SPARSE ISO-FLOP TRANSFORMATIONS FOR MAXIMIZING TRAINING EFFICIENCY

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ABSTRACT

Recent studies have explored the application of weight sparsity to enhance the training efficiency of DNNs in terms of test accuracy w.r.t training FLOPs. These studies have focused on reducing training FLOPs, but training with sparse weights often results in accuracy degradation or necessitates prolonged training schedules to attain performance similar to the original dense models; making the actual training efficiency gains less evident. In contrast, our work emphasizes leveraging sparsity to increase accuracy while maintaining the same FLOPs as the dense model, thereby demonstrating improved training efficiency through higher accuracy. We introduce Sparse-IFT, a family of Sparse Iso-FLOP Transformations that serve as drop-in replacements for dense layers, enhancing their representational capacity and FLOP efficiency. Each transformation is parameterized by a single hyperparameter (i.e., sparsity level), offering a broader search space for identifying optimal sparse masks. Substituting dense layers with Sparse-IFT, without altering any training hyperparameters, yields substantial improvements across a range of computer vision and natural language processing tasks; ResNet-18 on ImageNet (+3.5%) and GPT-3 Small on WikiText-103 (-0.4 PPL), both matching larger dense models that use 2x or more FLOPs. To our knowledge, this is the first work to demonstrate the use of sparsity for improving the accuracy of dense models, all while maintaining consistent training FLOPs budgets via a simple set of sparse transformations.

1 INTRODUCTION

Increases in model size and training data have led to many breakthroughs in deep learning (e.g., AlexNet (Krizhevsky et al., 2012), ResNet (He et al., 2016), Transformers (Vaswani et al., 2017), GPT (Radford et al., 2018; 2019), AlphaGo (Silver et al., 2017), etc.). Consequently, computational and memory demands for training and deploying deep neural networks (DNNs) have surged dramatically. To enable the deployment of large models, multiple techniques (e.g., distillation (Hinton et al., 2015), quantization (Han et al., 2015a), pruning (Han et al., 2015b)) have been introduced to reduce inference FLOPs and memory requirements. While these techniques improve inference efficiency (test accuracy w.r.t inference FLOPs), the associated training costs are still prohibitive. In this work, we focus on improving the training efficiency (test-accuracy w.r.t training FLOPs) of DNNs.

Recent works (Evci et al., 2020; Jayakumar et al., 2020) have explored using weight sparsity to reduce the FLOPs spent in training. Frankle & Carbin (2018) demonstrate that sparse subnetworks (termed "lottery tickets") exist at initialization and can be trained to match the accuracy of their original dense network. Inspired by this result, various dynamic sparse training (DST) methods (Ma et al., 2022; Evci et al., 2020; Liu et al., 2021b; Jayakumar et al., 2020) attempt to find optimal sparse subnetworks within a training run. While these methods primarily aim to improve training efficiency by reaching dense accuracy with fewer FLOPs, they often perform worse than their dense baselines or rely on longer training schedules (up to 2-5$\times$ training iterations) to close the gap (Yuan et al., 2021; Tai et al., 2022; Liu et al., 2021a). As a result, these techniques can sometimes even require more FLOPs than training the dense model (Ma et al., 2022; Evci et al., 2020; Jayakumar et al., 2020). Our aim is to highlight our unique contribution in utilizing sparsity to enhance standard dense model accuracy, distinguishing our work from previous research. While past studies focused on pruning techniques to improve accuracy of pre-trained dense models (Han et al., 2015b; Liu et al., 2017; Molchanov et al., 2017), our innovation lies in demonstrating sparsity's impact on accuracy when training from scratch within the same training FLOP budget as dense models. Specifically, we introduce a family of Sparse Iso-FLOP Transformations (Sparse-IFT) that can be used as drop-in replacements for dense layers in DNNs.

These transformations increase the representational capacity of layers and facilitate the discovery of optimal sparse subnetworks without changing the layer's underlying training and inference FLOPs (i.e., Iso-FLOP). For example, making a layer wider but sparser increases dimensionality while still maintaining FLOPs due to sparsity. All Sparse-IFT members are parameterized by a single hyperparameter, the sparsity level. Figure 1 summarizes the ImageNet performance with ResNet models, where our Sparse Wide IFT variants significantly increase the accuracy of matching Iso-FLOP dense models. In particular, Sparse Wide ResNet-18 at 90% sparsity improves the top-1 accuracy from 70.9% to 74.4% (+3.5%), and outperforms a dense ResNet-34 (74.2%) while using 2x fewer FLOPs. We emphasize that these gains were obtained by replacing dense layers with transformations from the Sparse-IFT family and required no changes to training hyperparameters. The main contributions of our work are:

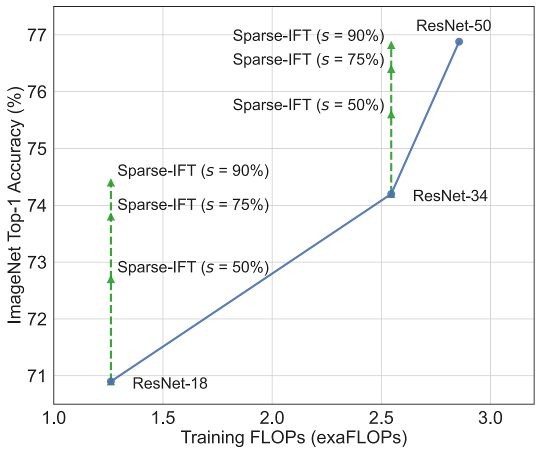

Figure 1: Accuracy vs. Training FLOPs for different variants of ResNet on ImageNet. Sparse-IFT provides significant accuracy gains across different models and sparsity levels while using the same FLOP budget as its dense counterpart.

1. We introduce Sparse Iso-FLOP Transformations (Sparse-IFTs), a family of techniques aimed at enhancing DNN training efficiency. These transformations boost accuracy while maintaining a constant FLOP count. Sparse-IFTs are characterized by a *single hyperparameter, sparsity level*, and can be seamlessly used as drop-in replacements for dense layers.
2. In the CV domain, using Sparse-IFT increases the top-1 accuracy of ResNet-18 and ResNet-34 by 3.5% and 2.6% respectively on ImageNet. Finetuning these pre-trained models for object detection (MS COCO) and segmentation (CityScapes) leads to an improvement of 5.2% mAP and 2.4% mIoU, respectively.
3. In the NLP domain, using Sparse-IFT with GPT-3 Small leads to a 0.4 perplexity improvement on the WikiText-103 language modeling task, and matches the PPL of a dense GPT-3 Medium while using 2.4x fewer training FLOPs.

2 Method

In this section, we present our method to improve training efficiency. We first explain our intuition and hypotheses, followed by our methodology.

