

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 VMF GUIDED LEARNING FOR BIOMEDICAL VISION- LANGUAGE MODELS

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## ABSTRACT

Effective adaptation of Vision-Language Models (VLMs) to biomedical tasks remains challenging due to a substantial semantic gap between general knowledge and domain-specific expertise. Domain-specific models such as BiomedCLIP narrow this gap; however, prevailing prompt-learning methods collapse diverse text embeddings into a single prototype, discarding distributional information. We introduce vMF Distribution Semantic Alignment (VDSA), which models each class with a von Mises–Fisher distribution on the unit hypersphere and aligns images to the entire distribution rather than a single prototype. We further derive a closed-form upper bound to the expected contrastive loss, yielding a sampling-free objective that is implicitly equivalent to aligning against an infinite prompt ensemble with minimal overhead. Experiments on multiple biomedical benchmarks show that VDSA consistently improves few-shot adaptation and generalization to unseen classes, providing a robust recipe for adapting specialized VLMs.

## 1 INTRODUCTION

Vision-Language Models (VLMs), such as CLIP (Radford et al., 2021) and Align (Jia et al., 2021), have established a powerful paradigm for learning unified image–text representations. By leveraging large-scale image–text training, they achieve robust semantic alignment and impressive zero-shot transfer, motivating extensive research on their adaptation to diverse downstream tasks (Zhang et al., 2021; Zhou et al., 2022b;a). However, transferring this success to highly specialized domains such as biomedicine reveals a fundamental challenge: a substantial semantic gap between general world knowledge and domain-specific biomedical expertise (Zhang et al., 2023; Eslami et al., 2021). This gap spans both visual and textual modalities. Unlike natural images, the visual language of biomedical imaging (e.g., X-rays, MRIs) is abstract and non-intuitive, characterized not by common semantic categories but by subtle, intricate patterns of texture, intensity, and anatomical structure that require specialized training to interpret (Koleilat et al., 2025). As a result, the generic visual concepts captured by standard VLMs often fail to align with these domain-specific patterns. Similarly, the medical terminology used to describe such findings (e.g., “pleural effusion,” “nodular sclerosis”) lies far beyond the common web-scale vocabulary, thereby limiting the applicability of general-purpose text encoders (Zhang et al., 2023).

To bridge the semantic gap in biomedicine, models pre-trained on domain-specific data are indispensable. Foundation models such as BiomedCLIP (Zhang et al., 2023), trained on 15 million biomedical image–text pairs, provide a far stronger starting point for downstream medical tasks than general-purpose VLMs. However, adapting these models through full fine-tuning is often computationally prohibitive and prone to overfitting or catastrophic forgetting (Gao et al., 2024). This challenge motivates the use of parameter-efficient adaptation strategies that update only a small subset of parameters while preserving the knowledge captured during pretraining (Zhou et al., 2022b; Bafghi et al., 2025).

Beyond the choice of adaptation mechanism, the effectiveness of VLMs depends critically on the design of text prompts. Early approaches rely on a single handcrafted prompt (e.g., “a photo of a [CLASS]”), which is sensitive to phrasing and limited in expressiveness (Radford et al., 2021). Prompt ensembling alleviates this by combining multiple templates to improve robustness (Kim et al., 2025), while recent advances leverage large language models (LLMs) to automatically generate diverse, domain-specific prompts (Pratt et al., 2023). These strategies enrich the semantic space

available to the model and have proven to benefit prompt learning by bringing diverse text representations (Zheng et al., 2024). Alternatively, approaches such as CoOp (Zhou et al., 2022b) and CoCoOp (Zhou et al., 2022a) learn continuous prompt embeddings directly by treating the prompt tokens as trainable parameters.

Despite these advances, current prompt ensembling methods share a fundamental limitation in how they aggregate semantic diversity. A finite set of  $N$  prompt embeddings is ultimately collapsed into a single prototype, a lossy compression that preserves only the semantic center while discarding crucial information about the dispersion and angular structure of the distribution (Radford et al., 2021; Allingham et al., 2023). This restricts the model to align with a narrow point estimate of class semantics rather than the full semantic space. Moreover, the choice of  $N$  presents a costly trade-off between semantic coverage and efficiency, with no principled way to determine an optimal value.

We attribute this limitation to representing semantic diversity as a finite set of discrete prompt embeddings that are merely averaged. A more principled alternative is to model the underlying continuous probability distribution that the discrete prompt set approximates (Ma et al., 2023). By shifting the alignment objective from a single prototype to the full distribution, the model can capture not only the semantic center but also the dispersion and angular structure that encode diversity and ambiguity. This perspective effectively enables the model to learn from the complete semantic space, as if integrating over an unlimited ensemble of text embeddings, while avoiding the prohibitive cost of explicitly sampling a large number of prompts.

To realize distributional alignment, we propose **vMF Distribution Semantic Alignment (VDSA)**, which interprets a class's  $N$  prompts as samples from a latent semantic distribution on the unit hypersphere and estimates its parameters by maximum likelihood. Instead of aligning image features to a single prototype, VDSA aligns them to the entire class distribution. The exact expected contrastive loss is analytically intractable because it involves high-dimensional surface integrals on the hypersphere induced by class-specific vMF distributions (Mardia & Jupp, 2009) for which no closed form exists. To address this challenge, we derive a closed-form Jensen upper bound that is fully differentiable and free of sampling, which encourages embeddings to the high-density regions of the semantic space. For computational efficiency, only a small subset of vision encoder weights is updated through LoRA (Hu et al., 2022). Conceptually, VDSA can be viewed as aligning against an infinite ensemble of prompts, providing implicit semantic augmentation that surpasses discrete prompt ensembling. Evaluations across diverse biomedical benchmarks demonstrate that VDSA achieves state-of-the-art performance in both few-shot adaptation and base-to-novel generalization.

## 2 RELATED WORK

### 2.1 VISION–LANGUAGE MODELS

Vision–Language Models (VLMs) such as CLIP (Radford et al., 2021) and ALIGN (Jia et al., 2021) have shown that large-scale image–text pretraining yields robust representations with strong zero-shot generalization. Recent extensions in the biomedical domain include MedCLIP (Wang et al., 2022), PubMedCLIP (Eslami et al., 2021), and BiomedCLIP (Zhang et al., 2023). BioMedCLIP (Zhang et al., 2023) incorporates millions of domain-specific image–text pairs, achieving notable gains over general-purpose VLMs on medical benchmarks. Despite these advances, their utility on specialized biomedical tasks remains limited, as capturing subtle, disease-specific semantics often requires additional adaptation beyond pretraining. This motivates the development of methods that can more effectively tailor biomedical foundation models to the demands of target clinical applications (Koleilat et al., 2025).

### 2.2 PARAMETER-EFFICIENT ADAPTATION

Adapting large-scale foundation models to downstream tasks through full fine-tuning is often computationally prohibitive and risks overfitting or catastrophic forgetting (Ding et al., 2022). To address this, a range of parameter-efficient adaptation (PEFT) strategies has been proposed. Adapter-based methods (Gao et al., 2024; Zhang et al., 2021) introduce lightweight modules between transformer layers, while low-rank adaptation (LoRA) (Hu et al., 2022) injects trainable rank-decomposed matrices into weight updates. Linear probing simply freezes the pretrained encoder and trains only a linear classifier on top of the fixed feature representation (Huang et al., 2024b). These techniques

108 preserve most pretrained weights, enabling efficient transfer across diverse domains. In biomedical  
 109 vision–language tasks, PEFT has also shown promise in tailoring domain-specific foundation  
 110 models with limited supervision (Peng et al., 2025). However, while PEFT mitigates the cost of  
 111 adaptation, it primarily operates on the model parameters themselves and does not directly address  
 112 how semantic prompts are constructed. Since the quality of prompts critically determines the alignment  
 113 between image and text, optimizing prompt design remains an orthogonal yet equally crucial  
 114 direction.

115

### 116 2.3 PROMPT ENSEMBLING AND LEARNING

117

118 The design of text prompts plays a decisive role in the effectiveness of vision–language models.  
 119 Early approaches relied on manually crafted templates, which are simple but sensitive to phrasing  
 120 and lack semantic richness (Radford et al., 2021). To improve robustness, prompt ensembling has  
 121 been widely adopted (Allingham et al., 2023; Roth et al., 2023), where multiple templates are used to  
 122 provide diverse textual descriptions. Recent work further leverages large language models (LLMs)  
 123 to automatically generate domain-specific prompt ensembles (Pratt et al., 2023; Koleilat et al., 2025),  
 124 alleviating the need for manual design.

