

000 001 002 003 004 005 006 007 008 009 010 FEATURE-FREE APPROACH FOR SAT SOLVER SELECTION

005 **Anonymous authors**

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009 ABSTRACT

011 Boolean Satisfiability Problem is a cornerstone in computer science and artifi-
 012 cial intelligence, underpinning numerous applications through its ability to solve
 013 complex computational problems. However, existing SAT solvers face significant
 014 limitations, including the complexity and domain expertise required for feature
 015 design, the static nature of many feature sets that limit adaptability to evolving
 016 problem structures, and the poor generalization of handcrafted features to new
 017 instances, thereby constraining performance across diverse SAT problem distribu-
 018 tions. To address these challenges, we introduce a Feature-Free SAT Solver Selec-
 019 tion (F2S3), which integrates the Sensitive-Associative Cascade Forest (SACF),
 020 Correlation Refinement Factor Graph (CRFG), and Dual-Proximity Graph Repre-
 021 sentation (DPGR) to address the complexities of SAT problems. F2S3 method
 022 transforms problem instances into graph data, utilizes CRFG to maintain the
 023 higher-order nature of the graph structure and node relationships, and uses DPGR
 024 to enhance the graph data features and map them to low-dimensional vectors. This
 025 approach effectively captures the structural intricacies of graph data and improves
 026 feature representation in low-dimensional spaces, overcoming the limitations of
 027 previous methods regarding feature sparsity and generalization ability. Experi-
 028 ments conducted on datasets from the ASlib database demonstrate that F2S3
 029 outperforms existing solutions, particularly in scenarios where previous meth-
 030 ods were hindered by challenges such as feature sparsity and computational in-
 031 efficiency. The method’s performance is evaluated across multiple competitive
 032 datasets, showing high gap values and consistent robustness.

033 1 INTRODUCTION

035 Given a Boolean formula, the Boolean satisfiability (SAT) problem determines whether there ex-
 036 ists a satisfying assignment. As the first problem recognized as NP-complete, the SAT problem
 037 exerts significant influence in the fields of computer science and artificial intelligence. Numer-
 038 ous problems from various domains—including logic, graph theory, operations research, automated
 039 planning, formal verification, and more—can be transformed into SAT problems or require the use
 040 of a SAT oracle Alyahya et al. (2022). Consequently, any advancements in SAT solving can cre-
 041 ate a ripple effect across these related fields. Despite its intrinsic complexity, the development of
 042 the Davis–Putnam–Logemann–Loveland (DPLL) algorithm Davis & Putnam (1960); Govindasamy
 043 et al. (2024), along with the subsequent Conflict-Driven Clause Learning (CDCL) framework Zhang
 044 & Malik (2002); Marques-Silva et al. (2021), has enabled current state-of-the-art solvers to effec-
 045 tively handle SAT instances with millions of variables.

046 While SAT solving is a hallmark of symbolic artificial intelligence, recent years have witnessed
 047 a revolutionary impact on computer science from another branch of artificial intelligence: data-
 048 dependent algorithms or machine learning approaches. This raises a natural question: Can machine
 049 learning enhance SAT solving? There are two main lines of research that can be explored to address
 050 this problem. The relatively new line, which has attracted more attention recently, is the end-to-end
 051 SAT solving, exemplified by NeuroSAT Selsam et al. (2019). NeuroSAT is an experimental SAT
 052 solver that learns to solve SAT problems after being trained as a classifier to predict satisfiability.
 053 It utilizes a message-passing neural network to predict the satisfiability of random SAT problems
 and learns to search for satisfying assignments to explain that bit of supervision. When it guesses
 satisfiable, the satisfying assignment can often be decoded from its activations. However, the scale

054 and complexity of the problems addressed by this method are relatively small, and it often exhibits
 055 unstable performance on instances that deviate from the distribution of its training data.
 056

057 The second line of research, leveraging machine learning to improve CDCL SAT solving, is more
 058 practical in applications. Key approaches include SAT algorithm configuration and algorithm selec-
 059 tion. The former involves automatically identifying parameter settings that optimize a given SAT
 060 solver’s performance on a specific set or distribution of problem instances, while the latter refers to
 061 selecting the most suitable solver from a “portfolio” of different algorithms for a given SAT instance.
 062 This paper focuses on the latter. Although creating a single, universally effective SAT solver seems
 063 intuitive, it is well known in the community that no one solver dominates all others across diverse
 064 problem instances. SAT Solver Selection (SSS) Kerschke et al. (2019); Alissa et al. (2023) exploits
 065 machine learning techniques to dynamically select the most appropriate solver and thereby com-
 066 bine the complementary strengths of different algorithms across problem instances to improve over-
 067 all performance. By dynamically selecting the most appropriate solver based on instance-specific
 068 features, SSS can substantially enhance solving efficiency. SATzilla Xu et al. (2008) is the first
 069 successful and best known portfolio-based SAT solver. Its initial version innovatively employed
 070 ridge regression techniques to accurately predict the efficiency of the solver when dealing with un-
 071 known SAT instances. Leveraging this technological edge, SATzilla has won multiple gold medals
 072 in SAT competitions, setting a significant milestone in the development of the SAT solving field.
 073 Additionally, methods such as 3S Kadioglu et al. (2011), MapleCOMSPS Liang et al. (2016), and
 074 Kissat_MAB Cherif et al. (2021) have also emerged as winners in relevant competitions. Lingeling
 075 ayv algorithmBiere (2014) outperformed 34 other solvers on 300 benchmark instances in the 2014
 076 SAT competition with a 77% completion rate. GraSS Zhang et al. (2024) builds upon traditional SAT
 077 solver selection methods by incorporating graph structures, enabling more effective use of the struc-
 078 tural information in SAT problems with promising results. In addition to SSS, algorithm selection
 079 has also been successfully applied in areas like the DelfiKatz et al. (2018), QBF PortfolioHoos et al.
 080 (2018), and automated machine learning (AutoML)Feurer et al. (2018).
 081

082 A critical limitation in the existing SSS methods is their heavy reliance on handcrafted features,
 083 which are manually designed based on expert knowledge. While these features can be effective, they
 084 introduce several challenges. First, the process of designing features is often complex and requires
 085 domain expertise. Second, many feature sets have remained unchanged for years, limiting their
 086 adaptability to evolving problem structures. This limited adaptability is sometimes compounded by
 087 the non-negligible cost of computing such features, especially when dealing with large or complex
 088 instances. Third, handcrafted features may not generalize well to novel instances, thereby constrain-
 089 ing model performance across diverse SAT problem distributions. To address these challenges,
 090 feature-free approaches that automatically learn hidden structures from raw problem instances have
 091 emerged as a promising alternative. By directly capturing complex patterns without manual fea-
 092 ture engineering, feature-free methods reduce the reliance on expert knowledge and improve model
 093 generalization across varied SAT instances.
 094

095 This paper proposes a Feature-Free method for SAT Solver Selection, F2S3, which integrates Corre-
 096 lation Refinement Factor Graph, Dual-Proximity Graph Representation, and Sensitive-Associative
 097 Cascade Forest. By leveraging deep graph embedding techniques and an advanced cascade forest
 098 model, our method optimizes both the representation of SAT instances and the decision-making pro-
 099 cess in solver selection. This results in a more flexible and effective algorithm selection strategy that
 100 is capable of automatically adapting to problem variations and providing more accurate SAT-solving
 101 recommendations.
 102

103 The major contributions of this work are as follows:
 104

- 105 • We propose F2S3, a feature-free SAT solver selection method that eliminates manual fea-
 106 ture engineering, mitigates graph sparsity, and improves prediction accuracy and robust-
 107 ness.
- 108 • We introduce Correlation Refinement Factor Graph, Dual-Proximity Graph Representa-
 109 tion, and the Sensitivity-Correlation Cascaded Forest to enhance graph representation and
 110 decision accuracy, addressing sparsity and refining ensemble decision-making.
- 111 • Experimental results show that F2S3 outperforms existing methods, improving selection
 112 accuracy and adaptability across various task settings.

