

YOU CAN LEARN TOKENIZATION END-TO-END WITH REINFORCEMENT LEARNING

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ABSTRACT

Tokenization is a hardcoded compression step which remains in the training pipeline of Large Language Models (LLMs), despite a general trend towards architectures becoming increasingly end-to-end. Prior work has shown promising results at scale in bringing this compression step inside the LLMs’ architecture with heuristics to draw token boundaries, and also attempts to learn these token boundaries with straight-through estimates, which treat the problem of drawing discrete token boundaries as a continuous one. We show that these token boundaries can instead be learned using score function estimates, which have tighter theoretical guarantees due to directly optimizing the problem of drawing discrete token boundaries to minimize loss. We observe that techniques from reinforcement learning, such as time discounting, are necessary to reduce the variance of this score function sufficiently to make it practicable. We demonstrate that the resultant method outperforms prior proposed straight-through estimates, both qualitatively and quantitatively at the 100 million parameter scale.

1 INTRODUCTION

Tokenization is a crucial pre-processing step in the training and inference pipelines of modern LLMs. Standard practice compresses text into symbols representing commonly occurring substrings. This is typically done using algorithms such as Byte-Pair Encoding (BPE), which recursively groups frequently co-occurring byte sequences into individual tokens. State-of-the-art open-source models further augment BPE with numerous hand-crafted decisions. For example, the Gemma-series tokenizers (Rivière et al., 2024) explicitly split digits and preserve whitespace, reflecting the extent to which tokenizer design remains largely artisanal.

A growing line of work seeks to eliminate this BPE step entirely by operating directly on UTF-8 bytes (Xue et al., 2022; Wang et al., 2024; Zheng et al., 2025). This can be viewed as using a tokenizer with a maximally small vocabulary and an effective downsampling rate of 1 token per byte. Recent scaling analyses suggest that downstream loss is optimized by increasing vocabulary size, and thus downsampling rate, with model scale (Tao et al., 2024).

We thus focus our investigation onto methods which admit a growing downsampling rate. Interestingly, BPE itself experiences harsh diminishing returns in downsampling rate as vocabulary size scales. Further, a large vocabulary size can have undesirable downstream effects, such as the emergence of very rare “glitch tokens” (Rumbelow & Watkins, 2023). Marginally increasing the achieved downsampling rate to vocabulary size tradeoff with curricula (Liu et al., 2025), has thus proved fruitful in improving LLM performance.

An alternative family of approaches processes text at the byte level for several transformer layers before downsampling into a shorter sequence of latent tokens (Nawrot et al., 2022; Yu et al., 2023). Some of these methods facilitate a simple modification of the downsampling rate as a hyperparameter. When token boundaries are chosen heuristically—e.g., using whitespace (Slagle, 2024) or spikes in next-byte entropy (Nawrot et al., 2023)—these models have been reported to achieve superior performance to pure BPE transformer models at large scales (Pagnoni et al., 2024).

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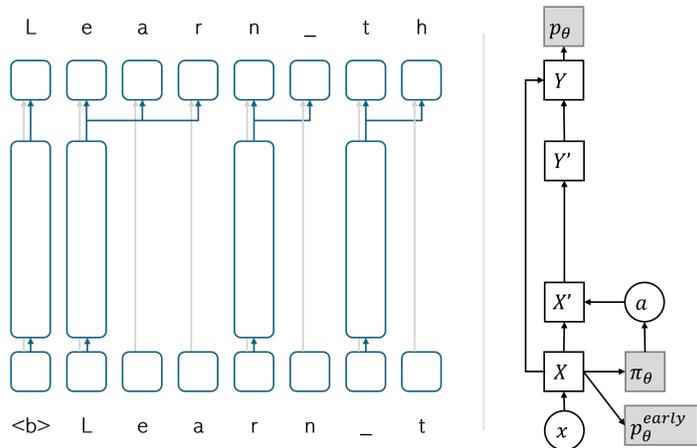


Figure 1: [Left] An example of the autoregressive U-net architecture (Nawrot et al., 2022), computed with the values $x_{0:7} = \langle b \rangle \text{Learn } t$ and $a_{0:7} = 11001010$. Blocks with rounded edges represent transformer blocks, arrows represent flow of representations. [Right] the stochastic computation graph (Schulman et al., 2015) of a forward pass of our method, deterministic nodes in squares, stochastic nodes in circles, distributions in gray.

However, such heuristics raise a natural question: can we improve on these boundary rules by learning tokenization itself from end-to-end training? Prior approaches to this question have focused on straight-through estimators (Nawrot et al., 2023), which craft rules for backpropagating gradients from representations internal to the model. We instead investigate score function estimators, which directly approximate the gradient of the expected loss with respect to the token boundaries. Score function estimators have stronger theoretical guarantees, at the cost of a higher variance.

Our main contributions are as follows:

- We show that we can learn tokenization strategies that align closely with semantic boundaries without any explicit prior structure or inductive bias with a score-function estimator, equipped with variance-reduction techniques from reinforcement learning.
- We further find that our method qualitatively and quantitatively outperforms prior approaches using straight-through estimators.
- We demonstrate robust performance across a range of downsampling rates.

2 THEORY & METHOD

In this section, we motivate our method from first principles. We first present some desiderata for an end-to-end tokenisation method §2.1, then show that these desiderata necessarily lead to an autoregressive U-net style architecture §2.2. We then present score functions as the canonical way to learn tokenization under this framework §2.3 and present our method for making them practicable §2.4 - 2.7

2.1 DESIDERATA FOR DESIGNING A END-TO-END TOKENIZATION METHOD

For the reasons outlined in §1, we are interested in bringing the tokenization process inside the architecture and training of an LLM. We propose the following desiderata for such a method to be practicable and general:

- **End-to-end tokenizer training:** the token boundary decisions should be learned to minimise loss, in favor of hand-crafted methods and heuristics.

