
Learned 1-D advection solver to accelerate air quality modeling

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Abstract

1 Accelerating the numerical integration of partial differential equations by learned
2 surrogate model is a promising area of inquiry in the field of air pollution modeling.
3 Most previous efforts in this field have been made on learned chemical operators
4 though machine-learned fluid dynamics has been a more blooming area in machine
5 learning community. Here we show the first trial on accelerating advection operator
6 in the domain of air quality model using a realistic wind velocity dataset. We de-
7 signed a convolutional neural network-based solver giving coefficients to integrate
8 the advection equation. We generated a training dataset using a 2nd order Van Leer
9 type scheme with the 10-day east-west components of wind data on 39°N within
10 North America. The trained model with coarse-graining showed good accuracy
11 overall, but instability occurred in a few cases. Our approach achieved up to 12.5×
12 acceleration. The learned schemes also showed fair results in generalization tests.

13 1 Introduction

14 Numerical integration of partial differential equations (PDEs) is a core element of air quality model.
15 To run the air quality model one should solve coupled PDEs within many grid boxes and multiple
16 time steps. Since solving many PDEs requires a huge amount of computational cost, the invention of
17 a fast and accurate solver with has been always welcomed. Recent advancement in physics-informed
18 machine learning [Karniadakis et al., 2021, Kashinath et al., 2021] has gained popularity to emulate
19 existing solvers and researchers are seeking a *pareto* optimum between speed and accuracy.

20 So far, research efforts on learning air quality models for acceleration have mostly focused on
21 chemistry solvers. Kelp et al. [2020] developed the encoder-operator-decoder structure neural
22 network and achieved ×260 speedup in emulation of Carbon Bond Mechanism Z coupled to the
23 Model for Simulating Aerosol Interactions and Chemistry. Huang and Seinfeld [2022] showed their
24 neural integrator could solve two benchmarking problems (the H₂O₂/OH/HO₂ System and the Verwer
25 System) with acceleration in at least one order of magnitude.

26 Although this type of research is an emerging area, the potential of a learned solver for transport
27 operators in air quality modeling has not been actively investigated. However, there could be possible
28 speedup by using a learned transport operator since researchers in computational fluid dynamics
29 (CFD) already showed machine-learned acceleration. Kochkov et al. [2021] and Stachenfeld et al.
30 [2021] showed convolutional neural network (CNN) based coarse-graining solver could accelerate
31 CFD solvers. Zhuang et al. [2021]’s learned discretization to estimate coefficients in advection solver
32 achieved 1.8× faster computing by 4× coarsening. The more examples of machine-learned fluid
33 dynamics could be found from the review papers by Brunton et al. [2020] andKutz [2017].

34 Our study examines the potential of a learned advection operator for computational acceleration of air
35 quality modeling. To build a foundation for future analysis, we first explore the potential of learned

36 solver in 1-D advection. We used a realistic wind velocity instead of a synthetic velocity to generate
 37 our baseline dataset. Since the modeling domain in the global air quality model is not rectangular, but
 38 spherical, evaluation of the model skills in different grid sizes is critical to generalization. We tested
 39 the model’s generalization ability in different latitudes which have different grid sizes. Also, we
 40 tested the model’s ability to integrate the wave from an initial condition shape out of training regime.

41 2 Numerical advection

42 We simulated a passive scalar advection in a horizontal line passing 39.00°N of North America
 43 (130°W - 60°W). The spatial grid size was 0.3125°. We simulated advection using east-west
 44 components of wind data from 1 to 10 January 2019 with 5-minute intervals. We obtained wind data
 45 from GEOS-FP of NASA Global Modeling and Assimilation Office [NASA, 2022]. We used a square
 46 initial condition with a 10^{-7} on the central 1/3 of domain, while other areas have 0. We implemented
 47 the L94 advection scheme [Lin et al., 1994] in the Julia computing language [Bezanson et al., 2012].