108 **2 RELATED WORK**
109110 **2.1 GRAPH NEURAL NETWORKS IN SAT SOLVING**
111112 Graph neural networks (GNNs) have recently attracted attention in the SAT community, as Boolean
113 formulas can be naturally represented as graphs. NeuroSAT Selsam et al. (2019) first demonstrated
114 the potential of message-passing networks to capture satisfiability patterns directly from formula
115 structure. Later work extended this idea with NsNet Li & Si (2022), which incorporated probabilistic
116 inference, and GraSS Zhang et al. (2024), which introduced heterogeneous representations with
117 task-sensitive objectives. These studies show that graph-based models can complement traditional
118 handcrafted statistics. They highlight the promise of GNNs in learning structural properties of SAT
119 instances, while also pointing to opportunities for closer integration with efficient CDCL-based
120 solving procedures.
121122 **2.2 SAT SOLVER SELECTION**
123124 Solver selection has become an important topic in SAT research. SATzilla Xu et al. (2008) pi-
125oneered performance prediction using instance-level features, which inspired a variety of subse-
126 quent approaches, including cost-sensitive clustering Malitsky et al. (2013), nearest-neighbor strate-
127 gies Nikolić et al. (2013), and automated machine learning pipelines Malone et al. (2017). More
128 recent frameworks such as Sunny-as2 Liu et al. (2021) refine feature selection, solver scheduling,
129 and pre-solver strategies, achieving strong results across benchmarks. While these approaches have
130 demonstrated substantial success, they typically rely on handcrafted features. It led to excessive
131 computation time and hindered further improvement in generalization due to the lack of introduc-
132 ing new features, which motivates exploration of representation learning methods to remove the
133 dependence on manual feature engineering in this study.
134135 **2.3 GRAPH EMBEDDING AND CASCADE FOREST**
136137 The main idea of this study is to integrate graph embedding and deep random forest to achieve
138 feature-free SAT solver selection. Graph embedding provides a framework for learning low-
139 dimensional representations of graph-structured data, using techniques such as random walks, ma-
140 trix factorization, and deep autoencoders Zheng et al. (2023). These methods capture structural
141 relationships, which are especially beneficial for SAT, where formulas are represented as bipartite or
142 heterogeneous graphs. The Node-Similarity Factor Graph Liu et al. (2023) is particularly effective
143 for SAT due to its ability to preserve both local and global structural dependencies. Random For-
144 est Rigatti (2017) leverages an ensemble of decision trees for prediction, while Cascade Forest Zhou
145 & Feng (2019) enhances prediction accuracy through hierarchical optimization.
146147 **3 APPROACH**
148149 This section introduces our approach, and Figure 1 illustrates the overall workflow.
150151 **3.1 PROBLEM DEFINITION**
152153 The satisfiability (SAT) problem involves deciding whether a satisfying assignment exists for a Con-
154 junctive Normal Form (CNF) formula as $F = c_1 \wedge c_2 \wedge \dots \wedge c_m$, where each clause c_i is a disjunction
155 of literals, and each literal is either a Boolean variable x or its negation $\neg x$. Given a fixed set of SAT
156 solvers $\{A_1, A_2, \dots, A_K\}$, the problem of *SAT Solver Selection* is to construct a mapping from a
157 given SAT problem instance to its optimal solver. The task of this paper is to use feature-free method
158 to learn this mapping by leveraging a training set of SAT instances paired with their optimal solvers,
159 enabling the model to predict the most suitable solver for unseen instances during the test phase.
160161 **3.2 CORRELATION REFINEMENT FACTOR GRAPH CONSTRUCTION**
162163 Inspired by the Node-Similarity Factor Graph Liu et al. (2023), we construct an initial factor graph
164 for each SAT instance based on its Conjunctive Normal Form (CNF) representation. The resulting
165

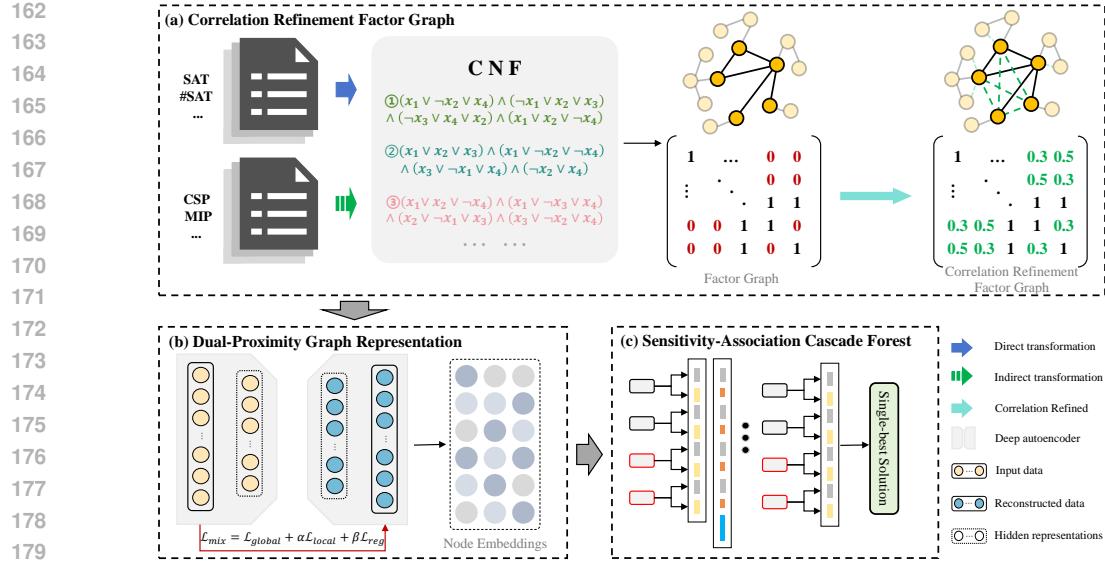


Figure 1: Overview of the proposed F2S3 framework. A SAT instance is first transformed into a Correlation Refinement Factor Graph (CRFG), then mapped into low-dimensional embeddings via the Dual-Proximity Graph Representation (DPGR), and finally fed into the Sensitive-Associative Cascade Forest (SACF) to predict the optimal solver.

factor graph consists of two types of nodes: variable nodes x_j corresponding to the Boolean variables, and factor nodes c_i corresponding to the clauses. An edge is established between a variable node and a factor node whenever the variable x_j or its negation $\neg x_j$ appears in the clause c_i . This graph is represented by an adjacency matrix $\mathbf{S} \in \{0, 1\}^{m \times n}$, where the entry S_{ij} is 1 if the variable x_j or its negation $\neg x_j$ appears in clause c_i , and 0 otherwise.