- **End-to-end architecture:** learned representations at the byte level should be re-used at the token level.
- **Efficiency:** use less than 0.1% additional pretraining compute, on existing hardware, than byte-pair-encoding guided tokenization in the forward & backward passes.

2.2 AUTOREGRESSIVE U-NET ARCHITECTURE AND SETUP

In this section, we define notation for the autoregressive U-net architecture (Nawrot et al., 2022), depicted in Figure 1 as the canonical architecture which satisfies the **Efficiency** and **End-to-end architecture** desiderata.

For autoregressive models, tokenization is a fundamentally *discrete* question as it dictates how much compute is spent on a given (sub)sequence at inference time. Furthermore, modern GPUs cannot efficiently perform sparse memory accesses (Yuan et al., 2025), meaning that tokenization for feed-forward models needs to be *consistent across layers*.

Thus, bringing tokenization inside an autoregressive feedforward architecture necessarily have some amount of layers applied at the byte-level with some amount of layers applied at the token-level, in contrast to BPE tokenizer models which operate solely on the token-level. The result is a forward pass structured as follows. Let $x_1 \dots x_N$ be a sequence of input bytes and d_{enc} , d_{mid} , d_{dec} be hyperparameters for the model dimensions.

1. Autoregressively encode the input bytes into byte-level representations $X \in \mathbb{R}^{N \times d_{enc}}$:

$$X = \text{encode}(x) \tag{1}$$

2. (Potentially stochastically) predict token boundaries from the byte-level representations, where $a_i = 1$ if we wish to draw a token boundary at x_i and $a_i = 0$ if not.

$$a \sim \pi(X, x) \tag{2}$$

3. Downsample the byte-level representations into token-level representations $X' \in \mathbb{R}^{M \times d_{mid}}$ where $M = \sum_{i=0}^N a_i$:

$$X' = \text{downsample}(X, a) \tag{3}$$

4. Enrich token-level representations $Y' \in \mathbb{R}^{M \times d_{mid}}$ with an autoregressive feedforward network:

$$Y' = \text{mid}(Y) \tag{4}$$

5. Upsample into updated byte-level information $Y \in \mathbb{R}^{N \times d_{dec}}$, potentially carrying encoded byte-level information:

$$Y = \text{upsample}(Y', X, a) \tag{5}$$

6. Decode the resulting byte-level representations into predictions of the next bytes $y_i = x_{i+1}$

$$y \sim \text{decode}(Y) \tag{6}$$

All the above operations must be autoregressive, for example the `decode` function must be formulated such that X'_j , the downsampled representation at position j , depends only on the preceding bytes $X_{\leq i}$, where i is the minimum token index such that $j = \sum_{k=0}^i a_k$.

The score function estimate we detail in the following sections can be flexibly applied to any implementation of the above functions, in contrast to straight-through estimate based approaches which require specialized up/downsamplers. We make the following standard choices for our experiments in §4: `mid` is a decoder-only transformer with full attention, and `encode`, `decode` are decoder-only transformers with sliding window attention and linear embedding/unembedding matrices respectively. Our implementation of `downsample` simply selects the values of X' which correspond to $a_i = 1$ values:

$$\text{downsample}(X, a)_i = X'_j \quad \text{for } j \text{ the minimum value such that } j = \sum_{k=0}^i a_k \quad (7)$$

For upsampling, we employ a simple distribute-then-add:

$$\text{upsample}(Y', X, a)_j = X_j + Y_i \quad \text{for } i = \sum_{k=0}^j a_k \quad (8)$$

2.3 SCORE FUNCTION ESTIMATION FOR TOKENIZATION

As tokenization is discrete, to satisfy the **End-to-end tokenizer training** desideratum, our model must explore strategies for drawing token boundaries stochastically.

The model next-token cross-entropy loss outputted is thus conditional on a sampled tokenization strategy $a \sim \pi_\theta$, so we can formulate the problem of simultaneously learning the (potentially shared) parameters of the autoregressive model p_θ and the gating strategy π_θ as minimizing the next-token cross-entropy marginalized over all a :

$$\log p_\theta(y|x) = \mathbb{E}_{a \sim \pi_\theta} \log p_\theta(y|a, x). \quad (9)$$

This is a case of a stochastic computation graph as in Schulman et al. (2015), who show that the gradient of this likelihood can be computed as the expectation of the sum of two separate gradients, one being the standard next-token cross-entropy loss conditioned on the tokenization strategy and a correction term which can be interpreted as applying REINFORCE (Williams, 1992) to the tokenization strategy π_θ with the next-token cross-entropy as the reward:

$$\nabla_\theta \mathbb{E}_{a \sim \pi_\theta} \log p_\theta(y|a, x) = \mathbb{E}_{a \sim \pi_\theta} \underbrace{(\nabla_\theta \log p_\theta(y|a, x))}_{\text{conditional loss gradient}} + \underbrace{\log p_\theta(y|a, x) \nabla_\theta \log \pi_\theta(a|x)}_{\text{policy gradient}}. \quad (10)$$

See Appendix A.1 for a complete derivation. This type of estimator is known as a score function estimate. In particular this means that, in the large compute and data limit, performing gradient descent on the policy gradient term above will yield a locally optimal tokenization strategy. No such theoretical guarantee exists for the straight-through estimates we discuss in §3.

2.4 REDUCING THE VARIANCE OF THE SCORE FUNCTION ESTIMATE

In practice, we wish to use a Monte-Carlo estimate of the policy gradient term in equation 10 with a single sample of a per sequence: using more would break our **Efficiency** desideratum. We empirically find that the naïve REINFORCE policy gradient is too noisy to efficiently learn in this setting. In this section, we show how standard techniques from reinforcement learning can be used to denoise this estimate, solving the corresponding “reward attribution problem”: associating which token boundary decisions are responsible for increasing or decreasing the loss in the succeeding bytes.

