Learning Value Functions from Undirected State-only Experience

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Abstract

This paper tackles the problem of learning value functions from undirected state-only experience (state transitions without action labels *i.e.* (s, s', r) tuples). We first theoretically characterize the applicability of Q-learning in this setting. We show that tabular Q-learning in discrete Markov decision processes (MDPs) learns the same value function under any arbitrary refinement of the action space. This theoretical result motivates the design of Latent Action Q-learning or LAQ, an offline RL method that can learn effective value functions from state-only experience. Latent Action Q-learning (LAQ) learns value functions using Q-learning on discrete latent actions obtained through a latent-variable future prediction model. We show that LAQ can recover value functions that have high correlation with value functions learned using ground truth actions. Value functions learned using LAQ lead to sample efficient acquisition of goal-directed behavior, can be used with domain-specific low-level controllers, and facilitate transfer across embodiments. Our experiments in 5 environments ranging from 2D grid world to 3D visual navigation in realistic environments demonstrate the benefits of LAQ over simpler alternatives, imitation learning oracles, and competing methods.

1 Introduction

Offline or batch reinforcement learning focuses on learning goal-directed behavior from pre-recorded data of undirected experience in the form of (s_t, a_t, s_{t+1}, r_t) quadruples. However, in many realistic applications, action information is not naturally available (*e.g.* when learning from video demonstrations), or worse still, isn't even well-defined (*e.g.* when learning from the experience of an agent with a different embodiment). Motivated by such use cases, this paper studies if, and how, intelligent behavior can be derived from undirected streams of observations: (s_t, s_{t+1}, r_t).[2]

Our key conceptual insight is that while an observation-only dataset doesn't tell us the precise action to execute, *i.e.* the policy $\pi(a|s)$; it may still tell us which states are more likely to lead us to the goal than not, *i.e.* the value function $V(s)$. For example, simply by looking at someone working in the kitchen, we can infer that approaching the microwave handle is more useful (*i.e.* has higher value) for opening the microwave than to approach the buttons. Thus, we can still make use of observation-only data, if we focused on learning value functions as opposed to directly learning goal-directed policies. Once we have learned a good value function, it can be used to quickly acquire or infer behavior. Using learned value functions as dense rewards can lead to quick policy learning through some small amount of interaction in the environment. Alternatively, they could be used to directly guide the behavior of low-level controllers that may already be available for the agent (as is often the case in robotics) without any further training. Furthermore, decoupling the learning of value functions

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[2]We assume r_t is observed. Reward can often be sparsely labeled in observation streams with low effort.

from policy learning enables deriving behavior for agents with a different embodiment as long as the overall strategy to solve the task remains similar.

Thus, the central technical question is how to learn a good value function from undirected observation streams. Is it even possible? If so, under what conditions? This paper tackles these questions from a theoretical and practical perspective.

We start out by characterizing the behavior of tabular Q-learning from [45] under missing action labels. We note that Q-learning with naively imputed action labels is equivalent to the TD(0) policy evaluation, which serves as a simple baseline method for deriving a value function. However, depending on the policy that generated the data, the learned values (without any action grounding) can differ from the optimal values. Furthermore, it is possible to construct simple environments where the behavior implied by the learned value function is also sub-optimal.

Next, we present a more optimistic result. There are settings in which Q-learning can recover the optimal value function even in the absence of the knowledge of underlying actions. Concretely, we prove that if we are able to obtain an action space which is a strict refinement of the original action space, then Q-learning in this refined action space recovers the optimal value function.

This motivates a practical algorithm for learning value functions from the given undirected observation-only experience. We design a latent-variable future prediction model that seeks to obtain a refined action space. It operates by predicting s_{t+1} from s_t and a discrete latent variable $\hat{a}$ from a set of actions $\hat{\mathbf{A}}$ (Section 4.1). Training this latent variable model assigns a discrete action $\hat{a}_t$ to each (s_t, s_{t+1}) tuple. This allows us to employ Q-learning to learn good value functions (Section 4.2). The learned value function is used to derive behavior (Section 4.3) either through some online interaction with the environment, or through the use of domain specific low-level controllers.

The use of a latent action space for Q-learning allows us to exceed the performance of methods based on policy evaluation [9], which will learn the value of the demonstration policy, not the optimal value function. Additionally, it side-steps the problem of reconstructing high-dimensional images faced by other state-only value learning methods [10]. Other approaches for learning from state-only data rely on imitating the demonstration data, which renders them unable to improve on sub-optimal demonstration data. See Section 7 for more discussion.