However, the adjacency matrix \mathbf{S} is often highly sparse, as each clause c_i involves only a small subset of variables. Consequently, many variable and clause nodes lack direct connections and are only related through long indirect paths. This confines information propagation to local neighborhoods, weakens the capture of global dependencies, and ultimately leads to insufficient graph representations. Therefore, we introduce the *Correlation Refinement Factor Graph* (CRFG), which refines the original factor graph to alleviate sparsity while maintaining its structural integrity, as detailed in the following.

Factor node correlation optimization. For each factor node c_i , we define its associated set of variable nodes as $L(c_i) = \{x_j \mid x_j \in c_i \text{ or } \neg x_j \in c_i\}$. If two factor nodes c_i and c_k share at least one variable, i.e., $L(c_i) \cap L(c_k) \neq \emptyset$, we introduce a *direct similarity* defined by

$$P_{c_i, c_k}^{\text{direct}} = \frac{|L(c_i) \cap L(c_k)|}{\sqrt{|L(c_i)| \cdot |L(c_k)|}}.$$

Moreover, if two factor nodes are not directly connected but are indirectly related through other nodes, we define an *indirect similarity* as

$$P_{c_i, c_k}^{\text{indirect}} = \frac{N(c_i) \cdot N(c_k)}{\|N(c_i)\| \cdot \|N(c_k)\|},$$

where $N(c_i)$ and $N(c_k)$ denote the neighborhood vectors of factor nodes c_i and c_k , respectively.

Variable node correlation extension. For each variable node x_j , we define its associated factor set as $C(x_j) = \{c_i \mid x_j \in c_i \text{ or } \neg x_j \in c_i\}$. If two variables x_p and x_q co-occur in common clauses, i.e., $C(x_p) \cap C(x_q) \neq \emptyset$, they are connected with a *direct similarity*:

$$P_{x_p, x_q}^{\text{direct}} = \frac{|C(x_p) \cap C(x_q)|}{\sqrt{|C(x_p)| \cdot |C(x_q)|}}.$$

216 For variables without direct co-occurrence but with similar neighborhoods, we introduce an *indirect*
 217 *similarity* defined as

$$218 \quad 219 \quad P_{x_p, x_q}^{\text{indirect}} = \frac{N(x_p) \cdot N(x_q)}{\|N(x_p)\| \cdot \|N(x_q)\|},$$

220 where $N(x_p)$ and $N(x_q)$ denote the neighborhood vectors of variables x_p and x_q , respectively.
 221

222 **Adjacency matrix optimization.** Finally, we integrate the direct and indirect similarities of both
 223 factor nodes and variable nodes to obtain the optimized bipartite adjacency matrix:
 224

$$225 \quad \mathbf{S}' = \lambda \mathbf{P}_{\text{direct}} + \mu \mathbf{P}_{\text{indirect}},$$

226 where λ and μ are trade-off parameters balancing local and global similarity. Compared to the
 227 original matrix $\mathbf{S} \in \{0, 1\}^{m \times n}$, the optimized matrix $\mathbf{S}' \in \mathbb{R}^{m \times n}$ is denser, significantly reduces
 228 zero elements, and enhances latent correlations among nodes.
 229

230 3.3 DUAL-PROXIMITY GRAPH REPRESENTATION

231 After obtaining the optimized factor graph from the *Correlation Refinement Factor Graph (CRFG)*,
 232 we derive a refined bipartite adjacency matrix $\mathbf{S}' \in \mathbb{R}^{m \times n}$ that encodes richer structural correlations
 233 between clauses and variables. While \mathbf{S}' alleviates the sparsity inherent in the original factor
 234 graph, it still represents a purely structural enhancement and does not directly yield compact feature
 235 representations suitable for downstream learning tasks. To bridge this gap, we propose the *Dual-
 236 Proximity Graph Representation (DPGR)*. DPGR takes \mathbf{S}' as input, employs a deep auto-encoder
 237 to learn low-dimensional embeddings, and jointly integrates global and local proximity constraints.
 238 This ensures that the learned representations preserve local pairwise correlations while simultane-
 239 ously capturing global structural dependencies.
 240

241 **Auto-encoder representation learning.** Given the refined bipartite adjacency matrix $\mathbf{S}' \in \mathbb{R}^{m \times n}$,
 242 each row $\mathbf{z}_i^{(c)} \in \mathbb{R}^n$ corresponds to the neighborhood vector of a clause c_i , and each column $\mathbf{z}_j^{(x)} \in$
 243 \mathbb{R}^m corresponds to the neighborhood vector of a variable x_j . For notational simplicity, we use \mathbf{z}_i
 244 to denote the adjacency vector of a node, which can be either a clause or a variable. These vectors
 245 serve as the input to a deep auto-encoder that maps them into a low-dimensional latent space.
 246

247 The encoder applies a sequence of nonlinear transformations:

$$248 \quad \mathbf{y}_i^{(1)} = \sigma(W^{(1)} \mathbf{z}_i + b^{(1)}), \quad \mathbf{y}_i^{(k)} = \sigma(W^{(k)} \mathbf{y}_i^{(k-1)} + b^{(k)}), \quad k = 2, \dots, K,$$

249 where $\mathbf{y}_i^{(k)}$ denotes the hidden representation of node i at the k -th layer, and $\sigma(\cdot)$ is a nonlinear
 250 activation function. $W^{(k)}$ and $b^{(k)}$ are the weight matrix and bias vector of the k -th encoder layer,
 251 respectively. The decoder reconstructs the original adjacency vector, producing $\hat{\mathbf{z}}_i$, which is com-
 252 pared with \mathbf{z}_i to compute the reconstruction error.
 253

254 **Local proximity preservation.** To preserve local structural correlations, we introduce a local
 255 proximity loss. If two nodes exhibit strong correlation in the optimized adjacency matrix \mathbf{S}' , their
 256 embeddings should be closely aligned in the latent space. Formally:
 257

$$258 \quad 259 \quad \mathcal{L}_{\text{local}} = \sum_{i,j} S'_{ij} \|\mathbf{y}_i^{(K)} - \mathbf{y}_j^{(K)}\|_2^2,$$

260 where S'_{ij} denotes the correlation strength between clause c_i and variable x_j derived from CRFG.
 261

262 **Global proximity preservation.** To capture global structural dependencies, DPGR incorporates
 263 a global proximity loss based on the reconstruction error of the auto-encoder. Even if two nodes
 264 are not directly connected, their embeddings should be close if they share similar neighborhood
 265 structures:
 266

$$267 \quad 268 \quad \mathcal{L}_{\text{global}} = \sum_{i=1}^m \|\hat{\mathbf{z}}_i - \mathbf{z}_i\|_2^2,$$

269 where \mathbf{z}_i is the input adjacency vector and $\hat{\mathbf{z}}_i$ is its reconstruction. This constraint ensures that global
 270 topological patterns are preserved in the latent space.
 271

Regularization. To improve generalization and stabilize training, we add Frobenius norm penalties on both encoder and decoder weights. Let $W^{(k)}$ and $b^{(k)}$ denote the parameters of the k -th encoder layer, and $\widehat{W}^{(k)}$ and $\widehat{b}^{(k)}$ those of the corresponding decoder layer. The regularization term is defined as

$$\mathcal{L}_{reg} = \frac{1}{2} \sum_{k=1}^K \left(\|W^{(k)}\|_F^2 + \|\widehat{W}^{(k)}\|_F^2 \right),$$

where $\|\cdot\|_F$ denotes the Frobenius norm. This term constrains parameter magnitudes, reduces overfitting, and encourages smoother latent representations.