For this section, we will let d_{model} be the model dimension, `vocab_size` be the number of unique `utf-8` bytes and special characters forming our vocabulary. We will also use the token index of i and the batch index of b , which will be omitted where unused.

Early exit relative rewards As our model is autoregressive, we restrict our analysis to the case where the token boundary decision at byte a_i may only depend on the preceding bytes and token boundary decisions, $x_{<i}$ and $a_{<i}$. Thus, the policy gradient term in eq. (10) treats the token boundary policy $\pi_\theta(a_i|x_{<i}, a_{<i})$ as having corresponding rewards $\log p(x_j|a_{<j}, x_{<j})$ for $j > i$.

As $\mathbb{E}_{a \sim \pi_\theta} \nabla_\theta \log \pi_\theta(a_i|x_{<i}, a_{<i}) = \mathbf{0}$, we may add any term independent of a_i to these rewards and get a valid policy gradient estimate. The feed-forward nature of the autoregressive U-net architecture

presents a natural method to estimate the baseline, tokenization-independent difficulty of predicting the next token by using the early byte-level embeddings to estimate the next-token probability:

$$\log p_{\theta}^{\text{early}}(x_i = t_j | x_{<i}) = \log \text{softmax}(W_{\text{early}} X_{i-1})_j. \quad (11)$$

Where $W_{\text{early}} \in \mathbb{R}^{d_{\text{model}} \times \text{vocab.size}}$ is an unembedding matrix. This gives a reward with a large part of the tokenization-independent noise subtracted out:

$$R_i = \log p_{\theta}(x_i | x_{<i}, a_{<i}) - \log p_{\theta}^{\text{early}}(x_i | x_{<i}). \quad (12)$$

We initialize the weights of W_{early} to match the final output head to facilitate easy transfer.

Time discounting Summing the above rewards to produce advantages still suffers from too high a variance and due to the reward attribution problem. A common solution to this problem for long-horizon RL is to apply time discounting to the rewards when computing advantages: introducing a small amount of bias into the policy gradient to massively reduce the variance. Intuitively, this decouples the advantages given to far away parts of the sequence, giving us many approximately independent training stimuli for the token boundaries per sequence.

$$G_i = \sum_{j=0}^{N-i-1} \gamma^j R_{i+j+1} \quad (13)$$

In our experiments, we use a discount factor of $\gamma = 0.99$.

Batch-relative advantages Our G_i values tend to be positive as the final-layer model p_{θ} tends to outperform the early-exit model $p_{\theta}^{\text{early}}$. This bias depends on the token index, with the gap being larger for later token indices. To remedy this, we leverage that during batched training we in fact have a batch of B such values $G_{i,1} \dots G_{i,B}$ for a given forward/backward pass, for B separate sequences. These can be used to center the advantage estimates:

$$A_{i,b} = G_{i,b} - \bar{G}_i \quad \text{where} \quad \bar{G}_i = \frac{1}{B} \sum_{b=1}^B G_{i,b}. \quad (14)$$

This gives the final policy loss, whose gradient approximates the right hand term of equation (10):

$$\mathcal{L}^{\pi} = - \sum_{i=0}^N \log \pi_{\theta}(a_i | x_{\leq i}, a_{<i}) \cdot \text{detach}(A_i). \quad (15)$$

Where we use `detach` to emphasize that we do *not* allow gradients propagate through A_i directly.

2.5 DEFINING THE TOKEN BOUNDARY FUNCTION

We define a token boundary policy π_{θ} as a function which gives the probability that our model draws a token boundary at a given index, conditioned on the preceding bytes and token boundaries. We parameterize this probability as the sigmoid of the corresponding logit l_i .

$$a_i | x_{\leq i}, a_{<i} \sim \text{Bernoulli}(p_i), \quad p_i = \pi_{\theta}(a_i = 1 | x_{\leq i}, a_{<i}) = \sigma(l_i). \quad (16)$$

We design the computation of l_i to be of negligible computational cost relative to the total forward pass and sufficiently expressive that our model could learn the token boundary heuristics that have been explored by prior work. Even without explicit training, models already encode rich information about the sequence in their internal representations: for example entropy (Nawrot et al., 2022; Pagnoni et al., 2024) has been shown to be mediated by directions in the internal representations of

models solely trained on next-token prediction (Stolfo et al., 2024). Nonetheless, to express fixed striding strategies, as in MEGABYTE (Yu et al., 2023), we need to give the model access to some sliding window of preceding token boundaries.

Concretely, we compute the raw logit l_i^{raw} by applying a set of linear projections $W_j \in \mathbb{R}^{1 \times d_{model}}$ to the byte-level representation X_i to get a base value for the logit and a series of terms conditional on each token boundary in the window. This operation is amenable to fast computation on modern hardware by pre-computing $W_k X_i$ for all i, k in a single matrix multiply and then performing a fast scan operation.

$$l_i^{raw} = W_0 X_i + \sum_{j=1}^w a_{i-j} W_j X_i. \quad (17)$$

In our experiments, we set a window size of $w = 8$.

With no constraint on the downsample rate, the model will elect to use the computationally expensive strategy of separating every byte with a token boundary. To avoid this, we need to push our model towards a target downsample rate $\bar{\pi}_{target}$, which we discuss further in §2.6. As in attention, we need to scale the raw logits to be approximately uniform at initialization, we further aid stability by adding a $\sigma^{-1}(\bar{\pi}_{target})$ term so that $p_i \approx \bar{\pi}_{target}$ at initialization.

$$l_i^{scaled} = \frac{l_i^{raw}}{D} + \sigma^{-1}(\bar{\pi}_{target}) \quad (18)$$

We choose a scaling factor of $D = 16$ and a target downsample rate of $\bar{\pi}_{target} = \frac{1}{5}$. To avoid numerical issues caused by exploding logits, we finally apply softcapping (Rivière et al., 2024).