2.1 Training with Dense Matrices is FLOP Inefficient

Prior research indicates that modern DNNs are overparameterized, and they exhibit sparsity in both features and weights across layers. The Lottery Ticket Hypothesis (LTH) Frankle & Carbin (2018) demonstrates that sparse DNNs can achieve the same accuracy as dense counterparts when initialized with an effective sparsity mask ("lottery ticket"). These findings emphasize the advantage of sparse weight configurations over dense matrices during training. While sparse training methods are theoretically more efficient, their practical application often results in lower accuracy compared to dense baselines. This discrepancy may be attributed to the challenges of identifying "lottery tickets" within a single training run. While sparse models reduce the FLOPs needed per step, we hypothesize that existing sparse training methods make sub-optimal use of these computational savings. For example, state-of-the-art sparse training methods Jayakumar et al. (2020); Evci et al. (2020); Yuan et al. (2021); Tai et al. (2022); Liu et al. (2021a) invest these FLOP savings into longer training

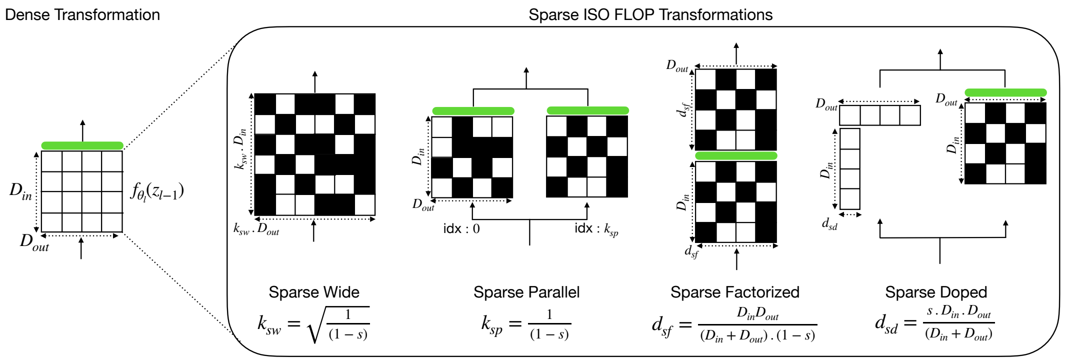
Dense Transformation
Sparse ISO FLOP Transformations
D_{in}
D_{out}
$f_{\theta_l}(z_{l-1})$
$k_{sw} . D_{in}$
$k_{sw} . D_{out}$
idx : 0
idx : k_{sp}
d_{sf}
d_{sd}
Sparse Wide
$k_{sw} = \sqrt{\frac{1}{(1-s)}}$
Sparse Parallel
$k_{sp} = \frac{1}{(1-s)}$
Sparse Factorized
$d_{sf} = \frac{D_{in}D_{out}}{(D_{in}+D_{out}).(1-s)}$
Sparse Doped
$d_{sd} = \frac{s . D_{in} . D_{out}}{(D_{in}+D_{out})}$

Figure 2: Different members of the Sparse-IFT family. Transformation of all members is parameterized by a single hyperparameter (i.e., sparsity level (s)). Black and white squares denote sparse and active weights, respectively. Green block indicates a non-linear activation function (e.g., BatchNorm, ReLU, LayerNorm). All transformations are derived with sparsity set to 50% as an example, are Iso-FLOP to the dense feedforward function f_{θ_l}, and hence can be used as a drop-in replacement of f_{θ_l}. See Section 2.4 for more details about each member.

schedules to close the accuracy gap and compensate for the inability to discover an optimal mask earlier in training. This setup is inefficient since it ultimately requires more training FLOPs than the dense baseline to reach the same target accuracy. In our work, we take an orthogonal approach and invest these FLOP savings into (a) increasing the representational capacity of a layer and (b) increasing its search space, which we hypothesize can facilitate the discovery of an optimal sparse mask Ramanujan et al. (2020); Stosic & Stosic (2021). While utilizing larger sparsity-enabled models has exhibited accuracy improvement potential, the challenge lies in designing an appropriate architecture. For instance, when aiming to surpass the ResNet-18 performance on ImageNet, finding the right sparsity and larger network design is crucial. Many studies explore diverse combinations to balance sparsity and network size for outperforming dense models. However, these methods often lack FLOP efficiency, requiring multiple iterations for optimal settings and hyperparameter tuning. Therefore, we propose replacing dense transformations with FLOP-equivalent sparse transformations. We denote these transformations as the Sparse Iso-FLOP Transformation (Sparse-IFT) family.

2.2 Setup

For clarity, we will explain our method for a fully connected neural network. In Appendix A.1, we detail the straightforward extension of our method to convolutional layers. Let $\mathcal{N}$ denote a L layered DNN parameterized by $\Theta_{\mathcal{N}}$. Let $\Theta_{\mathcal{N}} \in \{\theta_1, ..., \theta_L\}$ denote the parameters of the DNN. The output of the l-th layer is defined as: $z_l = \sigma(f_{\theta_l}(z_{l-1}))$ for some activation function σ (e.g., ReLU Nair & Hinton (2010)) and feedforward function f_{θ_l}. Specifically, let $f_{\theta_l}(z_{l-1}) = \theta_l^T z_{l-1}$, where $\theta_l \in \mathbb{R}^{D_{in} \times D_{out}}$, $z_{l-1} \in \mathbb{R}^{D_{in} \times B}$ and B, D_{in}, D_{out} denote the batch-size, input, and output dimensionality of features respectively. The total FLOPs needed for f_{θ_l} are given by $B \cdot D_{in} \cdot D_{out}$.

2.3 Sparse Iso-FLOP Transformations

In the standard setup, the feedforward function f_{θ_l} computes the output features as a linear transformation of input features. From a theoretical perspective, the feedforward function can make use of arbitrary non-linear transformations. However, in practice, most transformations are expressed as dense matrix multiplications due to widespread support on GPUs (Nvidia, 2023). As stated before, we are interested in improving the training efficiency of DNNs, by enhancing the representational capacity of the feedforward function. Naively increasing the representational capacity by stacking more layers Lin et al. (2014a), increasing width Zagoruyko & Komodakis (2016), mixture of experts Shazeer et al. (2016), etc. increases the computational FLOPs. In our work, we use unstructured sparsity in weight matrices and ensure that the FLOPs of the transformation are the same as that of a dense feedforward function. Let Ψ_l denote the set of Sparse Iso-FLOP Transformations (Sparse-IFT) for a particular layer l:

$$\Psi_l : \{\psi_l(s), 0 \leq s < 1, g(\psi_l) \approx g(f_{\theta_l})\},$$

where ψ_l is a transformation, s represents the sparsity level, and $g(\cdot)$ returns the computational FLOPs. Each transformation in this set satisfies the following properties: (1) the computational FLOPs of the transformation ψ_l are same as that of dense transformation f_{θ_l}, and (2) the transformation is parameterized by a single hyperparameter - the sparsity level. Since these transformations are Iso-FLOP to the dense feedforward function, we can use them as drop-in replacements without affecting the FLOPs of a layer. While there may be other FLOP-invariant transformations, in this work, we detail four different members: Sparse Wide, Sparse Parallel, Sparse Factorized, and Sparse Doped.

2.4 Members of Sparse-IFT

Sparse Wide The sparse wide transformation augments the representational capacity of a layer by increasing the number of output features while keeping s fraction of weights sparse. When using this transformation, we widen the input and output features for all the L layers of the network with the same widening factor, k_{sw}, to avoid a mismatch in feature dimensionality across layers. Let $\theta_l^{sw} \in \mathbb{R}^{k_{sw} \cdot D_{in} \times k_{sw} \cdot D_{out}}$ denote the transformation matrix, with s fraction of weights being sparse. Since the fraction of non-sparse weights is given by $1 - s$, the FLOPs required by this transformation are $B \cdot (k_{sw} \cdot D_{in}) \cdot (k_{sw} \cdot D_{out}) \cdot (1 - s)$. Setting these equal to the FLOPs of the original dense f_{θ_l}, we obtain the widening factor $k_{sw} = \sqrt{\frac{1}{(1-s)}}$. If we set the sparsity s to 0, we obtain k_{sw} as 1 and recover the original dense feedforward function.