Overall objective. The complete optimization objective integrates direct proximity, indirect proximity, and regularization as follows:

$$\mathcal{L} = \mathcal{L}_{global} + \alpha \cdot \mathcal{L}_{local} + \beta \cdot \mathcal{L}_{req};$$

where α and β are hyperparameters controlling the balance among different loss components. The optimization procedure of DPGR follows the paradigm of Structural Deep Network Embedding Wang et al. (2016), and the detailed derivations are provided in the Appendix A.1.1.

After training, the decoder generates the reconstructed adjacency vector \hat{z}_i for each node. These reconstructed adjacency vectors represent the learned graph structure in the latent space and are used for evaluating the model's reconstruction quality. The learned node embeddings from the encoder, which summarize the structural properties of the entire SAT instance, are then used as the input to the subsequent supervised learning module.

3.4 SENSITIVE-ASSOCIATIVE CASCADE FOREST

We propose the *Sensitive-Associative Cascade Forest* (SACF), a supervised learning module built on the cascade forest Zhou & Feng (2019). SACF incorporates *feature sensitivity optimization* and *feature correlation enhancement* to mitigate the bias of conventional splitting criteria and explicitly capture feature interactions. As illustrated in Figure 2, the instance embedding \mathbf{z} is progressively refined into class distributions, sensitivity-adjusted distributions, correlation-enhanced vectors, and updated inputs, until producing the final solver prediction \hat{y} .

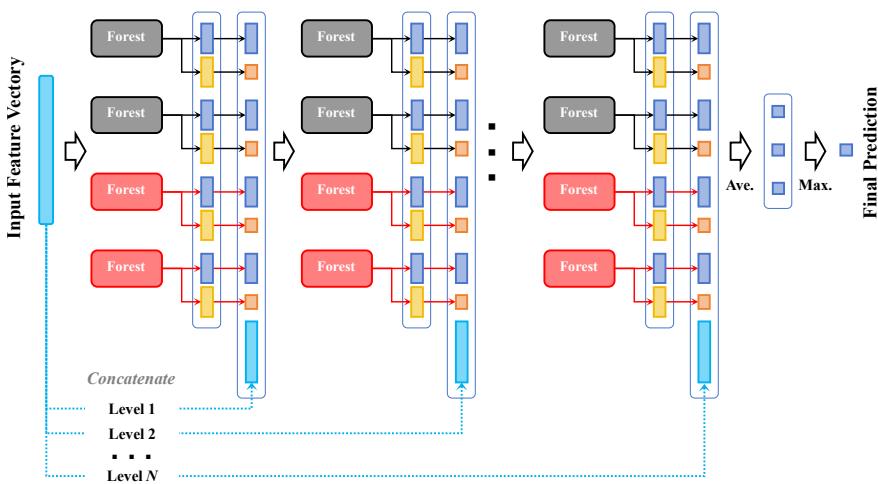


Figure 2: Architecture of the Sensitive-Associative Cascade Forest (SACF). Each layer includes two random forests (black) and two completely random forests (red). Outputs are optimized by feature sensitivity (yellow) and correlation enhancement (brown).

Layer-wise class distribution. At the ℓ -th level, each forest $\mathcal{F}_m^{(\ell)}$ produces a class distribution vector:

$$\mathbf{p}^{(\ell,m)}(\mathbf{z}) = (p_1^{(\ell,m)}, \dots, p_K^{(\ell,m)}),$$

324 where $p_k^{(\ell,m)}$ denotes the probability that solver A_k is optimal for the input \mathbf{z} . Concatenating the
 325 outputs of all forests gives:

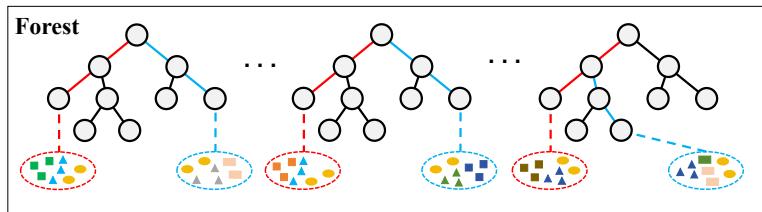
$$326 \quad \mathbf{v}^{(\ell)}(\mathbf{z}) = \text{concat}_m \mathbf{p}^{(\ell,m)}(\mathbf{z}).$$

329 **Feature sensitivity optimization.** The Gini index in decision tree splitting tends to favor high-
 330 cardinality but less informative features and fails to explicitly capture interactions among dimensions,
 331 which weakens the utilization of discriminative signals. To address this issue, SACF introduces a
 332 feature sensitivity optimization mechanism, where the term ‘‘feature’’ refers to embedding
 333 dimensions derived from the learned representation rather than handcrafted SAT statistics. This
 334 mechanism identifies critical dimensions by measuring their frequency in splitting chains and ex-
 335 plicitly reinforces their contribution in the leaf-node class distributions.

336 Specifically, each decision tree generates a *splitting chain*, i.e., the sequence of dimensions used
 337 from the root to the leaf. Based on all splitting chains, the sensitivity score of dimension f_j is
 338 defined as

$$339 \quad s_{f_j} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \mathbf{1}[f_j \in \text{chain}(t)],$$

341 where \mathcal{T} denotes the set of trees in the forest and $\mathbf{1}[\cdot]$ is the indicator function. The resulting vector
 342 $\mathbf{s} = (s_{f_1}, \dots, s_{f_d})$ quantifies the relative importance of each embedding dimension. The process is
 343 illustrated in Figure 3, where the original cascade forest paths (red) are complemented with addi-
 344 tional feature-sensitive branches (blue).



346 Figure 3: Feature sensitivity optimization in SACF. Red paths represent the original cascade forest,
 347 and blue paths denote the optimization branches.

348 For an input instance \mathbf{z} , suppose its corresponding class distribution at a leaf node is \mathbf{p} . SACF
 349 generates alternative paths dominated by highly sensitive dimensions and obtains a set of auxiliary
 350 class distributions $\{\hat{\mathbf{p}}^{(1)}, \hat{\mathbf{p}}^{(2)}, \dots\}$. The optimized distribution is then obtained by averaging the
 351 original and auxiliary distributions:

$$352 \quad \tilde{\mathbf{p}} = \text{mean}(\{\mathbf{p}\} \cup \{\hat{\mathbf{p}}^{(1)}, \hat{\mathbf{p}}^{(2)}, \dots\}).$$

353 **Feature correlation enhancement.** If two features f_i, f_j co-occur in the same splitting chain,
 354 they are considered correlated. The correlation strength is defined as:

$$355 \quad C_{ij} = \frac{\#\{\text{chains containing } f_i, f_j\}}{\#\{\text{chains}\}},$$

356 which yields a correlation matrix $\mathbf{C} \in \mathbb{R}^{d \times d}$.

