LACONIC: LENGTH-AWARE CONSTRAINED REINFORCEMENT LEARNING FOR LLM

Anonymous authors
Paper under double-blind review

ABSTRACT

Reinforcement learning (RL) has enhanced the capabilities of large language models (LLMs) by enabling self-evolution through reward-driven training. Nevertheless, this process can introduce excessively long responses that inflate inference latency and computational overhead. To address this issue, existing RL-based length control methods often incorporate fixed penalties or heuristic reward shaping to encourage outputs of a desired length. However, such strategies may misalign the optimization objective with the underlying task, resulting in suboptimal performance and limited generalization across model architectures and datasets. In this work, we propose `LACONIC`, a lightweight reinforcement learning method that enforces a target token budget during training. Specifically, we update policy models using an augmented objective that combines the task reward with a length-based cost applied only to tokens exceeding the specified budget. Furthermore, to balance brevity and task performance, the cost scale is adjusted online throughout training. This formulation directly optimizes task reward subject to an explicit token budget constraint, delivering precise and performance-preserving length control. Across mathematical reasoning models and datasets, `LACONIC` preserves or improves `pass@1` while reducing output length by up to 43%. It maintains out-of-domain performance on general knowledge and multilingual benchmarks with a 44% reduction in tokens. Moreover, `LACONIC` integrates into standard RL fine-tuning with no inference changes and minimal deployment overhead.

1 INTRODUCTION

Large language models (LLMs) such as GPT [OpenAI et al., 2024; OpenAI, 2025], Gemini [Comanici et al., 2025], DeepSeek [DeepSeek-AI, 2025], and Claude[Anthropic, 2025] have witnessed unprecedented success in its applications from software agents to enterprise analytics [Team et al., 2025; Li et al., 2025b; Jin et al., 2025; Feng et al., 2025]. The impressive capabilities of LLMs have been significantly enhanced by reinforcement learning based fine-tuning [Li et al., 2025a; Wu et al., 2025; Li et al., 2025b], a procedure that aligns pretrained models with task-specific rewards through interaction with an environment. This process has been pivotal in refining LLM reasoning skills, enhancing generalization, and achieving state-of-the-art performance across diverse benchmarks [Wang et al., 2024; Hsiao et al., 2025; Shi et al., 2025; Qu et al., 2025]. However, RL-tuned language models often suffer from generating unnecessarily long thinking traces. This problem is particularly acute on reasoning and mathematics tasks, where the model is asked to spell out logical steps [Chen et al., 2025; Sui et al., 2025]. In practice, excessive verbosity inflates training and inference time, increases memory pressure, and ultimately degrades user experience.

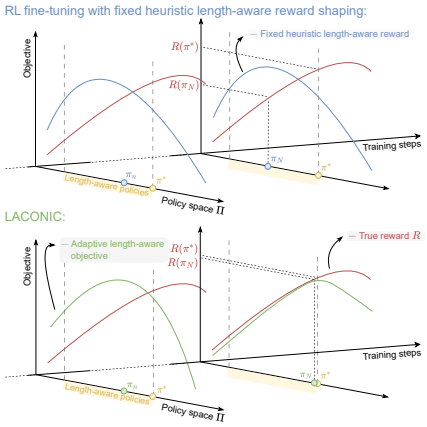


Figure 1: Heuristic reward shaping yields suboptimal task rewards, while adaptive length-aware objective realigns optimization with true rewards, preserving performance under length control.

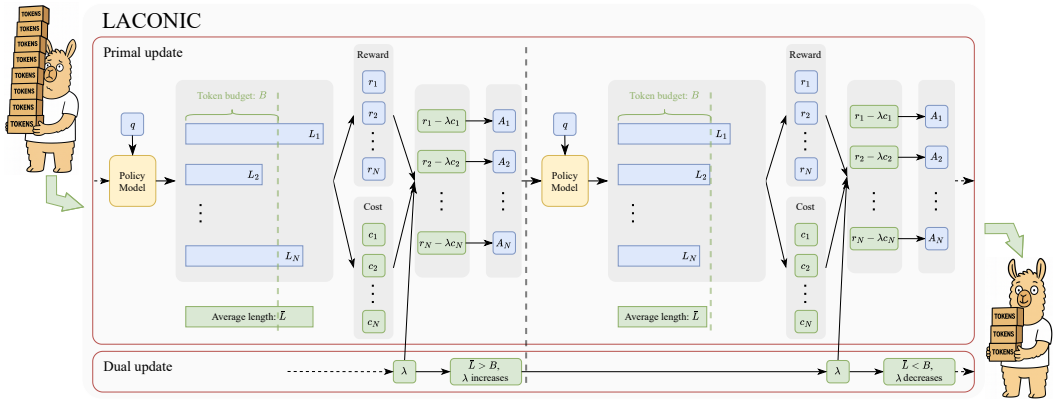


Figure 2: Illustration of LACONIC. LACONIC alternates two steps: (1) in a primal update, the policy model is updated on an augmented objective that trades off task reward r with a length-aware cost c scaled by the dual variable λ; (2) in a dual update, λ is adaptively updated to enforce a token budget constraint B by increasing when the average length $\bar{L}$ exceeds the budget B and decreasing otherwise. Together, these updates maximize task reward while meeting the budget on average.

Recent work on length-aware LLMs has explored positional encoding, prompt engineering, and post-generation truncation [Li et al., 2025a; Wu et al., 2025; Li et al., 2025b]. A straightforward method is to design new reward functions to incorporate response length signals into RL-tuning [Aggarwal & Welleck, 2025; Cheng et al., 2025; Huang et al., 2025; Yuan et al., 2025]. These methods hard-code a length-aware reward, typically with a fixed penalty or heuristic shaping that stays fixed throughout training. Fine-tuning with these rewards optimizes a surrogate objective that is misaligned with true task reward, and often demands per-setting hyperparameter tuning. Figure 1 (top) visualizes this issue by showing a sketch of training process. The fixed heuristic objective differs from the true objective, so optimizing it yields policies with suboptimal task rewards.

In this paper, we address length control in RL-tuning by maximizing task reward subject to an average token budget constraint. We introduce LACONIC (**L**ength-**A**ware **Con**strained Pol**ic**y Optimization), a primal-dual algorithm. During training, the model samples candidate responses for each prompt. Besides the task reward (e.g., correctness or usefulness) as in the standard RL-tuning, we assign to each candidate response also a length-aware cost proportional to its budget violation. We then construct a learning signal that combines task reward with length cost, scaled by an adaptively learned multiplier. We alternatively update the policy model and the multiplier. The policy is updated by a policy optimization step where advantages and objectives are calculated from the constructed signal. Then we update the multiplier based on the average response length of the current batch. We raise the multiplier if the current batch violates the token budget constraint on average, and lower it if the current batch falls short. This feedback automatically steers the model's average output length towards the token budget. Particularly, when the model consistently stays within the token budget, the multiplier naturally drops to zero and our training steps reduce to standard RL-tuning steps, allowing the model to recover task rewards with shortened responses. As illustrated in figure 1, LACONIC adopts an adaptive objective that dynamically align the optimization with true rewards, steering the policy towards the length-aware optimum.

We conduct extensive experiments to evaluate our method LACONIC and present the evaluation results in section 3. We apply LACONIC to fine-tune two reasoning models DeepScaleR-1.5B-Preview [Luo et al., 2025] and Qwen2.5-Math-1.5B-Instruct [Yang et al., 2024]. The experimental results show that our LACONIC-tuned models can significantly outperform existing length control baseline L1 [Aggarwal & Welleck, 2025] and match GRPO on pass@1 across common mathematics benchmarks, while effectively shortening response lengths by using 44% and 26% fewer tokens than the base model and L1-Max respectively. LACONIC also preserves accuracy on benchmarks outside our RL-tuning domain while reducing response length by 44% compared to GRPO. Furthermore, we perform ablation analysis on the hyperparameters of our method in section 4. Ablation experiments show that LACONIC provides robust and length control.

2 METHODOLOGY

2.1 PRELIMINARY BACKGROUND

RL fine-tuning casts text generation as a sequential decision process, where the prompt q together with the partial output sequence constitutes the state, selecting the next token is the action, and the language model parameterized by θ serves as the policy π_θ that maps states to action probabilities. After generating the response, the model receives task rewards $r(q, o)$ assigned by a reward model. Policy gradient algorithms such as Proximal Policy Optimization (PPO) [Schulman et al., 2017] and Group Relative Policy Optimization (GRPO) [Shao et al., 2024a] then translate such rewards into token-level gradients, so that the model can be updated to increase the corpus-level expected rewards, i.e, $\max_\theta \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q, o)]$. In practice, GRPO updates the policy model's parameters θ by optimizing the following surrogate objective

$$\mathcal{J}(\theta) = \mathbb{E}_{q,\{o_i\}_{i=1}^G}\left[\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\min(\rho_{i,t}A_{i,t},\ \text{clip}(\rho_{i,t}, 1-\varepsilon, 1+\varepsilon)A_{i,t}) - \beta\mathbb{D}_{\text{KL}}[\pi_\theta\|\pi_{\text{ref}}]\right], \quad (1)$$

where $\rho_{i,t} = \frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t}|q,o_{i,<t})}$ is the likelihood ratio, and $A_{i,t}$ is the group-relative advantage. The likelihood ratio clipping and an extra KL-divergence penalty are adopted to stabilize policy updates.