Sparse Parallel The sparse parallel transformation replaces the feedforward function with a sum of k_{sp} non-linear functions. Let $\theta_l^{sp} \in \{\theta_l^{sp,1}, ..., \theta_l^{sp,k_{sp}}\}$ denote the parameters of this transformation, where $\theta_l^{sp,j} \in \mathbb{R}^{D_{in} \times D_{out}}$ denotes the transformation matrix of j^{th} function, where s fraction of weights are sparse. The sparse parallel transformation in this case is $\psi_l^{sp} = \sum_{j=1}^{k_{sp}} \sigma((\theta_l^{sp,j})^T z_l)$, where σ is a non linear function. In practice, ψ_l^{sp} is implemented as a layer with k_{sp} parallel branches. The computational FLOPs of this transformation is $k_{sp} \cdot B \cdot D_{in} \cdot D_{out} \cdot (1 - s)$. Setting these FLOPs equal to FLOPs of f_θ, we obtain $k_{sp} = \frac{1}{(1-s)}$. Note, at $s = 0$, the number of parallel branches k_{sp} is 1. If we replace the non-linear function σ with Identity, we can recover the original dense feedforward transformation.

Sparse Factorized The transformation matrix of the feedforward function f_{θ_l} is denoted by $\theta_l \in \mathbb{R}^{D_{in} \times D_{out}}$. Multiple works have explored matrix factorization techniques to express the transformation matrix θ_l as a product of two matrices $\theta_l = UV^T$, where $U \in \mathbb{R}^{D_{in} \times d}$, $V \in \mathbb{R}^{D_{out} \times d}$. Khodak et al. (2020); Tai et al. (2016) and Chen et al. (2021b) have explored low-rank factorization ($d << D_{out}$) as a form of structured sparsity to improve training and inference efficiency, while Arora et al. (2018) and Guo et al. (2020a) have explored overparameterized factorizations for better generalization and faster convergence. In contrast, we use factorization to augment the representational capacity without decreasing or increasing the FLOPs. More precisely, let $\theta_l^{sf} \in \{U_l, V_l\}$ denote the parameters of this transformation, where $U_l \in \mathbb{R}^{D_{in} \times d_{sf}}$, $V_l \in \mathbb{R}^{d_{sf} \times D_{out}}$ are sparse matrices with s fraction of their weights being sparse. The functional transformation in this case is $\psi_l^{sf} = V_l^T \sigma(U_l^T z_l)$. The computational FLOPs of this transformation is $d_{sf} \cdot B \cdot (D_{in} + D_{out}) \cdot (1 - s)$. Setting these FLOPs equal to FLOPs of f_{θ_l}, we obtain $d_{sf} = \frac{D_{in} \cdot D_{out}}{(D_{in} + D_{out}) \cdot (1-s)}$. Note, setting sparsity $s = 0$, we recover a non-linear low-rank factorization with dense matrices.

Sparse Doped family of transformation is inspired by works Chen et al. (2021a); Thakker et al. (2021); Udell & Townsend (2019); Candès et al. (2011) which approximate a dense matrix with a combination of low-rank factorization and sparse matrix. In our work, we replace the feedforward function with low-rank factorization (with rank d_{sd}) and an unstructured sparse weight matrix (with sparsity s). Let $U_l \in \mathbb{R}^{D_{in} \times d_{sd}}$, $V_l \in \mathbb{R}^{d_{sd} \times D_{out}}$ denote the low-rank matrices, and $\theta_l^{sd} \in \mathbb{R}^{D_{in} \times D_{out}}$ denote the matrix with unstructured sparsity. The functional transformation, in this case, is given by $\psi_l^{sd} = V_l^T (U_l^T z_l) + \sigma((\theta_l^{sd})^T z_l)$. The computational FLOPs associated with this transformation are $B \cdot d_{sd} \cdot (D_{in} + D_{out}) + (1 - s) \cdot B \cdot D_{in} \cdot D_{out}$. Setting these FLOPs equal to FLOPs of f_{θ_l}, we obtain $d_{sd} = \frac{s \cdot D_{in} \cdot D_{out}}{(D_{in} + D_{out})}$. Note, as $s \to 0$ and $d_{sd} \to 0$, the low-rank component of the transformation disappears, and we can recover the dense feedforward function as a special case by setting σ to Identity.

2.5 Cardinality of Search Space

One of our hypotheses is that increasing the search space of the sparsity mask via Sparse-IFT can make training more efficient. Results from past work support this hypothesis. Ramanujan et al. (2020) demonstrate that the odds of finding a lottery ticket in a randomly initialized network increase with the width of a network. Liu et al. (2022b) and Stosic & Stosic (2021) show that increasing the search space by increasing width or depth improves accuracy. In our work, we define the cardinality of a search space as the number of weights a sparse training method can explore. Table 1 characterizes the cardinality of search space for each member of the Sparse-IFT family. The search space for Sparse Wide, Sparse Parallel, and Sparse Factorized transformations increase proportional to the width scaling factor, number of parallel branches, and size of intermediate hidden dimension, respectively. Sparse Doped transformation splits its computational FLOPs between low-rank factorization and unstructured sparse weight matrix. The size of the unstructured weight matrix is invariant to sparsity; thus cardinality of search space for this transformation is constant.

Table 1: Cardinality of search space of sparsity mask for different members of the Sparse-IFT family.

Transformation	Cardinality of Search Space
Sparse Wide	$(k_{sw})^2 \cdot (D_{in} \cdot D_{out})$
Sparse Parallel	$k_{sp} \cdot (D_{in} \cdot D_{out})$
Sparse Factorized	$d_{sf} \cdot (D_{in} + D_{out})$
Sparse Doped	$D_{in} \cdot D_{out}$

3 Experiments

In this section, we demonstrate how transformations from the Sparse-IFT Family lead to improvements across a variety of different tasks in the CV and NLP domains. First, in Section 3.2, we describe the experimental setups and validate the design choices through multiple ablation studies on CIFAR-100 Krizhevsky et al. (2009), followed by results on ImageNet Krizhevsky et al. (2012). Then, in Section 3.5, we highlight the advantages of pre-training with Sparse-IFT through gains on downstream tasks. Next, we present the benefits of Sparse-IFT in the NLP domain by demonstrating results on GPT Brown et al. (2020) in Section 3.6. Unless stated otherwise, the results presented below are obtained by replacing all dense layers with a given transformation from the Sparse-IFT family while only tuning the sparsity level. All sparse models are trained using a uniform sparsity distribution (i.e., all layers have the same sparsity level). We adopt the default hyperparameters from RigL Evci et al. (2020) for dynamic sparsity. More details about the setup can be found in Appendix B.2.

3.1 Implementation Details

Computer Vision We evaluate our method on CIFAR-100 and ImageNet using CNNs and hybrid Vision Transformer (ViT) networks. We follow published training settings for CIFAR-100 DeVries & Taylor (2017) and ImageNet Nvidia (2019b). For both datasets, we follow the standard evaluation procedures and report the top-1 accuracy. Details for model architectures, datasets, and training hyperparameters are given in Appendix B.2. All standard deviation was reported over 3 random seeds. However, for a select few computationally expensive experiments, we report results from a single run due to limited computational budget.