48 After generating this dataset by numerical integration, we down-sampled those wave datasets in lower
 49 resolutions in both space and time. To conserve mass, we averaged the scalar values in down-sampling.
 50 The sample resolutions are $\times 1$, $\times 2$, $\times 4$, $\times 8$, and $\times 16$ in space and $\times 1$, $\times 2$, $\times 4$, $\times 8$, $\times 16$, $\times 32$, $\times 64$
 51 in time, so we have 35 different cases from one scenario.

52 3 Learned advection using a convolutional neural network

53 **Figure 1** illustrates the design of the CNN-based surrogate advection solver. Our surrogate equation
 54 has information of $\Delta t/\Delta x$, which is critical in earth system modeling domain since grid spacing Δx
 55 can change along with latitude. The three-layer CNN receives scalar and velocity fields at n^{th} time
 56 step as inputs and yields six coefficients to construct surrogate numerical equation. We used two
 57 GeLU activations [Hendrycks and Gimpel, 2016] to resolve sharp gradients [Kim et al., 2021], and
 58 one hypertangent since the temporal gradient can have both positive and negative signs. We used
 59 gradient scaling by adopting k_1 and k_1 to make $k_1(\Delta t/\Delta x)$ and $k_2(\Delta t/\Delta x)^2$ be in the order of 10^0 .

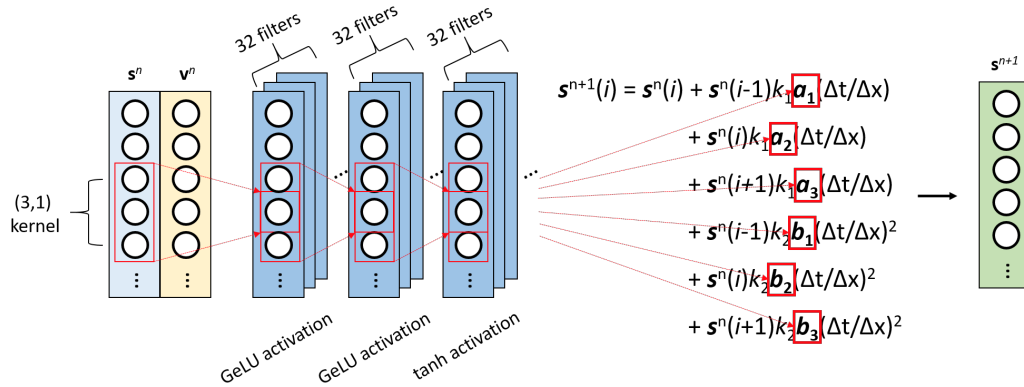


Figure 1: Illustrative diagram of convolutional neural net advection operator

60 We used mean absolute error (MSE) in 10-time steps as a loss function to reflect the dynamic nature
 61 of advection. To prevent error accumulation, we introduced random noise with 4×10^{-5} magnitude of
 62 initial scalar intensity. We used the ADAM optimizer [Kingma and Ba, 2014] with default parameters
 63 in Flux.jl [Innes, 2018], except for the learning rate. We used a decaying learning rate to reach the
 64 optimum. After training the model, we evaluated the model performance by feeding only the initial
 65 condition and velocity field and assessed if the model could integrate the advection process till a
 66 given period. We used a single CPU core from an HPE Apollo 6500 system with dual 6248 Cascade
 67 Lake CPUs to evaluate computational time.

68 **Figure 2** shows the performance of the learned solver in integrating the training dataset. We
 69 normalized MSE and root mean square error (RMSE) by the initial magnitude. Here we should note

70 that the model training was not successful in $(1\Delta x, 8\Delta t)$, $(1\Delta x, 16\Delta t)$, $(1\Delta x, 32\Delta t)$, and $(1\Delta x,$
 71 $64\Delta t)$ as the outputs exploded out. Except for those cases, the learned schemes showed fair fidelity
 72 since error was usually less than 10 % (**Figure 2A** and **2B**) and r^2 was higher than 0.9 (**Figure 2C**).
 73 Maximum acceleration was achieved in $(16\Delta x, 64\Delta t)$ and this scheme was $12.5\times$ faster than the
 74 original solver (**Figure 2D**). As seen in **Figure 2D**, the acceleration in failure cases would be lower
 75 than the maximal acceleration case, so we may consider those resolutions to be out of our scope.

