the error bars are capturing should be clearly stated (for
628 example, train/test split, initialization, random drawing of some parameter, or overall
629 run with given experimental conditions).
- 630 • The method for calculating the error bars should be explained (closed form formula,
631 call to a library function, bootstrap, etc.)

- 632 • The assumptions made should be given (e.g., Normally distributed errors).
- 633 • It should be clear whether the error bar is the standard deviation or the standard error
- 634 of the mean.
- 635 • It is OK to report 1-sigma error bars, but one should state it. The authors should
- 636 preferably report a 2-sigma error bar than state that they have a 96% CI, if the hypothesis
- 637 of Normality of errors is not verified.
- 638 • For asymmetric distributions, the authors should be careful not to show in tables or
- 639 figures symmetric error bars that would yield results that are out of range (e.g. negative
- 640 error rates).
- 641 • If error bars are reported in tables or plots, The authors should explain in the text how
- 642 they were calculated and reference the corresponding figures or tables in the text.

643 8. Experiments Compute Resources

644 Question: For each experiment, does the paper provide sufficient information on the com-
645 puter resources (type of compute workers, memory, time of execution) needed to reproduce
646 the experiments?

647 Answer: [Yes]

648 Justification: We have provided those detailed information on the computer resources needed
649 to reproduce the experiments.

650 Guidelines:

- 651 • The answer NA means that the paper does not include experiments.
- 652 • The paper should indicate the type of compute workers CPU or GPU, internal cluster,
653 or cloud provider, including relevant memory and storage.
- 654 • The paper should provide the amount of compute required for each of the individual
655 experimental runs as well as estimate the total compute.
- 656 • The paper should disclose whether the full research project required more compute
657 than the experiments reported in the paper (e.g., preliminary or failed experiments that
658 didn't make it into the paper).

659 9. Code Of Ethics

660 Question: Does the research conducted in the paper conform, in every respect, with the
661 NeurIPS Code of Ethics <https://neurips.cc/public/EthicsGuidelines?>

662 Answer: [Yes]

663 Justification: We have read NeurIPS Code of Ethics and we do not find any concern related
664 to our submission.

665 Guidelines:

- 666 • The answer NA means that the authors have not reviewed the NeurIPS Code of Ethics.
- 667 • If the authors answer No, they should explain the special circumstances that require a
668 deviation from the Code of Ethics.
- 669 • The authors should make sure to preserve anonymity (e.g., if there is a special consid-
670 eration due to laws or regulations in their jurisdiction).

671 10. Broader Impacts

672 Question: Does the paper discuss both potential positive societal impacts and negative
673 societal impacts of the work performed?

674 Answer: [Yes]

675 Justification: We have discussed the potential societal impacts of our work.

676 Guidelines:

- 677 • The answer NA means that there is no societal impact of the work performed.
- 678 • If the authors answer NA or No, they should explain why their work has no societal
679 impact or why the paper does not address societal impact.
- 680 • Examples of negative societal impacts include potential malicious or unintended uses
681 (e.g., disinformation, generating fake profiles, surveillance), fairness considerations
682 (e.g., deployment of technologies that could make decisions that unfairly impact specific
683 groups), privacy considerations, and security considerations.

- 684
- 685
- 686
- 687
- 688
- 689
- 690
- 691
- 692
- 693
- 694
- 695
- 696
- 697
- 698
- The conference expects that many papers will be foundational research and not tied to particular applications, let alone deployments. However, if there is a direct path to any negative applications, the authors should point it out. For example, it is legitimate to point out that an improvement in the quality of generative models could be used to generate deepfakes for disinformation. On the other hand, it is not needed to point out that a generic algorithm for optimizing neural networks could enable people to train models that generate Deepfakes faster.
 - The authors should consider possible harms that could arise when the technology is being used as intended and functioning correctly, harms that could arise when the technology is being used as intended but gives incorrect results, and harms following from (intentional or unintentional) misuse of the technology.
 - If there are negative societal impacts, the authors could also discuss possible mitigation strategies (e.g., gated release of models, providing defenses in addition to attacks, mechanisms for monitoring misuse, mechanisms to monitor how a system learns from feedback over time, improving the efficiency and accessibility of ML).

699 **11. Safeguards**

700 Question: Does the paper describe safeguards that have been put in place for responsible
701 release of data or models that have a high risk for misuse (e.g., pretrained language models,
702 image generators, or scraped datasets)?