125

126 In parallel, learnable prompt methods such as CoOp (Zhou et al., 2022b) and CoCoOp (Zhou et al.,  
 127 2022a) replace fixed templates with continuous embeddings that are optimized end-to-end, enabling  
 128 more flexible adaptation to downstream tasks. Building on this direction, KgCoOp (Yao et al.,  
 129 2023) and ProGrad (Zhu et al., 2023) further refine textual prompts by incorporating external knowl-  
 130 edge and gradient-based guidance, respectively, to enhance model generalizability. Building on this  
 131 line of work, BiomedCoOp (Koleilat et al., 2025) adapts prompt learning to biomedical images  
 132 by combining a BiomedCLIP backbone with joint semantic–knowledge modeling through context-  
 133 ual mapping and selective prompt distillation, enabling more effective and robust domain-specific  
 134 prompt context learning. Despite these advances, existing prompt ensembling and learnable prompt  
 135 methods ultimately aggregate semantic information into a single prototype representation (Khattak  
 136 et al., 2025; Huang et al., 2024a). Such compression inevitably discards distributional properties of  
 137 the prompt set, including its dispersion and angular structure on the hypersphere. This limitation  
 138 motivates approaches that move beyond discrete or mean-based representations toward principled  
 139 distributional formulations of class semantics.

140

## 141 3 VON MISES-FISHER DISTRIBUTION SEMANTIC ALIGNMENT

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143 In this work, we introduce vMF Distribution Semantic Alignment (VDSA), a framework that ad-  
 144 vances contrastive language–image learning by enriching the representation of class semantics.  
 145 VDSA generalizes beyond these formulations by modeling each class with a probability distribu-  
 146 tion over the hypersphere, thereby capturing its intrinsic semantic diversity. Specifically, we employ  
 147 the von Mises–Fisher (vMF) distribution Mardia & Jupp (2009) to explicitly represent the semantic  
 148 space of each class, and derive a closed-form upper bound of the expected contrastive loss under  
 149 these distributions. This enables end-to-end optimization that implicitly achieves semantic augmen-  
 150 tation and yields robust alignment.

151

### 152 3.1 PRELIMINARIES

153

**154 Vision-Language Models** CLIP (Radford et al., 2021) jointly trains an image encoder  $\mathcal{E}_v$  and a  
 155 text encoder  $\mathcal{E}_t$  to learn a shared embedding space where paired images and texts are aligned. Given  
 156 a batch of images, the encoder produces  $\{\mathbf{z}_i\}_{i=1}^B$ , with each  $\mathbf{z}_i \in \mathbb{R}^D$ . For a classification task with  
 157  $C$  classes, a class-specific prompt template (e.g., “a photo of a {class\\_name}”) is instantiated for  
 158 each class and encoded into text features  $\{\boldsymbol{\mu}_c\}_{c=1}^C$ , where  $\boldsymbol{\mu}_c \in \mathbb{R}^D$ . All image and text features are  
 159  $\ell_2$ -normalized to lie on the unit hypersphere.

160

161 The alignment is enforced by a contrastive loss. For zero-shot classification, the probability of  
 162 assigning image feature  $\mathbf{z}_i$  to class  $c$  is:

$$p(c|\mathbf{z}_i) = \frac{e^{s \cdot \mathbf{z}_i^\top \boldsymbol{\mu}_c}}{\sum_{j=1}^C e^{s \cdot \mathbf{z}_i^\top \boldsymbol{\mu}_j}}, \quad (1)$$

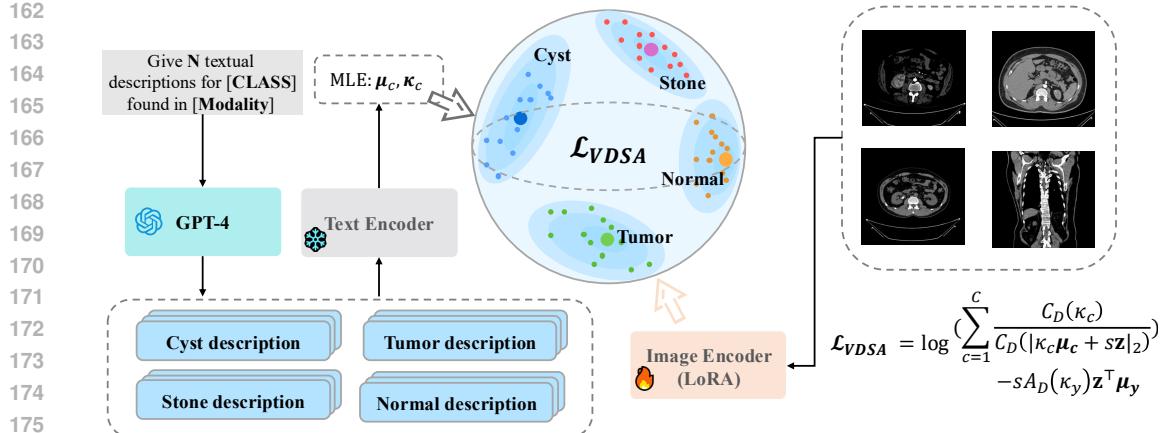


Figure 1: **Overview of our VDSA framework.** Our method models the semantics of each class as a von Mises–Fisher distribution on a hyperspherical feature space. We first leverage GPT-4 to generate  $N$  diverse textual descriptions for each class, which are encoded into a set of feature vectors. These vectors are then used to fit a class-conditional vMF distribution  $p(\mathbf{u} | \boldsymbol{\mu}_c, \kappa_c)$  via maximum likelihood estimation. An image encoder, fine-tuned with LoRA, is trained using our novel  $\mathcal{L}_{VDSA}$  objective, which aligns image embeddings with their corresponding class distribution.

where  $s > 0$  is a learnable temperature. The model is trained by minimizing the cross-entropy loss:

$$\mathcal{L}_{CE} = -\log p(y_i | \mathbf{z}_i). \quad (2)$$

In this formulation, each class  $c$  is represented by a *single prototype vector*  $\boldsymbol{\mu}_c$  on the hypersphere.

**Prompt Ensembling Method** To address the limitations of a single prompt representation, *Prompt Ensembling* has been shown to benefit prompt learning by incorporating diverse textual descriptions for each class (Kim et al., 2025). By leveraging multiple prompts, ensembling provides a more robust and comprehensive semantic representation. Recent advances employ Large Language Models (LLMs) to automatically generate high-quality, domain-specific prompt ensembles, alleviating the need for manual template design (Khattak et al., 2025; Pratt et al., 2023). Following Biomed-CoOp (Koleilat et al., 2025), we adopt GPT-4 (Achiam et al., 2023) to synthesize class-specific prompts. For each dataset with  $C$  classes, we query GPT-4 (Achiam et al., 2023) with the instruction:

Give  $N$  textual descriptions of visual discriminative features for distinct medical cases of [CLASS] found in [MODALITY].

This process yields  $N$  diverse descriptions per class that capture characteristic lesions, anatomical patterns, and imaging cues, ensuring that the resulting prompts encode the necessary clinical semantics. For each class  $c$ , we generate  $N$  diverse prompts, which are encoded by the text encoder  $\mathcal{E}_t$  into a set of feature vectors  $\{\boldsymbol{\mu}_c^{(i)}\}_{i=1}^N$ , with each  $\boldsymbol{\mu}_c^{(i)} \in \mathbb{R}^D$  lying on the unit hypersphere.

Following standard practice, these features are aggregated into a single prototype by computing their mean and subsequently re-normalizing to unit length:

$$\bar{\boldsymbol{\mu}}_c = \frac{\frac{1}{N} \sum_{i=1}^N \boldsymbol{\mu}_c^{(i)}}{\left\| \frac{1}{N} \sum_{i=1}^N \boldsymbol{\mu}_c^{(i)} \right\|_2}. \quad (3)$$

This ensemble prototype  $\bar{\boldsymbol{\mu}}_c$  then replaces the single-prompt feature  $\boldsymbol{\mu}_c$  in the contrastive loss (Eq. 1) for model adaptation.

As discussed earlier, the simple aggregation collapses  $N$  prompt embeddings into prototypes; we instead model each class as a probability distribution on the unit hypersphere.

216 3.2 DISTRIBUTIONAL CLASS REPRESENTATION WITH vMF  
217

218 To overcome the limitations of discrete prototypes, we lift the semantic representation of each class  
219 from a finite set of prompt embeddings to a continuous probability distribution, denoted  $p(\mathbf{u} \mid \boldsymbol{\theta}_c)$ ,  
220 where  $\mathbf{u} \in \mathcal{S}^{D-1}$  and  $\boldsymbol{\theta}_c = (\boldsymbol{\mu}_c, \kappa_c)$ . Here  $\mathcal{S}^{D-1} = \{\mathbf{u} \in \mathbb{R}^D : \|\mathbf{u}\|_2 = 1\}$ . This formulation  
221 provides a richer characterization of class semantics by capturing inherent diversity and ambiguity.  
222 Since CLIP features are  $\ell_2$ -normalized and thus lie on the unit hypersphere  $\mathcal{S}^{D-1}$ , the *von Mises-Fisher* (vMF)  
223 distribution is a natural choice, playing a role analogous to the Gaussian distribution  
224 in Euclidean space. See Appendix E for why Gaussian-based alternatives are unsuitable here.

