

000 001 SCALABLE SPATIO-TEMPORAL SE(3) DIFFUSION FOR 002 LONG-HORIZON PROTEIN DYNAMICS 003 004

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007 008 ABSTRACT 009

010
011 Molecular dynamics (MD) simulations remain the gold standard for studying
012 protein dynamics, but their computational cost limits access to biologically rel-
013 evant timescales. Recent generative models have shown promise in accelerat-
014 ing simulations, yet they struggle with long-horizon generation due to architec-
015 tural constraints, error accumulation, and inadequate modeling of spatiotemporal
016 dynamics. We present **STAR-MD** (Spatio-Temporal Autoregressive Rollout for
017 Molecular Dynamics), a scalable $SE(3)$ -equivariant diffusion model that gener-
018 ates physically plausible protein trajectories over microsecond timescales. Our
019 key innovation is a causal diffusion transformer with joint spatiotemporal atten-
020 tion that efficiently captures complex space-time dependencies while avoiding the
021 memory bottlenecks of existing methods. On the standard ATLAS benchmark,
022 STAR-MD achieves state-of-the-art performance across all metrics—substantially
023 improving conformational coverage, structural validity, and dynamic fidelity com-
024 pared to previous methods. STAR-MD successfully extrapolates to generate sta-
025 ble microsecond-scale trajectories where baseline methods fail catastrophically,
026 maintaining high structural quality throughout the extended rollout. Our com-
027 prehensive evaluation reveals severe limitations in current models for long-horizon
028 generation, while demonstrating that STAR-MD’s joint spatiotemporal modeling
029 enables robust dynamics simulation at biologically relevant timescales, paving the
030 way for accelerated exploration of protein function.

031 1 INTRODUCTION 032

033 Protein functions emerge from conformational dynamics – the continuous structural changes that
034 underlie important biological processes such as catalysis, binding, and allosteric regulations (Mc-
035 Cammon, 1984; Berendsen & Hayward, 2000). Classical molecular dynamics (MD) simulation
036 remains the gold standard and relies on physical models to **integrate** atomic motions over time us-
037 ing Newtonian mechanics. However, the need for small integration steps (\sim femtoseconds) severely
038 limits its practicality for exploring the microsecond–millisecond timescales often required to capture
039 biologically relevant events. Recent advances in protein structure prediction (Jumper et al., 2021;
040 Abramson et al., 2024; Lin et al., 2023) and generative modeling for conformational dynamics (Jing
041 et al., 2024b; Cheng et al., 2025; Shen et al., 2025; Costa et al., 2024) offer promising data-driven
042 approaches to accelerate simulations by learning the dynamics directly from data and generating
043 trajectories at coarser temporal resolutions. However, existing methods are still constrained by short
044 time horizons (typically up to nanoseconds) and struggle to scale to larger proteins. These limi-
045 tations highlight an urgent need for generative models capable of producing physically plausible
046 protein trajectories over extended timescales while remaining computationally efficient.

047 Scaling protein dynamics modeling to long timescales requires both *an efficient structural repre-
048 sentation* and *an architecture capable of capturing complex spatiotemporal dependencies*. Current
049 approaches often rely on pairwise residue representations or computationally expensive architectures
050 (e.g., AlphaFold2-style triangular attention), leading to quadratic memory growth and cubic com-
051 putational cost with respect to protein size. These challenges become even more pronounced when
052 accounting for spatiotemporal coupling across multiple conformations during trajectory modeling.
053 As a result, most existing models treat spatial and temporal components separately, employing inter-
leaved “spatial” and “temporal” modules (Jing et al., 2024b; Cheng et al., 2025; Shen et al., 2025),
which limits their expressiveness in capturing coupled dynamics. These limitations confine current

054 models to small proteins and short simulation horizons, ultimately hindering their ability to learn
 055 long-range temporal dependencies and generate high-quality conformations over extended rollouts.
 056

057 We introduce **Spatio-Temporal Autoregressive Rollout for Molecular Dynamics** (STAR-MD), an
 058 **SE(3)**-equivariant autoregressive diffusion model for generating physically plausible protein trajec-
 059 tories over microsecond timescales, even for large protein systems. At its core, STAR-MD employs
 060 *a causal diffusion transformer with joint spatio-temporal attention*, enabling improved autoregres-
 061 sive generation by dynamically computing attention over historical context during diffusion and
 062 capturing long-range dependencies through expressive yet memory-efficient spatiotemporal model-
 063 ing.

064 Our main contributions are summarized as follows:

- 065 • We present STAR-MD, a novel autoregressive diffusion transformer that leverages efficient spa-
 066 tiotemporal attention to model the complex dependencies underlying protein dynamics.
- 067 • Through several key technical improvements, including historical context noise addition dur-
 068 ing training and inference, block-diffusion-style causal training, and architectural optimizations,
 069 STAR-MD achieves efficient, scalable training and stable trajectory generation, significantly im-
 070 proving over current state-of-the-art models.
- 071 • We perform an exhaustive evaluation across multiple different simulation timescales ranging from
 072 100 ns to 1 μ s, providing a comprehensive assessment of conformation quality, coverage, and dy-
 073 namic fidelity. Our analyses yield new insights into the limitations of current state-of-the-art
 074 models in long-horizon generation and offer valuable guidance for future model design.

075 2 RELATED WORK

077 **Protein Conformation Generation.** Several models (Jing et al., 2024a; Lewis et al., 2024; Wang
 078 et al., 2024) build on advances in protein structure prediction (Jumper et al., 2021; Abramson et al.,
 079 2024; Lin et al., 2023) and diffusion models (Ho et al., 2020) to directly generate time-independent
 080 conformations. These methods provide an efficient alternative to MD by enabling parallel sampling.
 081 However, they capture only the equilibrium distribution of conformations and cannot model the
 082 temporal evolution of protein dynamics.

083 **MD Trajectory Generation.** To model the temporal evolution, operator-based methods Klein et al.
 084 (2024); Costa et al. (2024) aim to learn transport operators that predict conformations at lagged
 085 times. These methods approximate evolution as a Markovian process, and thus fail to capture the
 086 non-Markovian properties often present in partially observed systems such as protein dynamics data.
 087 In contrast, methods generating trajectories, akin to video generations, consider dependencies across
 088 multiple time frames. AlphaFolding (Cheng et al., 2025) incorporates higher-order information
 089 through additional “motion frames” and generates multiple future frames simultaneously. Similarly,
 090 MDGen (Jing et al., 2024b) models the joint distribution of frames across 100-ns trajectories. How-
 091 ever, both methods only capture the dependencies in a fixed context and prediction window. When
 092 generating longer trajectories through extension, they discard memory of earlier windows, breaking
 093 temporal consistency. To address this, ConfRover (Shen et al., 2025) adopts language model-style
 094 autoregression that can generate trajectories of arbitrary lengths while maintaining full memory via
 095 KV-caching. STAR-MD follows this approach to avoid the “memory break” in block-based models.

096 **Scalable Structural and Temporal Modeling.** Current structural models, including AlphaFolding
 097 and ConfRover, represent protein structures using single and pairwise features and process them
 098 with specialized architectures such as Pairformer and Invariant Point Attention (IPA) (Jumper et al.,
 099 2021). While these modules are highly expressive and preserved the required roto-translational
 100 symmetries, they are computationally and memory intensive. This limits model efficiency and scal-
 101 ability, especially when extended to modeling trajectories across multiple time frames. MDGen
 102 sidesteps this limitation by anchoring trajectories to key frames, encoding only single embeddings
 103 with standard transformers. While this allows modeling of long trajectories (e.g., 250 frames for AT-
 104 LAS), the reliance on key frames limited flexibility and model performance is suboptimal. Lastly,
 105 all the above approaches model dynamics between frames using interleaved spatial and temporal
 106 modules. While this decoupled design reduces computational cost, it limits the model’s expressive
 107 power to capture complex spatiotemporal relationships. In contrast, STAR-MD proposes to em-
 108 ploy joint spatiotemporal attentions on the single **embeddings**’ direct space-time processing while
 109 keeping the memory footprint manageable.

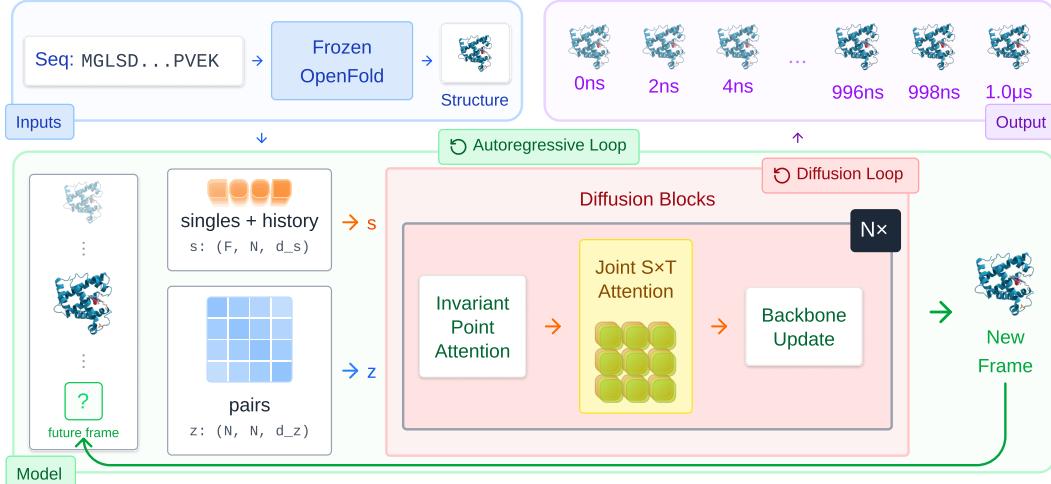


Figure 1: [Updated Figure] **Overview of STAR-MD generation.** Input contains protein sequence and a starting conformation. In autoregressive diffusion generation, structural information of previously generated conformations and current noisy conformations are encoded into single and pair representations. A joint spatiotemporal attention block is employed to capture context information to update the single representation of the current frame. The main model block iterated to diffuse a clean conformation for the current frame and added to history for generating next frames.

3 METHOD

To generate long and physically plausible protein trajectories, we propose **Spatio-Temporal Autoregressive Rollout for Molecular Dynamics** (STAR-MD), a spatio-temporal $SE(3)$ -diffusion model that operates within an autoregressive framework. In the following sections, we first describe the overall autoregressive diffusion framework, then detail the architecture of our diffusion model, and finally explain the training and inference procedures.

3.1 AUTOREGRESSIVE $SE(3)$ -DIFFUSION MODELS FOR PROTEIN TRAJECTORIES

Autoregressive Trajectory Generation. We aim to generate protein conformation *trajectories* $\mathbf{x}_{1:L}$ in an autoregressive fashion $\prod_{\ell=1}^L p(\mathbf{x}_\ell | \mathbf{x}_{<\ell}, \Delta t_\ell)$, where Δt_ℓ represents the time interval between frames. This formulation amortizes trajectory generation into a frame-level process conditioned on the entire past history, enabling the model to generate future dynamics of arbitrary length while maintaining a flexible memory representation learned from data. Causal transformers, widely used in language models for efficient autoregressive sequence modeling (Touvron et al., 2023), provide a natural architectural choice for this task.

$SE(3)$ Diffusion Models. To model the generation process of each frame $p(\mathbf{x}_\ell | \mathbf{x}_{<\ell}, \Delta t_\ell)$, we use a (conditional) diffusion model on the Riemannian manifold $SE(3)$, where $\mathbf{x} = [\mathbf{T}, \mathbf{R}]$ which captures translation positions $\mathbf{T} = \{T^i\}_{i=1}^N \in \mathbb{R}^{3N}$ and rotations $\mathbf{R} = \{R^i\}_{i=1}^N \in SO(3)^N$ for each amino-acid residue in a protein sequence of length N (Yim et al., 2023; Wang et al., 2024; Shen et al., 2025). The forward diffusion process independently corrupts translations and rotations across the diffusion time τ

$$\mathbf{T}_\tau = \sqrt{\alpha_\tau} \mathbf{T}_0 + \sqrt{1 - \alpha_\tau} \epsilon, \quad \epsilon \sim \mathcal{N}(0, I_{3N}) \quad (1)$$

$$\mathbf{R}_\tau \sim \text{IGSO}_3(\mathbf{R}_0, \sigma_\tau^2), \quad (2)$$

where IGSO_3 is the isotropic Gaussian on $SO(3)$. Given a noisy structure \mathbf{x}^τ , diffusion time τ , and condition \mathbf{c} , a denoising score network

$$s_\theta(\mathbf{x}^\tau, \tau, \mathbf{c}) = [s_\theta^{\mathbf{T}}, s_\theta^{\mathbf{R}}](\mathbf{x}^\tau, \tau, \mathbf{c}) \quad (3)$$

is trained via denoising score matching to predict the noise added to both components. During inference, the learned score function can be used to sample from the reverse diffusion process and generate protein structures.

To capture temporal dependencies needed for autoregressive generation, the condition \mathbf{c} for generating the current frame should incorporate all past history $\mathbf{x}_{<\ell}$. Specifically, we design an efficient

162 and expressive *causal diffusion transformer* network to compute $\mathbf{c}(\mathbf{x}_{<\ell}^0, \mathbf{x}_\ell^\tau, \tau)$ from the preceding
 163 clean frames and the current noisy frame \mathbf{x}_ℓ^τ . This design differs from prior works such as MDGen
 164 (Jing et al., 2024b) & AlphaFold (Cheng et al., 2025) which do not include autoregressive
 165 conditioning, and ConfRover (Shen et al., 2025) which compresses all preceding frames into a static
 166 condition $\mathbf{c}(\mathbf{x}_{<\ell})$. In the following section, we detail the design of $\mathbf{c}(\mathbf{x}_{<\ell}^0, \mathbf{x}_\ell^\tau, \tau)$ and how it effi-
 167 ciently integrates spatiotemporal information from preceding frames.

168
 169 **3.2 ARCHITECTURE**

170 In this section, we describe the architecture choices underlying STAR-MD. The main features of
 171 this architecture lie in the diffusion decoder, which uses *spatio-temporal attention* for autoregressive
 172 conditioning on previous frames and *block diffusion* allowing for efficient training and KV caching.

173 **Input Module.** We mirror the FrameEncoder module in Shen et al. (2025), starting with
 174 a frozen OpenFold FoldingModule for sequence-level single-residue and pairwise features
 175 $\mathbf{s}, \mathbf{z} = \{\mathbf{s}_i, \mathbf{z}_{ij}\}_{i,j=1}^N$. We incorporate these time-independent features with amino-acid metadata
 176 and pairwise C_β distance within each frame to obtain time-dependent features $\mathbf{s}_\ell^{\text{init}}, \mathbf{z}_\ell^{\text{init}}$, which are
 177 used as initial input to the diffusion blocks.

178 **Diffusion Blocks.** Our diffusion module contains submodules stacked into blocks:

179 1. Invariant Point Attention layers (Jumper et al., 2021) update single features \mathbf{s}_ℓ using pair features
 180 \mathbf{z}_ℓ and noisy frames \mathbf{x}_ℓ^τ . This step is independent for each frame.
 181 2. Joint Spatio-Temporal Attention layers update the single-residue features \mathbf{s}_ℓ by attending to those
 182 from previous frames $\mathbf{s}_{\leq \ell}$, which allows for exchange of temporal information.
 183 3. Pair features \mathbf{z}_ℓ and noisy frames \mathbf{x}_ℓ^τ are updated using single features \mathbf{s}_ℓ via the
 184 [EdgeTransition layer from Jumper et al. \(2021\)](#).

185 After stacking the above blocks, the final update of the coordinates \mathbf{x}_ℓ^τ is used as the score function
 186 prediction and fed to the denoising score matching loss.

187 **Joint Spatio-Temporal Attention.** Our joint spatio-temporal (S×T) attention mechanism integrates
 188 information across the temporal dimensions, departing from previous models that employ factorized,
 189 “space-then-time” attention which imposes a restrictive bias that spatial and temporal dependencies
 190 are separable. Instead, our S×T attention operates on tokens representing residue-frame pairs (i, ℓ) ,
 191 allowing it to directly model non-separable relationships, such as how motion at one residue is
 192 coupled to past motion at a distant site. 2D Rotary Position Embedding (2D-RoPE) (Heo et al.,
 193 2024) is used to embed residue and frame indices, which allows extrapolation past the training
 194 number of frames.

195 [This architectural choice is key to our model’s scalability. We analyze the per-layer computational](#)
 196 complexity of our S×T attention ($\mathcal{O}(N^2L^2)$) against general S+T (i.e., “space-then-time”) [architectural paradigms](#) (see Appendix B for details). Most baseline architectures employ a Pairformer (Jumper et al., 2021) backbone, incurring a cubic spatial complexity $\mathcal{O}(N^3L)$ due to expensive triangular attention operations. We consider two representative baseline configurations: (1) **Pairformer + Pair Temporal Attention** (e.g., ConfRover (Shen et al., 2025)): This configuration scales as $\mathcal{O}(N^3L + N^2L^2)$. Here, the pairwise temporal attention alone incurs $\mathcal{O}(N^2L^2)$ cost, matching that of STAR-MD, but the additional cubic spatial overhead makes it significantly more expensive. Furthermore, this approach incurs a quadratic memory cost $\mathcal{O}(N^2L)$ for caching pairwise key-value states during autoregressive generation, which becomes a severe bottleneck for long rollouts. In contrast, STAR-MD only requires $\mathcal{O}(NL)$ memory for caching single-residue features. (2) **Pairformer + Single Temporal Attention**: This configuration scales as $\mathcal{O}(N^3L + NL^2)$. While the temporal scaling is theoretically superior to STAR-MD’s $\mathcal{O}(N^2L^2)$ in the limit $L \gg N$, the cubic spatial cost $\mathcal{O}(N^3L)$ dominates in practical regimes. STAR-MD thus offers a favorable trade-off, enabling efficient modeling of long trajectories for large proteins without the cubic spatial overhead.

211 **3.3 ADDITIONAL MODEL DETAILS**

212 **Block-causal Attention for Efficient Training.** Traditional video diffusion models (Peebles &
 213 Xie, 2023; Ma et al., 2024) denoise all frames simultaneously, using a fixed window size. For
 214 autoregressive generation with variable context length, the model denoises one frame at a time,
 215 attending to clean history contexts. This is straightforward during inference, where we cache clean

frames for efficient autoregressive generation. However, given the sequential nature of this process, extra work is needed for parallel training. To simultaneously denoise all tokens with clean history context, we follow Arriola et al. (2025); Teng et al. (2025); Deng et al. (2024) to concatenate clean and noisy frames as input sequence and employ a special block-wise attention pattern: all frames only attend to clean history frames, preserving the causal structure during training. At the cost of doubling the sequence length, the model can optimize the training loss for all frames in a single forward pass, aligning parallel teacher-forcing training with sequential autoregressive inference.

Continuous-time Conditioning. To handle the vast range of timescales in protein motion, we draw the physical stride $\Delta t \sim \text{LogUniform}[10^{-2}, 10^1]$ ns independently for every training example. By conditioning the network on Δt through adaptive layernorm (AdaLN, Peebles & Xie (2023)), the model learns to modulate its internal activations as a function of both diffusion progress and physical timescale. This approach yields remarkable temporal extrapolation: at inference, the model stably generates trajectories an order of magnitude longer than the training window without additional fine-tuning.

Contextual Noise Perturbation for Robust Rollouts. To mitigate compounding errors during long autoregressive rollouts, we apply a contextual noise perturbation technique inspired by Diffusion Forcing (Chen et al., 2024; Song et al., 2025). During training, we perturb historical context frames $\mathbf{x}_{\leq \ell}$ by applying the forward diffusion process (Eqs. 1-2) with a small, randomly sampled noise level $\tau \sim \mathcal{U}[0, 0.1]$ to obtain noisy contexts $\mathbf{x}_{\leq \ell}^\tau$. The model then learns to predict frame ℓ conditioned on this perturbed history. Critically, at inference time, we apply the same noise perturbation: after generating frame ℓ , we add noise at level τ before using it as context for subsequent frames. This training-inference consistency ensures that the model experiences similar input distributions during deployment as during training, making it robust to its own prediction errors and preventing compounding drift in long rollouts.

3.4 THEORETICAL JUSTIFICATION FOR STAR-MD ARCHITECTURE

In this subsection, we use the Mori-Zwanzig formalism (Mori, 1965; Zwanzig, 1961) (See Appendix A for more details) to provide theoretical justification for two key aspects of our architecture: the necessity of temporal history in coarse-grained modeling, and the specific requirement for *joint* spatio-temporal attention arising from the removal of explicit pairwise features.

Atomistic molecular dynamics simulations evolve according to Hamiltonian mechanics and are Markovian in the full phase space of atomic positions and momenta. However, practical generative models must operate on coarse-grained representations, such as per-residue coordinates sampled at nanosecond intervals. The Mori-Zwanzig formalism shows that projecting Markovian dynamics onto coarse-grained variables yields a Generalized Langevin Equation with three terms (Eq. (6)): a Markovian component, a memory kernel encoding history dependence, and a stochastic force representing eliminated degrees of freedom. Consequently, accurate modeling of coarse-grained protein dynamics requires non-Markovian models that incorporate temporal history to capture memory effects arising from eliminated degrees of freedom. Our work makes this connection explicit, providing the theoretical justification underlying recent temporal architectures in trajectory modeling (Jing et al., 2024b; Cheng et al., 2025; Shen et al., 2025).

