

# Fairshare Data Pricing for Large Language Models

Luyang Zhang<sup>1 \*</sup> Cathy Jiao<sup>2 \*</sup> Beibei Li<sup>1</sup> Chenyan Xiong<sup>2</sup>

## Abstract

Training data is a pivotal resource for building large language models (LLMs), but unfair pricing in data markets poses a serious challenge for both data buyers (e.g., LLM builders) and sellers (e.g., human annotators), which discourages market participation, reducing data quantity and quality. In this paper, we propose a *fairshare* pricing framework that sets training data prices using *data valuation* methods to quantify their contribution to LLMs. In our framework, buyers make purchasing decisions using data valuation and sellers set prices to maximize their profits based on the anticipated buyer purchases. We theoretically show that pricing derived from our framework is tightly linked to data valuation and buyers’ budget, optimal for both buyers and sellers. Through market simulations using current LLMs and datasets (math problems, medical diagnosis, and physical reasoning), we show that our framework is *fairshare* for buyers by ensuring their purchased data is reflective of model training value, leading to higher LLM task performances per-dollar spent on data, and *fairshare* for sellers by ensuring they sell their data at optimal prices. Our framework lays the foundation for future research on equitable and sustainable data markets for large-scale AI.<sup>1</sup>

## 1. Introduction

Training data is a foundation for building effective and reliable large language models (LLMs). LLMs need an abundant amount of high-quality training data to excel at complex tasks – such as coding (Chen et al., 2021) and instruction-following (Bach et al., 2022) – which requires human knowledge. In particular, human-annotated training

data for LLMs critically affects model performance not only for NLP tasks (Wang et al., 2023), but also significantly impact the real-life capabilities of AI applications such as virtual assistants (Xi et al., 2023), recommendation systems (Wu et al., 2024b), and healthcare systems (He et al., 2024).

Given the critical role of data in distinguishing LLM performance, LLM builders have intense demands for high-quality data to achieve desired training outcomes (Reuters, 2024). However, the current market for LLM training data acquisition is characterized by a lack of fairness and transparency in data pricing (Paul & Tong, 2024; Zhang et al., 2024a), which negatively affects all participants. For data sellers or contributors (i.e., human annotators), unfair data pricing leads to inadequate compensation that does not accurately reflect the true value of their contributions (Mason & Watts, 2009; Hara et al., 2018; CBS News, 2024). In an interview with *60 Minutes*<sup>2</sup>, a civil rights activist described the exploitation of annotators working for large LLM builders as:

“They don’t pay well ... they could pay whatever, and have whatever working conditions”

— Nerima Wako-Ojiwa on 60 Minutes

Unfair training data pricing also harms buyers (i.e., LLM builders) by reducing annotation quantity and quality (TechCrunch, 2024), which ultimately hinders their long-term gains. Consequently, unfair data pricing ultimately discourages participation in the data market, risks market failure, (Ater & Rigbi, 2023; Akerlof, 1978), and hampers the sustainable development of high-quality AI systems.

To tackle these issues, we propose a *fairshare* pricing framework where data prices reflect their value for all participants. Our pricing framework leverages *data valuation* methods, which quantify the contribution of training data to model performances. In our framework, buyers use *data valuation* to make informed purchases under budget constraints. By ensuring *transparent data valuation* for all participants, our framework provides procedural justice (Konovsky, 2000) and allows data sellers to make informed pricing to maximize their profits. This practice eventually improves seller participation and increases the quality of data supplies.

We simulate buyer-seller interactions and empirically show

<sup>\*</sup>Equal contribution <sup>1</sup>Heinz College of Information Systems and Public Policy, Carnegie Mellon University <sup>2</sup>Language Technologies Institute, Carnegie Mellon University. Correspondence to: Chenyan Xiong <cx@cs.cmu.edu>.

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<sup>1</sup>Our code will be released.

<sup>2</sup>CBS, *60 Minutes*: “Labelers training AI say they’re overworked, underpaid and exploited by big American tech companies”

that our framework achieves *fairshare* pricing. We create markets for sourcing data to train current open-source LLMs on complex NLP tasks, including math problems (Amini et al., 2019), medical diagnosis (Jin et al., 2021), and physical reasoning (Bisk et al., 2020). We then analyze our framework from two perspectives: *data valuation* and *pricing*. Our findings show that adopting *data valuation* enables buyers to achieve higher model performance at competitive costs, making our framework particularly advantageous for those with limited budgets.

Furthermore, both our empirical and theoretical results show that it is the buyers’ best interest to accept the sellers’ *fairshare* prices through a mechanism that models the seller’s decrease in participation over time if compensated unfairly. Our findings show that pricing methods which follow data market norms (undervaluing sellers’ contributions) offer buyers short-term utility gain. However, this approach is unsustainable—persistent unfair compensation will drive sellers away, reducing available datasets (hindering even the most resourceful buyers), and stall LLM development. This phenomenon is supported by research in organizational justices theory (Folger et al., 1998; Folger & Cropanzano, 2001; Colquitt et al., 2013; Adamovic, 2023), which highlights that individuals’ motivation depends not only on the *fairness* of compensation but also on the *transparency* of information and processes.

In contrast, our findings show that by adopting our *fairshare* pricing framework, where buyers accept the sellers’ *fairshare* price, buyers (regardless of budget size) maximize seller participation and, in turn, achieve greater long-term utility and create a win-win scenario for both parties. We encourage future research on equitable pricing methods for LLM training data.

In summary, our contributions are as follows:

1. We propose a *fairshare* pricing framework where data prices are reflected through *data valuation* methods.
2. We formalize our market dynamics with sequentially arrived sellers and characterize the optimal data pricing derived from our framework.
3. We show that our framework provides a win-win situation: sellers sell their datasets at optimal prices, and buyers simultaneously achieve greater long-term gains.

## 2. Related Work

**Data Acquisition for LLMs:** Multiple sources are used to supply training data for LLMs, including web-scraped data (often used for LLM pre-training) and crowd-sourced data (often used for LLM fine-tuning) (Wang et al., 2024). To create datasets from these sources, human annotators are often employed to give domain-specific annotations (e.g., labels for expert-level math and coding tasks (Tan et al., 2024)),

but may face unfavorable labor conditions. For instance, annotators are often underpaid on crowd-sourcing platforms (Mason & Watts, 2009; Hara et al., 2018; Toxtli et al., 2021; CBS News, 2024), and in regions without strong worker rights, annotators are exposed to toxic digital material during data labeling (Muldoon et al., 2023). While recent works aimed to reduce unfair wages (Singer & Mittal, 2013; Wang et al., 2013), their methods are based on task completion quantity, which is a misleading factor for compensation, and yields low quality annotations (Huynh et al., 2021).

**Data Valuation:** *Data valuation* is the process of measuring the contribution of data towards a goal (i.e., LLM performance). Influence-based methods, which rely on influence functions (Hampel, 1974; Koh & Liang, 2017), perform data valuation by measuring the contribution of training samples to a model’s outputs using learning gradients. Recent works have approximated influence functions for LLMs by estimating the computationally costly inverse-Hessian (Grosse et al., 2023; Choe et al., 2024; Bae et al., 2024). Other influence-based methods omit this calculation entirely (Pruthi et al., 2020; Xia et al., 2024). Given the effectiveness of influence-based methods towards LLM training data selection (Yu et al., 2024; Xia et al., 2024), recent works such as Trak (Park et al., 2023), LOGIX (Choe et al., 2024), and ICP (Jiao et al., 2024) examine the trade-offs between efficiency/accuracy in influence function approximations.

Other *data valuation* methods measures semantic similarity between training samples and target text. For instance, is BM25 (Trotman et al., 2014), a ranking algorithm based on word-term frequency. BM25 is used as a common baseline for benchmarking new data valuation techniques (Akyurek et al., 2022; Wu et al., 2024a) given its transparency.

**Game Theory and Data Pricing:** In dynamic games with complete information, such as *Stackelberg* games (Von Stackelberg, 2010; Maharjan et al., 2013; Zhang et al., 2024b) and *bi-level* optimization (Colson et al., 2007; Sinha et al., 2017; Bard, 2013), participants aim to maximize their objectives in a sequential manner: a leader acts first, then the followers. This has been used for pricing goods (Pei, 2020; Böhnlein et al., 2021), where the seller (*leader*) first sets prices for items to maximize profit, and buyers (*followers*) aim to buy a subset of items with minimal cost.

Previous studies have explored this dynamic for different data pricing settings. Agarwal et al. (2019) proposed an auction-based data pricing framework where the data platform allocates sellers’ data to buyers. Yang (2022) uses historical transactions to conduct personalized pricing for future consumers, assuming that buyers are risk-averse. These frameworks focused pricing via market mechanism design rather than potential LLM gains.

### 3. Data Market Framework

In this section, we present a theoretical framework for a *fairshare* data market. Beginning in Section 3.1, we provide an overview of the market participants—buyers and sellers—and explain how data valuation informs their objectives and decision-making processes. Then, we present the market dynamics, outlining the interactions between buyers and sellers as well as the conditions guiding their decision-making processes. Finally, we propose a pricing strategy derived from our framework, characterize its key properties, and demonstrate how it safeguards the welfare of both sellers (Section 3.2) and buyers (Section 3.3).

#### 3.1. Market Participants and Dynamics

In this section, we describe the objectives of buyers and sellers within our market framework, and their interactions. First, we denote a set of buyers  $\{B_k\}_{k=1}^M$  (e.g., LLM builders), whose aim is to purchase datasets that maximize performance gains for their LLMs, denoted as  $\mathcal{M}_k$ . Next, sellers  $\{S_j\}_{j=1}^N$  (e.g., human annotators) arrive to the market and set prices for their datasets  $D_j$  to maximize profits based on buyers' anticipated purchasing decisions.

Our framework adopts the well-established *Stackelberg* dynamic game (Von Stackelberg, 2010; Maharjan et al., 2013; Zhang et al., 2024b) under complete information, where players in the market move in sequence and possess full knowledge of all other players. Such a market enforces *information transparency*, allowing buyers and sellers to access estimates of each dataset's contribution to LLM performance (e.g., provided by platforms or marketplace organizers). In Section 5, we show that this transparency is beneficial for both parties. We provide a detailed explanation of the buyers' and sellers' objectives below.

**Buyer's Objective.** When a buyer  $B_k$  enters the market, they optimize their purchasing decisions based on three factors: (i) the dataset prices, represented by the price vector  $\mathbf{p} := [p_1, \dots, p_N]$ , (ii) their budget constraint  $b_k$ , and (iii) their *utility* gain  $u_k$  from acquiring a dataset  $D_j$ . Specifically,  $u_k : D_j \rightarrow \mathbb{R}_+$  denotes the utility (e.g., economical value, which may be expressed in currency) tied to the performance improvement of  $\mathcal{M}_k$  after training on dataset  $D_j$ , estimated using a *data valuation method*, such as influence functions (Koh & Liang, 2017). Data valuation methods can be applied to the buyer's objectives in various ways for different downstream LLM applications (see Appendix C).

Next, given datasets  $\{D_j\}_{j=1}^N$  in the market, price vector  $\mathbf{p}$ , budget  $b_k$ , and the *utility*  $u_k$ , the buyer makes a *purchase decision*, denoted as a binary vector  $\mathbf{x} \in \{0, 1\}^N$ , where  $x_j = 1$  indicates that dataset  $D_j$  is selected and  $x_j = 0$  indicates it is not. Given a decision  $\mathbf{x}$ , the *net utility* for a

decision  $\mathbf{x}$  is the utility,  $u_k$ , minus the data's listed price:

$$g_{k,N}(\mathbf{x}) := u_k(\mathbf{x}) - \mathbf{x}^T \mathbf{p}, \quad (1)$$

Finally,  $B_k$ 's purchasing problem is formulated as selecting an optimal collection of datasets to maximize its *net utility*:

$$\tilde{\mathbf{x}}^{k,N} := \arg \max_{\mathbf{x} \in \mathcal{X}_{k,N}} g_{k,N}(\mathbf{x}), \quad \text{s.t.} \quad (2)$$

$$\mathcal{X}_{k,N} := \{\mathbf{x} \mid g_{k,N}(\mathbf{x}) \geq 0, \mathbf{x}^T \mathbf{p} \leq b_k\}, \quad (3)$$

where  $\tilde{\mathbf{x}}^{k,N}$  is the optimal solution, and  $\mathcal{X}_{k,N}$  is the set of all feasible solutions. Note that  $\mathbf{x}^T \mathbf{p} \leq b_k$  ensures that the total paid price is under the budget  $b_k$ , and  $g_{k,n}(\mathbf{x}) \geq 0$  ensures a non-negative improvement in *net utility*,  $g_{k,N}$ .

**Seller's Objective.** The objective of seller  $S_j$  is to set a price  $p_j \in \mathbb{R}_+$  for its listed dataset  $D_j$ , maximizing its profit. There are two factors that  $S_j$  must consider for pricing its data: (i) a fixed cost  $c_j$  reflecting the effort that creates dataset  $D_j$ , and (ii) the *anticipated* purchases of buyers for a given price  $p_j \in \mathbb{R}_+$  (i.e., for all  $k \in [M]$ , the solution  $\tilde{\mathbf{x}}^{k,N}$  to  $B_k$ 's purchasing problem). In practice, due to *information transparency*, sellers can use buyers' data valuations to make reasonable estimates, as noted in previous works such as Ghorbani et al. (2020), which characterizes data valuation in the context of underlying data distributions, and Chen et al. (2022) which examines price data via costly signaling.