225 The vMF distribution is parameterized by a mean direction  $\boldsymbol{\mu}_c \in \mathcal{S}^{D-1}$  and a concentration parameter  
226  $\kappa_c \geq 0$ . The mean direction  $\boldsymbol{\mu}_c$  captures the semantic center of a class, while  $\kappa_c$  controls its  
227 dispersion: larger  $\kappa_c$  corresponds to a narrower, more coherent concept, whereas smaller  $\kappa_c$  reflects  
228 broader variability.

229 Formally, the density for a random unit vector  $\mathbf{u}$  is  
230

$$231 \quad p(\mathbf{u} \mid \boldsymbol{\mu}_c, \kappa_c) = C_D(\kappa_c) e^{\kappa_c \boldsymbol{\mu}_c^\top \mathbf{u}} \quad (4)$$

232 with normalization constant  
233

$$234 \quad C_D(\kappa_c) = \frac{\kappa_c^{\frac{D}{2}-1}}{(2\pi)^{\frac{D}{2}} I_{\frac{D}{2}-1}(\kappa_c)}, \quad (5)$$

235 where  $I_\nu(\cdot)$  is the modified Bessel function of the first kind. Here  $C_D(\kappa)$  is fixed by the normalization  
236  $\int_{\mathcal{S}^{D-1}} p(\mathbf{u} \mid \boldsymbol{\mu}, \kappa) d\sigma(\mathbf{u}) = 1$ , where  $d\sigma$  denotes the uniform surface-area measure on the  
237 sphere; evaluating this surface integral yields as shown in Equation 5. For efficiency, we compute  
238  $\log C_D(\cdot)$  via a numerically stable asymptotic approximation to  $\log I_\nu(\cdot)$  (rather than explicit Bessel  
239 evaluations).

240 Given  $N$  unit-normalized prompt embeddings  $\{\mathbf{u}_c^{(i)}\}_{i=1}^N$  for a class  $c$ , we estimate its von Mises-Fisher (vMF)  
241 distribution parameters  $(\boldsymbol{\mu}_c, \kappa_c)$  via Maximum Likelihood Estimation (MLE). The detailed derivation is provided in Appendix F.  
242

243 • **MLE of Mean Direction ( $\hat{\boldsymbol{\mu}}_c$ ):** The estimate is the normalized direction of the sum of the  
244 embedding vectors.

$$245 \quad \hat{\boldsymbol{\mu}}_c = \frac{\sum_{i=1}^N \mathbf{u}_c^{(i)}}{\left\| \sum_{i=1}^N \mathbf{u}_c^{(i)} \right\|_2}. \quad (6)$$

246 • **MLE of Concentration ( $\hat{\kappa}_c$ ):** The estimate is obtained by computing the empirical mean  
247 resultant length,  $\bar{R}_c$ , which serves as a sufficient statistic for the concentration parameter.

$$248 \quad \bar{R}_c = \left\| \frac{1}{N} \sum_{i=1}^N \mathbf{u}_c^{(i)} \right\|_2 \in [0, 1).$$

249 The estimate  $\hat{\kappa}_c$  is then the unique solution to the equation:  
250

$$251 \quad A_D(\hat{\kappa}_c) = \bar{R}_c, \quad \text{where} \quad A_D(\kappa) = \frac{I_{D/2}(\kappa)}{I_{D/2-1}(\kappa)}.$$

252 Since  $A_D(\kappa)$  is a strictly increasing function on  $[0, \infty)$  with range  $[0, 1)$ , the inverse  $A_D^{-1}$  is well-defined,  
253 yielding a unique solution  $\hat{\kappa}_c = A_D^{-1}(\bar{R}_c)$ . For simplicity, we replace the true parameters  
254 with their MLE and continue to denote them as  $(\boldsymbol{\mu}_c, \kappa_c)$ .

255 3.3 THE VDSA OPTIMIZATION OBJECTIVE  
256

257 We represent each class  $c$  by a vMF distribution  $p(\mathbf{u}_c \mid \boldsymbol{\mu}_c, \kappa_c)$  on the unit hypersphere and define  
258 the stochastic logit  
259

$$S_c = s \mathbf{z}^\top \mathbf{u}_c, \quad \mathbf{u}_c \sim \text{vMF}(\boldsymbol{\mu}_c, \kappa_c), \quad \|\mathbf{z}\|_2 = \|\boldsymbol{\mu}_c\|_2 = 1.$$

270 The ideal learning objective, therefore, is to optimize the expected cross entropy over the joint  
 271 distribution of all random prototypes:

$$272 \quad \mathcal{L}_{\text{ideal}} = \mathbb{E} \left[ \underbrace{\log \sum_{c=1}^C e^{S_c}}_{\text{(A)}} - \underbrace{S_y}_{\text{(B)}} \right]. \quad (7)$$

278 **A Jensen upper bound.** Directly computing the expectation of term **(A)** is intractable, as it  
 279 involves the expectation of a logarithm over a sum of exponentials, for which no closed form  
 280 solution exists. We therefore derive a tractable upper bound using Jensen's inequality. Let  
 281  $Y := \sum_{c=1}^C e^{S_c} > 0$ . Since  $\log(\cdot)$  is a concave function,

$$283 \quad \mathbb{E} \left[ \log \sum_c e^{S_c} \right] = \mathbb{E}[\log Y] \leq \log \mathbb{E}[Y] = \log \left( \sum_c \mathbb{E}[e^{S_c}] \right). \quad (8)$$

286 Applying this bound to term **(A)**, while noting that the expectation of term **(B)** can be computed  
 287 directly, yields an upper bound on the ideal objective:

$$288 \quad \mathcal{L}_{\text{ideal}} \leq \underbrace{\log \sum_{c=1}^C \mathbb{E}[e^{S_c}]}_{\text{(I)}} - \underbrace{\mathbb{E}[S_y]}_{\text{(II)}}.$$

292 **Closed forms under vMF.** We derive closed forms for **(I)** and **(II)** by viewing vMF as an expo-  
 293 nential family. With natural parameter  $\theta_c = \kappa_c \mu_c$ , the partition function is

$$295 \quad Z(\theta) := \int_{S^{D-1}} \exp(\theta^\top \mathbf{u}) d\sigma(\mathbf{u}) = \frac{1}{C_D(\|\theta\|_2)},$$

297 where  $C_D(\cdot)$  is the vMF normalizer.

299 To evaluate the sum inside term **(I)**, we use the moment generating function identity. For any  
 300  $\mathbf{h} \in \mathbb{R}^D$ ,  $\mathbb{E}_{\mathbf{u} \sim p(\cdot|\theta)} [e^{\mathbf{h}^\top \mathbf{u}}] = Z(\theta + \mathbf{h})/Z(\theta)$ . Setting  $\mathbf{h} = s\mathbf{z}$  gives the expectation of each  
 301 exponentiated logit:

$$302 \quad \mathbb{E}[e^{S_c}] = \mathbb{E}[e^{s \mathbf{z}^\top \mathbf{u}_c}] = \frac{C_D(\kappa_c)}{C_D(\|\kappa_c \mu_c + s\mathbf{z}\|_2)}. \quad (9)$$

304 For term **(II)**, we compute the expectation of the logit for the ground-truth class  $y$ . This corresponds  
 305 to the mean of the vMF distribution scaled by  $s\mathbf{z}^\top$ :

$$306 \quad \mathbb{E}[S_y] = s \mathbf{z}^\top \mathbb{E}[\mathbf{u}_y] = s A_D(\kappa_y) \mathbf{z}^\top \mu_y,$$

308 where  $A_D(\kappa) = I_{D/2}(\kappa)/I_{D/2-1}(\kappa)$  is the ratio of modified Bessel functions of the first kind.

309 **Objective.** Substituting the closed-form results for terms **(I)** and **(II)** into our upper bound yields  
 310 the final Von Mises–Fisher Distributional Semantic Alignment (VDSA) training objective:

$$312 \quad \mathcal{L}_{\text{VDSA}} = \log \left( \sum_{c=1}^C \frac{C_D(\kappa_c)}{C_D(\|\kappa_c \mu_c + s\mathbf{z}\|_2)} \right) - s A_D(\kappa_y) \mathbf{z}^\top \mu_y. \quad (10)$$

315 This loss is a differentiable upper bound to the ideal objective in equation 7 that requires no sampling.  
 316 In the limits, as  $\kappa_c \rightarrow \infty$  for all  $c$ , it recovers the standard cross-entropy loss, while for  $\kappa_c = 0$  (or  
 317  $s=0$ ), it yields the constant  $\log C$ .