2.2 LACONIC: LENGTH-AWARE CONSTRAINED POLICY OPTIMIZATION

To add explicit control over response length, we extend standard RL fine-tuning to a constrained setting that maximizes task reward under an average token count constraint B, a pre-specified budget reflecting deployment targets such as latency and computational resources in practice. We treat the token budget as a given hyperparameter and learn a policy whose average response length respects the budget B. Formally, letting $L(o)$ denote the length of response o, we address

$$\max_\theta \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q, o)],\ \text{s.t.}\ \ \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)] \le B. \quad (2)$$

In this formulation, we enforce a corpus-level average token budget rather than a more strict per-sequence length constraint. Response lengths naturally vary across prompts. As an example, a math olympiad problem typically requires more tokens to solve than a simple arithmetic query. The average budget constraint in equation (2) allows the model to generate more tokens where they improve reward and shorten responses on easier cases while still meeting the overall budget constraint.

Length-aware cost. In order to enforce the budget constraint during training, we assign each response a sequence-level cost that measures its budget violation. Specifically, for a prompt q and a response o generated by the policy model π_θ, we define the cost as

$$c(q, o) = \max\left\{\frac{L(o) - B}{B}, 0\right\}. \quad (3)$$

The cost is zero for responses no longer than B, and increases linearly with the number of over-budget tokens. This design discourages unnecessary verbosity while preserving exploration as the model can freely use up to B tokens without penalty. While task rewards often lie in $[0, 1]$, response lengths can range widely from a few hundreds to over 100K across backbones, prompt distributions, and output length caps in practice. We add the $1/B$ normalization to keep the cost on a comparable scale across different magnitudes of lengths and budgets.

Lagrangian reward. We incorporate the cost into the learning objective via a Lagrangian relaxation. With a nonnegative multiplier $\lambda \ge 0$, also known as a dual variable, we define the Lagrangian reward

$$\ell_\lambda(q, o) = r(q, o) - \lambda \cdot c(q, o). \quad (4)$$

Primal updates. To tackle the constrained optimization in equation (2), we alternate a primal (policy) update and a dual (multiplier) update. In the primal update, we hold λ fixed and take a policy gradient step to optimize expected Lagrangian reward, i.e., $\max_\theta \mathbb{E}_{q,o}[\ell_\lambda(q, o)]$. This is essentially the standard RL-tuning objective with the task reward r replaced by the Lagrangian reward ℓ_λ. Therefore,

Algorithm 1: LACONIC (Length-Aware Constrained Policy Optimization)

Input: initial policy model $\pi_{\theta_{\text{init}}}$; reward models r_φ; task prompts $\mathcal{D}$; token budget B; step size η; initial dual variable λ_{init}

policy model $\pi_\theta \leftarrow \pi_{\theta_{\text{init}}}$
dual variable $\lambda \leftarrow \lambda_{\text{init}}$
for *iteration* = $1, \ldots, I$ **do**
  reference model $\pi_{\text{ref}} \leftarrow \pi_\theta$
  for *step* = $1, \ldots, M$ **do**
    Sample a batch $\mathcal{D}_b$ from $\mathcal{D}$
    Update the old policy model $\pi_{\theta_{\text{old}}} \leftarrow \pi_\theta$
    Sample G outputs $\{o_i\}_{i=1}^{G} \sim \pi_{\theta_{\text{old}}}(\cdot|q)$ for each question $q \in \mathcal{D}_b$
    Compute rewards $\{r_i\}_{i=1}^{G}$ for each sampled output o_i by running r_φ
    Compute costs $\{c_i\}_{i=1}^{G}$ for each sampled output o_i by equation (3)
    Compute Lagrangian rewards $\{\ell_{\lambda,i}\}_{i=1}^{G}$ for each sample output by equation (4)
    Compute advantages $A_{i,t}$ for the t-th token of o_i by equation (5)
    `// Primal update`
    Update the policy model π_θ by maximizing the GRPO-style objective
    `// Dual update`
    Update the dual variable λ by equation (6)
  Update r_φ
Output: π_θ

we adapt the GRPO-style policy update with the objective computed from ℓ_λ. Specifically, for each prompt q, we sample a group of candidate outputs $\mathbf{o} = \{o_1, o_2, \ldots, o_G\}$ from the current policy model π_θ, and compute their task rewards $\mathbf{r} = \{r_1, r_2, \ldots, r_G\}$ and costs $\mathbf{c} = \{c_1, c_2, \ldots, c_G\}$ by equation (3). We then compute the Lagrangian rewards $\boldsymbol{\ell}_\lambda = \{\ell_{\lambda_1}, \ell_{\lambda,2}, \ldots, \ell_{\lambda,G}\}$ where $\ell_{\lambda,i} = r_i - \lambda c_i$. For each token $o_{i,t}$, we construct the GRPO-style advantage as the normalized Lagrangian reward, i.e.,

$$A_{i,t} = \widetilde{\ell}_{\lambda,i} = \frac{\ell_{\lambda,i} - \text{mean}(\boldsymbol{\ell}_\lambda)}{\text{std}(\boldsymbol{\ell}_\lambda)}. \tag{5}$$

The policy model is optimized by maximizing the GRPO objective in equation (1) where advantages are calculated by equation (5).

Dual updates. In the dual update, we adjust the dual variable λ based on how the batch compares to the token budget. Let $\bar{L}$ denote the average response length of the current batch. We update

$$\lambda \leftarrow \max\left\{\lambda + \eta\left(\frac{\bar{L}}{B} - 1\right), 0\right\}, \tag{6}$$

with the step size $\eta > 0$. When the batch violates the token budget on average ($\bar{L} > B$), the update increases λ, raising the effective price of tokens in ℓ_λ. Longer responses then receive lower (often negative) advantages than shorter responses with similar task rewards, so the next primal update shifts the policy toward shorter outputs. If the batch falls within the budget, λ relaxes towards 0. Notably, when $\lambda = 0$, ℓ_λ reduces to the task reward r, and the next primal update is exactly a GRPO step. This feedback adapts λ to track the budget constraint throughout training as the policy and length distribution evolve.

We present LACONIC in algorithm 1 and illustrate the workflow with two sample steps in figure 2.

3 Experiment

3.1 Experimental Setup

Models and Datasets. For the training dataset, we use DeepScaleR-Preview-Dataset [Luo et al., 2025], a math dataset containing 40.3K rows of question-answer pairs sampled from AIME (prior to 2023), AMC (prior to 2023), Omni-MATH [Gao et al., 2024], and STILL [Min et al., 2024]. For

Table 1: Evaluation results across five math benchmarks.

Model	AIME2024		MATH		Minerva		Olympiad		Macro Average	
	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens
DeepScaleR-1.5B										
+ GRPO	26.25	4462	83.44	1657	28.81	1982	**46.59**	2600	46.27	2675
+ L1-Exact	21.88	3754	82.44	3614	28.70	3612	44.55	3687	44.39 ↓1.88	3667 ↑37%
+ L1-Max	25.00	2879	83.50	1794	28.28	1624	44.96	2025	45.44 ↓0.83	2080 ↓22%
+ LACONIC	**27.50**	**2451**	**83.72**	**1000**	**28.84**	**1113**	45.82	**1496**	**46.47** ↑0.2	**1515** ↓43%
Qwen-Math-1.5B										
+ GRPO	11.46	952	**74.86**	570	25.14	656	**39.84**	808	**37.89**	747
+ L1-Exact	9.79	1309	69.69	988	22.52	1277	37.17	1128	34.79 ↓3.1	1176 ↑57%
+ L1-Max	11.04	1229	70.39	779	22.93	1239	37.07	1063	35.36 ↓2.53	1078 ↑44%
+ LACONIC	**11.46**	**674**	73.85	**464**	**25.25**	**524**	38.39	**603**	37.24 ↓0.65	**566** ↓24%

base models, we use DeepScaleR-1.5B-Preview [Luo et al., 2025] (**DeepScaleR-1.5B** for short) and Qwen2.5-Math-1.5B-Instruct [Yang et al., 2024] (**Qwen-Math-1.5B** for short). DeepScaleR-1.5B is a 1.5B-parameter reasoning model fine-tuned from DeepSeek-R1-Distilled-Qwen-1.5B [DeepSeek-AI, 2025] on DeepScaleR-Preview-Dataset, and Qwen-Math-1.5B is a 1.5B-parameter instruction-tuned math model. In line with previous works, we set the maximum response length to 4K tokens per prompt during training and 8K tokens during evaluation for DeepScaleR-1.5B. We set the maximum response length to 2K tokens per prompt during training and evaluation for Qwen-Math-1.5B, because the model supports only up to 4K context lengths.