Existing models typically rely on explicit pairwise features to capture spatial structure, but this incurs prohibitive $\mathcal{O}(L^2 N^2 + LN^3)$ or $\mathcal{O}(LN^3)$ costs. STAR-MD circumvents this by projecting out pairwise features, a design choice we analyze through our *Memory Inflation* proposition (Proposition 1, Appendix A). We show that removing explicit spatial correlations “inflates” the memory kernel for the remaining residues, necessitating a significantly richer temporal history to compensate. Crucially, this inflated kernel exhibits non-separable spatio-temporal coupling (Corollary 2), meaning spatial and temporal modes cannot be factorized. This theoretical insight directly motivates our architecture: instead of the interleaved spatial and temporal blocks used in prior work, STAR-MD employs *joint* spatio-temporal attention to approximate this complex, non-separable memory kernel, balancing physical fidelity with computational scalability.

4 EXPERIMENTS

We conduct a comprehensive set of experiments to evaluate STAR-MD’s ability to generate long-horizon protein dynamics. First, we benchmark STAR-MD against state-of-the-art models on the

270 standard 100 ns trajectory generation task (Section 4.2). Next, we assess its extrapolation capabilities by generating trajectories over longer, microsecond-scale horizons not seen during training (Section 4.3). Finally, we perform a series of analyses and ablations to investigate model stability, scalability, and the contributions of our key design choices (Section 4.4).

275 4.1 EXPERIMENTAL SETUP

276 **Dataset.** ATLAS dataset (Vander Meersche et al., 2024) contains 100 ns MD trajectories for 1390
 277 structurally diverse proteins. We use the standard train/val/test splits from prior works (Jing et al.,
 278 2024b; Shen et al., 2025) to assess model performance in a transferable setting. To evaluate long-
 279 horizon generation—a key focus of our work—we extend the benchmark by running new MD sim-
 280 ulations to produce 250 ns and 1 μ s trajectories for selected proteins, using the original ATLAS
 281 simulation protocols for consistency. Further details are provided in Appendix C.1.

282 **Implementations.** We follow the training procedure described in Section 3.3 and generate trajec-
 283 tories using the configurations listed in Table 4. We compare STAR-MD with three state-of-the-art
 284 trajectory generative models trained on ATLAS: AlphaFolding (Cheng et al., 2025), MDGen (Jing
 285 et al., 2024b), and ConfRover (Shen et al., 2025). All models parametrize proteins using $SE(3)$
 286 backbone rigidts (translations and rotations) with torsional angles for side-chain atoms. To stan-
 287 dardize evaluation, trajectories from variable-stride models (STAR-MD, ConfRover) are generated
 288 directly at the required intervals. Trajectories from fixed-stride models (AlphaFolding, MDGen) are
 289 first generated at their native resolution and then subsampled. Finally, we include an oracle reference
 290 based on MD simulations run independently to represent the target performance. All trajectories are
 291 aligned to the starting frame of the reference ATLAS trajectory via $C\alpha$ superposition prior to anal-
 292 ysis (Appendix C.3). Specific parameters for each benchmark are detailed in the relevant sections
 293 below, with full implementation details in Appendix C.2.

- 294 • **Structural Quality:** We assess the physical plausibility of generated conformations using a hi-
 295 erarchy of geometric and stereochemical metrics. First, we perform coarse-grained checks for
 296 $C\alpha$ - $C\alpha$ clashes (non-bonded atoms too close) and chain breaks (consecutive $C\alpha$ atoms too far
 297 apart). Second, we use the MolProbity suite (Chen et al., 2010) for fine-grained, all-atom analysis
 298 of backbone Ramachandran and side-chain rotamer outliers. We report three distinct validity met-
 299 rics based on different criteria: **$C\alpha$ -level Validity**, passing only $C\alpha$ checks; **All-Atom Validity**,
 300 passing only MolProbity checks; and **Combined Validity**, passing all checks simultaneously. See
 301 Appendix C.3 for details on the thresholds.
- 302 • **Conformational Coverage:** To evaluate how well generated trajectories explore the confor-
 303 mational space of the reference MD simulation, we follow the protocol of Shen et al. (2025). We
 304 project all conformations into a low-dimensional space defined by the principal components of
 305 the reference trajectory and compute the Jensen-Shannon Divergence (JSD) and conformation re-
 306 call between the distributions. To ensure that coverage reflects physically plausible exploration,
 307 we report these metrics computed exclusively on structurally valid conformations (“Cov Valid”).
- 308 • **Dynamic Fidelity:** We assess temporal coherence using four metrics. First, we use **tICA lag-time**
 309 **correlation** to quantify the preservation of slow collective variables (Molgedey & Schuster, 1994;
 310 Pérez-Hernández et al., 2013; Shen et al., 2025); crucially, this is computed only on valid transi-
 311 tions to avoid artifacts from broken structures. Additionally, we evaluate **RMSD** to measure the
 312 magnitude of structural change over varying intervals, **autocorrelation** to assess temporal mem-
 313 ory, and **VAMP-2 score** to evaluate how well slow dynamical modes of the system are captured.
 314 For last three metrics, we report the deviation from MD reference, with details in Appendix C.3.

315 4.2 STANDARD BENCHMARK: 100 NS TRAJECTORY GENERATION

316 We first evaluate STAR-MD on the standard 100 ns ATLAS benchmark. Following the protocol of
 317 Shen et al. (2025), we generate trajectories of 80 frames at a 1.2 ns interval. Results are summarized
 318 in Table 1. It is important to note that due to its high computational cost, AlphaFolding failed to
 319 run on the four largest proteins in the test set, highlighting its scalability limitations even on this
 320 standard benchmark. On the full test set, STAR-MD, achieves a superior balance of conformational
 321 coverage, dynamic fidelity, and structural quality.

322 The results demonstrate clear performance deficiencies in all baseline models. Baselines like MD-
 323 Gen and ConfRover exhibit low conformational coverage (0.30 and 0.42 Recall, respectively) and
 324 produce a substantial number of structurally invalid frames, with validity rates of only 64.9% and

Table 1: Quantitative results on 100 ns trajectory generation (ATLAS test set). STAR-MD achieves state-of-the-art performance across all metrics, with particularly significant improvements in conformational coverage (JSD, F1) and dynamic fidelity (tICA, [average difference of RMSD](#), autocorrelation and VAMP-2 score to MD reference).

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Model	Cov Valid		Dynamic Fidelity				Validity		
	JSD \downarrow	Rec \uparrow	tICA \uparrow	RMSD \downarrow	AutoCor \downarrow	VAMP-2 \downarrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
MD (Oracle)	0.31	0.67	0.17	0.00	0.00	0.02	98.37	98.07	96.43
MDGen	0.56 \pm 0.01	0.28 \pm 0.01	0.12 \pm 0.00	0.38 \pm 0.01	0.05 \pm 0.00	0.38 \pm 0.01	71.83 \pm 1.90	95.03 \pm 0.59	68.31 \pm 2.20
AlphaFolding ¹	0.59 \pm 0.01	0.20 \pm 0.01	N/A	3.31 \pm 0.06	0.12 \pm 0.01	1.56 \pm 0.01	10.58 \pm 0.09	0.82 \pm 0.10	0.47 \pm 0.04
ConfRover	0.52 \pm 0.01	0.36 \pm 0.01	0.15 \pm 0.01	0.20 \pm 0.00	0.08 \pm 0.00	0.47 \pm 0.02	56.94 \pm 0.52	92.47 \pm 0.25	52.06 \pm 0.36
STAR-MD	0.43 \pm 0.01	0.54 \pm 0.01	0.17 \pm 0.00	0.07 \pm 0.02	0.02 \pm 0.00	0.10 \pm 0.02	86.81 \pm 0.64	98.18 \pm 0.05	85.29 \pm 0.62

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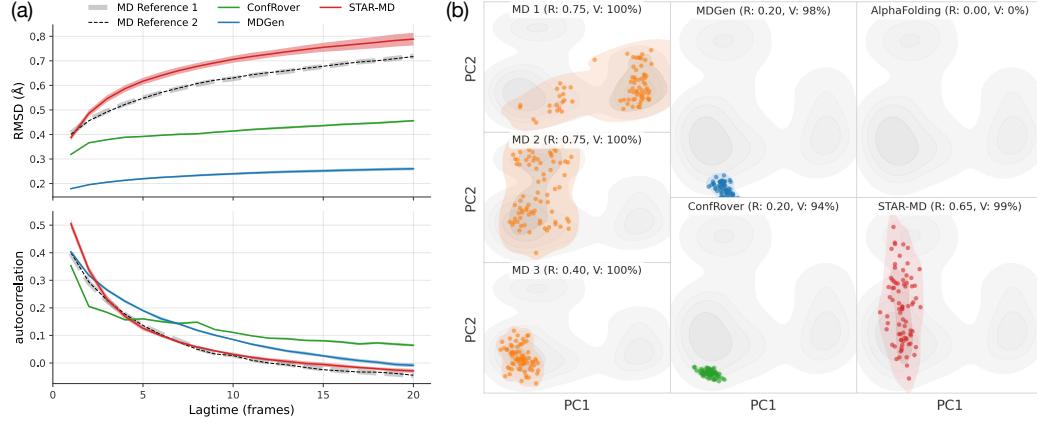
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Figure 2: [\[UPDATED FIGURE\]](#) **Kinetic fidelity and conformational coverage on the ATLAS 100 ns benchmark.** (a) Comparing $\text{C}\alpha$ coordinate RMSD (top) and autocorrelation (bottom) at varying lagtime for different models. STAR-MD better captures the overall trend and characteristic magnitudes, similar to the MD reference runs (dashed lines). Shaded bands represent ± 1 standard deviation. The small size of the shaded bands demonstrate the robustness of this metric. (b) Conformational coverage comparison for 6XB3-H for all models and 3 MD simulations. Generated trajectories are projected onto the first two principal components (PCs) of the reference MD simulation (gray contours). Only structurally valid frames are considered for this plot. Baseline methods (MDGen, ConfRover) exhibit limited diversity, becoming confined to a small region of the conformational landscape. AlphaFolding’s generated trajectories consist of all structurally implausible frames, with a validity of 0%. In contrast, STAR-MD demonstrates significantly broader exploration (with a recall value of 0.65), visiting two of the major modes observed in the MD reference, matching the diversity seen in independent MD runs.

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49.7%, respectively. AlphaFolding achieves a higher raw recall (0.51), but this metric is misleading as a negligible 0.11% of its generated frames are structurally valid; when controlled for valid structures, this recall drops to 0.01, rendering AlphaFolding as a very unreliable model for this task. In contrast, STAR-MD achieves the highest conformational coverage (0.58 recall) and structural validity (86.36%) of any generative model, significantly narrowing the performance gap to the ground-truth MD simulation which serves as the oracle for this task.

In Figure 2(a) and 7, we compare the level of conformation changes at different lag times, as an indicator how well the model captures the ground truth motion changes. STAR-MD shows improved coherence with MD reference trajectories, while AlphaFolding significantly overestimates the motion and MDGen and ConfRover tend to underestimate the dynamics level.

Finally, Figure 2(b) provides a qualitative assessment of conformational coverage using the principal component projection for an example ATLAS 100 ns protein. STAR-MD exhibits better significantly better exploration and diversity than comparison methods and is able to populate two distinct modes.

4.3 EXTRAPOLATING FOR LONG-HORIZON TRAJECTORIES

The ability to generate stable, physically realistic trajectories over extended time horizons is critical for modeling functionally relevant protein dynamics. Most conformational transitions of biological

¹AlphaFolding results are evaluated on 78/82 proteins due to out-of-memory error for 4 large proteins.

Table 2: **Results on molecular dynamics trajectory generation at 240 ns and 1 μ s timescales.** STAR-MD demonstrates competitive performance across different temporal scales, with particularly strong quality metrics at both timescales.

Model	Time	Cov Valid		Dynamic Fidelity		Validity		
		JSD \downarrow	Rec \uparrow	RMSD \downarrow	AutoCor \downarrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
MD (Oracle)	240 ns	0.26	0.75	0.01	0.00	99.53	96.83	96.36
	1 μ s	0.23	0.91	0.00	0.00	96.25	86.50	82.75
MDGen	240 ns	0.52 \pm 0.01	0.38 \pm 0.01	0.48 \pm 0.01	0.25 \pm 0.01	63.25 \pm 2.10	87.83 \pm 1.13	56.60 \pm 2.10
	1 μ s	0.56 \pm 0.01	0.36 \pm 0.03	0.37 \pm 0.02	0.39 \pm 0.01	36.11 \pm 7.34	56.99 \pm 4.52	24.81 \pm 4.30
AlphaFolding	240 ns	0.57 \pm 0.03	0.20 \pm 0.03	1.76 \pm 0.05	0.14 \pm 0.01	8.96 \pm 0.25	0.94 \pm 0.12	0.63 \pm 0.16
	1 μ s	0.65 \pm 0.00	0.20 \pm 0.00	0.78 \pm 0.03	0.04 \pm 0.01	9.64 \pm 0.02	0.19 \pm 0.00	0.06 \pm 0.00
ConfRover-W	240 ns	0.51 \pm 0.01	0.42 \pm 0.02	0.35 \pm 0.01	0.39 \pm 0.01	44.71 \pm 1.55	73.13 \pm 0.84	36.51 \pm 1.22
	1 μ s	0.55 \pm 0.02	0.45 \pm 0.02	0.33 \pm 0.02	0.38 \pm 0.03	54.74 \pm 1.79	62.32 \pm 3.43	36.91 \pm 1.39
STAR-MD	240 ns	0.44 \pm 0.01	0.59 \pm 0.01	0.20 \pm 0.02	0.03 \pm 0.01	85.16 \pm 1.91	97.57 \pm 0.13	83.15 \pm 1.99
	1 μ s	0.46 \pm 0.01	0.61 \pm 0.02	0.13 \pm 0.02	0.10 \pm 0.02	88.47 \pm 1.09	89.81 \pm 0.65	79.93 \pm 1.04

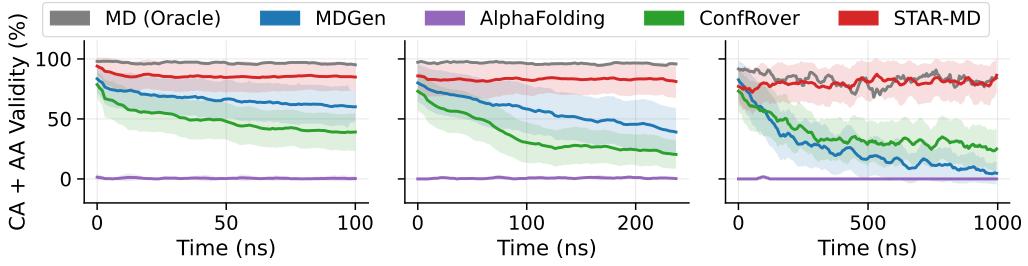


Figure 3: **[Updated Figure] Long-horizon stability and error accumulation across different time scales.** We plot the structural validity percentage over time for trajectories generated by STAR-MD and baseline models, evaluated at 100 ns (left), 240 ns (middle), and 1 μ s (right) horizons. Shaded bands represent ± 1 standard deviation across 5 different repeats. While most models exhibit clear error accumulation over simulation time, STAR-MD maintains high structural validity, regardless of the simulation time scale.

interest occur beyond the 100 ns timescale, where existing generative models begin to fail. We assess STAR-MD’s long-horizon capability through two increasingly challenging settings: 240 ns simulation with 32 proteins and 1 μ s simulation with 8 proteins, with all proteins unseen during training. **To rigorously evaluate temporal extrapolation, all models were trained exclusively on the 100 ns ATLAS trajectories and received no fine-tuning for longer horizons. This setup directly tests the ability to generalize to dynamics far beyond the training data distribution.** Since the ATLAS benchmark only provides trajectories up to 100 ns, we generated our own molecular dynamics simulations for longer timescales to create proper reference data for evaluation. These extended simulations follow the same simulation protocols as ATLAS but continue the dynamics to longer timescales.

Due to its scaling issues, ConfRover could not be evaluated on either 240ns or 1 μ s generation tasks. This is because ConfRover performs temporal attention over pair features, requiring previous frames’ pair features to be stored in memory as KV cache (see Appendix B for more detailed analysis). As such, even with CPU offloading of the KV cache, ConfRover’s memory requirements exceeded our hardware limits (1869 GB CPU RAM, 8 \times H100 GPUs with 80GB VRAM each). As a result and for fair comparison, we utilize a variant of ConfRover with windowed attention with attention sink tokens (as described in Xiao et al. (2023b)) to reduce memory usage. We report results for this windowed variant, labeled ConfRover-W, in Table 2.

Table 2 summarizes the results for these two extended timescales. Remarkably, STAR-MD maintains stable and competitive performance at both the 240 ns timescale and the challenging 1 μ s timescale, demonstrating the effectiveness of our joint spatio-temporal attention and contextual noise techniques for long-horizon stability. Further, STAR-MD exhibits controlled error accumulation, with low clash and break rates that remain consistent throughout the extended trajectory (Figure 3 (right), Section 4.4). Among the methods achieving reasonable structural quality, STAR-MD attains the best balance of coverage and fidelity, nearing the oracle MD performance. In both the 240 ns and

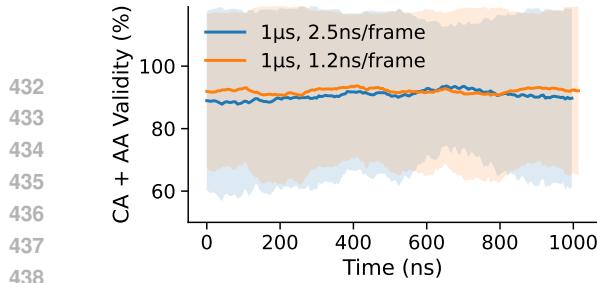


Figure 4: [Updated Figure] **Stability of STAR-MD across different temporal strides for 1 μ s generation.** We plot the structural validity over time for two 1 μ s trajectories generated with different strides: 2.5 ns/frame (400 steps) and a more challenging 1.2 ns/frame (~ 833 steps). **Solid lines show mean validity, while shaded bands represent ± 1 standard deviation across test proteins.** Our models remain stable and maintain high structural quality even when generating much longer sequences of frames than seen in training.

1 μ s case, AlphaFolding produces physically implausible, high-error structures despite good coverage metrics, while ConfRover-W and MDGen degrade significantly over the microsecond horizon. We further report kinetic metrics for the long trajectory cases in Fig. 7, where STAR-MD shows the best alignment with the RMSD and autocorrelation of the MD reference from among baseline methods.

4.4 ADDITIONAL ANALYSIS

Error Accumulation. A key challenge in long-horizon generation is error accumulation, where minor inaccuracies compound over time, leading to trajectory degradation. Figure 3 plots the average validity percentage over simulation time for the 100 ns, 240 ns, and 1 μ s simulations. As the simulation horizon extends, baseline models exhibit rapidly deteriorating performance, with structural validity declining sharply. In contrast, STAR-MD remains stable, maintaining high structural validity close to the MD oracle across all timescales. This demonstrates our model’s robustness against error accumulation, a quality largely attributable to the historical-context noise mechanism, which enables stable long-horizon autoregressive generation.

Varying temporal resolution for long-horizon generation. The 1 μ s generation task above uses a stride of 2.5 ns, resulting in 400 frames. Thanks to our continuous-time conditioning, we can generate at different temporal resolutions without retraining. We test this by generating a 1 μ s trajectory with a 1.2 ns stride, nearly doubling the number of frames to ~ 850 . Figure 4 shows that STAR-MD remains stable even with this much longer sequence of frames, maintaining high structural quality. This further underscores our model’s robustness to different sampling intervals and trajectory lengths, showing strong capability for stable, long-horizon autoregressive generation.

KV Cache Analysis. A critical limitation of ConfRover is the need to maintain the KV-cache of single and pair embedding for temporal attentions. In contrast, STAR-MD only requires maintaining the KV cache for singles. By only maintaining attentions among single representations, STAR-MD has a magnitudes lower memory footprints (see Fig. 5 for a comparison). This advantage allows our model to maintain a full KV caches in GPU memory without resorting to CPU offloading (e.g, additional overhead) or sliding window style KV caches (compromises temporal history).

4.5 ABLATION STUDIES ON KEY COMPONENTS

To validate our key design choices, we conduct systematic ablations of the main components of STAR-MD. Table 3 summarizes the quantitative results on the 100 ns benchmark. Appendix D contains full ablation tables for the 100 ns, 240 ns, and 1 μ s benchmarks.

S \times T attention vs. separable attention. We compare our joint S \times T attention with a separable alternative that processes spatial and temporal dimensions sequentially. The separable model suffers

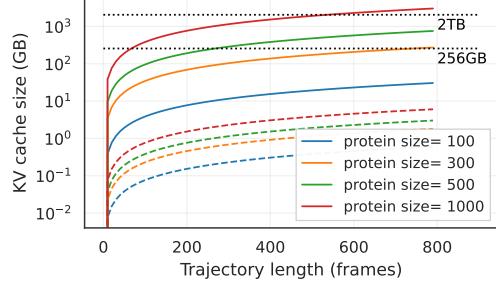


Figure 5: **Memory footprint of KV caches** with varying protein sizes and trajectory lengths. ConfRover is shown in solid lines and STAR-MD is shown in dashed lines. Both models contain 8 attention layers with hidden dimension of 128 (ConfRover) or 256 (STAR-MD). STAR-MD show much smaller memory cost compared to ConfRover. Horizontal lines marks the 256 GB and 2 TB memory caps.