Next, based on the seller's estimates, we denote  $\mathbb{1}_{\{B_k, D_j, p_j\}}$  as an indicator function which returns 1 if buyer  $B_k$  purchases dataset  $D_j$  at price  $p_j$  (i.e.,  $j$ -th element of  $\tilde{\mathbf{x}}^{k,N}$  is 1) and 0 otherwise (i.e.,  $j$ -th element of  $\tilde{\mathbf{x}}^{k,N}$  is 0). Then, the *net profit* for seller  $S_j$  defined as:

$$r(p_j) := \sum_{k=1}^M \mathbb{1}_{\{B_k, D_j, p_j\}} p_j - c_j. \quad (4)$$

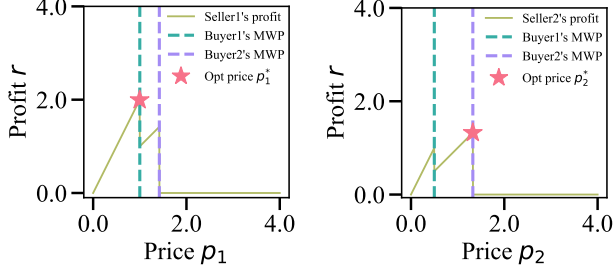
Finally,  $S_j$ 's pricing problem is formulated as choosing the optimal price for their dataset to maximize its net profit:

$$p_j^* := \arg \max_{p_j \in \mathcal{P}_{j,M}} r(p_j), \quad \text{s.t.} \quad (5)$$

$$\mathcal{P}_{j,M} := \{p_j \in \mathbb{R}_+ \mid r(p_j) \geq 0\}, \quad (6)$$

where  $p_j^*$  is the optimal price and  $r(p_j) \geq 0$  ensures that seller  $S_j$ 's net profit must be non-negative.

**Market Dynamics.** We now discuss the market dynamics (i.e., the interactions between buyers and sellers). To mirror a real-life scenario, we assume a fixed number of buyers  $\{B_k\}_{k=1}^M$  in the market while sellers  $\{S_j\}_{j=1}^N$  arrive sequentially, each with a dataset  $D_j$ . Only the number of sellers in the market changes over time. When a new seller  $S_j$  arrives, it prices  $D_j$  by solving for  $p_j^*$  (Equation (5)), based on buyers' updated purchases  $\tilde{\mathbf{x}}^{k,j}$  (Equation (2)) for all  $k \in [M]$ . Algorithm 2 in Appendix F.1 shows a detailed example of our market dynamic.



(a) Seller  $S_1$ ' profit over price. (b) Seller  $S_2$ ' profit over price.

Figure 1. The change in sellers  $S_1$ 's and  $S_2$ 's profit as their datasets' prices increase, alongside buyers  $B_1$ 's and  $B_2$ 's MWP (buyer's *Maximum Willingness to Pay*). Note: the market contains 2 sellers ( $S_1, S_2$ ) and 2 buyers ( $B_1, B_2$ ).

### 3.2. Optimal Pricing for Sellers

Previously, we described our framework's buyer-seller dynamics. A follow-up question is: what factors influence the buyer's decision to purchase newly arrived data, and does it affect the seller's optimal price? In this section, we show these factors that characterize our framework's *fairshare* pricing. Specifically, we show that the optimal price  $p_j^*$  for a dataset rests on each buyer's *Maximum Willingness to Pay* (MWP), which depends on two factors: (1) the utility gain of a dataset for the buyer's LLM and (2) the buyer's budget.

We begin by discussing the first factor: the utility gain of a dataset towards the buyer's LLM. We denote the set of all feasible purchase decisions for a buyer  $B_k$  before the arrival of a dataset  $D_j$  as  $\mathcal{X}_{k,j-1}$ . For each feasible purchase decision (i.e., a collection of datasets), represented by  $\mathbf{x} \in \mathcal{X}_{k,j-1}$ , let  $\mathbf{x}^{\text{new}}$  denote its union with  $D_j$ . Then the utility that  $D_j$  provides to  $B_k$ 's LLM is defined as:

$$\Delta u_k(\mathbf{x}^{\text{new}}) = g_{k,j}(\mathbf{x}^{\text{new}}) - g_{k,j-1}(\tilde{\mathbf{x}}^{k,j-1}). \quad (7)$$

In other words,  $\Delta u_k(\mathbf{x}^{\text{new}})$  represents the additional *utility* that gained from  $\mathbf{x}$  and  $D_j$  together, compared to the *utility* provided by the optimal decision  $\tilde{\mathbf{x}}^{k,j-1}$ .

Second, given buyer  $B_k$ 's purchase decision  $\tilde{\mathbf{x}}^{k,j-1}$  before the arrival of  $D_j$ , we define the *budget surplus* as

$$\Delta b_k(\tilde{\mathbf{x}}^{k,j-1}) = b_k - (\tilde{\mathbf{x}}^{k,j-1})^T \mathbf{p}. \quad (8)$$

That is, the *budget surplus* is the difference between the budget  $b_k$  and the total price of  $\tilde{\mathbf{x}}^{k,j-1}$ . Then we have the formal definition of *buyer's maximum willingness to pay*:

**Definition 3.1** (*Buyer's Maximum Willingness to Pay*). The buyer  $B_k$ 's *buyer's maximum willingness to pay* (MWP) for dataset  $D_j$  is defined as the maximum of the minimum of (1) additional utility  $\Delta u_k(\mathbf{x}^{\text{new}})$  that  $D_j$  provides and (2) the budget surplus  $\Delta b_k(\tilde{\mathbf{x}}^{k,j-1})$  across all feasible collection of datasets  $\mathbf{x} \in \mathcal{X}_{k,j-1}$  (before the arrival of  $D_j$ ):

$$\text{MWP}_k := \max_{\mathbf{x} \in \mathcal{X}_{k,j-1}} \{\min\{\Delta u_k(\mathbf{x}^{\text{new}}), \Delta b_k(\tilde{\mathbf{x}}^{k,j-1})\}\}. \quad (9)$$

With respect to the buyer's MWP, the seller  $S_j$ 's optimal price,  $p_j^*$ , for dataset  $D_j$  can be characterized as follows:

**Lemma 1** (*Characterization of optimal price  $p_j^*$* ). Given Definition 3.1 (MWP), seller  $S_j$ 's optimal price for  $D_j$  is characterized as the MWP of one of the buyers in  $\{B_k\}_{k=1}^M$ :

$$p_j^* \in \cup_{k=1}^M \max_{\mathbf{x}^{\text{new}} \in \mathcal{X}_{k,j-1}} \{\min\{\Delta u_k(\mathbf{x}^{\text{new}}), \Delta b_k(\tilde{\mathbf{x}}^{k,j-1})\}\}. \quad (10)$$

See Appendix D for proof.

Lemma 1 states that among all buyers  $\{B_k\}_{k=1}^M$ 's MWP,  $D_j$ 's optimal price is set at the one that maximizes seller  $S_j$ 's profit. Since the optimal price  $p_j^*$  depends on the *minimum* of the change in utility  $\Delta u_k(\mathbf{x}^{\text{new}})$  and budget surplus  $\Delta b_k(\tilde{\mathbf{x}}^{k,j-1})$ , when *either* buyer's budget or its *utility* of  $D_j$  increases,  $D_j$ 's price increases.

In Figure 1, we simulate a scenario of 2 buyers and 2 sellers. The figure illustrates how buyers' MWP determine the optimal price  $p_1^*, p_2^*$ . Specifically,  $p_1^*$  matches buyer  $B_1$ 's MWP and  $p_2^*$  matches buyer  $B_2$ 's MWP. These values are "break-points" for the profit function  $r(p_j)$ . Each buyer's MWP represents the highest price that seller  $S_j$  could set without reducing the total number of sales (i.e.,  $\sum_{k=1}^M \mathbb{1}_{\{B_k, D_j, p_j\}}$ ). If  $p_j$  exceeds  $B_k$ 's MWP, the number of sales drops by 1, creating a discontinuity in  $r(p_j)$ .

### 3.3. Optimal Pricing for Buyers

In this section, we show that buyers in *fairshare* framework benefit by accepting sellers' optimal prices, ensuring seller participation and long-term utility gains. This alignment of incentives between both parties fosters a sustainable, mutually beneficial market.

To demonstrate this, we consider a single seller  $S$  and a single buyer  $B$  in an infinite time-step setting. At each time step  $t \geq 0$ , seller  $S$  sets a price  $p_t$  for dataset  $D_t$  and buyer  $B$  decides whether to purchase based on utility  $u_t$  and budget  $b_t$ . We assume that  $\mathbb{E}[b_t] < \mathbb{E}[u_t], \forall t$ , where expected utility always exceeds buyer's expected budget.

Next, we make assumptions regarding the buyer's and seller's behaviors: (1) a seller reduces future market participation under unfair compensation, and (2) a buyer values future gains, reflecting a forward-looking decision-making process. These assumptions are formally defined below.

**Assumption 1.1** (*Seller's participation is reduced under unfair compensation*). Suppose at any time period  $t$ , the buyer has bargaining power and can negotiate a price  $p_t \leq p_t^*$ . Then, there exists a strictly increasing function  $\pi: (p_t, p_t^*) \rightarrow [0, 1]$  that models the probability of the seller's participation in all future periods, such that  $\pi(0, p_t^*) = 0$  and  $\pi(p_t^*, p_t^*) = 1$ . In addition, the probability that the seller  $S$  participates in the market at time  $T$  is  $\prod_{t=0}^{T-1} \pi(p_t, p_t^*)$ .



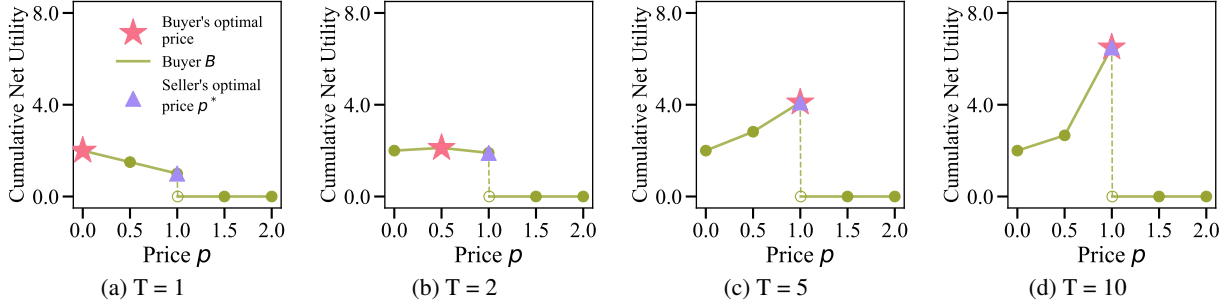


Figure 2. Analysis of the buyer’s cumulative net utility as a function of the transaction prices over different time steps ( $T = 1, 2, 5, 10$ ). Note: we simulate with a single buyer  $B$  and a single seller  $S$  with a single dataset  $D$ . The function of cumulative net utility over price is a discontinuous piecewise linear function which breaks at  $p = 1$ .

It states that if seller  $S$  receives fair compensation at time  $t$ , its probability of continued participation remains one; otherwise, it declines. As shown in (Akerlof, 1978), underpaid sellers are less likely to produce high-quality goods or remain active in the market in the future, inevitably leading to a “Lemon Market” for datasets. The assumption 1.1 is aligned with organizational theory (Folger et al., 1998; Folger & Cropanzano, 2001; Colquitt et al., 2013), which highlights that unfair compensation causes distributive injustice that significantly undermines motivation and discourages participation over time. Moreover, seller’s participation probability is assumed to be:

**Assumption 1.2** (Seller reaction to unfair compensation is sufficiently strong). The Lipschitz continuity of participation probability function  $\pi$  over  $p_t^*$  is lower-bounded:

$$|\pi(p_{t,1}, p_t^*) - \pi(p_{t,2}, p_t^*)| \geq L |p_{t,1} - p_{t,2}|, \quad (11)$$

for some constant  $L > 0$  and all  $p_{t,1}, p_{t,2} \in \mathbb{R}_+$  for all  $t$ .

This assumption ensures that when seller  $S$  receives unfair compensation, its probability of participation decreases by a margin that is lower-bounded, and proportional to the difference between the unfair compensation and the *fairshare* compensation.

Next, we use the participation function to model the value function of  $B$  with the following Bellman equation (Bellman & Kalaba, 1957):

$$G(u_t, b_t) = \max_{p_t \in [0, \infty)} [\mathbb{E}[u_t - p_t] + \delta \mathbb{E}[\pi(p_t, p_t^*) G(u_{t+1}, b_{t+1}) | u_t, b_t]], \quad (12)$$

where  $G$  denotes the buyer  $B$ ’s cumulative *net utility* over infinite horizon, as the sum of the expected *net utility* at current time step and discounted future expected *net utility*. Here,  $\delta$  is the buyer’s discount factor, which has the following assumption:

**Assumption 2** (Discount factor is lower bounded). The discount factor  $\delta$  satisfies the following inequality:

$$\delta \geq \frac{1}{1 + L \min_{t \in [0, \infty)} \mathbb{E}[\max\{u_t - b_t, 0\}]}, \quad \forall t. \quad (13)$$

A higher discount factor indicates a greater emphasis on future gains. This assumption indicates that the buyer assigns weight to its future utility gains (as shown in Equation (12)) to adopt a forward-looking perspective. Finally, combining assumptions 1.1 to 2, note the following result regarding buyer  $B$ ’s cumulative *net utility* under  $p_t^*$ :

**Lemma 2** (The optimal price for the buyer  $B$  is also  $p_t^*$ ). The *fairshare* price for the seller  $S$  under our framework is

$$p_t^* := \min\{u_t, b_t\}, \quad \forall t. \quad (14)$$

With assumptions 1.1 to 2,  $p_t^*$  gives the buyer  $B$  the maximum cumulative *net utility* over infinite horizon. Proof is in Appendix D.

