A Budget-Balanced Mechanism for Siting Noxious Facilities with Identity-Dependent Externalities

Younghwan In* and Yunkyeong Seo†

May 2017

(preliminary and incomplete)

Abstract

We apply Minehart and Neeman's (2002) auction-like procedure for siting noxious facilities to the environments where there are identity-dependent externalities, *i.e.* the negative externality depends on who becomes the host. We show that it is an ϵ-equilibrium for any small ϵ for each community to bid its true disutility minus the minimum externality if there are a sufficient number of communities in a general asymmetric environment. We also show that the bidding in a symmetric Bayesian Nash equilibrium converges to the true disutility minus the average externality as the number of communities gets larger in a symmetric environment. Both equilibria tend to hurt efficiency by choosing a socially undesirable host. We also prove that the mechanism still guarantees the ex-post individual rationality of every community as in the case without the externalities.

JEL Classification: D44, D62, D82, Q53.

Keywords: NIMBY, externalities, budget balance, efficiency, auctions, mechanism design.

* KAIST College of Business, 85 Hoegiro, Dongdaemun-gu, Seoul, 130-722, Republic of Korea. Tel: +82-2-958-3314. e-mail: cyin@business.kaist.ac.kr

† KAIST College of Business, 85 Hoegiro, Dongdaemun-gu, Seoul, 130-722, Republic of Korea. Tel: +82-2-958-3649. e-mail: tjdbsrud@business.kaist.ac.kr

1 Introduction

Over the past decades, the NIMBY (Not in My Backyard) phenomena have been deeply entrenched in our society. Despite governmental efforts on negotiating with the potential hosts, the strong opposition to bringing noxious facilities into one's own communities persisted and brought about serious social problems. They delayed the construction of the necessary facilities, generated the social costs on negotiating and over-compensating the hosts, and aggravated the regional conflicts. Especially the siting issues of the waste disposal plants such as waste water treatment centers, garbage incinerators, and nuclear waste storages drew even stronger backlash from the local communities, every time the issue surfaced, since they concerned the local people with their health problem.

The fundamental causes that generate and aggravate the NIMBY phenomena are the pursuit of human selfishness and the inefficiency in the siting and compensation system. The efficiency can be achieved when the community who suffers the least actually hosts it. However, the true damage each community actually sustains when hosting the facility is its private information, which is unknown to the other communities and the government. Thus, each community has an incentive to cheat on its willingness to accept (WTA) from the government for hosting the facility out of its self-interest. Therefore, without an efficient siting and compensating mechanism to reveal the true damages and the most efficient potential host, these deceptions will be continued and worsen the NIMBY phenomena.

For the theoretical solutions, there had been a number of studies on developing a mechanism to reveal the true WTAs, identify the least suffering host, and compensate it under the budget constraint. Researchers, modeling the situations as Bayesian games, sought the most efficient, budget-balanced, and individually rational mechanism as far as possible. They showed how their equilibria with each player's best response strategy lead to the efficient outcomes. However, in all the previously suggested budget-balanced mechanisms, it has been intentionally assumed that each player's value of not hosting the facility but enjoying the benefit from it remains the same, no matter which community other than itself becomes the host. This strong assumption simplified their models and helped them clearly reach the equilibria. However in reality, each community not only suffers the disutility as a host but is also affected by the negative externalities that vary according to whom the real host is. The examples for the negative externalities include the polluted air or water inflow from the neighbor host, the noise, and the house price decline around the host. These externalities tend to be aggravated as a community gets closer to the host.

Among the budget-balanced mechanisms for siting noxious facilities, Minehart and

Neeman's (2002) second price-auction-like-procedure has been proven to be almost efficient and individually rational, thus quite ideal. This study aims to see if the mechanism still achieves almost-efficiency[3] when the communities suffer identity-dependent negative externalities. In Section 2, we review the previously suggested siting mechanisms, makes it clear why budget balance is required but is difficult to be achieved with the efficiency, and checks whether the previous studies have dealt with the externalities before. In Section 3, we clarify the research question by pointing out the probable inefficiency in Kunreuther's low-bid auction when taking the asymmetric externalities into account. Then in Section 4, we describe Minehart and Neeman's siting mechanism and remodel the utility functions considering the identity-dependent externalities. In Section 5, we then seek the equilibria of the model and check if the main properties siting procedures should satisfy hold. Then in Section 6, we interpret the results, compare them with the other mechanisms, and remark the academic contributions and possible policy implications of the study. Finally in Section 7, we conclude the main findings and suggest some probable further studies.

2 Literature Review

There have been a number of economic, environmental, and political studies on efficiently siting noxious facilities. In the economic ground, the researchers have regarded noxious facilities as economic bads that generate disutility for the host while bringing benefits to all the other members in the society, in other words, the non-excludable public bads. They designed several mechanisms for efficiently siting such public bads and properly compensating the host with taxes collected by the non-hosts.

2.1 Incomplete Information versus Complete Information

Numerous economists have modeled this NIMBY problem as a Bayesian game with incomplete information in their studies. They argued that the disutility each community suffers as a host is its private information that both the government and the other communities do not know. O'Hare (1997) has first realized that this incomplete information setting might lead to each community's willingness-to-accept (WTA) misrepresentation when seeking higher profits. He emphasized the importance of revealing the true preferences for the social efficiency and suggested Vickrey-type auction procedure for the solution, but did not actually model his idea.

[3]When we say that a mechanism is "almost-efficient" in this paper, we mean that the efficiency loss in the equilibrium of the mechanism is bounded and converges to zero when there exists an sufficiently large number of the participants.

Three main mechanisms form the mainstream of the studies on revealing the private information for solving NIMBY. First, the demand revealing mechanism developed by Clarke (1971), better known as the Clarke mechanism, a special form of the Vickrey-Clarke-Groves (VCG) mechanism, chooses the potential site which brings the biggest net social benefit as the host and collects the Clarke taxes from the non-hosts. This mechanism succeeded in revealing the true private disutilities, thus, proven to be incentive compatible and as a result, efficient. In addition, it is ex post individually rational, that is, it induces voluntary participation of every community. However, since each community pays the amount of total social welfare loss caused by the participation of its own to the game, the host neither pays a tax nor receives a reward and the non-hosts pay taxes. This thus, generates some budget surplus that has to be redistributed, causing another inefficiency afterwards (Clarke, 1971 and 1972).