486
 487 **Table 3: Ablation study on 100 ns simulation.** The full STAR-MD model is compared against
 488 variants with key components removed (no contextual noise, separate spatial and temporal attention,
 489 and placing S×T attention outside the diffusion block). These modifications affect model coverage
 490 and conformation quality. Metrics degraded in each ablation setting are highlighted in red. See
 491 complete ablation results on longer trajectories in Appendix D.

Model	Cov Valid		Dynamic Fidelity				Validity		
	JSD↓	Rec↑	tICA↑	RMSD↓	AutoCor↓	VAMP-2↓	CA %↑	AA %↑	CA+AA %↑
STAR-MD	0.42	0.57	0.17	0.09	0.02	0.12	86.62	98.28	85.18
w/o Noise	0.43	0.54	0.18	0.04	0.11	1.22	77.82	97.76	76.12
w/ Sep Attn	0.46	0.46	0.17	0.09	0.05	0.49	87.95	98.34	86.70
w/ Preproc Attn	0.47	0.48	0.17	0.09	0.05	0.54	84.55	97.81	82.56

500 a significant drop in conformational coverage and a modest drop in dynamic fidelity. This con-
 501 firms that joint S×T attention is crucial for capturing the complex, non-separable spatio-temporal
 502 dependencies inherent in protein dynamics, which directly impacts the model’s ability to explore the
 503 correct conformational space.

504 **S×T layer placement.** We evaluated placing the spatio-temporal attention module outside the dif-
 505 fusion decoder as a static conditioner which compresses the full trajectory history into a single
 506 conditioning vector, mirroring Shen et al. (2025). This variant underperforms our full model in
 507 both coverage and fidelity, confirming that integrating spatio-temporal attention directly within the
 508 diffusion module is essential for effective context utilization.

510 **Historical-context noise.** Removing the contextual noise perturbation leads to a substantial drop
 511 in structural quality. This highlights the importance of this technique for maintaining generation
 512 stability, a finding that is further supported by our long-horizon experiments in Section 4.3. Figure 11
 513 compares the CA+AA validity over time for models with and without contextual noise on the 240ns
 514 and 1 μ s benchmarks, showing that the noise helps maintain structural quality over long horizons.

515 These ablation studies demonstrate that each component of STAR-MD addresses a specific challenge
 516 in protein dynamics modeling. The S×T attention provides the expressivity needed for complex
 517 spatio-temporal dependencies. The historical-context noise ensures stable long-horizon generation
 518 by preventing error accumulation.

5 CONCLUSION

523 We introduced STAR-MD, a novel causal diffusion transformer model for generating long-horizon
 524 protein dynamics trajectories. STAR-MD addresses the challenges of long-horizon generation
 525 through several key innovations: joint spatiotemporal attention that efficiently models complex
 526 space-time couplings, continuous-time conditioning for generation at arbitrary timescales, and a
 527 noisy-context training scheme that mitigates error accumulation. Our comprehensive evaluations
 528 demonstrate that STAR-MD achieves new state-of-the-art performance on the standard 100 ns AT-
 529 LAS benchmark, outperforming previous methods in conformational coverage, structural quality,
 530 and kinetic fidelity. More importantly, our model successfully extrapolates to generate stable, high-
 531 fidelity trajectories up to the microsecond regime, where prior models often fail due to computational
 532 costs or compounding errors. By balancing expressiveness and scalability, STAR-MD establishes an
 533 efficient and empirically strong foundation for modeling protein dynamics at biologically relevant
 534 scales, paving the way for accelerated exploration of complex biological processes.

535 **Limitations and future work.** While STAR-MD represents a significant step forward, there are av-
 536 enues for future improvement. The temporal consistency of the generated trajectories, while strong,
 537 does not yet perfectly match that of oracle MD simulations. The model’s performance could be
 538 further enhanced by training on larger and more diverse MD simulation datasets, such as MD-
 539 CATH (Mirarchi et al., 2024). Additionally, a promising direction for future work is to extend the
 540 model’s capabilities to simulate the dynamics of protein complexes or their interactions with small
 541 molecules, which are crucial for understanding biological processes and drug design.

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702 A THEORETICAL FOUNDATION

704 A.1 FORMAL JUSTIFICATION FOR JOINT SPATIO-TEMPORAL MODELING

706 Any practical generative model must therefore adopt a coarse-grained representation, such as per-
 707 residue coordinates, to remain tractable. In this section, we analyze the formal consequences of this
 708 coarse-graining. We first use the Mori-Zwanzig formalism (Mori, 1965; Zwanzig, 1961) to show
 709 how moving to a coarser representation introduces complex temporal memory. We then specialize
 710 this result to protein representations and demonstrate that simplifying from a singles-and-pairs
 711 representation to a singles-only one both “inflates” this memory and induces non-separable spatio-
 712 temporal coupling, providing a rigorous justification for our architectural choices.

713 We formalize this with the **Memory Inflation** proposition. The proposition analyzes the effect of
 714 simplifying a protein’s state representation from a detailed one containing both per-residue features
 715 (“singles”) and inter-residue features (“pairs”), to a coarser one containing only singles. It shows
 716 that the memory kernel for the simpler singles-only dynamics, $\tilde{K}^{(s)}$, is “inflated” compared to the
 717 singles-part of the more detailed kernel, $\tilde{K}_{ss}^{(sp)}$:

718 **Proposition 1.** *[Memory Inflation] Under linearization, the memory kernel for the singles-only*
 719 *representation relates to the singles-and-pairs memory kernel by:*

$$720 \tilde{K}^{(s)}(p) = \tilde{K}_{ss}^{(sp)}(p) + \underbrace{(\Omega_{sz} + \tilde{K}_{sz}^{(sp)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(sp)}(p)]^{-1}(\Omega_{zs} + \tilde{K}_{zs}^{(sp)}(p))}_{\text{Inflation Term}}, \quad (4)$$

723 where $\tilde{K}(p)$ is the Laplace transform of the memory kernel, p is the Laplace variable, I is the
 724 identity matrix, Ω represents instantaneous (Markovian) dynamics, and the subscripts s and z refer
 725 to singles and pairs features, respectively.

727 The inflation term in Eq. 4 quantifies how the dynamics of the eliminated pair features are folded into
 728 the temporal evolution of the singles, forcing the system’s dynamics into a more complex memory
 729 structure. Crucially, this term is non-separable; it cannot be factored into independent spatial and
 730 temporal components, meaning the memory kernel itself couples spatial locations and temporal
 731 frequencies in an intricate way. These insights lead to our core design principle: a scalable and
 732 accurate protein dynamics model that avoids expensive pairwise representations *must* employ an
 733 architecture capable of learning complex, non-separable spatio-temporal dependencies. This directly
 734 motivates our departure from prior “space-then-time” factorized models and our adoption of a joint
 735 spatio-temporal attention mechanism.

736 A.2 PROJECTION OPERATOR FORMALISM AND GENERALIZED LANGEVIN EQUATION

738 The Mori-Zwanzig formalism provides a rigorous mathematical framework for deriving reduced
 739 dynamical models from high-dimensional systems. We begin with a high-dimensional system de-
 740 scribed by variables $\Gamma(t)$ evolving according to:

$$742 \frac{d\Gamma}{dt} = \mathcal{L}\Gamma(t) \quad (5)$$

745 where \mathcal{L} is the Liouville operator. When we project onto a lower-dimensional subspace of “slow”
 746 variables $A(t)$ using a linear projection operator \mathcal{P} ¹, the Mori-Zwanzig theorem provides the exact
 747 generalized Langevin equation (GLE) for their evolution:

$$749 \frac{dA(t)}{dt} = \mathcal{P}\mathcal{L}A(t) + \int_0^t K(t - \tau)A(\tau)d\tau + F(t) \quad (6)$$

751 where:

753 • $A(t)$ represents our chosen variables (the projection of $\Gamma(t)$)

754 ¹In the context of the Mori-Zwanzig formalism, \mathcal{P} is a linear operator that projects onto a chosen subspace
 755 of the full phase space.

756 • $\mathcal{PLA}(t)$ captures Markovian (instantaneous) dynamics
 757 • $\int_0^t K(t-\tau)A(\tau)d\tau$ is the memory term encoding history dependence
 758 • $F(t)$ is the random force, a rapidly-fluctuating noise term arising from the orthogonal
 759 degrees of freedom (e.g., solvent molecules, fast internal motions) that were projected out.
 760 By definition, it is orthogonal to the projected variables.
 761
 762 • Let $\mathcal{Q} = 1 - \mathcal{P}$ be the projection operator onto the orthogonal subspace. \mathcal{Q} isolates the
 763 “fast” degrees of freedom that are not explicitly modeled in the reduced representation.

764 The memory kernel $K(t)$ emerges as:

$$766 \quad K(t) = \mathcal{PL}e^{(1-\mathcal{P})\mathcal{L}t}(1 - \mathcal{P})\mathcal{L} \quad (7)$$

767 which involves the propagation of orthogonal dynamics through the operator $e^{(1-\mathcal{P})\mathcal{L}t}$. This formula
 768 reveals that the memory kernel encodes how information flows through degrees of freedom not
 769 explicitly modeled.

771 A.3 ALTERNATIVE PROJECTIONS FOR PROTEIN DYNAMICS

772 For protein systems, we consider two specific projections:

773 A.3.1 SINGLES-AND-PAIRS REPRESENTATION

774 We project onto both per-residue features $s(t) \in \mathbb{R}^{N \times d_s}$ and pairwise features $z(t) \in \mathbb{R}^{N \times N \times d_z}$,
 775 yielding state $A^{(1)}(t) = (s(t), z(t))$. The resulting dynamics can be written in block form:

$$776 \quad \begin{pmatrix} \dot{s}(t) \\ \dot{z}(t) \end{pmatrix} = \begin{pmatrix} \Omega_{ss} & \Omega_{sz} \\ \Omega_{zs} & \Omega_{zz} \end{pmatrix} \begin{pmatrix} s(t) \\ z(t) \end{pmatrix} + \int_0^t \begin{pmatrix} K_{ss}^{(1)}(t-\tau) & K_{sz}^{(1)}(t-\tau) \\ K_{zs}^{(1)}(t-\tau) & K_{zz}^{(1)}(t-\tau) \end{pmatrix} \begin{pmatrix} s(\tau) \\ z(\tau) \end{pmatrix} d\tau + \begin{pmatrix} F_s(t) \\ F_z(t) \end{pmatrix} \quad (8)$$

777 where the memory kernel $K^{(1)}$ has block structure corresponding to singles-singles ($K_{ss}^{(1)}$), singles-
 778 pairs ($K_{sz}^{(1)}$), pairs-singles ($K_{zs}^{(1)}$), and pairs-pairs ($K_{zz}^{(1)}$) interactions.

779 A.3.2 SINGLES-ONLY REPRESENTATION

780 We project directly onto per-residue tifeatures $s(t)$ alone, yielding the more compact state $A^{(2)}(t) =$
 781 $s(t)$. The dynamics become:

$$782 \quad \dot{s}(t) = \Omega^{(2)}s(t) + \int_0^t K^{(2)}(t-\tau)s(\tau)d\tau + F^{(2)}(t) \quad (9)$$

783 A.4 COMPLETE PROOF OF THE MEMORY INFLATION PROPOSITION

784 **Proposition 1.** [Memory Inflation] Under linearization, the memory kernel for the singles-only
 785 representation relates to the singles-and-pairs memory kernel by:

$$786 \quad \tilde{K}^{(s)}(p) = \tilde{K}_{ss}^{(sp)}(p) + \underbrace{(\Omega_{sz} + \tilde{K}_{sz}^{(sp)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(sp)}(p)]^{-1}(\Omega_{zs} + \tilde{K}_{zs}^{(sp)}(p))}_{\text{Inflation Term}}, \quad (4)$$

787 where $\tilde{K}(p)$ is the Laplace transform of the memory kernel, p is the Laplace variable, I is the
 788 identity matrix, Ω represents instantaneous (Markovian) dynamics, and the subscripts s and z refer
 789 to singles and pairs features, respectively.

790 Proof. We begin by taking the Laplace transform of the singles-and-pairs dynamics:

$$791 \quad \begin{pmatrix} p\tilde{s}(p) - s(0) \\ p\tilde{z}(p) - z(0) \end{pmatrix} = \begin{pmatrix} \Omega_{ss} & \Omega_{sz} \\ \Omega_{zs} & \Omega_{zz} \end{pmatrix} \begin{pmatrix} \tilde{s}(p) \\ \tilde{z}(p) \end{pmatrix} + \begin{pmatrix} \tilde{K}_{ss}^{(1)}(p)\tilde{s}(p) & \tilde{K}_{sz}^{(1)}(p)\tilde{z}(p) \\ \tilde{K}_{zs}^{(1)}(p)\tilde{s}(p) & \tilde{K}_{zz}^{(1)}(p)\tilde{z}(p) \end{pmatrix} + \begin{pmatrix} \tilde{F}_s(p) \\ \tilde{F}_z(p) \end{pmatrix} \quad (10)$$

810 From the second row, we can express $\tilde{z}(p)$ in terms of $\tilde{s}(p)$:
 811

$$812 \quad 813 \quad \tilde{z}(p) = [pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1} [z(0) + (\Omega_{zs} + \tilde{K}_{zs}^{(1)}(p))\tilde{s}(p) + \tilde{F}_z(p)] \quad (11)$$

814 Substituting this into the first row equation:
 815

$$816 \quad 817 \quad p\tilde{s}(p) - s(0) = \Omega_{ss}\tilde{s}(p) + \tilde{K}_{ss}^{(1)}(p)\tilde{s}(p) + \tilde{F}_s(p) + \quad (12)$$

$$818 \quad 819 \quad (\Omega_{sz} + \tilde{K}_{sz}^{(1)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1} [z(0) + (\Omega_{zs} + \tilde{K}_{zs}^{(1)}(p))\tilde{s}(p) + \tilde{F}_z(p)] \quad (13)$$

820 Collecting terms with $\tilde{s}(p)$ and comparing with the Laplace transform of the singles-only representation:
 821

$$822 \quad 823 \quad p\tilde{s}(p) - s(0) = \Omega^{(2)}\tilde{s}(p) + \tilde{K}^{(2)}(p)\tilde{s}(p) + \tilde{F}^{(2)}(p) \quad (14)$$

824 We identify:
 825

$$826 \quad \Omega^{(2)} = \Omega_{ss} \quad (15)$$

$$827 \quad \tilde{K}^{(2)}(p) = \tilde{K}_{ss}^{(1)}(p) + (\Omega_{sz} + \tilde{K}_{sz}^{(1)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1}(\Omega_{zs} + \tilde{K}_{zs}^{(1)}(p)) \quad (16)$$

$$828 \quad \tilde{F}^{(2)}(p) = \tilde{F}_s(p) + (\Omega_{sz} + \tilde{K}_{sz}^{(1)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1}[z(0) + \tilde{F}_z(p)] \quad (17)$$

834 This completes the proof, showing that the memory kernel in the singles-only representation incorporates additional terms that account for the eliminated pair dynamics. \square
 835

836 A.5 IMPLICATIONS OF MEMORY INFLATION

837 From the Memory Inflation result in Proposition 1, we can derive two important corollaries that
 838 guide our architectural design:

839 A.5.1 MEMORY ENRICHMENT

840 **Corollary 1** (Memory Enrichment). *When spatial detail is removed from a dynamical system, the
 841 memory kernel must become more complex to preserve the system’s physical accuracy.*

842 This follows directly from the inflation term in the Memory Inflation Proposition. By eliminating
 843 the explicit representation of pairwise features $z(t)$, we force the memory kernel $K^{(2)}$ to incorporate
 844 additional dynamics that were previously captured through the direct modeling of pairs. This
 845 represents a fundamental trade-off between:

- 846 1. **Spatial complexity:** Using $O(N^2)$ variables to explicitly represent all pairwise relationships
 847
- 848 2. **Temporal complexity:** Using a richer memory structure that implicitly encodes these relationships through time
 849

850 The theorem quantifies exactly how much additional memory structure is required: it must include
 851 all dynamical information that would have flowed through the pairs variables in the more complex
 852 representation.

853 A.5.2 SPATIO-TEMPORAL COUPLING

854 **Corollary 2** (Spatio-Temporal Coupling). *The memory kernel in the singles-only representation
 855 cannot be factorized as independent spatial and temporal components, requiring models that capture
 856 non-separable coupling between spatial indices and temporal frequencies.*

864 The inflation term:
 865

$$866 (\Omega_{sz} + \tilde{K}_{sz}^{(1)}(p))[pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1}(\Omega_{zs} + \tilde{K}_{zs}^{(1)}(p)) \quad (18)$$

868 has a critical property: it cannot be factorized as a product of purely spatial and purely temporal
 869 operators. To see this, consider its structure:
 870

- 871 1. $(\Omega_{sz} + \tilde{K}_{sz}^{(1)}(p))$ couples spatial indices (i, j) with temporal frequency p
 872
- 873 2. The matrix inverse $[pI - \Omega_{zz} - \tilde{K}_{zz}^{(1)}(p)]^{-1}$ mixes these couplings in a non-separable way
 874
- 875 3. The final term $(\Omega_{zs} + \tilde{K}_{zs}^{(1)}(p))$ further couples the result with additional spatial indices
 876

877 In the time domain, this corresponds to a memory kernel with structure:
 878

$$879 K_{ij}^{(2)}(t - \tau) \neq u_{ij}v(t - \tau) \quad (19)$$

881 where i, j are residue indices. This non-separability means that the memory kernel cannot be decom-
 882 posed into independent spatial and temporal components—spatial relationships evolve with time,
 883 and temporal patterns differ across spatial relationships.
 884

885 A.6 CONNECTION TO MACHINE LEARNING ARCHITECTURES

887 These theoretical results directly inform architectural design for protein dynamics models:
 888

- 889 1. **Factorized attention** (spatial then temporal, or vice versa) cannot capture the non-
 890 separable coupling revealed by Corollary 2.
- 891 2. **Joint spatio-temporal attention** with tokens indexing both residue and time provides ex-
 892 actly the structure needed to learn the inflated memory kernel demanded by Corollary 1.
- 893 3. **Expanded receptive field** in the temporal dimension compensates for the missing explicit
 894 pairwise information.
 895

896 This explains why architectures that model protein dynamics without explicit pairwise features re-
 897 quire sophisticated temporal modeling capabilities. The mathematical structure of the memory ker-
 898 nel dictates the minimum expressivity requirements for any machine learning model that aims to
 899 capture the underlying physics accurately.
 900

901 B COMPLEXITY ANALYSIS OF SPATIOTEMPORAL ATTENTION

903 To understand the computational complexity and scaling behavior of different protein dynamics
 904 models, we analyze the fundamental operations in both ConfRover (the previous state-of-the-art)
 905 and our STAR-MD approach.
 906

907 B.1 PROBLEM FORMULATION AND NOTATION

909 For a protein with N residues and a trajectory with L frames, we define:
 910

- 911 • $\mathbf{s}_t \in \mathbb{R}^{N \times d_s}$: Per-residue (singles) features at frame t
 912
- 913 • $\mathbf{z}_t \in \mathbb{R}^{N \times N \times d_z}$: Pairwise features at frame t

914 The full representation across all frames would be:
 915

$$916 \mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_L] \in \mathbb{R}^{L \times N \times d_s} \quad (20)$$

$$917 \mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_L] \in \mathbb{R}^{L \times N \times N \times d_z} \quad (21)$$

918 B.2 COMPUTATIONAL COMPLEXITY OF DIFFERENT ATTENTION MECHANISMS
919920 B.2.1 FULL SPATIOTEMPORAL ATTENTION (THEORETICAL)
921922 The most comprehensive approach would be to apply attention across all residues and all time frames
923 for both singles and pairs, which would have computational complexity:
924

$$\mathcal{O}((L \times (N + N^2))^2) = \mathcal{O}(L^2 N^4) \quad (22)$$

925 This is computationally prohibitive for any realistic protein system.
926927 B.2.2 CONFROVER'S APPROACH: TWO SOURCES OF COMPUTATIONAL COMPLEXITY
928929 ConfRover faces computational challenges from two distinct sources:
930931 **1. Channel-factorized temporal attention.** The primary limitation in ConfRover is its temporal
932 attention mechanism. It applies temporal attention independently to each single feature and each
933 pair feature:
934

- For singles: N separate $L \times L$ temporal attention operations, complexity $\mathcal{O}(N \times L^2)$
- For pairs: N^2 separate $L \times L$ temporal attention operations, complexity $\mathcal{O}(N^2 \times L^2)$

935 Total temporal attention complexity: $\mathcal{O}(N \times L^2 + N^2 \times L^2) = \mathcal{O}((N + N^2) \times L^2)$
936937 While this factorization makes computation manageable, it assumes that temporal dynamics can be
938 modeled independently per channel (whether single or pair), which fundamentally limits the model's
939 ability to capture complex spatio-temporal couplings that govern protein dynamics.
940941 **2. Pairformer operations for spatial modeling.** In addition to the temporal attention, ConfRover
942 employs a Pairformer layer for spatial interactions at each time step, which has complexity:
943

- $\mathcal{O}(N^3)$ per frame due to the interaction between all residues and all pairs
- $\mathcal{O}(N^3 \times L)$ for the entire trajectory

944 The combined time complexity is therefore $\mathcal{O}((N + N^2) \times L^2 + N^3 \times L)$. For large proteins, the
945 N^3 term dominates, making scaling to large systems prohibitive both in terms of computation time
946 and memory usage.
947948 B.2.3 STAR-MD APPROACH: SINGLE-RESTRICTED SPATIOTEMPORAL ATTENTION
949950 Our approach restricts spatiotemporal attention to only single-residue features:
951