$$l_i = \text{softcap}(l_i^{scaled}) \quad (19)$$

During evaluation, we skip this softcapping step, and simply set $l_i = l_i^{scaled}$

2.6 DOWNSAMPLE RATE TARGETING

We use a mechanism to keep the downsample rate close to $\bar{\pi}_{target}$ by applying an even pressure across all the logits $l_{i,b}$ in the batch. We prefer this to operating on the token boundary probabilities, which we find can have unstable results due to the uneven gradient magnitude of the sigmoid function. Specifically, we apply a negative or positive pressure to the whole batch mean logit $\bar{l} = \frac{1}{NB} \sum_{i,b} l_{i,b}$ if the mean token boundary probability $\bar{p} = \frac{1}{NB} \sum_{i,b} p_{i,b}$ exceeds or falls short of the target downsample rate respectively. We operationalize this with the loss:

$$\mathcal{L}^{target} = \bar{l} \cdot \text{detach}(\bar{p} - \bar{\pi}_{target}) \quad (20)$$

2.7 FULL LOSS FORMULA

We define the autoregressive losses, to learn the full model and the early exit model as:

$$\mathcal{L}^{auto} = - \sum_{i=0}^N \log p_{\theta}(x_i | x_{<i}, a_{<i}), \quad \mathcal{L}^{early} = - \sum_{i=0}^N \log p_{\theta}^{early}(x_i | x_{<i}). \quad (21)$$

This gives our total loss calculation:

$$\mathcal{L} = \mathcal{L}^{auto} + \lambda_{\pi} \mathcal{L}^{\pi} + \lambda_{target} \mathcal{L}^{target} + \lambda_{early} \mathcal{L}^{early} \quad (22)$$

In our experiments, we set $\lambda_{\pi} = \lambda_{target} = 10^{-2}$ to encourage exploration and $\lambda_{early} = 10^{-1}$.

Table 1: Performance of 147M parameter models on downstream natural language understanding tasks. Performance on `FineWeb Test` is in bits-per-byte, all others are in zero-shot accuracy.

	PIQA	HellaSwag	ARC-Easy	LAMBADA	FineWeb Test
Uniform	0.559	0.266	0.300	0.036	1.376
Dynamic (Nawrot et al.)	0.534	0.268	0.288	0.029	1.372
H-Net (Hwang et al.)	0.576	0.267	0.307	0.020	1.386
Ours	0.565	0.271	0.308	0.086	1.297

4 EXPERIMENTS

To evaluate our proposed score-function-based approach, we train autoregressive transformers on a filtered subset of the `FineWeb` dataset (Penedo et al., 2024), where we keep sequences of at least 4096 bytes. Subsequently, we truncate these sequences to exactly 4096 bytes. All models have approximately 147 million parameters and are trained with a target downsample rate of $\frac{1}{5}$. We select dataset size, batch size, and learning rate using the scaling laws derived by Porian et al. (2024). See Appendix D for further experimental details and hyperparameters.

We compare four dynamic tokenization strategies: a uniform baseline that selects exactly $4096\bar{\pi}_{target} = 819$, the straight-through estimators of Nawrot et al. (2023) Hwang et al. (2025) and our score function estimator.

Because our models are trained with several orders of magnitude less compute than contemporary frontier LLMs, the global downstream effects of learned tokenization state-of-the-art language modeling quality are difficult to assess directly. For example, the recently reported superiority autoregressive U-Nets over purely token-based architectures (Pagnoni et al., 2024; Hwang et al., 2025) was only observed to emerge at the $> 10^{21}$ -Flop scale. For these reasons, our analysis focuses on qualitative comparison to other dynamic tokenization strategies.

4.1 LEARNED TOKENIZATION STRATEGIES FOR NATURAL LANGUAGE

We train 147-million parameter models on our filtered version of `FineWeb`. Figure 2 illustrates the token boundaries produced by our method on held-out `FineWeb` samples. Remarkably, despite having no inductive bias toward linguistic structure, the model reliably learns to place boundaries at whitespace-like characters, such as “\n” and “ ”. Additional samples and tokenization strategies learned by straight-through estimates are provided in Appendices B.1 & B.2, B.3.

In appendix B.6, we further study the sensitivity of the learned tokenization strategy to the target downsample rate by varying the *tokenization aspect ratio*: we train models with $n = 2, 4, 6$ token-level transformer layers (sizes ranging from 20 to 40-million parameters) and a target downsample rate of $\bar{\pi}_{target} = \frac{1}{n}$, keeping the FLOPs-per-sequence fixed. The $n = 2$ model converges to a tokenization strategy of allocating almost exactly two bytes per token, with the exception of also drawing token boundaries at periods. By contrast, the $n = 4, 6$ models again discover whitespace-aligned boundaries and exhibit with varying degrees of chunking of longer words.

4.2 NATURAL LANGUAGE PERFORMANCE

In Appendix D we give further details on our experimental setup and FLOPs calculations. In Figure 10, we plot validation loss curves on our filtered `FineWeb` for the 147-million parameter training runs. We observe the qualitatively more semantically meaningful token boundaries that our method finds to translate to a consistent cross-entropy loss improvement as expended FLOPs varies over the training run. In table 1 we compare methods across a range of downstream natural language understanding benchmarks (Bisk et al., 2020; Zellers et al., 2019; Clark et al., 2018; Paperno et al., 2016), demonstrating improved language modelling performance.

4.3 PYTHON CODE

We train 90 million parameter models on the CodeParrot dataset (Hugging Face, 2022) of python code, using both our method and the straight-through estimate of Hwang et al. (2025). In Figure 2 we report that our model learns to draw token boundaries at the beginning of module names, space tokens to contain at least 2 bytes, and learns not to expand test-time compute on the Apache License, which is frequently repeated throughout the training dataset. Additional samples are provided in Appendix B.4 and tokenization strategies learned by H-Net are provided in appendix B.5. In Appendix D Figure 10 we plot the validation loss for both methods, finding that this intuitively appealing tokenization strategy leads to an improved downstream loss.