**Fairshare Pricing Ensures Win-win Outcomes:** The above lemma shows that  $p_t^*$  from our *fairshare* pricing framework is optimal for buyer  $B$ . Lower prices may boost short-term utility gains, but  $p_t^*$  ensures maximum long-term gains, making it the optimal choice.

In Figure 2, we demonstrate how the seller’s optimal price  $p_t^*$  also becomes optimal for the buyer  $B$ . We fix utility  $u_t = 2$ , budget  $b_t = 1$ , and the discount factor  $\delta = 0.95, \forall t$ , ensuring that buyer  $B$ ’s optimal price remains constant at  $p_t^* = 1$ . At each time step  $t$ , we identify the price that maximizes the buyer’s cumulative *net utility* up to  $t$ . Notably, if  $p_t > p_t^*$ , the buyer refrains from purchasing, as it would yield negative *net utility*.

Initially, the buyer’s optimal price is zero, benefiting from low prices to maximize *net utility*. Over time, the buyer’s optimal price shifts toward  $p_t^*$  and eventually converges to it. This occurs as sellers receiving unfair compensation reduce their market participation, leading to periods where no datasets are available. As a result, the buyer is unable to make purchases. The minimum time step  $t^*$  at which

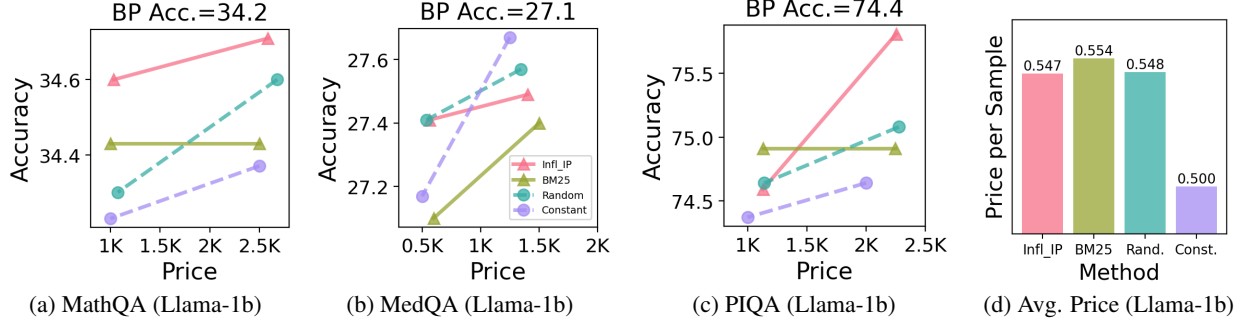


Figure 3. Left, middle-left, middle-right columns: Buyers’ model performance versus cost. Performance of models is shown *before purchasing* (BP) data, and after purchasing 2K and 4K data samples. Right column: Average price-per-sample cost of purchased data across math, medical, and physical reasoning data markets. Purchasing decisions were using the constant, random, BM25, Infl<sub>IP</sub> data valuation methods. Additional analysis on the Pythia-1b and Pythia-410m models are in Appendix F.

buyer’s optimal price becomes  $p_t^*$  is defined as:

$$t^* := \inf \left\{ T \in [0, \infty) : \mathbb{E} \left[ \sum_{t=0}^T \delta^t (u_t - p_t^*) \right] \geq \mathbb{E} \left[ \sum_{t=0}^T \delta^t \prod_{t=0}^{T-1} \pi(p_t, p_t^*) (u_t - p_t) \right], \forall p_t \in \mathbb{R}_+ \right\}, \quad (15)$$

which represents the equilibrium point where the buyer’s cumulative *net utility* under *fairshare* pricing exceeds that of other pricing methods.

## 4. General Experimental Setup

To ground the theoretical analysis of our fairshare framework to real-life scenarios, we *empirically* analyze our framework by running simulations on NLP tasks and datasets for training current LLMs. In this section, we discuss the general experimental setup for our analyzing our market framework, which is used for all subsequent experiments in Sections 5 and 6. In our simulations, each buyer has a single LLM, and each seller owns a single data sample. Buyers seek to buy training data to improve their model performance towards a specific tasks (e.g., math problem solving), which are described below.

**Buyers and Models:** We simulate three buyers, each using a different model: Llama-3.2-Instruct-1b (Grattafiori et al., 2024), Pythia-1b, and Pythia-410m (Biderman et al., 2023). Notably, these models are all common open-sourced LLMs. Since the Llama and Pythia models were pre-trained on different corpora, their preference for downstream post-training data can vary (Mai et al., 2024), reflecting buyers’ different data preferences in real life.

**Sellers and Datasets:** We focus on tasks that challenge current LLMs – math problems, medical diagnosis, and physical reasoning (Lu et al., 2023; Liu et al., 2023; Ahn et al., 2024) – which were curated with human annotations, adding to their complexity. For math problems, we use the MathQA (Amini et al., 2019) and GSM8K (Cobbe et al.,

2021) datasets. We use the MedQA (Jin et al., 2021) dataset for medical diagnosis, and PIQA (Bisk et al., 2020) for physical reasoning. Table 1 in Appendix F shows dataset splits and examples. We use the training splits of these datasets as the sellers’ data, and simulate the market dynamics in Section 3. Specifically, we simulate separate *math*, *medical*, and *physical reasoning* data markets (buyers and sellers do not purchase and price data across different markets).

Following this general setup, we examine the effects of incorporating data valuation into our framework in Section 5, and compare the optimal pricing obtained through our framework against other pricing strategies in Section 6.

## 5. Data Valuation Experiments

Since data valuation is a key component of our market framework — affecting buyers’ purchasing decisions sellers’ pricing — we begin by examining how different data valuation methods affect buyers and sellers. In particular, we use different data valuation methods to aid the buyers’ purchases, conduct supervised-finetuning on their models using their purchased data, and evaluate their model performance. We describe our data valuation methods in Section 5.1, and then discuss results in Section 5.2.

### 5.1. Data Valuation Experiment Setup

Using the previously mentioned models and dataset, we run separate simulations for each data market (e.g., math, medical, physical reasoning), each testing different data valuation methods for the buyer. Below, we describe our data valuation methods and our pricing setup.

**Data Valuation Methods:** In order to conduct data valuation, for each previously mentioned dataset (e.g. math, medical, physical reasoning), we randomly sample 200 demonstrations from their validation sets to form representative datasets<sup>3</sup>. We use data valuation to score each data sample

<sup>3</sup>Note: For PIQA we take 200 samples from the training set

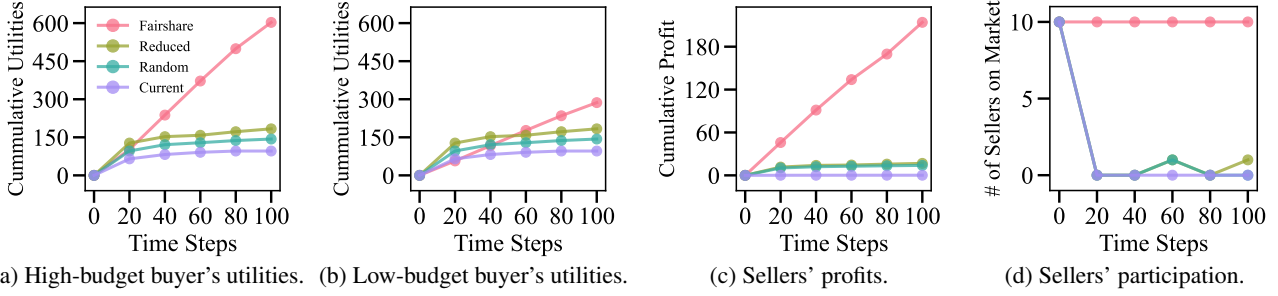


Figure 4. Analysis of (1) buyer’s cumulative utilities with high-budget buyer (Figure 4a) and low-budget buyer (Figure 4b), and (2) sellers’ average cumulative profits (Figure 4c) and number of sellers in the market (Figure 4d) over time ( $T = 100$ ). Model: Pythia-1b; Task: MedQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.

in the market according to its similarity with a representative dataset. We use the following methods for data valuation:

1. **Constant:** returns the same value for every data sample in the market. This is common in annotation platforms (e.g., MTurk, Remotasks<sup>4 5</sup>), where there is a flat-price-per-annotation.
2. **Random:** randomly assigns a number between  $[0, 1]$  for every data sample in the market.
3. **Semantic:** returns a score based on the avg. semantic similarity between each data sample and each sample in a representative dataset. Specifically, we use BM25 (Trotman et al., 2014), see Section 2.
4. **Influence-based:** returns a score which leverages learning gradients to estimate a data sample’s avg. contribution to model learning of a representative dataset. Specifically, we use  $\text{Infl}_{\text{IP}}$  (see Appendix A) (Pruthi et al., 2020; Xia et al., 2024).

We apply normalization so that each method returns a score between  $[0, 1]$  to better align it with monetary value (as discussed in Section 3). We note that both the semantic and influence-based methods fall under the category of *data valuation* given that it measures the potential impact of training their LLM on a data sample, which is not the case for the constant and random methods.

**Market/Pricing Setup:** We provide details following our market dynamics and sellers’ pricing values for our data valuation experiments in Appendix E.1.

**Model Training and Evaluation:** Fine-tuning and evaluation of the buyers’ models are described in Appendix E.1

## 5.2. Data Valuation Experiment Results

Figure 3 shows the results from using different data valuation methods for buyers, following the experiment setup previously described. We note that training on data purchased using BM25 and  $\text{Infl}_{\text{IP}}$  resulted in higher overall

since the validation set is commonly reserved for testing.

<sup>4</sup><https://www.mturk.com/>

<sup>5</sup><https://www.remotasks.com>

model performance across our models and tasks. When considering trade-offs between cost and performance, using  $\text{Infl}_{\text{IP}}$  provided reduced cost (lower than random and BM25, as shown in Figures 3d and 6d), while having high contribution towards model performance.

On the other hand, while constant valuation was the most cost-effective, its effect on model performance was lower than BM25 or  $\text{Infl}_{\text{IP}}$ . Meanwhile, using random valuation neither yielded the best model performance, nor was it the most cost effective. Given that constant valuation is a popular choice in data annotation platforms, our findings show that using data valuation in market frameworks provides a better alternative for buyers. In real-world scenarios, this is especially beneficial for financially constrained buyers seeking to improve their LLMs. Thus, our fairshare framework ensures the price of data is reflective of its value and provides a win-win situation: buyers receive greater value-per-price for data, and sellers get higher pay for their data.

## 6. Data Pricing Experiments

In this section, we evaluate our pricing framework in terms of buyer and seller welfare. Building on the theoretical analysis in Section 3.3, we empirically demonstrate that, compared to other pricing methods, our framework yields the highest utilities for buyers and the highest profits for sellers, ensuring mutually beneficial outcomes. Below, we outline our experimental setup Section 6.1, followed by findings in Section 6.2.

### 6.1. Data Pricing Experiment Setup

**Market Setup:** Following similar setups in Section 3.3, we simulate a market with multiple buyers ( $M = 2$ ) and sellers ( $N = 10$ ) across multiple time steps. To examine the impact of *fairshare pricing* on buyers with varying resources, we include a high-budget buyer (well-funded LLM builder) and a low-budget buyer (under-resourced one). Each buyer’s budget is generated randomly at each time step, with different mean values reflecting their resource disparity. Within each time step, (1) sellers arrive sequentially with a new dataset

(containing 300+ data samples) at fixed prices, and then (2) after all sellers arrive, buyers make purchase decisions based on Equation (2). Details of the experimental setup are provided in Appendix E.2.

**Participation Function:** Following assumptions 1.1 to 2, we simulate 100 time steps with a discount factor  $\delta = 0.999$ , reflecting buyers’ forward-looking behavior and prioritizing long-term value in the rapidly growing LLM industry (IBM, 2024). The participation function is  $\pi(p_{j,t}, p_{j,t}^*) = p_{j,t}/p_{j,t}^*$  for its simplicity and compliance with assumptions 1.1 and 1.2. When sellers receive unfair compensation (i.e.,  $p_{j,t} < p_{j,t}^*$ ), their probability of future participation declines.

**Pricing Methods:** We consider four methods to price  $p_{j,t}$ :

1. **Fairshare:** Prices are set by the *fairshare* pricing framework from Section 3, i.e.,  $p_{j,t} = p_{j,t}^*$ .
2. **Reduced:** Buyers and sellers negotiate reduced prices as a fixed fraction of the optimal price,  $p_{j,t} = c * p_{j,t}^*$  with  $c = 0.5$ .
3. **Random:** Prices are randomly determined within the range  $(0, p_{j,t}^*)$ .
4. **Current:** Buyers and sellers negotiate a low fixed price, reflecting current data market norms, (e.g., MTurk). We set  $p_{j,t}$  as 10% of the avg. utility contribution of each dataset to a LLM.

## 6.2. Pricing Experiment Results

Figure 4 compares the long-term welfare outcomes of different pricing methods for buyers and sellers using Pythia-1b on the MedQA task. For models’ performances on MathQA and PiQA (including Pythia-410m and Llama-3.2-Instruct-1b), see Appendix F, which shows similar trends.