Our experiments in five environments (2D grid world, 2D continuous control, Atari game Freeway, robotic manipulation, and visual navigation in realistic 3D environments) test our proposed ideas. Our method approximates a refinement of the latent space better than clustering alternatives, and in turn, learns value functions highly correlated with ground truth. Good value functions in-turn lead to sample efficient acquisition of behavior, leading to significant improvement over learning with only environment rewards. Our method compares well against existing methods that learn from undirected observation-only data, while being also applicable to the case of high-dimensional observation spaces in the form of RGB images. We are also able to outperform imitation learning methods, even when these imitation learning methods have access to privileged ground-truth action information. Furthermore, our method is able to use observation-only experience from one agent to speed up learning for another agent with a different embodiment. Code, models, simulation environments will be released.

2 Preliminaries

Following the notation from [38], our Markov decision process (MDP) is specified by $(\mathbf{S}, \mathbf{A}, p, \gamma)$, where $\mathbf{S}$ is a state space, $\mathbf{A}$ is an action space, γ is the discount factor, and $p(s', r|s, a)$ is the state/reward joint dynamics function. It specifies the probability distribution that the agent ends up in state s', receives a reward of r on executing action a from state s.

Offline or batch RL [22, 23] studies the problem of deriving high reward behavior when only given a dataset of experience in an MDP, in the form of a collection of quadruples (s, a, s', r). In this paper, we tackle a harder version of this problem where instead we are only given a collection of triplets (s, s', r), *i.e.* experience without information about intervening actions. In general, this dataset could contain any quality of behavior, from optimal, to actively adversarial. In contrast to some methods (see Section 7), we will not assume that demonstrations in the dataset are of high quality, and design our method to be robust to sub-optimal data.

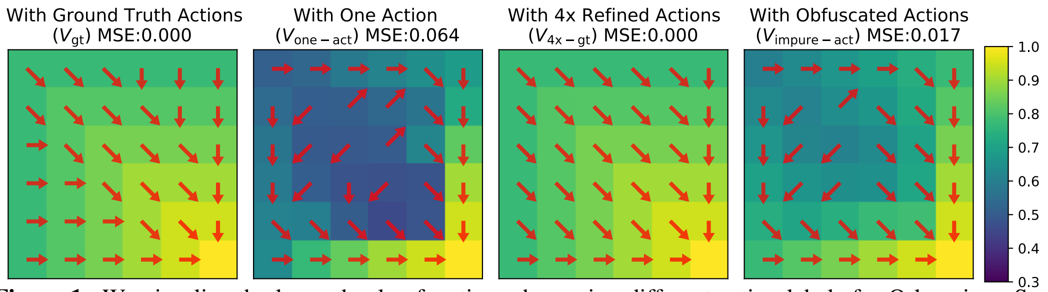


Figure 1: We visualize the learned value function when using different action labels for Q-learning. See Section 3.2 for more details.

In this paper, we will focus on methods based on Q-learning [45] for tackling this problem. Q-learning has the advantage of being *off-policy*, *i.e.*, experience from another policy (or task) can be used to learn or improve a different policy for a different task. Q-learning seeks to learn the optimal Q-function $Q^*(s,a)$ by iteratively updating $Q(s_t,a_t)$ to the Bellman equation. This process converges to the Q^* under mild conditions in many settings [45], and gives the optimal state-value function as $V^*(s) = \max_a Q^*(s,a)$.

3 Characterizing Q-learning without True Action Labels

We characterize the outcome of Q-learning in settings where we don't have ground truth intervening actions in the offline dataset being used for Q-learning. We first consider the case of ignoring the action altogether, which amounts to TD(0) policy evaluation. Next, we study if labeling (s,s',r) samples with actions from a different action space $\hat{\mathbf{A}}$ to construct a new MDP could aid learning. More specifically, can the optimal Q-function for this new MDP, as obtained through Q-learning on samples (s,s',r) labeled with actions from $\hat{\mathbf{A}}$, be useful in the original MDP? We show that under the right conditions the value function learned under the altered action space $\hat{\mathbf{A}}$ is identical to the value function learned for the original MDP.

Q-learning naive action labels (single action): Without action labels, one could simply assign all transitions the same label. In this case, Q-Learning becomes TD(0) policy evaluation. The induced value function isn't the optimal value function for the MDP, but rather the value according to the policy that generated the dataset. Depending on the dataset, this could be sub-optimal.