Kunreuther and Kleindorfer (1986) suggested a budget-balancing low-bid auction for the game. In this auction, the government chooses the lowest bidder as the host, collects the tax proportional to the reported WTA of each community, respectively, and provides the whole taxes to the host as a subsidy, leading to the budget balance. Each community's best response bidding under this auction has been proven to be the max-min strategy. With these strategies, the mechanism succeeds in choosing the host maximizing the social welfare, thus results in the efficiency, and also guarantees the voluntary participation. Nevertheless, it is not a Bayesian Nash equilibrium so players might deviate from the strategy and the result might not be efficient, consequentially (Kunreuther and Kleindorfer, 1986 and Kunreuther *et al.*, 1987).

Finally, Minehart and Neeman (2002) applied the k+1 auction suggested by Cramton *et al.* (1987) to the NIMBY problem. Cramton *et al.* seek a way of dissolving the partnership of which each player has the partial ownership. Similarly, Minehart and Neeman assumed that the information on how much share or responsibility each community takes for utilizing the public facility is commonly known in the society. For example, the share represents the amount of waste generated by each community over the total waste in the society in the case of the waste treatment center and the amount of energy transported to each community over the total energy generation in the case of noxious power plants. Now under their second price auction-like procedure, an example of the k+1 auction, the players bid their disutilites, the government chooses the lowest bidder for the host and charges each player a tax for the amount of the second lowest bid multiplied by its share. Providing the whole amount of collected taxes to the host, the mechanism achieves the budget balance. It also has been proven to be almost incentive compatible and efficient with the sufficiently many number of communities and to have bounded efficiency loss even when the least suffering community fails to be the host. Lastly, it guarantees every player's individual rationality. From 2002, since Minehart and Neeman's mechanism seemed to be almost ideal for simultaneously achieving the budget balance, the efficiency, and the individual rationality, most literatures on solving NIMBY have benchmarked it for their baselines.

On the other hand, some researchers have regarded this problem as a game with complete information and approached from a different point of view. Bellettini and Kempf (2013) found a mechanism for the government to decide the location and size of a public facility via social planner's social welfare maximization. Several researchers also studied on solving NIMBY via social planner's decentralization scheme (Romero and Paredes, 2013, Sakai, 2012, and Song *et al.*, 2013). They have developed a mathematical algorithm for allocating undesirable facilities from the computer science approach.

Between these two aspects, this study takes the former one, modeling it the game with the incomplete information, as our baseline. This is partly because it is the mainstream of the studies in this field and partly because the incomplete information on the disutilities seems to be a more realistic and rational setting. If exactly how much damage each community suffers from hosting the noxious facilities economically, environmentally, and psychologically were perfectly known to the government and the society, the government would have easily chosen the host and compensated it properly, never suffering from the delay of the construction or the afterward complaints. However the tremendous backlash from the potential hosts and the exaggerated compensation claims in the real world reflect the information asymmetry among the society.

2.2 Efficiency versus Budget Balance

While the standard auctions view the one maximizing the seller's revenue optimal, the allocation of the public bads takes the achievement of the efficiency for the top priority and requires the budget balance instead of the money outflows. It is intuitively easy to accept that the budget deficit requires some money inflow from the outside, leading to social inefficiency. Even in the case of budget surplus, Tideman and Tullock (1976) have pointed out that the redistribution of the surplus distorts the incentive of the players, thus distorts the Incentive-compatibility (IC) condition. In this sense, Groves and Ledyard (1977) criticized the budget-imbalanced VCG mechanism for the inefficient equilibrium. Thus both the efficiency and the budget balance are the important properties the public bad allocating mechanisms should take into account.

The trade-off between the efficiency and the budget balance has been burningly studied for a long time. Green *et al.* (1976) have shown that it is impossible for a dominant-strategy incentive compatible mechanism to achieve both the efficiency and the budget balance, even if restricted to the simple exchange setting, in which the agents with quasilinear utility functions trade a single identical unit of good. Then Myerson-Satterthwaite's Impossibility Theorem (1983) have proved that even there exists no Bayes-Nash incentive compatible mechanism that is efficient, weakly budget-balanced, and ex interim individually rational at the same time. The Arrow-d'Aspremont-Gerard-Varet (AGV) mechanism has been proven to be budget-balanced, efficient, and could be individually rational simultaneously but it only guarantees the ex ante rationality rather than the ex interim or ex post individual rationality (Arrow, 1979 and d'Aspremont and Gerard-Varet, 1979).

In consideration of this trade-off, Moulin and Shenker (2001) calculated how much budget imbalance VCG mechanism induces in return for being efficient via worst-case analysis. They also named the mechanism with the minimal budget imbalance in worst case among the whole group of Groves mechanism, the almost budget-balanced VCG mechanism or the optimal VCG mechanism. Moulin (2009) found an exact allocation and payment rule of such optimal VCG mechanism. In reverse, as mentioned above, Minehart and Neeman's mechanism solved the problem by deriving a perfectly budget-balanced but almost-efficient solution.

2.3 Externalities

It is obvious that non-excludable public bads generate various externalities to all the communities near the host. In the sense that the benefits the public facilities bring to the non-hosts are positive externalities, it seems rational to collect taxes from the whole communities in return. Some studies have explicitly stated these positive externalities each community suffers from another community hosting the facility and let them remain the same regardless of whom the host is (Kunreuther and Kleindorfer, 1986 and Kunreuther *et al.*, 1987). In other words, they assumed the identity-independent externalities. Some studies have defined the disutility as that each community suffers as a host over the positive externality it might have enjoyed when not being the host. However they also regarded the positive externality to be identity-independent (Minehart and Neeman, 2002).

3 Motivation

The lacunae of the previous economic studies on the NIMBY syndrome are the overlook on the negative externalities. As mentioned above, the previous studies take the positive externalities into account but failed to consider the negative ones each community suffers when the nearby community hosts the noxious facility. In reality, the value one gets when the other community hosting the facility has a great tendency to vary according to whom the real host is. (This is so-called the identity-dependent externalities.) Several studies have proved that the externality decreases as the distance from the host increases (Ahlfeldt and Maennig, 2012 and Bellettini and Kempf, 2013). For example, the air pollution generated during the waste disposal procedure tends to damage the other nearby communities more than the ones far away from the host.

