Utilizing Everything in History: Modeling Relation Inference Path and Entity Structure for Temporal Knowledge Graph Reasoning

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⁰⁰¹ Abstract

 Temporal Knowledge Graph (TKG) extrapola- tion fundamentally involves selecting the cor- rect answer from all entities based on histor- ical information. Current methods can easily eliminate most incorrect answers, narrowing the candidate pool to a tiny area called the candidate zone. However, these methods of- ten fail to find the correct answer within this zone, primarily because the entities within the candidate zone are similar in subgraph struc- ture or relational connectivity, causing signif- icant interference. These methods, which ei-014 ther model the graph structure of entities or the paths of relationships, can only address one type of similarity. To address this issue, we propose a model called the Relation Causal Logic Inference and Entity Structure Learning (RIES), which consists of two modules: rela- tion inference and entity structure. These two modules model the causal logic of relations over time and the temporal evolution of enti- ties' subgraph structure, respectively, allowing for the differentiation of candidates similar in 025 subgraph structure and relational connectivity. 026 When evaluated on five commonly used public datasets, the performance of RIES surpasses that of other state-of-the-art baselines.

⁰²⁹ 1 Introduction

 Predicting future facts accurately requires a com- prehensive analysis of historical data. Each times- tamp links entities through a variety of relations, constructing a knowledge graph characterized by intricate structural and causal logic. Methods like 035 CyGNet [\(Zhu et al.](#page-9-0) [\(2021\)](#page-9-0)), CENET [\(Xu et al.](#page-9-1) [\(2023\)](#page-9-1)), HGLS [\(Zhang et al.](#page-9-2) [\(2023\)](#page-9-2)), and EvoEx-**plore [\(Zhang et al.](#page-9-3) [\(2022\)](#page-9-3)) typically model histor-** ical facts based on repetitive patterns, primarily making predictions from these recurrences. In con- trast, some methods are entirely independent of entities, such as DaeMon [\(Dong et al.](#page-8-0) [\(2023\)](#page-8-0)) and TiPNN [\(Dong et al.](#page-8-1) [\(2024\)](#page-8-1)), which search for rela-tion paths that have occurred in history and learn

Figure 1: An illustration of temporal reasoning over a TKG.

entity-agnostic inference rules. The main issues **044** with these methods include: 045

Issue 1: The causal logic in the temporal order **046** of relations between pairs of entities is not cap- **047** tured. Some graph-structured TKGR methods like **048** CyGNet, CENET, HGLS, and EvoExplore do not **049** focus on the changes in relations of the same en- **050** tity pair across different timestamps, ignoring the **051** causal logic of these relations over time. In the ex- **052** ample of Figure [1,](#page-0-0) the variety of historical relations **053** between the entities China and the US President **054** do not contribute equally to answering queries. Fo- **055** cusing more on relations that are highly relevant **056** to the query can reduce semantic noise during the **057** reasoning process. **058**

Issue 2: The aforementioned approaches con- **059** sider only entities, or only relations, which have **060** limitations in some specific cases. If we focus **061** solely on relations, independent of the entities, it 062 becomes difficult to distinguish between entities **063** that share very similar historical relations with the **064** query subject s. For instance, in Figure [1,](#page-0-0) the en- **065** tities USA and the US President would be hard **066** to differentiate. If we only consider the subgraph **067** structure of the entities, such as USA and India, we find that the neighboring entities connected in the **069** subgraphs for these two countries at different times- **070** tamps are all other country entities. The subgraph **071** structures represented by these two entities are very **072** similar, making it difficult to distinguish between **073**

 them in the final prediction. To summarize, exist- ing models focus on only one type of information in entities and relations and ignore the other, which limits their performance in TKGR.

 To address the aforementioned issues, we model relations and entities information in a unified frame- work that allows these two types of information to be complementary in the reasoning process.

 To solve issue 1, we propose a relation inference module, which consists of two parts: RCL (Re- lation Causal Logic) and PCA (Path Confidence Aggregation). (1) RCL: This part focuses on learn- ing the temporal causal logic between historical relations and the query relation *rq*. (2) PCA: This part involves aggregating the confidence scores of all relation inference paths between query subject *s* and candidate entities. It calculates the probability 091 score that the query relation r_q will occur between the query subject *s* and the candidate entities at the timestamp *tq*, based solely on relation data.

 In order to tackle issue 2, we first propose an en-095 tity structure module, which models the structural dependencies between entities and concurrent facts. This enables us to generate a dynamic structural encoding of the query subject *s* and each candi- date entity. We then decode this information to determine the probability of interaction between the query subject *s* and each candidate entity at the query timestamp *t^q* and under the query relation *rq*. Subsequently, we combine the predictive probabil- ity scores from both the relation level and the entity level for each candidate entity to arrive at a final predictive probability score. By leveraging both re- lation and entity information, we can significantly improve the accuracy of our predictions.

109 In summary, our work makes the following con-**110** tributions:

- **111** 1) We have developed a relation inference mod-**112** ule that explores the causal logic of relations **113** in their temporal sequence by collecting infor-114 **114** mation about the interactions between query **115** entities and candidate entities from historical **116** data.
- **117** 2) To our knowledge, we are the pioneers in in-**118** tegrating modeling of relations and entities **119** within a unified framework, effectively lever-**120** aging both relation and entity information.
- **121** 3) Extensive experiments indicate that our model **122** substantially outperforms existing methods.