5 CONCLUSION

We propose a flexible parameterization of the problem of intra-architecture tokenization, which generalises prior proposed heuristics for this problem. We have demonstrated that applying our variance reductions yields a score function estimator that can optimize this parameterization to learn semantic boundaries §4.1 in text solely from optimizing the cross entropy and that doing so outperforms prior straight-through estimation based techniques §4.2.

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A PROOFS

A.1 LOSS BREAKDOWN

$$\begin{aligned}
\nabla_{\theta} \mathbb{E}_{a \sim \pi_{\theta}} \log p_{\theta}(y|a, x) &= \nabla_{\theta} \sum_a \log p_{\theta}(y|a, x) \pi_{\theta}(a|x) \\
&= \sum_a (\nabla_{\theta} \log p_{\theta}(y|a, x)) \pi_{\theta}(a|x) \\
&\quad + \sum_a \log p_{\theta}(y|a, x) (\nabla_{\theta} \pi_{\theta}(a|x)) \\
&= \nabla_{\theta} \sum_a (\nabla_{\theta} \log p_{\theta}(y|a, x) + \log p_{\theta}(y|a, x) \nabla_{\theta} \log \pi_{\theta}(a|x)) \pi_{\theta}(a|x) \\
&= \mathbb{E}_{a \sim \pi_{\theta}} \underbrace{(\nabla_{\theta} \log p_{\theta}(y|a, x))}_{\text{cross entropy loss}} + \underbrace{\log p_{\theta}(y|a, x) \nabla_{\theta} \log \pi_{\theta}(a|x)}_{\text{REINFORCE gradient}}
\end{aligned}$$

B.2 STRAIGHT-THROUGH ESTIMATOR FROM NAWROT ET AL. (2023)

Token boundary probability: Low to High
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Apparently, with the passage of time, the talented Asim Azhar has made quite a name in Pakistan's music industry. Having a massive following worldwide, the singer recently generously responded to one of his fan's requests which landed him in confusion. Last weekend, Asim Azhar promised to fulfill a fan's wish but it turned out that many others were already in line. Another Asim Azhar humbly replies to an excited fan on Twitter. Being a celebrity, the talented artist has also incurred some of the most controversial incidents until now. Whilst performing in a concert, someone hurled a shoe at Asim Azhar well, the singer then clarified the infamous shoe-throwing incident. However, recently, Asim Azhar's humble reply to a fan made Twitter demanding as the admirers now have a list ready. Check out what the singing star replied to his fan. I have a few green jackets. Which one do you like?? Send a picture & your address to firstname.lastname@example.org - we'll try and make it happen before 2019 ends. <https://t.co/jkM0PovKAg> Asim Azhar (@AsimAzhar) December 29, 2019 Since fans are never satisfied the adorable exchange of words sure made Twitter a hot letter for the time being. Quoting a fan's request, Asim Azhar responded, "I have a few green jackets. Which one do you like?? Send a picture & your address to email@example.com - we'll try and make it happen before 2019 ends." Bollywood waley to apny fans ko apni chahin gife kr dyte hain? Well, Asim Azhar sure made the fan's life as apparently, he will soon be getting the star's green colored jacket. Particularly, a Twitter user named Rabia Ali Raza tagged the singer and asked him that is she going to be treated like a Bollywood fan or not? Unexpectedly, Asim Azhar responded and assured his fan that she will get the "green jacket" before the world enters into 2020. Unspecifically tagging Asim Azhar with the hashtag #AsimWag, the Twitter user expressed her "jacket" wish. Moreover, she wrote, "Yar mujy (Asim Azhar) ki green colour wali jacket bohut pasand hai, Bollywood waley to apny fans ko apni chahin gife kr dyte hain. Gen 1 get this before 2019 ends." Asim Azhar's appreciative reply attracted a lot of Asim Azhar's reply. However, the lovely chat got twisted when other users also started demanding gifts from Asim Azhar, sarcastically. While the singer is soon gonna miss his funky jacket, the responses seem to be leaving him with nothing at the end. From jacket to slippers, Twitter-tatis are asking Asim Azhar for everything portraying an image of the famous game show "Jeeto Pakistan." "Nhai mujhey b chahye per main larka hun", samthay? "Nhai mujhey b chahye per main larka hun" - Abu Shalwar (@Abu Shalwar) December 30, 2019 Nro too puri qum ka masla hai. Asim bhai meri handfree ki ek side bhil kharab hai. Mail address du? nightmare.pj@gmail.com (I said) December 29, 2019 Nkur aur aur? They Pakistan ki gar wo white black wali wo harkat ko udh kr brown jacket bhil khun wo din to slipper khabay they so or so? glasses leaye they wo bhil or hn or kis hn bas is saal k liye ye bhil hai? - Hussain (@Shuhadesoflove) December 29, 2019 What's the spirit? You can send me anyone of your green one to me!!! Sending address in your email and will wait for reply via courier. Iqbal Ahmad (@IqbalAhmad) December 30, 2019 Famous media persons Iqbal Ahmad, Iqbal Hassan Syed also approached Asim Azhar's account. Unseen gesture Iqbal Ahmad (@IqbalAhmad) December 29, 2019 Seemingly, the singing star doesn't hesitate in keeping his foot ahead regarding different heated conditions. In the last edition of Bum Awards Asim Azhar showed support for the Indian Communist Party by wearing one of its (love) shoes. Well, hopefully, the fan will soon receive a call from the courier company as the "green jacket" might be on its way. What do you think of Asim Azhar's gesture? Let us know in the comments section below.