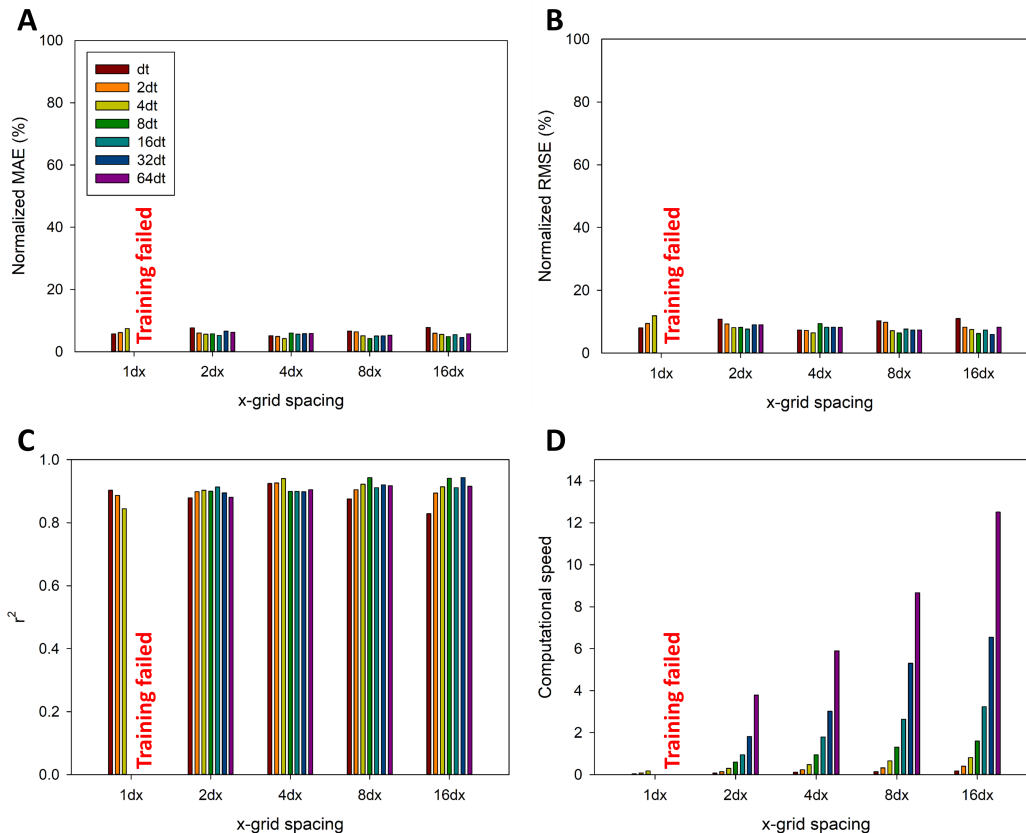


Figure 2: Performance indices of the surrogate learned solver in emulating the training dataset (A: normalized mean absolute error, B: normalized root mean square error, C: r^2 , and D: speed)

76 Time series plots of advection in the resolution with the best accuracy ($4\Delta x, 4\Delta t$; **Figure 3A**) and
 77 with the maximum acceleration ($16\Delta x, 64\Delta t$; **Figure 3B**) show the snapshots in the first step from
 78 initial condition, 1/3 of time span, 2/3 of time span, and the final step. As seen in **Figure 3**, we could
 79 confirm that the learned model could emulate the coarsened numerical results within fair accuracy.

80 4 Generalization

81 We tested if our model could integrate the waves from outside of the training regime. We implemented
 82 two tests: 1) integrating advection in the Northern area; 2) integrating advection from Gaussian
 83 shape initial condition. In the first test, we applied our surrogate solver in a horizontal line passing
 84 45°N . Though we used 0.3125° as grid spacing as we did on the training set, the exact spacing was
 85 shorter here because of the earth's spherical shape. In the second test, we fed a Gaussian shape initial
 86 condition to be integrated. Any conditions not mentioned are the same as the training.