2 Related Work **¹²³**

Depending on the type of historical information **124** that a model focuses on, existing models can be **125** divided into two categories: models based on his- **126** torical entity information and models based on his- **127** torical relation information. **128**

Models based on historical entity information **129** focus on modeling information about the entity **130** [\(Park et al.](#page-8-2) [\(2022\)](#page-8-2)[;Yang et al.](#page-9-4) [\(2023\)](#page-9-4)[;Wu et al.](#page-9-5) **131** [\(2020\)](#page-9-5)[;Jin et al.](#page-8-3) [\(2020\)](#page-8-3)[;Xiao et al.](#page-9-6) [\(2024\)](#page-9-6)[;Zhang](#page-9-2) **132** [et al.](#page-9-2) [\(2023\)](#page-9-2)). For instance, CyGNet [\(Zhu et al.](#page-9-0) **133** [\(2021\)](#page-9-0)) counts the frequency of entities occurring **134** repeatedly in history and uses a copy mechanism **135** to select prediction results from the entities that ap- **136** pear frequently. CENET [\(Xu et al.](#page-9-1) [\(2023\)](#page-9-1)) adopts **137** a comparative learning approach to capture the de- **138** pendency of queries on both historical and non- **139** historical entities. EvoExplore [\(Zhang et al.](#page-9-3) [\(2022\)](#page-9-3)) 140 implements a hierarchical attention mechanism to **141** model the intricate local and global structures of 142 entities. **143**

Models based on historical relation informa- **144** tion are completely independent of entities and **145** focus on modeling the temporal path of relations **146** [\(Sun et al.](#page-9-7) [\(2021\)](#page-9-7)[;Lin et al.](#page-8-4) [\(2023\)](#page-8-4)). For instance, **147** CluSTeR [\(Li et al.](#page-8-5) [\(2021\)](#page-8-5)) utilizes reinforcement **148** learning to develop cluster search strategies that **149** identify explicit and reliable relation clues for pre- **150** dicting future facts. DaeMon [\(Dong et al.](#page-8-0) [\(2023\)](#page-8-0)) **151** introduces a novel architecture that leverages time- **152** line relations to adaptively capture temporal path **153** information between query topics and candidate **154** objects. ALRE-IR [\(Mei et al.](#page-8-6) [\(2022\)](#page-8-6)) extracts rela- **155** tion paths from historical subgraphs, aligns these **156** paths with current events to formulate rules, and **157** then uses these rules to predict missing entities. **158**

3 Method **¹⁵⁹**

3.1 Preliminaries 160

Let ε , R , T denote the finite set of entities, rela- 161 tions, and timestamps, respectively. In the tem- **162** poral knowledge graph, each fact is represented by **163** a quaternion (s, r, o, t) , where $s \in \varepsilon$ is the subject 164 entity, $o \in \mathcal{E}$ is the object entity, and $r \in R$ is the **165** relation between *s* and *o* that occurs at timestamp **166** $t \in T$. Specifically, given a query $q = (s, r_q, ?, t_q)$, 167 we take the candidate object $o_i \in \mathcal{E}_c$ as an example, 168 where the subscript *c* of ε_c is the initial letter of 169 candidate, and ε_c is denoted as the set of all enti- **170** ties connected in the history of the query subject **171** *s*,which we take as the set of candidate entities. **172**

173 3.2 Model Overview

 For predicting queries, we can consider two levels: On the one hand, from the relation, for a specific re- lation *r^j* between a subject *s* and a candidate object *o_i* under the historical timestamp t_{τ} denoted as $r_j^{t_{\tau}}$, a relation inference path $path(r_j^{t_\tau}) = (r_j, t_\tau) \rightarrow$ (r_q, t_q) is formed between it and the relation r_q un- der the query timestamp *tq*. This relation inference path suggests that any pair of entities that have a 182 relation r_j under timestamp t_{τ} , that pair will have a relation *r^q* under timestamp *tq*. We explore the potential causal logic between (r_i, t_τ) and (r_q, t_q) to assess the confidence level that the relation inference path $path(r_j^{t_\tau})$ holds, and use it as a basis for reasoning that the query $q = (s, r_a, o_i, t_a)$ holds. After obtaining confidence scores for all relation inference paths between the subject *s* and the candi-190 date object o_i , we aggregate these scores to finally obtain the likelihood score for reasoning that the query $q = (s, r_q, o_i, t_q)$ holds from the relation level.

On the other hand, focusing on entities, we ex- amine the changes in the connectivity of the can- didate object *oⁱ* with neighboring entities across various historical timestamps. We achieve the dy- namic structural encoding of *oⁱ* by capturing the structural changes in the subgraphs where *oⁱ* **¹⁹⁸** is situated, which reflects the evolution of *oi*'s struc- tural semantics over time. Similarly, we can obtain the dynamic structural encoding for the subject *s*. Subsequently, we decode the dynamic structural [e](#page-8-7)ncodings of *s* and *oⁱ* using the ConvTransE [\(Shang](#page-8-7) [et al.](#page-8-7) [\(2019\)](#page-8-7)) decoder to determine the probability of interaction between *s* and *oⁱ* at the given query timestamp *t^q* and query relation *rq*.