3.1 Optimality of Action Refinement

Assume we have a Markov Decision Process (MDP) M specified by $(\mathbf{S},\mathbf{A},p,\gamma)$. Let the action space $\mathbf{A}$ be composed of actions $a_1,a_2,a_3,...,a_n \in \mathbf{A}$. We are interested in the value learned under a modified MDP, $\hat{M}$ composed of $(\mathbf{S},\hat{\mathbf{A}},\hat{p},\gamma)$. We will show that if the actions and transitions $\hat{\mathbf{A}}$ and $\hat{p}$ are a *refinement* of $\mathbf{A}$ and p, then the value function learned on $\hat{M}$, $V_{\hat{M}}$ is identical to the value function learned on M, V_M. We define actions and transitions in $\hat{M}$ to be a refinement of those in M when, a) in each state, for every action in $\hat{\mathbf{A}}$, there is at least one action in $\mathbf{A}$ which is functionally identical in the same state, and b) in each state, for each action in $\mathbf{A}$ is represented by at least one action in $\hat{\mathbf{A}}$ in that state.

Definition 3.1 Given a discrete finite MDP, M specified by $(\mathbf{S},\mathbf{A},p)$ and MDP, $\hat{M}$ specified by $(\mathbf{S},\hat{\mathbf{A}},\hat{p})$, $\hat{M}$ is a *refinement* of M when

$$\underset{\hat{a}\in\hat{\mathbf{A}},s\in\mathbf{S}}{\forall}\ \underset{a\in\mathbf{A}}{\exists}\ \underset{s',r}{\forall}\ \hat{p}(s',r|s,\hat{a}) = p(s',r|s,a), \text{ and } \underset{a\in\mathbf{A},s\in\mathbf{S}}{\forall}\ \underset{\hat{a}\in\hat{\mathbf{A}}}{\exists}\ \underset{s',r}{\forall}\ \hat{p}(s',r|s,\hat{a}) = p(s',r|s,a),$$

Note that this definition of refinement requires a *state conditioned* correspondence between action behavior. Actions do not need to have to correspond across states.

Theorem 3.1. *Given a discrete finite MDP, $\hat{M}$ which is a refinement of M (Definition 3.1) then both MDPs induce the same optimal value function, i.e.* $\forall_s V^*_{\hat{M}}(s) = V^*_M(s)$.

We prove this by showing that optimal policies under both MDPs induce the same value function.

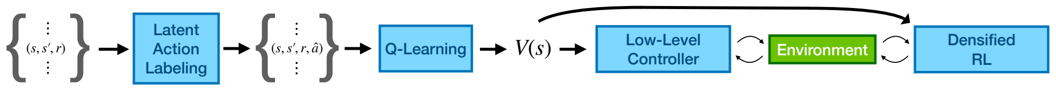


Figure 2: Approach Overview. Our proposed approach Latent Action Q-Learning (LAQ) starts with a dataset of (s, s', r) triples. Using the latent action learning process, each sample is assigned a latent action $\hat{a}$. Q-learning on the dataset of quadruples produces a value function, $V(s)$. Behaviors are derived from the value function through densified RL, or by guiding low-level controllers.

Lemma 3.2. *For any policy π_M on M, there exists a policy $\pi_{\hat{M}}$ on $\hat{M}$ such that $V_{\hat{M}}^{\pi_{\hat{M}}}(s) = V_M^{\pi_M}(s)$, $\forall s$, and for any policy $\pi_{\hat{M}}$ on $\hat{M}$ there exists a policy π_M on M such that $V_{\hat{M}}^{\pi_{\hat{M}}}(s) = V_M^{\pi_M}(s)$ $\forall s$.*

For this lemma we introduce the notion of *fundamental actions*, which are actions which correspond to sets of actions which have the same state and reward transition distributions in a given state. We utilize the equivalence of fundamental actions between MDPs to construct a policy in the new MDP which induces the same value function as a given policy in the original MDP. We provide proofs for Theorem 3.1 and Lemma 3.2 in Section A.1.

3.2 Gridworld Case Study

We validate these results in a tabular grid world setting. In particular, we measure the error in learned value functions and the induced behavior, when conducting Q-learning with datasets with different qualities of intervening actions. The agent needs to navigate from the top left of a 6×6 grid to the bottom right with sparse reward. We generate data from a fixed, sub-optimal policy to evaluate all methods in an offline fashion (additional details in Section A.5). We generate 20K episodes with this policy, and obtain value functions using Q-learning under the following 4 choices for the intervening actions: **(1)** Ground truth actions (V_{gt}), **(2)** One action ($V_{\text{one-act}}$, ammounts to TD(0) policy evaluation), **(3)** $4\times$ refinement of original action space ($V_{4\times\text{-gt}}$). We modify the data so that each sample for a particular action in the original action space is randomly mapped to one of 4 actions in the augmented space. **(4)** Obfuscated actions ($V_{\text{impure-act}}$). Original action with probability 0.5, and a random action with probability 0.5.