**Current Pricing Norm Leads to Lose-loss Results:** The *current* pricing method, which sets uniformly low, fixed prices to reflect real-world practices ((CBS News, 2024)), offers LLM developers short-term utility gains (see Figures 4a and 4b). However, this approach systematically undervalues datasets and unfairly compensates annotators. As a result, sellers gradually exit the market due to unsustainable returns, leading to a collapse in data supply. Over time, even well-funded LLM developers will struggle to acquire sufficient datasets, regardless of their budgets. If this pricing model persists, it will not only hurt the market but also hinder the long-term advancement of LLMs.

**Fairshare Pricing Creates Win-win Outcomes:** For sellers, the results (Figure 4c) demonstrate that our *fairshare* pricing framework consistently maximizes seller profits at all time steps. This aligns with the market setup described in Section 3.1, which aims to maximize sellers’ profits.

For buyers, the *fairshare* pricing framework is particularly

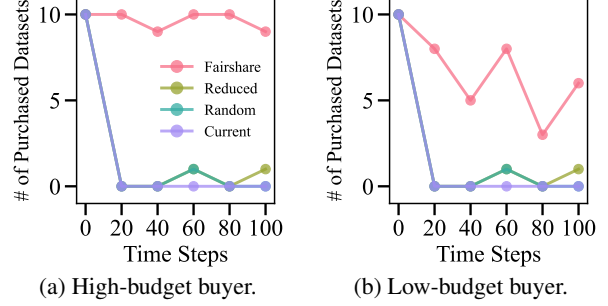


Figure 5. Number of purchased datasets for the buyer with high budget (Figure 5a) and low budget (Figure 5b) over time periods ( $T = 100$ ). Model: Pythia-1b; Task: MedQA.

effective for the high-budget buyer (Figure 4a), delivering the highest cumulative utility over time. Low-budget buyer (Figure 4b), however, experiences reduced short-term utility in exchange for long-term gains. Its limited budget prevents them from fully leveraging the increased dataset supply ensured by *fairshare* pricing, making other low-pricing methods initially more appealing. Yet, in the long run, *fairshare* pricing sustains seller participation, ensuring data supply. In contrast, low-pricing methods drastically reduce seller participation, leaving fewer datasets available.

**Analysis of the mechanism:** Unfair compensation drives sellers out of the market (see Figure 4d), ultimately depleting data availability. Except for *fairshare* pricing, all other methods lead to a rapid collapse in data transactions (see Figures 5a and 5b). This exposes a critical trade-off: buyers may initially benefit from cheaper prices, but unsustainable pricing methods erode market supply, leaving them with no datasets to purchase. *Fairshare* pricing, in contrast, sustains a healthy data market ecosystem, ensures long-term market viability, and is essential for the continued advancement of the LLM industry.

## 7. Conclusion

In this paper, we proposed a *fairshare* pricing framework using data valuation methods for transparent training data pricing for LLMs. Our results showed that buyers achieved higher gains for their models at reduced costs by leveraging data valuation methods, which promote buyer participation in the market, particularly for those with financial constraints. Simultaneously, sellers were able to sell their data at optimal prices, ensuring a win-win situation leading to long-term social welfare gain.

To the best of our knowledge, we are the first to combine the economics of optimal pricing and game theory, with a deep understanding of the LLM data valuation methods, to develop solutions that reflect real-world dynamics in the emerging LLM data market. Our approach provides policy makers and regulatory bodies potential guidelines for pricing training data in LLM markets to ensure fairness and



transparency. By fostering fair market access, our framework also empowers small businesses and startups, leading to more equitable technological advancements.

Future research can explore our framework from different angles, including additional data valuation methods, incomplete information game theoretical framework (e.g., Bayesian game), and applications across diverse data domains (e.g., pre-training data vs fine-tuning). We hope this work paves way for future research in equitable markets for AI and emerging technologies.

## Impact Statement

This paper addresses the critical issue of fairshare pricing in the data market for large language models (LLMs) by proposing a framework and methodologies for fair compensation of datasets from LLM developers to data annotators. Our work directly tackles the ethical and societal challenges in the current data market, where many data annotators are underpaid and receive compensation significantly disconnected from the true economic value their contributions bring to LLMs.

From ethical and societal perspectives, our framework prioritizes the welfare of both data annotators and LLM developers. Our methodology ensures that data annotators are fairly compensated for their labor, promoting equity and fairness in the data ecosystem. This contributes to mitigating the exploitation of vulnerable annotators in the data market and aligns the incentives of stakeholders toward a more ethical and sustainable practice. In addition, our framework also benefits LLM developers, by demonstrating that our framework maximizes their utilities and welfare in the long term. Fair compensation encourages ongoing participation of data annotators in the market, ensuring a steady supply of diverse, high-quality datasets essential for LLM development. By addressing existing inequities, our work lays the foundation for a more sustainable, equitable, and mutually beneficial ecosystem for all stakeholders in the LLM data market.

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## A. Influence-based Data Valuation

In Section 4, we introduced a gradient-based data attribution method, denoted as  $\text{Infl}_{\text{IP}}$ . In this section, we provide additional information on  $\text{Infl}_{\text{IP}}$ , which has been shown to be effective in training data selection in previous works (Pruthi et al., 2020; Xia et al., 2024). Suppose we have a LLM parameterized by  $\theta$ , and a train set  $D$  and a test set  $\mathcal{D}'$ . For a training sample  $d \in D$ , we wish to estimate its training impact on a test sample  $d' \in \mathcal{D}'$ . That is, we want to measure the impact of  $d$  on the model’s loss on  $d'$  (i.e.,  $\mathcal{L}(d'; \theta)$ ). A simple method of achieving this is to take training step – that is, a gradient descent step – on  $d$  and obtain:

$$\hat{\theta} = \theta - \eta \nabla \mathcal{L}(d; \theta) \quad (16)$$

where  $\eta$  is the learning rate. Then, in order to measure the influence of  $d$  towards  $d'$ , we wish to find the change in loss on  $d'$ :

$$\mathcal{L}(d'; \theta) - \mathcal{L}(d'; \hat{\theta}) \quad (17)$$

Instead of taking a single training step to measure the influence of  $d \in D$  on  $d'$ , we can instead approximate Equation (17) with using the following:

**Lemma 3.** Suppose we have a LLM with parameters  $\theta$ . We perform a gradient descent step with training sample  $d$  with learning rate  $\eta$  such that  $\hat{\theta} = \theta - \eta \nabla \mathcal{L}(d; \theta)$ . Then,

$$\mathcal{L}(d'; \theta) - \mathcal{L}(d'; \hat{\theta}) \approx \nabla \mathcal{L}(d'; \theta) \cdot \nabla \mathcal{L}(d; \theta)$$

See Appendix D for the proof.

Then, we set  $\text{Infl}_{\text{IP}}$  to be:

$$\text{Infl}_{\text{IP}} = \nabla \mathcal{L}(d'; \theta) \cdot \nabla \mathcal{L}(d; \theta) \quad (18)$$

which is the dot-product between the learning gradients of  $d'$  and  $d$ .

## B. Royalty model

So far, we have shown the case of *flat rate* (see Section 3.1), which is well-suited resource-rich buyers, such as leading tech companies whose LLMs generate significant economic value due to their wide-ranging impact and scalability. In this section, we introduce the *royalty model*, a contract framework that differs from the flat rate by offering a subscription-like structure. Under the royalty model, the price paid for training data is proportional to the future economic value generated by the LLM, providing a flexible and performance-based approach to data valuation. This scenario incorporates buyers in a less dominant position – those who are (1) uncertain about the prospective model outcome or (2) do not own a sufficient cash flow for purchasing data with full prices. We present updated decision-making models for buyers and sellers as follows.

**Buyers.** Unlike the flat pricing setting, the buyer  $B_k$  would alternatively pay with a *fractional price*. Suppose each dataset  $D_j$  is priced with an individual rate  $\alpha_j \in [0, 1]$  (as we denote  $\alpha = (\alpha_1, \dots, \alpha_N)$ ), then the price of an arbitrary data collection  $u_k(\mathbf{x})$  is a fraction of its future marginal gain, i.e.,  $\mathbf{x}^T \mathbf{p} = f(\alpha, \mathbf{x}) u_k(\mathbf{x})$ , where the overall rate function  $f : [0, 1]^{|\alpha|} \times \{0, 1\}^{|\mathbf{x}|} \rightarrow [0, 1]$  depends on specific contexts. We assume that  $f$  is a monotonically non-decreasing function of  $\alpha$ . In this sense, the buyer  $B_k$  reduces the risk of losing  $b_k$  from its cash flow while the seller is betting on the potential value of the LLM  $M_k$ . Then we obtain an updated objective function for  $B_k$ :

$$g_{k,N,\text{frac}}(\mathbf{x}) := (1 - f(\alpha, \mathbf{x})) u_k(\mathbf{x}), \quad (19)$$

On the other hand, similar to the budget constraint 3, here each buyer  $B_k$  has a maximum rate  $\bar{\alpha}_k$  that it is willing to pay. Then the buyer’s purchasing problem is given as

$$\tilde{\mathbf{x}}^{k,N,\text{frac}} := \arg \max_{\mathbf{x} \in \mathcal{X}_{k,N,\text{frac}}} g_{k,N,\text{frac}}(\mathbf{x}), \quad \text{s.t.} \quad (20)$$

$$\mathcal{X}_{k,N,\text{frac}} := \{\mathbf{x} \mid g_{k,N,\text{frac}}(\mathbf{x}) \geq 0, f(\alpha, \mathbf{x}) \leq \bar{\alpha}_k\}, \quad (21)$$

And  $\tilde{\mathbf{x}}^{k,N,\text{frac}}$  is the optimal solution to  $\max_{\mathbf{x} \in \mathcal{X}_{k,N,\text{frac}}} g_{k,N,\text{frac}}(\mathbf{x})$  with a given rate vector  $\alpha$ .

**Sellers.** In the fractional pricing setting, since the buyer  $B_k$  pays for the entire data collection, there should exist a fair and transparent allocation mechanism that distributes a portion of the total price charged to each individual dataset  $D_j$ . That is,

$\mathbf{x}_j p_j = \sum_{k=1}^M f_j(\boldsymbol{\alpha}, \tilde{\mathbf{x}}^{k,j-1,\text{frac}}) u_k(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})$ , where  $f(\cdot) = \sum_{j=1}^N f_j(\cdot)$ . And we assume that for all  $j \in [N]$ ,  $f_j(\cdot)$  is monotonically non-decreasing over  $\boldsymbol{\alpha}$ . Therefore, we have an updated profit function for  $Se_j$ :

$$r_{\text{frac}}(\alpha_j) := \sum_{k=1}^M f_j(\boldsymbol{\alpha}, \tilde{\mathbf{x}}^{k,N,\text{frac}}) u_k(\tilde{\mathbf{x}}^{k,N,\text{frac}}) - c_j. \quad (22)$$

which gives the following problem:

$$\alpha_j^* := \arg \max_{\alpha_j \in \mathcal{A}_{j,M,\text{frac}}} r_{\text{frac}}(\alpha_j), \quad \text{s.t.} \quad (23)$$

$$\mathcal{A}_{j,M} := \{\alpha_j \in [0, 1) \mid r_{\text{frac}}(\alpha_j) \geq 0\}, \quad (24)$$

From this point onward, the market dynamics stays the same as in the previous section. It is noted that, compared to  $\max_{p_j \in \mathcal{P}_{j,M}} r(p_j)$  where optimal flat rate is indirectly connected to the *utility*, the optimal rate of  $\max_{\alpha_j \in \mathcal{A}_{j,M,\text{frac}}} r_{\text{frac}}(\alpha_j)$  offers a more direct representation of the *utility*.

### B.1. Solving for optimal price of the royalty model

Similar to *flat rate*, in the case of *royalty model*, we need to solve buyer's problem  $\max_{\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}} g_{k,j-1,\text{frac}}(\mathbf{x})$  (before the arrival of  $S_j$ ) and  $\max_{\mathbf{x} \in \mathcal{X}_{k,j,\text{frac}}} g_{k,j,\text{frac}}(\mathbf{x})$  (after the arrival of  $S_j$ ) for all  $k \in [M]$  and seller's problem  $\max_{\alpha_j \in \mathcal{A}_{j,M,\text{frac}}} r_{\text{frac}}(\alpha_j)$ .

**Solve buyer's problems.** For each feasible collection of datasets  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}$  (before the arrival of  $S_j$ ), denotes it union with dataset  $D_j$  as  $\mathbf{x}^{\text{new}}$ . Then we run the check: (1) if the *net utility* of  $\mathbf{x}^{\text{new}}$  is larger than the one of  $\tilde{\mathbf{x}}^{k,j-1,\text{frac}}$ , i.e.,  $g_{k,j,\text{frac}}(\mathbf{x}^{\text{new}}) > g_{k,j,\text{frac}}(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})$ , and (2) if the rate for purchasing  $\mathbf{x}^{\text{new}}$  is still under the budget  $\bar{\alpha}_k$ , i.e.,  $f([\boldsymbol{\alpha}^T \alpha_j], \mathbf{x}^{\text{new}}) \leq \bar{\alpha}_k$ , where  $[\boldsymbol{\alpha}^T \alpha_j]$  denotes concatenating  $\alpha_j$  to  $\boldsymbol{\alpha}$ . If the answer is positive to both tests, then we can determine that the buyer  $B_k$  will change its decision and purchase  $S_j$  under the rate  $\alpha_j$ .