## 318 4 EXPERIMENTS

321 **Experimental Setup** We evaluate VDSA under two standard problem settings: (i) few-shot learn-  
 322 ing with varying numbers of shots (Section 4.1), and (ii) base-to-novel generalization (Section 4.2).  
 323 All experiments are built on the open-source BiomedCLIP (Zhang et al., 2023) backbone. Below  
 we describe datasets, evaluation protocols, baselines, and implementation details.

324  
 325 Table 1: Few-shot classification accuracy averaged over 10 biomedical datasets for different shot  
 326 numbers ( $K = 1 \sim 16$ ).

327 <b>Method</b>	<b>K=1</b>	<b>K=2</b>	<b>K=4</b>	<b>K=8</b>	<b>K=16</b>
<b>Zero-shot Methods</b>					
329 BiomedCLIP			42.38		
330 BiomedCLIP + Ensemble			52.14		
331 CLIP-Adapter	$45.52 \pm 2.28$	$44.70 \pm 1.54$	$45.30 \pm 1.54$	$46.54 \pm 1.40$	$48.45 \pm 1.32$
332 Tip-Adapter	$50.35 \pm 5.03$	$53.50 \pm 5.39$	$58.33 \pm 4.93$	$62.01 \pm 5.76$	$67.60 \pm 4.44$
333 Tip-Adapter-F	$52.54 \pm 7.18$	$54.16 \pm 6.00$	$62.30 \pm 6.30$	$68.11 \pm 3.75$	$72.61 \pm 2.28$
334 Standard LP	$50.72 \pm 8.06$	$55.94 \pm 7.50$	$62.83 \pm 6.63$	$67.78 \pm 5.04$	$71.22 \pm 2.83$
335 LP++	$50.27 \pm 7.95$	$55.66 \pm 6.68$	$61.85 \pm 6.66$	$66.14 \pm 4.82$	$70.52 \pm 3.70$
336 CoOp	$52.59 \pm 6.67$	$55.71 \pm 4.10$	$61.35 \pm 3.36$	$67.74 \pm 3.18$	$71.48 \pm 2.85$
337 CoCoOp	$50.88 \pm 4.41$	$53.91 \pm 5.30$	$57.63 \pm 4.70$	$63.15 \pm 3.58$	$67.51 \pm 2.20$
338 KgCoOp	$54.31 \pm 4.68$	$55.79 \pm 5.13$	$60.92 \pm 3.97$	$66.00 \pm 2.42$	$67.71 \pm 1.90$
339 ProGrad	$53.67 \pm 5.77$	$56.42 \pm 4.16$	$62.10 \pm 3.62$	$67.06 \pm 3.07$	$69.21 \pm 2.69$
340 BiomedCoOp	$56.87 \pm 2.53$	$59.32 \pm 3.80$	$64.34 \pm 2.47$	$68.96 \pm 2.77$	$73.41 \pm 1.68$
341 <b>VDSA (Ours)</b>	<b><math>60.50 \pm 2.06</math></b>	<b><math>64.25 \pm 1.75</math></b>	<b><math>69.48 \pm 2.50</math></b>	<b><math>74.77 \pm 1.20</math></b>	<b><math>79.41 \pm 1.28</math></b>

342 **Datasets** VDSA is assessed on ten publicly available biomedical datasets spanning ten organs  
 343 and eight imaging modalities: Computerized Tomography (CTKidney (Islam et al., 2022)), Endoscopy (Kvasir (Pogorelov et al., 2017)), Fundus Photography (RETINA (Köhler et al., 2013;  
 344 Porwal et al., 2018)), Histopathology (LC25000 (Borkowski et al., 2019), CHMNIST (Kather et al.,  
 345 2016)), Magnetic Resonance Imaging (BTMRI (Masoud, 2021)), Optical Coherence Tomography  
 346 (OCTMNIST (Kermany et al., 2018)), Ultrasound (BUSI (Al-Dhabayani et al., 2020)), and X-Ray  
 347 (COVID-QU-Ex (Tahir et al., 2021), KneeXray (Chen, 2018)). This diverse benchmark enables a  
 348 rigorous evaluation of robustness across heterogeneous biomedical tasks and imaging conditions.  
 349 Full dataset descriptions and train/val/test splits are provided in Appendix B. **Note** that DermaMNIST  
 350 (Codella et al., 2019; Tschanzl et al., 2018) was excluded from the comparison due to class  
 351 imbalance, which led to BiomedCoOp achieving unnaturally high scores by predicting the dominant  
 352 class. Further discussion on this issue can be found in the Appendix C.

353 **Evaluation Protocols** We adopt two challenging protocols:

354 **Few-shot Learning.** To emulate limited data scenarios, models are trained using  $K \in$   
 355  $\{1, 2, 4, 8, 16\}$  labeled examples per class. This directly tests the sample efficiency of the adaptation  
 356 method.

357 **Base-to-Novel Generalization.** Each dataset’s classes are partitioned into disjoint base and novel  
 358 sets. Models are trained on 16-shot samples from base classes and evaluated on both base and  
 359 unseen novel classes, measuring the ability to retain knowledge while adapting, a key indicator  
 360 against catastrophic forgetting.

361 **Baselines** We compare VDSA with three major families of lightweight adaptation approaches: (1)  
 362 *Prompt learning methods*: CoOp (Zhou et al., 2022b), CoCoOp (Zhou et al., 2022a), ProGrad (Zhu  
 363 et al., 2023), KgCoOp (Yao et al., 2023) and BioMedCoOp (Koleilat et al., 2025); (2) *Adapter-based*  
 364 *methods*: CLIP-Adapter (Gao et al., 2024), Tip-Adapter (Zhang et al., 2021), and Tip-Adapter-F  
 365 (Zhang et al., 2021); (3) *Linear probing methods*: standard Linear Probing and LP++ (Huang  
 366 et al., 2024b). Zero-shot and LLM-prompted zero-shot BiomedCLIP (Zhang et al., 2023) serve as  
 367 strong reference points. All baselines share the same BiomedCLIP (Zhang et al., 2023) backbone  
 368 and comparable parameter budgets.

369 **Implementation Details** We build on the official BiomedCLIP (Zhang et al., 2023) codebase with  
 370 a ViT-B/16 vision encoder. For VDSA, class-wise semantic distributions are estimated from  $N = 50$   
 371 GPT-4 (Achiam et al., 2023) generated prompts and kept fixed during training. Only the LoRA  
 372 modules in the vision encoder are optimized using SGD with a learning rate of  $1 \times 10^{-4}$  and a batch  
 373 size of 4. Few-shot experiments run for 100 epochs and base-to-novel for 50 epochs. Results are  
 374 averaged over three random seeds. All experiments are performed on a single NVIDIA RTX 4090  
 375 GPU (24 GB VRAM).

378

379

Table 2: Performance comparison on 9 biomedical datasets. Bold indicates the best in each row.

Dataset		BiomedCLIP	CoOp	CoCoOp	KgCoOp	ProGrad	BiomedCoOp	VDSA (Ours)
Average on 9 datasets	Base	49.27	76.71	75.52	71.91	75.69	78.60	<b>82.48</b>
	Novel	67.17	65.34	67.74	65.94	67.33	73.90	<b>80.12</b>
	HM	55.23	68.80	69.11	67.22	69.86	74.04	<b>78.55</b>
BTMRI	Base	40.88	82.25	77.88	78.03	82.13	82.42	<b>86.25</b>
	Novel	96.18	94.51	94.84	95.05	94.98	<b>96.84</b>	94.66
	HM	57.37	87.95	85.53	85.70	88.09	89.05	<b>90.26</b>
COVID-QU-Ex	Base	53.96	75.92	<b>77.28</b>	75.42	75.19	75.91	76.57
	Novel	89.43	90.07	87.61	89.61	90.34	<b>91.63</b>	91.57
	HM	67.31	82.39	82.12	81.90	82.07	83.03	<b>83.40</b>
CTKIDNEY	Base	38.55	82.24	81.96	81.67	83.86	86.93	<b>89.73</b>
	Novel	52.99	67.92	56.56	58.45	63.01	78.94	<b>85.79</b>
	HM	44.63	74.40	66.93	68.14	71.96	82.74	<b>87.72</b>
Kvasir	Base	75.00	86.22	85.94	81.56	82.89	86.50	<b>88.00</b>
	Novel	60.50	58.06	53.95	59.00	60.45	61.83	<b>69.50</b>
	HM	66.97	69.39	66.29	68.47	69.91	72.11	<b>77.66</b>
CHMNIST	Base	37.63	89.41	87.77	75.45	82.98	88.87	<b>92.42</b>
	Novel	40.69	35.11	42.51	38.70	44.19	42.73	<b>53.06</b>
	HM	39.10	50.42	57.28	51.16	57.67	57.71	<b>67.42</b>
LC25000	Base	59.73	90.12	88.33	88.13	90.29	93.77	<b>95.09</b>
	Novel	87.60	87.55	95.02	86.44	85.47	<b>97.00</b>	96.27
	HM	71.03	88.82	91.55	87.28	87.81	95.36	<b>95.68</b>
RETINA	Base	45.18	70.98	66.88	60.77	68.77	68.46	<b>80.57</b>
	Novel	55.28	56.90	65.56	54.91	58.43	67.72	<b>74.02</b>
	HM	49.72	63.16	66.21	57.69	63.18	68.09	<b>79.06</b>
KneeXray	Base	35.89	38.28	34.08	37.94	40.88	44.23	<b>48.26</b>
	Novel	71.90	47.69	63.14	61.19	59.12	78.35	<b>81.02</b>
	HM	47.88	42.47	44.27	46.84	48.34	56.54	<b>60.49</b>
OCTMNIST	Base	56.60	75.00	79.60	68.20	74.20	80.33	<b>83.40</b>
	Novel	50.00	50.23	50.47	50.13	50.02	50.07	<b>56.20</b>
	HM	53.10	60.17	61.77	57.79	59.76	61.69	<b>67.15</b>