Figure 1 shows the learned value functions under these different action labels, and reports the MSE from the true value function, along with induced behavior. In line with our expectations, $V_{4\times\text{-gt}}$ which uses a refinement of the actions is able to recover the optimal value function. $V_{\text{one-act}}$ fails to recover the optimal value function, and recovers the value corresponding to the policy that generated the data. $V_{\text{impure-act}}$, under noise in action labels (non-refinement) also fails to recover the optimal value function. Furthermore, the behavior implied by $V_{\text{impure-act}}$ and $V_{\text{one-act}}$ is sub-optimal. We also analyze the effect of the action impurity on learned values and implied behavior. Behavior becomes increasingly inaccurate as action impurity increases. More details in Section A.3.

4 Latent Action Q-Learning

Our analysis in Section 3 motivates the design of our approach for learning behaviors from state-only experience. Our proposed approach decouples learning into three steps: mining *latent* actions from state-only trajectories, using these latent actions for Q-learning to obtain value functions, and learning a policy to act according to the learned value function. As per our analysis, if learned latent actions are a *state-conditioned refinement* of the original actions, Q-learning will result in good value functions, that will lead to good behaviors. Refer to Algorithm 1 for details.

4.1 Latent Actions from Future Prediction

Given a dataset $\mathbf{D}$ of observations streams $\ldots, o_t, o_{t+1}, \ldots$, the goal in this step is to learn *latent* actions that are a refinement of the actual actions that the agent executed [3]. We learn these latent actions through future prediction. We train a future prediction model f_θ, that maps the observation o_t at time t, and a latent action $\hat{a}$ (from a set $\hat{\mathbf{A}}$ of discrete latent actions) to the observation o_{t+1} at time

[3]We use the terms state (s_t) and observation (o_t) interchangeably.

$t+1$, *i.e.* $f_\theta(o_t, \hat{a})$. f is trained to minimize a loss l between the prediction $f_\theta(o_t, \hat{a})$ and the ground truth observation o_{t+1}. $\hat{a}$ is treated as a latent variable during learning. Consequently, f_θ is trained using a form of expectation maximization [6]. Each training sample (o_t, o_{t+1}) is assigned to the action that leads to the lowest loss under the current forward model. The function f_θ is optimized to minimize the loss under the current latent action assignment. More formally, the loss for each sample (o_t, o_{t+1}) is: $L(o_t, o_{t+1}) \coloneqq \min_{\hat{a} \in \hat{\mathbf{A}}} l\left(f_\theta(o_t, \hat{a}), o_{t+1}\right)$. We minimize $\sum_{(o_t, o_{t+1}) \in \mathbf{D}} L(o_t, o_{t+1})$ over the dataset to learn f_θ.

Latent action $\hat{a}_t$ for observation pairs (o_t, o_{t+1}) are obtained from the learned function f_θ as: $\arg\min_{\hat{a} \in \hat{\mathbf{A}}} l\left(f_\theta(o_t, \hat{a}), o_{t+1}\right)$. Choice of the function f_θ and loss l vary depending on the problem. We use L2 loss in the observation space (low-dimensional states, or images).

4.2 Q-learning with Latent Actions

Latent actions mined from Section 4.1 allow us to complete the given (o_t, o_{t+1}, r_t) tuples into $(o_t, \hat{a}_t, o_{t+1}, r_t)$ quadruples for use in Q-learning [45]. As our actions are discrete we can easily adopt any of the existing Q-learning methods for discrete action spaces (*e.g.* [26]). Though, we note that this Q-learning still needs to be done in an *offline* manner from pre-recorded state-only experience. While we adopt the most basic Q-learning in our experiments, more sophisticated versions that are designed for offline Q-learning (*e.g.* [21, 14]) can be directly adopted, and should improve performance further. Value functions are obtained from the Q-functions as $V(s) = \max_{\hat{a} \in \hat{\mathbf{A}}} Q(s, \hat{a})$.

4.3 Behaviors from Value Functions

Given a value function, our next goal is to derive behaviors from the learned value function. In general, this requires access to the transition function of the underlying MDP. Depending on what assumptions we make, this will be done in the following two ways.