**Solve seller's problem.** First, we consider when  $\alpha_j = 0$ . We could first find all  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}$  such that  $g_{k,j,\text{frac}}(\mathbf{x}^{\text{new}}) > g_{k,j,\text{frac}}(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})$ . And we denote the set that contains such  $\mathbf{x}$  as  $\mathcal{X}_{k,j-1,\text{frac}}^1$ . If  $\mathcal{X}_{k,j-1,\text{frac}}^1$  is empty, then  $\mathbb{1}_{\{B_k, D_j, p_j\}} = 0$ , as  $S_j$  cannot bring positive value to  $B_k$ ; else, then for all  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1$ , thanks to the monotonicity of  $f_j$  over  $\alpha_j$ , we could gradually increase  $\alpha_j$  until the either of the two criterion are met first: (1) we find the largest  $\alpha_j$  such that  $g_{k,j,\text{frac}}(\mathbf{x}^{\text{new}}) > g_{k,j,\text{frac}}(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})$ , and (2)  $f_j([\boldsymbol{\alpha}^T \alpha_j], \mathbf{x}^{\text{new}}) \leq \bar{\alpha}_k$ . Then we have the following property about the optimal rate  $\alpha_j^*$  for Equation (23):

**Lemma 4** (Characterization of  $\alpha_j^*$  under *royalty model*). Define  $\alpha_j^{\mathbf{x}}$  as

$$\min \left\{ \sup_{\alpha_j \in [0,1)} \left\{ \alpha_j : f_j([\boldsymbol{\alpha}^T \alpha_j], \mathbf{x}^{\text{new}}) < 1 - (1 - f_j(\boldsymbol{\alpha}, \tilde{\mathbf{x}}^{k,j-1,\text{frac}})) \frac{u_k(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})}{u_k(\mathbf{x}^{\text{new}})} \right\}, \right. \\ \left. \sup_{\alpha_j \in [0,1)} \left\{ \alpha_j : f_j([\boldsymbol{\alpha}^T \alpha_j], \mathbf{x}^{\text{new}}) \leq \bar{\alpha}_k \right\} \right\}. \quad (25)$$

For every  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1$  and all  $k \in [M]$ , we obtain  $\alpha_j^{\mathbf{x}}$  and their union  $\cup_{k=1}^M \cup_{\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1} \{\alpha_j^{\mathbf{x}}\}$ . Then we have  $\alpha_j^* \in \cup_{k=1}^M \cup_{\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1} \{\alpha_j^{\mathbf{x}}\}$ .

**Remark B.1** (Similarities between *flat rate* and *royalty model*). Observing from 4 and 1, we see that the both the optimal price  $p_j^*$  and the optimal rate  $\alpha_j^*$  are closely tied to  $B_k$ 's *maximum willingness to pay*. That is, compared to the market prior to the arrival of  $S_j$ , the optimal values are characterized by the minimum of two factors: (1) *marginal utility* that  $S_j$  provides to  $B_k$  and (2)  $B_k$ 's *budget surplus*. It is also noted that, under *royalty model*, the rate function  $f$  also plays an important role as it determines the how the single rate  $\alpha_j$  affects the total rate that  $B_k$  pays.

## C. Applications for Real-Life Scenarios

In real-life settings, the relationship between the data valuation of a training sample and the buyer's utility  $u_k$  (i.e., the economical value, which may be expressed in dollar amounts) can have different mappings, as mentioned in Section 3.1. Suppose the data valuation function is denoted as  $v_k : D \rightarrow \mathbb{R}$  for a dataset  $D$ . Then, a buyer may expect a linear relationship between  $v_k$  and  $u_k$ , where the utility increases as the data valuation score increases. Alternatively, a buyer may prefer to

only purchase data beyond a certain threshold for  $v_k$ . In this section, we present three types mappings between  $v_k$  and  $u_k$  to reflect these scenarios: *linear*, *discrete*, and *zero-one* mappings. We show that these mappings can be easily adapted to our proposed framework in Section 3. We only present the updated buyer’s purchasing problem (Equation (2)) since the seller’s pricing problem (Equation (5)) stays the same.

### C.1. Linear Outcome

In practice, there are many applications where  $u_k$  is an affine function of  $v_k$ . As previously mentioned, training LLMs on data with higher valuation scores  $v_k$  can result in better economic value towards downstream model performance, as shown in previous works (Xia et al., 2024; Yu et al., 2024). In this outcome setting, in addition to considering  $u_k$  to be an affine function of  $v_k$ , we also include a bias variable  $\beta$  to account for other potential other factors that are independent of  $v_k$ . Therefore, we can set  $u_k = \gamma v_k(\mathbf{x}) + \beta$  into Equation (1), where  $\gamma \in \mathbb{R}_+$  is a known coefficient, and obtain buyer  $B_k$ ’s net utility function for the linear outcome:

$$g_{k,N}(\mathbf{x}) = \gamma v_k(\mathbf{x}) + \beta - \mathbf{x}^T \mathbf{p}, \quad (26)$$

To obtain optimal price  $p^*$ , we can directly refer to same procedure described in Section 3 using set values for  $\gamma$  and  $\beta$ .

### C.2. Discrete Outcome

There are also many applications where  $u_k$  is discrete. For instance, if the data buyers are participating in an LLM benchmark challenge, such as MMLU (Hendrycks et al., 2021), then training on data that falls within various ranges  $v_k$  may lead to drastically different model performance, and hence leaderboard rankings.

To mirror this, consider  $u_k$  to be a category variable. We denote  $\{c_h\}_{h=1}^H$  as a strictly increasing set of numbers such that when  $v_k \in [c_h, c_{h+1})$ , the buyer will receive reward  $u_{k,h}$ . We also assume that  $u_{k,h+1} > u_{k,h}$  since higher data valuation scores may lead to a larger reward. Therefore, we could set  $u_k = \sum_{h=1}^H \mathbb{1}_{\{v_k(\mathbf{x}) \in [c_h, c_{h+1})\}} u_{k,h}(\mathbf{x})$  and rewrite buyer  $B_k$ ’s net utility function as

$$g_{k,N}(\mathbf{x}) = \sum_{h=1}^H \mathbb{1}_{\{v_k(\mathbf{x}) \in [c_h, c_{h+1})\}} u_{k,h}(\mathbf{x}) - \mathbf{x}^T \mathbf{p}. \quad (27)$$

We again apply the same procedure in Section 3 to solve for the optimal pricing.

### C.3. Zero-One Outcome

There are scenarios where the data buyers are risk-averse and focus on the effects of rare events. In these cases, suppose that  $v_k$  is normalized between  $[0, 1]$ . Then buyers may wish to purchase training data with higher values of  $v_k$ , assuming that purchasing data with lower  $v_k$  may result in severe adverse effects. For instance, data buyers who are building AI for healthcare should not purchase data with incorrect medical information, and even a small amount of contaminated data can result in severe real-life consequences such as mis-diagnosis (Jin et al., 2021; Zhou et al., 2023) or unsuitable medical protocols in emergency situations (Sun et al., 2024). Therefore, in this context, we consider  $u_k$  as a generalized Bernoulli distribution. The downstream outcome has a small positive reward  $\underline{u}$  with probability  $v_k$  (normal events) and a massive negative reward  $\bar{u}$  with probability  $1 - v_k$  (undesirable rare events). And we assume that  $\mathbb{E}(u_k) > 0$ . Therefore, we can plug in and obtain buyer  $B_k$ ’s net utility function:

$$g_{k,N}(\mathbf{x}) = \mathbb{E}[u_k(\mathbf{x})] - \mathbf{x}^T \mathbf{p} \quad (28)$$

$$= v_k(\mathbf{x})(\underline{u} - \bar{u}) + \bar{u} - \mathbf{x}^T \mathbf{p}, \quad (29)$$

which is an affine function of  $v_k$ . Therefore, we again apply same procedure in Section 3 to solve for the optimal pricing.

### C.4. Multiple tasks

In practice, many LLMs are evaluated over multiple tasks (Hendrycks et al., 2021). To this end, we consider the context where buyer  $B_k$  wishes their model  $\mathcal{M}_k$  to perform well across multiple tasks, denoted as  $Q$ . Each data valuation score for a task is denoted by  $v_1^k, \dots, v_Q^k$  and the vector of all task valuations is denoted as  $\mathbf{v}_k = (v_{k,1} \dots v_{k,Q})$ . Then we consider that the utility  $u_k$  is an affine function of the utility in each task, denoted by  $\mathbf{u}_k = (u_{k,1} \dots u_{k,Q})$  that is,  $u_k = \boldsymbol{\theta}^T \mathbf{u}_k + \epsilon$ ,

where  $\theta \in \mathbb{R}^Q$  is a coefficient vector and  $\epsilon \in \mathbb{R}$  denotes other factors independent from  $\mathbf{u}_k$ . We also assume that the each task is one of three categories mentioned in the last section. Therefore, we can rewrite  $\mathbf{u}_k$  as a function of  $\mathbf{v}_k$ , which gives  $u_k = \theta^T \mathbf{u}_k(\mathbf{v}_k) + \epsilon$ . Therefore, the buyer's net utility function becomes

$$g_{k,N}(\mathbf{x}) = \theta^T \mathbf{u}_k(\mathbf{v}_k(\mathbf{x})) + \epsilon - \mathbf{x}^T \mathbf{p}. \quad (30)$$

whose solution could adopt the same procedure as described in Section 3 to solve for the optimal pricing.

## D. Proofs

**Lemma 1** Seller  $S_j$ 's optimal price for  $D_j$  is characterized as the MWP of one of the buyers in  $\{B_k\}_{k=1}^M$ :

$$p_j^* \in \bigcup_{k=1}^M \max_{\mathbf{x}^{\text{new}} \in \mathcal{X}_{k,j-1}} \{\min\{\Delta u_k(\mathbf{x}^{\text{new}}), \Delta b_k(\tilde{\mathbf{x}}^{k,j-1})\}\}.$$

*Proof.* Recall that before the arrival of dataset  $D_j$ , each buyer  $B_k$  has already solved  $\max_{\mathbf{x} \in \mathcal{X}_{k,j-1}} g_{k,j-1}(\mathbf{x})$  according to our market dynamics in Section 3, where  $\mathcal{X}_{k,j-1}$  is the set of all feasible purchase decisions. Next, after seller  $S_j$  (with  $D_j$ ) has arrived on the market, we analyze the conditions in which  $B_k$  will purchase  $D_j$  at a potential price  $p_j$ . For each feasible purchase decision (i.e., a collection of datasets), represented by  $\mathbf{x} \in \mathcal{X}_{k,j-1}$ , let  $\mathbf{x}^{\text{new}}$  denote its union with  $D_j$ . For buyer  $B_k$  to change their previous decision to purchase  $D_j$ , there are two requirements that need to be satisfied. First, we must have:

$$g_{k,j}(\mathbf{x}^{\text{new}}) > g_{k,j-1}(\tilde{\mathbf{x}}^{k,j-1}). \quad (31)$$

That is, the *net utility*  $g_{k,j}(\mathbf{x}^{\text{new}})$  of purchasing decisions  $\mathbf{x}^{\text{new}}$ , must be larger than the *net utility*  $g_{k,j-1}(\tilde{\mathbf{x}}^{k,j-1})$  of a previous optimal purchasing decision  $\tilde{\mathbf{x}}^{k,j-1}$ . It is also noted that  $g_{k,j}(\tilde{\mathbf{x}}^{k,j-1}) = g_{k,j-1}(\tilde{\mathbf{x}}^{k,j-1})$ . Second, for buyer  $B_k$  to purchase  $\mathbf{x}^{\text{new}}$  at price  $p_j$ , we must fulfill the budget constraint:

$$p_j \leq b_k - \left(\tilde{\mathbf{x}}^{k,j-1}\right)^T \mathbf{p} = \Delta b_k(\tilde{\mathbf{x}}^{k,j-1}). \quad (32)$$

Which ensures that purchasing  $D_j$  does not exceed the buyer's budget  $b_k$ . If both requirements are satisfied, then the buyer  $B_k$  will change their previous purchasing decision in order to purchase  $D_j$  under the price  $p_j$ . This procedure is presented in detail in Algorithm 1 in Appendix F.1.

Next, given the conditions for the buyer  $B_k$  to purchase  $D_j$ , the seller must solve  $\max_{p_j \in \mathcal{P}_{j,M}} r(p_j)$  to find the optimal price  $p_j^*$ . First, we consider an edge case where the price of dataset  $D_j$  is set as  $p_j = 0$ . For a buyer  $B_k$ , we denote  $\mathcal{X}_{k,j}^1$  as the set of all purchasing decisions where including  $D_j$  in the purchase improves the buyer's previous net utility  $g_{k,j}(\tilde{\mathbf{x}}^{k,j-1})$ . That is, for every  $\mathbf{x}^{\text{new}} \in \mathcal{X}_{k,j}^1$ , we have  $g_{k,j}(\mathbf{x}^{\text{new}}) > g_{k,j}(\tilde{\mathbf{x}}^{k,j-1})$ . If  $\mathcal{X}_{k,j}^1$  is empty, then  $B_k$  will not purchase  $D_j$  at any price, since  $D_j$  cannot bring positive improved *net utility* to  $B_k$ . Then, when  $p_j$  gradually increases and exceeds  $\max_{\mathbf{x}^{\text{new}} \in \mathcal{X}_{k,j-1}} \{\min\{\Delta u_k(\mathbf{x}^{\text{new}}), \Delta b_k(\tilde{\mathbf{x}}^{k,j-1})\}\}$ , then  $B_k$  will decide not to purchase  $D_j$ , causing the value of  $\sum_{k=1}^M \mathbb{1}_{\{B_k, D_j, p_j\}}$  to drop by one. Since the profit function  $r(p_j)$  is a piecewise linear function, the its optimal point must be one of its breakpoints.  $\square$

**Lemma 2** The optimal price for the seller  $S$  under our framework is

$$p_t^* := \min\{u_t, b_t\}, \forall t. \quad (33)$$

With assumptions 1.1 to 2,  $p_t^*$  gives the buyer  $B$  the maximum cumulative *net utility* over infinite horizon.