Baselines. We fine-tune the base models with the following algorithms to serve as baselines and compare with our algorithm LACONIC: (i) GRPO [Shao et al., 2024a]: the standard RL-tuning algorithm originally used in the post-training of DeepScaleR-1.5B and Qwen math models; (ii) L1-Exact [Aggarwal & Welleck, 2025]: a length-controlled policy optimization algorithm that fine-tunes models to satisfy exact token-length constraints; (iii) L1-Max [Aggarwal & Welleck, 2025]: a variant of L1-Exact that further fine-tunes models to observe a maximum token budget.

Evaluation. We evaluate models on 4 common mathematics benchmarks: AIME2024, MATH [Hendrycks et al., 2021], Minerva [Lewkowycz et al., 2022], and Olympiad-Bench [He et al., 2024]. To assess the mathematical reasoning ability of the models, we report pass@1, the fraction of questions for which the model's first response matches the correct answer. To quantify the verbosity of the model's output, we report the average response length.

3.2 Main Results

In this section, we first present in table 1 the main results of all baselines and LACONIC on the 4 mathematics benchmarks. Then we present in table 2 the results on out-of-domain benchmarks, GPQA, LSAT, and MMLU, which probe general knowledge and logic reasoning.

LACONIC outperforms existing length-control methods and matches vanilla RL-tuning while significantly reducing response lengths. On DeepScaleR-1.5B, after fine-tuning, LACONIC outperforms GRPO and both L1 variants on pass@1 while significantly shortening its answers by emitting 43%, 58%, and 26% fewer tokens per prompt on average than GRPO, L1-Exact, and L1-Max respectively. Specifically, LACONIC-tuned model outperforms the three baselines with the highest pass@1 in AIME2024, MATH, and Minerva benchmarks while using significantly fewer tokens. On Qwen-Math-1.5B, after fine-tuning, LACONIC virtually preserves the pass@1 performance of GRPO and outperforms both L1 variants, while substantially shortening its answers by emitting 24%, 52%, and 47% fewer tokens per prompt on average than GRPO, L1-Exact, and L1-Max respectively. Notably, LACONIC generates least tokens across all benchmarks on both base models with comparable or higher pass@1.

LACONIC preserves out-of-domain (OOD) capabilities. LACONIC preserves GRPO's macro average accuracy while generating 44% fewer tokens on average. Our method matches vanilla RL-

Table 2: Evaluation results across out-of-domain (OOD) benchmarks.

Model	GPQA		LSAT		MMLU		Macro Average	
	Accuracy	# Tokens	Accuracy	# Tokens	Accuracy	# Tokens	Accuracy	# Tokens
DeepScaleR-1.5B								
+ GRPO	**28.88**	2229	24.6	3426	43.30	719	**32.26**	2125
+ L1-Exact	22.88	1475	25.19	**1374**	36.67	766	28.25 ↓4.01	1205 ↓43%
+ L1-Max	28.72	1773	**26.58**	2321	38.48	863	31.26 ↓1.00	1652 ↓22%
+ **LACONIC**	27.20	**1167**	24.53	1923	**43.36**	**497**	31.69 ↓0.57	**1196** ↓44%

tuning on MMLU and LSAT with 31% and 44% fewer tokens respectively, and trades 1.68 percentage points on GPQA for a 48% length reduction. Compared with both L1 variants, LACONIC attains higher macro pass@1 with fewer tokens. The results indicate strong OOD task reward preservation with substantially shorter outputs.

4 Further Analysis

In this section, we present additional ablation analysis related to hyperparameters, and examine the computational resources required by LACONIC, including runtime, FLOPs, and memory usage.

4.1 Ablation Analysis on Budget B

We vary the token budget $B \in \{3000, 2000, 1750, 1500\}$ on DeepScaleR-1.5B while keeping all other settings and hyperparameters (including the dual step size) fixed, and train for 300 steps. Figure 3 shows the training dynamics of (a) accuracy reward; (b) average response length; (c) dual variable λ; and (d) average response length to budget ratio $\bar{L}/B$. We evaluate the step-300 checkpoints on the four mathematics benchmarks. Table 3 reports pass@1 and average response lengths.

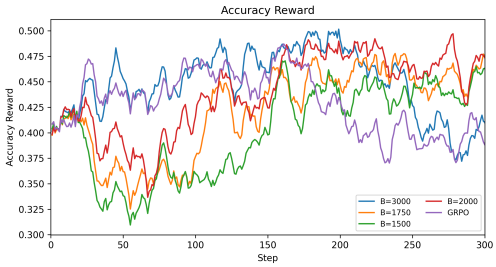
LACONIC provides reliable, hyperparameter-tuning-free length control. Across token budgets, training rapidly drives the average response length under the budget and maintains it near the budget once stabilized. Even under tight constraints on a backbone that naturally produces long responses, LACONIC keeps the average length near the budget. Although prompt distributions during evaluation differ from training, table 3 shows a clear monotonic relation between the token budget B and average response length. In practice, B acts as a single knob and no re-tuning of other hyperparameters is required to achieve effective length control.

LACONIC achieves better reward with less tokens than GRPO and existing length control methods across a wide range of token budgets. We compare the evaluation results of LACONIC with token budgets ranging from 1500 to 3000 in table 3 to GRPO and both L1 variants in table 1. Across a wide range of token budgets, LACONIC consistently outperforms all baselines (GRPO and L1) on most benchmarks while using substantially fewer tokens at the same time. This shows the superiority of LACONIC to preserve reward under constrained token budgets.

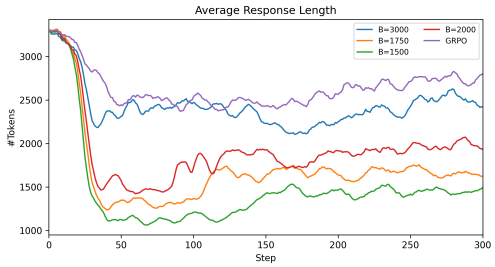
Overall, these results demonstrate LACONIC's controllability (the achieved lengths drop as the budget shrinks), stability (training lengths remain near the budget), and decoupling (changing token budgets B require no re-tuning of other hyperparameters).

4.2 Ablation Analysis on Dual Step Size η

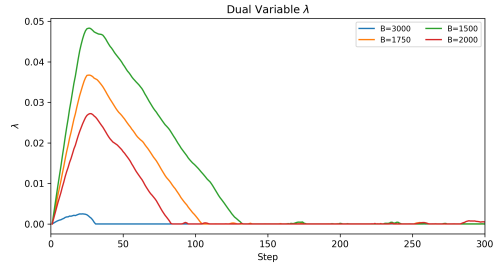
We vary the step size for dual updates $\eta \in \{0.002, 0.01, 0.02\}$ while keeping all other settings fixed and train Qwen-Math-1.5B for 350 steps under a token budget $B = 550$. We set the token budget $B = 550$ for all runs. For $\eta = 0.02$, we also set a ceiling $\lambda_{\text{max}} = 0.1$ for the dual variable. Figure 4 plots the training dynamics of (a) accuracy reward; (b) average response length; (c) dual variable λ; (d) average response length to budget ratio $\bar{L}/B$. We evaluate the step-350 checkpoints on the 4 mathematics benchmarks, and report the pass@1 and average response lengths in table 4.



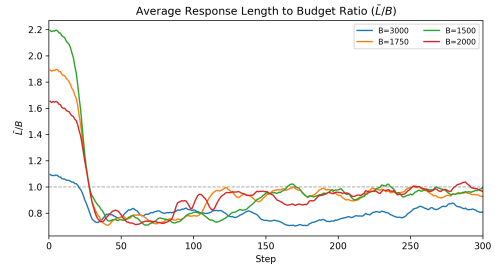
(a) Accuracy reward over training steps



(b) Average response length over training steps



(c) Dual variable λ over training steps



(d) Average length to budget $\bar{L}/B$ over training steps

Figure 3: Ablation on token budget B. All plots are smoothened over 10 steps except (c) dual variable.

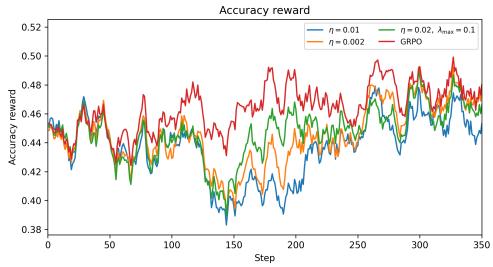
Table 3: Evaluation results across 4 math benchmarks.