Densified Reinforcement Learning. Learning a value function from state-only experience can be extremely valuable when a dense reward function for the underlying task is not readily available. In this case, using the learned value function can densify sparse reward functions, making previously intractable RL problems solvable. Specifically, we use the value function to create a *potential-based* shaping function $F(s, s') = V(s') - V(s)$, based on [27], and construct an augmented reward function $r'(s, a, s') = r(s, a, s') + F(s, s')$. Our experiments show that using this densified reward function speeds up behavior acquisition.

Domain Specific Low-level Controllers. In more specific scenarios, it may be possible to employ hand designed low-level controllers in conjunction with a model that can predict the next state s' on executing any of low-level controllers. In such a situation, behavior can directly be obtained by picking the low-level controller that conveys the agent to the state s' that has the highest value under the learned $V(s)$. Such a technique was used by [8]. We show results in their setup.

5 Experiments

We design experiments to assess the quality of value functions learned by LAQ from undirected state-only experience. We do this in two ways. First, we measure the extent to which value functions learned with LAQ without ground truth information agree with value functions learned with Q-learning with ground truth action information. This provides a direct quality measure and allows us to compare different ways of arriving at the value function: other methods in the literature (D3G [10]), and simpler alternatives of arriving at latent actions. Our second evaluation measures the effectiveness of LAQ-learned value functions for deriving effective behavior in different settings: when using it as a dense reward, when using it to guide low-level controllers, and when transferring behavior across embodiments. Where possible, we compare to behavior cloning (BC) with *privileged* ground truth actions. BC with ground truth actions serves as an upper bound on the performance of state-only imitation learning methods (BCO from [40], ILPO from [11], *etc.*) and allows us to indirectly compare with these methods.

Test Environments. Our experiments are conducted in five varied environments: the grid world environment from Section 3, the Atari game Freeway from [5], 3D visual navigation in realistic environments from [8, 33], and two continuous control tasks from [13]'s D4RL: Maze2D (2D

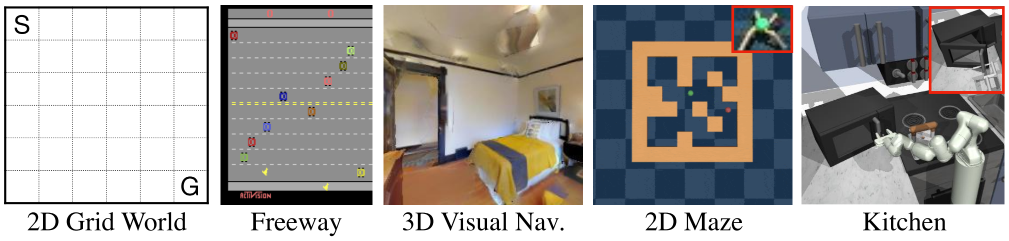


Figure 3: We experiment with five environments: 2D Grid World, Freeway (Atari), 3D Visual Navigation, Maze2D (2D Continuous Control), and FrankaKitchen. Top right corner of Maze2D and FrankaKitchen, shows the embodiments for cross-embodiment transfer (ant and hook, respectively).

Table 1: We report Spearman's correlation coefficients for value functions learned using various methods with DQN, against a value function learned offline using ground-truth actions (DQN for discrete action environments, and DDPG for continuous action environments). The *Ground Truth Actions* column shows Spearman's correlation coefficients between two different runs of offline learning with ground-truth actions. See Section 5.1. Details on model selection in Section A.8.

Environment	D3G	Single Action	Clustering	Clustering (Diff)	Latent Actions	Ground Truth Actions
2D Grid World	0.959	0.093	0.430	**1.000**	0.985	1.000
Freeway	– (image input)	0.886	0.945	0.902	**0.961**	0.970
3D Visual Navigation	– (image input)	0.641	0.722	0.827	**0.927**	0.991
2D Continuous Control	0.673	0.673	0.613	0.490	**0.844**	0.851
Kitchen Manipulation	0.854	0.858	0.818	0.815	**0.905**	0.901

continuous control navigation), and FrankaKitchen (dexterous manipulation in a kitchen). For Maze2D and FrankaKitchen environments, we also consider *embodiment transfer*, where we seek to learn policies for an ant and a hook respectively from the observation-only experience of a point mass and the Franka arm. Together, these environments test our approach on factors that make policy learning hard: continuous control, high-dimensional observations and control, complex real world appearance, 3D geometric reasoning, and learning across embodiments. See visualizations in Figure 3, details in Section A.4.

Experimental Setup For each setting, we work with a pre-collected dataset of experience in the form of state, next state and reward triplets, (o_t, o_{t+1}, r_t). We use our latent-variable forward model (Section 4.1) and label triplets with latent actions to obtain quadruples $(o_t, \hat{a}_t, o_{t+1}, r)$. We perform Q-learning on these quadruples to obtain value functions $V(s)$, which are used to acquire behaviors either through densified RL by interacting with the environment, or through use of domains-specific low-level controllers. We use the ACME codebase [17] for experiments.