 Ultimately, by integrating the scores from both the relation level and the entity structure level, we utilize this composite score as the final probabil- ity score for predicting the validity of the query $q = (s, r_q, o_i, t_q)$. The overall flow of our proposed model is shown in Figure [2.](#page-2-0) In the following, we elaborate on each part of the model.

214 3.3 Relation Inference

 We denote the set of relations connected to the subject s of a query *q* at timestamp t_{τ} as $R_{s\to\epsilon}^{t_{\tau}}$ $\mathbb{R}^{|\varepsilon_c| \times |R| \times d}$, where $|\varepsilon_c|$ is the base of the set of can- didate objects, |*R*| is the base of the set of relations, and *d* is the dimension of the relation embedding. Specifically, given a query $q = (s, r_q, ?, t_q)$, we con- sider all the connected relations between the subject s and the candidate entity o_i . Since our goal is to

Figure 2: Architecture of RIES Framework. The gray shaded area in the bottom left explores the causal logic over time in the connecting relations between the subject entity s and the candidate entity o_i ; the green shaded area in the upper right models each temporal subgraph of o_i to capture its dynamic structural semantics.

capture the causal logic of the relations between **223** *s* and *oⁱ* entity pairs across time, we need to ob- **²²⁴** tain all relations information $R_{s\to o_i}^{t_\tau} \in R_{s\to \varepsilon}^{t_\tau}$ within 225 the historical timestamp range of $[t_{q-len}, t_{q-1}], \tau =$ 226 q − *len*,..., q − 1, where the parameter *len* is the 227 length of the timestamp range of the historical in- **228** formation under consideration. Specifically for **229** a single relation $r_j^{t_\tau} \in R_{s \to o_i}^{t_\tau}(j = 1, ..., |R_{s \to o_i}^{t_\tau}|)$ at 230 timestamp t_{τ} , the confidence score of the relation 231 inference path $path(r_j^{t_\tau})$ corresponding to relation **232** $r_j^{t_{\tau}}$ is computed as follows: **233**

$$
con(path(r_j^{t_{\tau}})) = RCL(r_j, r_q, (t_{\tau}, t_q)) \qquad (1) \qquad \qquad \text{234}
$$

Where $RCL(\cdot)$ is a relation causal logic module, 235 which aims to mine the potential causal logic between the query relation r_q and relation r_j in terms 237 of temporal order. **238**

We then aggregate the confidence scores of these **239** relation inference paths to obtain the total confi- **240** dence score of all relation inference paths between **241** entity pairs *s* and o_i under timestamp t_τ : 242

$$
con(path(R_{s\to o_i}^{t_{\tau}})) = \sum_{j=1}^{|R_{s\to o_i}^{t_{\tau}}|} con(path(r_j^{t_{\tau}})) \quad (2)
$$

)) (2) **243**

Upon calculating the total confidence scores for the **244** relation inference paths between entities *s* and *oⁱ* **²⁴⁵** across the time horizon $[t_{q-len}, t_{q-1}]$, we utilize path 246 confidence aggregation (PCA) to aggregate these **247** total confidence scores. This aggregation provides **248** the historical relation inference path scores for $s \rightarrow 249$ o_i : : **250**

$$
score_r = PCA(con(path(R_{s \to o_i}^{[t_{q-len}, t_{q-1}]}))) \quad (3)
$$

Figure 3: The architecture of RCL module. Exploring the causal logic of the relations r_1 and r_2 at timestamp t_1 on the relation r_q at timestamp t_q in a temporal order.

252 In the following section, we provide a detailed de-**253** scription of the RCL module and the PCA module, **254** respectively.

255 3.3.1 Relation Causal Logic

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278

 The workflow of relation causal logic (RCL) is shown in Figure [3.](#page-3-0) We first encode the tempo- ral information as follows: At a specific historical timestamp t_{τ} , the relation $r_j^{t_{\tau}}$ occurring between the entity pairs *s* and *oⁱ* may lead to a query re-261 lation *r_q* occurring at timestamp $t_{\tau} + \Delta t$. There- fore, we encode the time interval ∆*t* between the query time t_q and the historical time t_τ . For a re-**lation** $r_j^{t_\tau} \in \mathbb{R}^{t_\tau}_{s \to o_i} (j = 1, ..., |\mathbb{R}^{t_\tau}_{s \to o_i}|)$ at timestamp t_{τ} , where the time interval from the query *q* is $\Delta t = t_q - t_\tau$, the time interval is encoded as a *d*- dimensional time-encoded vector using the follow-ing equation:

269
$$
T_{(\Delta t, 2\tau)} = \sin(\Delta t / 10000^{2\tau/d})
$$
 (4)

271
$$
T_{(\Delta t, 2\tau + 1)} = \cos(\Delta t / 10000^{2\tau / d})
$$
 (5)