*Proof.* We show that when the buyer pays the optimal price  $p_t^*$ , its total value is the largest, i.e.,

$$\mathbb{E}[u_t - p_t^* + \delta \mathbb{E}[r(p_t^*, p_t^*)G \mid u_t, b_t]] \geq \mathbb{E}[u_t - p_t + \delta \mathbb{E}[\pi(p_t, p_t^*)G \mid u_t, b_t]] \quad (34)$$

for all  $p_t \in [0, \infty)$ . Through some linear transformation, this is equivalent to show that

$$\mathbb{E}[p_t - p_t^* + \delta \mathbb{E}[G(\pi(p_t^*, p_t^*) - \pi(p_t, p_t^*)) \mid u_t, b_t]] \geq 0. \quad (35)$$



We first find the lower bound of  $G$ . We see that  $p_t^* = \min\{u_t, b\}$  is a feasible solution, which gives a payoff of

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \delta^t (u_t - \min\{u_t, b_t\}) \right] \geq \frac{\min_{t \in [0, \infty)} \mathbb{E}[\max\{u_t - b_t, 0\}]}{1 - \delta}. \quad (36)$$

Therefore, we must have  $G \geq \frac{\min_{t \in [0, \infty)} \mathbb{E}[\max\{u_t - b_t, 0\}]}{1 - \delta}$ . Along with assumptions 1.2 and 2, this gives us, for a given  $u_t$  and  $b_t$ ,

$$\frac{\delta G (\pi(p_t^*, p_t^*) - \pi(p_t, p_t^*))}{p_t^* - p_t} \geq \delta GL \geq 1, \quad (37)$$

implying that

$$\mathbb{E} [p_t - p_t^* + \delta \mathbb{E} [G(\pi(p_t^*, p_t^*) - \pi(p_t, p_t^*)) \mid u_t, b_t]] \geq 0. \quad (38)$$

□

**Lemma 3** Suppose we have a LLM with parameters  $\theta$ . We perform a gradient descent step with training sample  $d$  with learning rate  $\eta$  such that  $\hat{\theta} = \theta - \eta \nabla \mathcal{L}(d; \theta)$ . Then,

$$\mathcal{L}(d'; \theta) - \mathcal{L}(d'; \hat{\theta}) \approx \nabla \mathcal{L}(d'; \theta) \cdot \nabla \mathcal{L}(d; \theta)$$

*Proof.* First, we consider the change in loss of  $z'$  using a first-order approximation:

$$\mathcal{L}(d'; \hat{\theta}) = \mathcal{L}(d'; \theta) + \nabla \mathcal{L}(d'; \theta) (\hat{\theta} - \theta) + \mathcal{O}(\|\hat{\theta} - \theta\|^2) \quad (39)$$

$$\mathcal{L}(d'; \theta) - \mathcal{L}(d'; \hat{\theta}) = -\nabla \mathcal{L}(d'; \theta) (\hat{\theta} - \theta) + \mathcal{O}(\|\hat{\theta} - \theta\|^2) \quad (40)$$

Next, suppose a gradient descent step is taken on training sample  $d$ , and the model parameters are updated as:  $\hat{\theta} = \theta - \eta \nabla \mathcal{L}(d; \theta)$ . Thus, we have  $\hat{\theta} - \theta = -\eta \nabla \mathcal{L}(d; \theta)$ , and the change in loss can be written as

$$\mathcal{L}(d'; \theta) - \mathcal{L}(d'; \hat{\theta}) \approx \eta \nabla \mathcal{L}(d'; \theta) \cdot \nabla \mathcal{L}(d; \theta) \propto \nabla \mathcal{L}(d'; \theta) \cdot \nabla \mathcal{L}(d; \theta) \quad (41)$$

Given that  $\eta$  is a constant. □

#### Lemma 4

*Proof.* We show that, for every  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1$ ,  $\alpha_j^{\mathbf{x}}$  gives the largest revenue of  $\mathbf{x}^{\text{new}}$  for  $S_j$ . Recall that in the main text, we need to increase  $\alpha_j$  from zero until we find the largest  $\alpha_j$  such that either of:

1.  $g_{k,j,\text{frac}}(\mathbf{x}^{\text{new}}) > g_{k,j,\text{frac}}(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})$ ,
2.  $f_j([\alpha^T \alpha_j], \mathbf{x}^{\text{new}}) = \bar{\alpha}_k$ .

If we rewrite the first condition, we are essentially looking for  $\alpha_j$  such that

$$\sup_{\alpha_j \in [0,1)} \left\{ \alpha_j : f_j([\alpha^T \alpha_j], \mathbf{x}^{\text{new}}) < 1 - (1 - f_j(\alpha, \tilde{\mathbf{x}}^{k,j-1,\text{frac}})) \frac{u_k(\tilde{\mathbf{x}}^{k,j-1,\text{frac}})}{u_k(\mathbf{x}^{\text{new}})} \right\} \quad (42)$$

Then we see that the revenue that for each  $\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1$ , seller  $S_j$  can make from buyer  $B_k$  is

$$f_j([\alpha^T \alpha_j], \mathbf{x}^{\text{new}}) u_k(\mathbf{x}_j(\alpha)), \quad (43)$$

where  $f_j([\alpha^T \alpha_j], \mathbf{x}^{\text{new}})$  is a non-decreasing function over  $\alpha_j$  while other terms stays fixed. It indicates that  $\alpha_j^{\mathbf{x}}$  is the largest  $\alpha_j$  that the seller  $S_j$  could set for buyer  $B_k$  to purchase  $S_j$ . Therefore, the optimal rate  $\alpha_j^*$  is one of the rates  $\bigcup_{k=1}^M \bigcup_{\mathbf{x} \in \mathcal{X}_{k,j-1,\text{frac}}^1} \{\alpha_j^{\mathbf{x}}\}$ . □

## E. Additional Experimental Details

### E.1. Data Valuation Experiments

**Model Training:** After obtaining purchasing decisions for all data samples, the buyers train their models using the purchased data. In order to conduct a fair comparison across buyers, we sample a set number of data from the buyers' purchases

(shown in Figure 3). We train each model (i.e., buyer) on these samples separately using LoRA (Hu et al., 2021) for 3 epochs, with a learning rate of  $2e-7$  and batch size 32.

**Model Evaluation:** For evaluation, we use the test splits of the previously mentioned datasets. In particular, we use 5-shot evaluation on the MathQA test set, and 4-shot evaluation in on the MedQA test. Table 2 in Appendix F shows the demonstrations used for 5-shot and 4-shot evaluation.

**Market/Pricing Setup:** We reserve 1% of the samples from each dataset’s training split to represent the existing data in their respective markets. Each data sample was randomly priced between  $(0, 1]$ . Next, for each remaining data sample in the training set, we determine whether each buyer will purchase the data sample at potential price points  $[0.5, 0.625, 0.75, 0.875, 1.0]$  by solving Equation (2). The seller then sets their prices according to Equation (5). We price data separately for each data valuation method. This assesses the method’s ability to discern whether a new data sample is worth purchasing for each buyer given the existing market data, as noted in our analysis in Section 3.2.

## E.2. Data Pricing Experiments

**Experiment Setups:** We simulate two buyer budgets at each time step  $t$ . The first buyer (high budget) has a budget uniformly randomly generated between 95% and 100% of the total utilities of all 10 datasets listed in the market. The second buyer (low budget) has a budget uniformly randomly generated between 90% and 95% of the total utilities of all 10 datasets listed in the market.

## F. Additional Tables and Figures

### F.1. Algorithms

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**Algorithm 1** Determine if buyer  $B_k$  will purchase dataset  $D_j$  at price  $p_j$

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- 1: **Inputs:** prices  $\mathbf{p}$ , optimal solution  $\tilde{\mathbf{x}}^{k,j-1}$ , feasible solutions  $\mathcal{X}_{k,j-1}$ , price  $p_j$ .
  - 2: **Output:**  $\mathbb{1}_{\{B_k, D_j, p_j\}}$ .
  - 3: Initialize  $\mathbb{1}_{\{B_k, D_j, p_j\}} \rightarrow 0$ .
  - 4: **for**  $\mathbf{x} \in \mathcal{X}_{k,j-1}$  **do**
  - 5:     **if**  $g_{k,j}(\mathbf{x}^{\text{new}}) > g_{k,j-1}(\tilde{\mathbf{x}}^{k,j-1})$  and  $\mathbf{x}^T \mathbf{p} + p_j \leq b_k$  **then**
  - 6:          $\mathbb{1}_{\{B_k, D_j, p_j\}} \leftarrow 1$
  - 7:     **end if**
  - 8: **end for**
  - 9: **Return**  $\mathbb{1}_{\{B_k, D_j, p_j\}}$
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**Algorithm 2** Market Dynamic Procedure

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- 1: **Inputs:** Buyers  $\{B_k\}_{k=1}^M$  and sellers  $\{S_j\}_{j=1}^N$ .
  - 2: **Initialization:** Buyers  $\{B_k\}_{k=1}^M$  enter the market.
  - 3: **for**  $j \in [N]$  **do**
  - 4:      $S_j$  enters the market with potential prices  $\mathcal{P}_{j,M}$  for dataset  $D_j$
  - 5:     **for**  $p_j \in \mathcal{P}_{j,M}$  **do**
  - 6:         **for**  $k \in [M]$  **do**
  - 7:              $B_k$  solves for  $\tilde{\mathbf{x}}^{k,j-1} = \arg \max_{\mathbf{x} \in \mathcal{X}_{k,j}} g_{k,j}(\mathbf{x})$  to determine if they will purchase  $D_j$  at potential price  $p_j$ . (Eqn. 2)
  - 8:         **end for**
  - 9:         Seller calculates net profit  $r(p_j)$  if they sold potential price  $p_j$ . (Eqn. 4)
  - 10:     **end for**
  - 11:     Seller solves for  $p_j^* = \arg \max_{p_j \in \mathcal{P}_{j,M}} r(p_j)$ , and sets  $p_j^*$  as the price for  $D_j$  (Eqn. 5), which is fixed for subsequent rounds.
  - 12: **end for**
-

## F.2. Datasets

Dataset	# of Train/Valid/Test	Example
MathQA	29837/4475/2985	<b>Question:</b> A train running at the speed of 48 km / hr crosses a pole in 9 seconds . what is the length of the train? a ) 140 , b ) 130 , c ) 120 , d ) 170 , e ) 160 <b>Answer:</b> C
GSM8K	7473/1319	<b>Question:</b> Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May? <b>Answer:</b> 72
MedQA	10178/1272/1273	<b>Question:</b> A 27-year-old man presents to the emergency room with persistent fever, nausea, and vomiting for the past 3 days. While waiting to be seen, he quickly becomes disoriented and agitated. Upon examination, he has visible signs of difficulty breathing with copious oral secretions and generalized muscle twitching. The patient's temperature is 104°F (40°C), blood pressure is 90/64 mmHg, pulse is 88/min, and respirations are 18/min with an oxygen saturation of 90% on room air. When the nurse tries to place a nasal cannula, the patient becomes fearful and combative. The patient is sedated and placed on mechanical ventilation. Which of the following is a risk factor for the patient's most likely diagnosis? a) Contaminated beef b) Epiglottic cyst c) Mosquito bite d) Spelunking <b>Answer:</b> D
PIQA	16000/2000	<b>Question:</b> How do I ready a guinea pig cage for it's new occupants? a) Provide the guinea pig with a cage full of a few inches of bedding made of ripped paper strips, you will also need to supply it with a water bottle and a food dish. b) Provide the guinea pig with a cage full of a few inches of bedding made of ripped jeans material, you will also need to supply it with a water bottle and a food dish. <b>Answer:</b> A

Table 1. Dataset splits and demonstrations from the MathQA, GSM8K, MedQA, and PIQA datasets