Latent Action Quality. In line with the theory developed in Section 3, we want to establish how well our method learns a refinement of the underlying action space. To assess this, we study the *state-conditioned purity* of the partition induced by the learned latent actions. It computes the proportion of the most frequent action of all ground truth actions mapped to a latent action (for any given state). Overall, our method is effective at finding refinement of the original action space. It achieves higher state-conditioned purity than a single action and clustering. In high-dimensional image observation settings, it surpasses baselines by a wide margin. More details in Section A.6.

5.1 Quality of Learned Value Functions

We evaluate the quality of the value functions learned through LAQ. We use as reference the value function $V_{\text{gt-act}}$, obtained through *offline* Q-learning (DDPG for continuous action cases) with true ground truth actions *i.e.* (o_t, a_t, o_{t+1}, r_t).[4] For downstream decision making, we only care about the relative ordering of state values. Thus, we measure the Spearman's rank correlation coefficient between the different value functions. Table 1 reports the Spearman's coefficients of value functions obtained using different action labels: single action, clustering, latent actions (ours), and with ground

[4]Offline DDPG in the FrankaKitchen environment was unstable. To obtain a reference value function, we manually define a value function based on the distance between the end-effector and the microwave handle (lower better), and the angle of the microwave door (higher better). We use this as the reference value function.

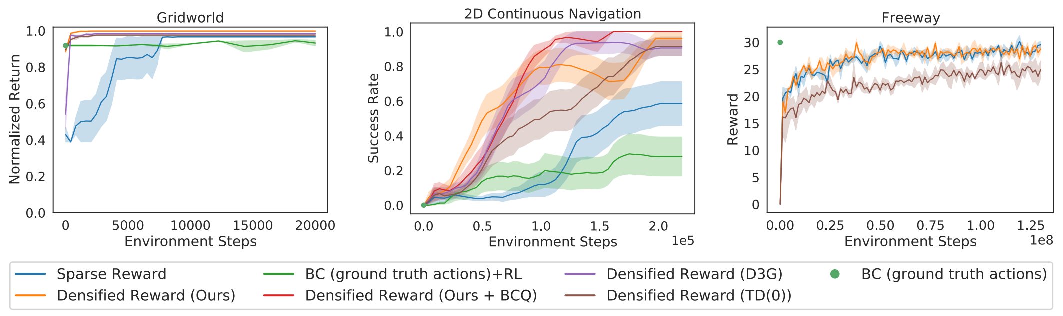


Figure 4: We show learning curves for acquiring behavior using learned value functions. We compare densified RL (Section 4.3) with sparse RL and BC/BC+RL. See Section 5.2 for more details. Results are averaged over 5 seeds and show ± standard error.

truth actions. We also report Spearman's correlations of value functions produced using D3G [10]. In all settings we do Q-learning over the top 8 dominant actions, except for Freeway, where using the top three actions stabilized training.

Our method out performs all baselines in settings with high-dimensional image observations (3D Visual Navigation, Freeway). In state based settings, where clustering state differences is a helpful inductive bias, we see that our method is still on-par with, or superior to clustering state differences and even D3G, which predicts state differences.

5.2 Using Value Functions for Downstream Tasks

Our next experiments test the utility of LAQ-learned value functions for acquiring goal-driven behavior. We first describe the 3 settings that we use to assess this, and then summarize our takeaways.

- **Using value functions as dense reward functions**. We combine sparse task reward with the learned value function as a potential function (Section 4.3). We scale up the sparse task rewards by a factor of 5 so that behavior is dominated by the task reward once policy starts solving the task. Figure 4 measures the learning sample efficiency. We compare to only using the sparse reward, behavior cloning (BC) with ground truth actions, and BC followed by spare reward RL.
- **Using value functions to learn behavior of an agent with a different embodiment.** Decoupling the learning of value function and the policy has the advantage that learned value functions can be used to improve learning across embodiment. We demonstrate this, we keep the same task, but change the embodiment of the agent in Maze2D and FrankaKitchen environments. Note that we do not assume access to ground truth actions in these experiments either. For Maze2D, the point-mass is replaced with a 8-DOF quadrupedal ant. For FrankaKitchen, the Franka arm is replaced with a position-controlled hook. We may need to define how we query the value function when the embodiment (and the underlying state space) changes. For the ant in Maze2D, the location (with 0 velocity) of the ant body is used to query the value function learned with the point-mass. For the hook in FrankaKitchen, the value function is able to transfer directly as both settings observe end-effector position and environment state. We report results in Figure 5.
- **Using value functions to guide low-level controllers.** Learned value functions also have the advantage that they can be used directly at test time to guide the behavior of low-level controllers. We do this experiment in context of 3D visual navigation in a scan of a real building and use the *branching environment* from [8]. We follow their setup and replace their value functions with ones learned using LAQ in their hierarchical policy, and compare the efficiency of behavior encoded by the different value functions.