Model	AIME2024		MATH		Minerva		Olympiad		Macro Average	
	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens
DeepScaleR-1.5B										
+ GRPO	26.25	4462	83.44	1657	28.81	1982	46.59	2600	46.27	2675
+ LACONIC (3000)	29.79	3642	84.46	1490	29.76	1921	47.07	2386	47.77 ↑1.50	2360 ↓12%
+ LACONIC (2000)	30.83	3558	85.70	1454	31.02	1839	48.94	2357	49.12 ↑2.85	2302 ↓14%
+ LACONIC (1750)	30.21	3477	84.62	1388	30.52	1659	48.78	2221	48.53 ↑2.26	2186 ↓18%
+ LACONIC (1500)	27.50	3124	84.39	1297	31.60	1494	48.65	2048	48.04 ↑1.77	1991 ↓26%

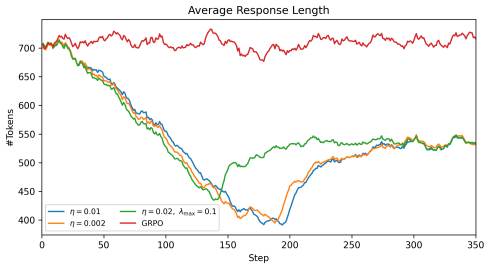
LACONIC is insensitive to the dual step size η. Across different dual step sizes, LACONIC delivers consistent length control. With $\eta = 0.002$ versus 0.01, the training dynamics of average response length are virtually identical throughout, and the final policies match after stabilization. With $\eta = 0.02$ (plus a λ-ceiling), the curves settle into the same stabilized dynamics and reach comparable final reward. This shows that LACONIC is robust to an order-of-magnitude change in the dual step size η.

LACONIC works consistently with a λ-ceiling. Across runs with and without a ceiling, the training curves and final policies are essentially the same once stabilized. In both cases LACONIC tracks the token budget and recovers reward. The λ-ceiling is a preference safeguard. Our primary goal is to favor correct responses over incorrect ones, even when correct answers are longer. With the Lagrangian reward, an excessively large λ can flip preferences and make very short but incorrect outputs score higher than correct long outputs. Specifically, for an indicator reward function $r(q, o) = \mathbb{1}_{\{o \text{ is correct}\}}$, we require $\ell_\lambda(q, o_c) = 1 - \lambda(L(o_c) - B)/B > 0$, where o_c is any correct response. This translates to an upper bound of $\lambda < B/(L(o_c) - B)$ for any correct response o_c. It suffices to set a ceiling of $\lambda_{\max} = B/(L_{\max} - B)$, where $L_{\max}$ is the maximum response length cap. In cases where a λ-ceiling $\lambda_{\max}$ is present, we augment the dual update to a projected version: $\lambda \leftarrow \text{clip}(\lambda + \eta \left(\frac{\bar{L}}{B} - 1\right), 0, L_{\max})$.

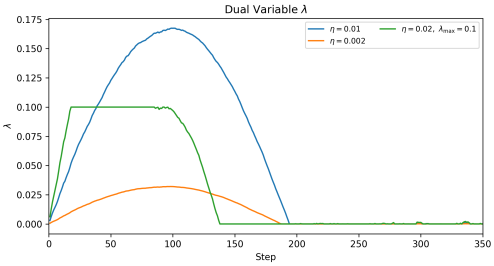
4.3 Computational Resource Analysis



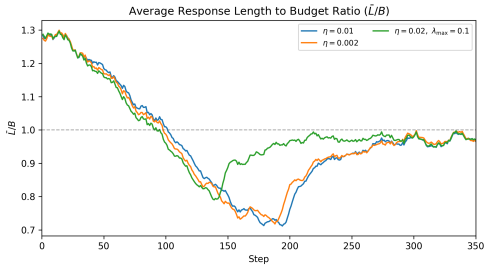
(a) Accuracy reward over training steps



(b) Average response length over training steps



(c) Dual variable λ over training steps



(d) Average response length to budget ratio $\bar{L}/B$ over training steps

Figure 4: Ablation of the length-aware fine-tuning: (a) reward accuracy, (b) mean response length, (c) length-penalty multiplier λ, (d) ratio of average response length over threshold.

Table 4: Evaluation results across 4 math benchmarks.

Model	AIME2024		MATH		Minerva		Olympiad		Macro Average	
	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens
Qwen-Math-1.5B										
+ GRPO	11.46	952	74.86	570	25.14	656	39.84	808	37.89	747
+ LACONIC (0.02)	11.04	661	73.73	466	25.34	539	37.72	601	36.96 ↓0.93	567 ↓24%
+ LACONIC (0.01)	11.46	674	73.85	464	25.25	524	38.39	603	37.24 ↓0.65	566 ↓24%
+ LACONIC (0.002)	10.42	652	73.74	464	25.32	527	38.43	609	36.98 ↓0.91	563 ↓25%

We report in figure 5 wall-clock step time, step time per token, and NVML GPU memory, averaged over training steps. LACONIC is end-to-end cheaper than vanilla RL-tuning. Our method is 19% faster and uses 22% less GPU memory. Per-token cost is nearly unchanged, with a small bookkeeping overhead for the length cost and dual update. Overall, LACONIC adds negligible kernel-level overhead and reduces overall runtime and memory by learning to generate fewer tokens.

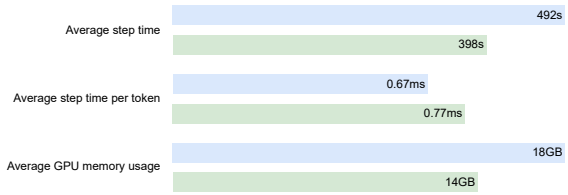


Figure 5: Average computational resource usage of LACONIC (green) and GRPO (blue).

5 Related Work

RL fine-tuning. Reinforcement learning has become a crucial component of LLM post-training, particularly for enhancing large-scale reasoning and aligning model behavior with human preferences or task-specific objectives. One of the most widely used approaches for policy optimization in this setting is the policy gradient algorithm [Sutton & Barto, 2018]. To address the instability of early methods such as REINFORCE [Williams, 1992], Proximal Policy Optimization (PPO) [Schulman et al., 2017] introduced a clipped importance sampling ratio to constrain policy updates, which has since become a standard in RL training pipelines for LLMs. More recently, GRPO [Shao et al., 2024b] proposed group-relative advantage estimation, which eliminates the need for a learned value

function, reducing variance and improving computational efficiency. This innovation has catalyzed a wave of follow-up methods aimed at improving sample efficiency, stability, and performance in RL fine-tuning.

Several GRPO-based extensions have been introduced to address specific challenges in LLM training. SRPO [Zhang et al., 2025] addresses the problem of ineffective samples through history resampling, enhancing credit assignment in sparse-reward settings. DAPO [Yu et al., 2025] introduces dynamic sampling and a token-level gradient loss to better handle complex, multi-step reasoning tasks such as chain-of-thought (CoT) generation. Other variants include VAPO [Yue et al., 2025], which adapts advantage estimation to better capture variance across different reasoning depths; GSPO [Zheng et al., 2025], which emphasizes group-level structure in sampling; GFPO [Shrivastava et al., 2025], which focuses on sample efficiency in long-horizon settings; and GMPO [Zhao et al., 2025], which explores geometric averaging of policy gradients for improved robustness. Collectively, these methods demonstrate the growing sophistication of RL fine-tuning techniques and the community's effort to make them more scalable, stable, and effective for large-scale LLM alignment.

Length-Aware LLMs. Recent work has investigated various approaches for making large language models (LLMs) aware of output length, including modifications to positional encoding, prompt engineering techniques, and post-hoc truncation methods [Li et al., 2025a; Wu et al., 2025; Li et al., 2025b]. A common strategy involves incorporating length preferences into reinforcement learning (RL) fine-tuning through manually designed reward functions that penalize or incentivize certain output lengths [Aggarwal & Welleck, 2025; Cheng et al., 2025; Huang et al., 2025; Yuan et al., 2025]. These methods typically rely on fixed heuristics or penalty terms that remain constant throughout training, and thus optimize a surrogate objective that may be misaligned with the true downstream task reward. This misalignment can lead to suboptimal performance and often requires extensive hyperparameter tuning to balance length control with task-specific quality. Recent efforts have also explored dynamic decoding strategies and curriculum learning to improve length adaptation, but these approaches still depend on manually specified schedules or heuristics. Our work differs in that it aims to align length control with task reward in a more adaptive and data-driven manner, avoiding the limitations of fixed shaping objectives.

6 Conclusion

We present LACONIC, a budgeted, feedback-driven formulation for length-aware reinforcement learning (RL) fine-tuning that integrates seamlessly into standard GRPO pipelines with minimal modifications. The method introduces a zero-penalty window up to a user-specified token budget and applies a single adaptive dual variable beyond that threshold, enabling the model to align incentives for concise reasoning without compromising accuracy. Unlike prior approaches that rely on fixed heuristics or extensive hyperparameter tuning, LACONIC learns to balance task reward and response length in a principled, data-driven manner. Across a suite of reasoning and code-generation benchmarks, the method consistently reduces average output length while preserving or even improving pass@1. Controlled ablations that vary only the token budget confirm precise and stable controllability: training lengths closely track the target, and evaluated models shorten proportionally as the threshold tightens—without the need to retune the dual learning rate or other hyperparameters. Furthermore, the method generalizes across tasks with minimal tuning, demonstrating robustness and scalability. In conclusion, LACONIC provides a simple and effective mechanism to trade off quality for cost, elevating length control to a first-class, reliable component of RL-based LLM fine-tuning.