87 The results of generalization test are summarized in **Figure 4**. **Figure 4A** and **4B** show the r^2 in
 88 the horizontal line passing 45°N and the integration Gaussian shape initial condition, respectively.
 89 As seen in **Figure 4**, we can use our learned scheme in a regime beyond the training set without
 90 substantial performance degradation. We did not test the coarsening cases in $(1\Delta x, 8\Delta t)$, $(1\Delta x,$
 91 $16\Delta t)$, $(1\Delta x, 32\Delta t)$, and $(1\Delta x, 64\Delta t)$ because the surrogate models in those cases failed in training.

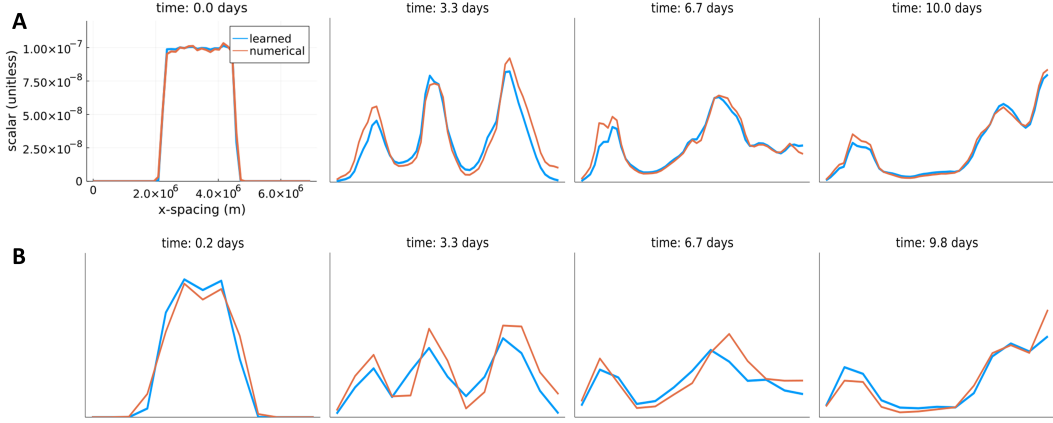


Figure 3: Time series display of both numerical (orange line) and learned (blue line) scalar advection (A: $(4\Delta x, 4\Delta t)$ and B: $(16\Delta x, 64\Delta t)$)

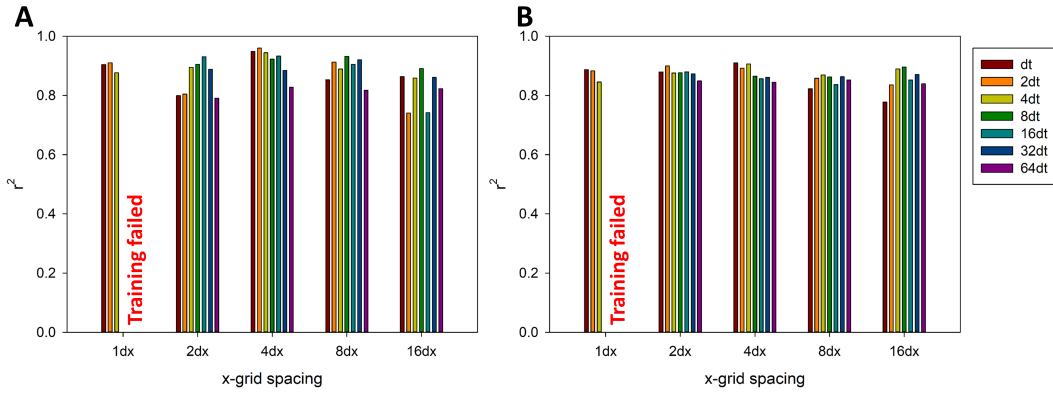


Figure 4: Generalization ability of the learned solver summarized by r^2 (A: The horizontal line passing 45°N , and B: Integration with Gaussian shape initial condition)