The identity-dependent externalities have been first considered by Jehiel *et al.* (1996). In their following study in Jehiel *et al.* (1998), they generalized the model by assuming that each player has a n-dimensional vector-type of its disutility and identity-dependent externalities, that is identically and independently distributed. Then they showed that the weakly dominant strategy of each player is to bid its true willingness to pay or utility minus the average externality one suffers in the general Vickrey auction.

While the efficient Groves mechanisms are well noted for internalizing the externalities, all the previous budget-balanced mechanisms for solving NIMBY phenomena have never dealt with the identity-dependent negative externalities yet. However, the consideration of these asymmetric externalities can change the equilibria of the mechanisms and even worsen the inefficiency. For example, Kunreuther and Kleindofer's low-bid auction sometimes fails to maximize the social welfare when combined with asymmetric externalities. One example is stated as below.

Table 1: An example of the efficient low-bid auction

Potential sites	**Communities**					**Net benefits**
	1	**2**	**3**	**4**	**5**	
1	-1000	775	1275	375	475	1900
2	575	-1500	1275	375	475	1200
3	575	775	-2400	375	475	-200
4	575	775	1275	-630	475	**2470**
5	575	775	1275	375	-800	2200
max-min bid	1260	1820	2940	**804**	1020	

Table 1 exhibits the true values the communities enjoy or suffer when one of them hosts the facility. For example, the cell in the second row and the third column represents the value community 3 enjoys when community 2 is the host. The last row states how each community bids its willingness to accept via the max-min strategy. The bidding strategy (community i's reported willingness to accept) is calculated as below.

$$X_i = \frac{N-1}{N}\Big[\min_{j\neq i} V_{ij} - V_i\Big],$$

where N is the total number of the communities participating in the game, V_i is the community i's value of hosting the facility ($V_i < 0$, *i.e.* true WTA = -V_i), and V_{ij} is community i's value when community j hosts the facility ($V_{ij} > 0$) for $i, j = 1, 2, ...N$. With these bids, community 4 with the lowest bid will be chosen as the host. This guarantees the efficiency of the mechanism since the net social benefit is maximized when community 4 hosts the facility.

However they assumed that the value one gets when another community hosts the facility are the same regardless of the host, *i.e.* $V_{ij} = V_{-i}$ for all $j \neq i$. This plays a key role for the low-bid auction's max-min strategies to lead to an efficient outcome. When we slightly alter the externalities to be asymmetric, the mechanism may pick a wrong community out for the host which does not maximize the social benefits. The counterexample is stated as below.

Table 2: A counterexample of the inefficiency caused by the asymmetric externalities in low-bid auction

Potential sites	**Communities**					**Net benefits**
	1	**2**	**3**	**4**	**5**	
1	-1000	775	1275	375	475	1900
2	575	-1500	1275	375	475	1200
3	575	775	-2400	375	475	-200
4	575	775	1275	-630	475	**2470**
5	0	775	1275	375	-800	1625
max-min bid	**800**	1820	2940	**804**	1020	

We only changed community 1's value when community 5 being the host from 575 to 0 in table 2. As a result, the lowest bid comes from community 1 but the total net benefits are maximized when community 4 is chosen for the host. Therefore, the low-bid auction fails to guarantee the efficient outcome under the asymmetric externalities.

This finding motivates us to study the probable efficiency loss in Minehart and Neeman's budget-balanced mechanism when identity-dependent externalities are introduced.

4 Model Description

The basic setting of our model is similar to that of Minehart and Neeman's model. Suppose that there exist n communities for potential hosts to site a single noxious facility. Each community has a concave disutility function, $d_i : R^+ \to R^+$ where the disutility of treating an amount w of the waste is $d_i(w)$. Define w_i as the amount of waste community i generates and α_i be its share of the amount of waste over the total amount of waste from the whole society ($i.e.$ $\alpha_i = \frac{w_i}{\sum_{j=1}^n w_j}$). We assume that the information on these waste shares is commonly known in the society. For simplicity, we refer to $d_i(w)$ as d_i where $w = \sum_{j=1}^n w_j$.

On top of that, we assume that each community suffers identity-dependent externalities that are proportional to its own disutility to be a host. To be specific, once the total economic, environmental, and psychological damages to a host are summed up to be a single value of disutility for each community, we assume that the negative externalities suffering from being a neighbor of the host are proportional to this original amount of disutility as a host. Also, according to the previous studies, it is reasonable to suppose that the externalities from the nearby host decrease as the distances from the host increase and vice versa.

Let us demonstrate these assumptions explicitly. The community i suffers the externalities, $e_i = (e_{i1}, e_{i2}, \cdots, e_{in})$ where e_{ij} represents the negative externality i suffers when j hosts the community. Also, the community i has a distance vector, $\gamma_i = (\gamma_{i1}, \gamma_{i2}, \cdots, \gamma_{in})$ where γ_{ij} represents the distance between the community i and j. Now, we assume that there exists a non-increasing function $g : R \to [0, 1]$, where $g(\gamma_{ij})$ represents the size of the influence between the communities i and j for all $i \neq j$ and $g(\gamma_{ii}) = 1$ for all i. Then, we assume the disutility-proportional externality by letting $e_{ij} = g(\gamma_{ij})d_i$.

Now, the community i bids b_i for $i = 1, 2, \cdots, n$. Lining up the bids in the increasing order, they are renamed to be $b_{[1]} < b_{[2]} < \cdots < b_{[N]}$. The government chooses the lowest bidder as the host and charges the non-hosts taxes of the second lowest bid multiplied by their shares of the waste. Thus, the community i pays the tax of $\alpha_i b_{[2]}$ if it is a non-host and receives the reward of $(1 - \alpha_i)b_{[2]}$ if it is a host.