Limitations and Future Work.

While LACONIC is effective and lightweight, it has several limitations. It currently enforces a global average token budget, which may not capture prompt-specific or context-dependent needs. Extending to per-prompt or adaptive budgets could improve flexibility. The method also assumes a fixed, reliable reward model, which may be unrealistic in noisy or subjective tasks—robust or uncertainty-aware variants are a promising direction. Our experiments are limited to math reasoning; future work could validate generality on dialogue, summarization, or code. Finally, LACONIC handles a single constraint, but the framework naturally extends to multi-constraint settings such as latency or safety.

REFERENCES

Pranjal Aggarwal and Sean Welleck. L1: Controlling how long a reasoning model thinks with reinforcement learning, 2025. URL https://arxiv.org/abs/2503.04697.

Anthropic. Claude opus 4.1. https://www.anthropic.com/news/claude-opus-4-1, August 2025. Accessed: 2025-08-06.

Xingyu Chen, Jiahao Xu, Tian Liang, Zhiwei He, Jianhui Pang, Dian Yu, Linfeng Song, Qiuzhi Liu, Mengfei Zhou, Zhuosheng Zhang, Rui Wang, Zhaopeng Tu, Haitao Mi, and Dong Yu. Do not think that much for 2+3=? on the overthinking of o1-like llms, 2025. URL https://arxiv.org/abs/2412.21187.

Xiaoxue Cheng, Junyi Li, Zhenduo Zhang, Xinyu Tang, Wayne Xin Zhao, Xinyu Kong, and Zhiqiang Zhang. Incentivizing dual process thinking for efficient large language model reasoning, 2025. URL https://arxiv.org/abs/2505.16315.

Gheorghe Comanici, Eric Bieber, Mike Schaekermann, Ice Pasupat, Noveen Sachdeva, Inderjit Dhillon, Marcel Blistein, Ori Ram, Dan Zhang, Evan Rosen, et al. Gemini 2.5: Pushing the frontier with advanced reasoning, multimodality, long context, and next generation agentic capabilities. *arXiv preprint arXiv:2507.06261*, 2025.

DeepSeek-AI. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning, 2025. URL https://arxiv.org/abs/2501.12948.

Jiazhan Feng, Shijue Huang, Xingwei Qu, Ge Zhang, Yujia Qin, Baoquan Zhong, Chengquan Jiang, Jinxin Chi, and Wanjun Zhong. Retool: Reinforcement learning for strategic tool use in llms. *arXiv preprint arXiv:2504.11536*, 2025.

Bofei Gao, Feifan Song, Zhe Yang, Zefan Cai, Yibo Miao, Qingxiu Dong, Lei Li, Chenghao Ma, Liang Chen, Runxin Xu, Zhengyang Tang, Benyou Wang, Daoguang Zan, Shanghaoran Quan, Ge Zhang, Lei Sha, Yichang Zhang, Xuancheng Ren, Tianyu Liu, and Baobao Chang. Omni-math: A universal olympiad level mathematic benchmark for large language models, 2024. URL https://arxiv.org/abs/2410.07985.

Chaoqun He, Renjie Luo, Yuzhuo Bai, Shengding Hu, Zhen Thai, Junhao Shen, Jinyi Hu, Xu Han, Yujie Huang, Yuxiang Zhang, Jie Liu, Lei Qi, Zhiyuan Liu, and Maosong Sun. OlympiadBench: A challenging benchmark for promoting AGI with olympiad-level bilingual multimodal scientific problems. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, 2024.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset. *NeurIPS*, 2021.

Vincent Hsiao, Morgan Fine-Morris, Mark Roberts, Leslie N Smith, and Laura M. Hiatt. A critical assessment of LLMs for solving multi-step problems: Preliminary results. In *AAAI 2025 Workshop LM4Plan*, 2025. URL https://openreview.net/forum?id=kFrqoVtMIy.

Chengyu Huang, Zhengxin Zhang, and Claire Cardie. Hapo: Training language models to reason concisely via history-aware policy optimization, 2025. URL https://arxiv.org/abs/2505.11225.

Bowen Jin, Hansi Zeng, Zhenrui Yue, Jinsung Yoon, Sercan Arik, Dong Wang, Hamed Zamani, and Jiawei Han. Search-r1: Training llms to reason and leverage search engines with reinforcement learning. *arXiv preprint arXiv:2503.09516*, 2025.

Aitor Lewkowycz, Anders Andreassen, David Dohan, Ethan Dyer, Henryk Michalewski, Vinay Ramasesh, Ambrose Slone, Cem Anil, Imanol Schlag, Theo Gutman-Solo, Yuhuai Wu, Behnam Neyshabur, Guy Gur-Ari, and Vedant Misra. Solving quantitative reasoning problems with language models. In *Advances in Neural Information Processing Systems*, 2022.

Xiaoxi Li, Guanting Dong, Jiajie Jin, Yuyao Zhang, Yujia Zhou, Yutao Zhu, Peitian Zhang, and Zhicheng Dou. Search-o1: Agentic search-enhanced large reasoning models. *arXiv preprint arXiv:2501.05366*, 2025a.

Xiaoxi Li, Jiajie Jin, Guanting Dong, Hongjin Qian, Yutao Zhu, Yongkang Wu, Ji-Rong Wen, and Zhicheng Dou. Webthinker: Empowering large reasoning models with deep research capability. *arXiv preprint arXiv:2504.21776*, 2025b.

Michael Luo, Sijun Tan, Justin Wong, Xiaoxiang Shi, William Y. Tang, Manan Roongta, Colin Cai, Jeffrey Luo, Li Erran Li, Raluca Ada Popa, and Ion Stoica. Deepscaler: Surpassing o1-preview with a 1.5b model by scaling rl, 2025. Notion Blog.

Yingqian Min, Zhipeng Chen, Jinhao Jiang, Jie Chen, Jia Deng, Yiwen Hu, Yiru Tang, Jiapeng Wang, Xiaoxue Cheng, Huatong Song, Wayne Xin Zhao, Zheng Liu, Zhongyuan Wang, and Ji-Rong Wen. Imitate, explore, and self-improve: A reproduction report on slow-thinking reasoning systems, 2024. URL https://arxiv.org/abs/2412.09413.

OpenAI. Gpt-5 system card. https://cdn.openai.com/gpt-5-system-card.pdf, August 2025. Accessed: 2025-08-13.