703 Answer: [NA]

704 Justification: We do not find any of those risk listed above for this submission.

705 Guidelines:

- 706
- 707
- 708
- 709
- 710
- 711
- 712
- 713
- 714
- 715
- The answer NA means that the paper poses no such risks.
 - Released models that have a high risk for misuse or dual-use should be released with necessary safeguards to allow for controlled use of the model, for example by requiring that users adhere to usage guidelines or restrictions to access the model or implementing safety filters.
 - Datasets that have been scraped from the Internet could pose safety risks. The authors should describe how they avoided releasing unsafe images.
 - We recognize that providing effective safeguards is challenging, and many papers do not require this, but we encourage authors to take this into account and make a best faith effort.

716 **12. Licenses for existing assets**

717 Question: Are the creators or original owners of assets (e.g., code, data, models), used in
718 the paper, properly credited and are the license and terms of use explicitly mentioned and
719 properly respected?

720 Answer: [NA]

721 Justification: This submission does not use existing assets.

722 Guidelines:

- 723
- 724
- 725
- 726
- 727
- 728
- 729
- 730
- 731
- 732
- 733
- 734
- 735
- The answer NA means that the paper does not use existing assets.
 - The authors should cite the original paper that produced the code package or dataset.
 - The authors should state which version of the asset is used and, if possible, include a URL.
 - The name of the license (e.g., CC-BY 4.0) should be included for each asset.
 - For scraped data from a particular source (e.g., website), the copyright and terms of service of that source should be provided.
 - If assets are released, the license, copyright information, and terms of use in the package should be provided. For popular datasets, paperswithcode.com/datasets has curated licenses for some datasets. Their licensing guide can help determine the license of a dataset.
 - For existing datasets that are re-packaged, both the original license and the license of the derived asset (if it has changed) should be provided.

736 • If this information is not available online, the authors are encouraged to reach out to
737 the asset’s creators.

738 **13. New Assets**

739 Question: Are new assets introduced in the paper well documented and is the documentation
740 provided alongside the assets?

741 Answer: [NA]

742 Justification: The paper does not release new assets.

743 Guidelines:

- 744 • The answer NA means that the paper does not release new assets.
- 745 • Researchers should communicate the details of the dataset/code/model as part of their
746 submissions via structured templates. This includes details about training, license,
747 limitations, etc.
- 748 • The paper should discuss whether and how consent was obtained from people whose
749 asset is used.
- 750 • At submission time, remember to anonymize your assets (if applicable). You can either
751 create an anonymized URL or include an anonymized zip file.

752 **14. Crowdsourcing and Research with Human Subjects**

753 Question: For crowdsourcing experiments and research with human subjects, does the paper
754 include the full text of instructions given to participants and screenshots, if applicable, as
755 well as details about compensation (if any)?

756 Answer: [NA]

757 Justification: The paper does not involve crowdsourcing nor research with human subjects.
758 All the data are generated from simulator.

759 Guidelines:

- 760 • The answer NA means that the paper does not involve crowdsourcing nor research with
761 human subjects.
- 762 • Including this information in the supplemental material is fine, but if the main contribu-
763 tion of the paper involves human subjects, then as much detail as possible should be
764 included in the main paper.
- 765 • According to the NeurIPS Code of Ethics, workers involved in data collection, curation,
766 or other labor should be paid at least the minimum wage in the country of the data
767 collector.

768 **15. Institutional Review Board (IRB) Approvals or Equivalent for Research with Human
769 Subjects**

770 Question: Does the paper describe potential risks incurred by study participants, whether
771 such risks were disclosed to the subjects, and whether Institutional Review Board (IRB)
772 approvals (or an equivalent approval/review based on the requirements of your country or
773 institution) were obtained?

774 Answer: [NA]

775 Justification: The paper does not involve crowdsourcing nor research with human subjects.

776 Guidelines:

- 777 • The answer NA means that the paper does not involve crowdsourcing nor research with
778 human subjects.
- 779 • Depending on the country in which research is conducted, IRB approval (or equivalent)
780 may be required for any human subjects research. If you obtained IRB approval, you
781 should clearly state this in the paper.
- 782 • We recognize that the procedures for this may vary significantly between institutions
783 and locations, and we expect authors to adhere to the NeurIPS Code of Ethics and the
784 guidelines for their institution.
- 785 • For initial submissions, do not include any information that would break anonymity (if
786 applicable), such as the institution conducting the review.