403

404

## 405 4.1 FEW-SHOT ADAPTATION

406

407 Table 1 summarizes few-shot classification accuracy averaged over 10 biomedical datasets. Our  
408 proposed **VDSA** consistently achieves the highest accuracy across all shot counts. In the most  
409 challenging **1-shot** and **2-shot** settings, VDSA reaches **60.50%** and **64.25%** accuracy, respectively,  
410 outperforming the strongest baseline *BiomedCoOp* (56.87%, 59.32%) by **3.6** and **4.9** percentage  
411 points, and surpassing other prompt-tuning methods such as *CoOp* (52.59%, 55.71%) and *ProGrad*  
412 (53.67%, 56.42%) by even larger margins. As the number of labeled examples grows, the advantage  
413 of VDSA remains clear: it attains **69.48%**, **74.77%**, and **79.41%** for  $K = 4, 8, 16$ , consistently  
414 exceeding *BiomedCoOp* (64.34%, 68.96%, 73.41%) and other baselines. The detailed per-dataset  
415 performance curves are provided in Appendix D. Across 10 diverse biomedical datasets covering  
416 eight imaging modalities, VDSA achieves the best or tied-best performance at nearly every shot  
417 count. These strong and uniform gains highlight the robustness of VDSA to both domain variation  
418 and data scarcity.

419 The key to this improvement lies in VDSA’s distributional semantic alignment. Instead of com-  
420 pressing multiple prompts into a prototype, VDSA models the entire von Mises–Fisher distribution  
421 of class semantics. This richer representation provides an implicit, infinite sample semantic aug-  
422mentation that optimize the vision–language alignment, maintaining effectiveness as supervision  
423 increases.

423

## 424 4.2 BASE-TO-NOVEL GENERALIZATION

425

426 We next evaluate VDSA on the challenging *Base-to-Novel Generalization* protocol. Table 2 presents  
427 base, novel, and harmonic mean (**HM**) accuracies across nine datasets and their overall averages,  
428 with HM reflecting the level of balanced generalization. Note that the BUSI Al-Dhabayani et al.  
429 (2020) dataset is excluded here since it contains only three classes, making a base–novel split infeasi-  
430 ble. VDSA consistently ranks among the best on Base, Novel, and HM metrics, with gains exceeding  
431 10% in HM on representative datasets such as CHMNIST Kather et al. (2016) and RETINA Köhler  
432 et al. (2013); Porwal et al. (2018). These results demonstrate that modeling each class as a von

432  
 433 Table 3: Ablation study of VDSA. The few-shot part reports average accuracy (%) over 10 biomedical  
 434 datasets under different shots (1, 2, 4, 8, 16). The base-to-novel part reports average Base, Novel,  
 435 and their harmonic mean (HM) accuracies (%).

Method	Few-shot Avg. Accuracy (%)					Base-to-Novel Avg. (%)		
	1-shot	2-shot	4-shot	8-shot	16-shot	Base	Novel	HM
CE (w/o VDSA)	55.77	58.72	66.50	72.37	78.19	81.26	77.50	77.51
<b>VDSA (Ours)</b>	<b>60.50</b>	<b>64.25</b>	<b>69.48</b>	<b>74.77</b>	<b>79.41</b>	<b>82.48</b>	<b>80.12</b>	<b>78.55</b>

441  
 442 Mises–Fisher semantic distribution, rather than a single prototype, mitigates overfitting to base con-  
 443 cepts and enhances transfer to unseen classes.  
 444

### 445 4.3 PARAMETERS SENSITIVITY

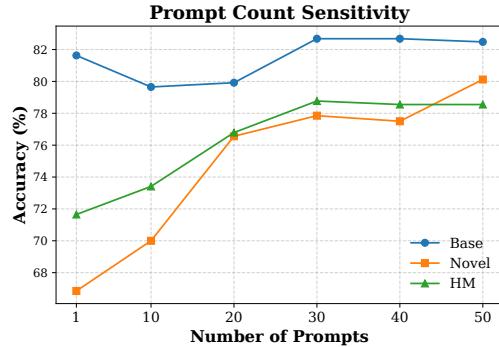
446 We analyze the impact of the number of prompts in Fig. 2. In our formulation, prompts are re-  
 447 garded as samples from the latent semantic distribution of each class, and the number of prompts  
 448 affects the reliability of the vMF parameter estimation. Increasing prompts from 1 to 20 leads to a  
 449 marked improvement in novel-class and HM accuracy. Beyond 20 prompts, performance changes  
 450 marginally: base accuracy stays around 82.5%–82.7%, and novel accuracy grows slightly to 80.12%  
 451 at 50 prompts. These results indicate that using about 30 prompts already captures most of the  
 452 achievable gains, and adding more prompts provides marginal additional benefit.  
 453

### 454 4.4 ABLATION STUDY

455 To evaluate the contribution of the proposed  
 456 VDSA module, we conduct an ablation study  
 457 by removing VDSA and training the model  
 458 with the conventional cross entropy classifi-  
 459 cation loss. This reduces the framework to  
 460 a prototype-based baseline, where images are  
 461 aligned only with the mean text embedding  
 462 of each class rather than with a full seman-  
 463 tic distribution. Table 3 reports the average  
 464 results for base-to-novel generalization, high-  
 465 lighting the consistent improvements brought  
 466 by VDSA. Across all shot levels from 1 to  
 467 16, VDSA consistently surpasses the CE base-  
 468 line. The improvements are most evident in the  
 469 low-shot regime, with gains of +4.7% at 1-shot  
 470 and +5.5% at 2-shot, showing that representing  
 471 each class as a von Mises–Fisher semantic dis-  
 472 tribution leads to stronger and more stable few-shot performance. Under the base-to-novel protocol,  
 473 VDSA also achieves consistent gains. The average HM rises from 77.51% to 78.55%, reflecting  
 474 better balance between base and novel classes and stronger transfer to unseen categories.  
 475

## 476 5 CONCLUSION

477 In this work, we identified a fundamental limitation in existing prompt ensembling methods for  
 478 Vision-Language Models: the collapse of rich semantic diversity into a single prototype. To ad-  
 479 dress this, we introduced VDSA, a novel parameter-efficient fine-tuning framework that shifts the  
 480 paradigm from prototype alignment to distributional alignment. By modeling class semantics with  
 481 the von Mises–Fisher distribution and deriving a tractable upper-bound objective, VDSA learns to  
 482 align image features with the entire semantic space of the concepts. Our extensive experiments on  
 483 10 biomedical datasets demonstrate that this principled approach yields significant improvements  
 484 in both few-shot adaptation and base-to-novel generalization, establishing a new state-of-the-art for  
 485 adapting VLMs to specialized domains.



476 Figure 2: Effect of the Number of Prompts on  
 477 base-to-novel generalization.

486 ETHICS STATEMENT  
487488 This study utilizes publicly available biomedical datasets, all of which have been anonymized to  
489 ensure privacy and comply with ethical standards. We adhered to ethical guidelines for data usage  
490 and have taken measures to avoid potential harmful insights. There are no conflicts of interest, and  
491 all research practices were in accordance with legal and ethical research standards.  
492493 REPRODUCIBILITY STATEMENT  
494495 To ensure the reproducibility of our results, we will provide the source code and datasets upon pub-  
496 lication. All theoretical results, assumptions, and proofs are detailed in the paper. Data processing  
497 steps necessary for reproducing the experiments are also included.  
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## APPENDIX

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## A LLM USAGE

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In this work, we use GPT-5 to assist in refining and polishing the paper. In addition, we use GPT-4 (Achiam et al., 2023) to generate textual descriptions for each medical class in the datasets. Specifically, for each dataset with  $C$  classes, we queried GPT-4 with the following instruction:

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Give  $N$  textual descriptions of visual discriminative features for distinct medical cases of [CLASS] found in [MODALITY].