357 We first select the top- k most sensitive features $\{f_1, \dots, f_k\}$ according to \mathbf{s} , and then retrieve their
 358 most correlated partners $\{f'_1, \dots, f'_k\}$ from \mathbf{C} . For each pair, we compute the mean squared devia-
 359 tion (MSD) of their class distributions, forming the correlation-enhanced vector:

$$360 \quad \mathbf{q}^{(\ell)}(\mathbf{z}) = \frac{1}{k} \sum_{i=1}^k (\mathbf{p}_{f_i} - \mathbf{p}_{f'_i})^2.$$

361 This operation not only captures feature interactions but also reduces sensitivity to the parameter k .

378 **Input update and cascade training.** The input to the next level is the concatenation of the previous
 379 representation, the optimized class distribution, and the correlation-enhanced vector:
 380

$$381 \quad \mathbf{h}^{(\ell)}(\mathbf{z}) = [\mathbf{h}^{(\ell-1)}(\mathbf{z}), \mathbf{v}^{(\ell)}(\mathbf{z}), \mathbf{q}^{(\ell)}(\mathbf{z})], \quad \mathbf{h}^{(0)}(\mathbf{z}) = \mathbf{z}.$$

382 Each layer is trained using cross-validation estimates to prevent overfitting, and the cascade continues
 383 to grow until validation performance no longer improves.
 384

385 **Final prediction.** During inference, the final prediction is made by aggregating the outputs of all
 386 forests at the last layer L :

$$388 \quad \hat{y} = \arg \max_k \frac{1}{M} \sum_{m=1}^M p_k^{(L,m)},$$

390 where M denotes the number of forests in the last level. The predicted label \hat{y} corresponds to the
 391 optimal solver from the pool $\{A_1, \dots, A_K\}$.
 392

393 4 EXPERIMENTAL RESULTS

395 We train and evaluate our approach on seven ASlib scenarios Bischl et al. (2016), selected for their
 396 overlap with the Open Algorithm Selection Challenge (OASC) Lindauer et al. (2017) and sunny-
 397 as2 Liu et al. (2021), covering SAT (*Sora*, *Svea*), MaxSAT (*Magnus*, *Monty*), CSP (*Caren*, *Camilla*),
 398 and MIP (*Mira*). Performance is measured using the *gap* metric relative to the virtual best solver.
 399 Details are in Appendix A.2.
 400

401 4.1 MAIN RESULTS

403 To assess the effectiveness of the F2S3 model, we evaluate it against several baselines on seven
 404 ASlib scenarios. As shown in Table 1, the baselines include AS-ASL and AS-RF Malone et al.
 405 (2017), ASAP.v3 Gonard et al. (2017), star-zilla Xu et al. (2012), sunny-based variants like sunny-
 406 as2-fk Liu et al. (2021) and sunny-autok Lindauer et al. (2019), and the neural model NeuroSAT
 407 (modified to a multi-class classifier for SAT solver selection) Selsam et al. (2019).

408 Table 1: Gap values for different scenarios of comparative experiments.
 409

410 Baselines	<i>Sora</i>	<i>Svea</i>	<i>Magnus</i>	<i>Monty</i>	<i>Caren</i>	<i>Camilla</i>	<i>Mira</i>	Avg.
412 AS-ASL	-0.6692	0.4385	-1.0528	-6.3895	-1.7325	0.4385	-0.4065	-1.3391
413 AS-RF	-0.3700	0.5853	-1.0521	-6.8992	-1.0617	0.5853	0.4947	-1.1025
414 ASAP.v3	0.0639	<u>0.6881</u>	0.4963	<u>0.7631</u>	0.3276	<u>0.6881</u>	<u>0.5091</u>	<u>0.5052</u>
415 star-zilla	0.1706	0.1715	<u>0.5751</u>	0.1731	-0.6409	0.1715	0.0328	0.0934
416 Sunny-autok	0.0021	0.5789	0.4924	0.6318	<u>0.6440</u>	0.5789	-0.0137	0.4163
417 sunny-as2-fk	0.3428	0.6643	0.4458	0.5846	0.0845	N/A	-0.1891	0.4244
418 NeuroSAT	0.2831	0.5922	0.3379	0.4620	0.3782	0.4216	0.1928	0.3811
419 F2S3 (ours)	<u>0.3374</u>	0.7786	0.6283	0.9230	0.8373	0.8921	0.6280	0.7178

420 *Note:* Bold indicates the best performance, and underline indicates the second best in each column.
 421

422 Table 1 summarizes the comparative results on seven ASlib scenarios. Our proposed **F2S3** achieves
 423 the best overall performance, with an average gap value of 0.7178, substantially outperforming the
 424 strongest baseline ASAP.v3 (0.5052). On most individual scenarios, F2S3 also attains the highest
 425 gap values, including *Magnus* (0.6283), *Monty* (0.9230), *Svea* (0.7786), *Caren* (0.8373), *Camilla*
 426 (0.8921), and *Mira* (0.6280). Notably, the gains in *Monty*, *Svea*, and *Mira* are particularly large
 427 compared with all baselines, underscoring the robustness of F2S3 across diverse datasets.

428 The superior performance of F2S3 can be interpreted from three perspectives. First, compared with
 429 traditional solver selection methods (e.g., AS-ASL, AS-RF, Sunny-autok), which rely on manu-
 430 ally designed SAT features, F2S3 is entirely feature-free. By constructing refined factor graphs
 431 and learning solver-oriented embeddings automatically, our approach avoids costly feature engi-
 neering and achieves more consistent performance across different problem distributions. Second,

relative to learning-based baselines such as NeuroSAT, F2S3 consistently yields higher gap values (e.g., 0.9230 vs. 0.4620 on *Monty*, 0.8373 vs. 0.3782 on *Caren*). This highlights the advantage of our CRFG+DPGR pipeline for extracting structure-preserving representations, together with SACF for capturing discriminative patterns that generic graph neural architectures overlook. Finally, although the experiments are conducted on SAT and MaxSAT scenarios, the feature-free design of F2S3 makes it broadly applicable. Preliminary studies on CSP and MIP datasets show similarly strong improvements, indicating that F2S3 generalizes beyond Boolean satisfiability to a wide range of combinatorial optimization domains. Together, these results demonstrate that F2S3 not only achieves state-of-the-art solver selection accuracy but also provides a general and extensible framework.

4.2 ABLATION STUDY

We conduct ablation studies on seven ASlib scenarios to evaluate the contribution of each component in F2S3. All results are reported as average precision over ten independent runs, with detailed per-run results provided in Appendix A.3.1. As shown in Table 2, the full model consistently achieves the best performance across all scenarios. Removing SACF leads to precision drops of 3–6 points, with the largest degradation observed on *Mira* (−6.54), highlighting its role in enhancing discriminative capability through feature sensitivity and correlation modeling. Removing DPGR causes even larger decreases, up to −8.66 on *Mira*, demonstrating its importance in preserving global-local structural information and learning robust embeddings.