Finally, we assume an additively separable quasilinear payoff function. The community i's payoff of bidding b_i (while the other communities bid $b_{-i} \equiv (b_j)_{j\neq i}$) with the disutility d_i, the externality vector e_i, and the waste share α_i, $U_i(b_i, b_{-i}, d_i, \alpha_i, e_i)$ can be stated as below:

$$U_i(b_i, b_{-i}, d_i, \alpha_i, e_i) = \begin{cases} -d_i + (1 - \alpha_i)b_{[2]} & \text{if } i = [1] \\ -\alpha_i b_{[2]} - e_{i[2]} & \text{if } i \neq [1]. \end{cases}$$

5 Analysis

Under the model setup, we derive the best-response bidding strategy for a community in the equilibrium and interpret the meaning of the result. Then we check whether the mechanism still meets the three important properties: the budget balance, the efficiency and the individual rationality.

5.1 Ex Post Perspectives

Before seeking the equilibria, we first analyze the structure of incentives from the ex post perspectives. That is, we assume that bids of the whole communities in the game are commonly known and see how much one specific community would like to bid.

Proposition 1. *Each community has no ex post incentive to bid more than its true disutiltiy minus the minimum externality. On the other hand, each community may have an ex post incentive to bid less than its true disutiltiy minus the minimum externality.*

Proof. For convenience, we define some of the main variables. Let $b_i^{opt} = d_i - \{\min_{m \neq i} g(\gamma_{im})\} d_i$, $Y_i^1 = \min_{j \neq i} b_j$, and $h = argmin_{j \neq i} b_j$ ($i.e.$ $b_h = Y_i^1$) Also, we abbreviate $U_i(b_i, b_{-i}, d_i, \alpha_i, \gamma_i)$ to $U_i(b_i)$ unless there is no risk of confusion.

First, we show that each community has no incentive to overstate its bid than 'its true disutility minus the minimum externality', $i.e.$ to bid more than b_i^{opt}. Suppose that $b_i > b_i^{opt}$. Obviously, $b_i^{opt} > 0$ since $g(\gamma_{im}) < 1$ for $\forall m \neq i$ for all i.

i) If $b_i^{opt} \leq b_i \leq Y_i^1$,

Since $i = [1]$ in either case of bidding b_i or b_i^{opt},

$$U_i(b_i) = U_i(b_i^{opt}) = -d_i + (1 - \alpha_i) Y_i^1.$$

Therefore, there is no difference between bidding b_i^{opt} or more than that.

ii) If $b_i^{opt} \leq Y_i^1 \leq b_i$,

$$\begin{aligned} U_i(b_i) - U_i(b_i^{opt}) &= (-\alpha_i b_{[2]} - g(\gamma_{i[2]}) d_i) - (-d_i + (1 - \alpha_i) Y_i^1) \\ &= -\alpha_i (b_{[2]} - Y_i^1) + (-Y_i^1 + (1 - g(\gamma_{i[2]}) f) d_i) \\ &\leq \alpha_i (b_{[2]} - Y_i^1) + (-(1 - \{\min_{m \neq i} g(\gamma_{im})\}) d_i + (1 - g(\gamma_{i[2]})) d_i) \\ &= -\alpha_i (b_{[2]} - Y_i^1) + (\min_{m \neq i} g(\gamma_{im}) - g(\gamma_{i[2]})) d_i \leq 0. \end{aligned}$$

Therefore, bidding more than b_i^{opt} is worse off.

iii) If $Y_i^1 \leq b_i^{opt} \leq b_i$,

Let $b_{[2]}$ denote the second lowest bid when the community bid b_i^{opt} and $b'_{[2]}$ denote the second lowest bid when the community i bid b_i.

$$\begin{aligned} U_i(b_i) - U_i(b_i^{opt}) &= (-\alpha_i b_{[2]} - g(\gamma_{ih})d_i) - (-\alpha_i b'_{[2]} - g(\gamma_{ih})d_i) \\ &= \alpha_i(b'_{[2]} - b_{[2]}) \leq 0. \end{aligned}$$

Therefore, again bidding more than b_i^{opt} is worse off.

Second, we show that each community may have an incentive to understate its bid than 'its true disutility minus the minimum externality', *i.e.* to bid less than b_i^{opt}. Suppose that $b_i < b_i^{opt}$.

i) If $b_i \leq b_i^{opt} \leq Y_i^1$,

Since $i = [1]$ in either case of bidding b_i or b_i^{opt},

$$U_i(b_i) = U_i(b_i^{opt}) = -d_i + (1 - \alpha_i)Y_i^1.$$

Therefore, there is no difference between bidding b_i^{opt} or less than that.

ii) If $b_i \leq Y_i^1 \leq b_i^{opt}$,

$$\begin{aligned} U_i(b_i) - U_i(b_i^{opt}) &= (-d_i + (1 - \alpha_i)Y_i^1) - (-\alpha_i b_{[2]} - g(\gamma_{ih})d_i) \\ &= d_i(g(\gamma_{ih}) - 1) + Y_i^1 + (b_{[2]} - Y_i^1)\alpha_i. \end{aligned}$$

Therefore, it is not sure whether bidding b_i^{opt} is better or worse than bidding less than that.

iii) If $Y_i^1 < b_i < b_i^{opt}$,

Let $b_{[2]}$ denote the second lowest bid when the community bid b_i^{opt} and $b'_{[2]}$ denote the second lowest bid when the community i bid b_i.

$$\begin{aligned} U_i(b_i) - U_i(b_i^{opt}) &= (-\alpha_i b_{[2]} - g(\gamma_{ih})fd_i) - (-\alpha_i b'_{[2]} - g(\gamma_{ih})d_i) \\ &= \alpha_i(b'_{[2]} - b_{[2]}) \geq 0. \end{aligned}$$

Therefore, bidding less than b_i^{opt} is better off. □

To conclude, the community never has an ex post incentive to bid more than its disutility minus the minimum externality. Additionally, even each community might have an incentive to bid less than its disutility minus the minimum externality, it cannot bid far less than that since it involves the risk of being the real host with little compensation.

5.2 Equilibrium

Now, we seek equilibria under two different settings of the environments.