Dataset	Prompts
MathQA	<p><b>Question:</b> the banker's gain of a certain sum due 3 years hence at 10 % per annum is rs . 36 . what is the present worth ? a ) rs . 400 , b ) rs . 300 , c ) rs . 500 , d ) rs . 350 , e ) none of these</p> <p><b>Answer:</b> A</p> <p><b>Question:</b> average age of students of an adult school is 40 years . 120 new students whose average age is 32 years joined the school . as a result the average age is decreased by 4 years . find the number of students of the school after joining of the new students . a ) 1200 , b ) 120 , c ) 360 , d ) 240 , e ) none of these</p> <p><b>Answer:</b> D</p> <p><b>Question:</b> sophia finished 2 / 3 of a book . she calculated that she finished 90 more pages than she has yet to read . how long is her book ? a ) 229 , b ) 270 , c ) 877 , d ) 266 , e ) 281</p> <p><b>Answer:</b> B</p> <p><b>Question:</b> 120 is what percent of 50 ? a ) 5 % , b ) 240 % , c ) 50 % , d ) 2 % , e ) 500</p> <p><b>Answer:</b> B</p> <p><b>Question:</b> there are 10 girls and 20 boys in a classroom . what is the ratio of girls to boys ? a ) 1 / 2 , b ) 1 / 3 , c ) 1 / 5 , d ) 10 / 30 , e ) 2 / 5</p> <p><b>Answer:</b> A</p>
MedQA	<p><b>Question:</b> A mother brings her 3-week-old infant to the pediatrician's office because she is concerned about his feeding habits. He was born without complications and has not had any medical problems up until this time. However, for the past 4 days, he has been fussy, is regurgitating all of his feeds, and his vomit is yellow in color. On physical exam, the child's abdomen is minimally distended but no other abnormalities are appreciated. Which of the following embryologic errors could account for this presentation? a) Abnormal migration of ventral pancreatic bud b) Complete failure of proximal duodenum to recanalize c) Abnormal hypertrophy of the pylorus d) Failure of lateral body folds to move ventrally and fuse in the midline</p> <p><b>Answer:</b> A</p> <p><b>Question:</b> A 53-year-old man comes to the emergency department because of severe right-sided flank pain for 3 hours. The pain is colicky, radiates towards his right groin, and he describes it as 8/10 in intensity. He has vomited once. He has no history of similar episodes in the past. Last year, he was treated with naproxen for swelling and pain of his right toe. He has a history of hypertension. He drinks one to two beers on the weekends. Current medications include amlodipine. He appears uncomfortable. His temperature is 37.100b0C (99.300b0F), pulse is 101/min, and blood pressure is 130/90 mm Hg. Examination shows a soft, nontender abdomen and right costovertebral angle tenderness. An upright x-ray of the abdomen shows no abnormalities. A CT scan of the abdomen and pelvis shows a 7-mm stone in the proximal ureter and grade I hydronephrosis on the right. Which of the following is most likely to be seen on urinalysis? a) Urinary pH: 7.3 b) Urinary pH: 4.7 c) Positive nitrites test d) Largely positive urinary protein</p> <p><b>Answer:</b> B</p> <p><b>Question:</b> A 48-year-old woman comes to the emergency department because of a photosensitive blistering rash on her hands, forearms, and face for 3 weeks. The lesions are not itchy. She has also noticed that her urine has been dark brown in color recently. Twenty years ago, she was successfully treated for Coats disease of the retina via retinal sclerotherapy. She is currently on hormonal replacement therapy for perimenopausal symptoms. Her aunt and sister have a history of a similar skin lesions. Examination shows multiple fluid-filled blisters and oozing erosions on the forearms, dorsal side of both hands, and forehead. There is hyperpigmented scarring and patches of bald skin along the sides of the blisters. Laboratory studies show a normal serum ferritin concentration. Which of the following is the most appropriate next step in management to induce remission in this patient? a) Pursue liver transplantation b) Begin oral thalidomide therapy c) Begin phlebotomy therapy d) Begin oral hydroxychloroquine therapy</p> <p><b>Answer:</b> C</p> <p><b>Question:</b> A 23-year-old pregnant woman at 22 weeks gestation presents with burning upon urination. She states it started 1 day ago and has been worsening despite drinking more water and taking cranberry extract. She otherwise feels well and is followed by a doctor for her pregnancy. Her temperature is 97.700b0F (36.500b0C), blood pressure is 122/77 mmHg, pulse is 80/min, respirations are 19/min, and oxygen saturation is 98% on room air. Physical exam is notable for an absence of costovertebral angle tenderness and a gravid uterus. Which of the following is the best treatment for this patient? a) Ampicillin b) Ceftriaxone c) Doxycycline d) Nitrofurantoin</p> <p><b>Answer:</b> D</p>

Table 2. Demonstrations included for 5-shot evaluation on the MathQA dataset and for 4-shot evaluation on the MedQA dataset. Demonstrations were randomly selected from their respective dataset's training sets.



## E.3. Additional Experimental Results

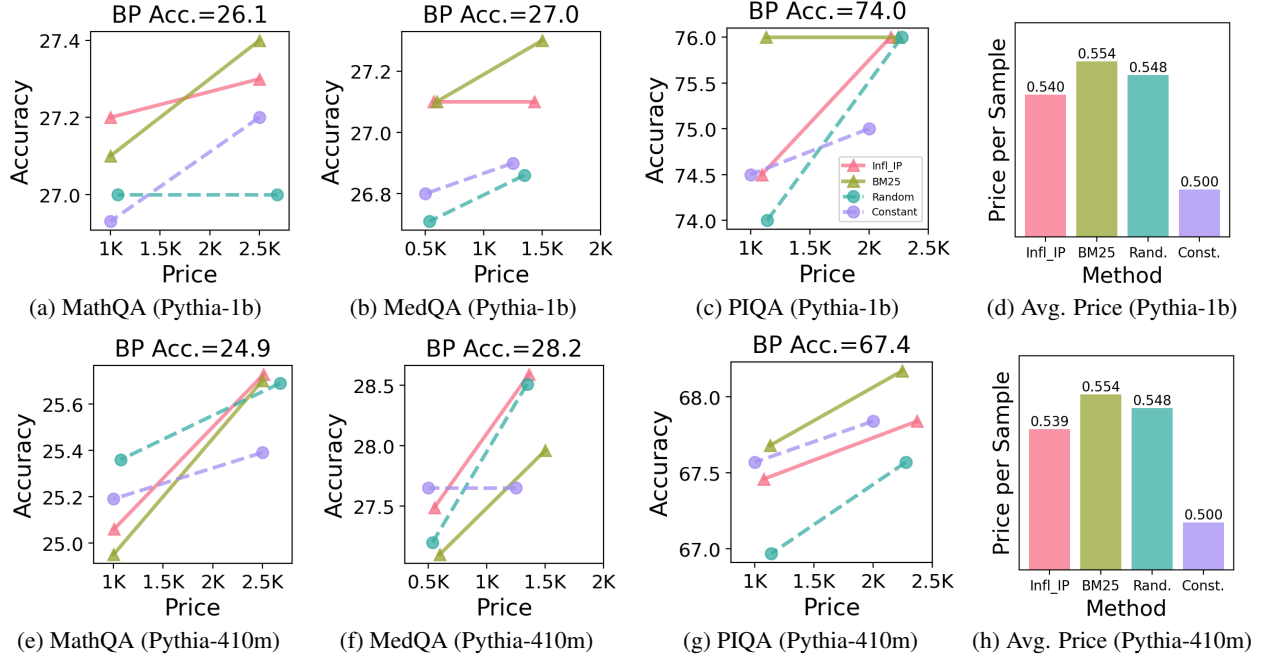


Figure 6. Buyers' model (Pythia-410m) performance and costs from their purchased data from math, medical, and physical reasoning data markets. Purchasing decisions were using the constant, random, BM25, Infl<sub>IP</sub> data valuation methods (see Section 5 for details).

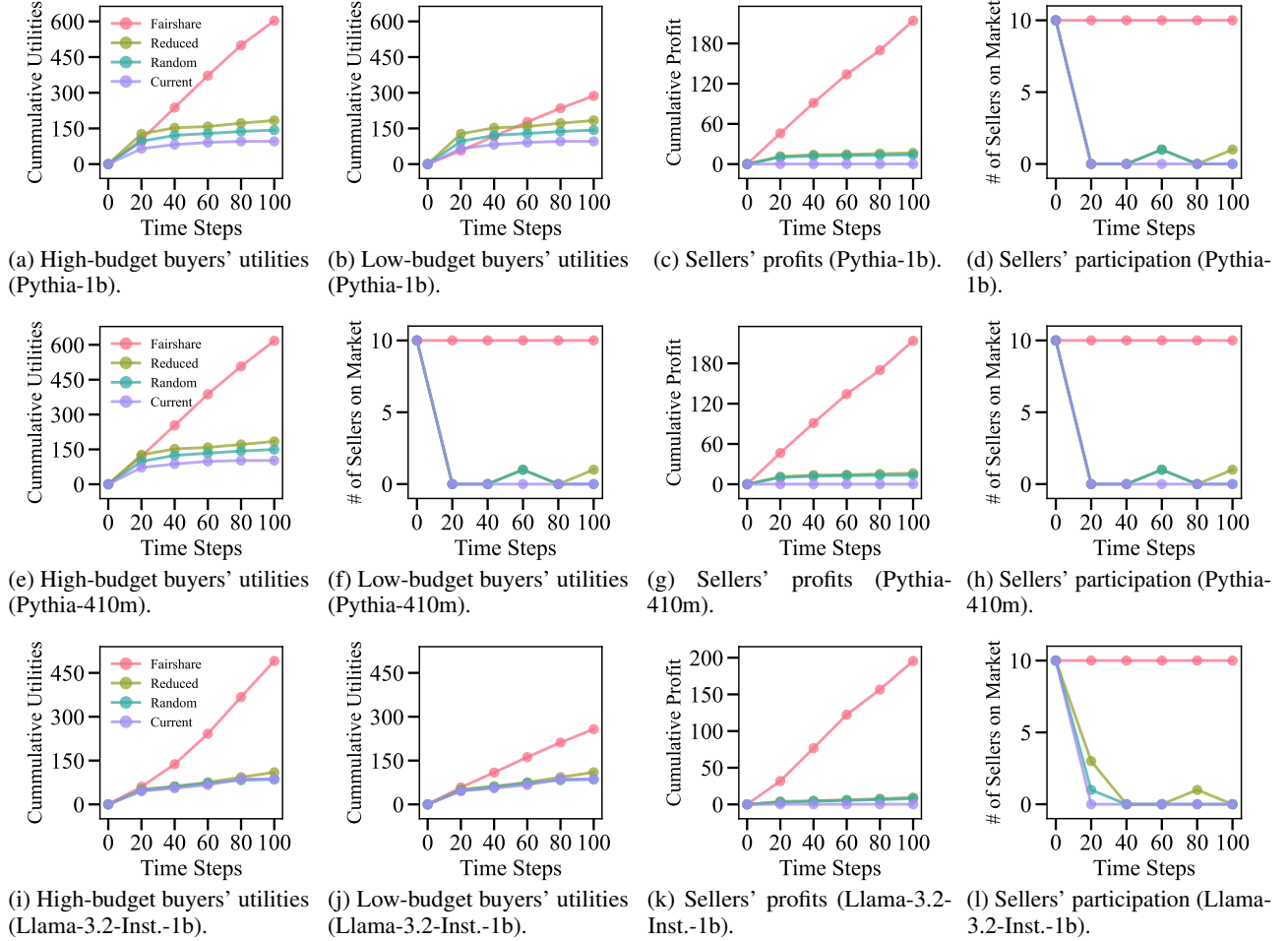


Figure 7. Analysis of (1) buyer’s cumulative utilities with high-budget buyer (Figures 7a, 7e and 7i) and low-budget buyer (Figures 7b, 7f and 7j), and (2) sellers’ average cumulative profits (Figures 7c, 7g and 7k) and number of sellers in the market (Figures 7d, 7h and 7l) over time ( $T = 100$ ). Model: Pythia-1b, Pythia-410m, and Llama-3.2-Inst.-1b; Task: medqaQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.

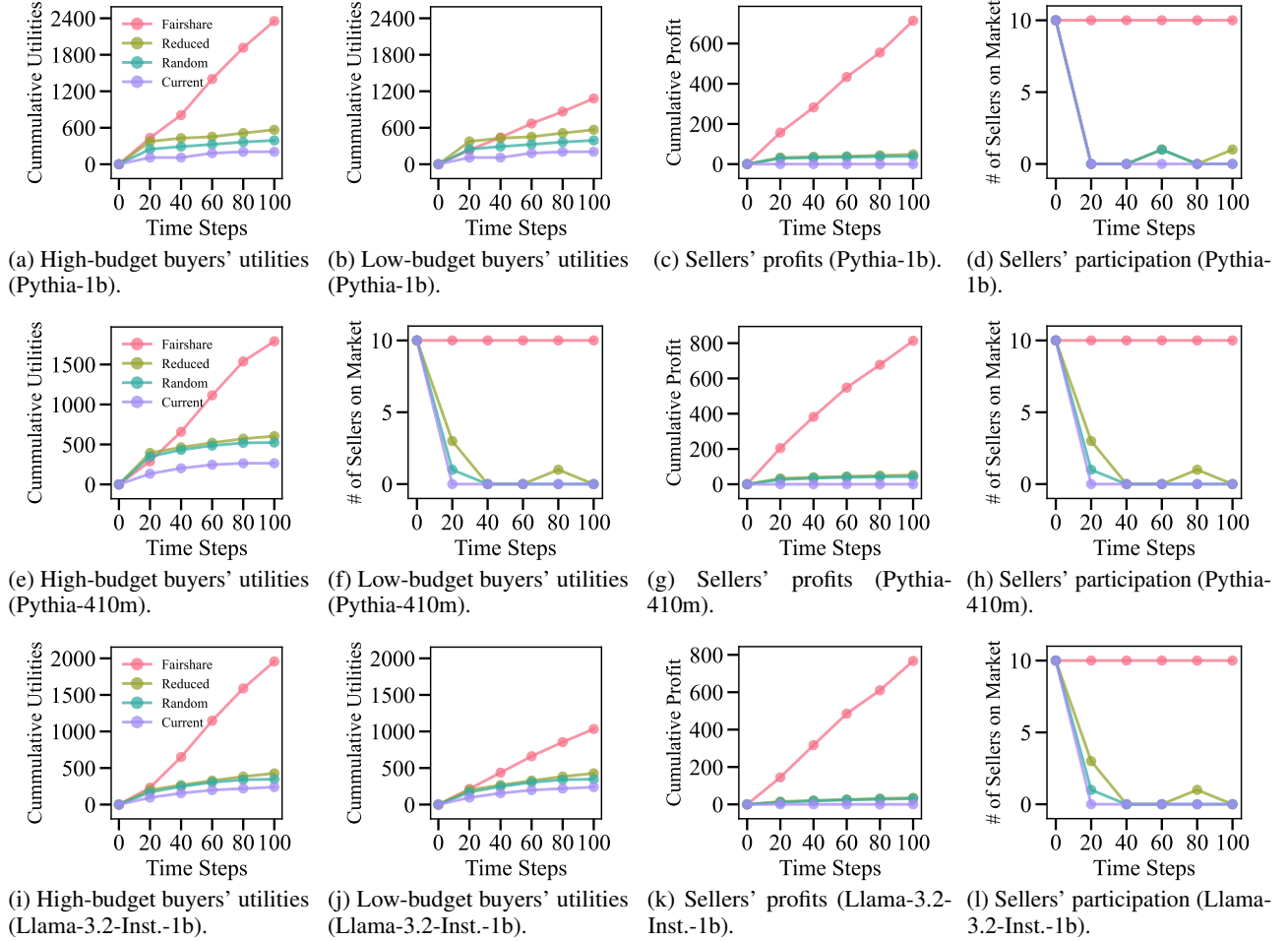


Figure 8. Analysis of (1) buyer's cumulative utilities with high-budget buyer (Figures 8a, 8e and 8i) and low-budget buyer (Figures 8b, 8f and 8j), and (2) sellers' average cumulative profits (Figures 8c, 8g and 8k) and number of sellers in the market (Figures 8d, 8h and 8l) over time ( $T = 100$ ). Model: Pythia-1b, Pythia-410m, and Llama-3.2-Inst.-1b; Task: MathQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.