LAQ value functions speed up downstream learning. Learning plots in Figure 4 show that LAQ-learned value functions speed up learning in the different settings over learning simply with sparse rewards (orange line *vs.* blue line). In all settings except Freeway, our method not only learns more quickly than sparse reward, but converges to a higher mean performance.

LAQ discovers stronger behavior than imitation learning when faced with undirected experience. An advantage of LAQ over other imitation-learning based methods such as BCO [40] and ILPO [11] is LAQ's ability to learn from sub-optimal or undirected experience. To showcase this, we compare the performance of LAQ with behavior cloning (BC) with ground truth actions. Since

Table 2: We report Spearman's correlation coefficients for value functions learned using either DQN or BCQ, against a value function learned offline using BCQ with ground-truth actions. The *Ground Truth Actions* column shows Spearman's correlation coefficients between two different runs of offline learning with ground-truth actions. See Section 5.1.

Environment	Single Action	Clustering	Clustering (Diff)	Latent Actions	Ground Truth Actions
2D Continuous Control (DQN)	0.664	0.431	0.312	**0.807**	0.765
2D Continuous Control (BCQ)	0.710	0.876	0.719	**0.927**	0.990

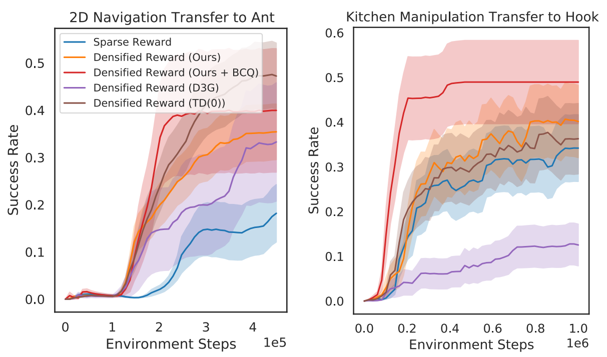
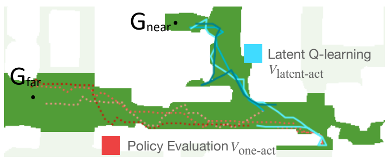


Figure 5: Learning curves for acquiring behavior with value functions across embodiment. Results averaged over 50 seeds and show $\pm$ standard error.


G_{near}
G_{far}
Latent Q-learning $V_{\text{latent-act}}$
Policy Evaluation $V_{\text{one-act}}$

	Interaction Samples	SPL
$V_{\text{one-act}}$ [8]	0	0.53
$V_{\text{cluster-act}}$	0	0.57
$V_{\text{latent-act}}$	0	0.82
$V_{\text{inverse-act}}$ [8]	40K	0.95

Figure 6: Visualization of trajectories and SPL numbers in the 3D visual navigation environment.

BCO and ILPO recover ground truth actions to perform behavior cloning (BC), BC with ground truth actions serves as an upper bound on the performance of all methods in this class. Learning plots in Figure 4 shows the effectiveness of LAQ over BC and BC followed by fine-tuning with sparse rewards for environments where the experience is undirected (Maze2D, and GridWorld). For Freeway, the experience is fairly goal-directed, thus BC already works well. A similar trend can be seen in the higher Spearman's coefficient for LAQ *vs.* $V_{\text{one-act}}$ in Table 1. LAQ discovers stronger behavior than imitation learning when faced with undirected data.

LAQ is compatible with other advances in batch RL. Although LAQ uses the most basic Q-Learning as our offline value learning method, it is compatible with recent more advanced offline RL value-learning methods (such as CQL [21] and BCQ [14]). We validate by simply swaping to using (discrete) BCQ with our latent actions. Figures. 4 and 5 show that LAQ with BCQ is the strongest method, outperforming ours with DQN, and D3G, on Maze2D and embodiment transfer environments. Analysis of Spearman's correlations in Table 2 shows the same trend as before with latent actions: better than single actions, and clustering variants. Note also that use of BCQ leads to value functions with better Spearman's correlations than DQN.