5.2.1 ϵ-Equilibrium in General Asymmetric Environment

To begin with, we assume that the disutility of each community is independently but not necessarily identically distributed. In this general asymmetric environment, the bidding function as a best response strategy for a community in the equilibrium should take n-dimensional information including the disutility to be a host as an input, and match it to a one-dimensional bid as an output. Thus it is impossible to define an inverse function of the original bidding function, which makes hard to find a Bayesian Nash equilibrium here. Another possible approach to finding the Bayesian Nash equilibrium not using the inverse function is suggested in Section 7 with concluding remarks. Instead we focus on finding an ϵ-equilibrium in this asymmetric setup here.

Proposition 2. *In the general asymmetric environment, for any $\epsilon > 0$, there exists an integer M such that if there are at least M communities, it is an ϵ-equilibrium for each community to bid its disutility as a host minus the minimum externality.*

Proof. We want to show that for $\forall \epsilon > 0$, $\exists$ M such that

$$EU_i(d_i, b_i, \alpha_i, \gamma_i) - EU_i(d_i, d_i - \{\min_{m \neq i} g(\gamma_{im})\}d_i, \alpha_i, \gamma_i) < \epsilon \qquad \forall n \geq M,$$

where $b_i \in [0, d_i - \{min_{m\neq i} g(\gamma_{im})\}d_i]$ and all the other players bid $d_j - \{min_{m \neq j} g(\gamma_{jm})\}d_j$ for all $j \neq i$. For convenience, let $p_i \equiv 1 - min_{m\neq i} g(\gamma_{im})$. Since the optimal bid for the community i to maximize its utility never exceeds $p_i d_i$ from Proposition 1, it is sufficient to prove Proposition 2 only for $b_i \in [0, p_i d_i]$. For $\forall j \neq i$, since $b_j = p_j d_j$ from the assumption and $d_j \sim F_j$ with probability density function f_j on the support $[\underline{d_j}, \overline{d_j}]$, b_j is distributed according to a continuously differentiable cumulative density function H_j and pdf h_j on the support $[p_j \underline{d_j}, p_j \overline{d_j}]$. Thus, the expected payoff that the community i maximizes is as below.

$$\begin{aligned} EU_i(d_i, b_i, \alpha_i, \gamma_i) &= \sum_{j \neq i} \int_{b_i}^{p_j \overline{v_j}} (-d_j + (1-\alpha_i)x) h_j(x) \prod_{l \neq i,j} (1 - H_l(x)) dx \\ &\quad + \sum_{j \neq i} (-\alpha_i b_i - g(\gamma_{ij}) d_i) H_j(b_i) \prod_{l \neq i,j} (1 - H_l(b_i)) \end{aligned}$$

$$+\sum_{j\neq i}\sum_{k\neq i,j}\int_{p_j\underline{v_j}}^{b_i}(-\alpha_i x - g(\gamma_{ij})d_i)h_k(x)H_j(x)\prod_{l\neq i,j,k}(1-H_l(x))dx.$$

Therefore,

$$\begin{aligned}
EU_i(d_i,b_i,\alpha_i,\gamma_i) \quad - \quad & EU_i(d_i,p_id_i,\alpha_i,\gamma_i) \\
&= \sum_{j\neq i}\int_{b_i}^{p_id_i}(-d_j+(1-\alpha_i)x)h_j(x)\prod_{l\neq i,j}(1-H_l(x))dx \\
&+ \sum_{j\neq i}(-\alpha_i b_i - g(\gamma_{ij})d_i)H_j(b_i)\prod_{l\neq i,j}(1-H_l(b_i)) \\
&+ \sum_{j\neq i}(\alpha_i d_i - \alpha_i\{\min_{m\neq i} g(\gamma_{im})\}d_i + g(\gamma_{ij})d_i)H_j(p_id_i)\prod_{l\neq i,j}(1-H_l(p_id_i)) \\
&+ \sum_{j\neq i}\sum_{k\neq i,j}\int_{b_i}^{p_id_i}(\alpha_i x + g(\gamma_{ij})d_i)h_k(x)H_j(x)\prod_{l\neq i,j,k}(1-H_l(x))dx \\
&\leq \sum_{j\neq i}2d_i\prod_{l\neq i,j}(1-H_l(p_id_i)) + \sum_{j\neq i}\sum_{k\neq i,j}\int_0^\infty (x+d_i)h_k(x)\prod_{l\neq i,j,k}(1-H_l(x))dx.
\end{aligned}$$

The inequality holds above since the first term in the right-hand side of the equation is negative because $b_i \leq p_id_i$ at first. Also, the second term and '$-\alpha_i\{\min_{m\neq i} g(\gamma_{im})\}d_i$' in the third term are negative. Lastly, because $\alpha_i, p_i, g(\gamma_{ij}) \leq 1$, the inequality finally holds.

Now, define a piecewise continuous function, $H^{min}(x) = \min_{j\in\{1,2,\ldots,n\}}\{H_j(x)\}$ on $x \in [0,\infty]$. Then, $H^{min}(x)$ has a piecewise continuous density function that is strictly positive on $\bigcap_j [\underline{d_j}, \overline{d_j}]$. Therefore, $H_j(x) \geq H^{min}(x)$ for $\forall$ x$\in [0,\infty)$. Thus,

$$\begin{aligned}
EU_i(d_i,b_i,\alpha_i,\gamma_i) - U_i(d_i,p_id_i,\alpha_i,\gamma_i) \quad &\leq \quad (n-1)2d_i(1-H^{min}(p_id_i))^{n-2} \\
&+ \quad (n-1)(n-2)\int_0^\infty (x+d_i)h^{min}(x)(1-H^{min}(x))^{n-3}dx.
\end{aligned}$$

As n approaches to infinity, the first term in the right-hand side simply converges to zero by the ratio test. Also by the Lebesque Convergence Theorem, since $(n-1)(n-2)(x+d_i)h^{min}(x)(1-H^{min}(x))^{n-3}$ converges to zero as n approaches to infinity, the second term also converges to zero. □

Thus, we conclude that for every community, the additional benefit from bidding less than its disutility minus the minimum externality to be a host is small, if there exist sufficiently many participating communities. In other words, as the number of communities approaches to infinity, the incentive to bid less disappears and each bid converges to the community's disutility minus the minimum externality it suffers.