The most severe performance deterioration occurs when both DPGR and SACF are removed, with precision drops exceeding 10 points in several scenarios (e.g., *Sora* and *Monty*). These results indicate that the two modules are not only individually effective but also complementary: DPGR preserves structural information while SACF refines embeddings for solver prediction. Overall, the ablation results clearly confirm that both modules are essential for achieving the high precision of F2S3.

Table 2: Ablation results of F2S3 on seven ASlib scenarios.

Variants	<i>Sora</i> (%)	<i>Svea</i> (%)	<i>Magnus</i> (%)	<i>Monty</i> (%)	<i>Caren</i> (%)	<i>Camilla</i> (%)	<i>Mira</i> (%)
F2S3	50.18	81.66	80.62	93.72	83.04	88.52	78.58
w/o SACF	44.22 ↓5.96	76.82 ↓4.84	75.32 ↓5.30	89.84 ↓3.88	79.46 ↓3.58	83.28 ↓5.24	72.04 ↓6.54
w/o DPGR	43.88 ↓6.30	76.82 ↓4.84	74.56 ↓6.06	89.12 ↓4.60	79.32 ↓3.72	81.12 ↓7.40	69.92 ↓8.66
w/o DPGR+SACF	39.32 ↓10.86	71.66 ↓10.00	72.00 ↓8.62	82.64 ↓11.08	75.52 ↓7.52	78.36 ↓10.16	69.06 ↓9.52

4.3 DISCUSSION

This study presents a feature-free approach for SAT solver selection, addressing the limitations of GraSS (which depends on unreleased feature computation methods) and NeuroSAT (which focuses primarily on local node-edge interactions). Our approach eliminates manual feature engineering, automatically captures structural information, and mitigates performance degradation due to feature sparsity or distribution shifts. As a result, it achieves improved generalization and stability across various scenarios. The key components-CRFG, DPGR and SACF-complement each other in alleviating graph sparsity, learning robust embeddings, and enhancing discriminative power. These combined strengths enable F2S3 to outperform existing methods, even when individual components alone are less effective. Additionally, the parameter sensitivity analysis in Appendix A.3.2 shows that F2S3 is robust to variations in embedding dimensions and hyperparameters, demonstrating its practical applicability without extensive parameter tuning.

5 CONCLUSION

We propose F2S3, a feature-free approach for SAT solver selection that integrates Correlation Refinement Factor Graph, Dual-Proximity Graph Representation, and Sensitive-Associative Cascade Forest. F2S3 outperforms existing methods, particularly in scenarios impacted by feature sparsity and computational inefficiency, demonstrating its effectiveness and robustness across diverse problem instances.

486 ETHICS STATEMENT
487488 We adhere to the ICLR Code of Ethics. No human subjects were involved in our research, and
489 all datasets used are publicly available. We have ensured that our research complies with ethical
490 guidelines, including privacy, fairness, and avoiding harmful applications. There are no conflicts of
491 interest or commercial sponsorship.
492493 REPRODUCIBILITY STATEMENT
494495 The model used in this research is available at <https://anonymous.4open.science/r/F2S3-5228/>, and
496 the datasets are from the public dataset ASlib.
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648 THE USE OF LARGE LANGUAGE MODELS (LLMs)
649650 We did not use any Large Language Models (LLMs) for research ideation or writing in the development
651 of this paper. All research ideas, analysis, and writing were carried out by the authors without
652 the assistance of LLMs.
653654 A APPENDIX
655656 A.1 APPROACH DETAILS
657658 A.1.1 DPGR OPTIMIZATION.
659660 The optimization of *Dual-Proximity Graph Representation* (DPGR) aims to jointly preserve both
661 local and global structural properties while preventing overfitting. The overall loss is defined as
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$$\mathcal{L} = \mathcal{L}_{global} + \alpha \cdot \mathcal{L}_{local} + \beta \cdot \mathcal{L}_{reg},$$

664 where \mathcal{L}_{global} enforces global proximity preservation, \mathcal{L}_{local} maintains local pairwise correlations,
665 and \mathcal{L}_{reg} controls model complexity through parameter regularization. The parameters of the en-
666 coder and decoder are optimized via backpropagation.
667668 **Gradient computation.** Let $\theta = \{W^{(k)}, \widehat{W}^{(k)}\}_{k=1}^K$ denote the learnable parameters of the auto-
669 encoder. The optimization objective is to minimize \mathcal{L} with respect to θ . The gradients of the loss
670 with respect to the decoder and encoder parameters are:
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$$\frac{\partial \mathcal{L}}{\partial \widehat{W}^{(k)}} = \frac{\partial \mathcal{L}_{global}}{\partial \widehat{W}^{(k)}} + \beta \frac{\partial \mathcal{L}_{reg}}{\partial \widehat{W}^{(k)}}, \quad (1)$$

673
$$\frac{\partial \mathcal{L}}{\partial W^{(k)}} = \frac{\partial \mathcal{L}_{global}}{\partial W^{(k)}} + \alpha \frac{\partial \mathcal{L}_{local}}{\partial W^{(k)}} + \beta \frac{\partial \mathcal{L}_{reg}}{\partial W^{(k)}}, \quad k = 1, \dots, K. \quad (2)$$

674 **Global proximity gradient.** The global proximity loss is measured by reconstruction error:
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$$\mathcal{L}_{global} = \sum_{i=1}^m \|\hat{\mathbf{z}}_i - \mathbf{z}_i\|_2^2.$$

677 Its gradient with respect to the decoder weight $\widehat{W}^{(k)}$ can be decomposed as
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$$\frac{\partial \mathcal{L}_{global}}{\partial \widehat{W}^{(k)}} = \frac{\partial \mathcal{L}_{global}}{\partial \hat{Z}} \cdot \frac{\partial \hat{Z}}{\partial \widehat{W}^{(k)}}, \quad (3)$$

680 where
681

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$$\frac{\partial \mathcal{L}_{global}}{\partial \hat{Z}} = 2(\hat{Z} - Z), \quad \hat{Z} = \sigma(\hat{Y}^{(K-1)} \widehat{W}^{(K)} + \hat{b}^{(K)}).$$

683 **Local proximity gradient.** The local proximity loss is defined as
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$$\mathcal{L}_{local} = \sum_{i,j=1}^n S'_{ij} \|\mathbf{y}_i - \mathbf{y}_j\|_2^2 = 2 \operatorname{tr}(Y^\top LY),$$

686 where $L = D - S'$, D is the diagonal degree matrix with $D_{ii} = \sum_j S'_{ij}$, and Y is the embedding
687 matrix. The gradient can be computed as
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$$\frac{\partial \mathcal{L}_{local}}{\partial W^{(K)}} = \frac{\partial \mathcal{L}_{local}}{\partial Y} \cdot \frac{\partial Y}{\partial W^{(K)}}, \quad (4)$$