Women in face veils detained as France enforces ban. At least two women have been briefly detained in France while wearing Islamic veils, after a law banning the garment in public came into force. Asim said they were held not because of their veils but for joining an unauthorized protest, and they were later released. France is the first country in Europe to publicly ban a form of dress some Muslims regard as a religious duty. A 100-euro fine (113/121) and a citizenship course for uncooperative foreign women to wear the veil face a much larger fine and a prison sentence of up to two years. Two women detained had taken part in a demonstration outside Notre Dame cathedral in Paris. Police said the protest had not been authorized and so people were asked to move on, when they did not they were arrested. Under the new law the ban is likely to be largely symbolic. It will be difficult to prove that a woman is being forced to wear a niqab because of her husband or family. One of the women, Kenza Drider, had arrived in Paris from the southern city of Arles, wearing a niqab, and unchanged in the police station. We were held for three and a half hours at the police station while the prosecutors decided what to do," she told AFP news agency. Three and a half hours later, they told us: "It's fine, you can go." Under the law any woman - French or foreign - walking on the street or in a park in France wearing a face-covering veil such as the niqab or burka can be stopped by police and given a fine. It is a small fine, but symbolically this is a huge change, says the BBC's Hugh Schofield in Paris. Guidelines issued to police say they should not ask women to remove their veils in the street, but should ensure that the women are completely hidden from the public. The French government says the face-covering veil undermines the basic standards required for living in a shared society and also relegates its wearers to an inferior status. France's ban on face covering is the most explicit of equality laws, but has angered some Muslims and libertarians. Exceptions to ban on public face covering: Motorcycle helmets, Face-masks for health reasons, Face-covering for sporting or professional activities, Religious activities, Face-covering for not completely hide the face: Niqab, Burka used in "traditional activities" such as carnivals or religious processions. Source: Radio France International. French Muslim property dealer, Rachid Nekkas, said he was creating a fund to pay women 50,000 euros and encouraged all French women who wear the veil in the street and reside in civil disobedience. Nekkas said he and a female friend wearing the niqab were arrested at a separate demonstration in front of President Nicolas Sarkozy's Elysee Palace. We wanted to be fined for wearing the niqab, but the police didn't want to issue a fine," he told AFP. Muslim opposition to protests by libertarians and libertarians are unlikely to raise much of an issue, but, our correspondent says. What is more open to question, he says, is whether an out-and-out legal ban is necessary when, on most estimates, only 2,000 or so women in France actually wear the niqab or burka. Critics of French President Nicolas Sarkozy say it suits him to play up the Muslim question because he is an unpopular president in need of an easy vote-winner. The word hijab comes from the Arabic for veil and is used to describe the headscarves worn by Muslim women. These scarves come in myriad styles and colours. The type most commonly worn in the West is a square scarf that covers the head and neck but leaves the face clear. The niqab is a veil for the face that leaves the area around the eyes clear. However, it may be worn with a separate eye veil. It is worn with an accompanying headscarf. The burka is the most concealing of all Islamic veils. It covers the entire face and body, leaving just a mesh screen to see through. The all-enclosing niqab is the least common.

Sheseeseeseeseese's back y'all and with a brand new set of bangs to boot! Rachel Zoe is finally back to Bravo for season 5 of the Rachel Zoe Project and I literally went Ba-Nanas!! I forgot how much I missed this crazy fashionista, her ever patient husband Roger Berman, and of course, uber adorable baby Taylor. Rachel lets us hear that her last prodigy, Jenayah, is no longer with the company standard introduction to the season's another employee has flown the coop. I have got to say it's feeling a little more than de ja vu - Brad Goreski, Taylor Jacobson anyone? Whilst I love me some RZ, her prodigies balling makes me want to lead to think about it. But then I see Mandana and Roger, and if they are still around then it can't be all bad. This is so NOT bananas. In fact, it's so far from being bananas it's practically a cantaloupe. Everyone's favorite 1970's inspired bag lady stylist turned designer returned to Bravo last night with a vengeance. That's right! The fifth season of the Rachel Zoe Project premiered, and it started off on a very bad note thanks to Page Six. Way back in August, Page Six published a scathing article saying that Rachel Zoe's new line was in trouble and was not attracting customers at high end department stores. The article speculated that her show would be canned before a fifth season could air. Well, we all know that last part isn't true! As you can imagine, Rachel wasn't too thrilled with the article, and we got to see her reaction play out in our living rooms. According to a major news outlet, Cameron Diaz has fired Rachel Zoe as her stylist. I know, I know - how can we possibly go on with our regularly scheduled Friday after reading such devastating news? It won't be easy - but we must try. The New York Post has reported that Cameron is no longer working with Rachel, adding, "Some sources snipe that Zoe's been too busy building her own empire with clothes, shoes, bags and accessories bearing her name, and that she's got less time for styling her celeb clients." (NS200) shoes - check! Fur chubby - check! Assistant that treats you like the queen you wish you were - check! Coffee as food replacement - oh, you know CHECK! Celebrity clients willing to look ridiculous at your behest - check! Husband that foresees any of his interests to accommodate your wardrobe and fashion wishes - CHECK, CHECK, CHECK! Baby fashionista as your best accessory - check again! Vahaaahhh, they're just Rachel Zoe's Top Ten Fashion Must Haves. Number one on the list is actually unlimited bank account. Rachel Zoe Project bitchasses recently shared her fashion antidotes with Bravo and it's a long list of um, things we all could live without. Oh, just kidding, I actually need them all! Hopefully Rachel will buy them for me. Behold Rachel's list is below! CLICK CONTINUE READING FOR RACHEL'S MUST HAVES LIST! Thank you, Bravo. Thank you, the network has announced it is renewing some of our favorite reality shows, as well as giving a new platform to a familiar face. We will now be able to find out what the fashionable peeps are doing thanks to the line up, I. D.I.D. The former besties from the Rachel Zoe Project realized that one show wasn't big enough for the both of them. After a falling out between the teeny vintage draped outfits and her uber-preppy get 'em ready aesthetic, we were treated to double the pleasure in styling shows. Bravo has now confirmed that both Rachel Zoe and Brad Goreski will be back for more. Over four seasons of the Rachel Zoe Project viewers have watched Rachel Zoe evolve from waifish, self-absorbed A-list celebrity stylist to totally demanding, self-important, self-important business and mom. Now some report claims that the bottom is about to drop out from America's worst boss. It seems that last season garnered very low ratings for Rachel's show, which focused predominately on how much she hated ex-assistant Brad Goreski and her propensity. Both of which really have nothing to do with fashion. You know, the real reason people even cared about the show.