5.2.2 Symmetric Bayesian Nash Equilibrium in Symmetric Environment

Next, we seek a symmetric Bayesian Nash equilibrium in the symmetric model setting. The 'symmetric model set up' requires three conditions as below.

$$\begin{array}{ll} i) & d_i \overset{\text{iid}}{\sim} F \text{ on } D = [\underline{d}, \overline{d}] \text{ with probability density function } f \\ ii) & \alpha_i = \frac{1}{n} \\ iii) & \sum_j g(\gamma_{ij}) = \sum_j g(\gamma_{kj}) \text{ for all } i, k = 1, 2, ...n. \end{array}$$

The third condition might seem too strong. However it applies to every symmetric geographic model that guarantees for every player the same sum of the distances from the other players. For example, the circular model with constant inbetween distances among the players naturally satisfies the last condition.

Proposition 3. *In the symmetric environment satisfying the three conditions above, it is a symmetric Bayesian Nash equilibrium for each community to bid according to the bidding function $\beta : D \to D$, where*

$$\beta(d) = d - \Big(\frac{1}{n-1}\sum_j g(\gamma_{ij})\Big)d - \frac{\int_{\underline{d}}^{d} (1 - \frac{1}{n-1}\sum_j g(\gamma_{ij}))F(z)^n dz}{F(d)^n}.$$

Proof. Under the symmetric model setting, the expected payoff the community i maximizes is as below.

$$\begin{aligned} \max_{b_i} EU_i(d_i, b_i, \gamma_i, \frac{1}{n}) &= (n-1)\int_{\beta^{-1}(b_i)}^{\overline{d_i}} (-d_i + \frac{n-1}{n}\beta(x))f(x)(1-F(x))^{n-2}dx \\ &\quad + \sum_{j \neq i}(-\frac{1}{n}b_i - g(\gamma_{ij})d_i)F(\beta^{-1}(b_i))(1 - F(\beta^{-1}(b_i)))^{n-2} \\ &\quad + \sum_{j \neq i}(n-2)\int_{\underline{d}}^{\beta^{-1}(b_i)} (-\frac{1}{n}\beta(x) - g(\gamma_{ij})d_i)f(x)F(x)(1-F(x))^{n-3}dx \\ &= (n-1)\int_{\beta^{-1}(b_i)}^{\overline{d}} (-d_i + \frac{n-1}{n}\beta(x))f(x)(1-F(x))^{n-2}dx \\ &\quad + (n-1)(-\frac{1}{n}b_i)F(\beta^{-1}(b_i))(1-F(\beta^{-1}(b_i)))^{n-2} \\ &\quad + (n-1)(n-2)\int_{\underline{d}}^{\beta^{-1}(b_i)} (-\frac{1}{n}\beta(x))f(x)F(x)(1-F(x))^{n-3}dx \\ &\quad - [F(\beta^{-1}(b_i))(1-F(\beta^{-1}(b_i)))^{n-2}]d_i(\sum_j g(\gamma_{ij})) \\ &\quad - [(n-2)\int_{\underline{d}}^{\beta^{-1}(b_i)} F(x)(1-F(x))^{n-3}f(x)dx]d_i(\sum_j g(\gamma_{ij})). \end{aligned}$$

Now differentiating the expected payoff with respect to b_i and letting $b_i = \beta(d_i)$ for symmetric Bayesian Nash Equilibrium,

$$n(d_i - \beta(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})d_i}{n-1})f(d_i) = F(d_i)\beta'(d_i). \tag{1}$$

We use Cramton and Gibbons' method for solving this differential equation here. Multiplying $F(d_i)^{n-1}$ to both sides in (1),

$$n(d_i - \beta(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})d_i}{n-1})f(d_i)F(d_i)^{n-1} = F(d_i)^n\beta'(d_i). \tag{2}$$

Differentiating $(d_i - \beta(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})d_i}{n-1})$ as below and substituting (2) into the result,

$$\begin{aligned}
&\frac{\partial(d_i - \beta(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})d_i}{n-1})F(d_i)^n}{\partial d_i} \\
= \ & (1 - \beta'(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})}{n-1})F(d_i)^n + n(d_i - \beta(d_i) - \frac{\sum_{j\neq i} g(\gamma_{ij})d_i}{n-1})f(d_i)F(d_i)^{n-1} \\
= \ & (1 - \frac{\sum_{j\neq i} g(\gamma_{ij})}{n-1})F(d_i)^n.
\end{aligned}$$

Thus, the optimal bidding function of i, which is a best response strategy in symmetric Bayesian Nash equilibrium is as below.

$$\beta(d) = d - \Big(\frac{1}{n-1}\sum_j g(\gamma_{ij})\Big)d - \frac{\int_{\underline{d}}^{d}(1 - \frac{1}{n-1}\sum_j g(\gamma_{ij}))F(z)^n dz}{F(d)^n}.$$

□

Interestingly, as n approaches to infinity, the last term in the bidding function in Proposition 3 converges to zero since $0 < 1 - \frac{1}{n-1}\sum_j g(\gamma_{ij}) < 1$. Therefore, again as the number of communities increases, the best response bidding in symmetric Bayesian Nash equilibrium converges to the average disutility subtracted from the disutility to be a host.

5.3 Properties

Now we check if the Minehart and Neeman's second price-auction-like-procedure satisfies three main properties for so-called 'good' mechanisms for allocating public bads: the budget balance, the efficiency, and the individual rationality. Since we designed a budget-balanced payment rule for the mechanism from the beginning, we only need to check whether the other two properties hold.