OpenAI, :, Aaron Jaech, Adam Kalai, Adam Lerer, Adam Richardson, Ahmed El-Kishky, Aiden Low, Alec Helyar, Aleksander Madry, Alex Beutel, Alex Carney, Alex Iftimie, Alex Karpenko, Alex Tachard Passos, Alexander Neitz, Alexander Prokofiev, Alexander Wei, Allison Tam, Ally Bennett, Ananya Kumar, Andre Saraiva, Andrea Vallone, Andrew Duberstein, Andrew Kondrich, Andrey Mishchenko, Andy Applebaum, Angela Jiang, Ashvin Nair, Barret Zoph, Behrooz Ghorbani, Ben Rossen, Benjamin Sokolowsky, Boaz Barak, Bob McGrew, Borys Minaiev, Botao Hao, Bowen Baker, Brandon Houghton, Brandon McKinzie, Brydon Eastman, Camillo Lugaresi, Cary Bassin, Cary Hudson, Chak Ming Li, Charles de Bourcy, Chelsea Voss, Chen Shen, Chong Zhang, Chris Koch, Chris Orsinger, Christopher Hesse, Claudia Fischer, Clive Chan, Dan Roberts, Daniel Kappler, Daniel Levy, Daniel Selsam, David Dohan, David Farhi, David Mely, David Robinson, Dimitris Tsipras, Doug Li, Dragos Oprica, Eben Freeman, Eddie Zhang, Edmund Wong, Elizabeth Proehl, Enoch Cheung, Eric Mitchell, Eric Wallace, Erik Ritter, Evan Mays, Fan Wang, Felipe Petroski Such, Filippo Raso, Florencia Leoni, Foivos Tsimpourlas, Francis Song, Fred von Lohmann, Freddie Sulit, Geoff Salmon, Giambattista Parascandolo, Gildas Chabot, Grace Zhao, Greg Brockman, Guillaume Leclerc, Hadi Salman, Haiming Bao, Hao Sheng, Hart Andrin, Hessam Bagherinezhad, Hongyu Ren, Hunter Lightman, Hyung Won Chung, Ian Kivlichan, Ian O'Connell, Ian Osband, Ignasi Clavera Gilaberte, Ilge Akkaya, Ilya Kostrikov, Ilya Sutskever, Irina Kofman, Jakub Pachocki, James Lennon, Jason Wei, Jean Harb, Jerry Twore, Jiacheng Feng, Jiahui Yu, Jiayi Weng, Jie Tang, Jieqi Yu, Joaquin Quiñonero Candela, Joe Palermo, Joel Parish, Johannes Heidecke, John Hallman, John Rizzo, Jonathan Gordon, Jonathan Uesato, Jonathan Ward, Joost Huizinga, Julie Wang, Kai Chen, Kai Xiao, Karan Singhal, Karina Nguyen, Karl Cobbe, Katy Shi, Kayla Wood, Kendra Rimbach, Keren Gu-Lemberg, Kevin Liu, Kevin Lu, Kevin Stone, Kevin Yu, Lama Ahmad, Lauren Yang, Leo Liu, Leon Maksin, Leyton Ho, Liam Fedus, Lilian Weng, Linden Li, Lindsay McCallum, Lindsey Held, Lorenz Kuhn, Lukas Kondraciuk, Lukasz Kaiser, Luke Metz, Madelaine Boyd, Maja Trebacz, Manas Joglekar, Mark Chen, Marko Tintor, Mason Meyer, Matt Jones, Matt Kaufer, Max Schwarzer, Meghan Shah, Mehmet Yatbaz, Melody Y. Guan, Mengyuan Xu, Mengyuan Yan, Mia Glaese, Mianna Chen, Michael Lampe, Michael Malek, Michele Wang, Michelle Fradin, Mike McClay, Mikhail Pavlov, Miles Wang, Mingxuan Wang, Mira Murati, Mo Bavarian, Mostafa Rohaninejad, Nat McAleese, Neil Chowdhury, Neil Chowdhury, Nick Ryder, Nikolas Tezak, Noam Brown, Ofir Nachum, Oleg Boiko, Oleg Murk, Olivia Watkins, Patrick Chao, Paul Ashbourne, Pavel Izmailov, Peter Zhokhov, Rachel Dias, Rahul Arora, Randall Lin, Rapha Gontijo Lopes, Raz Gaon, Reah Miyara, Reimar Leike, Renny Hwang, Rhythm Garg, Robin Brown, Roshan James, Rui Shu, Ryan Cheu, Ryan Greene, Saachi Jain, Sam Altman, Sam Toizer, Sam Toyer, Samuel Miserendino, Sandhini Agarwal, Santiago Hernandez, Sasha Baker, Scott McKinney, Scottie Yan, Shengjia Zhao, Shengli Hu, Shibani Santurkar, Shraman Ray Chaudhuri, Shuyuan Zhang, Siyuan Fu, Spencer Papay, Steph Lin, Suchir Balaji, Suvansh Sanjeev, Szymon Sidor, Tal Broda, Aidan Clark, Tao Wang, Taylor Gordon, Ted Sanders, Tejal Patwardhan, Thibault Sottiaux, Thomas Degry, Thomas Dimson, Tianhao Zheng, Timur Garipov, Tom Stasi, Trapit Bansal, Trevor Creech, Troy Peterson, Tyna Eloundou, Valerie Qi, Vineet Kosaraju, Vinnie Monaco, Vitchyr Pong, Vlad Fomenko, Weiyi Zheng, Wenda Zhou,

Wes McCabe, Wojciech Zaremba, Yann Dubois, Yinghai Lu, Yining Chen, Young Cha, Yu Bai, Yuchen He, Yuchen Zhang, Yunyun Wang, Zheng Shao, and Zhuohan Li. Openai o1 system card, 2024. URL https://arxiv.org/abs/2412.16720.

Changle Qu, Sunhao Dai, Xiaochi Wei, Hengyi Cai, Shuaiqiang Wang, Dawei Yin, Jun Xu, and Ji-rong Wen. Tool learning with large language models: a survey. *Frontiers of Computer Science*, 19(8), January 2025. ISSN 2095-2236. doi: 10.1007/s11704-024-40678-2. URL http://dx.doi.org/10.1007/s11704-024-40678-2.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017. URL https://arxiv.org/abs/1707.06347.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. Deepseekmath: Pushing the limits of mathematical reasoning in open language models, 2024a. URL https://arxiv.org/abs/2402.03300.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, YK Li, Y Wu, et al. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. *arXiv preprint arXiv:2402.03300*, 2024b.

Zhengliang Shi, Shen Gao, Lingyong Yan, Yue Feng, Xiuyi Chen, Zhumin Chen, Dawei Yin, Suzan Verberne, and Zhaochun Ren. Tool learning in the wild: Empowering language models as automatic tool agents, 2025. URL https://arxiv.org/abs/2405.16533.

Vaishnavi Shrivastava, Ahmed Awadallah, Vidhisha Balachandran, Shivam Garg, Harkirat Behl, and Dimitris Papailiopoulos. Sample more to think less: Group filtered policy optimization for concise reasoning, 2025. URL https://arxiv.org/abs/2508.09726.

Yang Sui, Yu-Neng Chuang, Guanchu Wang, Jiamu Zhang, Tianyi Zhang, Jiayi Yuan, Hongyi Liu, Andrew Wen, Shaochen Zhong, Hanjie Chen, and Xia Hu. Stop overthinking: A survey on efficient reasoning for large language models, 2025. URL https://arxiv.org/abs/2503.16419.

Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. The MIT Press, second edition, 2018. URL http://incompleteideas.net/book/the-book-2nd.html.

Kimi Team, Yifan Bai, Yiping Bao, Guanduo Chen, Jiahao Chen, Ningxin Chen, Ruijue Chen, Yanru Chen, Yuankun Chen, Yutian Chen, et al. Kimi k2: Open agentic intelligence. *arXiv preprint arXiv:2507.20534*, 2025.

Chaojie Wang, Yanchen Deng, Zhiyi Lyu, Liang Zeng, Jujie He, Shuicheng Yan, and Bo An. Q*: Improving multi-step reasoning for llms with deliberative planning, 2024. URL https://arxiv.org/abs/2406.14283.

Ronald J Williams. Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine learning*, 8(3-4):229–256, 1992.

Junde Wu, Jiayuan Zhu, Yuyuan Liu, Min Xu, and Yueming Jin. Agentic reasoning: A streamlined framework for enhancing llm reasoning with agentic tools. *arXiv preprint arXiv:2502.04644*, 2025.

An Yang, Beichen Zhang, Binyuan Hui, Bofei Gao, Bowen Yu, Chengpeng Li, Dayiheng Liu, Jianhong Tu, Jingren Zhou, Junyang Lin, Keming Lu, Mingfeng Xue, Runji Lin, Tianyu Liu, Xingzhang Ren, and Zhenru Zhang. Qwen2.5-math technical report: Toward mathematical expert model via self-improvement. *arXiv preprint arXiv:2409.12122*, 2024.

Qiying Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Weinan Dai, Tiantian Fan, Gaohong Liu, Lingjun Liu, Xin Liu, Haibin Lin, Zhiqi Lin, Bole Ma, Guangming Sheng, Yuxuan Tong, Chi Zhang, Mofan Zhang, Wang Zhang, Hang Zhu, Jinhua Zhu, Jiaze Chen, Jiangjie Chen, Chengyi Wang, Hongli Yu, Yuxuan Song, Xiangpeng Wei, Hao Zhou, Jingjing Liu, Wei-Ying Ma, Ya-Qin Zhang, Lin Yan, Mu Qiao, Yonghui Wu, and Mingxuan Wang. Dapo: An open-source llm reinforcement learning system at scale, 2025.

Danlong Yuan, Tian Xie, Shaohan Huang, Zhuocheng Gong, Huishuai Zhang, Chong Luo, Furu Wei, and Dongyan Zhao. Efficient rl training for reasoning models via length-aware optimization, 2025. URL https://arxiv.org/abs/2505.12284.

Yu Yue, Yufeng Yuan, Qiying Yu, Xiaochen Zuo, Ruofei Zhu, Wenyuan Xu, Jiaze Chen, Chengyi Wang, TianTian Fan, Zhengyin Du, Xiangpeng Wei, Xiangyu Yu, Gaohong Liu, Juncai Liu, Lingjun Liu, Haibin Lin, Zhiqi Lin, Bole Ma, Chi Zhang, Mofan Zhang, Wang Zhang, Hang Zhu, Ru Zhang, Xin Liu, Mingxuan Wang, Yonghui Wu, and Lin Yan. Vapo: Efficient and reliable reinforcement learning for advanced reasoning tasks, 2025.

Xiaojiang Zhang, Jinghui Wang, Zifei Cheng, Wenhao Zhuang, Zheng Lin, Minglei Zhang, Shaojie Wang, Yinghan Cui, Chao Wang, Junyi Peng, Shimiao Jiang, Shiqi Kuang, Shouyu Yin, Chaohang Wen, Haotian Zhang, Bin Chen, and Bing Yu. Srpo: A cross-domain implementation of large-scale reinforcement learning on llm, 2025.

Yuzhong Zhao, Yue Liu, Junpeng Liu, Jingye Chen, Xun Wu, Yaru Hao, Tengchao Lv, Shaohan Huang, Lei Cui, Qixiang Ye, Fang Wan, and Furu Wei. Geometric-mean policy optimization, 2025.

Chujie Zheng, Shixuan Liu, Mingze Li, Xiong-Hui Chen, Bowen Yu, Chang Gao, Kai Dang, Yuqiong Liu, Rui Men, An Yang, Jingren Zhou, and Junyang Lin. Group sequence policy optimization, 2025.