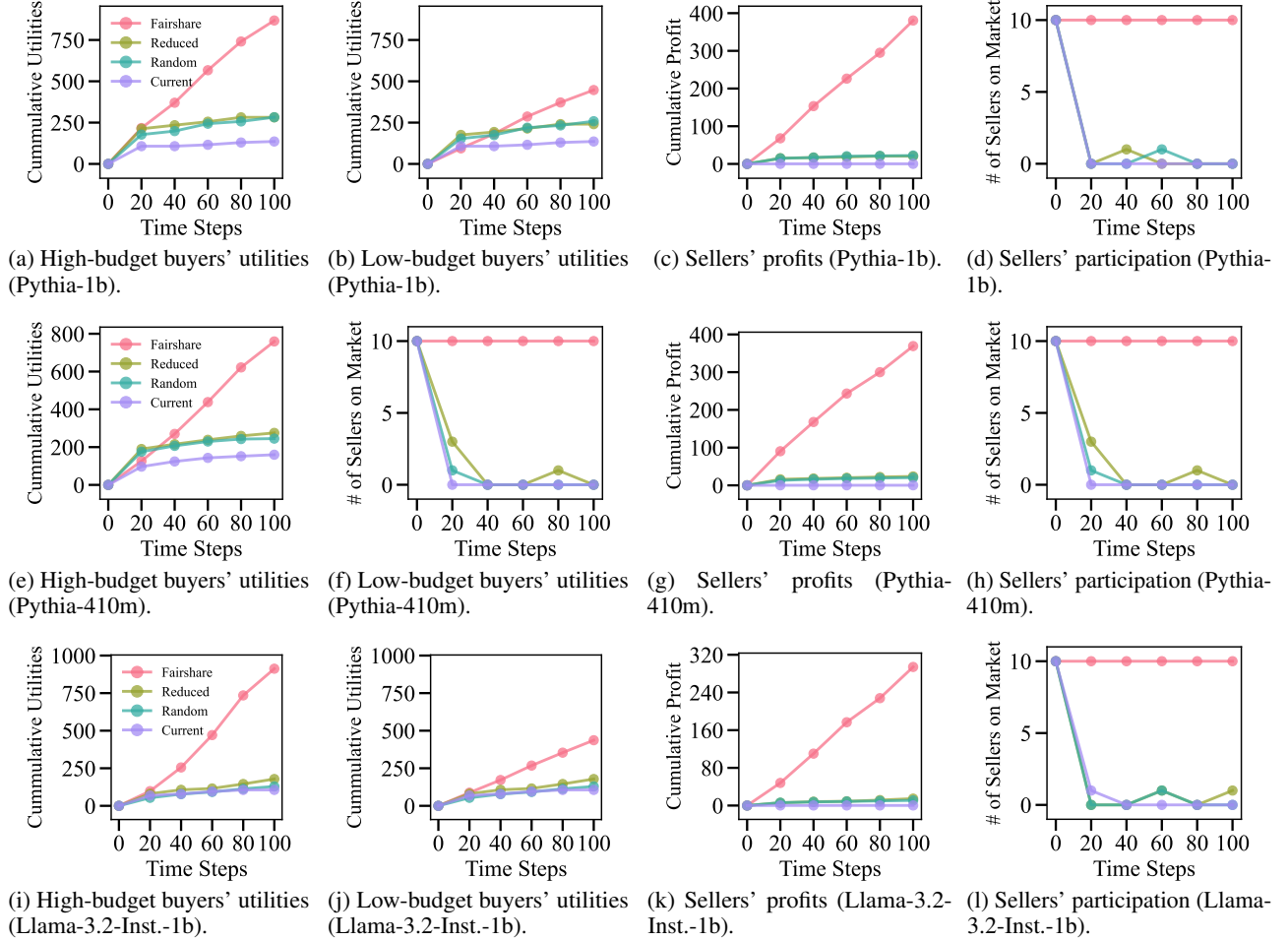


Figure 9. Analysis of (1) buyer’s cumulative utilities with high-budget buyer (Figures 9a, 9e and 9i) and low-budget buyer (Figures 9b, 9f and 9j), and (2) sellers’ average cumulative profits (Figures 9c, 9g and 9k) and number of sellers in the market (Figures 9d, 9h and 9l) over time ( $T = 100$ ). Model: Pythia-1b, Pythia-410m, and Llama-3.2-Inst.-1b; Task: PIQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.



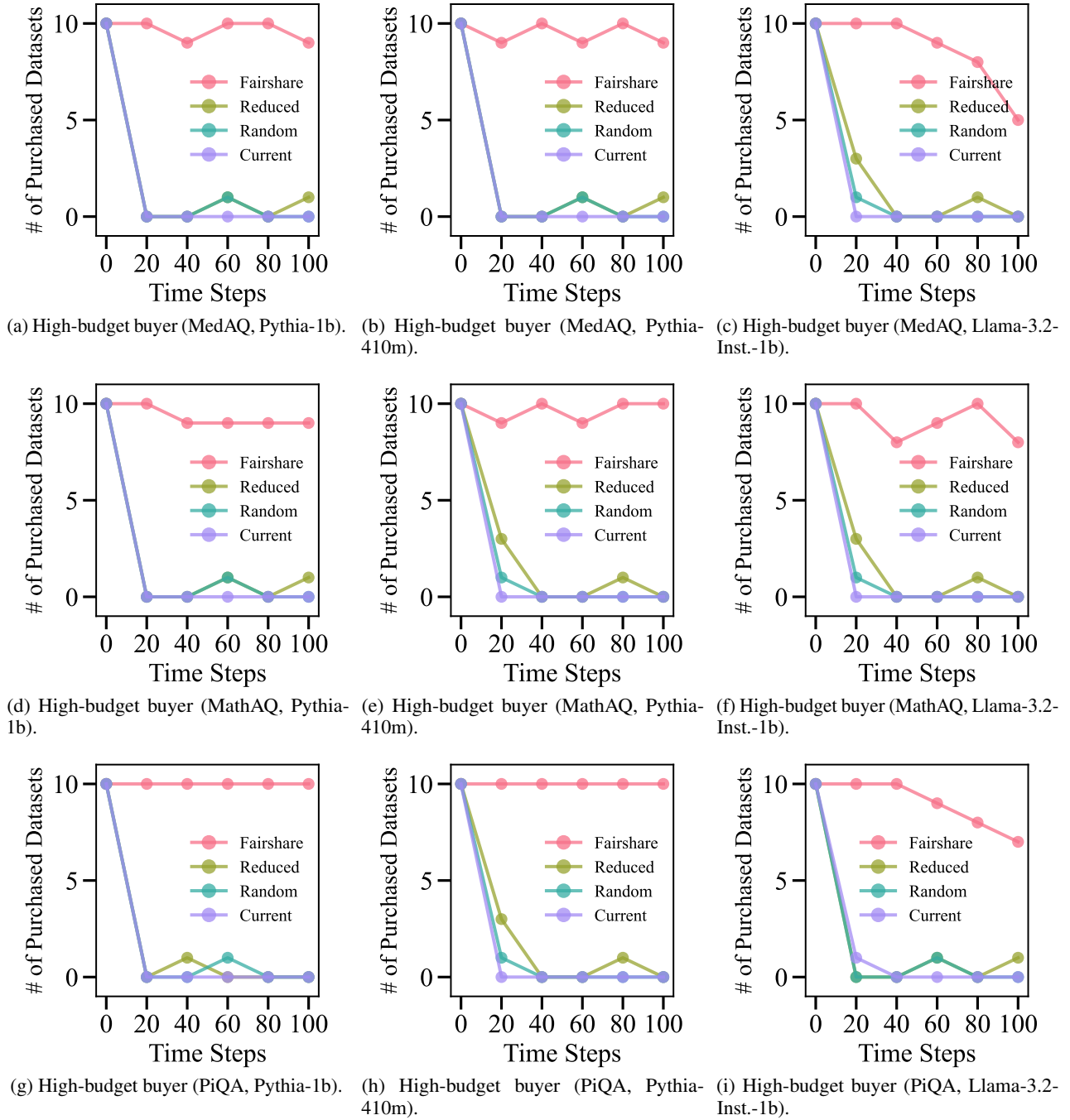


Figure 10. Number of purchased datasets for the buyer with high budget over time periods ( $T = 100$ ). Model: Pythia-1b, Pythia-410m, and Llama-3.2-Inst.-1b; Task: MedQA, MathQA, and PiQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.

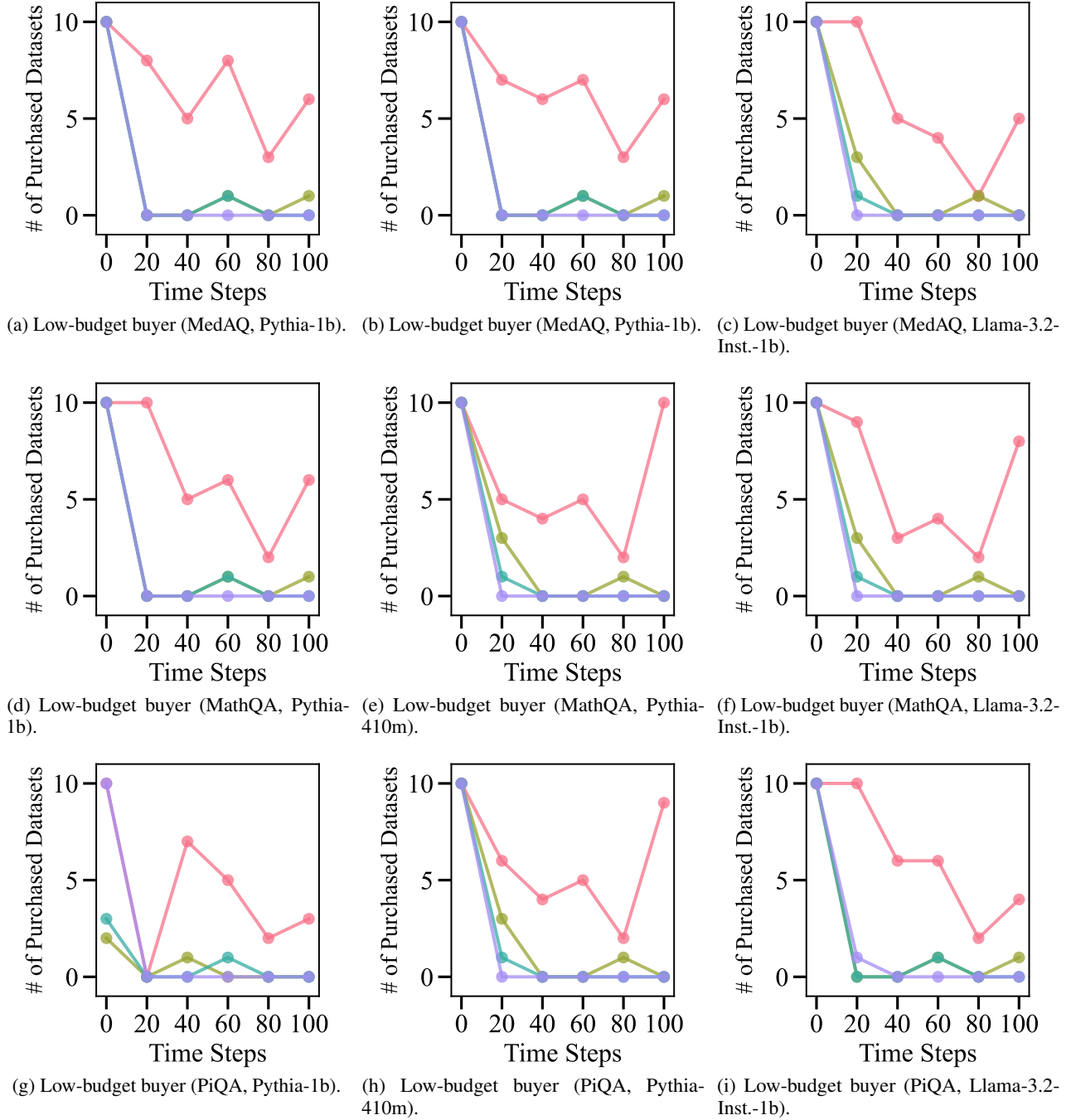


Figure 11. Number of purchased datasets for the buyer with low budget over time periods ( $T = 100$ ). Model: Pythia-1b, Pythia-410m, and Llama-3.2-Inst.-1b; Task: MedQA, MathQA, and PiQA. Experimental groups: (1) fairshare, (2) reduced, (3) random, and (4) current pricing.