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These generated descriptions were then used as prompts in the subsequent model training and evaluation process. Here we include one representative text prompt for each class across all datasets.

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**BTMRI**663  
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- normal brain: A normal brain in MRI appears with clearly defined structures, no abnormal growths, and symmetrical hemispheres.
- glioma tumor: A glioma tumor in MRI appears as an irregular, heterogeneous mass with poorly defined borders.
- meningioma tumor: A meningioma tumor in MRI appears as a well-circumscribed, extra-axial mass with a broad dural attachment.
- pituitary tumor: A pituitary tumor in MRI appears as a well-defined sellar mass, often with suprasellar extension.

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- benign tumor: an ultrasound image showing well-defined, smooth margins, indicating a benign breast tumor.
- malignant tumor: an ultrasound image showing irregular or spiculated margins, indicating a malignant breast tumor.
- normal scan: an ultrasound image showing homogenous echotexture throughout, indicating a normal breast ultrasound scan.

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683**CTKidney**684  
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- cyst kidney: a CT image showing a well-circumscribed lesion with thin, smooth walls, suggestive of a cyst kidney.
- kidney stone: a CT image showing a hyperdense focus within the renal pelvis, indicating a kidney stone.
- kidney tumor: a CT image showing a solid mass with heterogeneous enhancement, indicating a kidney tumor.
- normal kidney: a CT image showing well-defined renal contours and normal cortical thickness, indicating a normal kidney.

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694**COVID-QU-Ex**695  
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- covid lungs: an X-ray image showing bilateral ground-glass opacities, indicating covid lungs.
- lung opacity lungs: an X-ray image showing localized ground-glass opacity, indicating lung opacity lungs.
- normal lungs: an X-ray image showing clear lung fields, indicating normal lungs.
- viral pneumonia lungs: an X-ray image showing bilateral ground-glass opacities, indicating viral pneumonia lungs.

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703**Kvasir**704  
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- dyed lifted polyps: Dyed lifted polyps appear as raised lesions with a blue or green hue, indicating the application of dye to enhance visibility during endoscopy.
- dyed resection margins: Dyed resection margins appear as clearly delineated edges around a resected lesion, marked by the application of dye during endoscopy.
- esophagitis: Esophagitis presents as inflamed, reddened areas of the esophageal lining, often with visible erosions or ulcers in severe cases.
- normal cecum: The normal cecum appears as a blind-ended pouch located at the beginning of the large intestine, with a smooth and healthy mucosal surface.
- normal pylorus: The normal pylorus appears as a circular, well-defined opening at the distal end of the stomach, leading into the duodenum.
- normal z line: The normal Z line appears as a distinct, zigzagging line at the junction of the esophagus and stomach during endoscopy.
- polyps: Polyps appear as protrusions or growths on the mucosal surface of the gastrointestinal tract during endoscopy.
- ulcerative colitis: Ulcerative colitis appears as inflammatory changes in the mucosal lining of the colon during endoscopy.

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723**KneeXray**724  
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- healthy knee: An X-ray image showing a knee with a clear and even joint space, no bone spurs, indicating a healthy knee.
- doubtful osteoarthritis: An X-ray image showing a knee with slight joint space narrowing and the beginning of osteophyte formation, indicating doubtful osteoarthritis.
- minimal osteoarthritis: An X-ray image showing a knee with definite osteophytes and slight joint space narrowing, indicating minimal osteoarthritis.
- moderate osteoarthritis: An X-ray image showing a knee with multiple, well-defined osteophytes and moderate joint space narrowing, indicating moderate osteoarthritis.
- severe osteoarthritis: An X-ray image showing a knee with large osteophytes, severe joint space narrowing, and significant sclerosis, indicating severe osteoarthritis.

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**RETINA**737  
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- cataract: a retina image showing opacification of the lens with loss of transparency, indicative of cataract.
- diabetic retinopathy: a retina image showing microaneurysms, hemorrhages, and exudates in the retina, indicative of diabetic retinopathy.
- glaucoma: a retina image showing optic disc cupping, neuroretinal rim thinning, and retinal nerve fiber layer defects, indicative of glaucoma.
- normal retina: a retina image showing intact retinal layers with well-defined foveal depression and normal vasculature, indicating normal retina.

747  
748**CHMNIST**749  
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- adipose tissue: Adipose tissue in histology slides appears as clusters of large, clear cells with a thin rim of cytoplasm and a centrally located nucleus.
- complex stroma: Complex stroma in histology slides presents as a dense network of collagen fibers, fibroblasts, and extracellular matrix components, providing structural support to tissues.
- debris: Debris in histology slides often appears as irregular, darkly staining particles scattered throughout the tissue section, indicating areas of cellular breakdown.

756

- 757 empty background: Empty background in histology slides appears as clear or lightly stained
- 758 areas devoid of cellular or extracellular matrix components, providing contrast for identi-
- 759 fying tissue structures.
- 760 immune cells: Immune cells in histology slides appear as small, round cells with darkly
- 761 staining nuclei, often located within or surrounding areas of tissue inflammation.
- 762 normal mucosal glands: Normal mucosal glands in histology slides appear as well-
- 763 organized tubular structures lined by epithelial cells, often with a central lumen.
- 764 simple stroma: Simple stroma in histology slides consists of a loose network of connective
- 765 tissue, with sparse collagen fibers and scattered fibroblasts.
- 766 tumour epithelium: Tumor epithelium in histology slides appears as atypical epithelial cells
- 767 with irregular nuclei, prominent nucleoli, and increased mitotic activity.

768

### 769 **LC25000**

770

- 771 colon adenocarcinoma: a histopathological section showing malignant glands infiltrating
- 772 the colonic mucosa and submucosa, indicative of colon adenocarcinoma.
- 773 colon benign tissue: a histological slide showing normal colonic mucosa with intact crypt
- 774 architecture and absence of dysplastic changes, indicating colon benign tissue.
- 775 lung adenocarcinoma: a histopathological section showing glandular structures infiltrating
- 776 the lung parenchyma with stromal desmoplasia, indicative of lung adenocarcinoma.
- 777 lung benign tissue: a histological slide showing normal lung parenchyma with intact alve-
- 778 olar architecture and absence of dysplastic changes, indicating lung benign tissue.
- 779 lung squamous cell carcinoma: a histopathological section showing nests of squamous cells
- 780 with keratinization and intercellular bridges infiltrating the lung tissue, indicative of lung
- 781 squamous cell carcinoma.

### 782 **DermaMNIST**

783

- 784 actinic keratosis: Actinic keratosis presents as rough, scaly patches on sun-exposed areas
- 785 of the skin, often with a pink or red base.
- 786 basal cell carcinoma: Basal cell carcinoma (BCC) often appears as a pearly or translucent
- 787 nodule with visible blood vessels (telangiectasia) on the surface.
- 788 benign keratosis: Benign keratosis, such as seborrheic keratosis, presents as well-defined,
- 789 warty, or waxy growths on the skin.
- 790 dermatofibroma: Dermatofibromas present as firm, raised nodules on the skin, typically
- 791 with a hyperpigmented surface.
- 792 melanoma: a skin image showing an irregular, asymmetrical mole, indicating melanoma.
- 793 melanocytic nevus: Melanocytic nevi, or moles, are common benign skin lesions resulting
- 794 from the proliferation of melanocytes.
- 795 squamous cell carcinoma: a close-up of a lesion with irregular borders and a scaly surface,
- 796 indicating squamous cell carcinoma.
- 797 vascular lesion: Vascular lesions include a variety of conditions such as hemangiomas,
- 798 vascular malformations, and pyogenic granulomas.

### 799 **OCTMNIST**

800

- 801 choroidal neovascularization: OCT scan showing subretinal fluid accumulation due to ab-
- 802 normal blood vessels growth.
- 803 diabetic macular edema: OCT image showing retinal thickening with visible cystoid
- 804 spaces.
- 805 drusen: OCT showing small, hyperreflective deposits beneath the retinal pigment epithe-
- 806 lium.
- 807 normal OCT scan: OCT image showing a normal, uniform retinal structure with no signs
- 808 of fluid accumulation.

## 810 B DATASET

812 **Datasets.** We evaluate on eleven public datasets spanning nine biomedical imaging modalities and  
 813 ten organs. For each dataset, we state the modality and organ(s), list the class labels as given, and  
 814 report the train/val/test split.