Natural Language Processing We evaluate Sparse-IFT by training GPT-3 Small (Brown et al., 2020) from scratch on the WikiText-103 (Merity et al., 2017) language modeling task, a commonly used NLP benchmark dataset. The compute cost and resources for training quickly become prohibitive when transforming GPT models with Sparse-IFT. Hence, we train our GPT models on the Cerebras CS-2 (Lie, 2022a;b) and leverage its ability to accelerate training with unstructured sparsity.

3.2 Results and Ablations on CIFAR-100

In this section, we conduct various ablations to validate our design choices. Unless stated otherwise, all experiments below are with ResNet-18 architecture on CIFAR-100.

Importance of Dynamic Sparsity All members of the Sparse-IFT family utilize transformations with unstructured sparsity. This study investigates the importance of the sparse training method when training different configurations of Sparse-IFT architectures. For this analysis, we focus on the Sparse Wide IFT and evaluate it with transformations obtained with sparsity $\in \{50\%, 75\%, 90\%\}$ using three sparse training methods: static sparsity, SET Mocanu et al. (2018) and RigL Evci et al. (2020). RigL and SET are dynamic sparse training methods in which the sparsity mask evolves during training. The key difference is that RigL updates the mask based on gradient information, whereas SET updates the mask randomly. Results of our ablation are documented in Table 2. Here, the following trends can be observed: 1) the Sparse Wide IFT outperforms dense baselines across all operating points (sparsity and sparse training method), 2) dynamic sparse training methods (RigL and SET) obtain higher accuracies compared to training with static sparsity, and 3) gains with static sparsity plateau at lower levels of sparsity, while dynamic sparse training methods gain accuracy at higher sparsities. As mentioned in Section 2.5, Sparse-IFT transformations increase the search space $\propto$ sparsity. Dynamic sparse training methods can explore and exploit this increased search space Stosic & Stosic (2021) and therefore outperform training with static sparsity. Out of the two dynamic sparse training methods evaluated in our study, RigL consistently outperforms SET. Therefore, we use RigL as our sparse training method for all the experiments reported below.

Table 2: Sparse Wide IFT using various sparse training methods with ResNet-18 on CIFAR-100 across different levels of sparsity (columns). Best accuracy for each sparse training method is highlighted in bold.

Dense	Sparse Method	0.50	0.75	0.90
77.0 ± 0.2	Static	**78.5 ± 0.3**	78.3 ± 0.1	78.2 ± 0.3
	SET	78.8 ± 0.1	79.2 ± 0.2	**79.8 ± 0.2**
	RigL	79.1 ± 0.2	79.5 ± 0.1	**80.1 ± 0.2**

Importance of Using Non-Linear Activations Some members of the Sparse-IFT family are inspired by recent works which overparameterize the feedforward function during training and fold it back into a single dense matrix post training Ding et al. (2021b;a); Guo et al. (2020a); Ding et al. (2019). Although these works show the benefits of linear overparameterization, this comes at the cost of a significant increase in training FLOPs. In contrast, while we also increase the representational capacity of the feedforward function, we do so with an Iso-FLOP transformation. Since we remain Iso-FLOP to the original dense model, we do not require post-training modifications to collapse weight matrices for inference efficiency. This uniquely allows us to use non-linearities (e.g., ReLU) in members of the Sparse-IFT family to enhance the representational capacity of the network further. We validate the importance of this design choice by training ResNet-18 with Sparse Factorized IFT with and without non-linearities, and observe significant accuracy gains across all sparsity levels when using non-linear activations. For example, at 90% Sparse Factorized, using non-linearity, we see a 1.8% gain in test accuracy over the ResNet-18 CIFAR-100 dense baseline, compared to a drop of 0.5% without it. These findings hold for other members of the Sparse-IFT family as well (see Appendix B.1 for more details).

Sparse-IFT ResNet-18 Here, we evaluate different members of the Sparse-IFT family on ResNet-18 and CIFAR-100 across different sparsity levels. Table 3 highlights the best accuracy achieved by each member of the Sparse-IFT family. Compared to the accuracy of the dense baseline (77%), all Sparse-IFT members obtain significant accuracy improvements using the same FLOPs as the dense model. We note that the Sparse Doped transformation is the only member of the Sparse-IFT family which does not gain accuracy at higher levels of sparsity. We hypothesize that this phenomenon occurs due to two reasons: (a) cardinality of the search space of the sparsity mask does not increase with sparsity level (see Table 1), and (b) the number of active weights in the unstructured matrix decreases $\propto$ sparsity. In Appendix B.3.1, we compare Sparse-IFT against other baselines obtained with sparse training methods (e.g., RigL and SET) under the same training efficiency setup. Specifically, we train ResNet-18 model on CIFAR-100 at sparsity levels $\in \{50\%, 75\%, 90\%\}$, and ensure that these runs use the same FLOPs as the dense baseline by

Table 3: Sparse-IFT families on CIFAR-100 with ResNet-18 model across different levels of sparsity (columns). Best accuracy of each transformation is highlighted in bold.

Dense	Transformation	0.50	0.75	0.90
77.0 ± 0.2	Sparse Wide	79.1 ± 0.2	79.5 ± 0.1	**80.1 ± 0.2**
	Sparse Factorized	77.8 ± 0.2	78.4 ± 0.5	**78.9 ± 0.5**
	Sparse Parallel	77.9 ± 0.4	**79.1 ± 0.2**	78.2 ± 0.2
	Sparse Doped	**78.2 ± 0.1**	77.8 ± 0.1	76.9 ± 0.2

extending the training iterations. Our results show that Sparse-IFT outperforms these competitive baselines by a significant margin.

Sparse-IFT vs. Dense Overparametrization The success of Sparse-IFT members can be attributed to efficient exploration of large search space with sparsity. Training this large search space in a dense manner leads to consumption of more training FLOPs than the dense baseline, but provides us with the upperbound (in terms of accuracy) for a sparse subnetwork. In this section, we will charactetrize this gap between the Sparse-IFT members and their dense counterpart. In Table 4, we compare the sparse and dense counterparts of the two best performing Sparse-IFT members. For both members, training in dense or sparse manner, leads to similar accuracy across all sparsity levels. This result demonstrates that training with sparsity allows for efficient exploration and exploitation of overparameterized space without incurring the computational cost of dense training of large neural networks. For example, dense runs (with transformations achieved with 90% sparsity) consume 10x more FLOPs compared to sparse runs.

Table 4: Sparse-IFTs trained in a sparse and dense manner on CIFAR-100 with ResNet-18 for different levels of sparsity.

