Disentangling the role of heterogeneity and hyperedge overlap in explosive contagion on higher-order networks

Keywords: Higher-order networks, collective behaviours, phase transitions, spreading processes, epidemic modelling

Extended Abstract

In recent years, significant effort has been devoted to understanding the impact of higher-order interactions on dynamical processes [1]. In particular, much attention has been given to contagion dynamics, especially the SIS model [2–4] and its various extensions [5, 6]. Recent studies have demonstrated that the microscopic organization of higher-order interactions plays a crucial role in shaping the onset of epidemic processes [4, 7, 8].

Here, we introduce the Group-Based Compartmental Model (GBCM) for the Susceptible-Infected-Recovered (SIR) process in the presence of higher-order interactions across groups of arbitrary size. This framework explicitly accounts for heterogeneity in degree distributions and the overlap between hyperedges of different orders (inter-order hyperedge overlap) [9]. By incorporating these structural and dynamical correlations, the model bridges a critical gap in the study of heterogeneous higher-order contagion processes, enabling a detailed investigation of how their interplay shapes epidemic dynamics.

Focusing on systems with only 2- and 3-body interactions, we derive analytical expressions for the epidemic threshold. Our results show that increasing inter-order hyperedge overlap enhances the system's susceptibility to epidemics. Interestingly, we find that high heterogeneity in the degree distribution of 2-body interactions mitigates this effect. This phenomenon emerges from the intricate interplay between contagion pathways in pairwise and 3-body interactions, which our model disentangles analytically.

Furthermore, by analyzing both the final epidemic size and the temporal evolution of the infected population, our model predicts that high heterogeneity in group interactions can induce explosive epidemic phenomena. While this behavior has been observed in higher-order SIS models, it has not yet been reported in SIR processes. Specifically, we show that for a fixed strength of higher-order interactions, the pairwise contribution exhibits continuous transitions , whereas the 3-body component exhibits abrupt and explosive growth as heterogeneity increases.

Through analytical results, we predict and validate the emergence of explosive temporal outbreaks, driven by strong higher-order interactions and high levels of degree heterogeneity in group membership. This provides a pathway leading to the well-studied abrupt phase transitions observed in epidemic processes on higher-order networks [1, 2].

These findings, supported by Gillespie simulations on synthetic and empirical hypergraphs, highlight the profound impact of higher-order interactions on irreversible contagion processes and underscore the crucial role of structural and dynamical correlations in epidemic spreading. More broadly, our approach offers new insights into multiple interacting processes and provides a foundation for developing more sophisticated models that capture key structural features of real-world systems.

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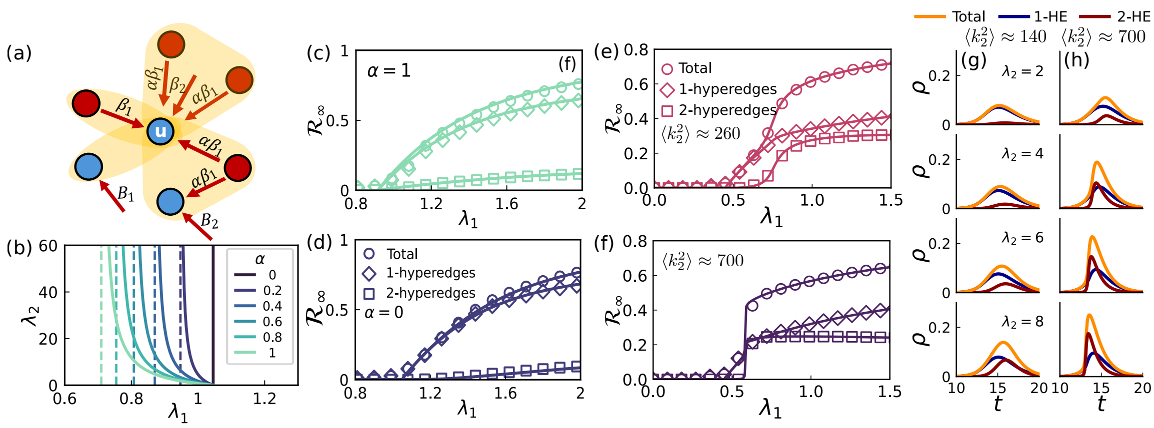

Figure 1: **Group-based approximation for two- and three-body interactions.** (a) Schematic representation of the model, where β_1 and β_2 denote the infection rates in two- and three-body interactions, respectively, and α represents the inter-order hyperedge overlap. B_1 and B_2 capture infection rates from external sources for nodes involved in 2- and 3-body interactions. The recovery rate is μ. (b) Epidemic thresholds in the $(\lambda_1 = \beta_1\langle k_1\rangle/\mu, \lambda_2 = \beta_2\langle k_2\rangle/\mu)$ plane predicted by the Group-Based Compartmental Model (GBCM) for Erdős-Rényi hypergraphs at different values of α. (c)-(d) Final epidemic sizes (R_∞) disaggregated by contributions from 1- and 2-hyperedges, comparing GBCM predictions (solid lines) with Gillespie simulations (dots) for ER-like hypergraphs ($N = 10^4$, $\langle k_1\rangle \approx 12$, $\langle k_2\rangle \approx 3$), at (c) $\alpha = 1$ and (d) $\alpha = 0$, both with $\lambda_2 = 3$. For $\alpha = 1$, the two processes coincide at the same λ_1 value, whereas for $\alpha = 0$, the 2-body process is delayed. (e)-(f) R_∞ disaggregated by contributions from 1- and 2-hyperedges in hypergraphs with negative binomial degree distributions ($N = 10^4$, $\langle k_1\rangle \approx 12$, $\langle k_2\rangle \approx 9$, $\alpha = 0$), with different levels of heterogeneity in three-body group membership. In (e), the second moment of the 3-body degree distribution is $\langle k_2^2\rangle \approx 260$, while in (f) $\langle k_2^2\rangle \approx 700$. In (f), an initially continuous growth transitions into an abrupt epidemic outbreak due to critical dynamics within the 2-hyperedge layer, despite $\lambda_2 = 6$ being fixed in both cases. (g)-(h) Temporal evolution of the infected density (orange), with separate contributions from pairwise (blue, 1-HE) and three-body (red, 2-HE) interactions, obtained from the GBCM. In panel (g) we consider the case for $\langle k_2^2\rangle \approx 140$, showing smooth epidemic progression. In panel (h) we show the case where $\langle k_2^2\rangle \approx 700$, revealing abrupt dynamics driven by higher-order interactions.