- We apply one $(N \times L) \times (N \times L)$ attention operation on the flattened singles tensor
- Computational complexity: $\mathcal{O}((N \times L)^2) = \mathcal{O}(N^2 L^2)$

952 Critically, we eliminate the Pairformer component and its $\mathcal{O}(N^3)$ complexity. Instead, we rely
953 on the spatiotemporal attention mechanism to implicitly learn the necessary pairwise relationships
954 through the Memory Inflation mechanism described in Section A.1.
955956 B.3 STAR-MD VS. FACTORIZED SPATIAL-TEMPORAL ARCHITECTURES.
957958 STAR-MD employs a joint spatio-temporal attention mechanism with complexity $\mathcal{O}(N^2 L^2)$. In
959 contrast, some architectures utilize spatial Pairformer layers ($\mathcal{O}(N^3 L)$) and temporal attention lay-
960 ers on singles ($\mathcal{O}(NL^2)$), resulting in a total complexity of $\mathcal{O}(N^3 L + NL^2)$.
961962 To identify the regime where each model is more efficient, we equate their complexities:
963

$$N^2 L^2 = N^3 L + NL^2 \quad (23)$$

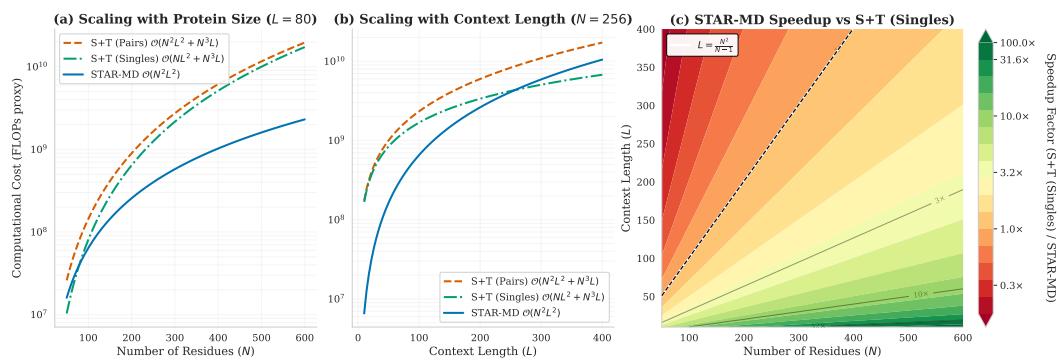
964 Assuming $N, L > 0$, we can simplify this to find the crossover point:
965

$$L \approx N \quad (\text{specifically } L = \frac{N^2}{N-1}) \quad (24)$$

972 This reveals two distinct regimes:
 973

974 • **Regime 1 ($L \gtrsim N$):** When the context length exceeds the protein size (e.g., small peptides
 975 with very long history), the $\mathcal{O}(NL^2)$ temporal scaling of baselines is advantageous.
 976 • **Regime 2 ($N \gg L$):** For realistic protein modeling, where system size N is large (hun-
 977 dreds to thousands of residues) and context history L is fixed (e.g., 50-100 frames), STAR-
 978 MD is significantly more efficient. In this regime, the baselines' cubic spatial scaling
 979 ($\mathcal{O}(N^3L)$) becomes a prohibitive bottleneck.
 980

981 As illustrated in Figure 6, for a standard context length of $L = 80$, STAR-MD's computational
 982 cost scales much more favorably with protein size than baseline methods, enabling the simulation
 983 of large protein complexes that are computationally intractable for $\mathcal{O}(N^3)$ methods.
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 997 **Figure 6: Computational complexity scaling.** Comparison of computational cost (FLOPs proxy)
 998 for STAR-MD ($\mathcal{O}(N^2L^2)$), Pairformer + Temporal Attention on Pairs ($\mathcal{O}(N^2L^2 + N^3L)$), and
 999 Pairformer + Temporal Attention on Singles ($\mathcal{O}(NL^2 + N^3L)$). **Left:** Scaling with protein size N
 1000 for a fixed context length $L = 80$. STAR-MD scales significantly better for large proteins due to
 1001 the absence of cubic spatial terms. **Middle:** Scaling with context length L for a fixed protein size
 1002 $N = 256$. While baselines have lower temporal complexity, their high spatial cost dominates in
 1003 the realistic regime where $N > L$. **Right:** Heatmap showing the speedup factor of STAR-MD over
 1004 Pairformer + Temporal Attention on Singles as a function of protein size N and context length L .
 1005 STAR-MD achieves significant speedups in the realistic regime where $N > L$.
 1006

1007 B.4 KV CACHE EFFICIENCY DURING INFERENCE

1008 An important aspect to consider in autoregressive protein trajectory generation is the memory cost
 1009 during inference, particularly related to the key-value (KV) cache used for efficient generation. This
 1010 becomes especially important for long-horizon generation where hundreds or thousands of frames
 1011 must be maintained in context.
 1012

1013 **ConfRover KV cache requirements.** During autoregressive generation, ConfRover must store in
 1014 its KV cache:
 1015

- 1016 • Singles features: $\mathcal{O}(N \times L \times d)$ memory
- 1017 • Pairs features: $\mathcal{O}(N^2 \times L \times d)$ memory

1018 Total KV cache memory: $\mathcal{O}((N + N^2) \times L \times d)$

1019 For a medium-sized protein with $N = 200$ residues, a context window of $L = 32$ frames, and
 1020 embedding dimension $d = 256$, this results in approximately:
 1021

$$\text{Memory for each layer} = (200 + 200^2) \times 32 \times 256 \times 4 \text{ bytes} \quad (25)$$

$$= (200 + 40000) \times 32 \times 256 \times 4 \text{ bytes} \quad (26)$$

$$\approx 1.3 \text{ GB} \quad (27)$$

	Simulation time	Intervals (snapshots)	Number of frames
1028	100 ns	120	80
1029	240 ns	120	200
1030	1 μ s	250	400

Table 4: Inference task configurations for ATLAS dataset.

STAR-MD KV cache requirements. In contrast, STAR-MD only needs to store keys and values for the singles features:

$$\text{Memory for each layer} = N \times L \times d \times 4 \text{ bytes} \quad (28)$$

$$= 200 \times 32 \times 256 \times 4 \text{ bytes} \quad (29)$$

$$\approx 6.6 \text{ MB} \quad (30)$$

This represents a **196 \times reduction** in KV cache memory requirements compared to ConfRover, which becomes increasingly significant as the protein size increases or when using longer context windows.

Scaling to larger proteins and longer trajectories. Figure 5 shows the estimated KV cache memory requirements for different protein sizes and trajectory lengths.

B.5 PRACTICAL SCALING BEHAVIOR

While the asymptotic complexity of STAR-MD with respect to protein size N is still quadratic ($\mathcal{O}(N^2 L^2)$), our approach offers significant practical advantages:

1. **Elimination of cubic scaling terms:** The most expensive $\mathcal{O}(N^3)$ operations from the Pairformer are eliminated.
2. **Efficient attention implementations:** Our attention pattern is amenable to highly optimized implementations like FlashAttention Dao et al. (2022), which provides significant practical speedups.
3. **KV cache efficiency:** As shown in Section B.4, our approach dramatically reduces memory requirements during inference by avoiding the need to store pair features in the KV cache.

C EXPERIMENTAL DETAILS

C.1 ATLAS DATASET

The ATLAS (Vander Meersche et al., 2024) dataset contains triplicated 100 ns simulation for 1390 proteins with diverse structures and dynamics, representative for different ECOD X-class domains. Following prior work (Jing et al., 2024a,b; Wang et al., 2024; Shen et al., 2025), we adopt a time-based split for ATLAS. Specifically, the train/validation/test sets are divided based on the release date of each protein, using cutoff dates of May 1, 2018 and May 1, 2019. Proteins released before May 1, 2018 are used for training, and proteins released after May 1, 2019 are used for testing. The diverse nature of ATLAS makes it a standard benchmark for evaluating protein conformation and trajectory generation under transferrable settings.

For each 100 ns trajectories, snapshots of atom coordinates were saved at 10 ps intervals. We use the coordinates for all heavy atoms in protein For model training and evaluation. Similar to Shen et al. (2025), we exclude the training proteins longer than 384 amino acids, leading to 1080 training proteins.

We include the inference task configurations for different simulation time in Table 4.

C.2 BASELINE IMPLEMENTATION

ATLAS MD oracles. In 100 ns simulation, we use one of the triplicated trajectories in the ATLAS dataset as the oracle to estimate the expected performance from an independent simulation run. To

1080 simulate longer trajectories, we follow the setup described in Vander Meersche et al. (2024) using
 1081 provided equilibrated structures and Gromacs .tpr files to preproduce production runs at the de-
 1082 sired lengths. All simulations are conducted using Gromacs (version 2023.2) on NVIDIA V100
 1083 GPUs with 40 GB memories. Independent reference and oracle trajectories are generated by repeat-
 1084 ing simulations with different random seeds.

1085 **AlphaFolding (Cheng et al., 2025).** We followed the official data preparation and in-
 1086 ference procedures provided by the authors ², using the recommended model weights
 1087 (frame16_step_95000.pth). Trajectories were generated using the recommended ex-
 1088 trapolation script (run_eval_extrapolation.sh) with parameters sample_step=1,
 1089 n_motion=2, and n_frames=16. This model employs a block-wise autoregressive scheme:
 1090 it generates a fixed-size block of 16 frames conditioned on the final frames of the preceding
 1091 block. This process is repeated to achieve the desired total simulation time by adjusting the
 1092 extrapolation_time parameter.

1093 A key characteristic of this baseline is its fixed, internal time step of 10 ps, which differs from our
 1094 model’s variable stride capability. Consequently, no time step normalization is performed at the
 1095 generation stage; instead, trajectories are sampled post-generation to match the frame rates required
 1096 for evaluation.

1097 **MDGen (Jing et al., 2024b).** We followed the official data preparation and inference procedures
 1098 provided by the authors ³, using the provided model weights (atlas.ckpt). The model was
 1099 trained on ATLAS trajectories preprocessed with 400 ps intervals (stride 40 with base interval
 1100 10 ps). For inference, we used the forward simulation script (sim_inference.py) with pa-
 1101 rameters num_frames=250 and suffix=_R1.

1102 This model generates trajectories autoregressively in blocks of 250 frames with an internal timestep
 1103 of 400 ps per frame. A single rollout produces 100 ns of simulation time (250 frames \times 400 ps
 1104 = 100 ns). For 100 ns trajectories, we used num_rollouts=1. For 240 ns and 1 μ s evaluations,
 1105 we extended generation by setting num_rollouts to the required number of sequential blocks.
 1106 Generated trajectories were then sampled post-generation to match the frame rates required for eval-
 1107 uation.

1108 **ConfRover (Shen et al., 2025).** We use the code and model weights provided by the authors.
 1109 ConfRover is a frame-level autoregressive model similar to STAR-MD, and we adopt the same
 1110 setup as our model to generate trajectories for each experiments. For 100 ns results, we enable
 1111 CPU-offloaded caching to store full key-value history in system memory, while for 240 ns and 1 μ s
 1112 results, we employ a sliding-window cache with attention sinks (Xiao et al., 2023a) (window size of
 1113 14 with first two frames as attention sinks) to prevent out-of-memory errors.

1114 All generative model baselines are evaluated on NVIDIA H100 GPUs with 80 GB GPU memory
 1115 and 2 TB system memory.

1117 C.3 DETAILS ON EVALUATION METRICS

1119 **Trajectory Alignment.** Prior to any quantitative analysis (including conformational coverage and
 1120 kinetic fidelity), all model-generated trajectories are aligned to the corresponding reference MD
 1121 simulation. This is performed via $C\alpha$ superposition to a common reference frame, ensuring that all
 1122 comparisons are independent of global rotational and translational differences.

1124 **Conformational coverage.** Conformational state recovery is evaluated by comparing the distri-
 1125 bution of model-generated and reference conformations in a PCA space. Each conformation is pa-
 1126 rameterized by the 3D coordinates of $C\alpha$ atoms. The PCA space is constructed using conformations
 1127 from reference MD simulations in ATLAS. To compare distributions, each principal component
 1128 is discretized into 10 evenly spaced bins. After projecting the conformations into this space, we
 1129 count their occurrences in each bin and compute the distribution similarity using Jensen–Shannon
 1130 Distance (JSD). We also binarize the occupancy counts to compute precision, recall, and F1-score
 1131 – evaluating whether sampled conformations fall within known states, following prior works (Lu
 1132 et al., 2024; Wang et al., 2024; Zheng et al., 2024; Shen et al., 2025).

1133 ²https://github.com/fudan-generative-vision/dynamicPDB/tree/main/applications/4d_diffusion

³<https://github.com/bjing2016/mdgen>

1134 **Kinetic fidelity.** Previous metrics operates on individual proteins and ensembles of proteins, independent of their temporal ordering. To characterize the temporal evolution of protein conformations, 1135 we use the correlation of leading Time-lagged Independent Component Analysis (tICA) components 1136 at varying lagtime similar to Shen et al. (2025), and additionally include two kinetic metrics: coordinate 1137 root mean squared distance (RMSD) and autocorrelation as functions of lagtime. 1138

1139 Given model-generated trajectory $\{\hat{x}_t\}_t \in \mathbb{R}^{N_{\text{atom}} \times 3}$ and MD reference $\{x_t\}_t \in \mathbb{R}^{N_{\text{atom}} \times 3}$, we project 1140 all trajectories onto the top 32 principal components computed using MD reference C_α coordinates. 1141 For a particular lagtime τ , the coordinate RMSD is computed via 1142

$$\mathbb{E}_t[\text{RMSD}(x_{t+\tau}, x_t)], \quad (31)$$

1143 which is an approximate average velocity highlighting the dynamical timescales of conformational 1144 changes. Autocorrelation is computed via 1145

$$\mathbb{E}_t \left[\frac{(x_t - \mu)(x_{t+\tau} - \mu)}{\sigma^2} \right], \quad (32)$$

1146 which is a measure of covariance within each trajectory. The aim of each metric is to approximate 1147 how the corresponding measure behaves in the MD reference data (dashed lines), capturing not only 1148 their overall trend but also characteristic magnitudes. 1149

1150 **tICA Correlation Metric.** Following Shen et al. (2025), we evaluate dynamic fidelity by comparing 1151 the principal dynamic modes of the generated and reference trajectories. Input trajectories (X_{ref} and 1152 X_{gen}) are aligned and represented by C_α coordinates flattened to $d = 3N$ features. We first compute 1153 a validity mask $M \in \{0, 1\}^T$ for the generated trajectory, retaining only frames that pass the “CA + 1154 AA Validity” checks defined in Appendix C.3. Only time-lagged pairs $(x_t, x_{t+\tau})$ where both frames 1155 are valid ($M_t = 1, M_{t+\tau} = 1$) are used for analysis. 1156

1157 We fit separate Time-lagged Independent Component Analysis (tICA) models to the reference and 1158 generated trajectories by solving $C_\tau \mathbf{v}_i = \lambda_i C_0 \mathbf{v}_i$, where C_0 and C_τ are the instantaneous and 1159 time-lagged covariance matrices, respectively. We use lagtimes $\tau \in [1, 5, 10, 20]$, apply kinetic map 1160 scaling, and set the regularization cutoff to $\epsilon = 10^{-6}$. 1161

1162 For the top k independent components (PC1 and PC2), we extract the left singular vector $\mathbf{v}^{(k)} \in 1163 \mathbb{R}^{3N}$ and compute a per-residue contribution score $S_i^{(k)} = \max(|v_{i,x}|, |v_{i,y}|, |v_{i,z}|)$ for each residue 1164 i . The final metric is the absolute Pearson correlation coefficient between the reference and generated 1165 scores: $|r_k| = |\text{Pearson}(S_{\text{ref}}^{(k)}, S_{\text{gen}}^{(k)})|$, averaged across PC1 and PC2. 1166

1167 Our evaluation procedure mirrors Shen et al. (2025), with the sole exception that we only consider 1168 valid pairs via M . For any given lagtime, we report the tICA correlation only if at least 30 time- 1169 lagged pairs are valid; otherwise, we denote the result as ‘N/A’. 1170

1171 **Structural Quality and Validity** To assess the physical plausibility of generated protein conformations, 1172 we use a suite of metrics that measure different aspects of structural integrity. A generated 1173 frame is considered **structurally valid** if it simultaneously satisfies all of the following conditions, 1174 which are based on thresholds derived from oracle MD simulations: 1175

- **No C_α Chain Breaks:** The distance between consecutive C_α atoms must be below a certain threshold.
- **No C_α Clashes:** The distance between any two non-adjacent C_α atoms must be above a certain threshold.
- **Plausible Backbone Geometry:** The percentage of residues that are outliers in the Ramachandran plot must be below a threshold.
- **Plausible Side-Chain Geometry:** The percentage of residues with outlier rotamers must be below a threshold.

1184 **Threshold Derivation.** The specific thresholds for these metrics were determined by analyzing the 1185 distribution of these quality metrics in the ground-truth 100ns MD simulation trajectories (our “oracle”). 1186 For each metric, we computed its value across all frames of the oracle trajectories and set the 1187 threshold at the 99th percentile (i.e., accepting 99% of the oracle frames). 1188

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Table 5: **Dataset and Training Configuration**

1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200 1201 1202 1203 1204 1205 1206 1207 1208 1209 1210 1211 1212 1213 1214 1215 1216 1217 1218 1219 1220 1221 1222 1223 1224 1225 1226 1227 1228 1229 1230 1231 1232 1233 1234 1235 1236 1237 1238 1239 1240 1241 <th>Parameter</th> <th>Value</th>	Parameter	Value
<i>Dataset Configuration</i>		
Training data	ATLAS train split (length ≤ 384)	
Global batch size	1	
Frames per trajectories	8	
Frame intervals	$1 \sim 2^{10}$ of 10 ps snapshots	
<i>Feature Representation</i>		
Single-residue feature dim	384	
Pairwise feature dim	128	
Number of recycles	3	
<i>Data Augmentation</i>		
Clean noise max magnitude	0.1	
Clean noise sampling prob	0.75	
Noise sampling	Frame-level	

An exception was made for the C_α chain break rate. The oracle MD simulations contained no chain breaks, resulting in a 99th percentile threshold of 0. This is too strict for current generative models. To allow for a more meaningful comparison, we set a small, non-zero tolerance for chain breaks.

The exact thresholds used for determining a valid frame are listed in Table 9. We define three sets of criteria for validity: **$C_\alpha + All-Atom$** , which requires passing all four checks; **All-Atom Only**, which checks only Ramachandran and rotamer criteria; and **C_α -Only**, which checks only for C_α clashes and breaks.

C.4 MODEL HYPERPARAMETERS

We include the details of our training and inference configuration as follows. The model was trained on the ATLAS dataset following the train/val/test split in the previous works(Shen et al., 2025; Jing et al., 2024a;b; Wang et al., 2024; Cheng et al., 2025). STAR-MD contains 4 diffusion blocks, each containing 1 IPA layer and 2 S \times T layers with hidden dimension 256 and 8 attention heads. We train on trajectory snippets with context length $L = 8$ frames and global batch size of 8.

We use the Adam optimizer and learning rate 5×10^{-5} with the similar loss setup as used in Shen et al. (2025).

For distributed training, we employed DeepSpeed Stage 2 optimization with gradient checkpointing to efficiently manage memory usage during training of large protein systems. All models are trained 8 NVIDIA H100 GPUs with 80 GB memory.

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Table 6: Diffusion Model Configuration

Parameter	Value
<i>SE(3) Diffusion</i>	
Coordinate scaling	0.1
Translation b_{\min}/b_{\max}	0.1 / 20.0
Translation schedule	Linear
Rotation $\sigma_{\min}/\sigma_{\max}$	0.1 / 1.5
Rotation schedule	Logarithmic
<i>Sampling</i>	
Method	Euler SDE
Diffusion steps	200
t_{\max} / t_{\min}	1.0 / 0.01

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Table 7: Model Architecture

Component	Configuration
<i>Encoder (PseudoBetaPairEncoder)</i>	
Residue index embedding size	128
Output size	128
<i>Invariant Point Attention</i>	
IPA blocks	4
Single channel size	256
Pair channel size	128
Hidden channel size	256
Skip channel size	64
Attention heads	4
Query/key points	8
Value points	12
<i>Spatiotemporal Attention</i>	
Model dimension	256
Number of layers	2
Number of heads	4
RoPE 2D	Enabled

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Table 8: Loss Functions and Optimization

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Table 9: **Structural quality thresholds for validity.** Thresholds were derived from the 99th percentile of the oracle 100ns MD simulations, except for the C_α break rate.