 After encoding the timing information, we add the time encoding to the initialized relation encoding $\mathbf{r}_{j,init}$ so that we obtain an embedding of the relation $r_j^{\tilde{t}_\tau}$:

$$
\mathbf{r}_j = \mathbf{r}_{j,init} + T_{\Delta t} \tag{6}
$$

277 Next, we obtain the relation inference path $path(r_j^{t_\tau}) = (r_j, t_\tau) \rightarrow (r_q, t_q)$ from the relation $r_j^{t_\tau}$ between the entity pairs *s* and o_i to the relation r_q 280 at the query time t_q . We consider $r_j^{t_\tau}$ as the cause 281 and r_q at t_q as the effect. Finally, we assess the confidence that the relation inference path $path(r_j^{t_{\tau}})$ holds by capturing the association between $r_j^{t_\tau}$ and 284 r_q at the query time t_q . To compute this, we directly **285** use the dot product method:

$$
con(path(r_j^{t_{\tau}})) = \mathbf{r}_j \ast \mathbf{r}_q \tag{7}
$$

287 Where \mathbf{r}_j is the relation $r_j^{t_\tau}$ embedding that contains 288 the time encoding and \mathbf{r}_q is the initial relation em-**289** bedding of the query *q* that does not contain the **290** time encoding.

Figure 4: The architecture of PCA module. Aggregating the confidence scores of all relation inference paths between query subject *s* and candidate entity *oⁱ* .

3.3.2 Path Confidence Aggregation **291**

The workflow of path confidence aggregation **292** (PCA) is shown in Figure [4.](#page-3-1) Calculation by means **293** of Equation [2,](#page-2-1) we obtain the total confidence level **294** score $con(path(R_{s\rightarrow o_i}^{t_{q-1}}))$,...,*con*(*path*($R_{s\rightarrow o_i}^{t_{q-len}})$) for 295 the relation inference path for $s \rightarrow o_i$ at each times- 296 tamp within the time range $[t_{q-\ell en}, t_{q-1}]$. In special 297 cases, when two inference paths, $path(r_i^{t_q - len})$ $j^{tq - len}$ and 298 $path(r_i^{t_{q-1}})$ j_j^{q-1}), under different historical timestamps 299 have the same relation r_j , we should assign differ- 300 ent weights to these paths to distinguish between **301** them. Due to the stability and simplicity of power **302** functions, we define a power function-based time **303** decay coefficient: **304**

$$
W_d(t_q, t_\tau) = (t_q - t_\tau)^{-\gamma} \tag{8}
$$

(8) **305**

The larger the value of γ in the above equation, 306 the faster the rate at which W_d decays over time. 307 The time decay coefficient W_d ensures that relation 308 inference paths closer in time to the query time t_q 309 are assigned higher weights. We weight the relation **310** inference path confidence scores at each timestamp **311** as follows: **312**

$$
PCA(con(path(R_{s\to o_i}^{[t_{q-len},t_{q-1}]}))) =
$$

$$
\sum_{\tau=q-len}^{q-1} W_d(t_q,t_\tau)con(path(R_{s\to o_i}^{t_\tau}))
$$
 (9)

3.4 Entity Structure **314**

This module explores the association between the **315** subject *s* of a query *q* and a candidate object o_i in 316 terms of dynamic structural semantics, determin- **317** ing the probability that the subject *s* of the query **318** interacts with candidate object o_i under the query 319 timestamp t_q and the query relation r_q . The entire **320** process is divided into two parts: encoding and **321** decoding. **322**

323 3.4.1 Entity Dynamic Structural Encoding

 For simultaneous facts, entities usually have strong semantic correlations with their neighboring enti- ties. To capture these semantics, we model them us- ing the ω-layer R-GCN [\(Schlichtkrull et al.](#page-8-8) [\(2018\)](#page-8-8)) as a structural encoder:

329
$$
\mathbf{h}_{s,t}^{l} = f\left(\frac{1}{|N_{s,t}|}\sum_{e_o^{t} \in N_{s,t}} W_1^{l}(\mathbf{h}_{o,t}^{l} + \mathbf{r}) + W_2^{l} \mathbf{h}_{s,t}^{l-1}\right) (10)
$$

330 Where $N_{s,t}$ is the set of neighbors of entity *s* in the **331** static subgraph at timestamp *t*, $f(\cdot)$ is the reflection **332** modified linear unit (RReLU [\(Xu et al.](#page-9-8) [\(2015\)](#page-9-8))) activation function, $W_1^l \in \mathbb{R}^{d \times d}$ is a relation-specific **334** parameter used for aggregating structural features **based on different edges,** $W_2^l \in \mathbb{R}^{d \times d}$ **denotes the 336** parameter that aggregates the self-loop features of all entities, $h_{o,t}^l$ and **r** denote the embedding of 338 the neighboring entity e^t in the *l*-th layer of the **339** R-GCN and the embedding of the connected rela-**³⁴⁰** tion, respectively. After ω layers of R-GCN, we $\sum_{s,t}$ can obtain a representation $h_{s,t}^{\omega}$ that only considers **342** semantic dependencies with neighboring nodes of **343** entity *s* at timestamp *t*.