690 with
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$$\frac{\partial \mathcal{L}_{local}}{\partial Y} = 2(L + L^\top)Y, \quad Y = \sigma(Y^{(K-1)} W^{(K)} + b^{(K)}).$$

702 **Training procedure.** DPGR combines the unsupervised component (reconstruction of neighborhoods to preserve global proximity) with the supervised component (local correlation preservation).
 703 These objectives are jointly optimized in a semi-supervised manner. To stabilize training and achieve
 704 effective initialization, the model is first pre-trained with a deep belief network Hinton (2009), fol-
 705 lowed by fine-tuning using stochastic gradient descent with backpropagation.
 706

707 This optimization procedure ensures that DPGR captures both fine-grained local dependencies and
 708 broader global structures, yielding robust and discriminative embeddings for solver selection.
 709

710 **A.1.2 CASCADE FOREST CONFIGURATION**
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712 For completeness, we provide additional details of the cascade forest implementation that were
 713 omitted in the main text. Each level of the cascade consists of two completely-random tree forests
 714 and two random forests, following the standard design of deep forest. Each forest contains 500
 715 trees. In a completely-random tree forest, every split is made by randomly selecting one embedding
 716 dimension until pure leaves are reached. In contrast, in a random forest, the best split is chosen
 717 among \sqrt{d} randomly sampled dimensions using the Gini index. These configurations remain fixed
 718 across all experiments.
 719

720 **A.2 EXPERIMENTAL DETAILS**
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722 **A.2.1 DATASET**
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724 Table 3 summarizes the selected benchmark scenarios, including the number of algorithms and
 725 instances. As noted in the main text, we excluded the BNSL scenario due to excessive informa-
 726 tion loss during transformation, which makes it unsuitable for solver selection. For consistency
 727 across domains, CSP and MIP scenarios were converted into CNF: CSP instances were Booleanized
 728 and encoded into clauses following Tamura et al. Tamura et al. (2010), while MIP instances were
 729 transformed by encoding bounded integer variables into binary form and translating linear in-
 730 equalities into pseudo-Boolean constraints, which were further reduced to CNF as in Sheini and
 731 Sakallah Sheini & Sakallah (2005).
 732

733 Table 3: Overview of the problem scenarios. $|A|$ and $|I|$ denote the number of algorithms and
 734 instances, respectively.
 735

Scenarios	Alias	$ A $	$ I $
<i>SAT scenarios</i>			
SAT03-16 INDU	Sora	10	2,000
SAT12-ALL*	Svea	31	1,614
<i>MaxSAT scenarios</i>			
MAXSAT-PMS-2016	Magnus	19	100
MAXSAT-WPMS-2016	Monty	18	100
<i>CSP scenarios</i>			
CSP-Minizinc-Obj-2016	Caren	8	100
CSP-Minizinc-Time-2016	Camilla	22	9,720
<i>MIP scenarios</i>			
MIP-2016	Mira	5	218

736 For completeness, we briefly summarize the transformation principles. CSP instances can be en-
 737 coded into SAT by Booleanizing domain variables and translating constraints into clauses, following
 738 the approach of Tamura et al. Tamura et al. (2010). MIP instances are transformed by representing
 739 bounded integer variables in binary form and converting linear inequalities into pseudo-Boolean
 740 constraints, which can be further encoded into CNF as in Sheini and Sakallah Sheini & Sakallah
 741 (2005). These transformations allow all selected scenarios to be consistently represented in SAT
 742 form.
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744 **A.2.2 BASELINES**
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- 747 • **AS-ASL** Malone et al. (2017): A SAT solver selection method that uses Auto-sklearn to
 748 identify key features and train a stacking model for solver selection.
 749

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- 756 • **AS-RF** Malone et al. (2017): A SAT solver selection method that uses random forests for
757 selecting the optimal solver based on problem instance features.
- 758
- 759 • **ASAP.v3** Gonard et al. (2017): A system that uses a sequential scheduler and algorithm
760 selector to optimize solver selection, with a pre-scheduler identifying easier instances and
761 selecting the best solver for more complex ones.
- 762
- 763 • **star-zilla** Xu et al. (2012): A solver portfolio that uses predictive modeling to select the
764 best SAT solver from a set of candidates, demonstrating the efficacy of ensemble-based
765 approaches.
- 766
- 767 • **sunny-as2-fk** Liu et al. (2021): A method that combines feature selection with k-nearest
768 neighbor configuration to optimize SAT solver selection, improving performance by jointly
769 tuning the neighborhood size and relevant features.
- 770

770 A.2.3 EVALUATION METRICS

771 To evaluate the effectiveness of algorithm selection methods, we use the *gap* metric, which compares
772 the performance of the selection system with the virtual best solver (VBS) and the single best solver
773 (SBS). Formally, it is defined as

$$775 \quad \text{gap} = \frac{m_{SBS} - m_s}{m_{SBS} - m_{VBS}}, \quad (5)$$

776 where m_s denotes the performance of the selection system, m_{SBS} is the performance of the SBS
777 (the single solver that performs best on average across all instances), and m_{VBS} is the performance
778 of the VBS (the oracle that always selects the best solver for each instance).

779 The gap value ranges from $-\infty$ to 1:

- 780 - $\text{gap} = 1$ means the selection system performs as well as the VBS, i.e., it always selects the best
781 solver for every instance.
- 782 - $0 < \text{gap} < 1$ indicates that the system improves over the SBS but has not yet reached the perfor-
783 mance of the VBS.
- 784 - $\text{gap} = 0$ means the system only matches the SBS, i.e., no better than simply using a single solver
785 across all instances.
- 786 - $\text{gap} < 0$ indicates that the system performs worse than the SBS, i.e., its selections on average lead
787 to higher cost than always choosing the single best solver.

790 A.3 ADDITIONAL EXPERIMENTAL RESULTS

791 A.3.1 DETAILED ABLATION RESULTS

792 Figure 4 illustrates the precision comparison of F2S3-DS, F2S3-S, F2S3-D, and F2S3 across five
793 random dataset samplings. Overall, the F2S3 model outperforms the other models in all 10 experi-
794 ments, particularly demonstrating significantly higher precision rates than F2S3-DS, F2S3-S, and
795 F2S3-D in most scenarios, indicating a clear advantage in task performance. Although the F2S3-D
796 model performs closely, it still falls slightly short of F2S3, suggesting that the additional enhance-
797 ments in the F2S3 model are effective in improving precision. The overall performance of F2S3-S
798 and F2S3-DS is relatively lower, especially the F2S3-DS model, which shows markedly lower preci-
799 sion rates than the others in most scenarios. The differences in performance across various scenarios
800 are also noteworthy. In certain scenarios, such as Magnus and Svea, the precision rates of F2S3 and
801 F2S3-D are particularly prominent, indicating that these two models can better capture the features
802 of these scenarios, exhibiting higher adaptability. In contrast, in scenarios like Camilla and Sora,
803 the precision rates of all models are closer, yet F2S3 still slightly outperforms, demonstrating its
804 consistent advantage across multiple types of scenarios. Furthermore, examining the fluctuations
805 in the results of the five experiments, F2S3 shows a more stable performance across different ran-
806 dom datasets with a smaller range of precision fluctuations, indicating strong robustness in multiple
807 experiments. In comparison, F2S3-DS and F2S3 exhibit greater precision fluctuations across differ-
808 ent experiments, especially showing instability in scenarios with lower precision rates. This further
809 emphasizes the superiority and consistency of F2S3 under conditions of random datasets.