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Parameter	Value
<i>Loss Weights</i>	
Rotation score loss	0.5
Translation score loss	1.0
Torsion loss	0.5
Backbone FAPE loss	0.5
Sidechain FAPE loss	0.5
Backbone coordinates loss	0.25 (diffusion $t \leq 0.25$)
Backbone distance map loss	0.25 (diffusion $t \leq 0.25$)
<i>Optimization</i>	
Optimizer	Adam
Learning rate	0.0002
LR schedule	Linear warmup + cosine decay
Warmup epochs	5
Total epochs	250
Warmup start LR factor	0.01
Minimum LR factor	0.1
Gradient clipping	1.0 (norm)
Precision	BF16 mixed precision

Metric	Threshold
<i>C_α + All-Atom Evaluation</i>	
Ramachandran Outliers (%)	≤ 4.12
Rotamer Outliers (%)	≤ 7.05
C_α Clash Rate (%)	≤ 1.29
C_α Break Rate (%)	≤ 0.2
<i>All-Atom Only Evaluation</i>	
Ramachandran Outliers (%)	≤ 4.12
Rotamer Outliers (%)	≤ 7.05
<i>C_α-Only Evaluation</i>	
C_α Clash Rate (%)	≤ 1.29
C_α Break Rate (%)	≤ 0.2

1350 C.5 ADDITIONAL RESULTS ON DYNAMIC FIDELITY
13511352 We visualize the corresponding kinetic metrics for 100ns, 240 ns, and $1\mu\text{s}$ in Figure 7. For each
1353 metric, we include 2 separate MD simulations for reference (dashed lines). In all but one case
1354 (autocorrelation for $1\mu\text{s}$), STAR-MD better approximates the trend and values of MD references,
1355 showing higher kinetic fidelity than the rest of models.1356 We also include the corresponding metrics for our ablation study (Section 4.5) in Figure 8, where
1357 STAR-MD outperforms other variants in all but two cases (100 ns C α coordinate RMSD and $1\mu\text{s}$
1358 C α autocorrelation at large lagtime).1359 We further provide a detailed breakdown of the tICA correlation metric in Figure 9 and Table 10.
1360 Figure 9 shows the per-component and per-lagtime correlations for all valid runs, highlighting the
1361 variability across different simulation instances. Table 10 summarizes these results, demonstrating
1362 that STAR-MD achieves the highest mean correlation among generative models, closely matching
1363 the reference MD baseline.1364 Table 10: **Aggregated tICA Correlation Statistics.** Mean and standard deviation of tICA correlations
1365 averaged over PC1, PC2, and all lagtimes. We report results on both valid samples (Valid) and
1366 all samples (Unfiltered).

Model	tICA (Valid)	tICA (Unfiltered)
MD (Oracle)	0.170	0.17
MDGen	0.130 ± 0.039	0.12 ± 0.00
AlphaFolding	N/A	<u>0.14</u>
ConfRover	0.162 ± 0.035	0.18 ± 0.01
STAR-MD	0.176 ± 0.033	0.18 ± 0.01

1376 C.5.1 VAMP-2 SCORE
13771378 The variational approach for Markov models (VAMP, Wu & Noé (2020)) is another metric to evaluate
1379 the kinetic fidelity for Markov state models, which measures how well a set of features captures
1380 the slow dynamical modes of the system. Given a sequence $\{x_t\}_t$, the Koopman matrix for the
1381 system is approximated via

1382
$$K = C_{00}^{-0.5} C_{0\tau} C_{\tau\tau}^{-0.5} \text{ where } C_{t_1 t_2} = \mathbb{E}_t[(x_{t_1} - \mu)(x_{t_1+t_2} - \mu)], \quad (33)$$

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1384 and the VAMP-2 score is the Frobenius norm of the Koopman matrix.

1385 To expand on the metrics for dynamic fidelity, we computed VAMP-2 score as a function of lagtime,
1386 where we first project all trajectories onto the top 32 principle components computed using MD
1387 reference C α coordinates, similar to RMSD and autocorrelation metrics. The results for 100 ns are
1388 included in Figure 10, and we observe that STAR-MD better approximates the VAMP-2 score of
1389 MD references for all lagtime than other models we compared against and other variants.1390
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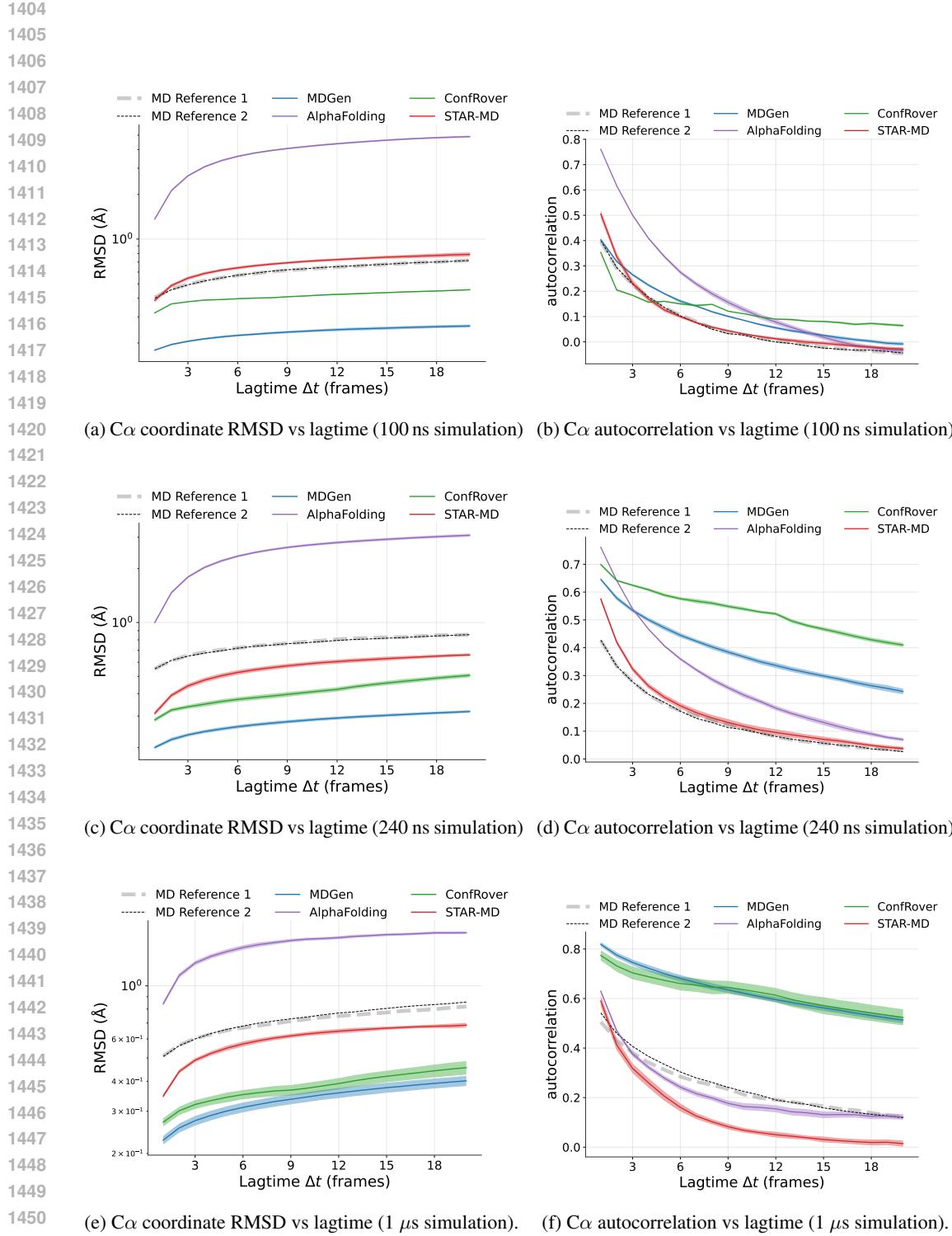


Figure 7: Kinetic metrics at different trajectory lengths: 100ns, 240ns, and 1 μ s. Dashed lines are MD references (in most cases MD reference metrics are very close, demonstrating the metrics are robust across independent MD reference simulations). STAR-MD better approximates the MD references, demonstrating higher kinetic fidelity than the other models.

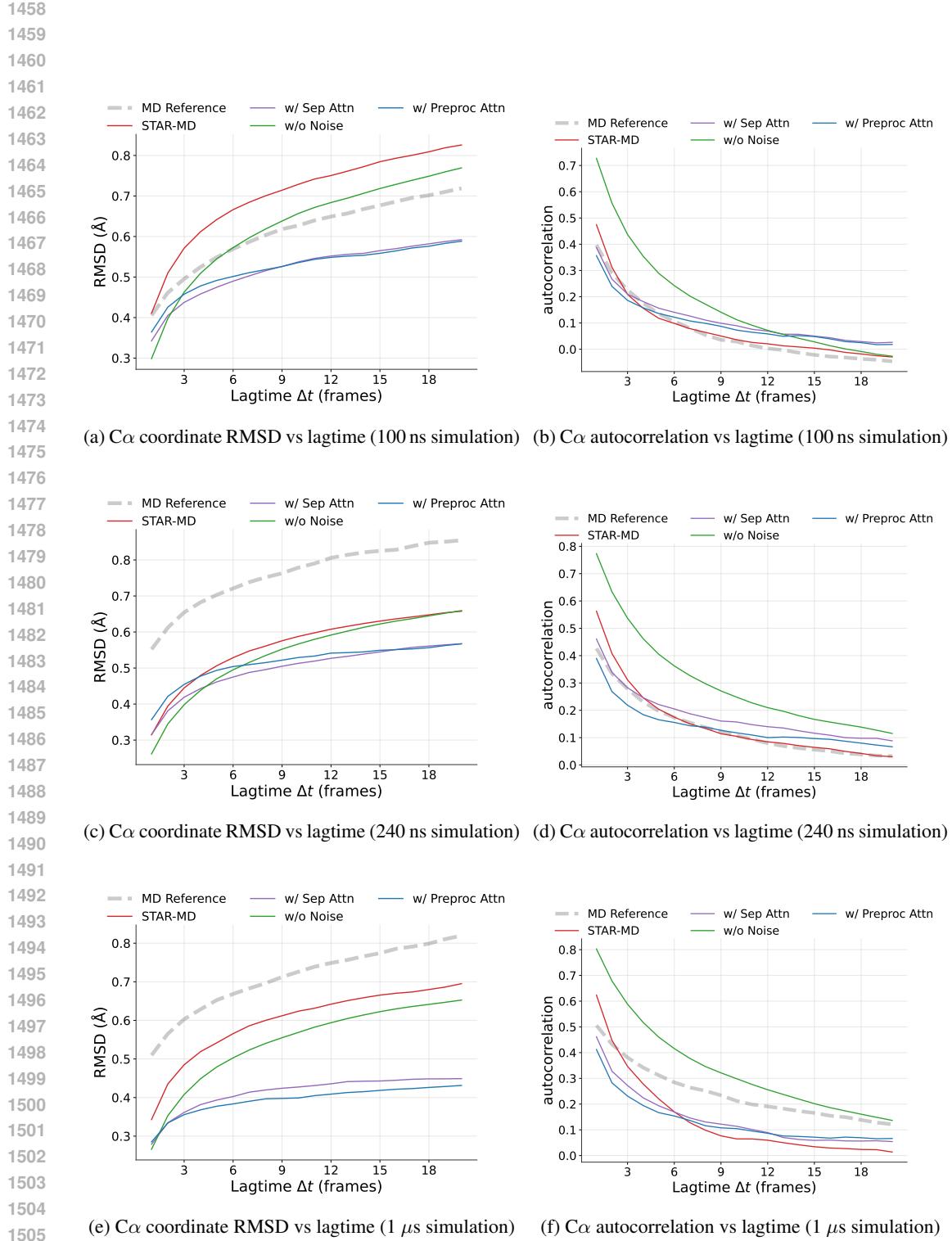
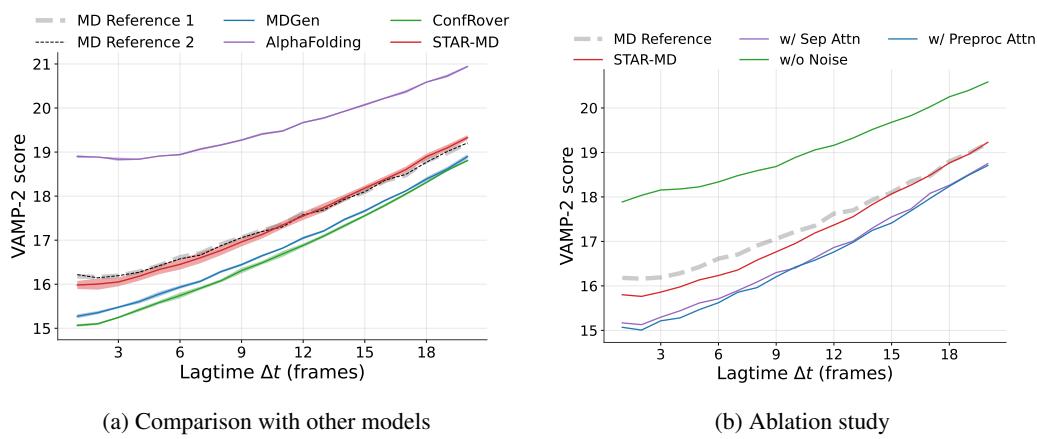
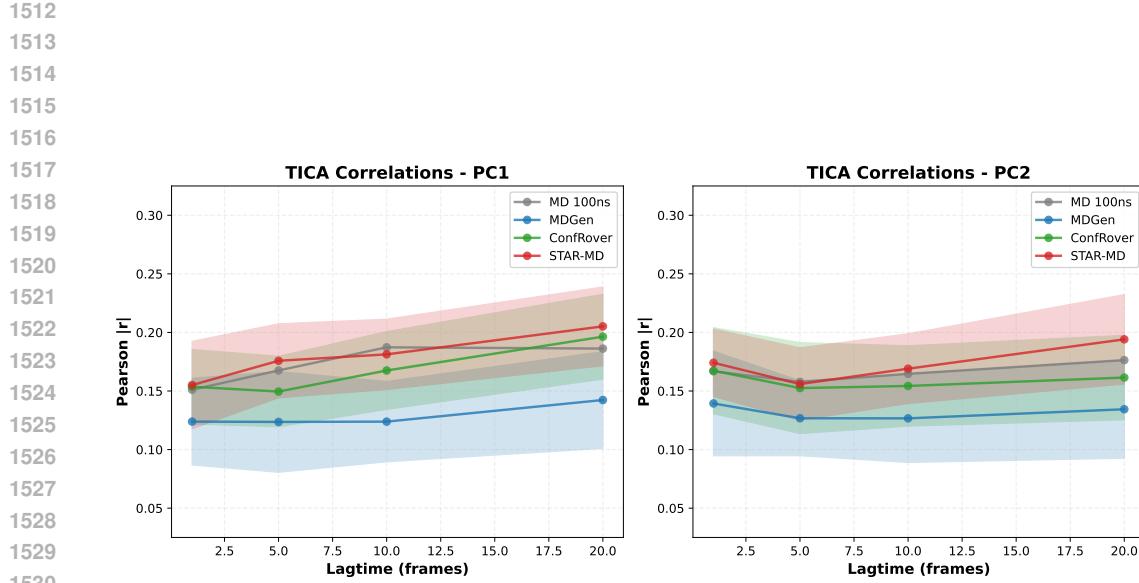


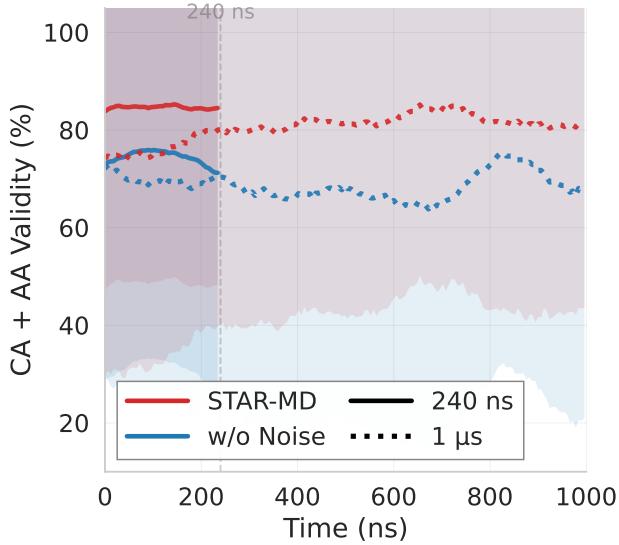
Figure 8: **Kinetic metrics for ablation study.** Dashed line is MD reference. STAR-MD has similar/better performances than other variants in all but two cases (100ns C α coordinate RMSD and 1 μ s C α autocorrelation at large lagtime).



1566 D FULL ABLATION RESULTS

1568 We include the full results Tables for ablation study in this section (Table 11,12). We observe
 1569 a consistent trend that contextual noise improves structure quality, and spatio-temporal attention
 1570 improves conformation coverage.

1571 Figure 11 compares the CA+AA validity over time for models with and without contextual noise on
 1572 the 240 ns and 1 μ s benchmarks, showing that the noise helps maintain structural quality over long
 1573 horizons.



1593 Figure 11: **Effect of Contextual Noise on Long-Horizon Stability.** Comparison of Structural
 1594 Validity ($\text{C}\alpha + \text{All-Atom}$) over time for STAR-MD with and without contextual noise on 240 ns and
 1595 1 μ s benchmarks.

1597 Table 11: **Complete ablation study results (240 ns).** We compare our full model, STAR-MD,
 1598 against variants with key components removed. All metrics are reported for the 240 ns generation
 1599 task. Metrics degraded in each ablation setting are highlighted in red.

1602 Model	Cov Valid		Dynamic Fidelity		Validity		
	JSD \downarrow	Rec \uparrow	RMSD \downarrow	AutoCor \downarrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
STAR-MD	0.45	0.59	0.20	0.02	86.44	97.61	84.62
w/o Noise	0.42	0.61	0.22	0.16	77.11	96.19	74.25
w/ Sep Attn	0.48	0.51	0.26	0.04	87.09	97.58	84.83
w/ Preproc Attn	0.47	0.52	0.25	0.03	87.25	96.72	84.28

1609 Table 12: **Complete ablation study results (1 μ s).** We compare our full model, STAR-MD, against
 1610 variants with key components removed. All metrics are reported for the 1 μ s generation task. Metrics
 1611 degraded in each ablation setting are highlighted in red.

1613 Model	Cov Valid		Dynamic Fidelity		Validity		
	JSD \downarrow	Rec \uparrow	RMSD \downarrow	AutoCor \downarrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
STAR-MD	0.45	0.61	0.11	0.11	91.00	91.44	83.59
w/o Noise	0.48	0.63	0.17	0.10	75.84	91.22	68.66
w/ Sep Attn	0.51	0.49	0.30	0.10	93.12	92.03	85.31
w/ Preproc Attn	0.52	0.49	0.32	0.11	94.12	92.19	86.44

1620 **E INFERENCE COST**
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1622 We evaluated the wall-clock time for generating 100 ns trajectories under our 100 ns benchmark
 1623 setting for each methods, across proteins of varying lengths (50~750 amino acids). To estimate the
 1624 runtime of molecular dynamic (MD) simulation, we used OpenMM with the Amber force field and
 1625 implicit solvent, running a 0.1 ns simulation and extrapolating the cost to 100 ns. MDGen, trained
 1626 with a time step of 400 ps and window of 250 frames, can directly generate 100 ns trajectories in
 1627 single joint diffusion process and therefore provides the greatest acceleration. AlphaFolding, trained
 1628 on a fixed time step of 10 ps and a window of 16 frames, requires substantially longer inference time
 1629 to complete a 100 ns simulation. Both ConfRover and STAR-MD support varying-stride simulation
 1630 and can generate under our evaluation configuration (stride=1.2 ns, 80 frames). While STAR-MD is
 1631 slightly slower than ConfRover for small proteins, it is more efficient when scaling to larger proteins
 1632 and is faster on 6SMS-A (724 AA). All methods are evaluated on a single NVIDIA H100 GPU.

1633 Table 13: **Inference cost for different methods.** The wall-clock time (in seconds) to generate
 1634 100 ns trajectories across proteins with varying lengths. AlphaFolding encounters out-of-memory
 1635 error for larger proteins and is reported as ‘N/A’. All methods are evaluated on a single NVIDIA
 1636 H100 GPU.

Method	6OKD-C (51)	6Q9C-A (155)	6XB3-H (241)	7AQX-A (364)	7P41-D (448)	7MF4-A (554)	6L8S-A (650)	6SMS-A (724)
MD	6322.8	9610.8	14420.3	24272.4	34850.8	49294.4	66296.7	78451.4
AlphaFolding	3135.7	5846.5	10024.8	19036.3	27340.3	40472.5	N/A	N/A
MDGen	4.1	8.1	13.0	20.5	28.2	35.3	41.2	54.4
ConfRover	404.5	423.6	463.5	573.7	710.4	934.4	1170.2	1617.7
STAR-MD	766.3	807.4	818.4	795.8	902.5	1055.5	1232.4	1381.2

1644 **F ARCHITECTURE DETAILS: PAIR FEATURES**
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1646 STAR-MD modifies Shen et al. (2025)’s standard AlphaFold-derived architecture by removing the
 1647 Pairformer module to reduce computational cost, while retaining explicit pairwise feature updates.

1648 **FrameEncoder.** STAR-MD takes as input the pre-trained pairwise features from a frozen Open-
 1649 Fold (Jumper et al., 2021) model. Then, it employs a **FrameEncoder** module Shen et al. (2025)
 1650 which incorporates pairwise geometric information into the pair features.