- 816 • **CTKidney** (Islam et al., 2022). *Modality: Computerized Tomography; Organ(s): Kidney.* Classes: Kidney Cyst, Kidney Stone, Kidney Tumor, Normal Kidney. Split: 6221/2487/3738.
- 819 • **DermaMNIST** (Codella et al., 2019; Tschandl et al., 2018). *Modality: Dermatoscopy; Organ(s): Skin.* Classes: Actinic Keratosis, Basal Cell Carcinoma, Benign Keratosis, Dermatofibroma, Melanocytic nevus, Melanoma, Vascular Lesion. Split: 7007/1003/2005.
- 822 • **Kvasir** (Pogorelov et al., 2017). *Modality: Endoscopy; Organ(s): Colon.* Classes: Dyed Lifted Polyps, Normal Cecum, Esophagitis, Dyed Resection Margins, Normal Pylorus, Normal Z Line, Polyps, Ulcerative Colitis. Split: 2000/800/1200.
- 825 • **RETINA** (Köhler et al., 2013; Porwal et al., 2018). *Modality: Fundus Photography; Organ(s): Retina.* Classes: Cataract, Diabetic Retinopathy, Glaucoma, Normal Retina. Split: 2108/841/1268.
- 828 • **LC25000** (Borkowski et al., 2019). *Modality: Histopathology; Organ(s): Lung, Colon.* Classes: Colon Adenocarcinoma, Colon Benign Tissue, Lung Adenocarcinoma, Lung Benign Tissue, Lung Squamous Cell Carcinoma. Split: 12500/5000/7500.
- 831 • **CHMNIST** (Kather et al., 2016). *Modality: Histopathology; Organ(s): Colorectal.* Classes: Adipose Tissue, Complex Stroma, Debris, Empty Background, Immune Cells, Normal Mucosal Glands, Simple Stroma, Tumor Epithelium. Split: 2496/1000/1504.
- 834 • **BTMRI** (Masoud, 2021). *Modality: Magnetic Resonance Imaging; Organ(s): Brain.* Classes: Glioma Tumor, Meningioma Tumor, Normal Brain, Pituitary Tumor. Split: 2854/1141/1717.
- 837 • **OCTMNIST** (Kermany et al., 2018). *Modality: Optical Coherence Tomography; Organ(s): Retina.* Classes: Choroidal Neovascularization, Drusen, Diabetic Macular Edema, Normal. Split: 97477/10832/1000.
- 840 • **BUSI** (Al-Dhabayani et al., 2020). *Modality: Ultrasound; Organ(s): Breast.* Classes: Benign Tumors, Malignant Tumors, Normal Scans. Split: 389/155/236.
- 843 • **COVID-QU-Ex** (Tahir et al., 2021). *Modality: X-Ray; Organ(s): Chest.* Classes: COVID-19, Lung Opacity, Normal Lungs, Viral Pneumonia. Split: 10582/4232/6351.
- 846 • **KneeXray** (Chen, 2018). *Modality: X-Ray; Organ(s): Knee.* Classes: No, Doubtful, Minimal, Moderate, and Severe Osteoarthritis. Split: 5778/826/1656.

## 848 C DERMAMNIST PERFORMANCE

850 Table 4 reports results on the 7-class DermaMNIST test set (2,005 images), which is highly imbalanced:  
 851 the majority class *melanocytic nevus* constitutes 1,341 samples (66.9%). Under this skew,  
 852 plain accuracy is unreliable because a trivial majority class predictor already attains 66.9%.

854 **Metrics.** We report (i) *Macro-Recall* (balanced accuracy), (ii) *Macro-F1*, (iii) the *Zero-Recall*  
 855 (*count*)—the number of classes with recall exactly 0, and (iv) *Minority Avg. Recall*, the mean recall  
 856 over the other 6 classes. All values are shown as percentages except the count.

858 **Findings.** At 1-shot, although BiomedCoOp reaches 61.7% accuracy, it still trails the 66.9%  
 859 majority baseline and exhibits poor balance (Macro-Recall 25.2%, Macro-F1 19.1%, two zero-recall  
 860 classes). Our method (1-shot) improves Macro-Recall/F1 to 30.5%/21.6%, removes zero-recall  
 861 classes, and nearly doubles minority class recall (14.6% → 28.9%). At 16-shot, BiomedCoOp  
 862 modestly improves (Macro-Recall 29.7%, Macro-F1 26.5%) but still has one zero-recall class and only  
 863 21.0% minority recall. In contrast, Ours (16-shot) delivers substantially better balance (Macro-  
 Recall 54.6%, Macro-F1 39.1%, zero-recall count 0, minority recall 52.3%). While its accuracy

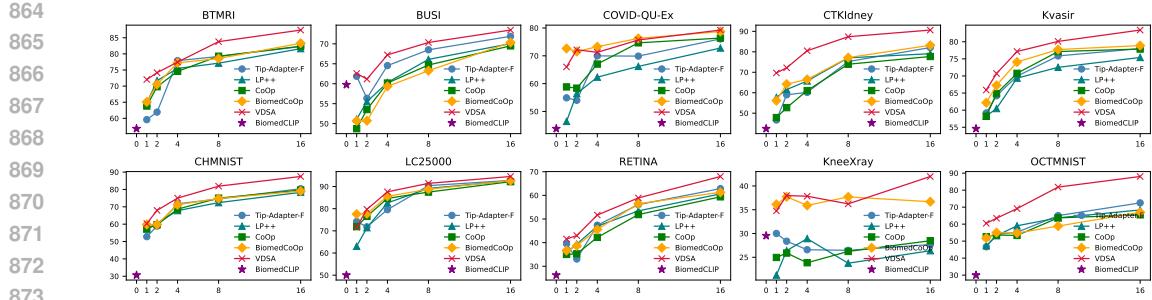


Figure 3: Per-dataset few-shot classification performance across 10 biomedical datasets.

(59.8%) is slightly below BiomedCoOp (61.5%), both are inferior to the 66.9% majority baseline underscoring that balanced metrics, not accuracy, reflect clinically meaningful performance under severe imbalance.

Table 4: DermaMNIST under severe class imbalance (values in %).

Metric	Majority	BiomedCoOp		Ours	
	–	1-shot	16-shot	1-shot	16-shot
Accuracy (%)	<b>66.9</b>	61.7	61.5	36.3	59.8
Macro-Recall (%)	–	25.2	29.7	30.5	<b>54.6</b>
Macro-F1 (%)	–	19.1	26.5	21.6	<b>39.1</b>
Zero-Recall (count)	–	2	1	<b>0</b>	<b>0</b>
Minority Avg. Recall (%)	–	14.6	21.0	28.9	<b>52.3</b>

## D PER-DATASET FEW-SHOT PERFORMANCE

Figure 3 reports the per-dataset few-shot classification performance across 10 biomedical datasets under different shot numbers ( $K = 1 \sim 16$ ). Consistent with the averaged results in the main text, VDSA attains the best or tied-best accuracy across nearly all datasets and shot counts, further confirming its robustness to diverse imaging modalities and limited-data settings.

## E LIMITATION OF GAUSSIAN-BASED DISTRIBUTION

**Setup.** Let image features be  $\ell_2$ -normalized,  $\|\mathbf{z}\|_2 = 1$ , and each class  $c$  be represented by a random prototype  $\mathbf{u}_c$ . We compare (i) an *Euclidean Gaussian* model  $\mathbf{u}_c \sim \mathcal{N}(\mu_c, \Sigma_c)$  in  $\mathbb{R}^D$  with (ii) an *on-sphere* model. We study the Jensen upper bound

$$\mathcal{L} = \log \sum_{c=1}^C \mathbb{E}[e^{s \mathbf{z}^\top \mathbf{u}_c}] - \mathbb{E}[s \mathbf{z}^\top \mathbf{u}_y], \quad s > 0.$$

### A. EUCLIDEAN GAUSSIAN VIOLATES THE UNIT-SPHERE COSINE GEOMETRY

**Closed form under Gaussian.** If  $\mathbf{u}_c \sim \mathcal{N}(\mu_c, \Sigma_c)$  in  $\mathbb{R}^D$ , then  $\mathbf{z}^\top \mathbf{u}_c \sim \mathcal{N}(\mathbf{z}^\top \mu_c, \mathbf{z}^\top \Sigma_c \mathbf{z})$ . Hence

$$\mathbb{E}[e^{s \mathbf{z}^\top \mathbf{u}_c}] = e^{s \mathbf{z}^\top \mu_c + \frac{s^2}{2} \mathbf{z}^\top \Sigma_c \mathbf{z}}, \quad \mathbb{E}[s \mathbf{z}^\top \mathbf{u}_y] = s \mathbf{z}^\top \mu_y,$$

and therefore

$$\mathcal{L}_{\text{Gauss}} = \log \sum_{c=1}^C e^{s \mathbf{z}^\top \mu_c + \frac{s^2}{2} \mathbf{z}^\top \Sigma_c \mathbf{z}} - s \mathbf{z}^\top \mu_y. \quad (11)$$