Transformation	Train Method	0.50	0.75	0.90
Sparse Wide	Sparse	79.1 ± 0.2	79.5 ± 0.1	**80.1 ± 0.2**
	Dense	78.9 ± 0.2	79.7 ± 0.1	**80.2 ± 0.3**
Sparse Parallel	Sparse	77.9 ± 0.4	**79.1 ± 0.2**	78.2 ± 0.2
	Dense	78.1 ± 0.2	**78.9 ± 0.1**	78.1 ± 0.1

Unstructured vs. Structured Sparsity We compare unstructured sparsity to structured sparsity with Sparse-IFT. In theory, for a fixed number of non-zero elements in a sparse mask, the use of unstructured sparsity can search over all the possible variations of the mask. However, since most hardware accelerators are not able to accelerate computations with unstructured sparsity, multiple works have investigated training with structured sparsity (e.g., low-rank and block-sparse matrices) to obtain wall-clock speed-ups Khodak et al. (2020); Tai et al. (2016); Chen et al. (2021b); Hubara et al. (2021); Dao et al. (2022); Chen et al. (2022a). We study structured sparsity by deriving Iso-FLOP configurations using low-rank and block sparsity with Sparse Wide IFT. We use the method proposed in Hubara et al. (2021) to search N:M transposable sparsity, which can accelerate training on GPUs with Tensor Cores. In our evaluation, the low-rank factorization results were worse than block sparsity (see more details in Appendix B.3.3). Table 5 compares unstructured sparsity to block sparsity. Although using Sparse-IFT with block sparse matrices lead to improvements over the dense baseline, unstructured sparsity achieves the highest gains. This result can be explained by the fact that block-sparse matrices have reduced mask diversity (Hubara et al., 2021) compared to unstructured sparse matrices.

Table 5: Sparse Wide IFT with unstructured and structured sparsity across different levels of sparsity (columns) on CIFAR-100 with ResNet-18.

Dense	Sparsity Pattern	0.50	0.75	0.90
77.0 ± 0.2	Unstructured	79.1	79.5	**80.1**
	N:M Block Sparse	77.1	**78.4**	78.1

3.3 Results with Efficient Architectures

To further understand the robustness of Sparse-IFT across different model families, we evaluate Sparse-IFT on architectures that are optimized for efficient inference (MobileNetV2 (Sandler et al., 2018) and MobileViT (Mehta & Rastegari, 2021)) and efficient training (BotNet (Srinivas et al., 2021)). We transform the dense layers in these architectures with Sparse Wide IFT and evaluate them at different sparsity levels. We observe a noticeable increase in test accuracy across all architectures (see Table 6). In addition, we demonstrate the robustness of the Sparse-IFT family by also applying the Sparse Parallel transformation and show consistent improvement across all architectures (see Appendix B.3.2). We evaluate the best performing architecture (BotNet-50) on ImageNet (see Section 3.4). The details of the experimental setup can be found in Appendix B.2.

Table 6: Sparse Wide IFT with various efficient architectures on CIFAR-100 across different levels of sparsity (columns).

Model	Dense	0.50	0.75
MobileNetV2	72.4 ± 0.2	73.4	**73.7**
MobileViT-S	73.5 ± 0.1	74.6	**74.8**
BotNet-50	79.8 ± 0.2	80.3	**80.6**

3.4 Results on ImageNet

We take the best performing Sparse-IFT transformations (i.e., Sparse Wide IFT and Sparse Parallel IFT) on CIFAR-100, and evaluate them on ImageNet using ResNet-18. Both families of Sparse-IFT obtain significantly higher accuracy compared to the dense baseline (refer to Table 7). Note, Sparse Wide IFT ResNet-18 at 90% sparsity improves over the dense baseline by 3.5%, and is able to match accuracy of dense ResNet-34 with 2× fewer training FLOPs (see Figure 1). We take the best performing transformation (Sparse Wide IFT) and apply it to ResNet-34 and BotNet-50. Increasing sparsity leads to a consistent increase in accuracy, indicating improved training efficiency at higher sparsities. On BotNet-50, a hybrid ViT model, we see a 1% improvement at 90% sparsity.

Table 7: Sparse-IFT on ImageNet. Best result for each transformation and architecture is highlighted in bold.

Model	Dense	Transformation	Sparsity		
			0.50	0.75	0.90
ResNet-18	70.9 ± 0.1	Sparse Wide	72.7	73.8	**74.4**
		Sparse Parallel	72.7	73.2	**74.0**
ResNet-34	74.2 ± 0.1	Sparse Wide	75.6	76.4	**76.8**
BotNet-50	77.5 ± 0.1	Sparse Wide	77.9	78.3	**78.5**

3.5 Transfer Learning with Sparse-IFT

To show the effectiveness of pre-training our Sparse-IFT classification backbones, we evaluate them on 1) object detection on MS COCO 2017 Lin et al. (2014b), and 2) semantic segmentation on CityScapes Cordts et al. (2016). For object detection, we adopt the RetinaNet Lin et al. (2017b) framework from the MMDetection open-source toolbox Chen et al. (2019) and report results in the standardized training setting. For semantic segmentation, we utilize DeepLabV3+ Chen et al. (2018) in the MMSegmenation open-source toolbox Contributors (2020). We evaluate ResNet-18 with Sparse Wide IFT (best-performing transformation on ImageNet). To ensure FLOP-equivalent comparisons with the dense backbone, the Sparse-IFT backbones remain sparse during fine-tuning. Appendix B.3.4 provides more details regarding the training setup. We summarize our findings in Table 8, where using Sparse Wide IFT ResNet-18 backbone leads to significant accuracy gains across all metrics on both downstream tasks.

Table 8: Sparse-IFT variants of ResNet-18 as backbones on downstream tasks : (a) Object detection on MS COCO, (b) Semantic segmentation on Cityscapes.

	Metric	Dense	Sparsity		
			0.50	0.75	0.90
MS COCO	AP	29.3	31.3	32.8	**34.5**
	AP_{50}	46.2	49.0	51.0	**53.5**
	AP_{75}	30.9	33.0	34.8	**36.5**
CityScapes	mIoU	76.7	77.9	78.9	**79.1**
	mAcc	84.4	85.1	85.7	**86.0**

3.6 Results on GPT End-to-End Training

We train the Sparse Wide IFT GPT-3 Small models at 50% and 75% sparsity levels, and compare against the standard dense GPT-3 Small and GPT-3 Medium models. Following Dao et al. (2022), we train all models from scratch on the WikiText-103 dataset and report the average test perplexity (PPL) over 3 random seeds in Table 9. We show that Sparse Wide IFT GPT-3 Small at 50% sparsity improves the perplexity by 0.4 over its dense counterpart. We also note that the Sparse Wide IFT GPT-3 Small model performs comparable to a dense GPT-3 Medium (20.5 ± 0.2 PPL) while using 2.4x fewer training FLOPs. In Appendix C.1, we provide details on the hyperparameters and how the total training FLOPs for the models in Table 9 were calculated.

Table 9: Sparse-IFT for pre-training GPT-3 Small from scratch on WikiText-103 and report the test perplexity (lower is better).

	Dense	0.50	0.75
GPT-3 Small	20.8 ± 0.3	**20.4**	22.1

GPT Pre-training and Fine-tuning While not the primary focus of our method, we note that Sparse-IFT can also be applied in a fine-tuning setup for NLP models. After pre-training sparse, the Sparse-IFT model can be fine-tuned as-is (i.e., remains sparse) or after densifying (i.e., allow the zeroed weights to learn) using a technique such as SPDF (Thangarasa et al., 2023). We perform some preliminary fine-tuning studies on BERT and GPT and those results can be found in Appendix C.2.

4 Related Work

Our work is similar to the body of work studying the role of overparameterization and sparsity for training DNNs. The modeling capacity needed to learn a task is often unknown. Hence, we often solve this by training overparameterized models to fully exploit the learning capability and then compress them into a smaller subnetwork.