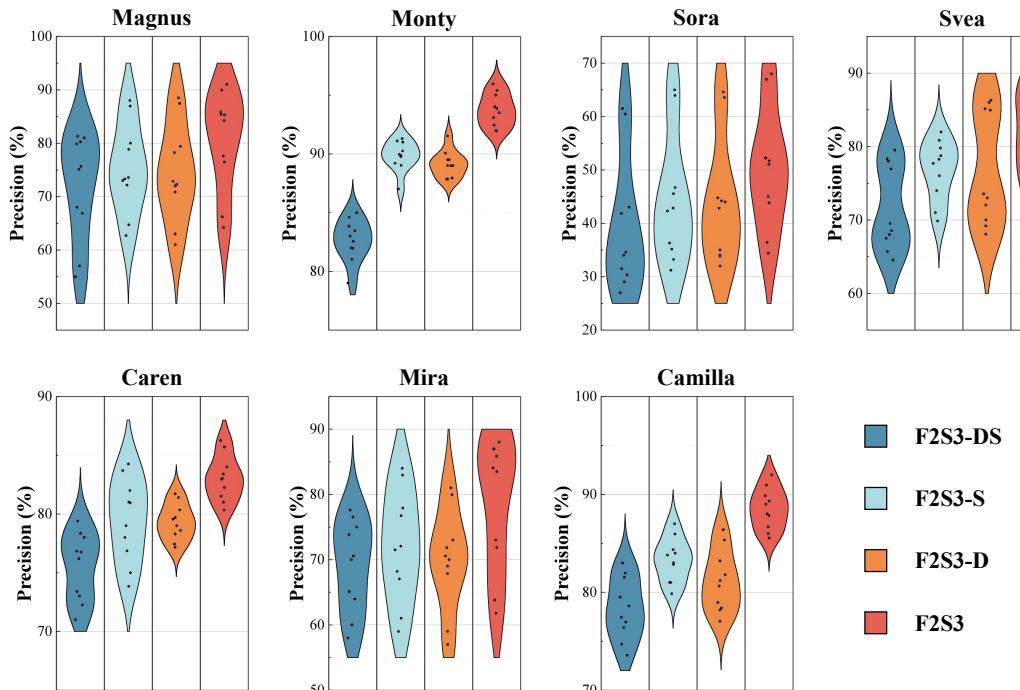


Figure 4: Results of ablation experiments for 10 runs.

A.3.2 PARAMETER SENSITIVITY ANALYSIS

Parameter sensitivity analysis is a crucial step in understanding the relationship between model performance and key parameters. In this study, we focused on analyzing the impact of embedding dimension, hyperparameters α and β on the performance of the network embedding model.

The investigation into embedding vector dimensionality reveals a notable influence on model performance. As depicted in Figure 5(a), performance initially rises and then falls with increasing dimensionality. This indicates that while an optimal dimension enhances information encoding, excessive dimensions introduce noise and degrade performance. Although our method shows low sensitivity to dimensionality, selecting an appropriate dimension is still essential.

The hyperparameter α , which balances direct and indirect similarities, exhibits scenario-dependent optimal values as shown in Figure 5(b). Generally, $\alpha \in [0.1, 0.2]$ yields superior performance, emphasizing the significance of both direct and indirect similarities. Notably, higher α values enhance performance in Monty, Magnus, Sora, and Svea scenarios, while Camilla and Caren scenarios benefit from α values between 0.05 and 0.1. In the Mira scenario, α monotonically increases within $[0, 0.2]$, further highlighting the importance of these similarities.

The hyperparameter β , controlling the reconstruction weight of non-zero elements in the training graph, shows an initial increase and subsequent decrease in model performance with increasing β across seven scenarios (Figure 5(c)). Optimal performance is typically achieved when $\beta \in [5, 7]$, with Monty and Mira scenarios peaking at $\beta = 8$. This indicates that moderate β values improve non-zero element reconstruction, while excessive values degrade performance. The F2S3 model's enhanced performance with higher β is attributed to its balanced reconstruction of non-zero and zero elements and optimization of the CRFG.

In summary, the parameter sensitivity analysis provides valuable insights into how to adjust model parameters to achieve optimal performance. In practical applications, these parameters should be selected and adjusted reasonably based on specific scenarios and requirements.

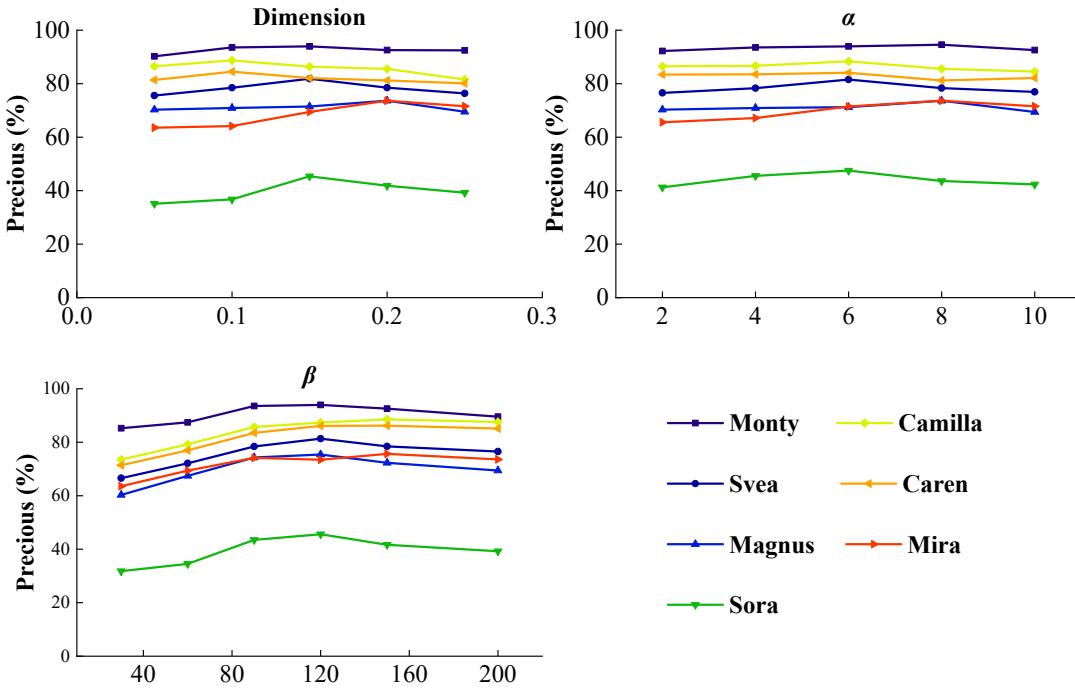


Figure 5: Parameter sensitivity comparison results. (a) Sensitivity comparison results for embedding dimension. (b) Sensitivity comparison results for parameter α . (c) Sensitivity comparison results for parameter β .

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