5.3.1 Efficiency

The change in the best response bidding functions in the equilibrium above, induced by the externalities, has a great potential to lower the efficiency. In the previous model without identity-dependent externalities, since bidding truthfully was both a Bayesian Nash equilibrium and ϵ-equilibrium when sufficiently many communities participate in the game, it leads to the efficiency directly here. This is because both the host selection in the equilibrium of the mechanism and that maximizing the social welfare (*i.e.* efficient host selection) minimize the same objective function as below.

$$i^*_{eff} = i^*_{equil} = argmin_i d_i.$$

However in the model with externalities, the externalities have to be subtracted from the disutility term to form an optimal bid of each community. First, in the symmetric environment, the optimal bid in the equilibrium converges to the true disutility minus the average externality. As a result, the government will choose the lowest bidder in the equilibrium, i^*_{sym} as below.

$$\begin{aligned} i^*_{sym} &= argmin_i\Big[d_i - \Big(\frac{1}{n-1}\Big)\sum_{j\neq i} g(\gamma_{ij})d_i\Big] \\ &= argmin_i\Big[\Big(1 - \frac{1}{n-1}\sum_{j\neq i} g(\gamma_{ij})\Big)d_i\Big]. \end{aligned}$$

Second, in the asymmetric environment, the optimal bid in the equilibrium converges to its disutility minus the minimum externality. The government will choose the lowest bidder in the equilibrium, i^*_{asy} as below.

$$\begin{aligned} i^*_{asy} &= argmin_i\Big[\ d_i - \Big(\frac{1}{n-1}\Big)\min_{m\neq i} g(\gamma_{ij})d_i\ \Big] \\ &= argmin_i\Big[\ \Big(1 - \frac{1}{n-1}\min_{m\neq i} g(\gamma_{ij})\Big)d_i\ \Big]. \end{aligned}$$

Meanwhile, the social-welfare maximizing, thus efficient, host selection must be as below.

$$i^*_{eff} = argmin_i\Big[d_i + \sum_{i\neq j} g(\gamma_{ij})d_j\Big].$$

Since none of the two host selection rules have the same objective function with the efficient allocation rule, the efficiency is hard to be achieved in consideration of the externalities.

5.3.2 Individual Rationality

The voluntary participation of every community in the game is still guaranteed in the model with externalities. The following proposition shows how it works.

Proposition 4. *Minehart and Neeman's auction-like procedure is still ex post individually rational even in consideration of the externalities.*

Proof. We should show that in both cases of being the host and one of the non-hosts, participating in the game is always better than nonparticipation, even from the ex-post perspective.

Lemma 1. *In the case of not hosting the noxious facility, it is strictly dominant to participate from the ex-post perspective. Thus, for all $j \neq i$,*

$$-\frac{w_i}{\sum_{j=1}^{n} w_j}\Big(d_{[2]}\Big(\sum_{j=1}^{n} w_j\Big)\Big) - g(\gamma_{ij})d_i \geq -d_i(w_i) - g(\gamma_{ij})d_i.$$

Since the negative externalities can be crossed out from both sides above, the inequality holds according to the previous proof in Minehart and Neeman's study.

Lemma 2. *In the case of hosting the noxious facility, it is strictly dominant to participate from the ex-post perspective. Thus, for all $j \neq i$,*

$$-d_i\Big(\sum_{j=1}^{n} w_j\Big) + \Big(1 - \frac{w_i}{\sum_{j=1}^{n} w_j}\Big)\Big(d_{[2]}\Big(\sum_{j=1}^{n} w_j\Big)\Big) \geq -d_i(w_i) - g(\gamma_{ij})d_i.$$

Even without the last negative externality term in the right side above, the inequality holds according to Minehart and Neeman's study. Thus, the inequality above definitely holds. □

6 Implications

6.1 Economic Implications in Terms of Efficiency

Bidding 'the true disutility minus the average externality' in the symmetric model has a reasonable economic meaning. If a player suffers serious externalities from the neighbor even when not hosting the facility, it might lose its strong incentive not to be the host but rather lowers its bid to rather focus on lowering its potential tax payment. On the other hand if a player suffers little externalities, it might have greater incentive to avoid the risk of being the host so that it would bid its disutility with small degradation. Thus the decrease in the bid would be proportional to the amount of the externalities one suffers.

Also, taking a closer view on this Bayesian Nash equilibrium, we can realize that it has a great tendency to lower the efficiency. In other words, it mostly fails to choose the socially desirable host. For example, imagine two different cities; one surrounded by a lot of neighbor communities and the other isolated one. Absolutely, the average distance from the other communities would be smaller in the case of surrounded one rather than in that of isolated one. Thus, the isolated city's bid tends to be lowered by a smaller amount than the crowded one, enhancing the possibility for the crowded city to be the lowest bidder. In other words, in this model, the noxious facility tends to be located in the center of the town rather than in the countryside, which is socially inefficient. This finding supports our previous analysis in Section 5.3.1.

Likewise, in case of the ϵ-equilibrium, 'its disutility minus the minimum externality' bidding strategy has great tendency to choose a socially undesirable host, leading to an inefficient outcome. For example, in the case of a circular city or a linear city (the asymmetric environment), or even in the case of city with randomly scattered communities, the host tend to be chosen from the center of the city rather than the circumference, the endpoints, or the outskirts of the city under these bidding strategies. This is because the minimum locational externality increase as the community moves toward the center. Since the center usually generates negative externalities to a wider range of the neighborhoods than the outskirts, this equilibrium again tends to be inefficient.

6.2 Policy Implications

The findings in this study can be interpreted as the failure of Minehart and Neeman's auction-like procedure for siting noxious facilities. Thus, in applying this mechanism to solve NIMBY phenomena, policy makers should be careful unless the size of the externalities are too ignorable to lower the efficiency. Also, even if the budget balance is one key property that influences the efficiency of a mechanism, if the externalities have any potential to increase the efficiency loss in the society, the VCG mechanism that always guarantee the efficiency even in consideration of the externalities should be preferentially applied rather than the budget-balanced mechanisms.

7 Conclusion

On the issue of the tradeoff between the efficiency and the budget balance for siting noxious public facilities, Minehart and Neeman have proved that their budget-balanced and ex-post individually rational second-price-auction-like-procedure can even almost achieve the efficiency in some sense. In detail, they proved that truthful bidding is an ϵ-equilibrium for any small ϵ and a Bayesian Nash equilibrium in the asymmetric and the symmetric environments, respectively.

However this study revealed the critical drawback of their mechanism in achieving the efficiency. In consideration of the negative externalities that vary according to how much influential the real host is to oneself, such as how far the host is away from oneself, the mechanism reached different equilibria from the truth-telling one. Specifically, bidding 'its disutility minus the minimum externality' has been shown to be ϵ-equilibrium for any positive ϵ in the asymmetric environments where the disutilities are independently but not identically distributed. Also, in the symmetric environment where the disutilities are identically and independently distributed, each community generates the same share of the waste, and the average influential powers that other communities can exert to oneself remains the same over the communities, bidding strategy in symmetric Bayesian Nash equilibrium has converged to 'the true disutility minus the average externality' as the number of the communities approaches infinity. Also, the voluntary participation of every community has been proved to be still valid like in the original case without the identity-dependent externalities.