Overparameterization Nakkiran et al. (2021) show that DNNs benefit from overparameterization. Following this, there have been many works that leverage overparameterization by scaling the size of models Rae et al. (2021); Goyal et al. (2022) and augmenting existing DNNs to increase modeling capacity and the accuracy of trained networks Guo et al. (2020b); Ding et al. (2019; 2021b); Cao et al. (2022); Vasu et al. (2022); Liu et al. (2022a). These methods use linear parameterizations of the model, making them highly inefficient to train, and are focused on improving inference throughput (reduced latency). In contrast, our work is focused on improving the modeling capacity using sparse non-linear parameterizations, which do not increase training FLOPs compared to the baseline model. While both approaches have the same inference FLOPs, our approach improves accuracy without increasing the training FLOPs.

Sparse Training LTH (Frankle & Carbin, 2018; Frankle et al., 2020) shows that accurate sparse subnetworks exist in overparameterized dense networks but require training a dense baseline to find. Other approaches have proposed frameworks for identifying lottery tickets (Zhou et al., 2019; Ma et al., 2022) but still require a lot of compute resources. Following this, various attempts have been made to find the optimal sparse subnetwork in a single training run. These methods either try to find the subnetworks at initialization Tanaka et al. (2020); Wang et al. (2020a); de Jorge et al. (2020); Lee et al. (2018) or dynamically during training Mocanu et al. (2018); Evci et al. (2020); Jayakumar et al. (2020); Raihan & Aamodt (2020). However, given a fixed model capacity, these methods tradeoff accuracy relative to the dense baseline to save training FLOPs. Stosic & Stosic (2021) and Ramanujan et al. (2020) increase the search space during sparse training to retain accuracy; however, do not guarantee FLOPs savings. In contrast to these methods, our work introduces a set of non-linear sparse transformations, which increase the representational capacity of the network. Our approach does not entail the introduction of a novel sparse training algorithm. Instead, it enhances the search space of existing methods, resulting in improved generalization without compromising training efficiency.

Iso-Parameter vs. Iso-FLOP Recent sparsity literature is focused on improving generalization at high sparsity levels. Hence, layer-wise sparsity distributions such as the Erdös-Rényi-Kernel Evci et al. (2020), Ideal Gas Quota Chen et al. (2022b), and parameter leveling Golubeva et al. (2021) are often used with sparse training to boost accuracies. However, these works target the setting where the models being compared have a fixed parameter budget (i.e., Iso-Parameter), which does not translate to similar training FLOPs to the original dense model (especially in CNNs). As a result, training models with these distributions often require different memory or computational resources per layer. Our approach does not focus on this Iso-Parameter setting but instead adopts the uniform sparsity distribution (i.e., every layer gets the same sparsity level), ensuring uniform FLOP reductions across the network. We also ensure the same computational FLOPs of a dense network by leveraging sparsity along with our Iso-FLOP transformations.

5 Conclusion

We introduce a new family of Sparse Iso-FLOP Transformations (Sparse-IFT) to improve the training efficiency of DNNs. These transformations can be used as drop-in replacements for dense layers and increase the representational capacity while using sparsity to maintain training FLOPs. This increase in capacity also translates to a larger search space allowing sparse training methods to explore better and identify optimal sparse subnetworks. For the same computational cost as the original dense model, Sparse-IFT improves the training efficiency (test accuracy w.r.t training FLOPS) across multiple model families in the CV and NLP domains for various tasks. We hope our work will open new investigations into improving the accuracy of DNNs by leveraging sparsity, particularly in light of advancements in hardware accelerators that offer improved support for weight sparsity during the training process.

REFERENCES

Sanjeev Arora, Nadav Cohen, and Elad Hazan. On the optimization of deep networks: Implicit acceleration by overparameterization. In *ICML*, 2018.

Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. In *NeurIPS*, 2020.

Emmanuel J Candès, Xiaodong Li, Yi Ma, and John Wright. Robust principal component analysis? *Journal of the ACM*, 2011.

Jinming Cao, Yangyan Li, Mingchao Sun, Ying Chen, Dani Lischinski, Daniel Cohen-Or, Baoquan Chen, and Changhe Tu. Do-conv: Depthwise over-parameterized convolutional layer. *IEEE Transactions on Image Processing*, 2022.

Beidi Chen, Tri Dao, Eric Winsor, Zhao Song, Atri Rudra, and Christopher Ré. Scatterbrain: Unifying sparse and low-rank attention approximation. In *NeurIPS*, 2021a.

Beidi Chen, Tri Dao, Kaizhao Liang, Jiaming Yang, Zhao Song, Atri Rudra, and Christopher Re. Pixelated butterfly: Simple and efficient sparse training for neural network models. In *ICLR*, 2022a.

Kai Chen, Jiaqi Wang, Jiangmiao Pang, Yuhang Cao, Yu Xiong, Xiaoxiao Li, Shuyang Sun, Wansen Feng, Ziwei Liu, Jiarui Xu, Zheng Zhang, Dazhi Cheng, Chenchen Zhu, Tianheng Cheng, Qijie Zhao, Buyu Li, Xin Lu, Rui Zhu, Yue Wu, Jifeng Dai, Jingdong Wang, Jianping Shi, Wanli Ouyang, Chen Change Loy, and Dahua Lin. MMDetection: Open mmlab detection toolbox and benchmark. *arXiv*, 2019.

Liang-Chieh Chen, Yukun Zhu, George Papandreou, Florian Schroff, and Hartwig Adam. Encoder-decoder with atrous separable convolution for semantic image segmentation. In *ECCV*, 2018.

Patrick Chen, Hsiang-Fu Yu, Inderjit Dhillon, and Cho-Jui Hsieh. Drone: Data-aware low-rank compression for large nlp models. In *NeurIPS*, 2021b.

Tianlong Chen, Zhenyu Zhang, pengjun wang, Santosh Balachandra, Haoyu Ma, Zehao Wang, and Zhangyang Wang. Sparsity winning twice: Better robust generalization from more efficient training. In *ICLR*, 2022b.

MMSegmentation Contributors. MMSegmentation: Openmmlab semantic segmentation toolbox and benchmark. `https://github.com/open-mmlab/mmsegmentation`, 2020.

Marius Cordts, Mohamed Omran, Sebastian Ramos, Timo Rehfeld, Markus Enzweiler, Rodrigo Benenson, Uwe Franke, Stefan Roth, and Bernt Schiele. The cityscapes dataset for semantic urban scene understanding. In *CVPR*, 2016.

Tri Dao, Beidi Chen, Nimit S Sohoni, Arjun Desai, Michael Poli, Jessica Grogan, Alexander Liu, Aniruddh Rao, Atri Rudra, and Christopher Ré. Monarch: Expressive structured matrices for efficient and accurate training. In *ICML*, 2022.

Pau de Jorge, Amartya Sanyal, Harkirat S Behl, Philip HS Torr, Gregory Rogez, and Puneet K Dokania. Progressive skeletonization: Trimming more fat from a network at initialization. *arXiv*, 2020.

Terrance DeVries and Graham W Taylor. Improved regularization of convolutional neural networks with cutout. *arXiv*, 2017.

Xiaohan Ding, Yuchen Guo, Guiguang Ding, and Jungong Han. Acnet: Strengthening the kernel skeletons for powerful cnn via asymmetric convolution blocks. In *ICCV*, 2019.

Xiaohan Ding, Xiangyu Zhang, Jungong Han, and Guiguang Ding. Diverse branch block: Building a convolution as an inception-like unit. In *CVPR*, 2021a.

Xiaohan Ding, Xiangyu Zhang, Ningning Ma, Jungong Han, Guiguang Ding, and Jian Sun. Repvgg: Making vgg-style convnets great again. In *CVPR*, 2021b.