 To capture the dynamic structural semantic changes of an entity *s* over a short period, the model needs to consider all temporally neighboring facts. Therefore, we use the structural semantic output of the entity from the previous timestamped sub- graph as input to the R-GCN model for the next timestamp:

$$
\mathbf{h}_{s,t+1}^1 = \mathbf{h}_{s,t}^\omega \tag{11}
$$

 We use the time-gate loop component to further model the temporal dependence of the entity struc- ture. The dynamic structural semantic embedding **e**_{s t+1} of the final entity *s* is determined by two com- ponents: the output of the last layer of the R-GCN, $h_{s,t+1}^{\omega}$, and the $e_{s,t}$ from the previous timestamp. The specific expressions are as follows:

$$
\mathbf{e}_{s,t+1} = U_{t+1} \otimes \mathbf{h}_{s,t+1}^{\omega} + (1 - U_{t+1}) \otimes \mathbf{e}_{s,t} \quad (12)
$$

360 The expression ⊗ denotes the dot product operation. 361 The time gate $U_{t+1} \in \mathbb{R}^{d \times d}$ undergoes a nonlinear **362** transformation as:

$$
U_{t+1} = \sigma (W_u \mathbf{e}_{s,t} + b) \tag{13}
$$

Where $\sigma(\cdot)$ is the sigmoid function and $W_u \in \mathbb{R}^{d \times d}$ **365** is the weight matrix of the time gate.

364

3.4.2 Entity Dynamic Structure Decoding **366**

We choose ConvTransE [\(Shang et al.](#page-8-7) [\(2019\)](#page-8-7)) as 367 the decoder to compute the degree of association **368** between the subject *s* of the query *q* and the can- **369** didate object o_i at the dynamic structural-semantic 370 level under the query timestamp *tq*, represented as **³⁷¹** follows: **372**

$$
score_e = \sigma(\mathbf{e}_{o_i, t_q}ConvTransE(\mathbf{e}_{s, t_q}, \mathbf{r}_q)) \qquad (14) \qquad \qquad \text{373}
$$

Where \mathbf{r}_q is the initial relation embedding of query 374 *q*. This function yields the probability that the **375** subject *s* interacts with a candidate object o_i at time 376 t_q and relation r_q . In other words, it represents the **377** probability that the query $q = (s, r_q, o_i, t_q)$ holds 378 from the perspective of the entity structure. **379**

3.5 Inference 380

To ensure that we can maximize the use of relation **381** and entity information, we introduce the coeffi- **382** cient α to adjust the weight between the relation 383 inference score and the entity structure score. The **384** final prediction that the missing object entity in **385** $q = (s, r_q, ?, t_q)$ will be the highest combined prob- 386 ability entity \hat{o} for both aspects: 387

$$
P(o|s, r_q, t_q) = \alpha * score_r + (1 - \alpha) * score_e
$$
 (15)

$$
\hat{o} = argmax_{o \in \varepsilon_c} P(o|s, r_q, t_q) \tag{16}
$$

Where $P(o|s, r_q, t_q)$ denotes the predicted probabil- 391 ity of all candidate object entities $o \in \mathcal{E}_c$. 392

3.6 Train 393

In the relation inference process, we compute the **394** similarity between the embedding \mathbf{r}_j of r_j^t and the 395 relation embedding \mathbf{r}_q of the query *q* in the em- 396 bedding space by using the dot product to obtain **397** the confidence score for the relation inference path **398** $path(r_j^{t_{\tau}}) = (r_j, t_{\tau}) \rightarrow (r_q, t_q)$. The challenge lies **399** in determining the correct inference path and as- 400 signing it a higher confidence score. To address 401 this, we design a positive and negative sample com- **402** parison training method. This method learns the $r_j^{t_i}$ relation embedding \mathbf{r}_j in the relation inference path 404 $path(r_j^{t_{\tau}})$ so that when the relation inference path 405 is correct, the historical relation embedding \mathbf{r}_j is 406 spatially close to the relation embedding \mathbf{r}_q of the 407 query *q*. Conversely, when the relation inference **408** path is incorrect, \mathbf{r}_j is spatially distant from \mathbf{r}_q . 409

First, we negatively sample and generate the 410 error quaternion. Specifically, given a cor- **411** rect quaternion $pos = (s, r, o, t)$, we randomly 412

Datasets	Entities	Relations	Training	Validation	Test	Time Granules
ICEWS14	7128	230	63685	13823	13222	365
ICEWS0515	10488	251	322958	69224	69147	4017
ICEWS18	23033	256	373018	45995	49545	304
WIKI	12554	24	539286	67538	63110	232
YAGO	10623	10	161540	19523	20026	189

Table 1: Statistical data for the datasets.