Figure 4: Token boundaries learned by a 147M-parameter model using the straight-through estimator of Nawrot et al. (2023), on held-out samples of the FineWeb dataset. Instead of the probabilities, here we plot the soft boundaries at the output of the Gumbel-Sigmoid. Red and blue characters indicate high or low values of \hat{b}_t respectively at the corresponding bytes.

B.4 OUR METHOD ON CODEPARROT



Figure 6: Token boundaries learned by our 90M-parameter model on a held-out sample of the CodeParrot dataset. Red and blue characters characters indicate high or low values of $\pi_\theta(a)$ respectively at the corresponding bytes.

B.5 H-NET HWANG ET AL. (2025) ON CODEPARROT



Figure 7: Token boundaries learned by a 90M-parameter model using the straight-through estimator of Hwang et al. (2025), on held-out samples of the CodeParrot dataset. Instead of the probabilities, here we plot the hard token boundaries. Red and blue characters characters indicate high or low values of \hat{b}_i respectively at the corresponding bytes.

B.6 VARYING THE DOWNSAMPLE RATE

Token boundary probability: Low to high

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

<bos>Women in face veils detained as France enforces ban\n<at least two women have been briefly detained in France while wearing Islamic veils, after a law banning the garment in public came into force.&span style="color: red;">>\nPolice said they were held not because of their veils but for joining an unauthorised protest, and they were later released.>\nFrance is the first country in Europe to publicly ban a form of dress some Muslims regard as a religious duty.>\nOffenders face a fine of 150 euros (€133/\$217) and a citizenship course.>\nPeople forcing women to wear the veil face a much larger fine and a prison sentence of up to two years.>\nThe two women detained had taken part in a demonstration outside Notre Dame cathedral in Paris. Police said the protest had not been authorised and so people were asked to move on. When they did not, they were arrested.>\nAvignon's Europe>\nThe law is likely to be largely symbolic... It will be difficult to prove that a woman is being forced to wear a niqab because of her husband or family.>\nOne of the women, Kenna Brider, had arrived in Paris from the southern city of Avignon, boarding a train wearing a niqab, and unchallenged by police.>\nWe were held for three and a half hours at the police station while the prosecutors decided what to do," she told AFP news agency.>\nThree and a half hours later they told us: "It's a fine, you can go.">\nUnder the law, any woman - French or foreign - walking on the street or in a park in France and wearing a face-concealing veil such as the niqab or burka can be stopped by police and given a fine.>\nIt is a small fine, but symbolically this is a huge change, says the BBC's Hugh Schofield in Paris.>\nGuidelines issued to police say they should not ask women to remove their veils in the street, but should escort them to a police station where they would be asked to uncover their faces for identification.>\nThe French government says the face-covering veil undermines the basic standards required for living in a shared society and also relegates its wearers to an inferior status incompatible with French notions of equality.>\nThe ban on face coverings - which does not explicitly mention Islamic veils, but exempts various other forms - has angered some Muslims and libertarians.>\nExceptions to ban on public face covering>\nMotorcycle helmets>\nFace-masks for health reasons>\nFace-covering for sporting or professional activities>\nSunglasses, hats etc which do not completely hide the face>\nMasks used in "traditional activities", such as carnivals or religious processions>\nSource: Radio France International>\nA French Muslim property dealer, Rachid Nekkas, said he was creating a fund to pay women's fines, and encouraged "all free women who so wish to wear the veil in the street and engage in civil disobedience".>\nNekkas said he and "a female friend wearing the niqab" were arrested at a separate demonstration in front of President Nicolas Sarkozy's Elysee Palace.>\nWe wanted to be fined for wearing the niqab, but the police didn't want to issue a fine," he told AFP.>\nBut opposition protests by Islamists and libertarians are unlikely to make much of an impression, our correspondent says.>\nWhat is more open to question, he says, is whether an out-and-out legal ban is necessary when, on most estimates, only 2,000-or-so women in France actually wear the niqab or burka.>\nCritics of French President Nicolas Sarkozy say it suits him to play up the Muslim question because he is an unpopular president in need of an easy vote-winner.>\nThe word hijab comes from the Arabic for veil and is used to describe the headscarves worn by Muslim women. These scarves come in myriad styles and colours. The type most commonly worn in the West is a square scarf that covers the head and neck but leaves the face clear.>\nThe niqab is a veil for the face that leaves the area around the eyes clear. However, it may be worn with a separate eye veil. It is worn with an accompanying headscarf.>\nThe burka is the most concealing of all Islamic veils. It covers the entire face and body, leaving just a mesh screen to see through.>\nThe al-amir>\n

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Figure 8: Token boundaries learned by an array of models using our method with varying target downsample rates $\bar{\pi}_{target} = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ (top, middle, bottom, respectively) on a held-out sample of the FineWeb dataset. Red and blue characters indicate high or low values of $\pi_{\theta}(\alpha)$ respectively at the corresponding bytes.