1651 **EdgeTransition.** Pairwise features $\mathbf{z} \in \mathbb{R}^{N \times N \times d_z}$ are initialized by the **FrameEncoder**
 1652 and updated via **EdgeTransition** layers. Unlike the global attention in **Pairformer**,
 1653 **EdgeTransition** uses a local MLP update after each spatio-temporal attention block. For
 1654 residues i and j :

$$\mathbf{z}_{ij} \leftarrow \mathbf{z}_{ij} + \text{MLP}(\mathbf{s}_i, \mathbf{s}_j, \mathbf{z}_{ij}) \quad (34)$$

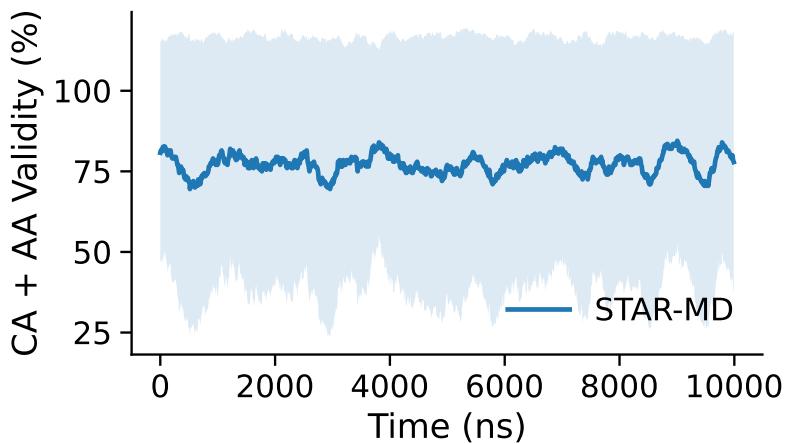
1655 This maintains spatial context without $\mathcal{O}(N^3)$ complexity.

1656 **Singles-Only Spatio-Temporal Attention.** Joint spatio-temporal attention is applied exclusively to
 1657 single-residue features \mathbf{s} . This avoids the $\mathcal{O}(N^2L)$ memory cost of temporal attention over pairs.
 1658 Pair features modulate the attention mechanism solely through bias terms, similar to Invariant Point
 1659 Attention (IPA).

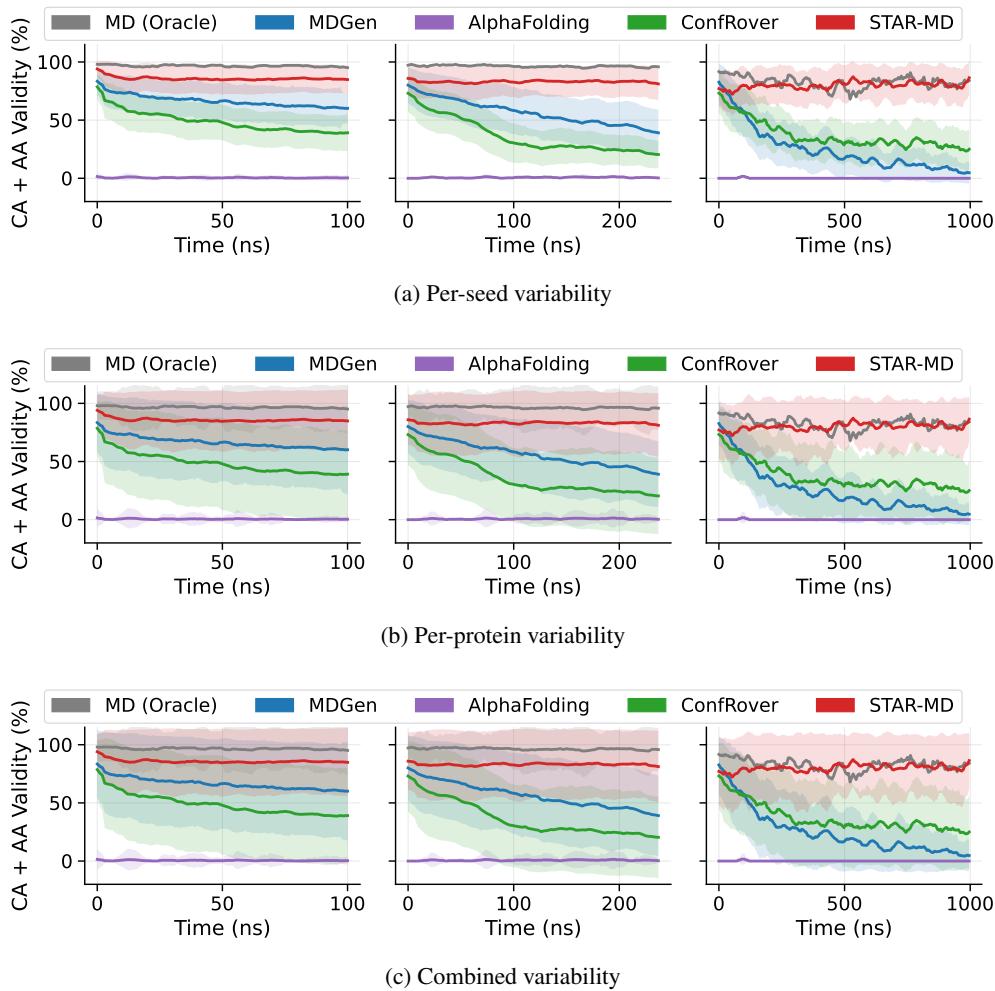
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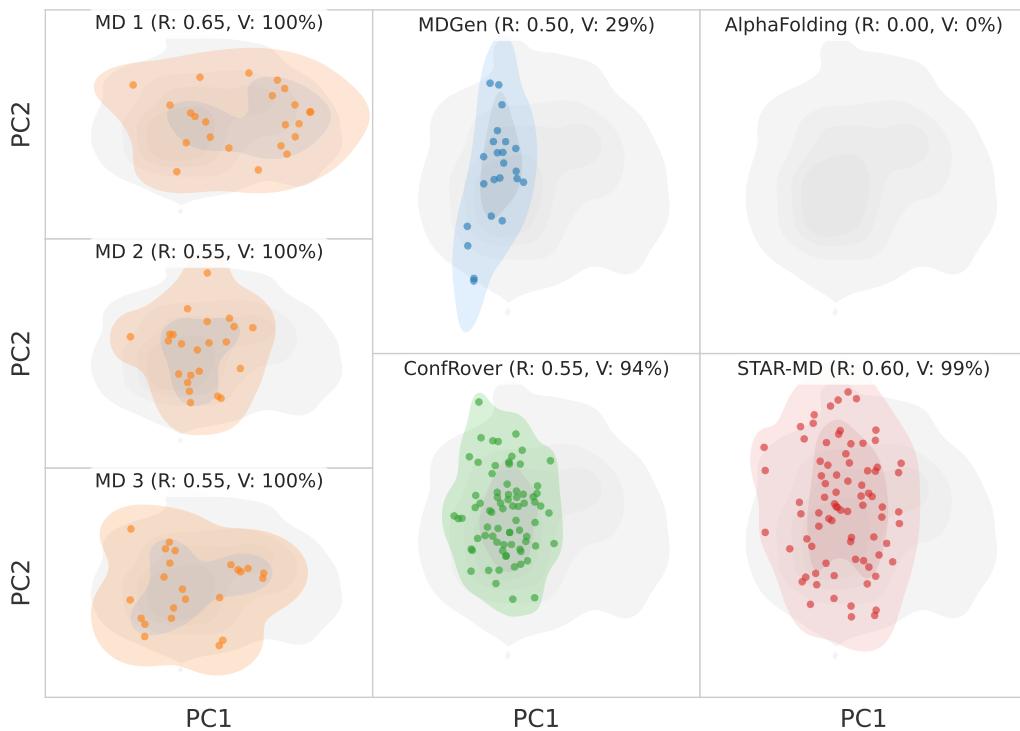
1674 G EXTREME-HORIZON GENERATION (10 μ s)
16751676 To probe the limits of STAR-MD’s stability, we generated trajectories using the maximum training
1677 stride (~ 10 ns) for 1000 steps, resulting in a total simulation time of approximately 10 μ s. Table 14
1678 reports the structural validity metrics averaged over the full 10 μ s trajectory for the ATLAS test set.
1679 Figure 12 visualizes the stability of structural validity over the course of the generation.
16801681 Table 14: **Structural validity for 10 μ s trajectories.** Metrics are averaged over 1000 steps (~ 10 ns
1682 stride) for the ATLAS test set.
1683

Metric	Validity (%)
CA Validity	85.21
AA Validity	90.93
CA + AA Validity	77.28

1707 Figure 12: **Structural validity over 10 μ s.** The average CA + AA Validity remains stable through-
1708 out the 1000-step generation process.
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1728 **H EXTENDED STABILITY ANALYSIS**
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1730 To further characterize the stability of STAR-MD, we provide a detailed breakdown of the variability
1731 in structural validity ($C\alpha$ + All-Atom) for the 100ns, 240 ns, and $1\mu s$ settings. Figure 13 decomposes
1732 the total variability into inter-seed and inter-protein components.
1733


1782 I ADDITIONAL VISUALIZATIONS OF CONFORMATIONAL COVERAGE
17831784 Figures 14 to 23 show conformational coverage of 10 proteins from the test set. These visualiza-
1785 tions follow the same format as Figure 2(b) in the main text, with one key difference: we filter the
1786 generated conformations to show only those that are structurally valid (passing all validity checks).
1787 This provides a more rigorous view of the useful conformational space explored by each model.
17881813 Figure 14: Conformational coverage for $6kty_A$ (Valid conformations only).
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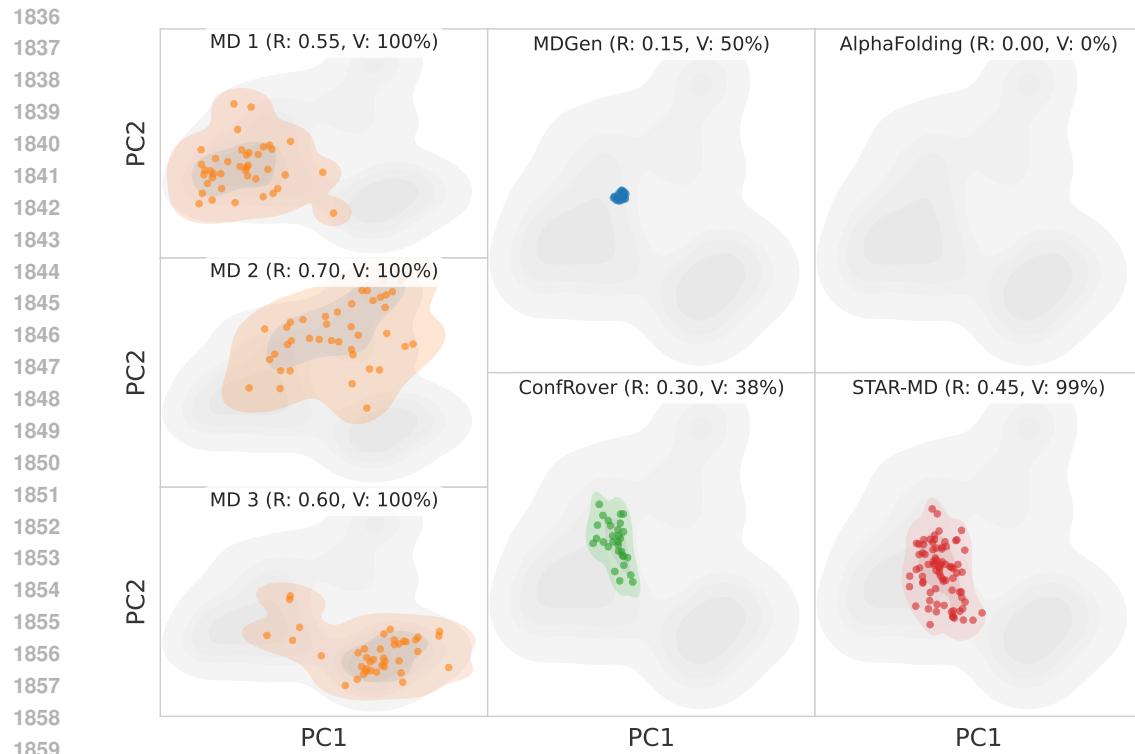


Figure 15: Conformational coverage for 6jwh_A (Valid conformations only).

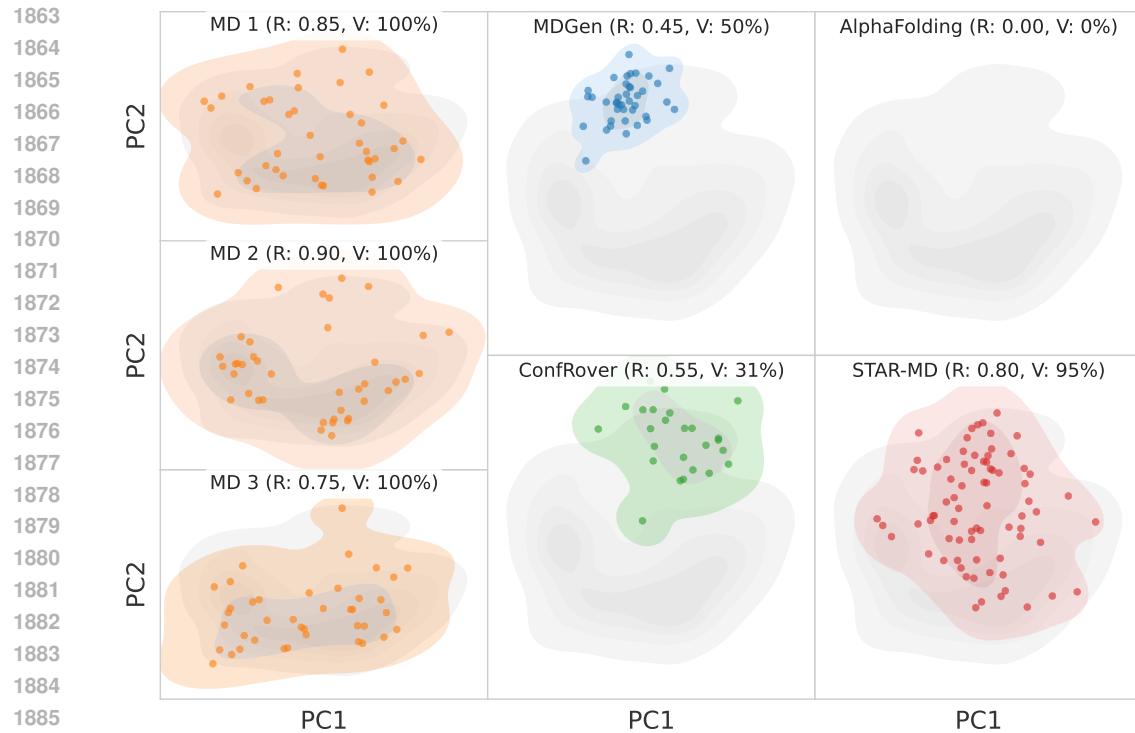


Figure 16: Conformational coverage for 6jpt_A (Valid conformations only).

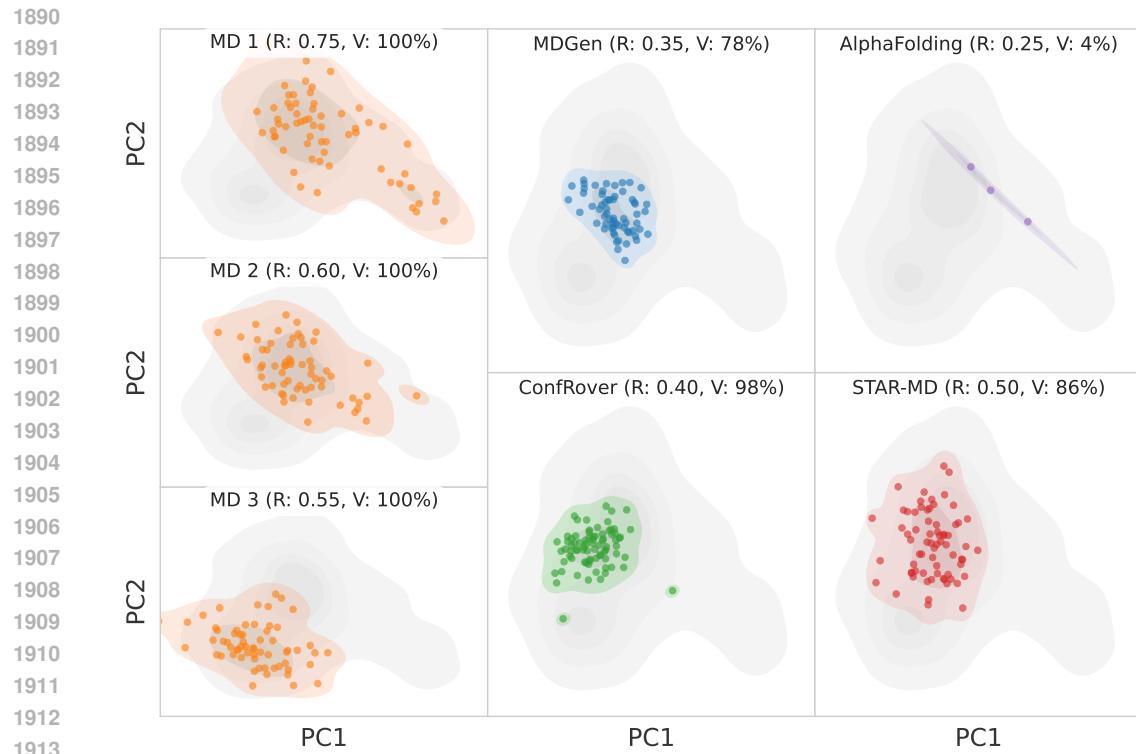


Figure 17: Conformational coverage for 6okd_C (Valid conformations only).

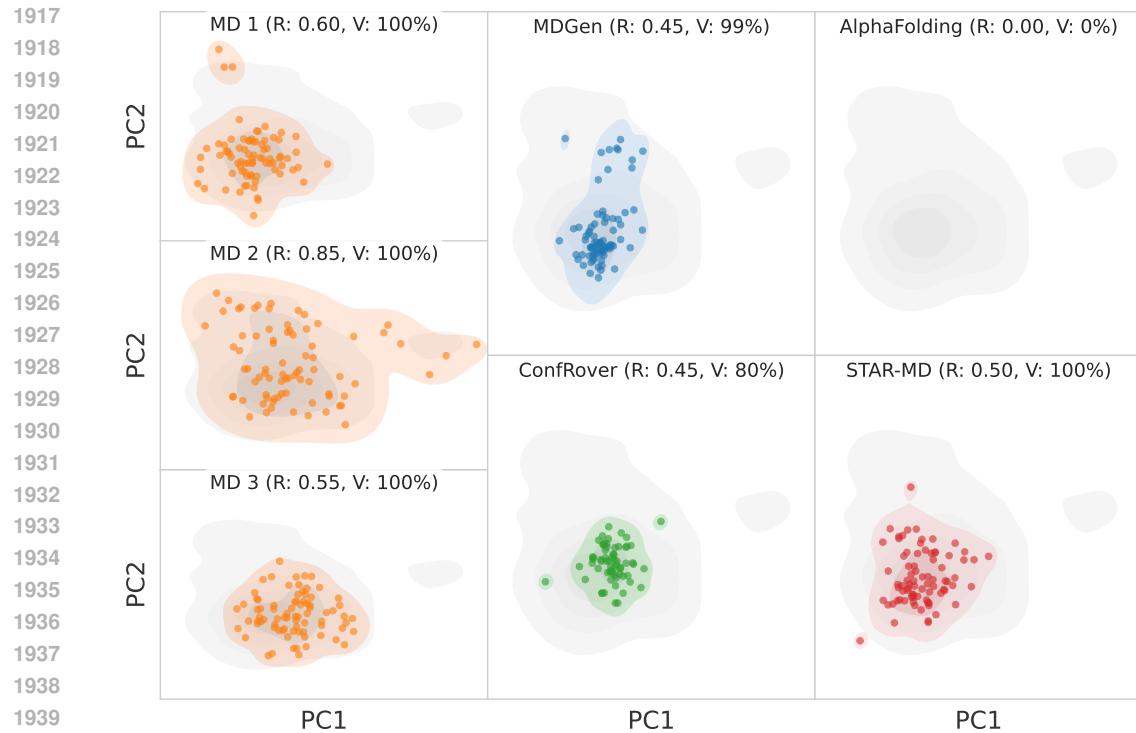


Figure 18: Conformational coverage for 6odd_B (Valid conformations only).

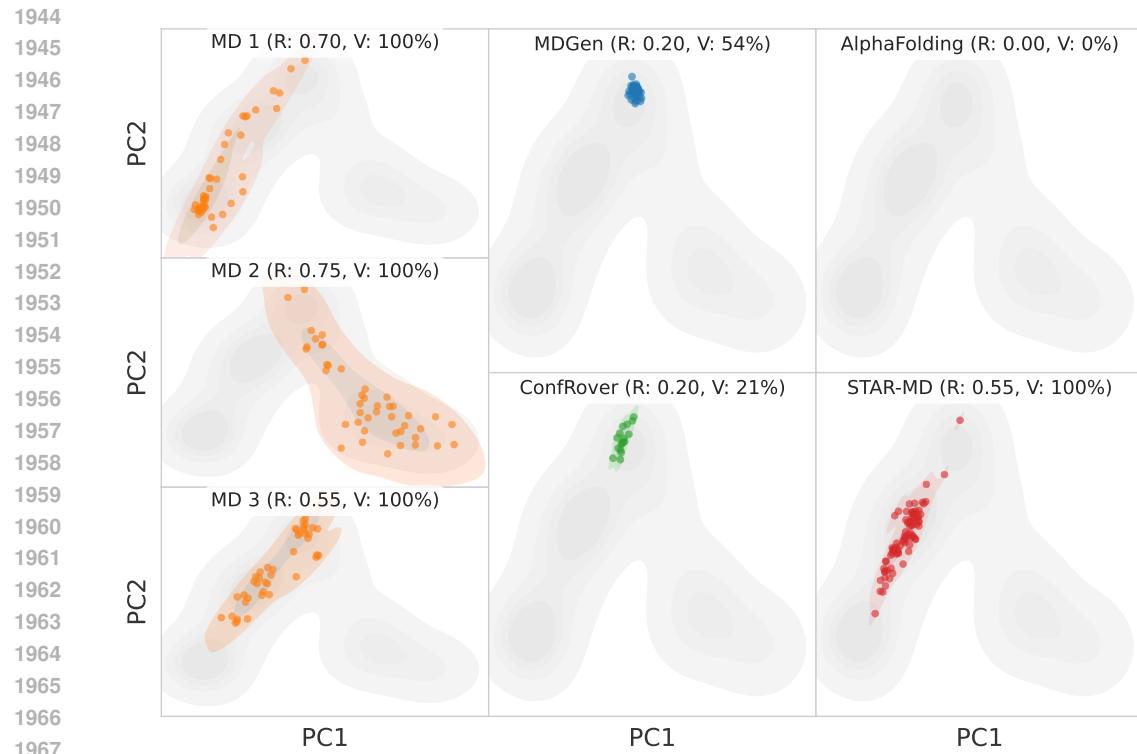


Figure 19: Conformational coverage for 7dmn_A (Valid conformations only).