LLM USAGE

We used LLMs as general-purpose writing and debugging assistants. Specifically, LLMs were employed to help polish the writing (e.g., improving sentence clarity, grammar, and flow) and occasionally to assist with debugging minor implementation issues (e.g., identifying syntax errors or suggesting code refactoring). However, all core ideas, research questions, methodological designs, codebase implementations, experiments, and analyses were entirely conceived, developed, and conducted by the authors. No part of the intellectual contribution, experimental framework, or scientific reasoning was generated by an LLM.

APPENDIX

A ABLATION ANALYSIS ON COST FUNCTION

In this section, we discuss an alternative cost function

$$\widetilde{c}(q,o) = \frac{L(o) - B}{B}. \tag{7}$$

Recall the constrained optimization problem of length-aware LLMs formulated in equation (2):

$$\max_{\theta} \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q,o)],\ \text{s.t.}\ \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)] \le B.$$

The corresponding Lagrangian is

$$\mathcal{L}(\theta,\lambda) = \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q,o)] - \lambda\,(\frac{\mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)]}{B} - 1), \quad \lambda \ge 0, \tag{8}$$

and the problem equivalently translates to $\min_{\lambda\ge 0}\max_\theta \mathcal{L}(\theta,\lambda)$. The standard primal-dual approach solves the primal variable θ and dual variable λ iteratively with partial derivatives. Specifically, this gives us the gradient-descent-style dual update rule:

$$\lambda \leftarrow \max\{\lambda + \eta\frac{\partial\mathcal{L}(\theta,\lambda)}{\partial\lambda}, 0\} = \max\{\lambda + \eta(\frac{\mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)]}{B} - 1), 0\}. \tag{9}$$

Empirically we estimate the expectation $\mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)]$ with the minibatch mean $\bar{L}$, and we derive exactly equation (6):

$$\lambda \leftarrow \max\{\lambda + \eta(\frac{\bar{L}}{B} - 1), 0\}.$$

The immediate primal update rule is given by

$$\theta \leftarrow \arg\max_{\theta} \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q,o)] - \lambda\,(\frac{\mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[L(o)]}{B} - 1). \tag{10}$$

By linearity of expectations and removing constants inside $\arg\max$, we have

$$\theta \leftarrow \arg\max_{\theta} \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q,o) - \lambda\frac{L(o) - B}{B}]. \tag{11}$$

By definition of $\widetilde{c}(q,o)$, we have

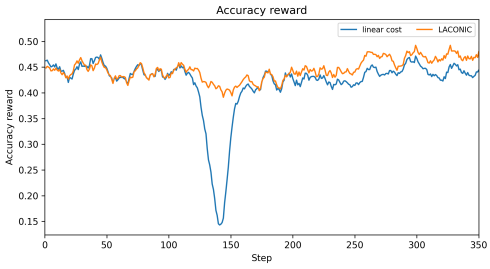
$$\theta \leftarrow \arg\max_{\theta} \mathbb{E}_{q\sim P(Q),o\sim\pi_\theta(\cdot|q)}[r(q,o) - \lambda\cdot\widetilde{c}(q,o)]. \tag{12}$$

Therefore, the linear cost function $\widetilde{c}(q,o)$ serves as a natural cost function for our constrained RL formulation.

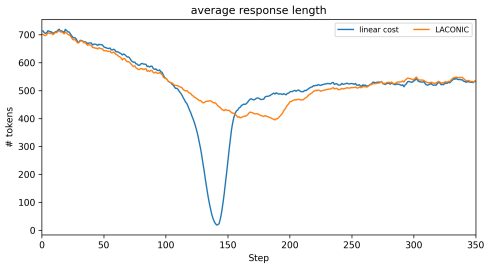
However, using the purely linear cost function $\widetilde{c}(q,o) = \frac{L(o)-B}{B}$ inside the primal (actor model) update in equation (12) is problematic in practice. Whenever $\lambda > 0$, the actor model is incentivized to shorten responses on all samples, and collapses the policy to extremely short outputs. This leads to highly unstable training dynamics, which is undesirable in practice. We conduct ablation experiments

Table 5: Ablation results on linear cost functions.

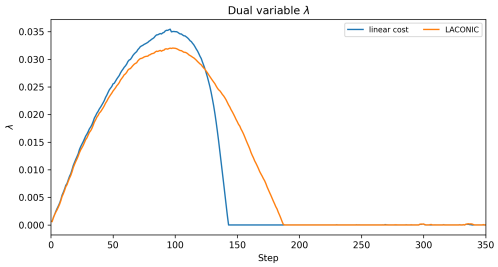
Model	AIME2024		MATH		Minerva		Olympiad		Macro Average	
	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens	Pass@1	# Tokens
Qwen-Math-1.5B										
+ **LACONIC**	11.46	674	73.85	464	25.25	524	38.39	603	37.24	566
+ linear costs	11.35	665	73.42	479	25.53	531	38.42	627	37.18	576



(a) Accuracy reward over training steps



(b) Average response length over training steps



(c) Dual variable λ over training steps

Figure 6: Ablation of the cost functions: (a) reward accuracy, (b) mean response length, (c) length-penalty multiplier λ,

by replacing the cost function with $\widetilde{c}$ while keep all other settings the same. We present the evaluation results of resulting checkpoints in table 5.

We also plot the training dynamics in figure 6. The experiment shows that with linear cost function $\frac{L(o)-B}{B}$, the average accuracy plummets from 40% to 8%, and the mean response lengths fall below 10 tokens, indicating that the model is no longer producing meaningful responses during training. Although as shown in table 5, the primal-dual framework with linear cost function $\widetilde{c}$ recovers the model's performance after the model is stabilized, the unstable update steps can introduce great risk and instability to LLM fine-tuning.

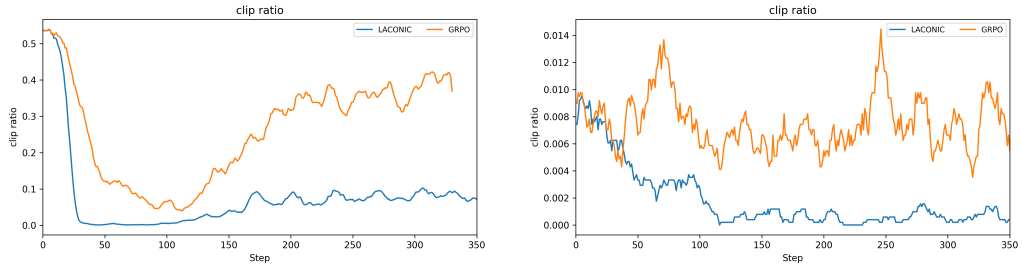
B Training Details

B.1 Training setup

For the main results, we train DeepScaleR-1.5B with a 2000-token budget for 150 steps, then further train the saved checkpoint with a 1000-token budget for another 100 steps. We train Qwen-Math-1.5B with a 550-token budget for 350 steps. We set batch size to 128, lambda initialized to 0, lambda ceiling to 0.03, step size η to 0.002, and rollout number to 7.

B.2 Training dynamics

We present the training dynamics of the clip ratio in appendix B.2. The logged data of clip ratio show that 35-40% responses generated by GRPO are clipped, while LACONIC is able to control the clip ratio under 10%. This partially explains the "counterintuitive" results where LACONIC yields better



(a) Clip ratio on DeepScaler-1.5B. (b) Clip ratio on Qwen-Math-1.5B.

Figure 7: Training dynamics of clip ratio. The clip ratio is the ratio of clipped responses in the current minibatch, and a response is clipped when its length reaches the maximum response length, in our experiment, 4096 tokens for DeepScaler-1.5B and 2048 for Qwen-Math-1.5B.

pass@1 on many benchmarks than GRPO, despite the token budget constraints, as high clip-ratio of GRPO causes insufficient training on corresponding prompts.

C Case Study

Prompt: Every morning Aya goes for a 9-kilometer-long walk and stops at a coffee shop afterwards. When she walks at a constant speed of s kilometers per hour, the walk takes her 4 hours, including t minutes spent in the coffee shop. When she walks $s+2$ kilometers per hour, the walk takes her 2 hours and 24 minutes, including t minutes spent in the coffee shop. Suppose Aya walks at $s+\frac{1}{2}$ kilometers per hour. Find the number of minutes the walk takes her, including the t minutes spent in the coffee shop. You FIRST think about the reasoning process as an internal monologue and then provide the final answer. The reasoning process MUST BE enclosed within <think> </think> tags. The final answer MUST BE put in \boxed{}.

Ground truth answer: 204.