Model	ICEWS14			ICEWS18			ICEWS0515					
	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
ComplEX	30.84	21.51	34.48	49.58	21.01	11.87	23.47	39.87	31.69	21.44	35.74	52.04
$R-GCN$	28.03	19.42	31.95	44.83	15.05	8.13	16.49	29.00	27.13	18.83	30.41	43.16
DE-SimplE	32.67	24.43	35.69	49.11	19.30	11.53	21.86	34.80	35.02	25.91	38.99	52.75
CyGNet	32.73	23.69	36.31	50.67	24.93	15.90	28.28	42.61	34.97	25.67	39.09	52.94
xERTE	40.79	32.70	45.67	57.30	29.31	21.03	33.51	46.48	46.62	37.84	52.31	63.92
CEN	42.40	32.08	47.46	61.31	31.05	21.70	35.44	50.59	٠	-	۰	٠
TECHS	43.88	34.59	49.36	61.95	30.85	21.81	35.39	49.82	48.38	38.34	54.69	68.92
DaeMon					31.85	22.67	35.92	49.80	٠		۰	
HGLS	47.00	35.06	\sim	70.41	29.32	19.21		49.83	46.21	35.32	٠	67.12
RPC	44.55	34.87	49.80	65.08	34.91	24.34	38.74	55.89	51.14	39.47	57.11	71.75
TiPNN					32.17	22.74	36.24	50.72				
DLGR	46.72	36.67	51.61	٠	35.48	25.11	40.03			-	۰	٠
RIES	54.34	41.88	61.49	77.84	39.12	26.28	45.02	64.69	56.52	44.50	63.47	79.03
Absolute Boost	7.34	5.21	9.88	7.43	3.64	1.17	4.99	8.80	5.38	5.03	6.36	7.28
Relative Boost	15.62	14.21	19.14	10.55	10.26	4.66	12.47	15.75	10.52	12.74	11.14	10.15

Table 2: Performance (in percentage) on ICEWS14, ICEWS18, ICEWS0515. Best results are bolded, sub-optimal results are underlined.

 sample an object entity from historical events and disrupt the quaternion to generate an incor- rect quaternion *neg* that satisfies the condition *neg* = { (s, r, o', t) | $o' \in \varepsilon - o$ }. We ensure that the correct quaternions (positive samples) receive higher scores and the incorrect quaternions (neg- ative samples) receive lower scores by using the *So ftMarginLoss* function, expressed as follows:

$$
L = \sum_{(s,r,o,t)\in P\cup N} log(1 + exp(-y \cdot score_r(s,r,o,t)))
$$
\n⁴²² (17)

423 $y = \begin{cases} 1, & (s,t,0,t) \in I \\ -1, & (s,t,0,t) \in N \end{cases}$ (18)

424 In Equation [18,](#page-5-0) *P* is the set of correct quaternions **425** and *N* is the set of incorrect quaternions.

 $y = \begin{cases} 1, & (s, r, o, t) \in P \\ 1, & (s, r, s, t) \in P \end{cases}$

 The training task based on the *So ftMarginLoss* function is to assign higher scores to correct quater- nions and lower scores to incorrect quaternions, with these scores derived from the confidence level of the relation inference paths. From the perspec- tive of the embedding space, this task brings the historical relation embeddings of the positive exam- ples closer to the query relation embedding, while moving the historical relation embeddings of the negative examples further away from the query re-lation embedding.

437 In short, this training task is to enable correct re-**438** lation inference paths to achieve higher confidence **439** scores.

4 Experiment 440

4.1 Experimental Setup **441**

4.1.1 Datasets **442**

[W](#page-8-9)e use five benchmark datasets (ICEWS14 [\(Li](#page-8-9) 443 [et al.](#page-8-9) [\(2022b\)](#page-8-9)), ICEWS0515 [\(Ren et al.](#page-8-10) [\(2023\)](#page-8-10)), **444** [I](#page-9-9)CEWS18 [\(Boschee et al.](#page-8-11) [\(2015\)](#page-8-11)), WIKI [\(Vran-](#page-9-9) **445** [d](#page-9-10)ečić and Krötzsch [\(2014\)](#page-9-9)), and YAGO [\(Suchanek](#page-9-10) 446 [et al.](#page-9-10) [\(2007\)](#page-9-10))) to evaluate the performance of the **447** model on the temporal knowledge graph reasoning **448** task. To ensure a fair comparison, we follow the **449** data partition provided in the reference TECHS **450** [\(Lin et al.](#page-8-4) [\(2023\)](#page-8-4)) to divide each dataset into train- **451** ing, validation, and test sets. Table [1](#page-5-1) provides **452** statistics for these data sets. **453**

To assess the validity of our proposed model, we **454** have thoroughly compared the experimental results **455** with various static and temporal models. 456

4.1.2 Assessment Indicators and Training **457** Settings **458**

In our experiments, we used MRR and Hits@1,3,10 **459** as evaluation indicators. For the configuration of **460** the model, we use random initialization to gen- **461** erate relation embeddings of dimension 200. To **462** optimize all model parameters, we used the Adam **463** [\(Kingma](#page-8-12) [\(2014\)](#page-8-12)) optimizer and set the initialized **464** learning rate to 0.001. For the entity structure mod- **465** ule, we set the number of layers ω of R-GCN to **⁴⁶⁶** 2. For each R-GCN layer, the dropout rate is set **467** to 0.2 and the history length is set to 10. For Con- **468**