92 **5 Limitation**

93 The most noticeable limitation of this work is that the learned scheme could not work in certain
 94 resolutions. One possible explanation is that violations of the CFL condition were most extreme in
 95 those cases. Though we increased the stencil size in an attempt to resolve this issue, this approach
 96 did not fix the problem. The reason for this instability is an area for future study. Another limitation
 97 is that we did not optimize the model training in the light of hyperparameter tuning or rigorous code
 98 optimization. We would argue, however, our approach can be promising since it showed robust
 99 performance without optimization and there is still room to achieve acceleration. Finally, to use this
 100 scheme on the whole globe, we would need more generalization tests on different latitudes to see if
 101 the current scheme could work or would need more training on different regimes.

102 **6 Conclusion**

103 Our study revealed the learned advection scheme can be used to emulate passive scalar advection in
 104 the domain of air quality modeling using real wind-field data. The learned advection solver showed
 105 up to $\times 12.5$ with fairly accurate integration. The learned solver was robust in generalization even
 106 though we only used a single training dataset. This robustness may come from the randomness
 107 of realistic wind data, implying choice of a dataset can be crucial in physics-informed machine
 108 learning. There could be more potential acceleration by code optimization or high-performance
 109 computing. With an appropriate splitting technique, we could extend this learned 1-D advection
 110 solver to multi-dimensions and eventually accelerate air quality models.

111 **Supplementary materials**

112 The codes and data can be accessed at [https://github.com/manozzing/Learned-1-D-advection-solver-](https://github.com/manozzing/Learned-1-D-advection-solver-with-grid-spacing-physics)
113 [with-grid-spacing-physics](https://github.com/manozzing/Learned-1-D-advection-solver-with-grid-spacing-physics)

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152 **Checklist**

- 153 1. For all authors...
- 154 (a) Do the main claims made in the abstract and introduction accurately reflect the paper's
155 contributions and scope? [Yes]
- 156 (b) Did you describe the limitations of your work? [Yes]
- 157 (c) Did you discuss any potential negative societal impacts of your work? [N/A]
- 158 (d) Have you read the ethics review guidelines and ensured that your paper conforms to
159 them? [Yes]
- 160 2. If you are including theoretical results...
- 161 (a) Did you state the full set of assumptions of all theoretical results? [N/A]
- 162 (b) Did you include complete proofs of all theoretical results? [N/A]
- 163 3. If you ran experiments...
- 164 (a) Did you include the code, data, and instructions needed to reproduce the main experi-
165 mental results (either in the supplemental material or as a URL)? [Yes]
- 166 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they
167 were chosen)? [Yes]
- 168 (c) Did you report error bars (e.g., with respect to the random seed after running experi-
169 ments multiple times)? [N/A]
- 170 (d) Did you include the total amount of compute and the type of resources used (e.g., type
171 of GPUs, internal cluster, or cloud provider)? [Yes]
- 172 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- 173 (a) If your work uses existing assets, did you cite the creators? [Yes] We cited NASA
174 GMAO webpage to cite GEOS-FP.
- 175 (b) Did you mention the license of the assets? [N/A]
- 176 (c) Did you include any new assets either in the supplemental material or as a URL? [N/A]
177
- 178 (d) Did you discuss whether and how consent was obtained from people whose data you're
179 using/curating? [N/A]
- 180 (e) Did you discuss whether the data you are using/curating contains personally identifiable
181 information or offensive content? [N/A]
- 182 5. If you used crowdsourcing or conducted research with human subjects...
- 183 (a) Did you include the full text of instructions given to participants and screenshots, if
184 applicable? [N/A]
- 185 (b) Did you describe any potential participant risks, with links to Institutional Review
186 Board (IRB) approvals, if applicable? [N/A]
- 187 (c) Did you include the estimated hourly wage paid to participants and the total amount
188 spent on participant compensation? [N/A]