Utku Evci, Trevor Gale, Jacob Menick, Pablo Samuel Castro, and Erich Elsen. Rigging the lottery: Making all tickets winners. In *ICML*, 2020.

Jonathan Frankle and Michael Carbin. The lottery ticket hypothesis: Finding sparse, trainable neural networks. In *ICLR*, 2018.

Jonathan Frankle, Gintare Karolina Dziugaite, Daniel Roy, and Michael Carbin. Linear mode connectivity and the lottery ticket hypothesis. In *ICML*, 2020.

Trevor Gale, Erich Elsen, and Sara Hooker. The state of sparsity in deep neural networks. *arXiv*, 2019.

Leo Gao, Stella Biderman, Sid Black, Laurence Golding, Travis Hoppe, Charles Foster, Jason Phang, Horace He, Anish Thite, Noa Nabeshima, et al. The pile: An 800gb dataset of diverse text for language modeling. *arXiv*, 2020.

Anna Golubeva, Guy Gur-Ari, and Behnam Neyshabur. Are wider nets better given the same number of parameters? In *ICLR*, 2021.

Priya Goyal, Quentin Duval, Isaac Seessel, Mathilde Caron, Mannat Singh, Ishan Misra, Levent Sagun, Armand Joulin, and Piotr Bojanowski. Vision models are more robust and fair when pretrained on uncurated images without supervision. *arXiv*, 2022.

Shuxuan Guo, Jose M Alvarez, and Mathieu Salzmann. Expandnets: Linear over-parameterization to train compact convolutional networks. In *NeurIPS*, 2020a.

Shuxuan Guo, Jose M. Alvarez, and Mathieu Salzmann. Expandnets: Linear over-parameterization to train compact convolutional networks. In *NeurIPS*, 2020b.

Song Han, Huizi Mao, and William J Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. *arXiv*, 2015a.

Song Han, Jeff Pool, John Tran, and William Dally. Learning both weights and connections for efficient neural network. In *NeurIPS*, 2015b.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Identity mappings in deep residual networks. In *ECCV*, 2016.

Tong He, Zhi Zhang, Hang Zhang, Zhongyue Zhang, Junyuan Xie, and Mu Li. Bag of tricks for image classification with convolutional neural networks. In *CVPR*, 2019.

Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the knowledge in a neural network. *arXiv*, 2015.

Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katherine Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Oriol Vinyals, Jack William Rae, and Laurent Sifre. An empirical analysis of compute-optimal large language model training. In *NeurIPS*, 2022.

Itay Hubara, Brian Chmiel, Moshe Island, Ron Banner, Joseph Naor, and Daniel Soudry. Accelerated sparse neural training: A provable and efficient method to find n:m transposable masks. In *NeurIPS*, 2021.

Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating deep network training by reducing internal covariate shift. In *ICML*, 2015.

Siddhant Jayakumar, Razvan Pascanu, Jack Rae, Simon Osindero, and Erich Elsen. Top-kast: Top-k always sparse training. In *NeurIPS*, 2020.

Peng Jiang, Lihan Hu, and Shihui Song. Exposing and exploiting fine-grained block structures for fast and accurate sparse training. In *NeurIPS*, 2022.

Mikhail Khodak, Neil A Tenenholtz, Lester Mackey, and Nicolo Fusi. Initialization and regularization of factorized neural layers. In *ICLR*, 2020.

Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. *Master's thesis, Department of Computer Science, University of Toronto*, 2009.

Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. In *NeurIPS*, 2012.

Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. Imagenet classification with deep convolutional neural networks. *Communications of the ACM*, 2017.

Namhoon Lee, Thalaiyasingam Ajanthan, and Philip HS Torr. Snip: Single-shot network pruning based on connection sensitivity. *arXiv*, 2018.

Sean Lie. Harnessing the Power of Sparsity for Large GPT AI Models. https://www.cerebras.net/blog/harnessing-the-power-of-sparsity-for-large-gpt-ai-models, 2022a.

Sean Lie. Cerebras architecture deep dive: First look inside the hw/sw co-design for deep learning : Cerebras systems. In *2022 IEEE Hot Chips 34 Symposium (HCS)*, 2022b.

Min Lin, Qiang Chen, and Shuicheng Yan. Network in network. In *ICLR*, 2014a.

Tsung-Yi Lin, Michael Maire, Serge Belongie, James Hays, Pietro Perona, Deva Ramanan, Piotr Dollár, and C Lawrence Zitnick. Microsoft coco: Common objects in context. In *ECCV*, 2014b.

Tsung-Yi Lin, Piotr Dollár, Ross Girshick, Kaiming He, Bharath Hariharan, and Serge Belongie. Feature Pyramid Networks for Object Detection. In *CVPR*, 2017a.

Tsung-Yi Lin, Priya Goyal, Ross Girshick, Kaiming He, and Piotr Dollár. Focal loss for dense object detection. In *ICCV*, 2017b.

Shiwei Liu, Tianlong Chen, Xiaohan Chen, Zahra Atashgahi, Lu Yin, Huanyu Kou, Li Shen, Mykola Pechenizkiy, Zhangyang Wang, and Decebal Constantin Mocanu. Sparse training via boosting pruning plasticity with neuroregeneration. 2021a.

Shiwei Liu, Decebal Constantin Mocanu, Yulong Pei, and Mykola Pechenizkiy. Selfish sparse rnn training. In *ICML*, 2021b.

Shiwei Liu, Tianlong Chen, Xiaohan Chen, Xuxi Chen, Qiao Xiao, Boqian Wu, Mykola Pechenizkiy, Decebal Mocanu, and Zhangyang Wang. More convnets in the 2020s: Scaling up kernels beyond 51x51 using sparsity. *arXiv*, 2022a.

Shiwei Liu, Tianlong Chen, Xiaohan Chen, Li Shen, Decebal Constantin Mocanu, Zhangyang Wang, and Mykola Pechenizkiy. The unreasonable effectiveness of random pruning: Return of the most naive baseline for sparse training. *arXiv*, 2022b.

Ze Liu, Yutong Lin, Yue Cao, Han Hu, Yixuan Wei, Zheng Zhang, Stephen Lin, and Baining Guo. Swin transformer: Hierarchical vision transformer using shifted windows. In *ICCV*, 2021c.

Zhuang Liu, Jianguo Li, Zhiqiang Shen, Gao Huang, Shoumeng Yan, and Changshui Zhang. Learning efficient convolutional networks through network slimming. In *ICCV*, 2017.

Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization. *arXiv*, 2017.

Xiaolong Ma, Minghai Qin, Fei Sun, Zejiang Hou, Kun Yuan, Yi Xu, Yanzhi Wang, Yen-Kuang Chen, Rong Jin, and Yuan Xie. Effective model sparsification by scheduled grow-and-prune methods. In *ICLR*, 2022.

Sachin Mehta and Mohammad Rastegari. Mobilevit: Light-weight, general-purpose, and mobile-friendly vision transformer. In *ICLR*, 2021.

Stephen Merity, Caiming Xiong, James Bradbury, and Richard Socher. Pointer sentinel mixture models. In *ICLR*, 2017.

Paulius Micikevicius, Sharan Narang, Jonah Alben, Gregory Diamos, Erich Elsen, David Garcia, Boris Ginsburg, Michael Houston, Oleksii Kuchaiev, Ganesh Venkatesh, and Hao Wu. Mixed precision training. In *ICLR*, 2018.