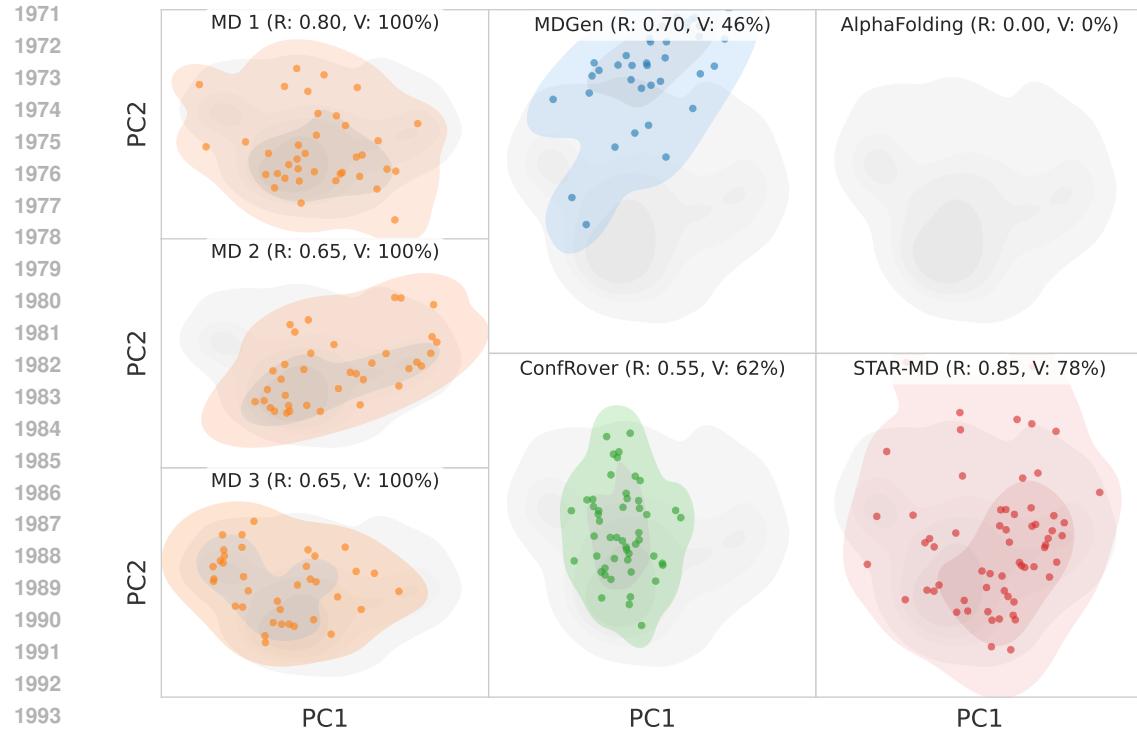


Figure 20: Conformational coverage for 7lp1_A (Valid conformations only).

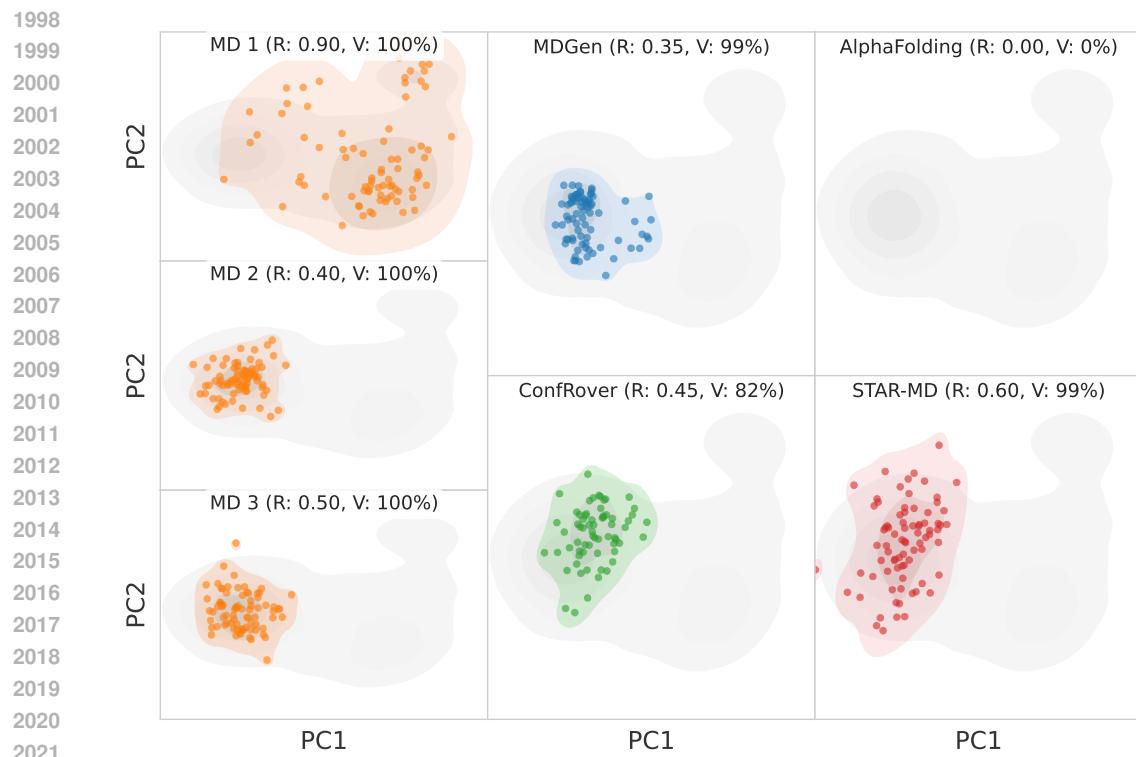


Figure 21: Conformational coverage for 6134_A (Valid conformations only).

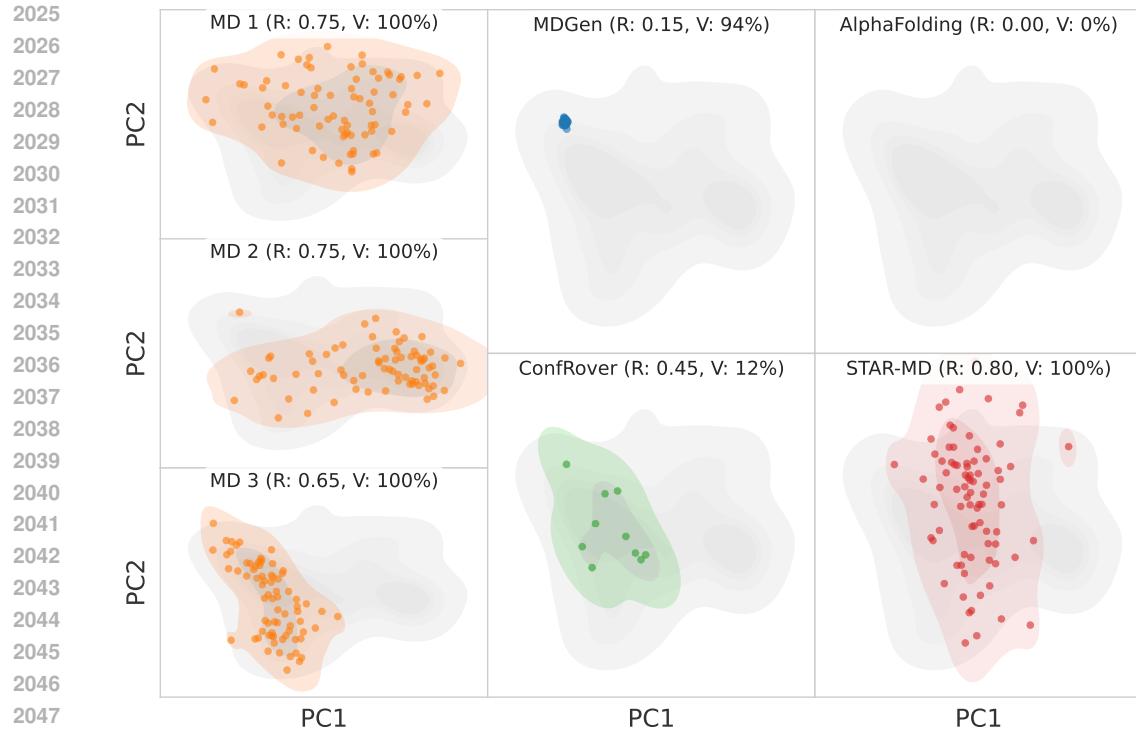


Figure 22: Conformational coverage for 5znj_A (Valid conformations only).

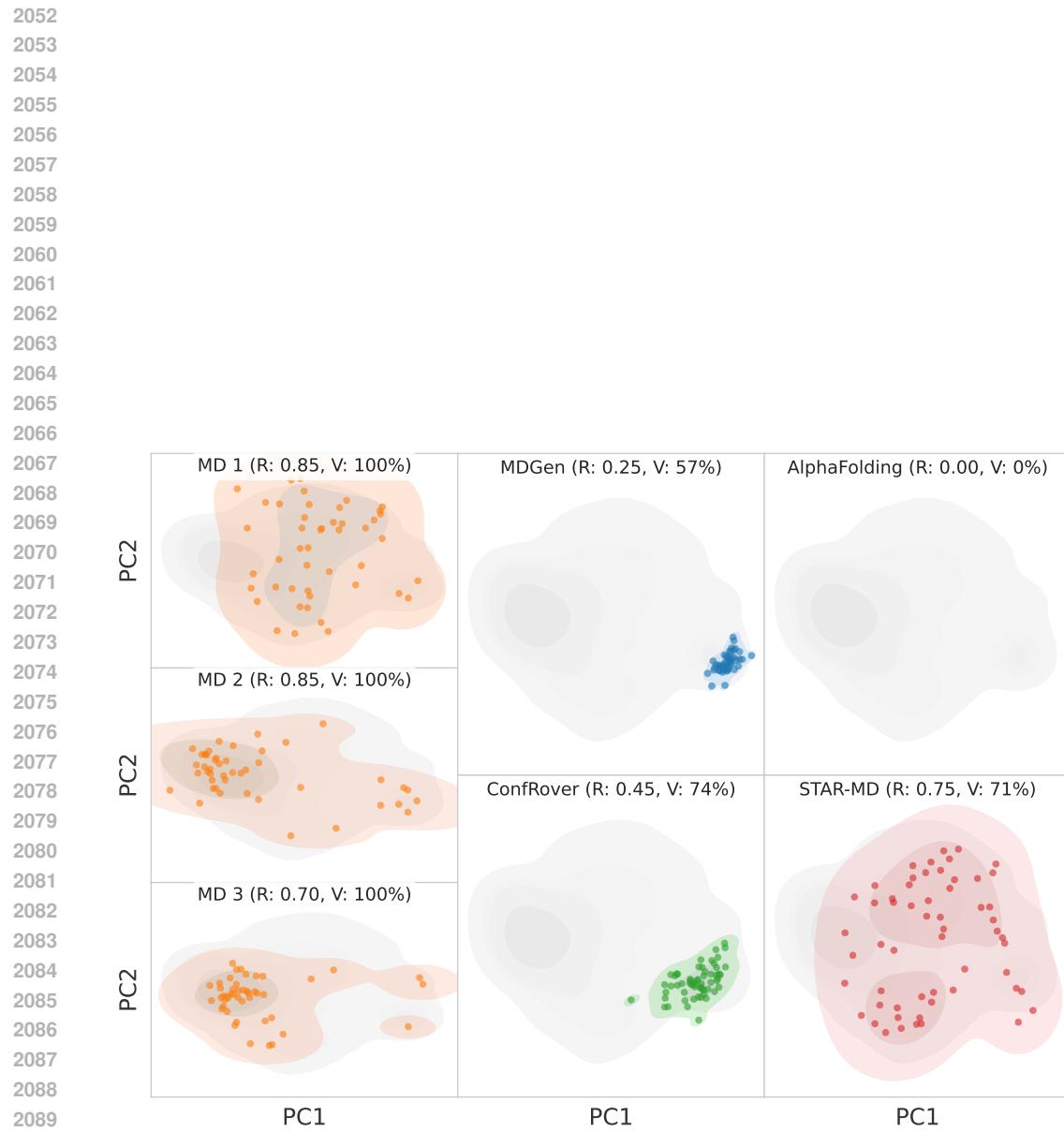


Figure 23: Conformational coverage for 6t1y_A (Valid conformations only).

2106 J ENSEMBLE GENERATION

2108 We examine the performance of STAR-MD to approximate target ensemble distributions from MD
 2109 simulation, and compare it against the state-of-the-art ensemble emulator BioEmu (Lewis et al.,
 2110 2024). Due to several differences in training dataset and paradigms, we evaluate them on two bench-
 2111 marks: (1) the standard ATLAS ensemble benchmark used in prior works (Jing et al., 2024a;b), and
 2112 (2) the CATH1 ensemble benchmark introduced in BioEmu.

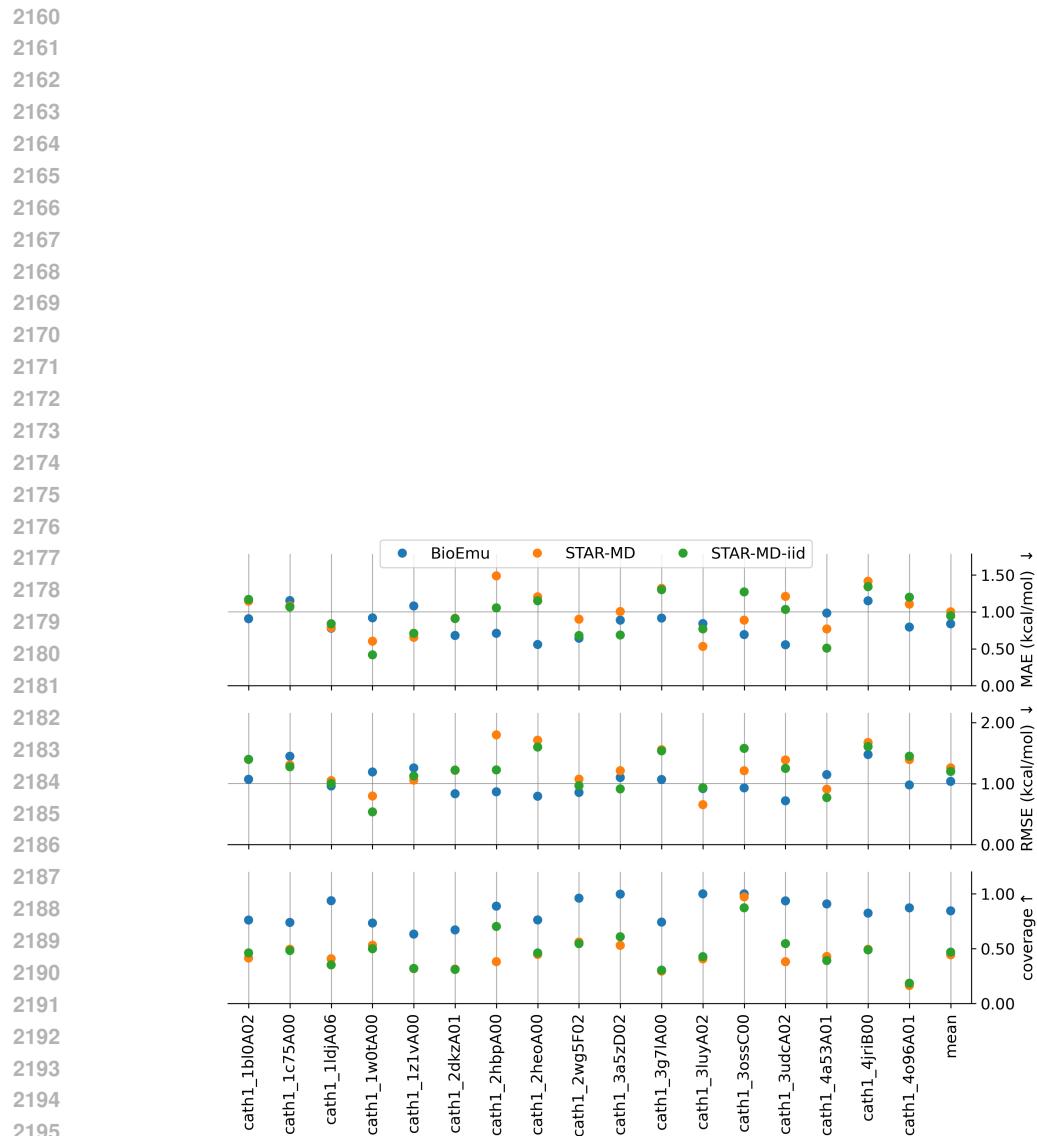
2113 We follow the standard protocol from Jing et al. (2024a), sampling 250 conformations for each of the
 2114 82 test proteins and comparing the generated ensembles with MD reference ensembles. For STAR-
 2115 MD, we repeat the 100 ns simulation procedure five times and randomly sample 250 conformations
 2116 from the collected set. As shown in Table 15, STAR-MD better captures the expected diversity,
 2117 more accurately reflects both global and local flexibility, and matches the reference ensemble at
 2118 distributional and ensemble-observable levels. In contrast, BioEmu tends to produce overly diverse
 2119 samples and shows lower similarity to the reference ensemble across most metrics.

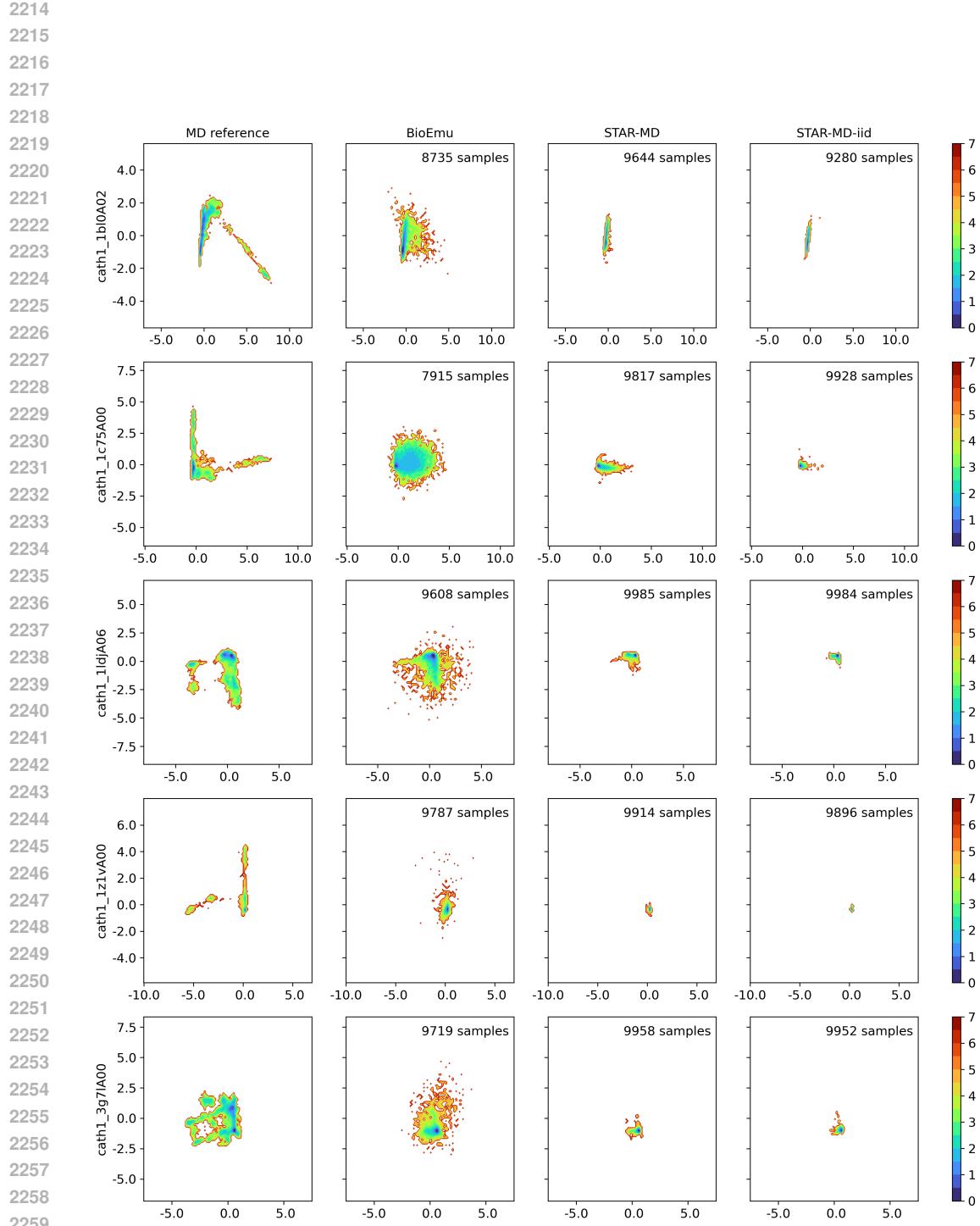
2120 We then evaluate ensemble generation performance on the CATH1 benchmark from BioEmu. This
 2121 benchmark contains the longer trajectories at microsecond level for 17 short proteins (< 75 amino
 2122 acids). We follow BioEmu’s setup by sampling 10,000 conformations per system and projecting
 2123 them onto the reaction coordinates to estimate free-energy and compute metrics using author’s eval-
 2124 uation code. For STAR-MD, we sample from five different starting frames of the initial MD runs
 2125 in the datasets, generating 80 frames at a stride of 1.28 ns. We repeat this process 25 times to col-
 2126 lect 10,000 samples per protein. As shown in Figure 24, BioEmu achieves higher accuracy with
 2127 lower error and higher coverage. When compared to the ground-truth distributions, STAR-MD con-
 2128 centrates around the low energy basins, while BioEmu explores a broader range of conformations,
 2129 occasionally extending beyond the region supported by the ground truth.