LAQ value functions allow transfer across embodiments. Figure 5 shows learning plots of agents trained with cross-embodiment value functions. LAQ-densified rewards functions, speed-up learning and consistently guide to higher reward solutions than sparse task rewards, or D3G.

LAQ compares favorably to D3G. We compare LAQ and D3G (a competing state-only method) in different ways. D3G relies on generating potential future states. This is particularly challenging for image observations, and D3G doesn't show results with image observations. In contrast, LAQ maps state transitions to discrete actions, and hence works with image observations as our experiments show. Even in scenarios with low-dimensional state inputs, LAQ learns better value functions than D3G, as evidences by Spearman's correlations in Table 1, and learning plots in Figure 4 and Figure 5.

LAQ value functions can guide low-level controllers for zero-shot control: We report the SPL for 3D navigation using value functions combined with low-level controllers in Figure 6. We report the efficiency of behavior induced by LAQ learned value functions as measured by the SPL metric from [3] (higher is better). The branching environment has two goal states, one optimal and one sub-optimal. The demonstrations there-in were specifically designed to emphasize the utility of knowing the intervening actions. Simple policy evaluation leads sub-optimal behavior (SPL of 0.53) and past work relied on using an inverse model to label actions [8] to derive better behavior. This inverse model itself required $40K$ interactions with the environment for training, and boosted the SPL to 0.95. LAQ is able to navigate to the optimal goal (w/ SPL 0.82) but without the $40K$ online interaction samples necessary to acquire the inverse model. It also performs better than clustering transitions, doing which achieves an SPL of 0.57. The improvement is borne out in visualizations in Figure 6. LAQ correctly learns to go to the nearer goal, even when the underlying experience came from a policy that preferred the further away goal.

6 Discussion

Our theoretical characterization and experiments in five representative environments showcase the possibility and potential of deriving goal-directed signal from undirected state-only experience. Here we discuss some scenarios which are fundamentally hard, and some avenues for future research.

Non-deterministic MDPs. Our theoretical result relies on a refinement where state-action transition probabilities are matched. However, the latent action mining procedure in LAQ results in deterministic actions. Thus, for non-deterministic MDPs (where executing the same action in the same state takes the agent to different next state), LAQ will be unable to achieve a strict refinement, leading to sub-optimal value functions. However, note that this limitation isn't specific to our method, but applies to *any* deterministic algorithm that seeks to learn from observation only data. We formalize this concept and provide a proof in Section A.2.

Constraining evaluation of $V(s)$ to within its domain. LAQ learns a value function $V(s)$ over the set of states that were available in the experience dataset, and as such its estimates are only accurate within this set. In situations where the experience dataset doesn't span the entire state space, we may need to assess where the predictions of $V(s)$ are valid to avoid degenerate solutions due to OOD observations. We discuss a density based model solution we used for this problem in Section A.4.

Offline RL Validation. Validation (*e.g.* when to stop training) is a known issue in offline RL [16]. Like other offline RL methods, LAQ suffers from it too. LAQ's use of Q-learning makes it compatible to recent advances [20] that tackle this validation problem.

7 Related Work

Our work focuses on batch (or offline) RL with state-only data using a latent-variable future prediction model. We survey works on batch RL, state-only learning, and future prediction.

Batch Reinforcement Learning. As the field of reinforcement learning has matured, batch RL [22, 23] has gained attention as a component of practical systems. A large body of work examines solutions the problem of extrapolation error in batch RL settings. Advances in these works are complementary to our approach, as substantiated by our experiments with BCQ. A more detailed discussion of batch RL methods can be found in Section A.0.

State-only Learning. Past works have explored approaches for dealing with the lack of actions in offline RL when given *goal-directed* or *undirected* state-only experience. Works in the former category rely on high quality behavior in the data, and suffer on sub-optimal data. Past work on state-only learning from undirected experience relies on either domain knowledge or state reconstruction and only show results with low dimensional states. See Section A.0 for continued discussion.

Future Prediction Models. Past work from [28, 2, 12] (among many others) has focused on building action conditioned forward models in pixel and latent spaces. Yet other work in computer vision studies video prediction problems [47, 7]. Given the uncertainty in future prediction, these past works have pursued variational (or latent variable) approaches to make better predictions. Our latent variable future model is inspired from these works, but we explore its applications in a novel context.

Latent MDP Learning One way to interpret our method is that of learning an approximate MDP homomorphism [39, 32]. Other works have explored learning latent homorphic MDPs. These methods tend to focus on learning equivalent latent state spaces [24, 15]. Most similarly to our work [44] also learns a latent action space, but relies on ground truth action data to do so.

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