Model			WIKI				YAGO	
	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
ComplEX	24.47	19.69	27.28	34.83	44.38	25.78	48.2	59.01
R-GCN	13.96		15.75	22.05	20.25		24.01	37.30
$DE-SimplE$	45.43	42.60	47.71	-	54.91	51.64	57.30	٠
CyGNet	58.78	47.89	66.44	78.70	68.98	58.97	76.80	86.98
xERTE	73.60	69.05	78.03	79.73	84.19	80.09	88.02	89.78
CEN	78.93	75.05	81.90	84.90	٠			۰
TECHS	75.98			82.39	89.24	$\overline{}$		92.39
DaeMon	82.38	78.26	86.03	88.01	91.59	90.03	93.00	93.34
HGLS	82.04	78.07	84.04	٠	87.48	83.17	89.76	٠
RPC	81.18	76.28	85.43	88.71	88.87	85.10	92.57	94.04
TiPNN	83.04	79.04	86.45	88.54	92.06	90.79	93.15	93.58
DLGR	82.98	80.14	80.14	-	88.87	84.60	92.35	٠
RIES	89.46	87.34	91.82	93.12	94.73	92.83	95.25	96.63
Absolute Boost	6.42	7.20	5.37	4.41	2.67	2.04	2.10	2.59
Relative Boost	7.73	8.98	6.21	4.97	2.90	2.25	2.25	2.75

Table 3: Performance (in percentage) on WIKI, YAGO. Best results are bolded, sub-optimal results are underlined.

469 vTransE, the kernel size is set to 2×3 and the dropout rate is set to 0.2. Specifically, we trained the model for 100 epochs, with early stopping if the validation loss did not decrease for 10 con- secutive epochs. All experiments were conducted on a single Tesla T4 GPU with 16GB of memory. The model has approximately 9 million parame- ters. The time required to run one epoch on the ICEWS14, ICEWS18, ICEWS0515, YAGO, and WIKI datasets is approximately 10, 60, 110, 10, and 20 minutes, respectively.

480 4.2 Experimental Results

 The experimental results of RIES and all the base- lines on TKG reasoning are presented in Tables [2](#page-5-2) and [3.](#page-6-0) The results are from the average of the ex- periments. We chose ComplEX [\(Trouillon et al.](#page-9-11) [\(2016\)](#page-9-11)) and R-GCN [\(Schlichtkrull et al.](#page-8-8) [\(2018\)](#page-8-8)) as static models for comparison. DE-SimplE [\(Goel et al.](#page-8-13) [\(2020\)](#page-8-13)), CyGNet [\(Zhu et al.](#page-9-0) [\(2021\)](#page-9-0)), xERTE [\(Han et al.](#page-8-14) [\(2020\)](#page-8-14)), CEN [\(Li et al.](#page-8-15) [\(2022a\)](#page-8-15)), TECHS [\(Lin et al.](#page-8-4) [\(2023\)](#page-8-4)), DaeMon [\(Dong et al.](#page-8-0) [\(2023\)](#page-8-0)), HGLS [\(Zhang et al.](#page-9-2) [\(2023\)](#page-9-2)), RPC [\(Liang](#page-8-16) [et al.](#page-8-16) [\(2023\)](#page-8-16)), TiPNN [\(Dong et al.](#page-8-1) [\(2024\)](#page-8-1)), and DLGR [\(Xiao et al.](#page-9-6) [\(2024\)](#page-9-6)) as comparative tempo-ral models.

 Static models such as ComplEX and R-GCN underperform compared to temporal models be- cause they fail to consider temporal information and dependencies across different snapshots. Simi- larly, the interpolation model DE-SimplE also per- forms poorly because such models struggle to han- dle events occurring in future timestamps. Among the extrapolation models, CyGNet, xERTE, CEN, HGLS, and DLGR focus on entity information and overlook the dynamic changes in relations between entity pairs over time. TECHS, DaeMon, RPC, and TiPNN start from relations, utilizing path-based

ICEWS14	ICEWS18	YAGO
54 34	39.12	94.73
$46.16(-8.2)$	$35.26(-3.9)$	$89.32(-5.4)$
$50.81(-3.5)$	$36.05(-3.1)$	$82.65(-12.1)$
$43.79(-10.6)$	$31.55(-7.6)$	$79.53(-15.2)$
$46.39(-8.0)$	$33.83(-5.3)$	$81.14(-13.6)$

Table 4: Results (in percentage) by different variants of our model on three datasets.

searches to extract potential logical rules within **506** the graph. These methods are limited by the ex- **507** isting paths, which restrict their search range and **508** impair their performance. Our proposed model 509 operates within a unified framework that models **510** relations and entities, exploring the causal logic **511** between relations over time and the dynamic struc- **512** tural changes of entities. By fully leveraging infor- **513** mation on relations and entities for prediction, our **514** model outperforms the state-of-the-art across all 515 metrics on five datasets. 516

4.3 Ablation Study 517

To test the contribution of each component in the **518** model, we performed ablation experiments. **519**

To further analyze the contribution that each part **520** of the model makes to the final prediction results, **521** we report in Table [4](#page-6-1) above the results of the MRR **522** metrics for the five sub-models on the test sets of **523** the three datasets. The five sub-models compared **524** are: 1. RIES, the full model. 2. RIES w/o R, **525** representing RIES without the relation inference **526** module. 3. RIES w/o E, representing RIES without **527** the entity structure module. 4. RIES w/o (E&R- **528** TE), representing RIES without the entity structure **529** module and without using time encoding in the re- **530** lation inference module. 5. RIES w/o (E&R-TD), **531** representing RIES without the entity structure mod- **532** ule and without using the time decay coefficient in **533** the relation inference module. **534**

query	relation		score-r	score-e	Target entity
	engage in negotiate,t-1 make statement.t-1 intent to cooperate,t-2 sign formal agreement, t-2	0.575 0.516 0.351 0.332	\Rightarrow 1.774	0.703	$USA(\sqrt{2})$
(China, engage in diplomatic cooperate, $?$, t)	engage in negotiate,t-1 praise,t-1 engage in negotiate,t-2 sign formal agreement, t-2	0.575 0.604 0.287 0.332	\Rightarrow 1.798	0.372	the US President
	host a visit, t-1 consult.t-1 make a visit, t-2 endorse _t -2	0.316 0.287 0.158 0.208	$\Rightarrow 0.969$	0.768	India

Table 5: A case demonstrating that entity and relation information can effectively complement each other in the reasoning process.