2130 To verify this performance difference is not caused by the forward simulation setup used for STAR-
 2131 MD, we also test unconditional generation (STAR-MD-iid) by inferring single frames from masked
 2132 token, similar to (Shen et al., 2025). The similar results between STAR-MD and STAR-MD-iid sug-
 2133 gest that the performance gap between STAR-MD and BioEmu on CATH1 likely arises from factors
 2134 other than the trajectory-generation architecture, such as differences in training datasets, training
 2135 objectives, or fine-tuning procedures.

	STAR-MD	BioEmu
RMSF (=1.63)	1.38	2.87
Pairwise RMSD (=2.76)	2.34	4.51
Pairwise RMSD r (\uparrow)	0.52	0.33
Global RMSF r (\uparrow)	0.56	0.52
Per target RMSF r (\uparrow)	0.86	0.82
RMWD (\downarrow)	2.85	4.33
MD PCA W2 (\downarrow)	1.50	1.64
Joint PCA W2 (\downarrow)	2.33	3.43
PC sim > 0.5 (\uparrow)	35.4 %	26.8 %
Weak contacts J (\uparrow)	0.52	0.49
Transient contacts J (\uparrow)	0.37	0.37
Exposed residue J (\uparrow)	0.57	0.54
Exposed MI matrix rho (\uparrow)	0.24	0.26

2151 Table 15: **ATLAS ensemble results.** Reported metrics include diversity, flexibility accuracy, dis-
 2152 tributional similarity, and ensemble observables. Parentheses indicate whether higher/lower values
 2153 are the better, or the expected value estimated from the ground truth trajectory. The better value is
 2154 shown in bold.





2260 Figure 25: **Example free energy surfaces from the BioEmu CATH1 ensemble benchmark.** Five
 2261 systems are randomly selected for visualization. The left columns shows the reference MD
 2262 distributions provided in BioEmu. For each model and system, the number of valid samples (out of 10,000)
 2263 that passes BioEmu's quality filter is indicated in the upper-right corner of each subplot.
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2268 **K CASE STUDIES ON FUNCTION-RELATED CONFORMATIONAL DYNAMICS**
22692270 **K.1 CRYPTIC-POCKET BINDING**
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2272 Sampling conformational changes during ligand binding is challenging, especially for proteins with
2273 cryptic pockets, where cooperative and complex pocket dynamics are required to enable binding. To
2274 assess whether STAR-MD can sample bound-state (*holo*) conformations from unbound-state (*apo*)
2275 structures in such settings, we selected three cases from BioEmu’s cryptic-pocket dataset: Adeny-
2276 losuccinate Synthetase (AdS), an enzyme has several allosteric or competitive binding substrates;
2277 TEM-1 β -Lactamase (TEM-1), forming complexes with allosteric inhibitors; and Adenylate Kinase
2278 (AdK), a highly flexible protein that undergoes large structural arrangement to each closed *holo*
2279 conformation with covered ‘lid’.

2280 For each of the case, we initialize from the provided *apo* structure and generate 200 frames at a
2281 stride of 2.56 ns, approximating a 500 ns simulation. We compare sampled conformations with
2282 the reference **holo** structure and compute RMSD over the local binding residues involved in the
2283 conformational changes⁴. The best *holo* samples with minimum local RMSD are shown in
2284 Figure 26. Across all three cases, STAR-MD successfully samples *holo*-like conformations with RMSD
2285 < 1.5 Å (Lewis et al., 2024). For AdS and AdK, the sampled conformations closely align with the
2286 target *holo* structures. For TEM-1, we observe remaining differences in the left small helix where the
2287 sampled structure does not fully form the helix. Nevertheless, these results demonstrate promising
2288 capability of STAR-MD in predicting complex, cooperative, and long-range conformational dynam-
2289 ics.

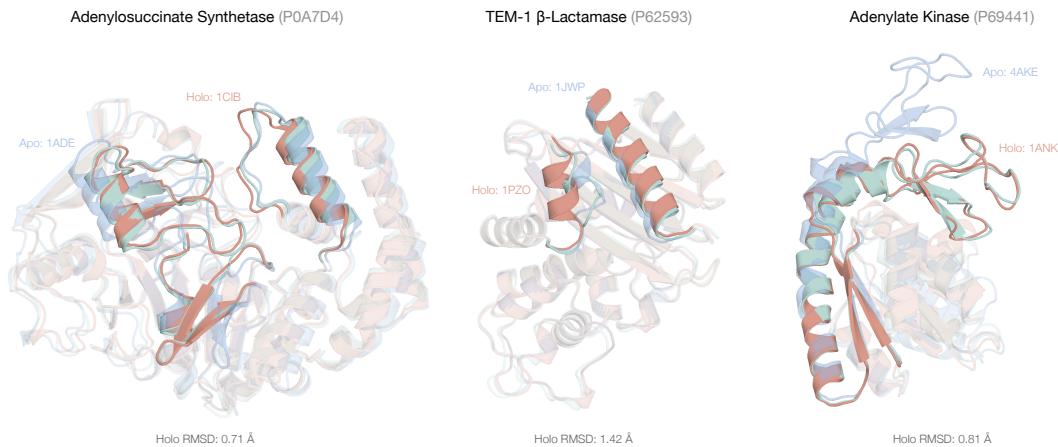


Figure 26: **STAR-MD samples *Holo* structures started from *Apo*.** Structures are superimposed and the local binding regions are highlighted. The Holo RMSD shown under each case is calculated based on the local binding residues.

2309 **K.2 KINASE ACTIVATION**
2310

2311 Abl kinase is a signaling protein related to leukemia and other cancers. It undergoes conformational
2312 transitions between active and inactive states, which regulate its activity. These transitions involve
2313 several functional domains (P-loop, α C-helix, and the Asp-Phe-Gly DFG motifs), imposing complex
2314 millisecond-scale dynamics (Xie et al., 2020). We investigate whether STAR-MD can sample the
2315 active-inactive transition when initialized from one of the two states. Specifically, we generate
2316 trajectories of 500 frames with a stride of 5.12 ns, corresponding to roughly 2.5 μ s of simulation.

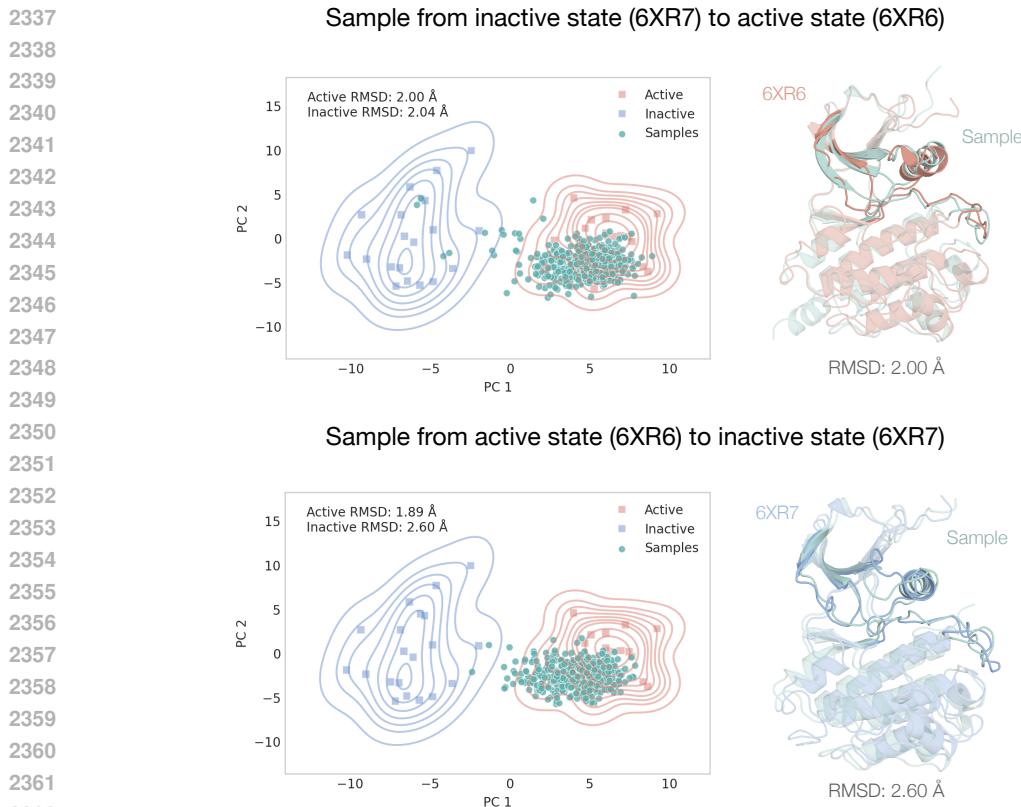
2317 To assess the states of generated conformations, we extract pairwise $C\alpha$ distances and backbone
2318 dihedral angles (ψ, ϕ) for residues around function domains involved in the transition (THR262-
2319 GLU277, VAL287-LEU317, LYS397-GLY417), from 20 active-state structures (PDB:6XR6) and

⁴Residue information: https://github.com/microsoft/bioemu-benchmarks/tree/main/bioemu_benchmarks/assets/multiconf_benchmark_0.1/crypticpocket/local_residinfo

2322 20 inactive-state structures (PDB:6XR7). We then apply principle component analysis to obtain a
 2323 2D low-dimensional projection that separates the active and inactive conformations. We also define
 2324 a core-RMSD metric measured as the $C\alpha$ -RMSD between sample and the reference structure over
 2325 the aforementioned residues of interest.

2326 As shown in Figure 27, when starting from the inactive state, STAR-MD is able to sample towards
 2327 the active state with the best core-RMSD to the active state as 2.00 Å. When sampling from the
 2328 active state, the majority of sampled conformations remain near the active-state region, while a
 2329 subset of samples explores the inactive region. This behavior is in line with the underlying energetic
 2330 profile, where approximately 88% of conformations correspond to the active state and only ~8%
 2331 correspond to the inactive state (Xie et al., 2020). The best RMSD to the inactive reference (2.60 Å)
 2332 is still smaller than the average pairwise RMSD between active and inactive conformations in PDB
 2333 (3.30 ± 0.34 Å).

2334 Overall, this case demonstrates the potential of STAR-MD to sample complex conformational dy-
 2335 namics such as active-inactive transition in kinases.



2363 **Figure 27: Results on ABI Kinase sampled from the inactive state (top) and active state (bot-
 2364 tom). Left: PCA projection of reference conformations from the active and inactive PDB ensembles,
 2365 with STAR-MD samples shown in green. Right: Superposed structures of the best sample and the
 2366 corresponding target state, with core dynamic regions highlighted.**

2376 L 100NS FILTERED RESULTS
23772378 L.1 PERFORMANCE BY SEQUENCE LENGTH
23792380 We break down the performance of STAR-MD and baselines across different protein sequence length
2381 buckets to analyze scalability and robustness. Tables 16 to 20 summarize these findings.
23822383 Table 16: Sequence Length Bucket: 0-100 (17 chains)
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Model	Cov Valid		tICA↑	Validity		
	JSD↓	Rec↑		CA %↑	AA %↑	CA+AA %↑
MD (Oracle)	0.28	0.71	0.10	100.00	93.24	93.24
MDGen	0.48 ± 0.01	0.38 ± 0.01	0.11 ± 0.01	85.66 ± 3.15	87.34 ± 1.76	75.91 ± 2.08
AlphaFolding	0.56	0.28	N/A	30.07	3.60	2.06
ConfRover	0.48 ± 0.02	0.39 ± 0.02	0.11 ± 0.01	79.59 ± 2.00	81.24 ± 0.18	65.26 ± 1.30
STAR-MD	0.37 ± 0.01	0.56 ± 0.02	0.11 ± 0.01	91.37 ± 1.87	93.75 ± 0.47	86.10 ± 1.88

2393 Table 17: Sequence Length Bucket: 100-150 (16 chains)
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Model	Cov Valid		tICA↑	Validity		
	JSD↓	Rec↑		CA %↑	AA %↑	CA+AA %↑
MD (Oracle)	0.30	0.69	0.12	98.98	98.44	97.42
MDGen	0.51 ± 0.02	0.35 ± 0.02	0.10 ± 0.01	74.50 ± 7.40	94.17 ± 1.39	70.62 ± 7.62
AlphaFolding	0.58	0.10	N/A	14.38	0.23	0.16
ConfRover	0.53 ± 0.01	0.35 ± 0.01	0.12 ± 0.01	55.89 ± 0.95	86.38 ± 0.78	49.08 ± 1.18
STAR-MD	0.49 ± 0.01	0.53 ± 0.01	0.12 ± 0.01	79.05 ± 3.41	98.25 ± 0.35	77.64 ± 3.49

2403 Table 18: Sequence Length Bucket: 150-225 (11 chains)
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Model	Cov Valid		tICA↑	Validity		
	JSD↓	Rec↑		CA %↑	AA %↑	CA+AA %↑
MD (Oracle)	0.27	0.66	0.16	90.57	99.66	90.23
MDGen	0.52 ± 0.02	0.32 ± 0.03	0.12 ± 0.01	66.86 ± 5.84	97.18 ± 0.91	65.23 ± 5.65
AlphaFolding	0.72	0.10	N/A	4.55	0.34	0.11
ConfRover	0.47 ± 0.02	0.39 ± 0.01	0.15 ± 0.01	73.39 ± 2.56	98.77 ± 0.34	72.43 ± 2.43
STAR-MD	0.37 ± 0.02	0.53 ± 0.03	0.16 ± 0.01	89.55 ± 1.70	99.82 ± 0.22	89.41 ± 1.81

2414 L.2 GENERALIZATION TO DISSIMILAR PROTEINS
24152416 To strictly evaluate generalization to unseen protein folds, we report results excluding two test pro-
2417 teins (7buy_A and 7e2s_A) that have high sequence similarity (> 80%) to the training set. Table 21
2418 summarizes these findings.
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Table 19: Sequence Length Bucket: 225-400 (15 chains)

Model	Cov Valid		Dyn.	Validity		
	JSD \downarrow	Rec \uparrow	tICA \uparrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
MD (Oracle)	0.33	0.65	0.22	100.00	99.90	99.90
MDGen	0.62 \pm 0.01	0.21 \pm 0.01	0.14 \pm 0.00	61.69 \pm 4.22	97.71 \pm 1.06	60.40 \pm 4.75
AlphaFolding	N/A	N/A	N/A	1.15	0.00	0.00
ConfRover	0.53 \pm 0.00	0.36 \pm 0.01	0.22 \pm 0.02	54.00 \pm 2.82	99.44 \pm 0.08	53.77 \pm 2.95
STAR-MD	0.44 \pm 0.02	0.53 \pm 0.02	0.22 \pm 0.02	82.38 \pm 1.62	99.98 \pm 0.04	82.37 \pm 1.63

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Table 20: Sequence Length Bucket: 400+ (23 chains)

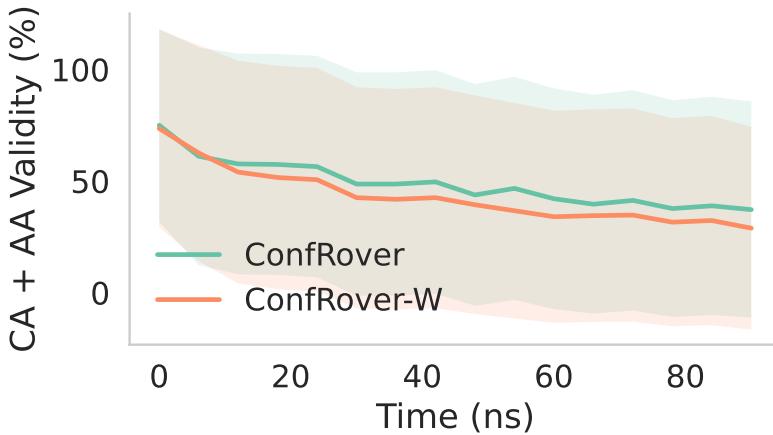
Model	Cov Valid		Dyn.	Validity		
	JSD \downarrow	Rec \uparrow	tICA \uparrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
MD (Oracle)	0.33	0.66	0.24	99.86	100.00	99.86
MDGen	0.65 \pm 0.01	0.16 \pm 0.01	0.14 \pm 0.01	66.75 \pm 3.69	99.81 \pm 0.17	66.67 \pm 3.67
AlphaFolding	N/A	N/A	N/A	0.97	0.00	0.00
ConfRover	0.57 \pm 0.01	0.32 \pm 0.01	0.26 \pm 0.01	28.54 \pm 1.14	99.61 \pm 0.31	28.54 \pm 1.14
STAR-MD	0.45 \pm 0.01	0.54 \pm 0.03	0.24 \pm 0.01	90.92 \pm 1.77	100.00 \pm 0.00	90.92 \pm 1.77

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Table 21: Non-similar Proteins, Excluding 7buy_A, 7e2s_A (80 chains)

Model	Cov Valid		Dyn.	Validity		
	JSD \downarrow	Rec \uparrow	tICA \uparrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
MD (Oracle)	0.31	0.67	0.17	98.37	98.07	96.43
MDGen	0.56 \pm 0.01	0.28 \pm 0.01	0.12 \pm 0.00	71.83 \pm 1.90	95.03 \pm 0.59	68.31 \pm 2.20
AlphaFolding ¹	0.58	0.22	N/A	10.98	0.92	0.52
ConfRover	0.52 \pm 0.01	0.36 \pm 0.01	0.15 \pm 0.01	56.94 \pm 0.52	92.47 \pm 0.25	52.06 \pm 0.36
STAR-MD	0.43 \pm 0.01	0.54 \pm 0.01	0.17 \pm 0.00	86.81 \pm 0.64	98.18 \pm 0.05	85.29 \pm 0.62

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2484 M CONFROVER-W: WINDOWED ATTENTION BASELINE
24852486 To enable long-horizon generation (240 ns and 1 μ s) within GPU memory limits, we implemented
2487 **ConfRover-W**, a variant using windowed attention (window size 14) with attention sinks (first 2
2488 frames) (Xiao et al., 2023a). This reduces temporal complexity from $\mathcal{O}(L^2)$ to $\mathcal{O}(L \times W)$.
24892490 Figure 28 and Table 22 validate this approach on the 100ns benchmark, showing that ConfRover-W
2491 maintains structural validity comparable to the full-attention model, confirming it as a fair baseline
2492 for longer horizons.
24932509 Figure 28: **Validation of ConfRover-W at 100ns.** Comparison of Structural Validity ($C\alpha$ + All-
2510 Atom) between the standard full-attention ConfRover and the windowed ConfRover-W. The win-
2511 dowed variant maintains comparable performance, validating its use as a proxy for long-horizon
2512 experiments.
25132514 Table 22: **Comparison of ConfRover and ConfRover-W at 100ns.** Quantitative metrics show that
2515 the windowed approximation (ConfRover-W) achieves performance comparable to the full-attention
2516 model (ConfRover), justifying its use for longer horizons.
2517

Model	Cov Valid		Dyn.		Validity	
	JSD \downarrow	Rec \uparrow	tICA \uparrow	CA % \uparrow	AA % \uparrow	CA+AA % \uparrow
ConfRover	0.52	0.36	0.15	56.43	92.37	52.02
ConfRover-W	0.51	0.37	0.15	50.08	89.05	45.70
STAR-MD	0.42	0.57	0.17	86.83	98.23	85.35

2538 N LLM USAGE DISCLOSURE
25392540 We utilized Large Language Models (LLMs) as a writing assistant during the preparation of this
2541 manuscript. For a few sections, our process involved authoring detailed outlines and initial drafts
2542 to establish the core scientific arguments and structure. Subsequently, we used LLMs to refine the
2543 verbiage, improve grammar, and enhance the overall clarity of the text in those specific sections. All
2544 scientific claims, experimental results, and theoretical assertions originated from the human authors,
2545 who take full responsibility for the final content of the paper, in accordance with ICLR 2026 policy.
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