These results brought some economic and political implications in applying Minehart and Neeman's mechanism for siting noxious facilities. The best response bids in the equilibria had potentials to cause inefficiency since the community surrounded by a lot of neighbors and the one near the center tend to be chosen as the host within these bids. Thus, the policy makers must take this problem carefully when putting the efficiency as the first priority over the budget balance, which is more rational.

This study has academically contributed by considering the identity-dependent externalities with the 'budget-balanced siting mechanisms'. While there had been numerous studies on identity-dependent externalities, these studies haven't required the budget balance (Jehiel *et al.*, 1996 and Jehiel *et al.*, 1998). Interestingly, bidding 'the true disutility minus the average externality' is regarded as the optimal choice in non-budget-balanced second price auction just as our budget-balanced second-price-auction-like-procedure. However, the important difference comes from that such bidding is weakly dominant strategy in the former one while it is only Bayesian Nash equilibrium strategy in the latter one.

For the further studies, firstly as mentioned before, it would be meaningful to model the bidding function that takes an n-dimensional type vector as an input and matches it to one-dimensional value. Here, we have endeavored to find a way of solving a model with n-dimensional vectors independently following the n-dimensional distribution. Instead of using an inverse function of the bidding function, which does not exist, one can use vector transformation and work on solving the vector-differential equations. The n-dimensional distribution analysis would be similar setting with Jehiel *et al.* (1998) but would make another contribution since it is budget-balanced. Second, within this budget-balanced model with asymmetric externalities, even though we proved that it might lead to the inefficient outcome here, one might be interested in checking whether such efficiency loss can be bounded and by how much. Lastly, as Moulin sought a Clarke-Groves mechanism with minimized budget-imbalance, it would be also meaningful to generalize this model by introducing the identity-dependent externalities and seek another Clarke-Groves mechanism with budget surplus minimization. This approach would differ from our approach in this paper since it perfectly guarantees the efficiency from the beginning and rather tries to minimize the budget imbalance.

References

Ahlfeldt, G., and W. Maennig (2012): "Voting on a NIMBY facility: proximity cost of an "iconic" stadium", *Urban Affairs Review*, 48, 205–237.

Arrow, K. (1979): "The property rights doctrine and demand revelation under incomplete information", Issue 580 of Discussion Paper, Harvard Institute of Economic Research.

Bellettini, G., and H. Kempf (2013): "Why not in your backyard? On the location and size of a public facility", *Regional Science and Urban Economics*, 43, 22–30.

Clarke, E. H. (1971): "Multipart pricing of public goods", *Public Choice*, 11, 17–33.

Clarke, E. H. (1972): "Multipart pricing of public goods: An example", in *Public Prices for Public Products*, ed. by S. J. Mushkin. Washington: The Urban Institute, 125–130.

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a Partnership Efficiently", *Econometrica*, 55, 615–632.

d'Aspremont, C., and L. Gerard-Varet (1979): "Incentives and Incomplete Information", *Journal of Public Economics*, 11, 25–45.

Green, J., E. Kohlberg, and J.-J. Laffont (1976): "Partial equilibrium approach to the free rider problem", *Journal of Public Economics*, 6, 375–394.

GROVES, T., AND J. O. LEDYARD (1977): “Some limitations of demand revealing process, *Public Choice*, 29, 107–124.

JEHIEL, P., B. MOLDOVANU, AND E. STACCHETTI (1998): “Multidimensional mechanism design for auctions with externalities”, *Journal of Economic Theory*, 85, 258–293.

JEHIEL, P., B. MOLDOVANU, AND E. STACCHETTI (1996): “How (not) to sell nuclear weapons”, *The American Economic Review*, 86, 814–829.

KRISHNA, V. (2010): *Auction Theory* (Second Ed.), San Diego: Academic Press.

KUNREUTHER, H., AND P. R. KLEINDORFER (1986): “A sealed-bid auction mechanism for siting noxious facilities”, *The American Economic Review*, 76, Papers and Proceedings, 295–299.

KUNREUTHER, H., P. KLEINDORFER, P. J. KNEZ, AND R. YAKSICK (1987): “A compensation mechanism for siting noxious facilities: Theory and Experimental Design”, *Journal of Environmental Economics and Management*, 14, 371–383.

MINEHART, D., AND Z. NEEMAN (2002): “Effective siting of waste treatment facilities”, *Journal of Environmental Economics and Management*, 43, 303–324.

MOULIN, H. (2009): “Almost budget-balanced VCG mechanisms to assign multiple objects” *Journal of Economic Theory*, 144, 96–119.

MOULIN, H., AND S. SHENKER (2001): “Strategyproof sharing of submodular costs: budget balance versus efficiency”, *Journal of Economic Theory*, 18, 511–533.

MYERSON, R. B., AND M. A. SATTERTHWAITE (1983): “Efficient mechanisms for bilateral trading”, *Journal of Economic Theory*, 29, 265–281.

TIDEMAN, T. N., AND G.TULLOCK (1976): “A new and superior process for making social choices”, *The Journal of Political Economy*, 84, 1145–1159.

O’HARE, M. (1977): “Not on my block you don’t: Facility siting and the strategic importance of compensation”, *Public policy*, 25, 409-458.

ROMERO, F. C., AND R. D. PAREDES (2013): “A decentralized mechanism for efficient and fair localization of NIMBYs”, Working Paper.

SAKAI, T. (2012): “Fair waste pricing: an axiomatic analysis to the NIMBY problem”, *Economic Theory*, 50, 499–521.

Shoham, Y., and K. Leyton-Brown (2009): *Multiagent Systems*, New York: Cambridge University Press.

Song, B., J. R. Morrison, and Y. Ko (2013): "Efficient location and allocation strategies for undesirable facilities considering their fundamental properties", *Computers & Industrial Engineering*, 65, 475–484.