LACONIC Output (#Tokens = 855)

Alright, let's tackle this problem step by step.
First, let's understand what's given. Aya walks a total distance of 9 kilometers. The time she takes depends on her walking speed. She stops at a coffee shop, and the time spent there is t minutes.
We are given two scenarios:
1. When her speed is s km/h, the total time (walking plus coffee) is 4 hours. 2. When her speed increases to $s+2$ km/h, the total time becomes 2 hours and 24 minutes, which is $2+\frac{24}{60}=2.4$ hours.
We need to find the total time when her speed is $s+0.5$ km/h.
Let's denote the time spent in the coffee shop as t minutes. Since the coffee shop time is the same in both scenarios, we can set up equations based on the total time.
In the first scenario: Total time = walking time + t minutes. Walking time = distance / speed $= 9/s$ hours. Therefore, $9/s+t=4$ hours.
In the second scenario: Walking time $= 9/(s+2)$ hours. Total time $= 9/(s+2)+t=2.4$ hours.
Now, we have two equations: 1. $9/s+t=4$ 2. $9/(s+2)+t=2.4$
We can subtract equation 2 from equation 1 to eliminate t:
$(9/s+t)-(9/(s+2)+t)=4-2.4$
Simplify: $9/s-9/(s+2)=1.6$
Factor out 9: $9[1/s-1/(s+2)]=1.6$
Compute the difference inside the brackets:
$1/s-1/(s+2)=(s+2-s)/(s(s+2))=2/(s(s+2))$ So, $9*(2)/(s(s+2))=1.6$
Simplify: $18/(s(s+2))=1.6$

Multiply both sides by $s(s+2)$: $18 = 1.6\,s(s+2)$
Divide both sides by 1.6: $s(s+2) = 18/1.6$
Calculate $18/1.6$: $18/1.6 = 11.25$
So, $s^2 + 2s - 11.25 = 0$
Now, solve the quadratic equation: $s^2 + 2s - 11.25 = 0$
Using quadratic formula: $s = [-2 \pm \sqrt{4 + 45}]/2$
Since speed can't be negative: $s = [-2 + \sqrt{49}]/2$ $\sqrt{49} = 7$
Thus, $s = (-2 + 7)/2 = 5/2 = 2.5$ km/h
Now, we can find t from equation 1: $9/s + t = 4$ $9/2.5 + t = 4$
Compute $9/2.5 = 3.6$ So, $3.6 + t = 4$
Thus, $t = 0.4$ hours $= 0.4 * 60 = 24$ minutes.
Now, when her speed is $s + 0.5 = 2.5 + 0.5 = 3$ km/h.
We need to find the total time including coffee shop.
First, compute walking time: $9/3 = 3$ hours.
Total time = walking time + coffee shop time $= 3 + 0.4 = 3.4$ hours.
Convert to minutes: $3.4 * 60 = 204$ minutes.
So, the total time is 204 minutes.
</think>
The total time Aya takes when walking at $s + \frac{1}{2}$ km/h is $\boxed{204}$ minutes.

DSR-4K (#Tokens = 1763)

Okay, so I have this problem about Aya's walking and coffee shop time. Let me try to figure it out step by step.
First, the problem says that Aya walks 9 kilometers every morning and stops at a coffee shop. When she walks at a speed of s km/h, it takes her 4 hours, including t minutes spent in the coffee shop. Then, when she increases her speed to $s + 2$ km/h, the walk takes her 2 hours and 24 minutes, but this time includes the same t minutes in the coffee shop.
I need to find the time it takes for Aya to walk at a speed of $s + \frac{1}{2}$ km/h, including the t minutes in the coffee shop.
Let me break down the information given:

1. When walking at speed s km/h, the total time taken is 4 hours, including t minutes in the coffee shop. So, the walking time is 4 hours minus t minutes. But since speed is in km/h, maybe it's better to convert everything into hours.
2. When walking at speed $s + 2$ km/h, the total time is 2 hours and 24 minutes, which is $2 + \frac{24}{60} = 2.4$ hours. Again, the walking time is total time minus t minutes.

So, let me denote:

- Walking speed: s km/h
- Walking speed in the second case: $s + 2$ km/h
- Total time in first case: 4 hours
- Total time in second case: 2.4 hours
- Both cases include t minutes in the coffee shop.

We need to find the total time when walking speed is $s + \frac{1}{2}$ km/h.
First, let's convert the walking times into hours to make calculations easier.
For the first scenario:
Total time $= 4$ hours
Time spent walking $= 4 - \frac{t}{60}$ hours (since t is in minutes)
Distance walked is 9 km, so using the formula:
Distance = speed $\times$ time
So, $9 = s \times \left(4 - \frac{t}{60}\right)$
Similarly, for the second scenario:
Total time $= 2.4$ hours
Time spent walking $= 2.4 - \frac{t}{60}$ hours
Distance walked is 9 km again:

$9 = (s+2) \times \left(2.4 - \frac{t}{60}\right)$
Now, we have two equations:
1) $9 = s \times \left(4 - \frac{t}{60}\right)$
2) $9 = (s+2) \times \left(2.4 - \frac{t}{60}\right)$
Since both equal 9, we can set them equal to each other:
$s \times \left(4 - \frac{t}{60}\right) = (s+2) \times \left(2.4 - \frac{t}{60}\right)$
Let me expand both sides:
Left side: $4s - s\left(\frac{t}{60}\right)$
Right side: $(s+2)\left(2.4 - \frac{t}{60}\right) = 2.4s - s\left(\frac{t}{60}\right) + 4.8 - 2\left(\frac{t}{60}\right)$
Simplify both sides:
Left: $4s - \dfrac{st}{60}$
Right: $2.4s - \dfrac{st}{60} + 4.8 - \dfrac{2t}{60}$
Now, let's write the equation:
$4s - \dfrac{st}{60} = 2.4s - \dfrac{st}{60} + 4.8 - \dfrac{2t}{60}$
Notice that the $-\dfrac{st}{60}$ terms on both sides cancel each other out. So we can subtract $2.4s$ from both sides:
$4s - 2.4s = -\dfrac{st}{60} + \dfrac{st}{60} + 4.8 - \dfrac{2t}{60}$
Simplify:
$1.6s = 4.8 - \dfrac{2t}{60}$
Simplify $1.6s = 4.8 - \dfrac{t}{30}$
Let me write that as:
$1.6s + \dfrac{t}{30} = 4.8$
Equation (A): $1.6s + \dfrac{t}{30} = 4.8$
Now, let's go back to the first equation:
$9 = s \times \left(4 - \frac{t}{60}\right)$
Let me write this as:
$9 = 4s - \dfrac{st}{60}$
Multiply both sides by 60 to eliminate denominators:
$9 \times 60 = 4s \times 60 - st$
$540 = 240s - st$
Let's write this as:
$st = 240s - 540$
Divide both sides by s (assuming $s \neq 0$):
$t = 240 - \dfrac{540}{s}$
Equation (B): $t = 240 - \dfrac{540}{s}$
Now, substitute Equation (B) into Equation (A):
$1.6s + \dfrac{240 - \frac{540}{s}}{30} = 4.8$
Simplify:
$1.6s + \dfrac{240}{30} - \dfrac{540}{30s} = 4.8$
Calculate:
$1.6s + 8 - \dfrac{18}{s} = 4.8$
Now, subtract 4.8 from both sides:
$1.6s + 8 - \dfrac{18}{s} - 4.8 = 0$
Simplify:

$1.6s + 3.2 - \frac{18}{s} = 0$
Multiply both sides by s to eliminate denominator:
$1.6s^2 + 3.2s - 18 = 0$
Now, we have a quadratic equation:
$1.6s^2 + 3.2s - 18 = 0$
Let me multiply both sides by 10 to eliminate decimals:
$16s^2 + 32s - 180 = 0$
Divide all terms by 4 to simplify:
$4s^2 + 8s - 45 = 0$
Now, solve for s using quadratic formula:
$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Where $a = 4$, $b = 8$, $c = -45$
Calculate discriminant:
$D = 8^2 - 4 \times 4 \times (-45) = 64 + 720 = 784$
Square root of D: $\sqrt{784} = 28$
So,
$s = \frac{-8 \pm 28}{2 \times 4} = \frac{-8 \pm 28}{8}$
We have two solutions:
1) $s = \frac{-8 + 28}{8} = \frac{20}{8} = 2.5$ km/h
2) $s = \frac{-8 - 28}{8} = \frac{-36}{8} = -4.5$ km/h
Since speed cannot be negative, $s = 2.5$ km/h
Now, find t using Equation (B):
$t = 240 - \frac{540}{s} = 240 - \frac{540}{2.5}$
Calculate $\frac{540}{2.5}$:
$540/2.5 = 216$
So,
$t = 240 - 216 = 24$ minutes
So, $t = 24$ minutes.
Now, when Aya walks at $s + \frac{1}{2}$ km/h, which is $2.5 + 0.5 = 3$ km/h.
We need to find the total time including t minutes in the coffee shop.
First, calculate the walking time at 3 km/h.
Distance is 9 km, speed is 3 km/h, so time is $\frac{9}{3} = 3$ hours.
But total time is walking time plus t minutes.
Walking time is 3 hours, which is 180 minutes.
Plus $t = 24$ minutes.
So total time is $180 + 24 = 204$ minutes.
Therefore, the walk takes her 204 minutes, including the t minutes spent in the coffee shop.
</think>
The number of minutes the walk takes her, including the t minutes spent in the coffee shop, is $\boxed{204}$.