C CONTINUED RELATED WORK

C.1 SCALING BYTE-LEVEL LANGUAGE MODELS WITH TOKENIZATION HEURISTICS

Byte-level language models have recently been shown to scale competitively with tokenized transformers, with ByT5 (Xue et al., 2022) demonstrating that a pure byte-level architecture can match token-level performance, though at the cost of quadratic attention that makes long-sequence scaling impractical. Subsequent work has addressed this limitation along two main directions: leveraging subquadratic attention mechanisms (Wang et al., 2024; Zheng et al., 2025), and reducing sequence length through hierarchical or pooled representations. The latter includes fixed-stride downsampling approaches (Tay et al., 2022; Nawrot et al., 2022; Yu et al., 2023), as well as methods that segment byte streams at more semantically meaningful boundaries. Nawrot et al. (2023) first explored pooling at whitespace characters, later scaled by Slagle (2024) to outperform fixed-stride schemes. In parallel, Nawrot et al. also introduced entropy-based boundary detection (Nawrot et al., 2023), leveraging spikes in next-byte entropy that correlate with semantic breaks; this idea was further scaled by Pagnoni et al. (2024), who showed that entropy-guided downsampling can enable byte-level models to surpass token-level baselines at the 10^{22} -FLOP scale.

While these methods demonstrate the promise of the autoregressive U-net architecture at scale and highlight some desirable properties that a tokenization mechanism for this architecture would have, all rely on heuristics, breaking the **End-to-end tokenizer training** desideratum in our setup.

C.2 ARCHITECTURAL INNOVATIONS

We focus on the method of deciding token boundaries, and thus use a simple architecture for the purpose of not confounding our experiments. Nonetheless, we would like to highlight the following architectural innovations that have been proposed to improve autoregressive U-nets:

- Using a vocabulary of byte-level n -grams which are added to the input embeddings to improve information flow Pagnoni et al. (2024).
- Using cross-attention between the byte-level and token-level transformers Pagnoni et al. (2024).
- Using subquadratic sequence-to-sequence blocks at the byte level Hwang et al. (2025).
- Using multiple levels of tokenization Hwang et al. (2025).
- Smoothing between tokenization levels in upsampling Hwang et al. (2025).

We would like to note that all of these methods could be used with our method to learn token boundaries.

Model size	147M	90M	20-40M
embedding_dim	768	768	512
num_heads	12	12	8
n_down_layers	4	4	4
n_mid_layers	12	6	n
n_up_layers	4	4	4
learning_rate	1.5×10^{-3}	2×10^{-3}	3×10^{-3}
effective_batch_size	128	128	64
warmup_bytes	4.5×10^8	2.5×10^8	1.1×10^8
training_bytes	6×10^9	3.4×10^9	1.5×10^9
downsample_rate_target	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{n}$

Table 2: The hyperparameters used for our runs varying the token boundary selection method used on natural language (left) python code (middle) and the varying tokenization aspect ratio (right)

D FURTHER EXPERIMENTAL DETAILS

We train models on a total of `training_bytes` bytes with a cosine learning rate schedule with a warmup of `warmup_bytes` and a maximum learning rate of `learning_rate`. We use the AdamW optimizer with default parameters $(\beta_1, \beta_2) = (0.9, 0.999)$. Per gradient update, we perform a forward & backward pass on `effective_batch_size` sequences of length 4096.

We use a decoder-only transformer architecture, with `n_down_layers` and `n_up_layers` byte-level transformer decoder layers with sliding window attention with window size 64 before and after the token-level backbone which consists of `n_mid_layers` token-level transformer decoder layers with full causal attention. All attention layers have `num_heads` heads. We make the model dimension the same for byte and token-level layers, such that $d_{enc} = d_{mid} = d_{dec} = \text{embedding_dim}$. Byte-level and token-level layers have an MLP hidden dimension of `embedding_dim` and $4 \times \text{embedding_dim}$ respectively, making the flops-per-layer roughly consistent. We closely follow the transformer architectural choices of Gemma 2 Rivière et al. (2024), with GeGLU non-linearity, Rotary Position Embeddings, post and pre- RMSNorm and Logit soft-capping.

See table 2 for the hyperparameters used in our 147-M parameter experiment and our experiments varying the token aspect ratio.

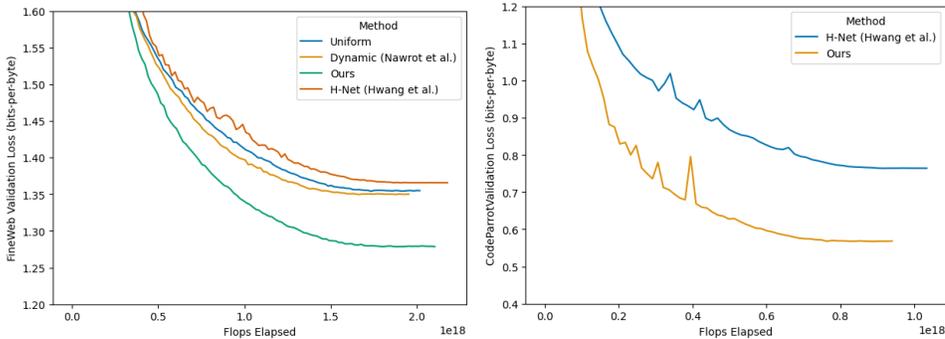


Figure 10: [left] Flops vs validation loss curves for 147M parameter models, trained on FineWeb, measured in bits-per-byte. Uniform random in blue, Straight through estimates in orange, ours in green. Validation loss values at 1.95×10^{18} FLOPs are 1.355, 1.350, 1.36 and 1.279 bits-per-byte respectively. [right] Flops vs validation loss curves for 90M parameter models, trained on CodeParrot, measured in bits-per-byte. H-Net in blue ours in orange. Validation loss values at 0.95×10^{18} FLOPs are 0.769 and 0.568 bits-per-byte respectively

Following Kaplan et al. (2020), we estimate the number of FLOPs in the model training run as approximately the number of FLOPs used for the matrix parameters in the forward/backward pass (which is the dominant source for LLMs), which takes a value of 6 FLOPs per parameter per byte

or token, the latter depending on which level the layer operates on (Bahdanau, 2022). We assume that the embedding operation is performed using an efficient lookup. Small deviations in `FLOPs` per batch for each method occur as a result of variations in the max token sequence length in the batch.