535 From the results in Table [4,](#page-6-1) we draw the follow-**536** ing findings:

 Effectiveness of combined use of relation and entity information. The full model RIES outper- forms RIES w/o R and RIES w/o E on all datasets, which confirms that relation and entity information complement each other well for future prediction.

 Validity of time encoding in relation inference modules. The experimental results of RIES w/o (E&R-TE) have a substantial decrease compared to RIES w/o E. This is because RIES w/o (E&R-TE) does not consider the dynamic change of causal logic between relations, and ignores the absolute temporal numerical information. What is learned in this case is a static relation inference path inde- pendent of temporal order, which is unsuitable for reasoning on temporal knowledge graphs.

 Validity of time decay coefficient in relation inference modules. The experimental results for RIES w/o (E&R-TD) have also decreased com- pared to RIES w/o E. This confirms the necessity of considering the relative temporal distance of the inference paths from the query. The value of histor- ical relation information decreases progressively as this relative temporal distance increases.

560 4.4 Case Study

 Considering the limited length of the paper, it is necessary to limit the number of relations between the subject entity and candidate entities. Therefore, we set the parameter *len* of the history time horizon to 2. For the query in the ICEWS14 test set (China, engage in diplomatic cooperate, ?, t), we selected the top three scoring entities among the candidates and presented them in Table [5.](#page-7-0)

 From the perspective of relation inference alone, relations such as engage in negotiate, make state- ment, and praise provide high scores for the candi-date entities the US President and USA. The scores

for USA (1.774) and the US President (1.798) are **573** very similar, but the incorrect answer, the US Pres- **574** ident, scores higher than the correct, USA. **575**

From the perspective of entity structure alone, 576 the subgraph structures of the candidate entities **577** USA and India are quite similar, with neighboring **578** nodes mostly being other national entities. How- **579** ever, the incorrect answer, India (0.768) , scores 580 higher than the correct, USA (0.703). This is primarily because India has a closer relationship with **582** China compared to USA, as both are Asian coun- **583** tries and their connected neighboring entities are **584** predominantly from Asia. **585**

The correct answer, USA, can only be deter- **586** mined by combining scores from both relation inference and entity structure. This shows that con- **588** sidering only relation or entity information alone is 589 not enough to distinguish similar candidate entities. **590** Optimal reasoning results can only be achieved by **591** effectively utilizing both types of information. **592**

5 Conclusion **⁵⁹³**

In this paper, we consider two types of information **594** in graphs: entity information and relation informa- **595** tion. For the first time, we model these two types **596** of information within a unified framework. We **597** further propose the RIES model, divided into two **598** components: relation inference and entity structure, **599** to handle relation and entity information. At the **600** relation level, the relation inference component ex- **601** plores the causal logic of different relations over **602** time and constructs reasonable inference paths. At 603 the entity level, the entity structure component en- **604** codes the dynamic structure of entities and discov- **605** ers their associations within subgraph structures. **606** Experiments on five benchmark datasets demon- **607** strate the effectiveness of our model in temporal 608 knowledge graph extrapolation tasks. **609**

⁶¹⁰ Limitations

 The timestamp range for historical information modeled by RIES is determined by the parameter *len*. Currently, selecting the *len* value requires man- ual intervention, with different datasets needing to be manually set to different values. This makes it challenging to determine the optimal parameter value. Future work could explore the automatic optimization of this parameter to further enhance the model's predictive capability.

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A Parameters Analysis **⁷⁷⁶**

Figure 5: Result on five dataset with different *len*.

In the relational inference module, we acquired **777** all relational information located within the his- **778** torical timestamp range $[t_{q-\text{len}}, t_{q-1}]$, where the pa- 779 rameter *len* represents the length of this historical **780** range. To determine the optimal value for *len*, we **781** conducted a detailed parameter tuning experiment **782** and tested the model's performance across different **783** *len* values on the metrics MRR and Hits@1. The **784** specific experimental results are shown in Figure [5.](#page-9-12) **785**

The *len* values for the ICEWS14, ICEWS18, and **786** WIKI datasets were set at 10, 20, 30, 40, 50, and 60. **787** For the ICEWS0515 dataset, they were set at 50, 788 100, 150, and 200. On the YAGO dataset, they were **789** set at 5, 10, 15, and 20. Across all five datasets, as **790** the value of *len* increased, both metrics, MRR and **791** Hits $@1$, initially improved and then declined. We **792** analyzed the reasons as follows: When the value **793** of *len* is too small, it considers too little historical **794** information, failing to capture enough relational **795** causal logic. Conversely, when *len* is too large, **796** it introduces history that is too distant from the **797**