918 **Geometric mismatch.** Differentiating equation 11 w.r.t.  $\mathbf{z}$  gives  
 919

920 
$$\nabla_{\mathbf{z}} \mathcal{L}_{\text{Gauss}} = \sum_{c=1}^C w_c (s \boldsymbol{\mu}_c + s^2 \boldsymbol{\Sigma}_c \mathbf{z}) - s \boldsymbol{\mu}_y, \quad w_c = \frac{e^{a_c}}{\sum_j e^{a_j}}, \quad a_c = s \mathbf{z}^\top \boldsymbol{\mu}_c + \frac{s^2}{2} \mathbf{z}^\top \boldsymbol{\Sigma}_c \mathbf{z}.$$
  
 921

923 Compared with the cosine geometry objective (which depends only on angles  $\mathbf{z}^\top \boldsymbol{\mu}_c$ ), the Gaussian  
 924 bound introduces the variance driven term  $s^2 \boldsymbol{\Sigma}_c \mathbf{z}$ . It biases  $\mathbf{z}$  toward directions of small  $\mathbf{z}^\top \boldsymbol{\Sigma}_c \mathbf{z}$   
 925 (low variance axes), making optimization depend on scale/variance in  $\mathbb{R}^D$  rather than purely on  
 926 directions on the unit sphere. This contradicts CLIP’s spherical cosine geometry.  
 927

928 **B. PROJECTED GAUSSIAN ON THE SPHERE IS NOT AN EXPONENTIAL FAMILY**

929 **Projected (normalized) Gaussian.** An on-sphere alternative is to sample  $\mathbf{g} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  and  
 930 project  $\mathbf{u} = \mathbf{g}/\|\mathbf{g}\| \in \mathcal{S}^{D-1}$ . The induced density on the sphere (with respect to surface mea-  
 931 sure  $d\sigma$ ) is

932 
$$p_{\text{PN}}(\mathbf{u} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) \propto \int_0^\infty r^{D-1} e^{-\frac{1}{2}(\mathbf{r}\mathbf{u}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{r}\mathbf{u}-\boldsymbol{\mu})} dr. \quad (12)$$
  
 933

935 Expanding the quadratic form inside the integral yields dependence on both  $\mathbf{u}^\top \boldsymbol{\Sigma}^{-1} \mathbf{u}$  and  $\mathbf{u}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$   
 936 in a *nonlinear* fashion after integrating out  $r$ .  
 937

938 **Consequence for the Jensen bound.** The vMF density is an exponential family on  $\mathcal{S}^{D-1}$ :  
 939  $p_{\text{vMF}}(\mathbf{u} \mid \boldsymbol{\theta}) \propto e^{\boldsymbol{\theta}^\top \mathbf{u}}$ , which implies the key identity

940 
$$\mathbb{E}_{\text{vMF}(\boldsymbol{\theta})} \left[ e^{\mathbf{h}^\top \mathbf{u}} \right] = \frac{Z(\boldsymbol{\theta} + \mathbf{h})}{Z(\boldsymbol{\theta})}.$$
  
 941

943 In contrast, equation 12 cannot be written in the form  $p(\mathbf{u} \mid \boldsymbol{\theta}) = e^{\boldsymbol{\theta}^\top \mathbf{u} - A(\boldsymbol{\theta})}$  with respect to  $d\sigma$  for  
 944 general  $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Therefore the above identity is unavailable, and the expectation  $\mathbb{E} \left[ e^{s \mathbf{z}^\top \mathbf{u}} \right]$  under  
 945 the projected Gaussian does not reduce to a tractable ratio of normalizers as in vMF. Hence one  
 946 cannot obtain an analytic Jensen-type upper bound of the same closed form used in our method.  
 947

948 **Summary.** Euclidean Gaussians introduce variance sensitive terms that break spherical cosine ge-  
 949 ometry; projected Gaussians live on the sphere but lack the exponential-family structure needed to  
 950 turn  $\mathbb{E} \left[ e^{s \mathbf{z}^\top \mathbf{u}} \right]$  into a closed-form expression. The vMF model satisfies both requirements simulta-  
 951 neously.  
 952

953 **F DERIVATION OF vMF MAXIMUM LIKELIHOOD ESTIMATORS**

956 Here, we derive the maximum likelihood estimators (MLE) for the parameters of the von Mises-  
 957 Fisher (vMF) distribution, given  $N$  i.i.d. samples  $\{\mathbf{u}_i\}_{i=1}^N$  from  $\text{vMF}(\boldsymbol{\mu}, \kappa)$ . The probability density  
 958 function is:

959 
$$p(\mathbf{u} \mid \boldsymbol{\mu}, \kappa) = C_D(\kappa) \exp(\kappa \boldsymbol{\mu}^\top \mathbf{u}),$$

960 where  $C_D(\kappa)$  is the normalization constant. The log-likelihood function  $\mathcal{L}$  for the  $N$  samples is:

961 
$$\log \mathcal{L}(\boldsymbol{\mu}, \kappa) = \log \prod_{i=1}^N p(\mathbf{u}_i \mid \boldsymbol{\mu}, \kappa) \quad (13)$$
  
 962

963 
$$= \sum_{i=1}^N \log [C_D(\kappa) \exp(\kappa \boldsymbol{\mu}^\top \mathbf{u}_i)] \quad (14)$$
  
 964

965 
$$= N \log C_D(\kappa) + \kappa \sum_{i=1}^N \boldsymbol{\mu}^\top \mathbf{u}_i \quad (15)$$
  
 966

967 
$$= N \log C_D(\kappa) + \kappa \boldsymbol{\mu}^\top \left( \sum_{i=1}^N \mathbf{u}_i \right). \quad (16)$$
  
 968

972 **Estimating Mean Direction  $\hat{\mu}$ .** To maximize the log-likelihood, we first focus on the term involving  $\mu$ , which is  $\kappa\mu^\top \left(\sum_{i=1}^N \mathbf{u}_i\right)$ . Let the resultant vector be  $\mathbf{R} = \sum_{i=1}^N \mathbf{u}_i$ . Since  $\kappa \geq 0$ , maximizing this term is equivalent to maximizing the dot product  $\mu^\top \mathbf{R}$ . By the Cauchy-Schwarz inequality, this dot product is maximized when the unit vector  $\mu$  is aligned with the vector  $\mathbf{R}$ . Therefore, the MLE for the mean direction is the normalized resultant vector:

$$\hat{\mu} = \frac{\mathbf{R}}{\|\mathbf{R}\|_2} = \frac{\sum_{i=1}^N \mathbf{u}_i}{\left\| \sum_{i=1}^N \mathbf{u}_i \right\|_2}.$$

981 **Estimating Concentration  $\hat{\kappa}$ .** We substitute the MLE  $\hat{\mu}$  back into the log-likelihood function  
982 (Eq. 16). The second term becomes:  
983

$$984 \kappa \hat{\mu}^\top \mathbf{R} = \kappa \frac{\mathbf{R}^\top}{\|\mathbf{R}\|_2} \mathbf{R} = \kappa \frac{\|\mathbf{R}\|_2^2}{\|\mathbf{R}\|_2} = \kappa \|\mathbf{R}\|_2. \\ 985 \\ 986$$

987 The log-likelihood is now a function of  $\kappa$  alone:

$$988 \log \mathcal{L}(\kappa) = N \log C_D(\kappa) + \kappa \|\mathbf{R}\|_2. \\ 989$$

990 To find the maximum, we differentiate with respect to  $\kappa$  and set the result to zero:

$$991 \frac{\partial \log \mathcal{L}}{\partial \kappa} = N \frac{C'_D(\kappa)}{C_D(\kappa)} + \|\mathbf{R}\|_2 = 0. \\ 992 \\ 993$$

994 Using the known identity for vMF distributions,  $A_D(\kappa) = -\frac{d}{d\kappa} \log C_D(\kappa) = -\frac{C'_D(\kappa)}{C_D(\kappa)}$ , we have:  
995

$$996 N(-A_D(\kappa)) + \|\mathbf{R}\|_2 = 0.$$

997 Rearranging gives the final equation for the MLE  $\hat{\kappa}$ :

$$998 A_D(\hat{\kappa}) = \frac{\|\mathbf{R}\|_2}{N} = \frac{\left\| \sum_{i=1}^N \mathbf{u}_i \right\|_2}{N} = \left\| \frac{1}{N} \sum_{i=1}^N \mathbf{u}_i \right\|_2. \\ 1000 \\ 1001$$

1002 This shows that the MLE for the concentration  $\hat{\kappa}$  is the solution to  $A_D(\hat{\kappa}) = \bar{R}$ , where  $\bar{R}$  is the  
1003 empirical mean resultant length.

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