Decebal Constantin Mocanu, Elena Mocanu, Peter Stone, Phuong H. Nguyen, Madeleine Gibescu, and Antonio Liotta. Scalable training of artificial neural networks with adaptive sparse connectivity inspired by network science. *Nature Communications*, 2018.

Pavlo Molchanov, Stephen Tyree, Tero Karras, Timo Aila, and Jan Kautz. Pruning convolutional neural networks for resource efficient inference. In *ICLR*, 2017.

Vinod Nair and Geoffrey E Hinton. Rectified linear units improve restricted boltzmann machines. In *ICML*, 2010.

Preetum Nakkiran, Gal Kaplun, Yamini Bansal, Tristan Yang, Boaz Barak, and Ilya Sutskever. Deep double descent: Where bigger models and more data hurt. *Journal of Statistical Mechanics: Theory and Experiment*, 2021.

Nvidia. Deep learning examples, language modeling using bert. 2019a. URL https://github.com/NVIDIA/DeepLearningExamples/tree/master/PyTorch/LanguageModeling/BERT.

Nvidia. Resnet v1.5 for pytorch. 2019b. URL https://catalog.ngc.nvidia.com/orgs/nvidia/resources/resnet_50_v1_5_for_pytorch.

Nvidia. Nvidia performance documentation. 2023. URL https://docs.nvidia.com/deeplearning/performance/dl-performance-matrix-multiplication/index.html.

Alec Radford, Karthik Narasimhan, Tim Salimans, Ilya Sutskever, et al. Improving language understanding by generative pre-training. *OpenAI Blog*, 2018.

Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, Ilya Sutskever, et al. Language models are unsupervised multitask learners. *OpenAI Blog*, 2019.

Jack W Rae, Sebastian Borgeaud, Trevor Cai, Katie Millican, Jordan Hoffmann, Francis Song, John Aslanides, Sarah Henderson, Roman Ring, Susannah Young, et al. Scaling language models: Methods, analysis & insights from training gopher. *arXiv*, 2021.

Md Aamir Raihan and Tor Aamodt. Sparse weight activation training. In *NeurIPS*, 2020.

Pranav Rajpurkar, Jian Zhang, Konstantin Lopyrev, and Percy Liang. Squad: 100, 000+ questions for machine comprehension of text. In *EMNLP*, 2016.

Vivek Ramanujan, Mitchell Wortsman, Aniruddha Kembhavi, Ali Farhadi, and Mohammad Rastegari. What's hidden in a randomly weighted neural network? In *CVPR*, 2020.

Mark Sandler, Andrew Howard, Menglong Zhu, Andrey Zhmoginov, and Liang-Chieh Chen. Mobilenetv2: Inverted residuals and linear bottlenecks. In *CVPR*, 2018.

Noam Shazeer, Azalia Mirhoseini, Krzysztof Maziarz, Andy Davis, Quoc Le, Geoffrey Hinton, and Jeff Dean. Outrageously large neural networks: The sparsely-gated mixture-of-experts layer. In *ICLR*, 2016.

David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *Nature*, 2017.

Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. *arXiv*, 2014.

Aravind Srinivas, Tsung-Yi Lin, Niki Parmar, Jonathon Shlens, Pieter Abbeel, and Ashish Vaswani. Bottleneck transformers for visual recognition. In *CVPR*, 2021.

Darko Stosic and Dusan Stosic. Search spaces for neural model training. *arXiv*, 2021.

Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jon Shlens, and Zbigniew Wojna. Rethinking the inception architecture for computer vision. In *CVPR*, 2016.

Cheng Tai, Tong Xiao, Yi Zhang, Xiaogang Wang, and E Weinan. Convolutional neural networks with low-rank regularization. In *ICLR*, 2016.

Kai Sheng Tai, Taipeng Tian, and Ser-Nam Lim. Spartan: Differentiable Sparsity via Regularized Transportation. In *NeurIPS*, 2022.

Hidenori Tanaka, Daniel Kunin, Daniel L Yamins, and Surya Ganguli. Pruning neural networks without any data by iteratively conserving synaptic flow. In *NeurIPS*, 2020.

Urmish Thakker, Paul N Whatmough, Zhigang Liu, Matthew Mattina, and Jesse Beu. Doping: A technique for efficient compression of lstm models using sparse structured additive matrices. In *MLSys*, 2021.

Vithursan Thangarasa, Abhay Gupta, William Marshall, Tianda Li, Kevin Leong, Dennis DeCoste, Sean Lie, and Shreyas Saxena. SPDF: Sparse pre-training and dense fine-tuning for large language models. In *ICLR Workshop on Sparsity in Neural Networks*, 2023.

Madeleine Udell and Alex Townsend. Why are big data matrices approximately low rank? *SIAM Journal on Mathematics of Data Science*, 2019.

Pavan Kumar Anasosalu Vasu, James Gabriel, Jeff Zhu, Oncel Tuzel, and Anurag Ranjan. An improved one millisecond mobile backbone. *arXiv*, 2022.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *NeurIPS*, 2017.

Chaoqi Wang, Guodong Zhang, and Roger Grosse. Picking winning tickets before training by preserving gradient flow. *arXiv*, 2020a.

Jingdong Wang, Ke Sun, Tianheng Cheng, Borui Jiang, Chaorui Deng, Yang Zhao, Dong Liu, Yadong Mu, Mingkui Tan, Xinggang Wang, et al. Deep high-resolution representation learning for visual recognition. In *TPAMI*, 2020b.

Greg Yang, Edward J Hu, Igor Babuschkin, Szymon Sidor, Xiaodong Liu, David Farhi, Nick Ryder, Jakub Pachocki, Weizhu Chen, and Jianfeng Gao. Tensor programs v: Tuning large neural networks via zero-shot hyperparameter transfer. In *NeurIPS*, 2022.

Yang You, Jing Li, Sashank Reddi, Jonathan Hseu, Sanjiv Kumar, Srinadh Bhojanapalli, Xiaodan Song, James Demmel, Kurt Keutzer, and Cho-Jui Hsieh. Large batch optimization for deep learning: Training bert in 76 minutes. In *ICLR*, 2020.

Geng Yuan, Xiaolong Ma, Wei Niu, Zhengang Li, Zhenglun Kong, Ning Liu, Yifan Gong, Zheng Zhan, Chaoyang He, Qing Jin, et al. Mest: Accurate and fast memory-economic sparse training framework on the edge. volume 34, 2021.

Sergey Zagoruyko and Nikos Komodakis. Wide residual networks. In *BMVC*, 2016.

Biao Zhang, Ivan Titov, and Rico Sennrich. Improving deep transformer with depth-scaled initialization and merged attention. In *EMNLP*, 2019.

Hengshuang Zhao, Jianping Shi, Xiaojuan Qi, Xiaogang Wang, and Jiaya Jia. Pyramid scene parsing network. In *CVPR*, 2017.

Hattie Zhou, Janice Lan, Rosanne Liu, and Jason Yosinski. Deconstructing lottery tickets: Zeros, signs, and the supermask. In *NeurIPS*, 2019.

Y. Zhu, R. Kiros, R. Zemel, R. Salakhutdinov, R. Urtasun, A. Torralba, and S. Fidler. Aligning books and movies: Towards story-like visual explanations by watching movies and reading books. In *ICCV*, 2015.